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Measuring Time-Varying Connectedness: A Bayesian Approach

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Abstract

Interconnectedness and structural changes. Among the many lessons that the Global Financial Crisis has taught us, these two might be the most crucial for financial modelling and their importance has been enhanced once again during the European Sovereign Debt Crisis and lately, in the Covid-19 Pandemic. Especially during turbulent, uncertain periods on the financial market, the risk spills over across markets and different asset classes. In this study, I propose a framework to simultaneously incorporate the two concepts to estimate the change of connectedness due to structural breaks in a network. I introduce a time varying parameter vector autoregression with stochastic volatility (TVP-VAR-SV) estimation method, to capture the dynamic nature of financial relationships, into the Diebold-Yilmaz spillover framework, that is widely used to measure connectedness within and across different levels of markets. I also examine the estimating efficiency of two different prior specifications (horseshoe and inverse gamma priors) across three Monte Carlo simulations. First, to analyze the properties of the priors, a univariate model is simulated with breaks in the parameters and the stochastic volatility process too. Then I generate a bivariate model with three different structural break regimes and analyze the model-fitting properties of the two prior specifications. Finally, I apply the Diebold-Yilmaz framework on the generated and the fitted bivariate time series too, which would allow us to assess the accuracy of the estimation of networks. I find that the accuracy of the different priors *ceteris paribus* depends on the variance level of parameters, on the ratio between the variance of parameters and the size of the structural breaks, and on the existence of structural breaks in the system. Furthermore, accuracy-patterns in the network estimation are similar to the case where I estimate the underlying TVP-VAR-SV model.

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I Introduction

The paradigm shift in markets after the Global Financial Crisis (GFC) has remarkably increased the importance of financial networks. During turbulent periods on the financial market, the risk spills over across industries markets and different asset classes. The European Sovereign Debt Crisis and the Covid-19 Pandemic reinforced the significance of the interconnectedness of market participants. Modelling and forecasting systemic risk became a not negligible part in the everyday work of policymakers, academics and investors alike ([Bisias et al., 2012](#)).

To analyze systemic risk in financial networks, several approaches have been presented in the literature since the GFC. Some of these methods are Conditional Value at Risk (CoVaR)([Adrian and Brunnermeier, 2011](#)), Systemic Expected Shortfall (SES) and Marginal Expected Shortfall (MES) ([Acharya et al., 2012](#)), the Granger-causality-based networks ([Billio et al., 2011](#)), and the Diebold-Yilmaz framework based on forecast error variance decompositions (FEVD) from Vector autoregressive (VAR) model presented by [Diebold and Yilmaz \(2009\)](#), [Diebold and Yilmaz \(2012\)](#), and [Diebold and Yilmaz \(2014\)](#). The latter two have the advantage over the CoVaR and MES approaches in examining the network at different levels ([Badics et al., 2023](#)), and the Diebold-Yilmaz framework seem to be the most popular one in recent years ([Badics, 2023](#)).

For the dynamic analysis of financial networks, the Diebold-Yilmaz spillover table can be derived from a rolling-window VAR model. However, these models might introduce several inconveniences and possible sources of uncertainty. Such as the arbitrary (and not necessarily appropriate) choice of window size, and the fact that the rolling-window VAR models tend to severely over or underestimate the real time-varying tendencies ([Antonakakis et al., 2020](#)). To deal with these issues, [Antonakakis et al. \(2020\)](#) introduces the time-varying parameter VAR (TVP-VAR) approach into the DY-framework. With this estimation method, (i) potential changes in the parameters can be more accurately estimated, (ii) outliers can be filtered out through the Monte Carlo simulations, (iii) there is no need for arbitrary choice of window size, (iv) and there is no loss of observation during the calculation of dynamic parameters. [Antonakakis et al. \(2020\)](#) use an approximate Kalman filtering method with forgetting factors proposed by [Koop and Korobilis \(2013\)](#). This is a simplification to avoid the computational difficulties imposed by the Markov Chain Monte Carlo (MCMC) simulations, which might arise in a Bayesian method. Without the complete information, this approach

tends to overestimate the actual variation of the underlying state during periods of uncertainty. A completely Bayesian estimation method is the time varying structural VAR of [Primiceri \(2005\)](#), but this is much less commonly used than the previously mentioned approach ([Liu et al., 2020](#); [Gong et al., 2021](#); [Gong and Xu, 2022](#); [Huang et al., 2022](#)).

Neither of the above-mentioned two approaches are able to capture abrupt structural breaks in time series, however, it is an important attribution of financial markets (see crises, announcements, news etc.). [Prüser \(2021\)](#) was one of the first ones who incorporated these abrupt changes with a shrinkage prior specification into a TVOP-VAR estimation. In this paper, we lean a lot on [Prüser \(2021\)](#)'s methodology and findings.

The aim of the following study is to examine the effect of [Prüser \(2021\)](#)'s and [Primiceri \(2005\)](#)'s method on the estimation of the DY-spillover table in case of structural breaks. My main contributions to the literature are (i) provide a more detailed and widespread simulation study than [Prüser \(2021\)](#)'s, and (ii) analyze the estimation efficiency of the DY-spillover table.

The rest of the paper is built as follows. In Section [II](#) we introduce the Bayesian estimation of a TVP-VAR model with stochastic volatility (TVP-VAR-SV) using two different prior specifications (Section [II.1](#)), as well as the DY-framework for time varying parameters (Section [II.2](#)). Then in Section [III](#) we present a Monte Carlo stimulation study of three parts: the first one is a univariate unobserved component model with stochastic volatility with independent structural breaks both in the unobserved parameter and in the stochastic volatility process (Section [III.1](#)). The second part of the simulation is a bivariate VAR model with stochastic volatility and structural breaks in the coefficient matrix (Section [III.2](#)). Finally, we use the generated time series to estimate the DY-spillover table and investigate the estimation efficiency of the different specifications (Section [III.3](#)).

II Econometric Approach

This study is based on two main econometric concepts. The first one is a time-varying parameter vector autoregression with stochastic volatility (TVP-VAR-SV) with an inverse gamma prior specification, and also with a global-local (horseshoe) prior specification ([Prüser, 2021](#)) to capture the dynamic nature of financial relationships between different time series. The other one is a gener-

alized vector autoregressive (GVAR) framework of [Diebold and Yilmaz \(2012\)](#) for forecast-error variance decomposition to analyze spillovers in financial networks. The combination of these methods would provide a better understanding and more realistic modeling opportunities of the evolution of interconnectedness among the participants of financial markets.

