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MNB INTÉZET
FENNTARTHATÓ PÉNZÜGYEK KÖZPONT

MODELING INFLATION EXPECTATION WITH FINANCIAL MARKET CHARACTERISTICS

AN EXPERIMENT WITH TIPS YIELDS BASED INFLATION COMPENSATION

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Abstract

Comparing the TIPS yield curve with the corresponding nominal yield curve allows to compute high frequency measure of inflation compensation. However, the TIPS-based inflation compensation term structure's usage is limited due to the relatively short time period for which data is available. For that reason the literature tried to extend the time series of inflation compensation to the past - mainly employing OLS, exploiting the relationship between nominal yields and inflation compensation. In an attempt to backcast inflation expectations more accurately this study uses return data for characteristic-managed portfolios (factors) to model inflation expectation with LightGBM, a gradient boosting framework that uses tree based learning algorithms. Factor-based out-of-sample estimations are compared to nominal yield based estimations which is widely used in the literature. Executing a rolling window backtest covering the period from 2010 to 2022 showed, that the factor-based model consistently underperformed compared to the nominal yield-based model. The mean absolute percentage error (MAPE) of the factor-based model was notably higher at 23.11%, in contrast to the 11.53% MAPE of nominal yield-based estimations. Additionally, factor-based estimations showed greater volatility and appeared to be unable to capture fluctuations in inflation expectations over time, often converging towards the mean. This inability to reflect deviations in breakeven inflation rates suggests that factors, in this form, are unsuitable for backcasting inflation expectations. Furthermore, an important finding of this study is that LightGBM-based predictions outperformed those derived from OLS, indicating that the introduction of LightGBM represents an improvement over existing backcasted inflation expectation methodologies.

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1 Introduction

Inflation expectations are a crucial component of macroeconomic analysis, with significant implications for monetary and fiscal policy. Several factors influence these expectations, including changes in monetary policy, changes in fiscal policy, and exchange rate fluctuations.

Despite the numerous research done in this field to date, the questions of how households form their inflation expectations and how these expectations contribute to their spending decisions remain debated. According to Berger and Vavra (Berger and Vavra 2015), households' expectations regarding inflation regime play an important role in shaping their consumption patterns. While other studies have shown that households' inflation expectations have minimal or non-significant impact on spending habits (Bachmann et al. 2015).

Inflation expectations affect investment decisions and labor market outcomes. Coibion et al. (Coibion et al. 2020) have found that inflation expectations have a causal effect on firms' investment decisions by analyzing the impact of unexpected changes in inflation expectations on firms' propensity to invest in capital. Additionally, inflation expectations have a considerable effect on wage-setting behavior and labor market outcomes (Carlson and Parkin 1975). Thus, households' and firms' inflation expectations play a critical role in shaping the overall macroeconomic environment through many channels, such as consumption and investment decisions or labor market outcomes.

The relationship between inflation and asset returns has been widely studied in the literature. Evidence from the cross section of asset returns supports the conclusion that investors care mostly about persistent movements in inflation and less about transitory fluctuations. Fang et al. (Fang et al. 2022) analysed the effects of different inflation shocks by studying the pricing of core and energy inflation across asset classes, including stocks, bonds, real estate, and commodities. The core beta turns out to be consistently negative, supporting the hypothesis that these assets do not hedge core inflation risk, whilst energy shocks, instead, do not appear to drive a significant risk compensation. However, it's important not to interpret these findings as suggesting that energy shocks have no bearing on asset prices, especially if oil price shocks eventually impact core inflation, as demonstrated in prior studies (Hamilton 2013; Meltzer et al. 2005)

Due to the significant co-movement between inflation and asset returns, the literature

extensively analysed how inflation expectations impact asset prices. According to widely recognized studies, rising inflation expectations may lead to a decline in asset returns and lower risk premia for stocks and bonds (Fama and Schwert 1977; French et al. 1983). However, not all inflation expectations are bad for asset returns. (Cieslak and Pflueger 2023) Research has also shown that inflation expectations can be “good” if they are associated with higher future output, which, in turn, reduces risk premia and raises stock prices. Higher inflation expectations results in investors lowering the probabilities they assign to bad states of the world, which in turn lowers risk premia and raises stock prices (David and Veronesi 2013).

Based on the above listed findings measuring inflation expectations is essential for understanding how economic actors anticipate future inflation and how that impacts their decisions. When measuring households’ inflation expectations, survey-based approaches are typically employed in which respondents are asked to provide their expectations for inflation over different horizons. Such surveys are for example the Michigan Survey of Consumers and the Survey of Professional Forecasters. (Armantier et al. 2017)

Firms’ inflation expectations are often measured through data-based approaches, such as the inflation breakeven rate. Moreover, surveys conducted with firms, such as the National Federation of Independent Business (NFIB) survey, can be utilized to obtain information about their inflation expectations

For over ten years, the U.S. Treasury has issued Treasury inflation-protected securities (TIPS)—debt securities for which the coupon and principal payments are indexed to the Consumer Price Index (CPI)—in addition to conventional nominal bonds. TIPS allows for calculating high frequency inflation compensation (also known as breakeven inflation rate) by comparing the nominal and TIPS smoothed yield curves. These rates represent the hypothetical level of inflation which would result equivalent returns on both nominal securities and TIPS at maturity. The breakeven rate can be expressed through various yield curve metrics such as zero-coupon yields, par yields, or forward rates. (Gürkaynak et al. 2010) One should be cautious with interpreting every fluctuation in inflation compensation solely as indicative of shifts in inflation expectations. The variation in inflation compensation also reflects the behavior of inflation risk premium and the liquidity premium. While the liquidity premium demonstrates only gradual movements over time, evidence indicates that

the inflation risk premium may display significant volatility even across relatively short timeframes. (Gürkaynak et al. 2010),

Treasury Inflation-Protected Securities (TIPS), introduced in 1997, provide investors with protection against inflation by adjusting both principal and coupon payments based on realized inflation rates. The early years of TIPS saw market illiquidity resulting in discounted prices and higher borrowing costs relative to nominal Treasury issues. These liquidity issues led to a liquidity premium estimated around 30 basis points, impacting TIPS pricing significantly. Despite recommendations to discontinue the program in 2001, the Treasury reaffirmed its commitment, leading to improved liquidity and increased trading volumes. (Sack and Elsasser 2004)

However, evidence suggests that TIPS continue to be underpriced, indicating ongoing complexities in their valuation not solely attributable to liquidity issues. (J. Ahn and Y. Ahn 2023) Nevertheless, TIPS remain essential for investors seeking to safeguard their portfolios against inflation, underscoring their enduring relevance in investment strategies. TIPS trading volumes are comparable to nominal Treasury securities, marking a significant evolution in their market presence since their inception.

The TIPS yield curve and the associated inflation compensation term structure have many applications for practitioners and researchers, but their usefulness is limited due to the relatively short time period for which data is available. However, if the relationship between other indicators and inflation compensation is relatively stable over time it might be possible to backcast inflation compensation to the past.

Gürkaynak et al. (Gürkaynak et al. 2010) backcasted fitted five-to-ten-year forward inflation compensation back to 1971. The nominal yield curve's term structure is widely understood to be effectively described by a couple principal components, often associated with macroeconomic factors such as the business cycle, inflation expectations, and the underlying real interest rates. For that reason they used the first three principal components of the nominal yield for their analysis.

As cited above, daily volatility in inflation compensation on daily granularity might not be entirely caused by changes in perceptions about future inflation. For that reason they used month-end data for their analysis. The exercise is complicated by the poor liquidity of TIPS in the early years. Accordingly, they estimated a linear regression using data starting only in

2003. The obtained fitted inflation compensation is depicted on Figure 1.

They reported an R-squared of 79 percent, indicating a tight relationship between inflation compensation and the nominal term structure, which confirms the usefulness of their backcasted proxy. Their backcasted inflation expectation seems qualitatively reasonable. It rose in the late 1970s and exceeded 6 percent in the early 1980s, presumably reflecting both a high level of longer-term inflation expectations and a sizable inflation risk premium. It declined sharply in the mid 1980s and drifted lower over the next 15 years, likely because investors were revising down both their inflation expectations and the compensation that they demanded for inflation risk over that period.

