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**INFLÁCIÓS CÉLKÖVETÉS KONTRA ÁRSZINTCÉLZÁS
HISZTERÉZIS JELENLÉTÉBEN- EGY ÚJ-KEYNESI ELEMZÉS
INFLATION TARGETING VERSUS PRICE LEVEL TARGETING
UNDER HYSTERESIS – A NEW KEYNESIAN ANALYSIS**

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Abstract

The primary aim of my thesis is to compare two common monetary policy rules: inflation targeting and price level targeting, from a welfare point of view. The New Keynesian model framework will construct the baseline, which will be extended with hysteresis.

In economics, hysteresis is a phenomenon where a transitory shock exerts a permanent effect on the long-term equilibrium of a system. It has been proven to be able to explain high persistence in output, though its implications are widely overlooked. The New Keynesian model framework will be used as the baseline, which will be extended with hysteresis in the hope of achieving a model that better replicates the patterns seen in empirics. Two types of monetary policy rules will be applied, inflation targeting and price level targeting. To prime my analysis and to unravel the underlying mechanisms, I will first analyze impulse response functions. Then, to measure and assess the performance of the two monetary policy rules, an ad hoc social welfare loss function will be used, which quantifies the utility losses experienced by households during deviations from the optimal allocation.

My results indicate that price-level targeting is a better policy alternative than inflation targeting, as it showed better control over inflation and output variability under technology and demand shocks. However, the case of markup shock was less clear. While price-level targeting restricted inflation's variability better in all cases, its relative efficacy in curtailing output gap's volatility depended on the level of hysteresis.

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1. Introduction

As inflation and a high interest rate environment have been prevailing in the recent years, it has become increasingly relevant to assess the efficiency of monetary policy applied. Motivated by this turbulent period and its significance, I turn to compare two of the most important monetary policies, inflation targeting and price-level targeting. To make my analysis more realistic, I extend the model with hysteresis, the concept of which gained popularity in macroeconomics after the Global Financial Crisis of 2008.

There has already been some progress towards alternative monetary policies, in which the U.S. Federal Reserve has made major strides. In 2020, the Fed has introduced average inflation targeting, which can be viewed as a golden mean between inflation and price-level targeting. For this very reason, it is of great importance to investigate alternative monetary policies utilizing the New Keynesian model.

Based on ideas formulated in the literature, I expect that in the presence of higher degrees of hysteresis, price-level targeting promises a greater advantage compared to inflation targeting, than under lower degrees of hysteresis. The rationale behind it can be that price-level targeting allows for looser monetary policy that can keep aggregate demand high, which can stabilize the real economy or possibly eliminate some of the losses created by hysteresis.

In my analysis, I have found that price-level targeting irrefutably outperforms inflation targeting in the cases of technology and demand shocks, and it only falls short in the case of markup shock if the weight of output gap's standard deviation is high in the ad hoc welfare loss function.

My thesis consists of six chapters, the second of which is an introduction to monetary policy and hysteresis. In the third chapter, I present a basic empirical analysis about hysteresis in the United States, and the fourth is dedicated to the structure of the New Keynesian model. In the fifth section, I present my results, which entail interpretations of impulse response functions and an ad hoc welfare loss analysis. Finally, I present my conclusions in the last chapter.

2. Literature review

2.1. Monetary Policy

Monetary policy is one of the most important tools with which the pace and direction of the economy, the level of aggregate output, employment levels and the rate of price changes can be controlled (Friedman, 2000). For this very reason, the emphasis on monetary policy has been on the rise for the past decades. Monetary policy is performed by special institutions called central banks, and their fundamental goal is to maintain price stability – in other words, to avoid excessive inflation or deflation- and to foster the highest possible level of employment and output (Svensson, 1999; Friedman, 2000). To achieve this objective, central banks may use open market operations (purchasing and selling securities), alter the reserve requirement ratio (minimum deposit held in reserves) or conduct interest rate policy, the most common of which is inflation targeting (IT) (Micossi, 2015). There are other theoretically feasible and effective monetary policy rules, like price-level targeting (PLT) and average inflation targeting (AIT), which have recently gained more popularity. Price level stability is often interpreted as a stationary price level with moderate variation, whereas inflation targeting refers to low and stable inflation (Svensson, 1999).

2.1.1. Inflation targeting

In the 1990s there has been a growing awareness about interest rate setting as a monetary policy tool, which culminated in the adaptation of formal interest rate regimes. New Zealand was the first country to implement inflation targeting in 1990, which was followed by other developed and developing countries (Svensson, 2010). The idea was to set a numerical inflation target against which the central bank could be held accountable by political institutions and the public. The target for many industrialized economies is around 2 percent at annual rate for the Consumer Price Index (CPI) or core CPI, whereas in emerging economies, it is usually a few percentage points higher. In practice, inflation targeting is a flexible tool, meaning that central banks do not solely focus on keeping inflation around the target, but also consider indicators of the real economy, like the difference between actual and potential output.

The arguments in favor of inflation targeting are that it has a clear objective, it is easily communicated and it makes central banks accountable and transparent. Its advantages can also be justified by the fact that it ignores bygones, and that it does not cause any further disruptions like price-level targeting does (Gasper et al., 2007). Furthermore, unlike with the transition to

other monetary policies, keeping inflation targeting would mean no additional costs of communication or inherent discontinuity.

One of the main concerns about inflation targeting is that its low target restricts monetary policy, meaning that central banks can lose the ability to lower interest rates in times of crisis, leading to devastating consequences (Bernanke, 2019).

2.1.2. Price-level targeting

Virtually all central banks in the world follow inflation targeting regimes, Sweden being the only country that has explicitly adopted price-level targeting in the 1930s (Berg and Jonung, 1998). According to this targeting rule, the central bank reacts to inflation on an infinite time horizon. If the price level gets diverted away from the targeted level as a result of a shock, the central bank targets average inflation to guide it back. If the targeted a price level is a constant, it is maintained by seeking zero inflation, meaning that after a period of inflation, a deflationary period must follow to compensate.

Many economists believe that price-level targeting can be a more favorable monetary policy tool compared to inflation targeting (Svensson, 1999; Williams, 2017). The first advantage of price-level targeting is that it is also consistent with the price stability objective of central banks, and it might even be more efficient in achieving it than inflation targeting. As it promises to stabilize long-run average inflation, it results in less uncertainty about future price levels, which is essential for long-term planning like purchasing real estate and retirement (Gasger et al., 2007). Another benefit of price-level targeting is that it has a compensating characteristic, which is a feature of the theoretical optimal monetary policy (Bernanke, 2019). It may also be a better alternative near the zero lower bound, as the reversing of the price level can mitigate the constraint on nominal rates by boosting the intertemporal adjustment mechanism, namely, that expectations move in opposite directions with diversions from the price level (Gasger et al., 2007; Evans, 2012).

However, this strategy is not free of drawbacks either. First, central banks would have to change their policy-making processes and would need to amend their reaction functions, which could potentially cause a financial burden. Assessing the difficulty of getting the public and market agents to comprehend and accept the new approach is also challenging, as it could potentially necessitate great effort in education by policymakers. Furthermore, the question of how quickly the public could conform to the new targeting rule is unanswered. Also, due to the nature of the targeting framework, the central bank is unable to disregard supply shocks that

fuel inflation—instead, it is committed to implement stricter monetary policy to counteract the impact of the shock on the overall price level. Price-level targeting was also attacked for causing a higher inflation and output gap variability as a result of sticky wages, since the monetary policy would need to push output over or below potential levels to reach its objective (Gasper et al., 2007). Lastly, those who oppose price-level targeting sometimes point out that if expectations are backward-looking, its benefits are negligible compared to inflation targeting.

2.1.3. Average inflation targeting

In August of 2020, the U.S. Federal Reserve has announced the adoption of a new monetary policy framework, where instead of inflation targeting, average inflation targeting will be pursued. The Fed applies this targeting rule by aiming for 2 percent inflation on average over time (Naggert et al., 2021). This shift was motivated by the fact that inflation had stayed below the targeted 2 percent for many years, and since inflation expectations play a major role in the formation of inflation, it caused concern that a persistently lower inflation environment would lower inflation expectations, perpetuating low inflation rates. Furthermore, if inflation is lower than the targeted value during recessions, when monetary policy is constrained by the zero lower bound, and it rises above the target during expansions, people will begin to expect that inflation generally falls short of the target over extended periods. This shift can support the Federal Reserve to achieve and sustain inflation at 2 percent, as average inflation targeting could help better anchor inflation expectations of individuals, businesses, and financial markets. This approach can be interpreted as a golden mean between the previously presented targeting rules. The difference between price-level targeting and average inflation targeting is that the latter one does not operate on an infinite time horizon but on a narrower window.

2.2. Hysteresis

Hysteresis in macroeconomics refers to a concept of path dependence, where the history of the process has a permanent influence on its long-run steady state (Mark Setterfield, 2010). The popularity of this theory has been on the rise, as explaining the recovery process following the Global Financial Crisis of 2008 has baffled economists. It happened that a great number of economies have failed to return to their pre-crisis growth paths, settling on a permanently lower path level, and in many cases, on a path with a smaller growth rate. According to mainstream macroeconomics, the effects of temporary shocks are eliminated, as market forces guide the

economy back to the previous growth path (Woodford, 2003; Galí, 2008). This theory is often justified by the argument that the long term growth rate of output is defined by other factors than those that define short-term fluctuations.

However, analysis conducted by Váry (2018) has shed light on the possible manifestation of hysteresis. By extrapolating the trends, he obtained the growth that would have happened in the absence of hysteresis, and he discovered that a transitory negative demand shock has evidently affected the long-term growth rate of GDP in the instance of the U.S. An even more striking case has been revealed in Hungary, where the shock has not only pushed the economy to a permanently lower growth path, but has also affected the growth rate of the economy, which can be referred to as super-hysteresis. While other factors could also contribute to the explanation of the recovery, it is evident that some other mechanisms are also at play.

Proceeding studies have not only supported the presence of hysteresis after the global crisis, but also pointed out that it is not exclusive to the recovery. Ball (2009) contends that the natural rate of unemployment and inflation moves in opposite directions, which indicates hysteresis. He states that when a transitory negative demand shock hits the economy, unemployment increases, which, according to the Philips curve, results in a disinflationary pressure. He portended that the natural rate of unemployment will rise under disinflationary periods if hysteresis is present during transitory demand shocks. Through empirics, he confirmed just that, as higher than 3 percentage point increases in unemployment were accompanied by disinflation in the case of 20 OECD countries. Similarly, he found that higher than 3 percentage point decreases in the natural rate were followed by inflationary periods, showcasing the potential presence of hysteresis in employment dynamics.

Blanchard (2015) examined other recovery periods following crises, and concluded that hysteresis is indeed not a unique phenomenon of the Global Financial Crisis. According to his results which examined 22 developed countries between 1970 and 2010, hysteresis was evident in two-thirds of recovery periods, and super-hysteresis was present in one-third of the cases.

Galí (2015) argues that in hysteresis the natural rate of unemployment fluctuates together with the actual rate, which can risk the (natural rate) detachment of expectations. Stockhammer and Sturn (2012) have discovered that in crises where central bank easing was not sufficient or where tightening was conducted, a higher degree of hysteresis was observable. Along the thought process of Ball (2009), central banks should not raise their rates after a recovery period, but should maintain a high-pressure economy, where inflation can rise above its targeted rate and unemployment levels can decrease below the natural level. By doing so, the return of

unemployed workers to the labor market would be facilitated and GDP could find its way back to the initial growth path.