II.1 Bayesian Framework

II.1.1 TVP-VAR-SV

We start with a structural representation of the model, which provides a theoretical approach and allows us to analyze shocks:

$$\mathbf{B}_{0t}\mathbf{y}_t = \boldsymbol{\mu}_t + \mathbf{B}_{1t}\mathbf{y}_{t-1} + \cdots + \mathbf{B}_{pt}\mathbf{y}_{t-p} + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_t), \quad (1)$$

where $\mathbf{y}_t$ is the $n \times 1$ vector of endogenous variables at time t , $\boldsymbol{\mu}_t \in \mathbb{R}^{n \times 1}$ is the column vector of time-varying intercepts, $\mathbf{B}_{0t} \in \mathbb{R}^{n \times n}$ is a lower triangular matrix with ones in the diagonal representing the contemporaneous connection between dependent variables, $\mathbf{B}_{1t}, \dots, \mathbf{B}_{pt} \in \mathbb{R}^{n \times n}$ are the lagged VAR coefficient matrices, and $\boldsymbol{\Sigma}_t \in \mathbb{R}^{n \times n}$ is a diagonal matrix with time-varying variances (this term will introduce stochastic volatility).

For the sake of a better comparison and distinction of contemporaneous and lagged effects, we transform Equation 1 into the following reduced form:

$$\mathbf{y}_t = \mathbf{X}_t\boldsymbol{\beta}_t + \mathbf{W}_t\boldsymbol{\gamma}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_t). \quad (2)$$

The first term of Equation 2 captures the effect of lagged variables and the time-varying intercept, where $\boldsymbol{\beta}_t = \text{vec}((\boldsymbol{\mu}_t, \mathbf{B}_{1t}, \dots, \mathbf{B}_{pt})')$ is a $k_\beta \times 1$ vector into which we stack $\mathbf{B}_{it}$ matrices by rows, and $\mathbf{X}_t = \mathbf{I}_n \otimes (1, \mathbf{y}'_{t-1}, \dots, \mathbf{y}'_{t-p})$ is an $n \times k_\beta$ matrix, where $k_\beta = n^2p + p$ and $\otimes$ denotes the Kronecker-product. The second term represents the instantaneous effects, having $\boldsymbol{\gamma}_t$ as a $k_\gamma \times 1$ vector of the free elements (the lower triangular part) of $\mathbf{B}_{0t}$ stacked by rows, and $\mathbf{W}_t$ is an $n \times k_\gamma$ matrix with the appropriate elements of $-\mathbf{y}_t$, where $k_\gamma = n(n-1)/2$.

Finally, we transform the model into a generic state-space form proposed by [Eisenstat et al. \(2015\)](#):

$$\mathbf{y}_t = \mathbf{X}_t \boldsymbol{\theta}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_t), \quad (3)$$

where $\mathbf{X}_t = (\mathbf{X}_t, \mathbf{W}_t)$ is an $n \times k_\theta$ matrix and $\boldsymbol{\theta}_t = (\boldsymbol{\beta}_t', \boldsymbol{\gamma}_t')$ is a $k_\theta \times 1$ vector, where $k_\theta = k_\beta + k_\gamma$. This representation of the TVP-VAR-SV model improves the sampling efficiency by drawing $\boldsymbol{\beta}_t$ and $\boldsymbol{\gamma}_t$ jointly, and also allows us to integrate them analytically (Chan and Eisenstat, 2018).

II.1.2 Prior specification

To make the estimation of the TVP-VAR-SV model given in a state-space representation form of Equation 3 feasible, we must combine the information of the likelihoods with hierarchical prior assumptions. In prior specification, we follow Chan and Eisenstat (2018) which is widely used in the literature.

First, we assume that the $\boldsymbol{\theta}_t$ vector of time-varying parameters follows a random walk process. Thus, the j^{th} coordinate of $\boldsymbol{\theta}_t$ is the following:

$$\theta_{j,t} = \theta_{j,t-1} + \eta_{j,t}, \quad \eta_{j,t} \sim \mathcal{N}(0, V_{\theta_{j,t}}), \quad \forall j = 1, \dots, k_\theta. \quad (4)$$

We assume that the log variances in $\boldsymbol{\Sigma}_t = \text{diag}(\exp(h_{1,t}), \dots, \exp(h_{n,t}))$ follow a random walk process too:

$$h_{i,t} = h_{i,t-1} + \xi_{i,t}, \quad \xi_{i,t} \sim \mathcal{N}(0, V_{h_{i,t}}), \quad \forall i = 1, \dots, n. \quad (5)$$

We treat the initial conditions $\boldsymbol{\theta}_t$ and $\mathbf{h}_0$ ¹ as parameters to be estimated, both with Gaussian priors: $\boldsymbol{\theta}_0 \sim \mathcal{N}(\mathbf{0}, 10 \times \mathbf{I}_{k_\theta})$ and $\mathbf{h}_0 \sim \mathcal{N}(\mathbf{0}, 10 \times \mathbf{I}_n)$. This prior setting allows the parameters to take on a different value in each period, and also allows the detection of structural changes in the model. Note that, in Equation 4 and Equation 5, we implicitly assumed that the covariance matrices $\mathbf{V}_{\theta_t}$ and $\mathbf{V}_{h_t}$ are diagonal. For a solution with general covariance matrices, see the Appendix B of Chan and Eisenstat (2018)². In order to estimate the TVP-VAR model I use the Gibbs-sampler as described in Chan and Eisenstat (2018) and the appendix of Prüser (2021).

¹Let us denote the vector of log volatilities at time t as $\mathbf{h}_t = (h_{1,t}, \dots, h_{n,t})'$.

²However, Chan and Eisenstat (2018) is dealing with constant variances.

II.1.3 Inverse gamma prior

I use the inverse gamma prior specification proposed by [Chan and Eisenstat \(2018\)](#) and also used by [\(Prüser, 2021\)](#). This prior specification favors many small breaks (gradual changes) in the parameters. Assuming the prior variance of the θ_t parameters and of the h_t logvariances to be constant, the inverse gamma specification is the following:

$$V_{\theta_j} = IG(\nu_{\theta_j}, S_{\theta_j}), \quad V_{h_j} = IG(\nu_{h_j}, S_{h_j}) \quad (6)$$

I set the shape parameter (degree of freedom) $\nu_{\theta_j} = \nu_{h_j} = 5$ following [Chan and Eisenstat \(2018\)](#) and [Prüser \(2021\)](#). The scale parameters S_{θ_j} and S_{h_j} are set to produce implied posterior means of 0.01^2 and 0.1^2 (respectively for V_{θ_j} and V_{h_j}). [Prüser \(2021\)](#) uses also a variant of the inverse gamma prior, where the posterior means are not constant, instead, they are optimized according to the variance level

II.1.4 Horseshoe Prior

For the distribution of the diagonal covariance matrices, I also use the global-local horseshoe prior as proposed by [Carvalho et al. \(2010\)](#), [Ray and Bhattacharya \(2018\)](#), and [Prüser \(2021\)](#).³ For further properties of the horseshoe prior and comparison with other shrinkage priors, see [Carvalho et al. \(2010\)](#).