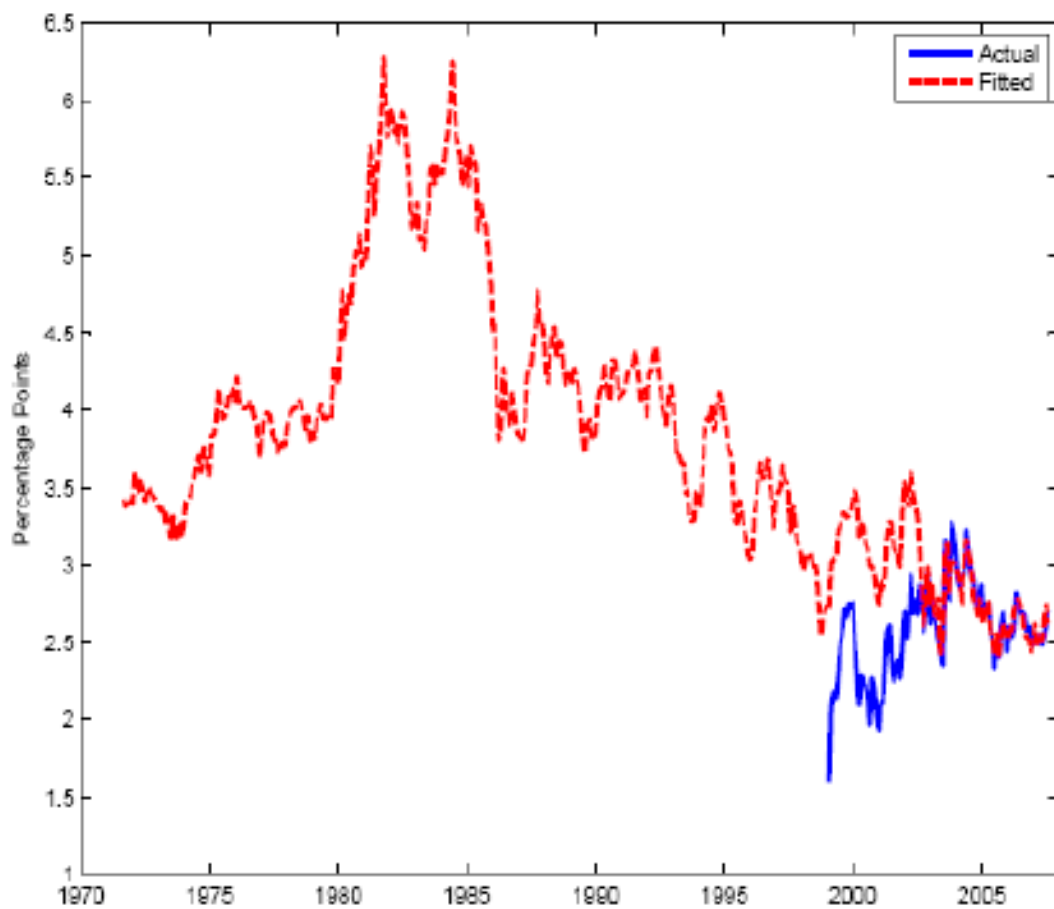


Figure 1: Actual and Fitted Five-to-ten Year Forward Inflation Compensation Figure 10 from the article of Gürkaynak et al. (Gürkaynak et al. 2010)

So far the literature concentrated to backcast inflation compensation using nominal yield data. In this study we aim to backcast inflation expectations using stock market informations.

Based on the relationship between inflation expectations and asset returns we hypothesise that inflation compensation could be modeled with return data for characteristic-managed portfolios, or "factors". Thanks to Jensen et al. (Jensen et al. 2023) these factors are available for 153 characteristics in 13 themes, using data from 93 countries and four regions.

In order to make our hypothesis measurable we use the extended inflation compensation from Gürkaynak et al. (Gürkaynak et al. 2010) as a baseline estimation. Our estimation and the baseline estimations for the inflation compensation could only be compared to the factual breakeven rate on the period after 1999 (since TIPS is only available from 1999). For that reason we carry out a rolling window backtest for the past to cross-validate that the factor-based estimation is better than the baseline estimation.

Please note, that being able to provide more accurate estimations after 1999 does not necessarily mean that the backcasted estimations are also better on the period before 1999. For this to be true, we need to accept the hypothesis, that the relationship between inflation compensation and the used variables has been stable over time.

2 Data

In this section we present in detail the database used in the analysis. First we describe the obtained data and the preprocessing steps then report the results of the exploratory data analysis.

2.1 Data source

The following data are used:

- **TIPS based inflation compensation:** Five-to-ten Year Forward daily inflation compensation downloaded from FED's website ¹. Calculation made by Gürkaynak et al. (ibid.)
- **Factors:** daily capped value weighted returns for US downloaded from Kelly's website². Calculation made by Jensen et al. (Jensen et al. 2023)

¹<https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm>, Data downloaded on: 10th of March 2024.

²<https://jkpfactors.com/>, Data downloaded on: 10th of March 2024.

- **Treasury yield curve estimates:** continuously compounded instantaneous forward nominal Treasury yield curve estimates on daily granularity downloaded from FED's website³. Calculation made by Gürkaynak et al. (Gürkaynak et al. 2007)

2.2 Summary statistics

This section provides summary statistics on our data. In this study we use three data sources: the TIPS based inflation expectation, the factors and the nominal yields. Table 1 provides summary statistics about these time series. As seen from Table 2 the factor data is only available until the end of 2022, for that reason we use the period between 1999 and 2022 in our analysis.

Table 1: Descriptive statistics

<i>Variable</i>	<i>Mean</i>	<i>Median</i>	<i>St. Dev.</i>	<i>Minimum</i>	<i>Maximum</i>
Five-to-ten Year Forward Inflation Compensation	2.39%	2.45%	0.44%	0.82%	3.59%
One Year Forward Nominal Treasury yield	5.05%	4.99%	3.29%	0.04%	16.14%
Five Year Forward Nominal Treasury yield	6.03%	5.8%	2.82%	0.51%	14.72%
Ten Year Forward Nominal Treasury yield	6.74%	6.54%	2.78%	0.89%	15.28%
Twenty Year Forward Nominal Treasury yield	6.35%	5.86%	2.97%	1.64%	18.36%
Thirty Year Forward Nominal Treasury yield	5.04%	4.51%	1.99%	0.41%	11.5%
Factor returns	0.01%	0.01%	0.56%	-14.27%	14.14%

³<https://www.federalreserve.gov/data/nominal-yield-curve.htm>, Data downloaded on: 10th of March 2024.

Table 2: Data availability

<i>Data</i>	<i>Data start</i>	<i>Data end</i>
Five-to-ten Year Forward Inflation Compensation	1999-01-04	2024-03-01
Nominal Treasury yield	1961-06-14	2024-03-01
Factor returns	1972-01-03	2022-12-30

2.2.1 Inflation compensation

Comparing the TIPS yield curve with the corresponding nominal yield curve allows measures of inflation compensation (or breakeven inflation rates) to be computed. (Gürkaynak et al. 2010) Gürkaynak et al. in their paper backcast the five-to-ten-year forward inflation compensation. In order to have comparable results we use the same metric in this study. The inflation compensation is displayed on Figure 2.