The study by Fatás and Summers (2016) found that ignoring hysteresis also has consequences on budgetary policy. They found that a one percent change in GDP induced by budgetary policy in EU countries have a one percent change in potential output in five years, and 1.4 percent change in ten years. This implies that measures conducted by fiscal policy have a permanent effect on GDP.

Hysteresis earned a greater importance thanks to Edmund Phelps, who presented that the natural rate of unemployment may depend on the path taken by the actual unemployment. Inspired by him, economists have attributed the high unemployment persistence of European economies to hysteresis, as after the shocks of the 1970s unemployment remained high, whereas it has long recovered in the United States (Blanchard and Summers, 1986). A possible solution to the problem was presented by Blanchard and Summers, who have designed the insider-outsider model, which can generate hysteresis in unemployment. In the context of their model, only employed individuals can participate in wage bargaining, and they do it in a way that solidifies their employment status. As a result, the wage will be so high that it prevents unemployed people from entering the labor market, and in the next period, a smaller employed circle will bargain for the wages, which perpetuates the issue.

In the 2000s, the idea of Kaldor (1957) was rediscovered, according to which interactions between the demand and supply sides can lead to path-dependence. In Dutt's (2006) model, this interaction emerges through the Kaldor-Verdoorn law, where a positive relationship is assumed between output and productivity. The intuition behind it is that innovation is incentivized in a developing economy, and „learning by doing” contributes to skill accumulation, which contributes to higher output. With this setup, a transitory positive (negative) demand shock can permanently increase (decrease) economic growth rate. Inspired by his seminal work, I will supplement the New Keynesian model with hysteresis, where the output gap influences the development of technology.

All these studies demonstrate that hysteresis is a real phenomenon with far-reaching implications for monetary policy design. It also urges central banks to reconsider their strict inflation targeting policy, as it can permanently reduce potential capacity of the economy, which the benefits of disinflation are unable to offset.

3. Evidence of hysteresis

To substantiate the importance of hysteresis, and to prime my analysis, I carried out data analysis using regressions in RStudio. The United States was chosen to carry this analysis out on, as the variables involved were most easily found in this case.

First, I considered total factor productivity (TFP) to be equivalent with the technology level (A_t) in the New Keynesian model and I began to collect data from the Federal Reserve Economic Data (FRED) database. I retrieved quarterly data for total factor productivity (TFP) from a database called „A Quarterly, Utilization-Adjusted Series on Total Factor Productivity”, which is a working paper by John Fernald (2014), and was published by the Federal Reserve Bank of San Francisco. The database consists of many types of TFPs, out of which I found „utilization-adjusted TFP” to be the most suited for my analysis. This variable was calculated in three steps. First, business sector TFP was estimated, by deducting the contribution of capital and labor from output. Then, utilization of labor and capital was calculated using the Basu, Fernald, and Kimball (2013) estimates. Finally, utilization of labor and capital was detracted from business sector TFP, which resulted in the final „Utilization-adjusted TFP” variable. The data was in percent changes from quarter to quarter, at annual rates (400 times the change in natural logs), which I have changed back to quarterly rates (by dividing with 400). The data was available from the first quarter of 1947 until the third quarter of 2023.

Then, I searched for data to estimate output gaps, for which I had to find data for potential and actual output. For potential output, „Real Potential Gross Domestic Product” variable was used, which was also retrieved from the FRED database (source: U.S. Congressional Budget Office). Data was available from the 1949 until 2023, in quarterly frequency, and the variable was chained to 2017 U.S. dollars.

For actual output, I used „Real Gross Domestic Product”, which was again retrieved from the FRED database (source: U.S. Bureau of Economic Analysis). The variable was available from 1947 until 2023, in quarterly frequency, and it was seasonally adjusted. Similarly to potential output, actual output was also chained to 2017 U.S. dollars.

After combining the variables, I estimated the output gap by subtracting potential output from actual output, and then dividing by potential output. The result is a variable that when multiplied by 100, it expresses deviation from the potential output as a percent, meaning that a

0.01 value would indicate that actual output is one percent higher than potential output. I created four-four lags for output gap and TFP, so that lagged effects could be estimated. To make the different variables compatible with each other, I left out the first two years of actual output and utilization-adjusted TFP, and as a result, my final dataset spanned from the first quarter of 1949 until the third quarter of 2023.

TFP		Output gap	
Mean	0,286498648	Mean	-0,427803115
Standard Error	0,048325703	Standard Error	0,138383536
Median	0,224735134	Median	-0,393302796
Standard Deviation	0,830021201	Standard Deviation	2,376815287
Sample Variance	0,688935195	Sample Variance	5,649250907
Kurtosis	0,001180244	Kurtosis	0,420041045
Skewness	0,320240455	Skewness	-0,224182029
Range	4,537179104	Range	14,8085958
Minimum	-1,7637575	Minimum	-9,153023289
Maximum	2,773421605	Maximum	5,655572507
Count	295	Count	295

1. figure: Descriptive statistics of TFP growth and Output gap

I created a table for descriptive statistics, which can provide some interesting facts about total factor productivity and output gap in the United States. The mean of TFP was 0.289, which means that over the observation period, TFP grew by 0.289 percent from quarter to quarter on average, and the negative mean of the output gap informs us about that on average, the U.S. economy operates below its potential level.

As the next step, I imported that dataset to RStudio, and began to build regressions. The idea was to estimate the same regression, which is consistent with the technology equation in the New Keynesian model. The dependent variable was TFP growth, and the explanatory variables were the lags of output gap and TFP.

$$TFP = \beta_0 + \beta_1 TFP_{t-1} + \beta_2 TFP_{t-2} + \beta_3 TFP_{t-3} + \beta_4 TFP_{t-4} + \beta_5 Output\ Gap_{t-1} + \beta_6 Output\ Gap_{t-2} + \beta_7 Output\ Gap_{t-3} + \beta_8 Output\ Gap_{t-4} + \varepsilon_t$$

1. equation: Regression with all the explanatory variables

In the beginning, I created regressions manually, but in order to try every possible combination of the four-four explanatory variables, I automated the process. In total, there were 255 possible combinations. To find the best model specification, I relied on three indicators: adjusted R-squared and Akaike and Bayesian information criterias. I printed out the five best models according to the three different indicators and compared the different models. The specification which appeared in all three indicator's top five models, was where TFP was explained by its first lag and by the fourth lag of output gap.

$$TFP = 0.354 - 0.115 * TFP_{t-1} + 0.077 * Output\ Gap_{t-4} + \varepsilon_t$$

2. equation: Final specification

In this specification, the inercept and the two explanatory variables were all significant on the 95 percent confidence interval. The fourth lag of output gap proved to be extremely significant, which signals the presence of hysteresis in the American economy. Its coefficient is 0.077, which can be interpreted as follows: a one percentage point increase in output gap, increases TFP's growth rate, on average, by 0.077 percentage points four quarters later. The intercept is also highly significant, which captures the trend growth rate of TFP. The regression suggests that if the explanatory variables were zero, TFP would grow by 0.354 percent from quarter to quarter, on average. The coefficient of the first lag of TFP was -0.115, which signals a mean-reverting behavior over time. Meaning that periods of high TFP growth are usually followed by periods of lower TFP growth, and vice versa. The coefficient of -0.115 means that if TFP's growth rate grows by one percentage point, on average, TFP' growth rate will be 0.115 percentage points lower in the next quarter.

Final model specification	
Intercept	0.354 (0.051)***
first lag of TFP	-0.115 (0.058)*
fourth lag of output gap	0.077 (0.020)***
N	295
*** p < 0.001; ** p < 0.01; * p < 0.05;	

2. figure: Results of the final regression specification

The results also suggest that the mechanisms of TFP and output gap operate on different timescales. Mean-reverting behavior in TFP can already be observed in the next quarter,

whereas hysteresis exerts its effect one year later. In summary, we can acknowledge that hysteresis plays a significant role in the development of TFP in the United States, which justifies the importance of its inclusion in the New Keynesian model.

4. The New Keynesian model

In this section, I will present how the basic New Keynesian model is structured, I will elaborate on the two different types of monetary policies, how hysteresis is incorporated and I will also show how the model is calibrated. The model consists of a representative household, an intermediate and final goods producer, the central bank and the government. My model will follow the structure presented by Galí (2008).

4.1. Model equations

New Keynesian Philips curve

We begin with assuming a production function, where the level of technology is common to all firms, and evolves exogenously. Firms face an identical isoelastic demand schedule, an aggregate price level and an aggregate consumption index that they take as given. We also assume monopolistic competition which is characterized according to the model of Dixit and Stiglitz (1977). Monopolistic competition arises in the goods market as firms produce differentiated goods, for which they are able to set their own prices. However, firms are only able to change their prices with probability $1 - \theta$ in a given period, while producers are stuck with their old prices with probability θ . This mechanism follows staggered price setting introduced by the Calvo model (1983). Consequently, price stickiness arises in the model, the measure of which is θ . Firms maximize profits by reoptimizing in period „t” and choosing a price level that offers the highest current market value while that price remains effective. Maximization is executed subject to the series of demand constraints. The solution of the firms’s problem yields us the first order condition, which is the optimal price-setting condition. After the log-linearization of the condition, the next step is expressing a given firm’s marginal cost in terms of the average marginal cost, the firm-specific marginal product of labor and the marginal cost gap. Finally, by expressing the real marginal cost gap as a function of the output gap, we can arrive at the New Keynesian Philips curve, which is the first equation of the New Keynesian model block.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t + c_t, \quad (1)$$

where c_t is the exogenous cost-push shock.

Dynamic IS curve

The DIS curve can be derived by first assuming a representative household, which aims to maximize expected lifetime utility (which is a function of consumption and leisure), subject to the Constant Elasticity of Substitution (CES) consumption index, the intertemporal budget constraint, and the transversality condition. The CES consumption index represents how consumption is allocated between the differentiated goods. While the budget constraint indicates that consumption expenditure and bond purchases must be covered by bond returns and labor income, less a lump-sum tax. Whereas the transversality condition specifies that the household's expected bond holding has to converge to zero on the infinite horizon. Utility maximization is executed with two-staged budgeting, according to which the household first chooses total consumption, labor supply and savings, then decides the optimal allocation of consumption between time periods. To proceed, an aggregate price index is proposed so that the product of the price and consumption index could be used for total consumption expenditure. After solving the maximization problem, one can obtain two optimality conditions, the first of which is the intratemporal optimality condition, which symbolizes that the marginal rate of substitution of leisure for consumption equals the real wage at optimum. Whereas the second optimality condition, namely, the Euler equation represents the trade-off between present and future consumption and the effect of interest rate on consumption. Finally, after expressing the output gap in the Euler equation and making use of the goods market clearing condition (consumption equals output because there are no investments, government purchases, and the economy is closed), we can obtain the Dynamic IS curve:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) + d_t, \quad (2)$$

where d_t is the exogenous demand shock.

Natural rate of interest

Also known as the interest rate under flexible prices, the equilibrium process of which can be expressed as:

$$r_t^n = \rho + \sigma \psi_{ya}^n \mathbb{E}_t \Delta a_{t+1} \quad (3)$$

These equations procreate the non-policy block of the New Keynesian model, which possesses a recursive structure. First, the New Keynesian Philips curve determines inflation for a given sequence of output gap. Second, the Dynamic IS curves determines output gap for a given sequence of natural and actual real interest rate.

To close the model, I consider two types of monetary policy frameworks for the central bank: inflation targeting and price-level targeting, which will determine how nominal interest rates evolve over time.