In the assumptions of the horseshoe prior specification, we define the variances of Equation 4 and Equation 5 as:

$$V_{\theta_{j,t}} = \tau_{\theta_j} \lambda_{\theta_{j,t}}, \quad \sqrt{\tau_{\theta_j}} \sim \mathcal{C}^+(0, 1), \quad \sqrt{\lambda_{\theta_{j,t}}} \sim \mathcal{C}^+(0, 1) \quad (7)$$

$$V_{h_{i,t}} = \tau_{h_i} \lambda_{h_{i,t}}, \quad \sqrt{\tau_{h_i}} \sim \mathcal{C}^+(0, 1), \quad \sqrt{\lambda_{h_{i,t}}} \sim \mathcal{C}^+(0, 1) \quad (8)$$

With such a setting, we can control for the amount of time variation with the τ global shrink-

³This prior specification is outstandingly useful for sparse problems because its flat tails allow larger signals to remain large, and its infinitely tall spike around zero enables a serious shrinkage for parameters very close to zero. This is the motivation behind the *horseshoe* name, since the prior is the densest when either there is total shrinkage or there is no shrinkage at all. No other commonly used shrinkage prior is so robust both to unobserved sparsity patterns and to large outlying signals.

age parameters, and also for abrupt changes with the λ local shrinkage parameters. Denote with $\mathcal{C}^+(0, 1)$ the half-Cauchy distribution⁴. Put it in other words, horseshoe prior offers a clear separation between global and local shrinkage effects, thus, it allows for many smaller gradual changes, few larger abrupt breaks or the mix of both.

II.2 Diebold-Yilmaz Framework

The second large part of this study's methodology is the Diebold-Yilmaz framework (hereafter DY framework) presented by [Diebold and Yilmaz \(2012\)](#). The aim of this method is the examination of financial network dynamics and evolution through directed graphs. The model allows one to track both pairwise connections on micro level and system-wide connections on macro level. In the very heart of the DY framework there is a VAR(p) model, on which we perform a generalized forecast error variance decomposition (GFEVD). To derive the necessary information, we need to estimate the coefficient matrices of the baseline VAR(p) model and the covariance matrices of its error terms. Because of the time-varying nature of our model, we need a slightly more serious discussion before the GFEVD calculation.

II.2.1 Time-varying vector moving average representation

We continue to work with the TVP-VAR-SV model defined in Section [II.1](#). Once we estimated all the parameters from the prior distributions, we can rewrite Equation [1](#) into a more compact form following [Antonakakis et al. \(2020\)](#):

$$\mathbf{y}_t = \tilde{\boldsymbol{\mu}}_t + \tilde{\mathbf{B}}_t \mathbf{z}_{t-1} + \mathbf{u}_t, \quad \mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \tilde{\boldsymbol{\Sigma}}_t) \quad (9)$$

where $\tilde{\boldsymbol{\mu}}_t = \mathbf{B}_{0t}^{-1} \boldsymbol{\mu}_t$, $\tilde{\mathbf{B}}_t = (\mathbf{B}_{0t}^{-1} \mathbf{B}_{1t}, \mathbf{B}_{0t}^{-1} \mathbf{B}_{2t}, \dots, \mathbf{B}_{0t}^{-1} \mathbf{B}_{pt})$ is a (hyper-) row vector⁵ containing the original VAR coefficient matrices transformed by an upper triangular matrix $\mathbf{B}_{0t}^{-1}$ (inverse of a lower triangular matrix), and $\mathbf{z}_{t-1} = (\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-p})'$ is a (hyper-) column vector⁶ containing the lagged vectors of the endogenous variables. Note that the error term $\mathbf{u}_t = \mathbf{B}_{0t}^{-1} \boldsymbol{\epsilon}_t$ is also drawn

⁴[Gelman \(2006\)](#) provides evidences for the half-Cauchy distribution's advantage over the inverse gamma distribution.

⁵If we write out every matrix in $\tilde{\mathbf{B}}_t$ by element, we get that $\tilde{\mathbf{B}}_t$ is an $n \times np$ matrix.

⁶If we write out every vector in $\mathbf{z}_{t-1}$ by element, we get that $\mathbf{z}_{t-1}$ is an $np \times 1$ vector.

from a multivariate normal distribution with zero mean, only the covariance matrix changed to $\tilde{\Sigma}_t = \mathbf{B}_{0t}^{-1} \Sigma_t (\mathbf{B}_{0t}^{-1})'$.

Using this compact form of the TVP-VAR-SV model, we can transform the model into its time-varying vector moving average representation (TVP-VMA) based on the Wold representation theorem. The next step on our way to the TVP-VMA form is the recursive substitution:

$$\mathbf{y}_t = \tilde{\boldsymbol{\mu}}_t + \mathbf{J}' \left(\mathbf{M}_t^{k-1} \mathbf{z}_{t-k-1} + \sum_{j=0}^k \mathbf{M}_t^j \boldsymbol{\eta}_{t-j} \right) \quad (10)$$

with

$$\mathbf{J} = \begin{pmatrix} \mathbf{I}_n \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix} \quad \mathbf{M}_t = \begin{pmatrix} \tilde{\mathbf{B}}_t & \\ \mathbf{I}_{n(p-1)} & \mathbf{0}_{n(p-1) \times n} \end{pmatrix} \quad \boldsymbol{\eta}_t = \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix} = \mathbf{J} \mathbf{u}_t \quad (11)$$

where having the hyperparameters written out by elements, we get that $\mathbf{J}$ is an $np \times n$ dimensional matrix, $\mathbf{M}_t$ is an $np \times np$ dimensional matrix, and $\boldsymbol{\eta}_t$ is an $np \times 1$ vector.

Converting to a VMA(∞) model, or in other words, taking the limit as k approaches ∞ :

$$\mathbf{y}_t = \tilde{\boldsymbol{\mu}}_t + \lim_{k \rightarrow \infty} \mathbf{J}' \left(\mathbf{M}_t^{k-1} \mathbf{z}_{t-k-1} + \sum_{j=0}^k \mathbf{M}_t^j \boldsymbol{\eta}_{t-j} \right) = \tilde{\boldsymbol{\mu}}_t + \sum_{j=0}^{\infty} \mathbf{J}' \mathbf{M}_t^j \boldsymbol{\eta}_{t-j} \quad (12)$$

Combining this result with the third term of Equation 11, the TVP-VMA representation of our model:

$$\mathbf{y}_t = \tilde{\boldsymbol{\mu}}_t + \sum_{j=0}^{\infty} \mathbf{A}_{j,t} \mathbf{u}_{t-j}, \quad \mathbf{A}_{j,t} = \mathbf{J}' \mathbf{M}_t^j \mathbf{J} \quad \forall j = 0, 1, \dots \quad (13)$$

where $\mathbf{A}_{j,t}$ is an $n \times n$ matrix $\forall j$.

II.2.2 Generalized forecast error variance decomposition

Once we have transformed the model into its vector moving average representation, we can deduce the metrics necessary for the measurement of connectedness in the network. To do so, we rely on variance decompositions, which allow us to compute the proportion of forecast error variance of a

variable y_i on a given time horizon due to shocks in every y_j variable where $j \neq i$. We apply the generalized framework proposed by [Koop et al. \(1996\)](#), [Pesaran and Shin \(1998\)](#), and [Diebold and Yilmaz \(2012\)](#) augmented with the time-varying specifications of [Antonakakis et al. \(2020\)](#). This generalized approach allows correlated shocks in an order-invariant way by appropriately using the historically observed distribution of the error terms (unlike other identification methods, such as the Cholesky factorization).