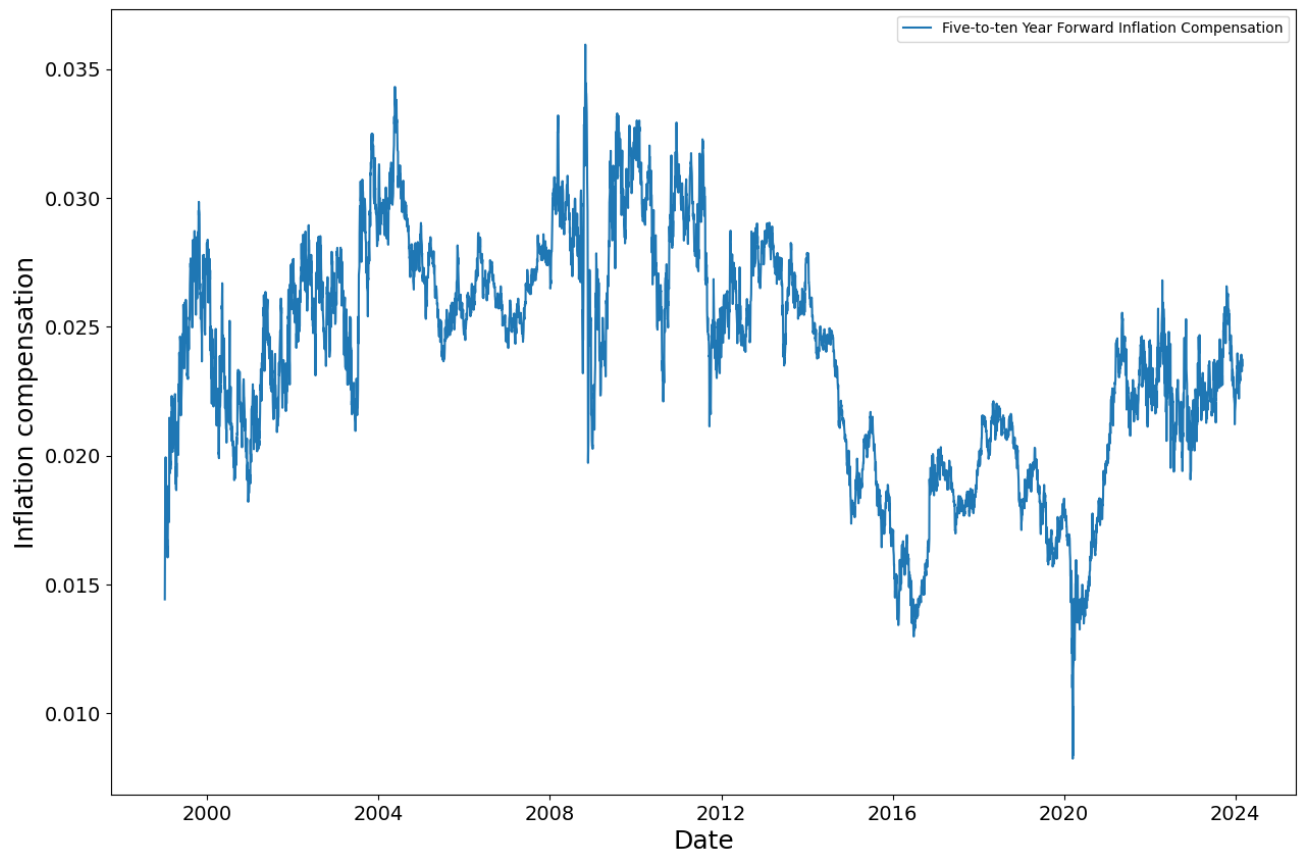


Figure 2: Five-to-ten Year Forward Inflation Compensation

2.2.2 Factors

Jensen et al. (Jensen et al. 2023) provides daily returns on the US market for 153 different characteristics-managed portfolios.⁴ As shown in Figure 3 the distribution of factors seems to be centered around zero, with most of the observations falling between the range of -1% and +1%. The light tails of the distribution reflect that daily factors may have extreme values, but these events happen only occasionally thus there are only a few of them in our sample. This is also represented in Figure 4: the daily return's distribution is light tailed relative to the normal distribution: less observations fall on the tails relative to the theoretical quantiles

⁴The exact calculation and meaning of these factors are available at Bryan Kelly's documentation: <https://jkpfactors.com/>

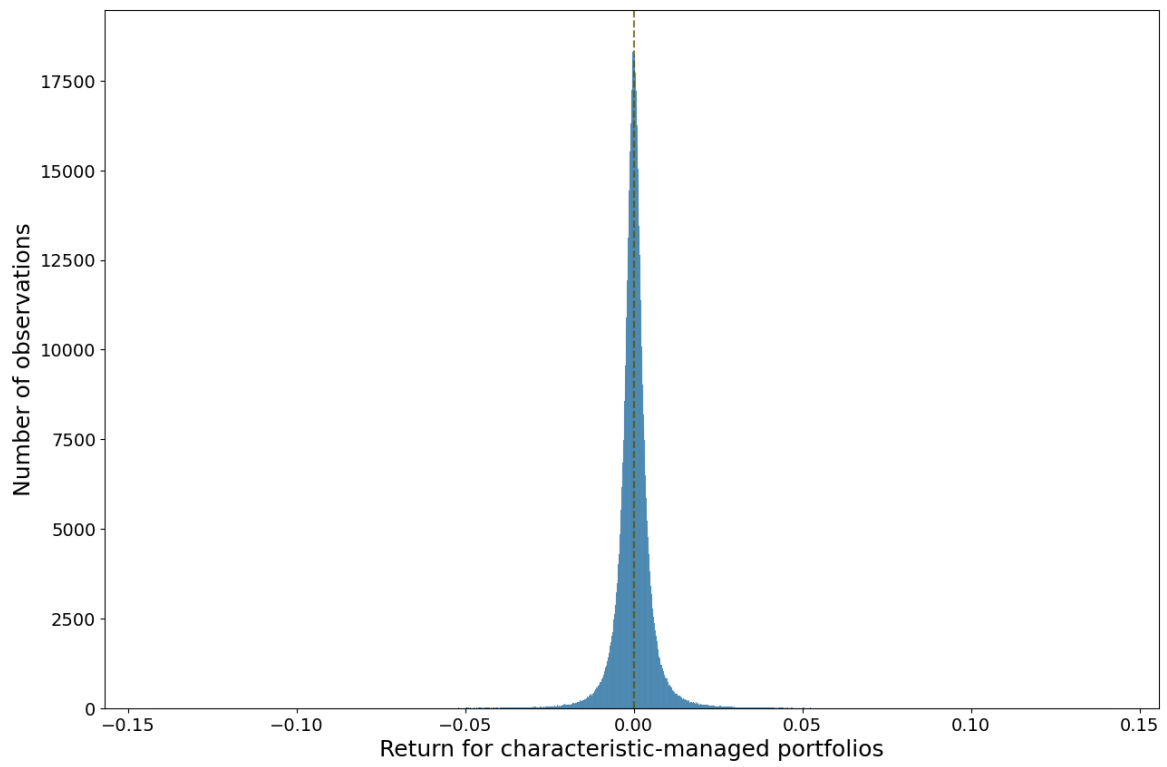


Figure 3: Factors' distribution (all factors)