Taylor rule

In the case of inflation targeting, the central bank adheres to the standard Taylor rule, where it sets nominal interest rates based on inflation and output gap. The parameters of ϕ_π and ϕ_y are non-negative coefficients, indicating the strength of the response to inflation and output gap, respectively. Thanks to the intercept ρ , the rule is consistent with the zero inflation environment prevailing in the steady state. When choosing the ϕ_π and ϕ_y values, one has to keep in mind that the output gap and inflation are nonpredetermined, thus a necessary and sufficient condition must hold for the solution to be locally unique, which requires that $\kappa(\phi_\pi - 1) + (1 - \beta) > 0$.

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t \quad (4/a)$$

Price-level targeting rule

In the case of price-level targeting, the central bank sets nominal interest rates based on the deviation from the targeted price level path and the output gap:

$$i_t = \rho + \phi_p \tilde{p}_t + \phi_y \tilde{y}_t \quad (4/b1)$$

Price-level gap equation

An additional equation is needed under price-level targeting, which measures the extent to which the actual price-level deviates from the central bank's desired level. It quantifies the price-level gap ($\tilde{p}$), which enters the model as an endogenous variable.

$$\tilde{p}_t = \pi_t + \tilde{p}_{t-1} \quad (4/b2)$$

Potential output

Also known as the output under flexible prices, which can be expressed as the following

$$y_t^n = \psi_{ya}^n a_t + \vartheta_y^n \quad (5)$$

Output gap

Naturally, the difference between the actual and potential output defines the output gap:

$$\tilde{y}_t = y_t - y_t^n \quad (6)$$

Fischer equation

The relationship between nominal and real interest rates is captured by the Fischer equation, which takes the following form:

$$r_t = i_t - \mathbb{E}_t \pi_{t+1} \quad (7)$$

Money demand function

For the money demand, an ad-hoc function is assumed, where η represent interest semi-elasticity:

$$l_t = y_t - \eta i_t \quad (8)$$

Money growth

The equation of money growth does not only consist of the change in money balances, but it also accounts for inflation:

$$\Delta m_t = m_t - m_{t-1} + \pi_t \quad (9)$$

Production function

Production follows a Cobb-Douglas functional form, and the only two inputs used are labor and technology. Clearing is also assumed in the labor market, meaning that the sum of labor supplied should equal the labor demanded by all firms. The production function combined with the labor market clearing condition yields us a relationship between aggregate production, employment and technology:

$$y_t = a_t + (1 - \alpha)n_t \quad (10)$$

Labor supply equation

The household's intratemporal optimality condition combined with the market clearing condition provides us the labor supply equation:

$$w_t = \sigma y_t + \varphi n_t \quad (11)$$

Technology

There are many ways to incorporate hysteresis in the New Keynesian model framework. For example, one can introduce hysteresis to the labor market, where persistence slows down the flow between employment and unemployment (Galí, 2018). Or another possibility is to alter the technology equation, where output gaps can have a direct effect on technology levels (Jordá et al., 2020). I opted for the latter approach and I also departed from the conventional stationary form of the technology process. The coefficient of the process took a value of one, meaning that it now follows a unit root process, where every shock leaves a permanent mark on the evolution of technology. Then, I added the term incorporating hysteresis by supplementing the equation with output gap. The parameter of η_y is the measure of hysteresis in the model, which was calibrated to be 0.08 based on earlier presented empirics.

$$a_t = a_{t-1} + \eta_y \tilde{y}_{t-1} + z_t \quad (12)$$

I incorporated three types of shocks that impede the economy: technology shock, cost-push shock, and demand shock. Evidently, the technology shock forms part of the technology equation, the cost-push shock is part of the Philips curve and the demand shock pertains to the DIS curve. All three shocks follow a first order autoregressive process, and take the following forms:

Technology shock process

$$z_t = \rho_z z_{t-1} + \varepsilon^z \quad (13)$$

Cost-push shock process

$$c_t = \rho_c c_{t-1} + \varepsilon^c \quad (14)$$

Demand shock process

$$d_t = \rho_d d_{t-1} + \varepsilon^d \quad (15)$$

The parameters of ρ_z , ρ_c and ρ_d stand for the persistence of the shock processes, and ε^z , ε^c and ε^d are the exogenous shocks, that were given a value of one percent.

4.2. Calibration of the model

The model was calibrated based on traditional values, for which I used the previously cited model of Galí (2008).

Parameters	Value	Description
α	1/3	degree of decreasing returns
β	0.99	subjective discount factor
ε	6	elasticity of substitution
η	4	interest semi-elasticity of money demand
σ	1	inverse intertemporal elasticity of substitution
φ	1	inverse Frisch elasticity of labor supply
θ	2/3	Calvo parameter
ϕ_π	1.5	inflation coefficient (inflation targeting)
ϕ_y	0.5/4	output gap coefficient (inflation targeting)
$\phi_{\hat{p}}$	1.5	price-level gap coefficient (price-level targeting)
$\eta_{\hat{y}}$	0.08	hysteresis parameter
ρ_z	1	persistence of technology shock
ρ_c	0.9	persistence of cost-push shock
ρ_d	0.9	persistence of demand shock

1. table: Parameters of the New Keynesian model

Endogenous variables	Description	Exogenous variables	Description
π	inflation	ε^z	standard deviation of technology shock
$\hat{y}$	output gap	ε^c	standard deviation of cost-push shock
r_n	natural rate of interest	ε^d	standard deviation of demand shock
i	nominal interest rate		
y_n	natural rate of output		
y	output		
r	real interest rate		
m	real money balances		
Δm	change in real money balances		
n	employment rate		
w	real wage		
a	technology level		
z	technology shock process		
c	cost-push shock process		
d	demand shock process		

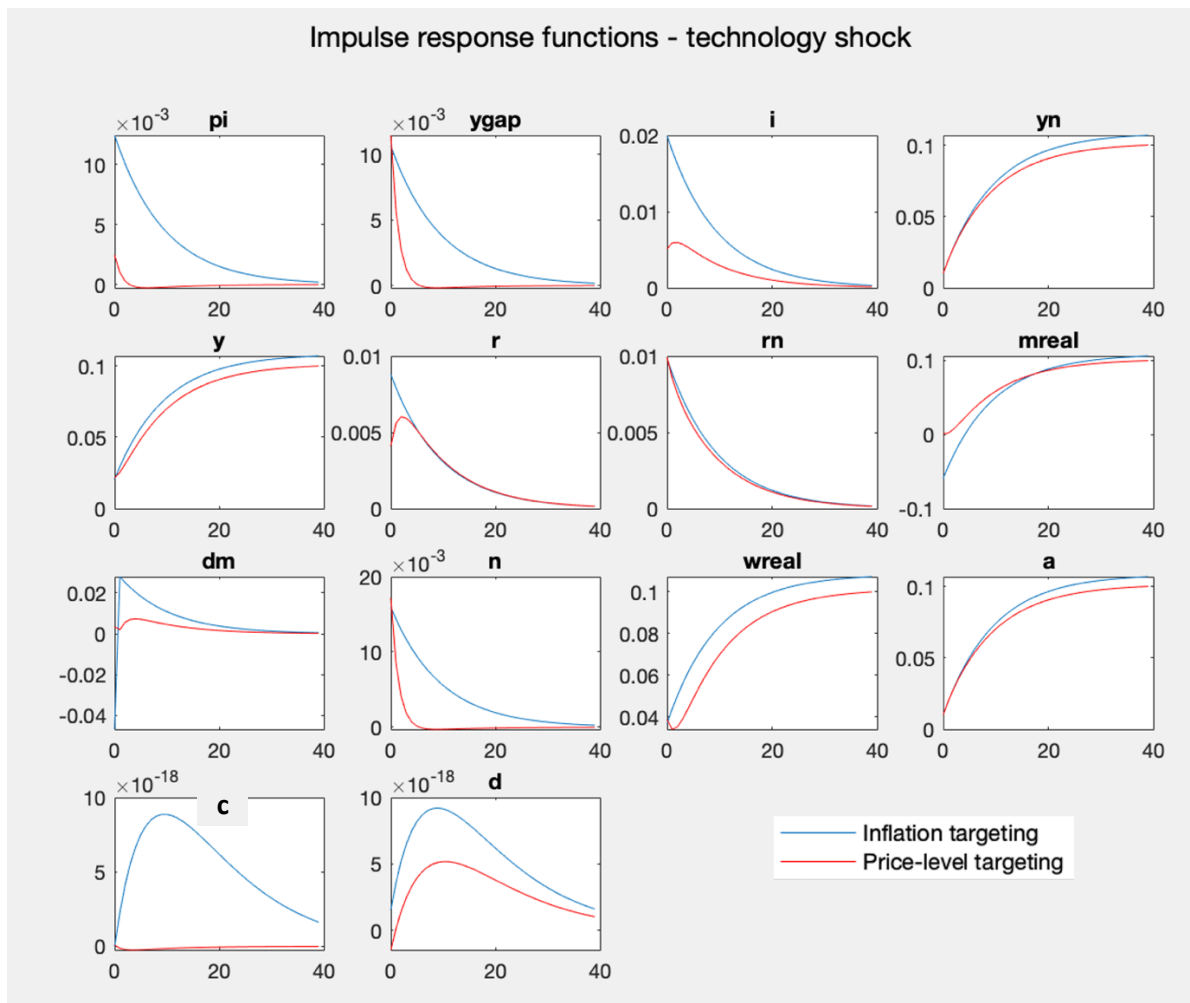
2. table: Endogenous and exogenous variables of the New Keynesian model

5. Results

5.1. Assessing the underlying dynamics of the model

The codes of the New Keynesian models were written in Dynare, and were ran using Matlab. The central bank of the first model follows inflation targeting, and the second model's follows price-level targeting, while hysteresis prevails in both simulations. Three different cases were analyzed: technology shock, cost-push shock and demand shock. First, to compare the underlying mechanisms and dynamics under inflation targeting and price-level targeting, I plotted the impulse response functions in the case of the previously mentioned three shocks. One time period in the model corresponds to one quarter, and the responses of the variables were followed for 40 periods, as it proved to be enough time to reach the newly formed steady states.