First, let us consider the generalized impulse response function (GIRF) of a shock in the j th endogenous variable of $\mathbf{y}_t$ in case of an H -step-ahead forecast:

$$\mathbf{GIRF}_t(H, \delta_{j,t}, \boldsymbol{\Omega}_{t-1}) = \mathbb{E}(\mathbf{y}_{t+H} | u_{j,t} = \delta_{j,t}, \boldsymbol{\Omega}_{t-1}) - \mathbb{E}(\mathbf{y}_{t+H} | \boldsymbol{\Omega}_{t-1}) \quad (14)$$

which is, by definition, the difference between a forecast once the variable j is shocked and once where it is not. $\delta_{j,t}$ denotes the shock in the j th coordinate of $\mathbf{u}_t$, while $\boldsymbol{\Omega}_{t-1}$ represents all information available until $t-1$. Since we saw earlier that $\mathbf{u}_t$ has a multivariate normal distribution, one can show the following (see also [Koop et al., 1996](#), and [Pesaran and Shin, 1998](#)):

$$\mathbb{E}(\mathbf{u}_t | u_{j,t} = \delta_{j,t}) = \frac{\tilde{\boldsymbol{\Sigma}}_t \mathbf{e}_j \delta_{j,t}}{\tilde{\Sigma}_{jj,t}} \quad (15)$$

where $\mathbf{e}_j$ is a selection vector with one in the j th coordinate and zero otherwise. Let us denote $\mathbf{GIRF}_t(H, \delta_{j,t}, \boldsymbol{\Omega}_{t-1})$ by $\boldsymbol{\Psi}_{j,t}(H)$ and apply Equation 13⁷:

$$\boldsymbol{\Psi}_{j,t}(H) = \frac{\mathbf{A}_{H,t} \tilde{\boldsymbol{\Sigma}}_t \mathbf{e}_j \delta_{j,t}}{\tilde{\Sigma}_{jj,t}} \quad (16)$$

By setting $\delta_{j,t} = \sqrt{\tilde{\Sigma}_{jj,t}}$, we obtain the scaled GIRF as:

$$\boldsymbol{\Psi}_{j,t}(H) = \tilde{\Sigma}_{jj,t}^{-\frac{1}{2}} \mathbf{A}_{H,t} \tilde{\boldsymbol{\Sigma}}_t \mathbf{e}_j \quad (17)$$

that measures the effect of a shock in the j th variable at time t on the expected (forecast) values of $\mathbf{y}_{t+H}$. We need to define H -step-ahead forecast error variance (FEV), that also can be interpreted as the mean squared error (MSE) of the forecast. Following [Diebold and Yilmaz \(2012\)](#), and [Buse and Schienle \(2019\)](#):

⁷Ignoring the vector of time-varying intercepts $\boldsymbol{\mu}_t$.

$$\mathbf{FEV}_t(H) = \mathbf{MSE}_t[\mathbf{y}_t(H)] = \mathbb{E}[(\mathbf{y}_{t+H} - \mathbf{y}_t(H)) \times (\mathbf{y}_{t+H} - \mathbf{y}_t(H))'] = \sum_{h=0}^{H-1} (\mathbf{A}_{h,t} \boldsymbol{\Sigma}_t \mathbf{A}_{h,t}', \quad (18)$$

where $\mathbf{y}_t(H) = \sum_{j=H}^{\infty} \mathbf{A}_{j,t} \mathbf{u}_{t-j}$ is the theoretical optimal predictor for $\mathbf{A}_{j,t}$, while $\mathbf{MSE}_t[\mathbf{y}_t(H)]$ is an $n \times n$ matrix.

These terms can be used in the derivation of the forecast error variance decompositions, which measures the pairwise directional connectedness from the variable (or member of the network) j to i . Hence, we analyze the proportion of the H-step-ahead forecast error variance of i which is due to innovations in j through the ij th generalized variance decomposition component:

$$\phi_{ij,t}(H) = \frac{\sum_{h=0}^{H-1} \Psi_{ij,h}^2}{\sum_{h=0}^{H-1} MSE_{ij,h}} = \frac{\tilde{\Sigma}_{jj,t}^{-1} \sum_{h=0}^{H-1} (\mathbf{e}_i' \mathbf{A}_{h,t} \tilde{\Sigma}_t \mathbf{e}_j)^2}{\sum_{h=0}^{H-1} (\mathbf{e}_i' \mathbf{A}_{h,t} \tilde{\Sigma}_t \mathbf{A}_{h,t}' \mathbf{e}_j)} \quad (19)$$

Since the generalized framework is used, the shocks to each variable are not orthogonalized, thus, the (row-wise) contributions to the FEV not necessarily add up to one in the variance decomposition table: $\sum_{j=1}^n \phi_{ij,t}(H) \neq 1$. For the sake of a better interpretation and usability, we normalize every entry of the variance decomposition table by the corresponding row sum of each:

$$\tilde{\phi}_{ij,t}(H) = \frac{\phi_{ij,t}(H)}{\sum_{k=1}^n \phi_{ik,t}(H)} \quad (20)$$

We call this measure the Generalized Forecast Error Variance Decomposition (GFEVD).

II.2.3 Measurement of volatility spillovers

Once we defined the very core measure in the DY framework, the GFEVD, we can progress by constructing several systemic (or network) connectedness measures. With the volatility contributions from Equation 20, we can define the total (SUM) volatility spillover index as:

$$S_t(H) = \frac{\sum_{i,j=1, i \neq j}^n \tilde{\phi}_{ij,t}(H)}{\sum_{i,j=1}^n \tilde{\phi}_{ij,t}(H)} = \frac{1}{n} \sum_{i,j=1, i \neq j}^n \tilde{\phi}_{ij,t}(H) \quad (21)$$

Note that $\sum_{i,j=1}^n \tilde{\phi}_{ij,t}(H) = n$ by definition. The total volatility spillover index is the average of off-diagonal elements of the GFEVD matrix. It measures the level of average propagation of a

shock in one time series to all others in the network.

The generalized approach enables us to analyze not only the amount of volatility spilled over a network, but the direction of these spillovers as well. First, we define the amount of spillovers that the i th time series receives from every other time series (FROM spillover index):

$$S_{i \leftarrow \bullet, t}(H) = \frac{\sum_{k=1, k \neq i}^n \tilde{\phi}_{ik, t}(H)}{\sum_{k=1}^n \tilde{\phi}_{ik, t}(H)} = \sum_{k=1, k \neq i}^n \tilde{\phi}_{ik, t}(H) \quad (22)$$

We can also study the reverse direction, the amount of volatility spillover received by the i th member of the network from all the others (TO spillover index):

$$S_{i \rightarrow \bullet, t}(H) = \frac{\sum_{k=1, k \neq j}^n \tilde{\phi}_{kj, t}(H)}{\sum_{k=1}^n \tilde{\phi}_{kj, t}(H)} = \sum_{k=1, k \neq j}^n \tilde{\phi}_{kj, t}(H) \quad (23)$$

The construction of the last two indices is very similar, and their derivation is very straightforward from the construction of the variance decomposition table. Since the rows of the matrix represent the volatility receivers and the columns represent the transmitters, in Equation 22, we summarize the off-diagonal elements of the i th row, while in Equation 23 we summarize the off-diagonal elements of the i th column.