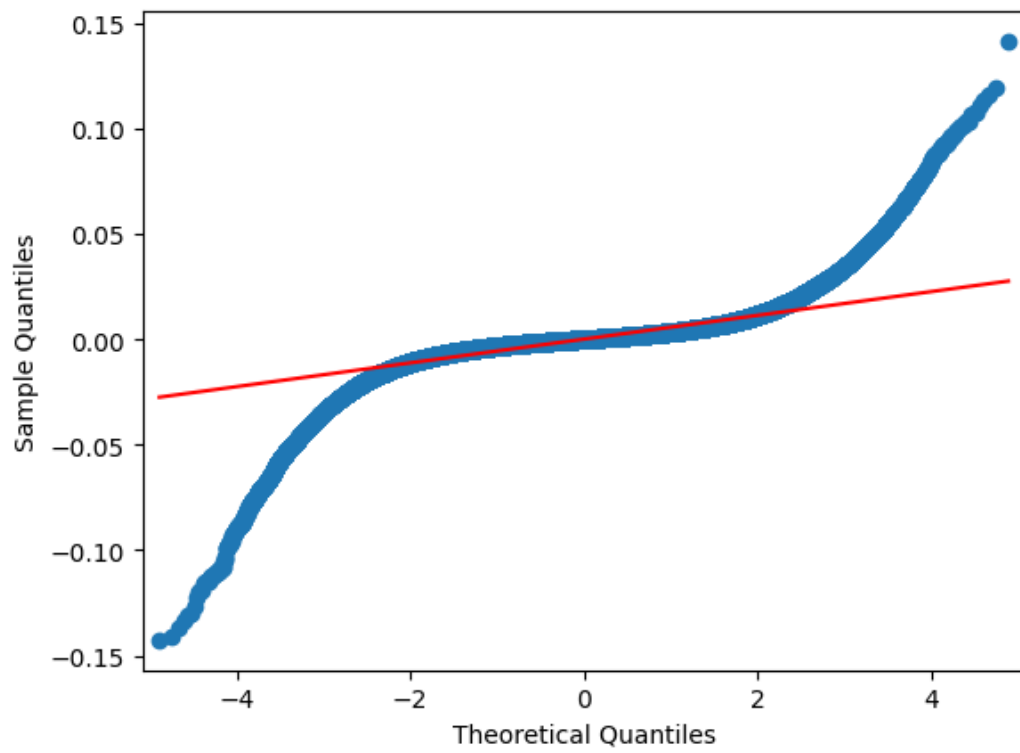


Figure 4: Q-Q plot of the daily factors

Figure 5 shows the monthly average return for each factor for each month since 1973. The more red (more blue) a tile is, the lower (the higher) the factor is in the given month. The Figure allows us to form two weak hypotheses. First, factor volatility seems to be associated with financial crises: extreme factor values seem to be present around 2000 (Dot-com bubble), 2007 (2007-2008 financial crisis) and since 2020 (COVID-crisis). This is also supported by the monthly average return and standard deviation calculated from all factors on Figure 6. Second, the standard deviation of the factors are different: there are factors which seem to be persistently more volatile, while others seem to be less deviated from zero.

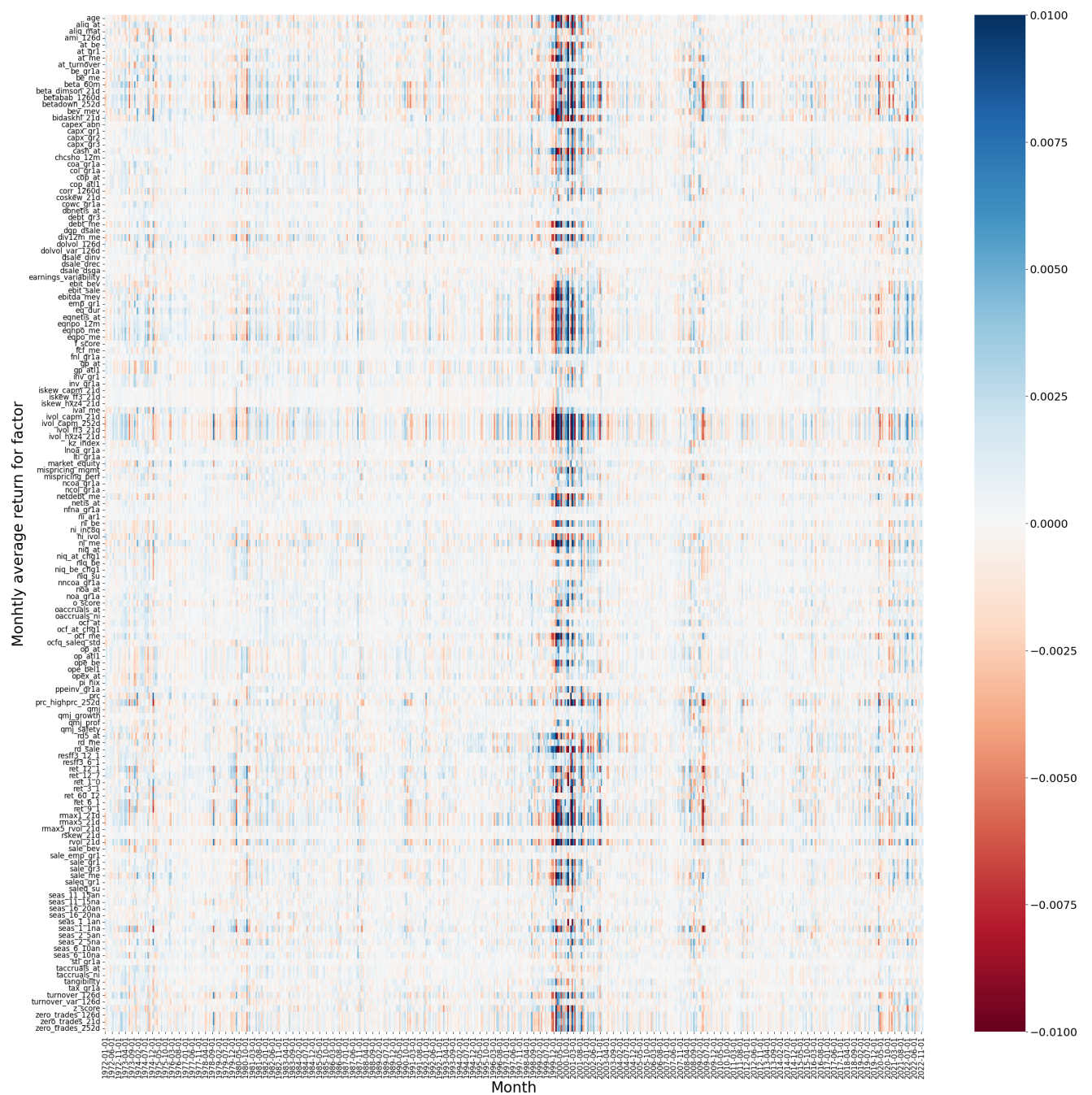


Figure 5: Monthly average factor returns since 1973

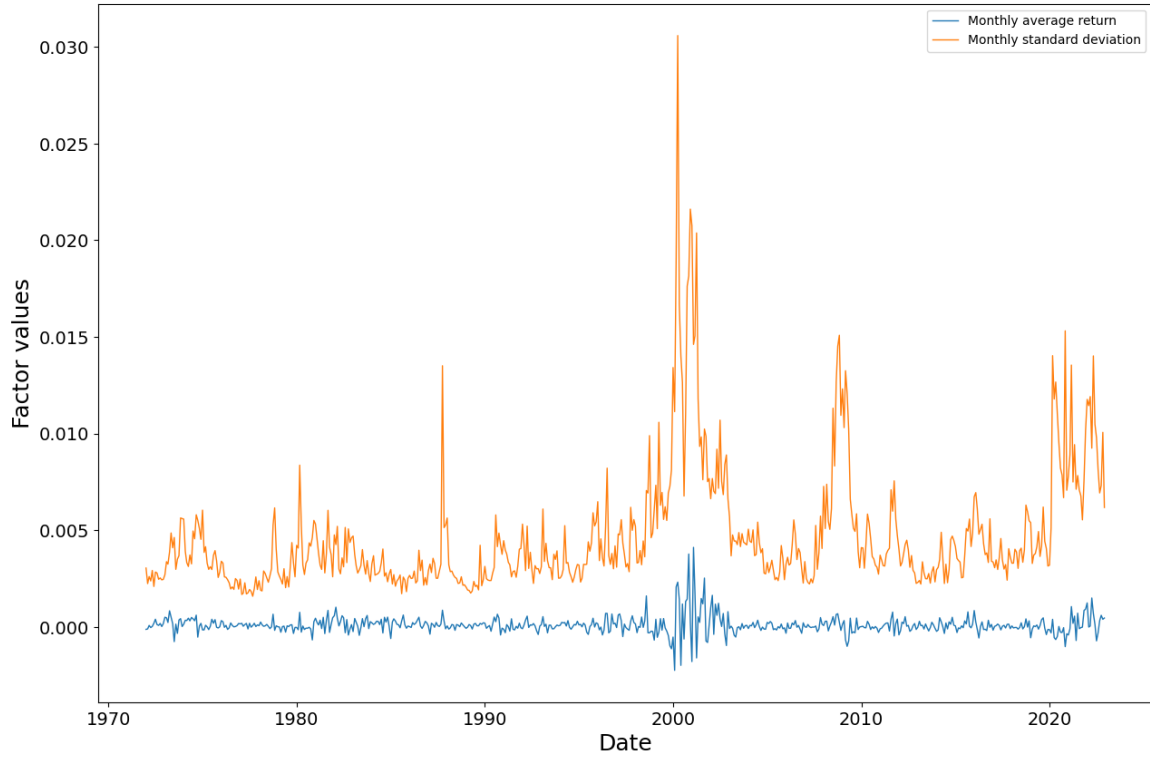


Figure 6: Monthly average of all factors' return

2.2.3 Nominal yields

Gürkaynak et al. (Gürkaynak et al. 2007) have estimated the U.S. Treasury yield curve at a daily frequency from 1961 to the present. The resulting yield curves are available in terms of zero-coupon yields, par yields, instantaneous forward rates, or n by- m forward rates (that is, the m -year rate beginning n years ahead) for any n and m . In this study we use the continuously compounded instantaneous 1 to 30 years forward rates. A subset of the available maturities are displayed on Figure 7.