5.1.1. Dynamics of technology shock

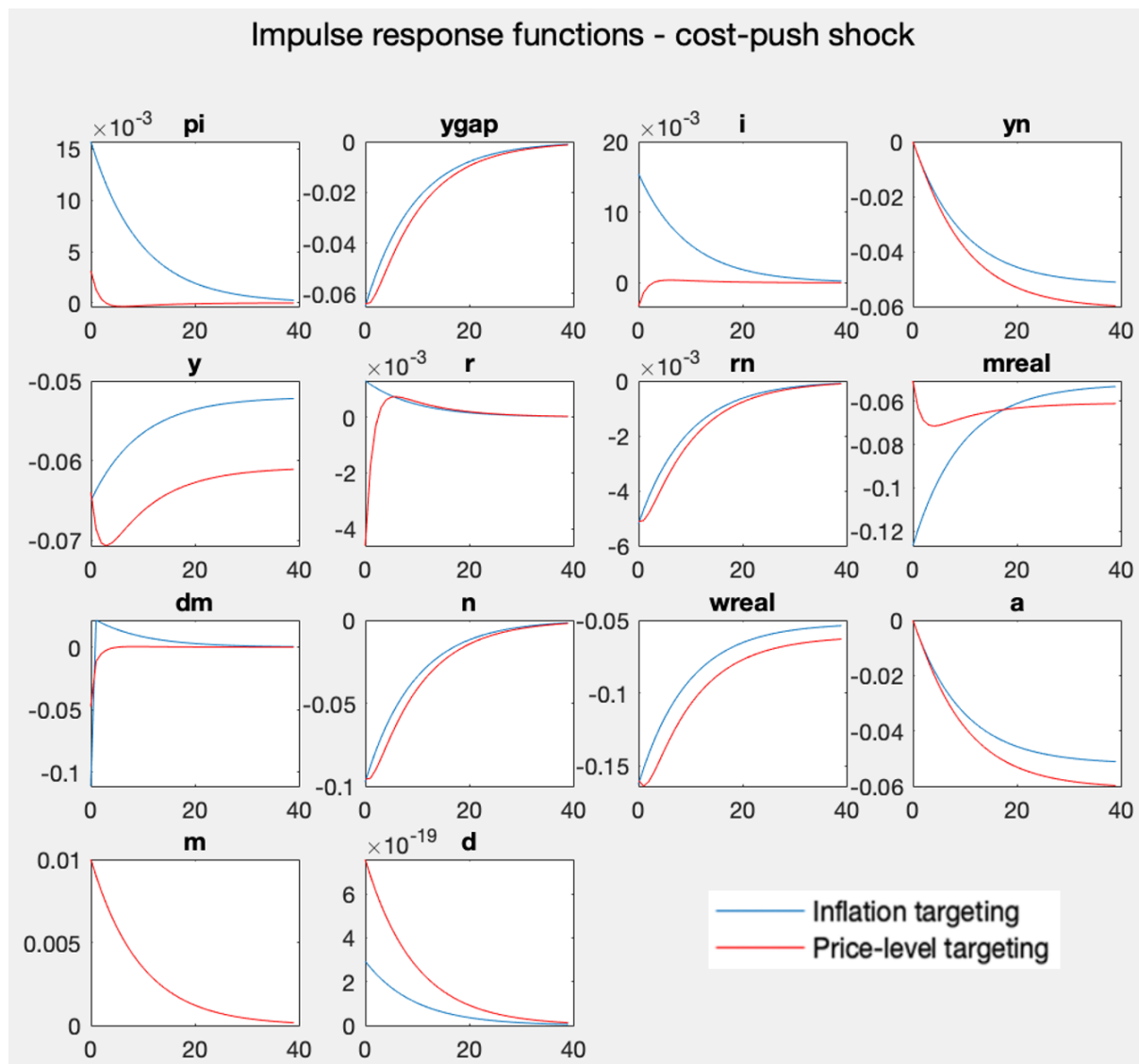


3. figure: Comparison of the impulse response functions under inflation targeting and price-level targeting in the case of technology shock

In the case of technology shock, a one percent technology shock impedes the economy. As a result of the modified technology process, the shock is not transitory, and it is known by the consumers. As given by the Euler equation, current consumption depends on future consumption, so if the future one increases so does the current one. Since consumers anticipate a permanent shock and in order to smooth the series of consumption, they overreact with their current consumption, and in the first period actual output overshoots potential output. Nevertheless, potential output increases immediately, as the technology has a direct influence through the production function. Consequently, a positive output gap opens, generating inflation under both targeting rules. Noticeably, the rise in inflation is much higher under inflation targeting than under price-level targeting, which can be explained as follows. Under price-level targeting, economic agents expect that a deflationary period will follow, so that the central bank can steer back the economy to the old price-level growth path, and their expectations will decrease inflation in the present through the Philips curve relationship. The rise in inflation, and the deviation from the targeted price level, is combated with increased nominal interest rates, but since inflation is lower under price-level targeting, the increase in nominal rate is also lower. As hysteresis unfolds in the technology, the potential output increases, which closes the output gap, eliminating inflationary pressures. Initially, employment rises because the increase in technology alone cannot create higher actual output, thus more labor is needed. As consumption is higher, it will increase the wages through the labor supply relationship. Due to the negative output gap in the case of price-level targeting, the technological level will converge to a slightly lower level than in the case of inflation targeting, as in the latter case there is no negative output gap, and hysteresis cannot exert its negative effect.

Under inflation targeting, inflation will eventually subside, but the price level is permanently higher due to the prevailing high inflation, whereas under price-level targeting, the price level is back on its original path, thanks to deflation. The high inflation in the case of inflation targeting will decrease real money balances, but as inflation cools down and as nominal interest rates decrease, real money balances will follow the increase in actual output.

5.1.2. Dynamics of cost-push shock



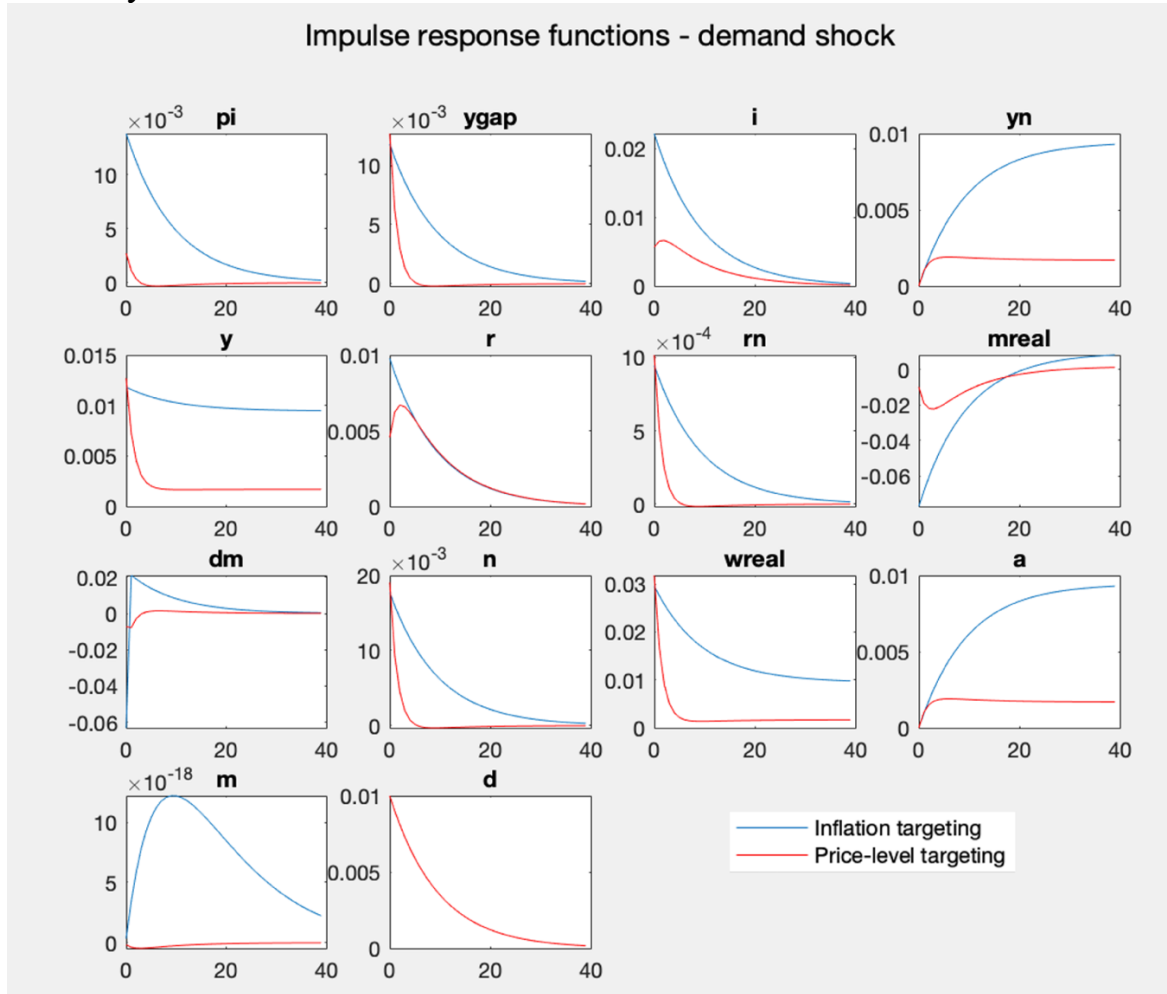
4. figure: Comparison of the impulse response functions under inflation targeting and price-level targeting in the case of cost-push shock

In the case of cost-push shock, a one percent cost shock hits the economy. This shock directly affects the New Keynesian Philips curve, raising inflation for any given level of output gap. As inflationary expectations are lower under price-level targeting due to the expected deflation in the future, inflation will increase less compared to inflation targeting. Actual output decreases, which is due to consumption smoothing and the spike in interest rates. Since households expect future consumption to decline, they decrease their current consumption, which decreases actual output with the same volume. To combat inflation and deviation from the targeted price level, central banks raise nominal interest rates. As a result, the real interest rate will be higher than

the natural real interest rate, which creates a negative output gap in both cases. Real interest rate increases sharply under price-level targeting, since deflation enhances the increase in the nominal rate. As a result, the gap between the real rate and the natural rate becomes even greater, further decreasing actual output, thus increasing output gap. The negative output gap will decrease technology through hysteresis, but as the output gap decrease is not as severe under inflation targeting, as under price-level targeting, technology will not decrease as much as under the former monetary policy. Similarly, potential output will also decrease following the same path as changes in technology. Employment levels decrease initially, as the lower level of actual output can be achieved with less labor. As actual output converges to potential, employment rates will rise back to their original levels. Thanks to the lower technology and output levels in steady state, the real wages also settle on a lower level.

This example demonstrates that inflation targeting has a relative advantage in dampening the effects of hysteresis, while price-level targeting is more effective in stabilizing inflation if the economy is subject to cost-push shocks,

5.1.3. Dynamics of demand shock



5. figure: Comparison of the impulse response functions under inflation targeting and price-level targeting in the case of demand shock

Similarly to the previous shocks, the demand shock also had a value of one percent. The shock has a direct effect on output gap, as the demand shock is a component of the Dynamic IS curve. Consumption increases under both monetary policies, and with a slightly larger volume than the demand shock, because expected consumption also increases. Excess demand elevates inflation in both cases, however, with much less intensity under price-level targeting compared to inflation targeting. As economic agents anticipate deflation under price-level targeting, they attenuate inflation with their expectations. Since a positive output gap has opened, hysteresis starts to operate, and technology starts to increase. As a bigger output gap opened under inflation targeting, the growth in technology will also be higher because of hysteresis. Following the rise in technology, potential output also increases. To fight inflation, central banks raise nominal interest rates, and as inflation is less severe under price-level targeting, the

spike of nominal interest rate is also lower. As nominal rates rise, so do real rates. Since real rates are higher than the natural real rates, output gaps will start to close. Under inflation targeting, consumers await higher future consumption, as they believe that a more positive output gap will cause higher technology levels through hysteresis, and to smooth their stream of consumption, they continue to consume more, contributing to elevated actual output levels. Whereas the opposite is true under price-level targeting. Thanks to lower future consumption expectations and a higher real interest rate and natural interest rate difference, a negative output gap will occur, which causes a deflationary period, forcing the economy back to its original price level growth path. Due to higher nominal rates, real money balances also decrease, as the increase in actual output cannot offset the negative effect. As technology is initially unaffected, more labor is hired to achieve the higher output, which will subside as output gaps close. Similarly, real wages grow due to an increase in employment, but as employment levels recover, real wages begin to fall. However, the decrease will wane thanks to increases in technology levels, and eventually, real wages will stay on a permanently higher level.

In summary, we can see that in the case of demand shock, price-level targeting is again more suitable for stabilizing inflation, output gap and the effects of hysteresis.