Once directional spillover indices are defined, we can compute a net spillover index (NET) too, that measures the influence of time series i on the whole network.

$$S_{i, t} = S_{i \rightarrow \bullet, t}(H) - S_{i \leftarrow \bullet, t}(H) \quad (24)$$

As Antonakakis et al. (2020) points out, if $S_{i, t}$ is positive (negative), then the i th member of the network is a net volatility transmitter (receiver), i.e. it rather influences (is being influenced by) the market. We can further break down the NET spillover index into a net pairwise spillover index (NETP), which measures the bidirectional relationship between time series i and j .

$$S_{ij, t} = \frac{\tilde{\phi}_{ij, t}(H)}{\sum_{k, l=1}^n \tilde{\phi}_{kl, t}(H)} - \frac{\tilde{\phi}_{ji, t}(H)}{\sum_{k, l=1}^n \tilde{\phi}_{kl, t}(H)} = \frac{1}{n}(\tilde{\phi}_{ij, t}(H) - \tilde{\phi}_{ji, t}(H)) \quad (25)$$

Similarly to the NET index, a positive (negative) NETP index implies the domination of time series i (j) over time series j (i).

The indices defined in Equation 21 - 25 are often multiplied by 100% for the sake of easier

interpretability.

III Simulation Study

In the first part of the Monte Carlo simulations, we use the data generating processes proposed by [Prüser \(2021\)](#) with some extensions: univariate unobserved component model with independent structural breaks in parameters and in stochastic volatility, then a TVP-VAR-SV DGP with three different type of structural breaks introduced to a system with coefficient matrices and breaks similar to those of [Antonakakis et al. \(2020\)](#). Finally, we apply the DY-framework to calculate the network of the generated time series. We measure the performance of fitting models using the inverse gamma and the horseshoe prior. We compare the performance of fitting models using the inverse gamma and the horseshoe prior in all three simulations.

III.1 Univariate examples

To demonstrate the difference between the horseshoe and the inverse gamma prior, I start with an extension of the univariate example of [Prüser \(2021\)](#) with an abrupt break in the unobserved component (Equation 27) and the logvariances (Equation 30). I expand the model by assuming that the variance of the observed component follows a stochastic process. This assumption allows an easier and more flowless continuation and model extension later in this study.

$$y_t = \tau_t + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \exp(h_t)) \quad (26)$$

$$\tau_t = \tau_{t-1} + c\mathbf{1}(t = t_b) + \epsilon_t^\tau, \quad \epsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2) \quad (27)$$

$$h_t = h_{t-1} + \epsilon_t^h, \quad \epsilon_t^h \sim \mathcal{N}(0, \sigma_h^2) \quad (28)$$

where $\mathbf{1}$ is an indicator function that equals to one if its argument is true and zero otherwise. This DGP has a structural break at t_b that is introduced by the parameter c . I follow [Prüser \(2021\)](#) in the parameter setting too, that is $c = 2$, $t_b = 100$, $T = 200$, $\sigma_h^2 = 0.1^2$ and $\sigma_\tau^2 = 0.1^2 = 0.01$ ⁸.

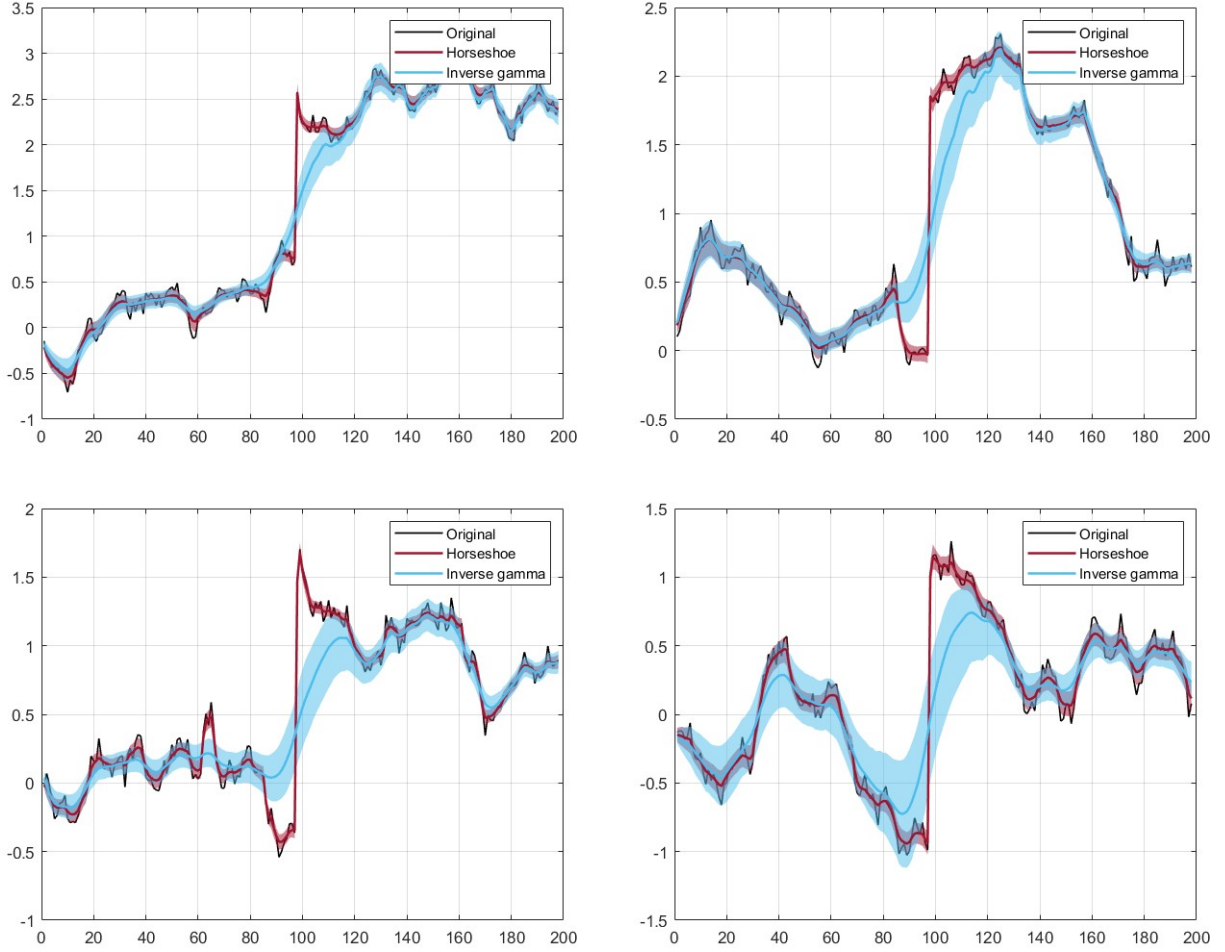
⁸However, [Prüser \(2021\)](#) sets $\sigma_\tau = 0.01$, but it might be just a typo, because the given realizations of τ can be obtained by setting $\sigma_\tau^2 = 0.1^2$, see Appendix A.