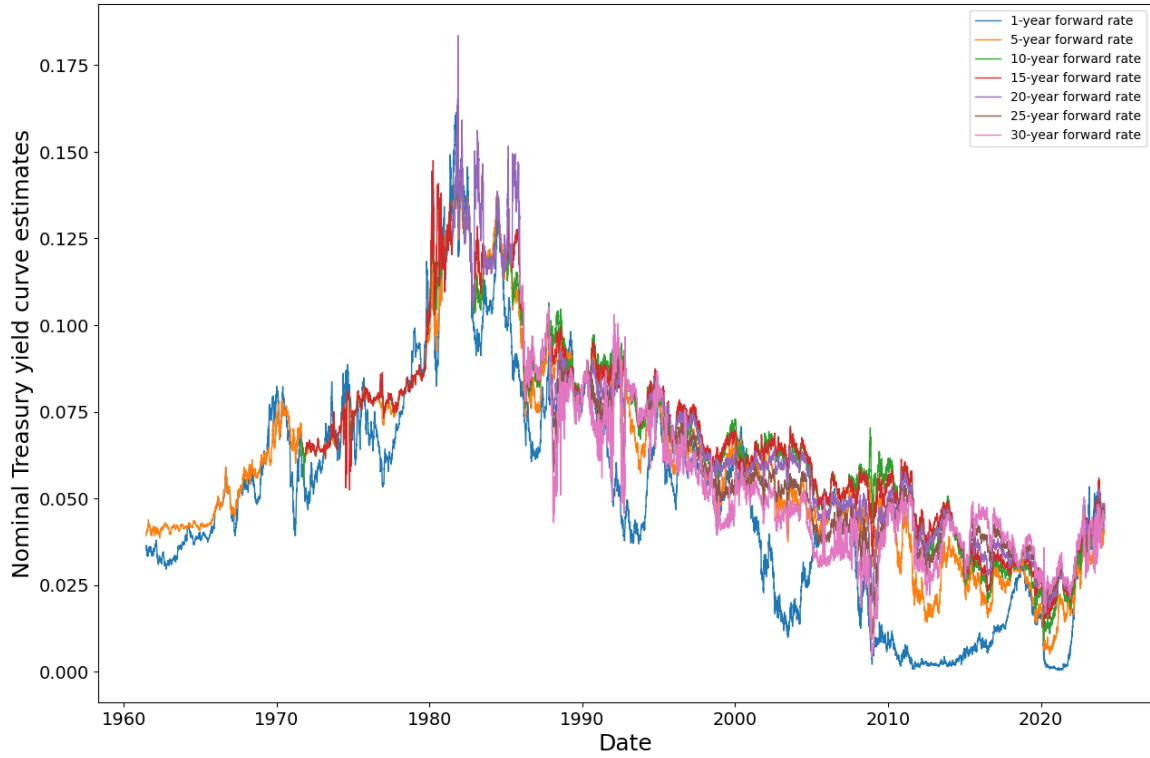


Figure 7: Continuously compounded instantaneous forward rate at various maturities

3 Methodology

The TIPS-based inflation compensation term structure's usage is limited due to the relatively short time period for which data is available (since 1999). Using the relationship between other indicators with longer data availability and inflation compensation the literature back-casted inflation compensation to the past.

Our hypothesis is that the factors provide an opportunity to obtain a more accurate backcast. To make this hypothesis measurable, the factor-based estimations are compared to the nominal yield based backcast (inflation expectation extended until back to 1971) which is widely used in the literature. Framing our hypothesis in this setting means, that with this study we analyse whether factor-based inflation expectation estimations are more accurate compared to the nominal yield based estimations.

As the TIPS-based inflation expectation is only available from 1999 the backcasted inflation compensation can not be compared to the factual values. For that reason two rolling window backtest is executed on the period between 2010 and 2022 using the factors and the

nominal yields respectively. The two out-of-sample predictions are then compared to the factual values. This way we can compare the out-of-sample performance of the factor-based and the nominal yield based approach.

The rolling window backtest scheme is displayed on Figure 8. In the first pass a factor-based and a nominal yield based model is trained using data from January 2003 to December 2009 and predict the monthly inflation expectation between January 2010 and December 2010. Thus we have two prediction for each month in 2010: one stemming from the model which used the factors as regressors, and another one which was created based on the nominal yields. In the next pass the training sample is extended to the December 2010, whilst the training sample start remains 2003. With the models trained in this pass the inflation expectation between January 2011 and December is predicted. These steps are repeated until we reach the last pass in which we train models using data between January of 2003 and December of 2021 and predict to 2022 (Table 3).

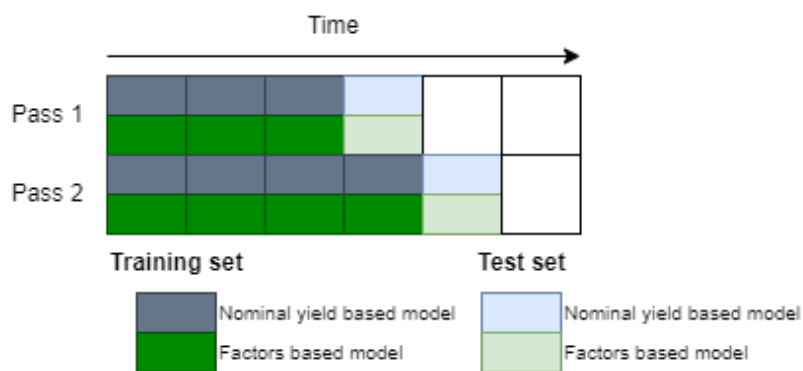


Figure 8: Backtesting setup

Table 3: Train and test periods during rolling window bactest

<i>Train period start</i>	<i>Train period end</i>	<i>Test period start</i>	<i>Test period end</i>
2003-01	2009-12	2010-01	2010-12
2003-01	2010-12	2011-01	2011-12
2003-01	2011-12	2012-01	2012-12
2003-01	2012-12	2013-01	2013-12
2003-01	2013-12	2014-01	2014-12
2003-01	2014-12	2015-01	2015-12
2003-01	2015-12	2016-01	2016-12
2003-01	2016-12	2017-01	2017-12
2003-01	2017-12	2018-01	2018-12
2003-01	2018-12	2019-01	2019-12
2003-01	2019-12	2020-01	2020-12
2003-01	2020-12	2021-01	2021-12

Gürkaynak et al. (Gürkaynak et al. 2010) in their study applied principal component analysis on the nominal yield structure and used the first three components for the backtesting estimation. Most of the variation in nominal yields can be explained by the first principal component which can be interpreted as the level of yields. Including two more principal components—commonly interpreted as the slope and curvature, respectively—accounts for nearly all of the variation in yields (ibid.). This analysis deviates from that and neglects the PCA and uses the raw nominal yields.

Following their study and due to the the poor liquidity of TIPS in the early years of the TIPS program we estimate the regression using monthly average data starting from 2003.⁵

The backtesting process results in two prediction for each month between 2010 and 2022,

⁵Due to the considerable noise in the factor data on daily granularity, the daily analysis was not feasible.

144 observations altogether. The two predictions are then compared using the error metrics described in Equations 1 to 3.

$$error = prediction - fact \quad (1)$$

$$absolute\ error = |error| \quad (2)$$

$$absolute\ percentage\ error = \frac{absolute\ error}{fact} \quad (3)$$

4 Results

This section describes the rolling window backtest results. First we executed the above described rolling window backtest using linear regression model. The unknown parameters were choosed with ordinary least squares (OLS) method. Next we execute the rolling window backtest with LGBM models .