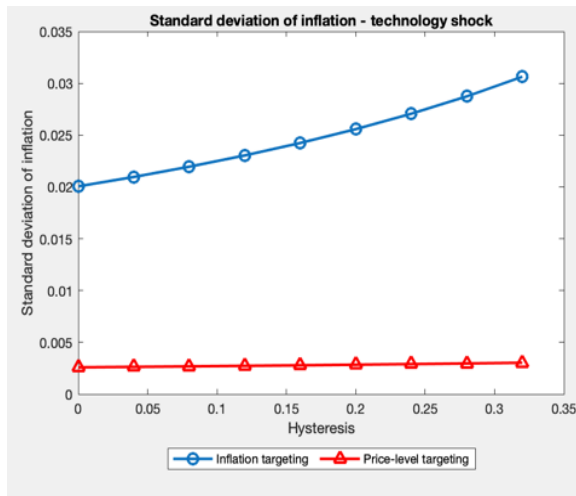
5.2. Assessing the standard deviation of inflation and output gap

After analyzing how inflation-targeting and price-level-targeting monetary policies behave in response to different kinds of shocks in the presence of hysteresis, I turn to examine its welfare implications. To prime my welfare analysis, in this section I will assess the standard deviation of inflation and output gap under the three shocks.

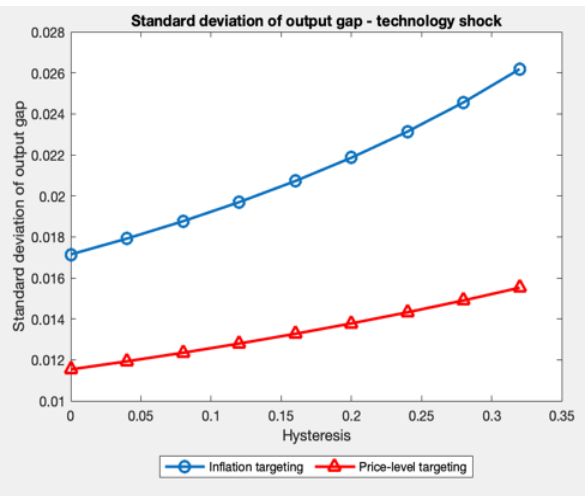
To compare the policies and to conduct a welfare analysis, the standard approach is to follow a welfare-based criterion, which depends on the second-order approximation of the utility losses beared by economic agents due to the deviations from the Pareto optimum. This method was applied by Rotemberg and Woodford (1999) and by Jordi Galí (2008), where they expressed average welfare losses in a given period by a combination of the inflation and output gap variances. The solution of the function quantifies welfare losses as a percent of the steady state consumption, and it can be interpreted as the level of consumption that should be provided so that utility levels return to their Pareto optimal levels. Unfortunately, my model setup has prevented me from using this method. In my model specification, where shocks can exert

permanent effects on the endogenous variables, the steady states change. Thus, the method of expressing losses as a percent of the former steady state's consumption is futile. However, I have come up with a similar way to analyze welfare losses inspired by this method. The idea was to compare the standard deviations of inflation and output gap under the two monetary policies, while also changing the hysteresis parameter in the models to assess the sensitivity. The highest degree of hysteresis under which both model's solutions were locally unique and both steady states were stable, was 0.32. As the baseline calibration of the model entailed a hysteresis parameter of 0.08, I chose to look at instances where hysteresis was changed with 0.04 increments, so that both the maximum and the baseline values could be plotted. In total, nine cases of hysteresis levels were observed, starting from 0 all the way until 0.32. To see shock-specific mechanisms, and to make the analysis more accessible, I structure this section similar to the previous one, where I separated the cases according to technology, cost-push, and demand shocks. For direct comparisons, I plotted inflation and output gap standard deviations on different graphs.

5.2.1. Standard deviation under technology shock



6. figure: Relationship between the standard deviation of output gap and the level of hysteresis under inflation targeting and price-level targeting in the case of technology shock



7. figure: Relationship between the standard deviation of inflation and the level of hysteresis under inflation targeting and price-level targeting in the case of technology shock

Under technology shock, we saw that both output gap and inflation increased under inflation targeting, as well as under price-level targeting. As consumers anticipated that the shock will be permanent, they overreacted with their consumption, which opened the output gap, and raised inflation. The magnitude of initial increase in output gap was similar under the two policies, but they followed different paths during later periods. Output gap stayed positive for

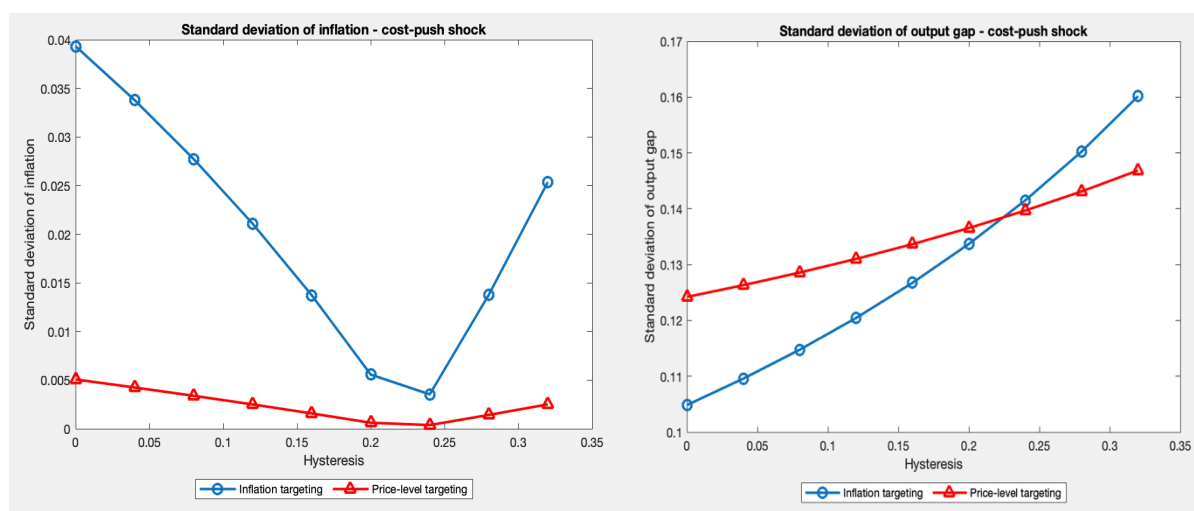
much longer under inflation targeting, which resulted in a higher standard deviation. In the case of price-level targeting, the output gap was much less persistent.

On the right side, we can see how the standard deviation of output gap compare under the two cases. It is apparent that price-level targeting can better attenuate output volatility under all hysteresis parameters. With increases in hysteresis, the standard deviation of output gap increases in both cases, due to the larger increases in actual output. This can be explained by the fact that under higher degrees of hysteresis, households expect larger increases in technology, thus higher levels of future consumption. In order to smooth consumption over the period, they increase their current consumption more, overshooting potential output even more, resulting in a greater output gap.

In the case of inflation, it increased with much greater volume under inflation targeting and it also decreased more slowly. As households expected deflation under price-level targeting, inflation grew more moderately, resulting in a lower standard deviation. Under inflation targeting, it can be inferred, that households expected higher inflation, which manifested in higher actual levels. The difference in inflation's standard deviation under the two cases is even more striking. Furthermore, as the level of hysteresis increased, inflation volatility under inflation targeting continued to grow even higher, whereas under price-level targeting, it barely increased.

To sum up, price-level targeting is not only superior to inflation targeting under the baseline calibration, but under all possible hysteresis values. Furthermore, with increases in hysteresis, the relative advantage of price-level targeting grows.

5.2.2. Standard deviation under cost-push shock



8. figure: Relationship between the standard deviation of inflation and the level of hysteresis under inflation targeting and price-level targeting in the case of cost-push shock

9. figure: Relationship between the standard deviation of output gap and the level of hysteresis under inflation targeting and price-level targeting in the case of cost-push shock

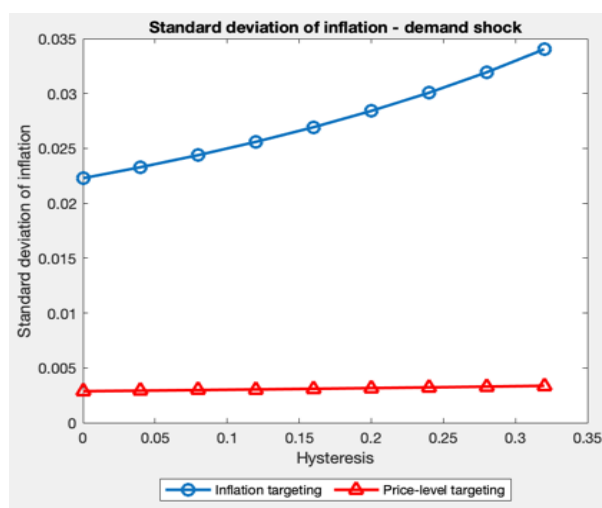
In the case of cost-push shock, the implied relationships do not seem as simple as they did in the case of technology shock.

The standard deviation of output gap is higher under price-level targeting, as actual output decreases with a larger volume and it also persists for longer. With higher degrees of hysteresis, output volatility increases in both cases, but with a higher rate under inflation targeting. The reason behind it is that consumption smoothing will happen at lower rates under higher degrees of hysteresis, which fuels output gap's variability. Around 0.23 hysteresis values, the two monetary policies seem to perform equally in controlling output gap, and under higher degrees, price-level targeting will start to dominate. However, in my empirical analysis, I did not find such high levels of hysteresis significant, so the probability of price-level targeting actually outperforming inflation targeting in limiting output gap variability is remote.

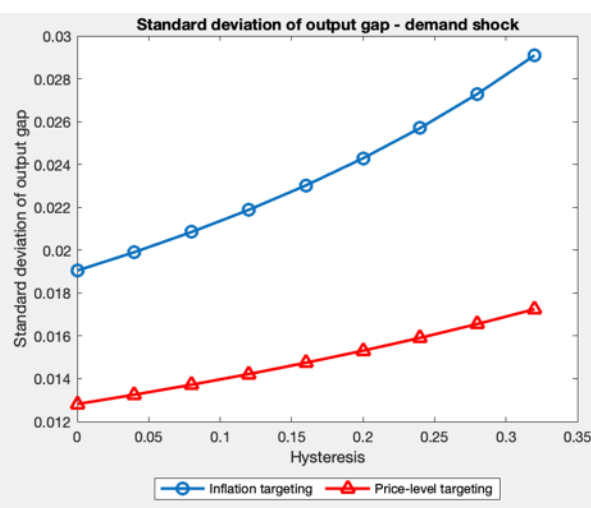
Under the baseline calibration, the standard deviation of inflation is higher in the case of inflation targeting. This can be again be attributed to the expectations of the households as they await a deflationary period under price-level targeting. The liaison describing the mechanism between hysteresis and inflation volatility seems more complex than previously. If hysteresis increases from low levels, the standard deviation of inflation does not only exhibit reluctance to increase, but it even decreases. It attenuates until reaching a hysteresis value of 0.23, from where any higher level of hysteresis, will induce a rise in inflation volatility. The unique “U”

shaped relationship can stem from the fact that divine coincidence does not prevail under cost-push shocks, as negative output gap and inflation arise simultaneously, and they move in opposite directions. Divine coincidence refers to a phenomenon where stabilizing inflation automatically stabilizes output as well. If we tighten monetary policy in order to control inflation, we destabilize output gap, and vice versa (Blanchard and Galí, 2007). This trade-off is clearly depicted on the graph: the stronger hysteresis is, the more potential output reacts to the shock, and the greater variability spreads over to output gap. However, due to the violation of the divine coincidence, as the variance of the output gap increases, inflation will decrease, which can be seen in the descending part of the function. However, it is possible that after a while, the output gap becomes so volatile that it will increase the variance of inflation through its direct impact on the Phillips curve. The turning point is around the hysteresis level of 0.23, from where any additional increase will result in the dominance of latter effect.