I present four randomly drawn realizations of τ in Figure 1 along with estimations using the TVP-VAR-SV model presented in Section II.1.1. The posterior distributions in the MCMC (Markov Chain Monte Carlo) algorithm are generated from a sample of 1000 trajectories with total number of simulations of 10000 and burn-in sample size of 1000⁹. The model fitting is done with both the horseshoe and the inverse gamma prior (see Section II.1.2 and Appendix II.1.3).

The results in Figure 1 support the advantages of the horseshoe prior over the inverse gamma, especially around the structural break at $t = 0$. The model with the horseshoe prior almost perfectly fits the original time series, while the inverse gamma prior makes the estimated values react slower to the abrupt change in the system. The figures below show us the 68% posterior confidence intervals of the horseshoe and the inverse gamma prior with ruby and light blue, respectively.

⁹In our estimation, we keep every 10th draw after discarding the burn-in sample.

Figure 1: Typical realizations of τ and the fitted TVP models with horseshoe and inverse gamma priors.



Notes: The figure shows typical realizations of the unobserved parameter τ presented in Equation 27 with a break of 2 at $t = 100$. The fitted values are the posterior means of MCMC simulations with an effective sample of 1000.

To further investigate the accuracy of the models, I compute the mean absolute error (MAE) for both prior specifications across 20 different realizations of τ . The measure is defined as:

$$MAE_{\tau} = \frac{1}{T \times S} \sum_{t=1}^T \sum_{s=1}^S |\tau_t - \hat{\tau}_{s,t}| \quad (29)$$

where τ_t is generated from Equation 27, while $\hat{\tau}_{s,t}$ is the posterior mean from the model of Equation 2. I choose the timeframe as $T = 200$ and the number of simulations as $S = 20$.

Table 1: Simulation results: Univariate example, abrupt break in a coefficient

	horseshoe	inverse gamma
$c = 0$	0.07	0.07
$c = 2$	0.05	0.11

Notes: The table shows the MAE between the τ and the corresponding estimates of the horseshoe prior and inverse gamma prior as defined in Equation 29. The degrees of time variation are $\sigma_\tau^2 = 0.1^2$ and $\sigma_h^2 = 0.1^2$. The length of the times series (T) is 200, and the simulation number (S) is 20.

The results in Table 1 confirm the better performance of the horseshoe prior specification when a structural break is introduced to the analyzed time series. Contrary to the results of Pr user (2021), the horseshoe prior tends to be less accurate when there is no break in the system. The theoretical argument behind this difference is not clear yet, and can be a question of further research.

I also conduct a similar simulation for the logvolatility h because this parameter would affect the covariance matrix which bears core importance in the DY-framework. I introduce a break to the stochastic volatility component defined in Equation 28 by:

$$h_t = h_{t-1} + c_h \mathbf{1}(t = t_b) + \epsilon_t^h, \quad \epsilon_t^h \sim \mathcal{N}(0, \sigma_h^2) \quad (30)$$

where $\mathbf{1}(\cdot)$ is the very same indicator function used in Equation 27, and $\sigma_h^2 = 0.1^{210}$.

I assess the accuracy of the fitted models with the MAE measure defined in Equation 29.

Table 2: Simulation results: Univariate example, abrupt break in a stochastic volatility

	horseshoe	inverse gamma
$c = 0$	0.07	0.08
$c = 0.5$	0.06	0.09

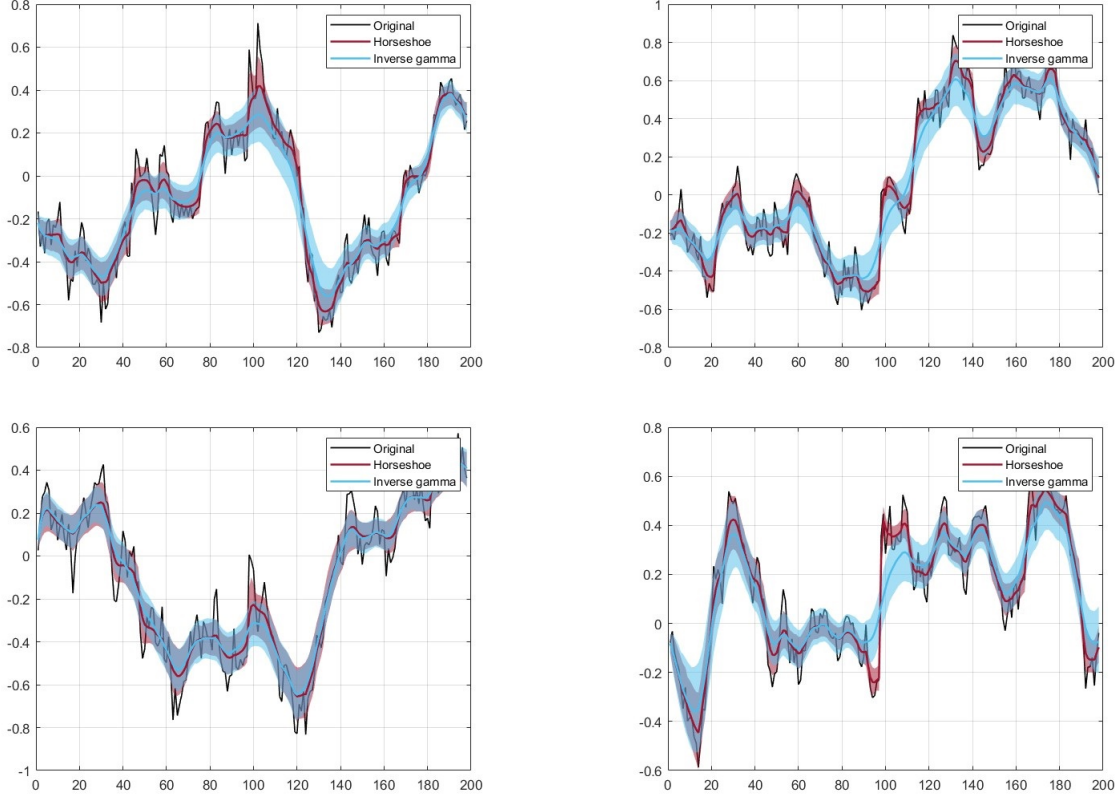
Notes: The table shows the MAE between the h and the corresponding estimates of the horseshoe prior and inverse gamma prior as defined in Equation 29. The degrees of time variation are $\sigma_\tau^2 = 0.1^2$ and $\sigma_h^2 = 0.1^2$. The length of the times series (T) is 200, and the simulation number (S) is 20.

The results in Table 2 a little bit differ from those of τ , because the advantage of the horseshoe prior is smaller even in the presence of breaks, which may happen because of the smaller effect of

¹⁰Which is the same as in Equation 28

these structural breaks. However, we still see a slower reaction of the inverse gamma prior around the jumps (approximately between $t = 95$ and $t = 105$) in the logvolatility process.