LightGBM (Light Gradient Boosting Machine) is a gradient boosting framework developed by Microsoft that has gained popularity in ML projects due to its speed and accuracy in handling large datasets. Unlike traditional linear regression models, LightGBM utilizes decision tree-based algorithms to iteratively improve predictive performance. In our hypothesis it should more effectively capture non-linear relationships through decision tree-based algorithms. (Ke et al. 2017) This makes it a preferred choice for the complex modeling task in this study.

4.1 OLS

Backtesting with linear regression showed that the nominal yield based predictions are superior to the factor based estimations. Over the analysed 12 years the monthly nominal yield based predictions had a mean absolute percentage errors (MAPE) of 12.75%, whilst the factor based predictions had a MAPE of 55.3% (Table 9).

Table 4: Monthly mean absolute percentage errors with OLS

<i>Year</i>	<i>Factor based MAPE</i>	<i>Nominal yield based MAPE</i>	<i>Winner</i>
2010	9.47%	6.12%	Nominal yield based
2011	11.23%	4.98%	Nominal yield based
2012	16.44%	11.21%	Nominal yield based
2013	16.55%	4.59%	Nominal yield based
2014	19.17%	8.11%	Nominal yield based
2015	79.42%	26.22%	Nominal yield based
2016	198.91%	50.14%	Nominal yield based
2017	48.13%	3.23%	Nominal yield based
2018	44.8%	7.78%	Nominal yield based
2019	50.6%	6.44%	Nominal yield based
2020	125.83%	6.59%	Nominal yield based
2021	60.93%	19.41%	Nominal yield based
2022	37.38%	11.8%	Nominal yield based
2010 - 2022	55.3%	12.75%	Nominal yield based

The yearly comparison reveals that the performance difference between the nominal yield based and factor based predictions were smaller from 2010 to 2014. From 2015 the factor based predictions became much more volatile as it is clearly displayed on Figure 9. As a result the magnitude of absolute percentage errors multiplied (Figure 10), reaching its "peak" in 2016 where the yearly MAPE of factor based predictions were 198.91% (Table 4)!

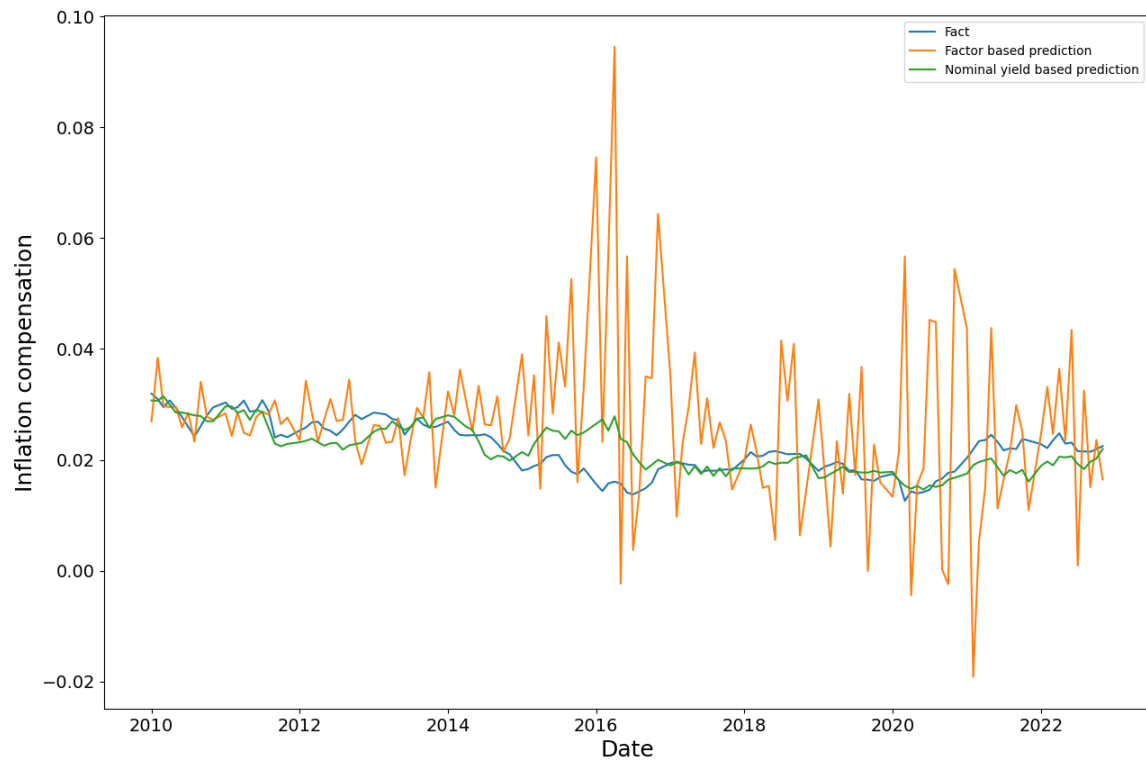


Figure 9: Rolling backtest results with OLS

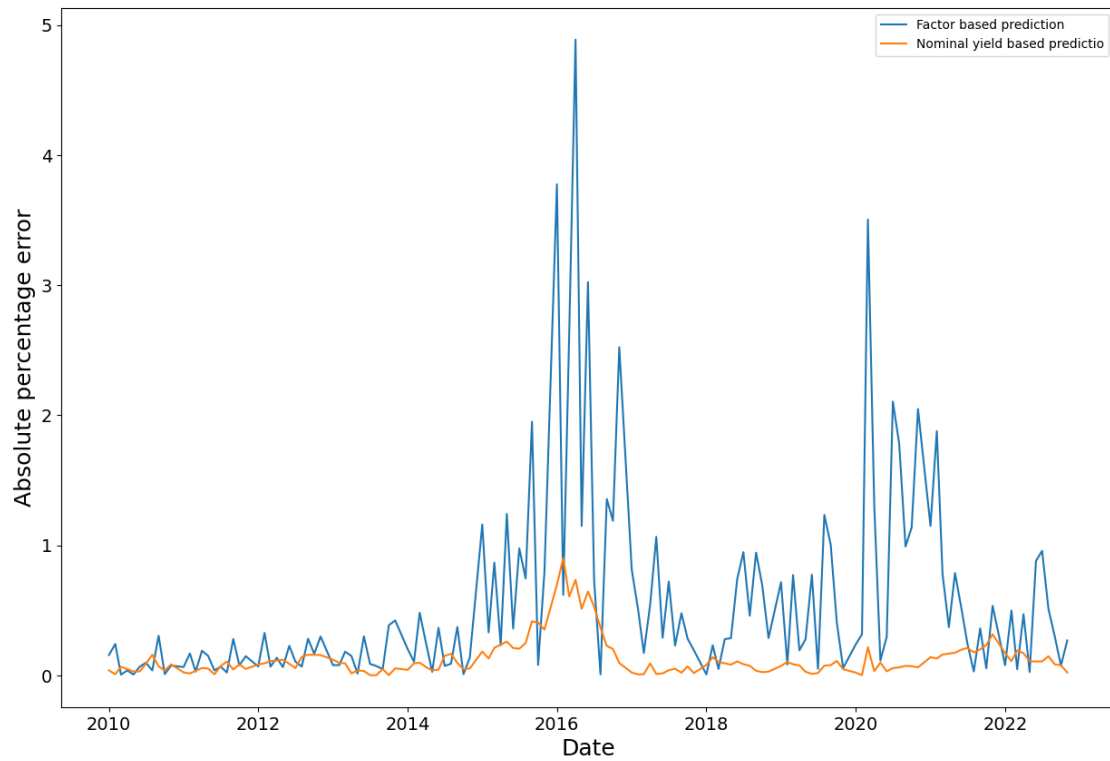


Figure 10: Daily absolute percentage errors with OLS

The nominal yield based predictions performance also fluctuates over time, but is much more stable compared to the factor based model. The nominal yield based model also showed elevated errors during 2016 (Figure 12), but their magnitude is lower than the factor based model's APEs. Thus based on the OLS based approach we can conclude that the nominal yield based model is clearly superior to the factor based model. The large difference in MAPE might be because the relationship between the factors and inflation compensation is non-linear, or would require the creation of feature interactions or the relationship is non-existent at all. For that reason in the next section we employ the more sophisticated LightGBM algorithm which is hypothesised to better capture the nonlinearity.