5.2.3. Standard deviation under demand shock



10. figure: Relationship between the standard deviation of inflation and the level of hysteresis under inflation targeting and price-level targeting in the case of demand shock



11. figure: Relationship between the standard deviation of output gap and the level of hysteresis under inflation targeting and price-level targeting in the case of demand shock

Under demand shock, a more straightforward picture can be seen. The demand shock directly increases actual output, raising output gaps in both cases. The initial effects are similar, however, households under inflation targeting continue to consume more than what is sustainable, keeping the positive output gap open for longer, and fueling output gap's standard deviation. With increases in hysteresis, consumers under inflation targeting can expect higher

future consumption levels due to the output gap's effect on technology, and in order to smooth consumption, they generate higher consumption, raising the output gap. However, under price-level targeting, due to the monetary policy's ability to curb hysteresis, consumers expect future consumption to grow less vigorously, and current consumption will also increase with less intensity compared to inflation targeting.

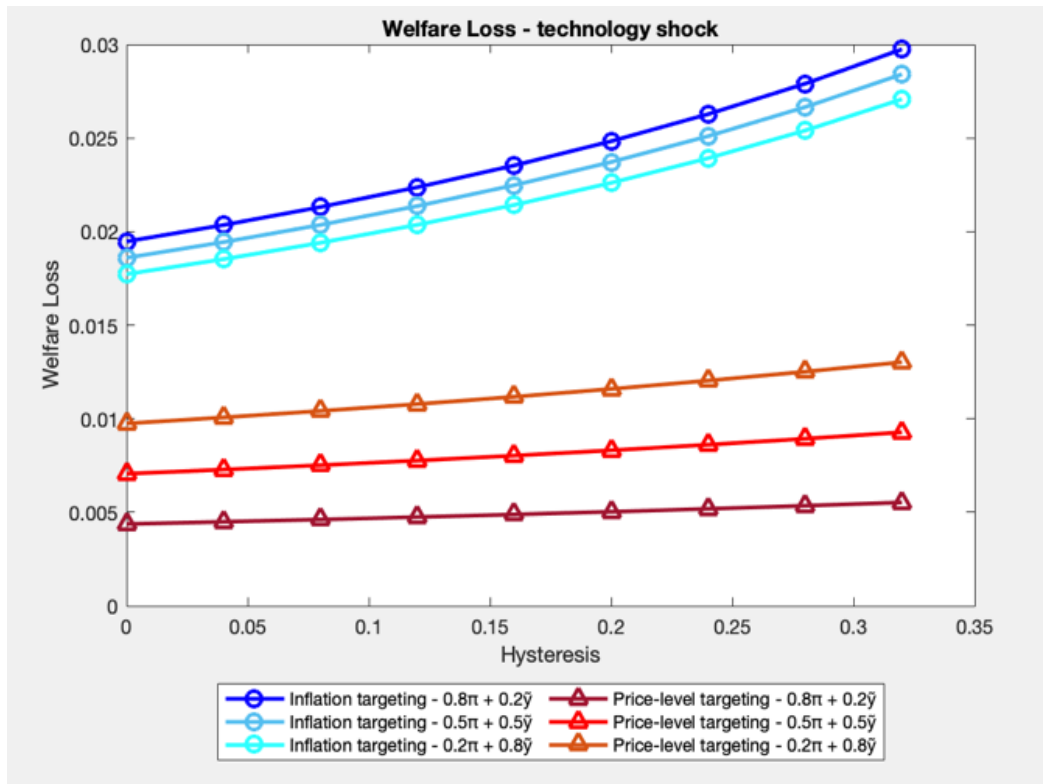
Through changes in output gaps, inflation levels rise. Nevertheless, under price-level targeting, as inflationary expectations are low, and output gaps stabilize quicker, inflation increases with a smaller margin. With increases in hysteresis, inflation's standard deviation rises under inflation targeting, as consumption smoothing will happen at a higher level, which opens the output gap which raises inflation. However, under price-level targeting, inflation's standard deviation only weakly responds to increments of hysteresis. This can again be attributed to the hysteresis attenuating characteristic of the monetary policy.

This case shows once more that price-level targeting's relative advantage increases with hysteresis.

5.3. Welfare loss functions

In this section, inspired by the welfare loss functions applied by pioneering economists, I turn to compare the welfare losses under inflation targeting and price-level targeting. The function is an ad-hoc representation and it does not rely on micro foundations for previously stated reasons. Using the results of the previous section, I summed up the standard deviation of inflation and output gap using three different weight pairs. The first weight pair attributes 80 percent of total welfare loss to inflation variability, and 20 percent to output gap variability. The second pair consists of equal weights, meaning that the proportion of the welfare loss arising from the two kinds of deviations is equal. Lastly the third pair ascribes 80 percent of the loss to output gap variability, and 20 percent to inflation variability. By comparing the welfare implications of the two monetary policies, this section aims to give a valuable insight about the relative strengths and weaknesses of these policy approaches.

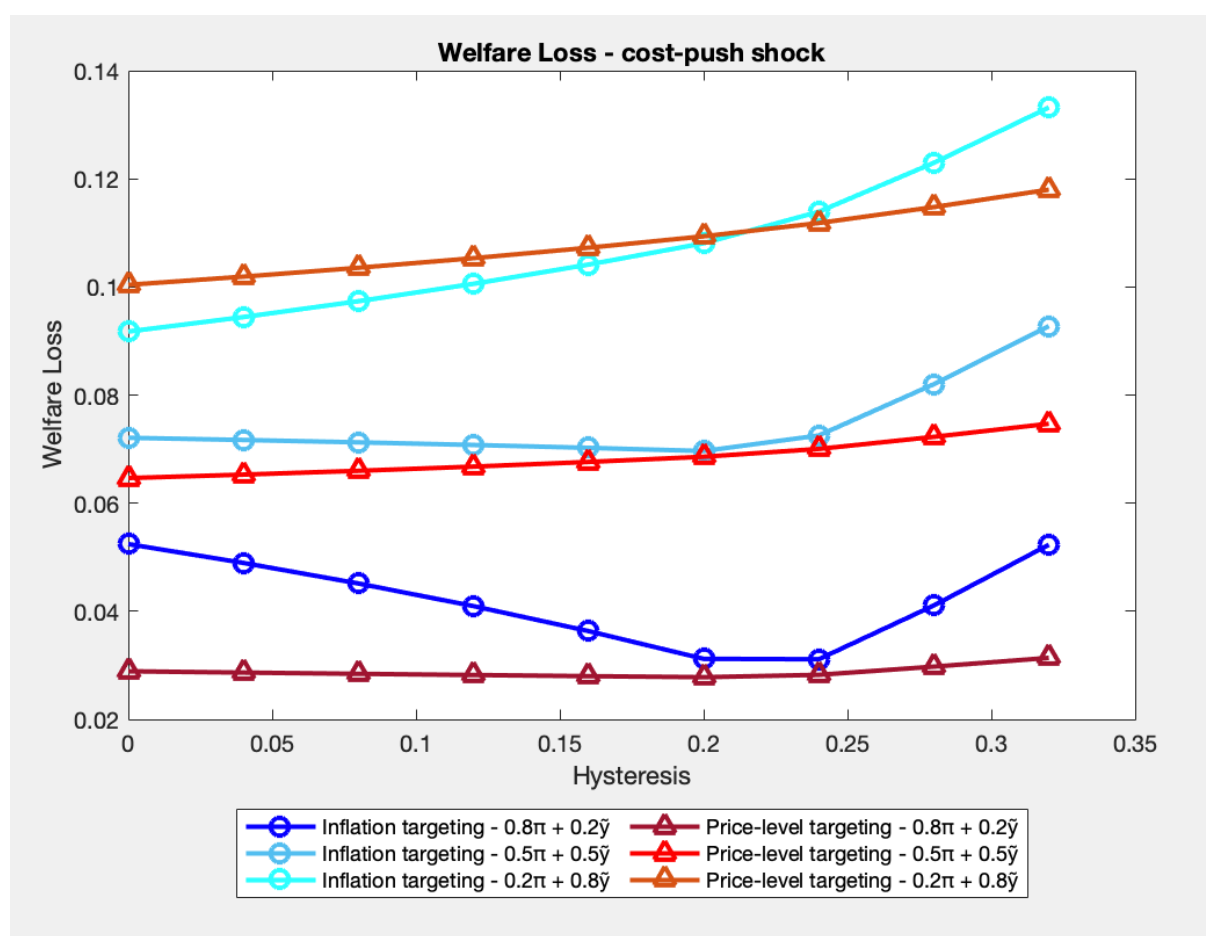
5.3.1. Welfare loss under technology shock



12. figure: Welfare loss functions under inflation targeting and price-level targeting in the case of technology shock

In the case of technology shock, both standard deviations were higher under inflation targeting, so it is not striking that price-level targeting reigns superior, irrespective of the welfare loss weights. The results do not change with increases in hysteresis, although, the relative disadvantage of inflation targeting seems to increase with higher levels. The graph is also informative about the variability difference between inflation and output gap under a given monetary policy. Interestingly, inflation's standard deviation is higher than that of output gap in the case of inflation targeting, while the opposite is true for price-level targeting, where the standard deviation of output gap has surpassed that of inflation.

5.3.2. Welfare loss under cost-push shock



13. figure: Welfare loss functions under inflation targeting and price-level targeting in the case of cost-push shock

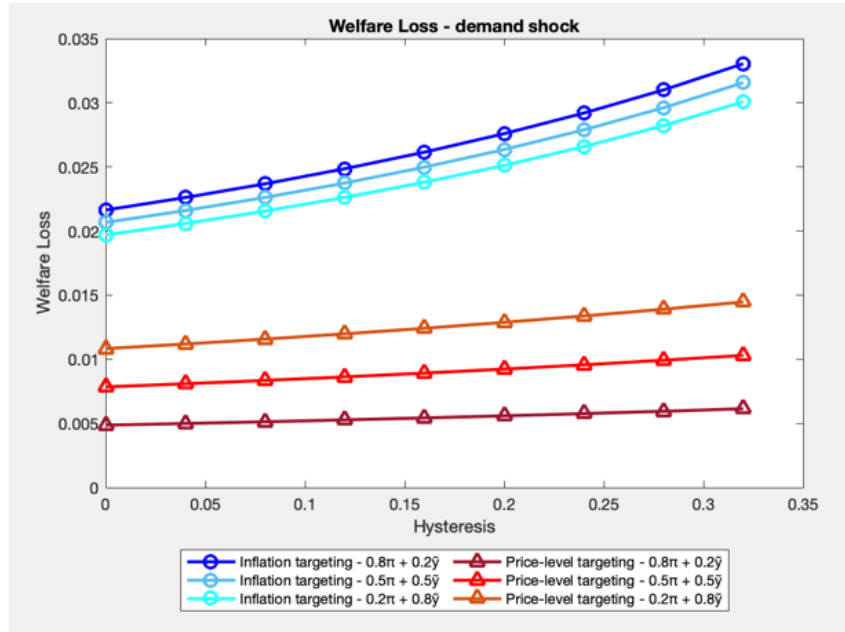
The case of the cost-push shock is slightly more complex as we saw that hysteresis exerts its effects through different channels. Price-level targeting seemed more effective than inflation targeting in curtailing inflation variability, while it was worse at controlling output gap's standard deviation, except for a few cases with high degrees of hysteresis. The „U” shaped relationship takes form again where the weight of inflation volatility is 50 and 80 percent.

An interesting difference compared to technology shock is that under inflation targeting, output's standard deviation is higher than that of inflation, which intuitively supports the monetary policy's relative efficiency under a cost-push shock.