Figure 2: Typical realizations of $\mathbf{h}$ and the fitted TVP models with horseshoe and inverse gamma priors.



Notes: The figure shows typical realizations of the logvolatility $\mathbf{h}$ presented in Equation 30 with a break of 0.5 at $t = 100$. The fitted values are the posterior means of MCMC simulations with an effective sample of 1000.

To summarize this section, I conclude that with a given level of variance in the coefficients and the stochastic volatility, it is the size of the break that determine the relative efficiency of the horseshoe and the inverse gamma prior. The larger the break, the more superior the horseshoe prior become because the inverse gamma prior captures the break more smoothly.

III.2 A VAR model

To be able to apply the DY-framework, we need to define a VAR model first. In the following section, I define TVP-VAR-SV data generating process and analyze the behavior of both priors

with three different state-switching regimes in the system (no break, break in one coefficient, break in all coefficients)¹¹. In the definition of time varying parameters, I follow the bivariate model of [Prüser \(2021\)](#):

$$y_{1,t} = y_{1,t-1}b_{11,t} + y_{2,t-1}b_{12,t} + \epsilon_{1,t}^y, \quad \epsilon_{1,t}^y \sim \mathcal{N}(0, \exp(h_{1,t})) \quad (31)$$

$$y_{2,t} = y_{1,t-1}b_{21,t} + y_{2,t-1}b_{22,t} + \epsilon_{2,t}^y, \quad \epsilon_{2,t}^y \sim \mathcal{N}(0, \exp(h_{2,t})) \quad (32)$$

$$b_{jk,t} = b_{jk,t-1} + c_{b,jk}\mathbf{1}(t = t_b) + \epsilon_{jk,t}^b, \quad \epsilon_{jk,t}^b \sim \mathcal{N}(0, \sigma_b^2) \quad (33)$$

$$h_{i,t} = h_{i,t-1} + \epsilon_{i,t}^h, \quad \epsilon_{i,t}^h \sim \mathcal{N}(0, \sigma_h^2) \quad (34)$$

where $i, j, k \in \{1, 2\}$. I set $\sigma_h^2 = 0.01$ ¹² and $t_b = 100$. The state-switching regimes are introduced through the $c_{jk,t}$ components. By organizing these into a matrix where $[\mathbf{C}_t]_{jk} = c_{jk,t}$, the matrices describing the regimes are as follows:

$$\mathbf{C}_t^0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{C}_t^1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{C}_t^4 = \begin{bmatrix} 0.2 & -0.2 \\ -0.2 & 0.2 \end{bmatrix} \quad (35)$$

where the upper indices denote the number of breaks in that given matrix. Furthermore, I also consider four different time variation regime of the VAR coefficients with $\sigma_b^2 \in \{0.02, 0.01, 0.001, 0.0001\}$. The initial values are set for each simulation as $b_{11,0} = b_{22,0} = 0.6$, $b_{12,0} = b_{21,0} = 0.3$ following [Antonakakis et al. \(2020\)](#), and $h_{1,0} = h_{2,0} = 0$. In the following parts of the study, I might refer to the VAR coefficients in their matrix form as $\mathbf{B}_t$, where $[\mathbf{B}_t]_{ij} = b_{ij,t}$. In all cases, I discard unstable draws of $\mathbf{y}$ and draws of $\mathbf{h}$ for which any of its coordinates having a larger absolute value than 1. To evaluate the simulation results, we need to redefine the MAE for this case as:

$$MAE_b = \frac{1}{T \times S \times 4} \sum_{s=1}^S \sum_{t=1}^T \sum_{k=1}^K \sum_{j=1}^J |b_{sjk,t} - \hat{b}_{sjk,t}| \quad (36)$$

¹¹Now, I ignore the jumps in the stochastic volatility process.

¹²The same as in the univariate examples.

where $\hat{b}_{sjk,t}$ is the posterior mean, $S = 5$, $T = 200$, and $K = J = 2$.

The results are presented in Table 3 for the cases with break in one entry of the coefficient matrix, and in Table 4 for the case with a structural break in each coefficient.

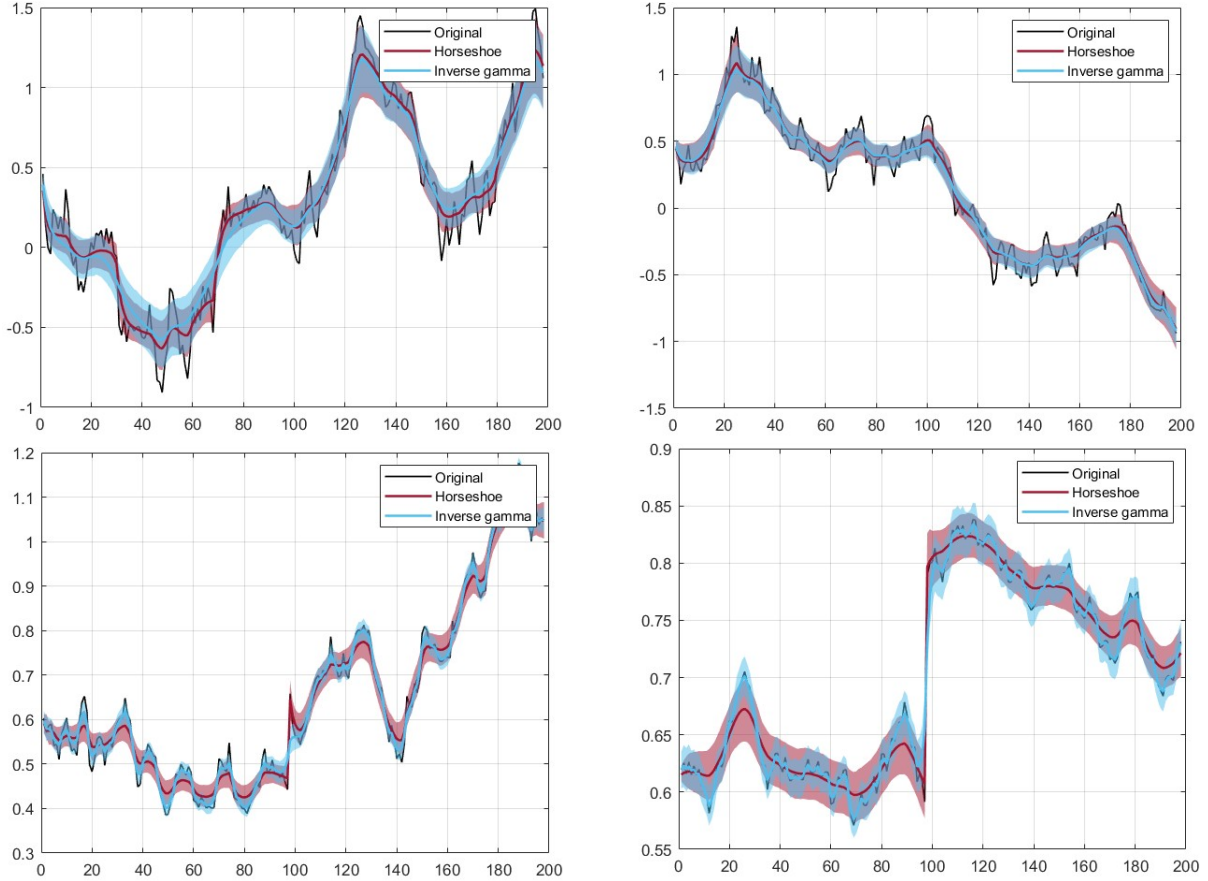
Table 3: Simulation results: VAR, abrupt break in one coefficient