4.2 LGBM

The LightGBM algorithm seemed to be able to mitigate the volatility of the factor based predictions: whilst the OLS predictions ranged between -1.92% and 9.46%, the LGBM predictions are less spread out ranging between 1.85% and 3.00% (Figure 11). As a result the factor based MAPE dropped from 55.3% to 23.11%. The LightGBM improved the overall performance of the nominal yield based predictions: the overall MAPE decreased by 1.22 percentage points from 12.75% to 11.53%.

Table 5: Monthly mean absolute percentage errors with LGBM

<i>Year</i>	<i>Factor based MAPE</i>	<i>Nominal yield based MAPE</i>	<i>Winner</i>
2010	7.67%	8.87%	Factor based
2011	10.17%	3.67%	Nominal yield based
2012	5.64%	6.11%	Factor based
2013	4.85%	5.47%	Factor based
2014	13.93%	6.49%	Nominal yield based
2015	40.38%	21.56%	Nominal yield based
2016	70.57%	23.62%	Nominal yield based
2017	35.53%	8.1%	Nominal yield based
2018	15.82%	9.17%	Nominal yield based
2019	31.52%	5.68%	Nominal yield based
2020	48.14%	12.43%	Nominal yield based
2021	7.99%	28.03%	Factor based
2022	10.83%	11.93%	Factor based
2010 - 2022	23.11%	11.53%	Nominal yield based

Despite, the improvements the nominal yield based predictions are still more accurate than the factor based predictions. The 11.58 percentage point MAPE difference clearly reflects the better performance of nominal yield based predictions as depicted in Table 5. Despite being inferior based on the overall performance the factor based predictions are more accurate in 2010, 2012, 2013 and 2021, 2022.

However, as Figure 11 shows the factor-based predictions seem to be unable to follow the fluctuations in inflation expectations over time but predict an inflation expectation close to the mean. ⁶ This somewhat casts a shadow over the years when the factor-based model was

⁶The inflation expectation mean over the period 1999 and 2010 was 2.62%.

superior: the factor-based predictions performed well in years when the inflation expectations were close to the average. However, the backcasted inflation expectation would be usable if they reflect the deviations of breakeven inflation rate. In that sense we can conclude, that the factor based out-of-sample predictions are not suitable for backcasting.

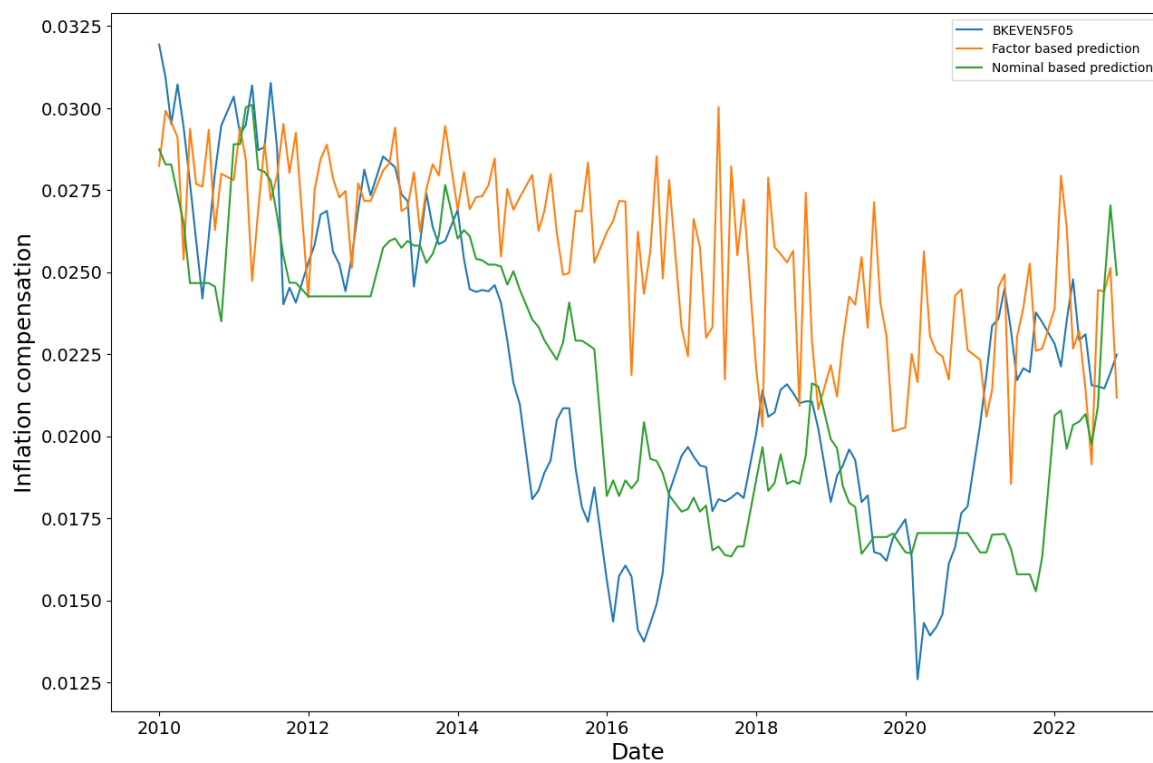


Figure 11: Rolling backtest results with LGBM

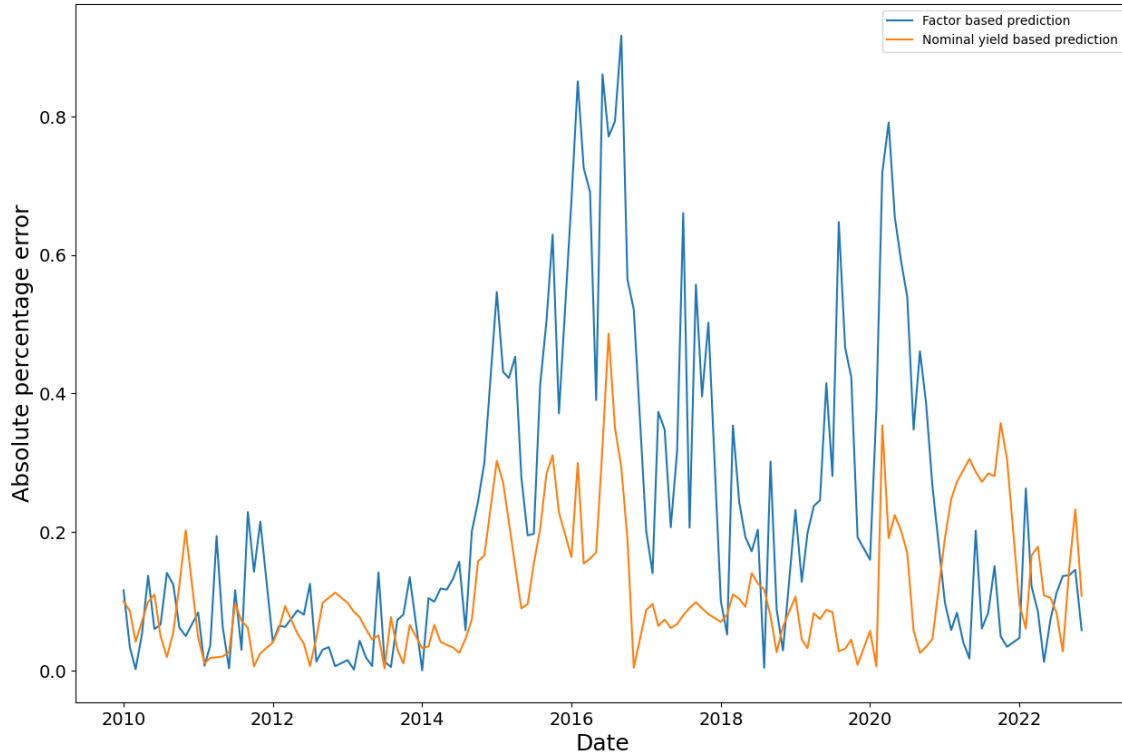


Figure 12: Daily absolute percentage errors with LGBM

5 Extending nominal yield based model with factors

As seen from previous sections the nominal yields provide more accurate out-of-sample predictions for inflation compensation than the factors. The factor-based predictions outperformed the nominal-based approach in the beginning and the end of the testing period (2010-2022). As our last effort we try to combine the advantages of the nominal and factor-based approaches and carry out a backtest using the factors and the nominal yields in the same model. The results are compared to the nominal-based model which proved to be superior in Section 4.2.