At first glance, this unique relationship could complicate the assessment of the two policies. However, if economic agents are consistent with their welfare loss weights under the two monetary policies, then one can realize that when considering the same weights, inflation

targeting only outperforms price-level targeting if the weight of output gap's standard deviation is high.

5.3.3. Welfare loss under demand shock



14. figure: Welfare loss functions under inflation targeting and price-level targeting in the case of demand shock

The case of demand shock is similar to that of technology shock, where the picture is rather straightforward. Both types of variability was higher under inflation targeting, which is clearly reflected in the weighted loss functions. The position of the graphs follow the same pattern as seen under technology shock, where inflation's standard deviation is greater than that of output gap's in the case of inflation targeting, while the opposite is true in the case of price-level targeting.

Thanks to the conducted welfare analysis, the relative efficacy of the two policies could be visualized and easily interpreted. Price-level targeting irrefutably outperformed inflation targeting under technology and demands shocks, while the case of cost-push shock was more involved, one could conclude that inflation targeting only outperformed inflation targeting if the weight of output gap volatility was high in the loss function. My initial hypothesis was confirmed, according to which the relative advantage of price-level targeting is higher under higher levels of hysteresis. Furthermore, the commonly used argument that price-level targeting permits higher inflation variability is refuted. These insights justify the growing support for price-level targeting, and can hopefully urge for greater awareness about its implications.

6. Summary

The research objective of my thesis was to conduct a welfare-oriented comparison between inflation targeting and price-level targeting in the presence of hysteresis.

I began by carrying out a simple empirical analysis to assess the presence and relevance of hysteresis in the case of the U.S. economy. To prime my later model setup in the New Keynesian model, I estimated the same equation which describes technology, where TFP growth was the dependent variable, and the lag of TFP growth and output gap were the explanatory variables. Model selection was based on adjusted R-squared and the Akaike and Bayesian information criterias, which all painted a similar pictures about the regressions. Eventually, a simple model specification was selected, where TFP was explained by the first lag of TFP and the fourth lag of output gap, which indicates a one-year long transmission mechanism in hysteresis. The coefficient of output gap was highly significant and took a value of 0.077, which was rounded to 0.08 as the hysteresis parameter in the modified New Keynesian model.

In the following section I outlined the structure and parameters of the model, and introduced the two different monetary policy frameworks. Following the chapters dedicated to construct a foundation, I began to assess the performance of the models under three different shocks. The cases of technology and demand shock all painted the same picture: price-level targeting is superior to inflation targeting. It controlled inflation's and output gap's standard deviation better under the baseline calibration and under higher degrees of hysteresis. In the case of the markup shock with a realistic value of hysteresis, price-level targeting could better curtail inflation, while inflation targeting could better limit output gap's standard deviation. However, under high levels of hysteresis, price-level targeting was again better, which supported my initial hypothesis, according to which the relative advantage of price-level targeting is higher under greater degrees of hysteresis.

A possible extension of the research could be to analyze the efficiency of the monetary policies under simplified targeting rules, where the central banks do not react to changes in output gap, only to inflation and price-level gaps. The case of average inflation targeting could also be explored under hysteresis, and its performance could be compared to the other two policies. Lastly, the model could aslo be extended with the concept of endogenous expectation forming, which could yield us valuable isnight about how expectations truly form under price-level targeting.

7. References

- Ajello, A., Cairo Blanco, I., Curdia, V., Lubik, T. A., & Queralto, A. (2020). *Monetary Policy Tradeoffs and the Federal Reserve's Dual Mandate*. Finance and Economics Discussion Series, 2020(066), 1–33
- Ball, L. M. (2009). *Hysteresis in Unemployment: Old and New evidence*. National Bureau of Economic Research, No. 14818. Retrieved from <https://doi.org/10.3386/w14818>
- Basu, S., Fernald, J., Fisher, J., & Kimball, M. (2013). *Sector-Specific Technical Change*. National Bureau of Economic Research, Retrieved from https://www.ecb.europa.eu/home/pdf/research/compnet/20140630/basu_paper1.pdf?5d03292e688ee5ee53905a623077d417
- Berg, C., & Jonung, L. (1999). *Pioneering price level targeting: The Swedish experience 1931–1937*. Journal of Monetary Economics, Vol. 43. No. 3., pp 525–551. Retrieved from <https://www.diva-portal.org/smash/get/diva2:327968/FULLTEXT01.pdf>
- Bernanke, B. S. (2019). *Monetary Policy in a New Era*. In O. Blanchard & L. H. Summers, *Evolution or Revolution?* The MIT Press, Retrieved from <https://www.brookings.edu/articles/monetary-policy-in-a-new-era/>
- Blanchard, O. J., Cerutti, E., & Summers, L. H. (2015). *Inflation and Activity. Two Explorations and Their Monetary Policy Implications*. National Bureau of Economic Research Working Paper, No. 21726. Retrieved from <https://doi.org/10.3386/w21726>
- Blanchard, O. J., Summers, L. H. (1986). *Hysteresis and the European Unemployment Problem*. National Bureau of Economic Research Macroeconomics Annual, Vol. 1. pp 15–78. Retrieved from <https://doi.org/10.1086/654013>
- Blanchard, O. J., Jordi, G. (2007). *Real Wage Rigidities and the New Keynesian Model*. Journal of Money, Credit and Banking, Retrieved from <https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1538-4616.2007.00015.x>
- Clarida, R. H. (2022). *The Federal Reserve's New Framework: Context and Consequences*. Finance and Economics Discussion Series, Retrieved from <https://www.federalreserve.gov/econres/feds/files/2022001pap.pdf>

Cúrdia, V. (2022). *Average Inflation Targeting in the Financial Crisis Recovery*. San Francisco Fed, Retrieved from <https://www.frbsf.org/research-and-insights/publications/economic-letter/2022/01/average-inflation-targeting-in-financial-crisis-recovery/>

Dutt, A. K. (2006). *Aggregate Demand, Aggregate Supply and Economic Growth*. *International Review of Applied Economics*, Vol. 20. No. 3. pp 319–336. Retrieved from <https://doi.org/10.1080/02692170600736094>

Evans, C. L. (2012). *Monetary Policy in a Low-Inflation Environment: Developing a State-Contingent Price-Level Target*. *Journal of Money, Credit and Banking*, Vol. 44. No. 1. pp 147–155. Retrieved from <https://www.google.com/search?client=safari&rls=en&q=CHARLES+L.+EVANS+Monetary+Policy+in+a+Low-Inflation+Environment%3A+Developing+a+State-Contingent+Price-Level+Target&ie=UTF-8&oe=UTF-8>

Fatás, A., Summers, L. H. (2016). *The Permanent effects of Fiscal Consolidations*. National Bureau of Economic Research Working Paper, No. 22374. Retrieved from <https://doi.org/10.3386/w22374>

Fernald, J. (2014). *A quarterly, utilization-adjusted series on total factor productivity*. Federal Reserve Bank of San Francisco, Retrieved from <https://ideas.repec.org/p/fip/fedfwp/2012-19.html>

Fischer, J. J. (2022). *De-anchored Inflation Expectations and Monetary Policy*. European University Institute, Retrieved from https://scholar.google.com/citations?view_op=view_citation&hl=en&user=rBHwzq0AAAAJ&citation_for_view=rBHwzq0AAAAJ:u-x6o8ySG0sC

Friedman, B. M. (2000). *Monetary Policy*. National Bureau of Economic Research, 57, Retrieved from <https://ssrn.com/abstract=254018>

Galí, J. (2008). *Monetary policy, inflation, and the business cycle: An introduction to the new Keynesian framework*. Princeton University Press, pp 41-94. Retrieved from https://perhuaman.wordpress.com/wp-content/uploads/2014/06/gali_polc3adtica_monetaria.pdf

- Galí, J. (2015). *Hysteresis and the European Unemployment Problem revisited*. National Bureau of Economic Research, Retrieved from <https://doi.org/10.3386/w21430>
- Galí, J. (2020). *Insider-Outsider Labor Markets, Hysteresis and Monetary Policy*. National Bureau of Economic Research, Retrieved from <http://www.nber.org/papers/w27385>
- Gaspar, V., Smets, F., & Vestin, D. (2007). *Is Time Ripe for Price Level Path Stability?* European Central Bank, Retrieved from <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp818.pdf>
- Jordà, O., Singh, S., & Taylor, A., (2020). *The Long-Run Effects of Monetary Policy*. Federal Reserve Bank of San Francisco, Retrieved from <https://www.frbsf.org/research-and-insights/publications/working-papers/2023/05/the-long-run-effects-of-monetary-policy/>
- Kaldor, N. (1957). *A Model of Economic Growth*. The Economic Journal, Vol. 67. No. 268. pp 591–624. Retrieved from <https://doi.org/10.2307/2227704>
- Micossi, S. (2015). *The Monetary Policy of the European Central Bank (2002-2015)*. Centre for European Policy Studies, No. 109. Retrieved from [http://aei.pitt.edu/64202/1/SR_109_SM_Monetary_Policy_of_the_ECB_0_\(1\).pdf](http://aei.pitt.edu/64202/1/SR_109_SM_Monetary_Policy_of_the_ECB_0_(1).pdf)
- Naggert, K., Rich, R., & Tracy, J. (2021). *Flexible Average Inflation Targeting and Inflation Expectations: A Look at the Reaction by Professional Forecasters*. Economic Commentary, Retrieved from <https://doi.org/10.26509/frbc-ec-202109>
- Rotemberg, J., Woodford, M. (1999). *Interest Rate Rules in an Estimated Sticky Price Model*. University of Chicago Press, pp 57-126. Retrieved from <https://www.nber.org/system/files/chapters/c7414/c7414.pdf>
- Setterfield, M. (2010). *Hysteresis*. Trinity College Department of Economics, Retrieved from <https://www3.trincoll.edu/repec/WorkingPapers2010/wp10-04.pdf>
- Sturn, S., Stockhammer, E. (2012). *The Impact of Monetary Policy on Unemployment Hysteresis*. Applied Economics, Vol. 44. No. 21. pp 2743–2756. Retrieved from <https://doi.org/10.1080/00036846.2011.566199>
- Svensson, L. E. O. (2010). *Inflation Targeting*. National Bureau of Economic Research, Retrieved from <https://larseosvensson.se/files/papers/HandbookIT.pdf>

Svensson, L. E. O. (1999). *Price-Level Targeting versus Inflation Targeting: A Free Lunch?* Journal of Money, Credit and Banking, Vol. 31. No. 3. pp 277-295. Retrieved from <https://www.jstor.org/stable/2601112>

Váry, M. (2018). *A hiszterézis közgazdasági jelentőségéről posztkeynesi szemléletben.* Közgazdasági Szemle, Vol. 65. No. 10. pp 1006–1047. Retrieved from <https://real.mtak.hu/87006/1/02%20Váry%20MiklósA.pdf>

Williams, J. (2017). *Preparing for the Next Storm: Reassessing Frameworks and Strategies in a Low R-star World.* Federal Reserve Bank of San Francisco, Retrieved from https://fraser.stlouisfed.org/files/docs/historical/frbsf/frbsf_let/frbsf_let_20170508.pdf

Woodford, M. (2003). *Interest and Prices: foundations of a theory of monetary Policy.* Princeton University Press, Retrieved from <http://www.nviegi.net/teaching/master/wood1.pdf>

8. Appendix:

```
Code of the model with inflation targeting
// The New Keynesian Model
// Monetary Policy Rule: Inflation targeting
// Written by Bence Pálfalvi

// Endogenous variables
var pi ygap rn i yn y r mreal dm n wreal a c d z;

// Exogenous variables
varexo eps_z eps_c eps_d;

// Parameters
parameters alpha beta eps eta sigma varphi theta phi_pi phi_y rho_c rho_d rho_z eta_ygap rho
mue psi_ya_n vartheta_y_n Theta lambda kappa;

// Deep parameter values
alpha = 1/3; // Degree of decreasing returns
beta = 0.99; // Subjective discount factor
eps = 6; // Elasticity of substitution
eta = 4; // Interest semi-elasticity of money demand
sigma = 1; // Inverse intertemporal elasticity of substitution
varphi = 1; // Inverse Frisch elasticity of labor supply
theta = 2/3; // Calvo parameter (probability of no reoptimization)
phi_pi = 1.5; // Inflation coefficient (Taylor rule)
phi_y = 0.5/4; // Output gap coefficient (Taylor rule)
rho_c = 0.9; // Persistence of cost-push shocks
rho_d = 0.9; // Persistence of demand shocks
rho_z = 0.9; // Persistence of technology shocks
eta_ygap = 0.08; // Hysteresis parameter

// Implied parameter values
rho = -log(beta);
mue = log(eps/(eps-1));
psi_ya_n = (1+varphi)/(sigma*(1-alpha)+varphi+alpha);
vartheta_y_n = -(((1-alpha)*(mue-log(1-alpha)))/(sigma*(1-alpha)+varphi+alpha));
Theta = (1-alpha)/(1-alpha+alpha*eps);
lambda = (((1-theta)*(1-beta*theta))/theta)*Theta;
kappa = lambda*(sigma+(varphi+alpha)/(1-alpha));

// Length of a simulation
```

```

T = 500;

// Steady state
piss = 0;
ygapss = 0;
rnss = rho;
iss = rho;
ynss = vartheta_y_n;
yss = vartheta_y_n;
rss = rho;
mrealss = yss - eta*iss;
dmss = 0;
nss = (1/(1-alpha))*yss;
wrealss = sigma*yss + varphi*nss;
ass = 0;
css = 0;
dss = 0;
zss = 0;

// The main model block
model(linear);
// 1. (pi) New Keynesian Phillips curve
pi = beta*pi(+1) + kappa*ygap + c;
// 2. (ygap) Dynamic IS curve
ygap = ygap(+1) - (1/sigma)*(i - pi(+1) - rn) + d;
// 3. (rn) Natural rate of interest
rn = rho + sigma*psi_ya_n*(a(+1)-a);
// 4. (i) Taylor rule
i = rho + phi_pi*pi + phi_y*ygap;
// 5. (yn) Natural level of output
yn = psi_ya_n*a + vartheta_y_n;
// 6. (y) Output gap
ygap = y - yn;
// 7. (r) Fisher equation
r = i - pi(+1);
// 8. (mreal) Money demand function
mreal = y - eta*i;
// 9. (dm) Money growth
dm = (mreal-mreal(-1)) + pi;
// 10. (n) Production function
y = a + (1-alpha)*n;
// 11. (wreal) Labor supply equation
wreal = sigma*y + varphi*n;
// 12. (a) Technology

```

```

a = a(-1) + eta_ygap*ygap(-1) + z;
// 13. (c) Cost-push shock process
c = rho_c*c(-1) + eps_c;
// 14. (d) Demand shock process
d = rho_d*d(-1) + eps_d;
// 15. (z) Technology shock process
z = rho_z*z(-1) + eps_z;
end;

```

```

// Initial values
initval;
pi = piss;
ygap = ygapss;
rn = rnss;
i = iss;
yn = ynss;
y = yss;
r = rss;
mreal = mrealss;
dm = dmss;
n = nss;
wreal = wrealss;
a = ass;
c = css;
d = dss;
z = zss;
end;

```

```

// Steady state and its stability
steady;
check;

```

```

// Variances of the exogenous shocks
shocks;
var eps_z = 0.01^2;
var eps_c = 0.01^2;
var eps_d = 0.01^2;
end;

```

```

// Stochastic simulation and IRFs
stoch_simul(periods=500,irf=40);

```

```

// Plotting the simulated time series
t = (0:499)';

```

```

figure;
subplot(4,4,1), plot(t,pi-piss,t,zeros(T,1),'--'), title('pi');
subplot(4,4,2), plot(t,ygap-ygapss,t,zeros(T,1),'--'), title('ygap');
subplot(4,4,3), plot(t,i-iss,t,iss*zeros(T,1),'--'), title('i');
subplot(4,4,4), plot(t,yn-ynss,t,zeros(T,1),'--'), title('yn');
subplot(4,4,5), plot(t,y-yss,t,zeros(T,1),'--'), title('y');
subplot(4,4,6), plot(t,r-rss,t,rss*zeros(T,1),'--'), title('r');
subplot(4,4,7), plot(t,rn-rnss,t,zeros(T,1),'--'), title('rn');
subplot(4,4,8), plot(t,mreal-mrealss,t,zeros(T,1),'--'), title('mreal');
subplot(4,4,9), plot(t,dm-dmss,t,zeros(T,1),'--'), title('dm');
subplot(4,4,10), plot(t,n-nss,t,zeros(T,1),'--'), title('n');
subplot(4,4,11), plot(t,wreal-wrealss,t,zeros(T,1),'--'), title('wreal');
subplot(4,4,12), plot(t,a-ass,t,zeros(T,1),'--'), title('a');
subplot(4,4,13), plot(t,z-zss,t,zeros(T,1),'--'), title('z');
subplot(4,4,14), plot(t,c-css,t,zeros(T,1),'--'), title('c');
subplot(4,4,15), plot(t,d-dss,t,zeros(T,1),'--'), title('d');

```

Code of the model with price-level targeting

// The New Keynesian Model

// Monetary Policy Rule: Price-level targeting

// Written by Bence Pálfalvi

// Endogenous variables

var pi ygap rn i yn y r mreal dm n wreal a c d z pgap;

// Exogenous variables

varexo eps_z eps_c eps_d;

// Parameters

parameters alpha beta eps eta sigma varphi theta phi_pi phi_y rho_c rho_d rho_z eta_ygap rho
mue psi_ya_n vartheta_y_n Theta lambda kappa;

// Deep parameter values

alpha = 1/3; // Degree of decreasing returns

beta = 0.99; // Subjective discount factor

eps = 6; // Elasticity of substitution

eta = 4; // Interest semi-elasticity of money demand

sigma = 1; // Inverse intertemporal elasticity of substitution

varphi = 1; // Inverse Frisch elasticity of labor supply

theta = 2/3; // Calvo parameter (probability of no reoptimization)

phi_pi = 1.5; // Price gap coefficient

phi_y = 0.5/4; // Output gap coefficient

```

rho_c = 0.5; // Persistence of cost-push shocks
rho_d = 0.9; // Persistence of demand shocks
rho_z = 0.9; // Persistence of technology shocks
eta_ygap = 0.08; // Hysteresis parameter

// Implied parameter values
rho = -log(beta);
mue = log(eps/(eps-1));
psi_ya_n = (1+varphi)/(sigma*(1-alpha)+varphi+alpha);
vartheta_y_n = -((1-alpha)*(mue-log(1-alpha)))/(sigma*(1-alpha)+varphi+alpha);
Theta = (1-alpha)/(1-alpha+alpha*eps);
lambda = (((1-theta)*(1-beta*theta))/theta)*Theta;
kappa = lambda*(sigma+(varphi+alpha)/(1-alpha));

// Length of a simulation
T = 500;

// Steady state
piss = 0;
ygapss = 0;
rnss = rho;
iss = rho;
ynss = vartheta_y_n;
yss = vartheta_y_n;
rss = rho;
mrealss = yss - eta*iss;
dmss = 0;
nss = (1/(1-alpha))*yss;
wrealss = sigma*yss + varphi*nss;
pgapss = 0;
ass = 0;
css = 0;
dss = 0;
zss = 0;

// The main model block
model(linear);
// 1. (pi) New Keynesian Phillips curve
pi = beta*pi(+1) + kappa*ygap + c;
// 2. (ygap) Dynamic IS curve
ygap = ygap(+1) - (1/sigma)*(i - pi(+1) - rn) + d;
// 3. (rn) Natural rate of interest
rn = rho + sigma*psi_ya_n*(a(+1)-a);

```

```

// 4. (i) Price level targeting
i = rho + phi_pi*pgap + phi_y*ygap;
// 5. (yn) Natural level of output
yn = psi_ya_n*a + vartheta_y_n;
// 6. (y) Output gap
ygap = y - yn;
// 7. (r) Fisher equation
r = i - pi(+1);
// 8. (mreal) Money demand function
mreal = y - eta*i;
// 9. (dm) Money growth
dm = (mreal-mreal(-1)) + pi;
// 10. (n) Production function
y = a + (1-alpha)*n;
// 11. (wreal) Labor supply equation
wreal = sigma*y + varphi*n;
// 12. (a) Technology
a = a(-1) + eta_ygap*ygap(-1) + z;
// 13. (c) Cost-push shock process
c = rho_c*c(-1) + eps_c;
// 14. (pgap) Price gap
pgap = pi + pgap(-1);
// 15. (d) Demand shock process
d = rho_d*d(-1) + eps_d;
// 16. (z) Technology shock process
z = rho_z*z(-1) + eps_z;
end;

// Initial values
initval;
pi = piss;
ygap = ygapss;
rn = rnss;
i = iss;
yn = ynss;
y = yss;
r = rss;
mreal = mrealss;
dm = dmss;
n = nss;
wreal = wrealss;
pgap = pgapss;
a = ass;
c = css;

```

```

d = dss;
z = zss;
end;

// Steady state and its stability
steady;
check;

// Variances of the exogenous shocks
shocks;
var eps_z = 0.01^2;
var eps_c = 0.01^2;
var eps_d = 0.01^2;
end;

// Stochastic simulation and IRFs
stoch_simul(periods=500,irf=40);

// Plotting the simulated time series
t = (0:499)';
figure;
subplot(4,4,1), plot(t,pi-piss,t,zeros(T,1),'--'), title('pi');
subplot(4,4,2), plot(t,ygap-ygapss,t,zeros(T,1),'--'), title('ygap');
subplot(4,4,3), plot(t,i-iss,t,iss*zeros(T,1),'--'), title('i');
subplot(4,4,4), plot(t,yn-ynss,t,zeros(T,1),'--'), title('yn');
subplot(4,4,5), plot(t,y-yss,t,zeros(T,1),'--'), title('y');
subplot(4,4,6), plot(t,r-rss,t,rss*zeros(T,1),'--'), title('r');
subplot(4,4,7), plot(t,rn-rnss,t,zeros(T,1),'--'), title('rn');
subplot(4,4,8), plot(t,mreal-mrealss,t,zeros(T,1),'--'), title('mreal');
subplot(4,4,9), plot(t,dm-dmss,t,zeros(T,1),'--'), title('dm');
subplot(4,4,10), plot(t,n-nss,t,zeros(T,1),'--'), title('n');
subplot(4,4,11), plot(t,wreal-wrealss,t,zeros(T,1),'--'), title('wreal');
subplot(4,4,12), plot(t,a-ass,t,zeros(T,1),'--'), title('a');
subplot(4,4,13), plot(t,z-zss,t,zeros(T,1),'--'), title('z');
subplot(4,4,14), plot(t,c-css,t,zeros(T,1),'--'), title('c');
subplot(4,4,15), plot(t,d-dss,t,zeros(T,1),'--'), title('d');
subplot(4,4,16), plot(t,pgap-pgapss,t,zeros(T,1),'--'), title('pgap');

```