As seen on Figure 13 and Figure 14 the two approaches provide very similar outputs. The addition of factors as new features do not seem to improve the out-of-sample estimations during those years when the errors are large. During 2015 and 2016 the nominal-yield based predictions were larger than the factual inflation compensation, whilst from 2020 to 2022 the predictions were below the facts. The addition of factors did not improve the performance during these periods.



Figure 13: Rolling backtest results with and without factors

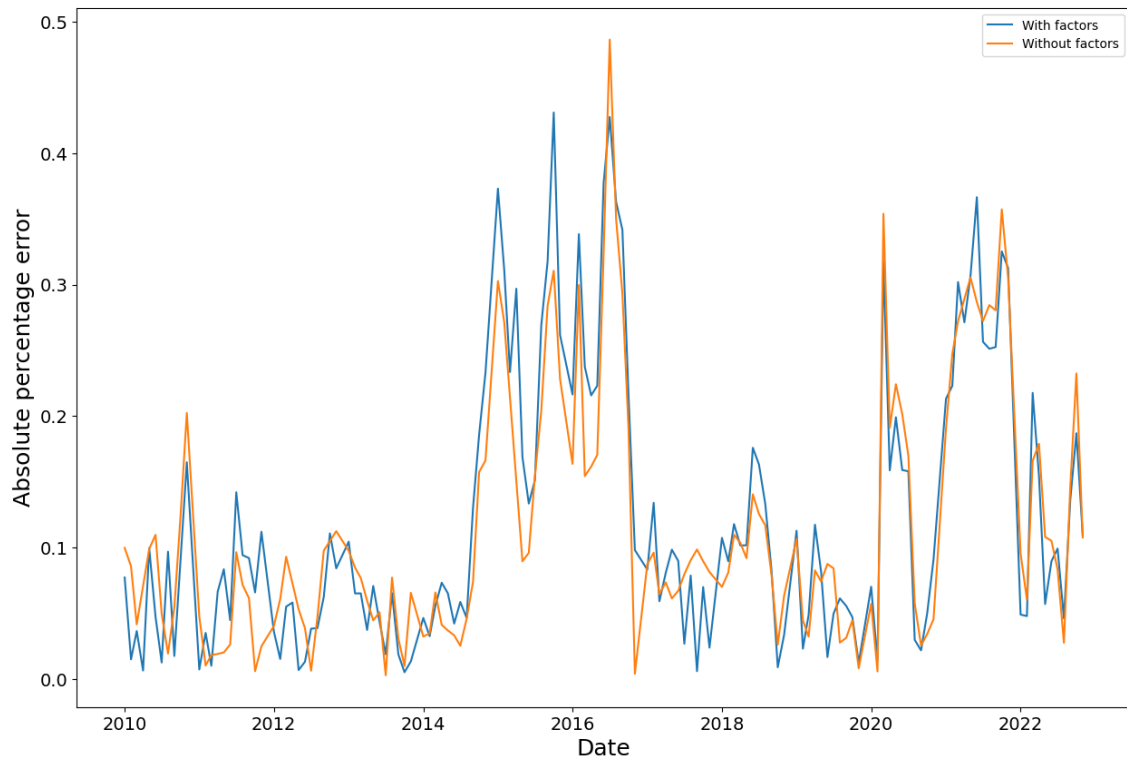


Figure 14: Daily absolute percentage errors with and without factors

Seeing the yearly MAPEs there are 6 years from the 12 analysed years in which the performance is better with the factors included. Although, as discussed above, the new features were not able to increase performance during those years when the nominal yields seemed to lack predictive power.

Table 6: Monthly mean absolute percentage errors with and without factors

<i>Year</i>	<i>MAPE with factors</i>	<i>MAPE without factors</i>	<i>Winner</i>
2010	6.04%	8.87%	With factor
2011	6.86%	3.67%	Without factors
2012	4.73%	6.11%	With factors
2013	4.62%	5.47%	With factors
2014	8.82%	6.49%	Without factors
2015	26.8%	21.56%	Without factors
2016	27.78%	23.62%	Without factors
2017	6.84%	8.1%	With factors
2018	10.14%	9.17%	Without factors
2019	5.71%	5.68%	Without factors
2020	11.54%	12.43%	With factors
2021	28.02%	28.03%	With factors
2022	10.85%	11.93%	With factors
2010 - 2022	12.11%	11.53%	Without factors

6 Conclusion

The inflation compensation term structure associated to TIPS yield curve have many applications for practitioners and researchers, but their usefulness is limited due to the relatively

short time period for which data is available. For that reason the literature tried to extend the inflation compensation time series: find stable relationship between inflation compensation and indicators with longer availability and use that relationship to backcast inflation expectations.

So far the literature concentrated to backcast inflation compensation using nominal yield data. In this study we aim to backcast inflation expectations using stock market informations. Based on the relationship between inflation expectations and asset returns we hypothesise that inflation compensation could be modeled with return data for characteristic-managed portfolios, or "factors".

In order to make our hypothesis measurable we carry out a rolling window backtest for the past to cross-validate that the factor-based out-of-sample estimation are more accurate than the estimations from the nominal yield based model. Framing our hypothesis in this setting means, that with this study we analyse whether factor-based inflation expectation estimations are more accurate compared to the nominal yield based estimations.

Executing the rolling window backtest using linear regression model revealed that the factors based predictions were much more volatile than the predictions using the nominal yields as regressors. As a result during 2010-2022 the overall MAPE of the factor-based model was 55.3%, whilst the nominal-based prediction had a mean absolute percentage error of 12.75%. Seeing the MAPEs and the instability of factor-based predictions we can conclude that the nominal yield based model is clearly superior to the factor based model. The large difference in MAPE might be because the relationship between the factors and inflation compensation is non-linear, or would require the creation of feature interactions or the relationship is non-existent at all.

Next we employed the more sophisticated LightGBM algorithm which might help tackle the above-mentioned problems. The overall MAPE of the factor-based model declined sharply to 23.11%, however still fell short of the nominal yield based estimations' MAPE of 11.53%. The introduction of the more sophisticated LGBM improved the performance of both models compared to the OLS.

The factor-based predictions seem to be unable to follow the fluctuations in inflation expectations over time but seem to have predicted an inflation expectation close to the mean. As a result the factor-based predictions performed better compared to the baseline in

years when the inflation expectations were close to the average. Being unable to reflect the deviations of breakeven inflation rates we can conclude that the factors, in this form, are not suitable for backcasting.

Despite the limited accuracy of the proposed approach, this study might help further research. In this study we included all the available 153 factors. It is possible that some factors are useful to model inflation expectation while others are not. In that sense one might attempt to employ a variable selection for the factors in order to mitigate the noise. Instrumented Principal Component Analysis (Kelly et al. 2020), an approach of factor analysis, might be a suitable methodology to select the most relevant factors.

Also, another extension of this current analysis could be to compare in-sample-estimations stemming from the factors-based and the nominal yields-based models. Periods when the factor based in-sample estimations were more accurate than the nominal yield based predictions might sign periods when inflation expectations were fueled by stock market changes. Using SHapley Additive exPlanations (SHAP values) (Lundberg and Lee 2017) - a game theoretic approach to explain the output of any machine learning model - one can obtain the factors which were the most important ones during the period in question. This analysis could better help understand the underlying causes of driving inflation expectations in the past.

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