σ_b^2	c = 0.2				c = 0			
	0.02	0.01	0.001	0.0001	0.02	0.01	0.001	0.0001
horseshoe	0.0100	0.0061	0.0019	0.0008	0.0098	0.0061	0.0020	0.0009
inverse gamma	0.0123	0.0065	0.0010	0.0003	0.0118	0.0063	0.0009	0.0002

Notes: The table shows the MAE between the B_t matrices and the corresponding estimates of the horseshoe prior and inverse gamma prior as defined in Equation 36. The initial ($t=0$) VAR coefficient matrix is the following $B_0 = [0.6 \ 0.3 ; 0.3 \ 0.6]$, similar to Antonakakis et al. (2020). The break is related to the first coefficient of the B_t matrix. The number of observations (T) is 200, and the simulation number (S) is 5.

The results differ from those of Prüser (2021)’s because the dominance of the horseshoe prior is not that straightforward. With relatively higher parameter variance (σ_b), the horseshoe prior’s estimate is closer to the real data, while with the lower volatility settings, the inverse gamma prior is the more accurate one. It implies that with a given size of break, the relative efficiency of the horseshoe compared to the inverse gamma prior depends on the variance of the VAR coefficients (σ_b^2). Note that, the variance of the coefficients in the inverse gamma prior specification was defined in Section A in such a way as to have a theoretical mean of 0.01^2 . As we set this variance higher and higher (further and further away from the theoretical mean) in the DGP, the inverse gamma prior become less and less effective. The realizations of b_{11} presented in Figure 3 also reinforce these results. In the upper row of figures (where $\sigma_b^2 = 0.02$ and $\sigma_b^2 = 0.01$), the inverse gamma prior is seemingly slower and softer than the horseshoe prior, while in the lower row (where $\sigma_b^2 = 0.001$ and $\sigma_b^2 = 0.0001$), the horseshoe prior performs better only around the structural breaks (between $t = 95$ and $t = 105$), apart from that, the inverse gamma prior is better at capturing small changes.

Figure 3: Typical realizations of b_{11} with a break of 0.2 and different variance levels



Notes: The figures show typical realisations of the VAR coefficients b_{11} (Equation 31 and Equation 33) with a break of 0.2 at $t = 100$. The four charts represent the four level of variances, from the top left by rows: $\sigma_b^2 = 0.02$, $\sigma_b^2 = 0.01$, $\sigma_b^2 = 0.001$, $\sigma_b^2 = 0.0001$ respectively. The original time series are generated with a data generating process defined in Equation 33, while the fitted values are the posterior means of MCMC simulations with an effective sample of 1000.

Table 4: Simulation results: VAR, abrupt break in all coefficients

σ_b^2	$c = 0.2$			
	0.02	0.01	0.001	0.0001
horseshoe	0.0091	0.0058	0.0017	0.0007
inverse gamma	0.0120	0.0071	0.0012	0.0005

Notes: The table shows the MAE between the B_t matrices and the corresponding estimates of the horseshoe prior and inverse gamma prior as defined in Equation 36. The initial ($t=0$) VAR coefficient matrix is the following $B_0 = [0.6 \ 0.3 ; 0.3 \ 0.6]$, similar to Antonakakis et al. (2020). The break is related to all coefficients of the B_t matrix ($c = 0.2$ for the diagonal elements, and $c = -0.2$ for the off diagonal elements). The length of the times series (T) is 200, and the simulation number (S) is 5.

One can discover a similar pattern in the third case, when we have a structural break in each entry of the coefficient matrix. The horseshoe prior performs better with the higher-volatility setups, and the inverse gamma is better in the lower parameter volatility environment. However, when the time variation of the VAR coefficients is the lowest, the difference between the accuracy of the two specifications is smaller than in the two other cases, which is in line with the finding that the horseshoe prior is more effective in the presence of breaks in the time series.

To sum up this part of the simulation, I find that with a given state-switching regime, the horseshoe prior performs better when the variance of the coefficients σ_b^2 is higher, but once this variance approaches the theoretical mean of the inverse gamma prior's variance, the inverse gamma prior becomes the more efficient setup. This latter result might provide interesting questions for further research, and makes it worthwhile to consider using the augmented inverse gamma prior with hyperparameter optimization, as mentioned in Section II.1.3.

III.3 Estimating connectedness

Once we have the data from the above defined TVP-VAR-SV data generating process, we can analyze the connectedness between the time series. To do so, I use the Diebold-Yilmaz spillover framework presented in Section II.2. I compute the spillover table (defined in Equation 20) for the generated dataset, for the estimation with the inverse gamma prior, and also for the estimation with the horseshoe prior for the last four variance-break setup (when the all the coefficients incorporate a structural break in B_t) defined in Section III.2. Then the mean absolute errors are calculated to compare the prior setups once again. The MAE is defined as follows

$$MAE_{\tilde{\phi}} = \frac{1}{T \times 4} \sum_{t=1}^T \sum_{k=1}^K \sum_{j=1}^J |\tilde{\phi}_{sjk,t} - \hat{\phi}_{sjk,t}| \quad (37)$$

where T, J, K is the same as in Equation 36, $\tilde{\phi}_{sjk,t}$ is an entry of the spillover table (Equation 20) and $\hat{\phi}_{sjk,t}$ denotes its posterior means. The forecast window (H) is set to 10. The results are presented in Table 5 similarly to how it was done in the previous section.

Table 5: Simulation results: VAR, abrupt break in all coefficients, Diebold-Yilmaz spillover table

	$c = 0.2$			
σ_b^2	0.02	0.01	0.001	0.0001
horseshoe	2.13	1.22	0.99	0.84
inverse gamma	2.73	1.42	0.86	0.74

Notes: The table shows the MAE between the theoretical DY spillover table and the corresponding estimates based on the horseshoe prior and inverse gamma prior as defined in Equation 37. The initial ($t=0$) VAR coefficient matrix is the following $B_0 = [0.6 \ 0.3 ; 0.3 \ 0.6]$, similar to Antonakakis et al. (2020). The break is related to all coefficients of the B_t matrix ($c = 0.2$ for the diagonal elements, and $c = -0.2$ for the off diagonal elements). The length of the times series (T) is 200, and the simulation number (S) is 5. The time horizon of the DY spillover table (H) is 10.

When the structural break is introduced to every coefficient in the B_t coefficient matrix¹³, the patterns of Table 4 still hold between the two specifications. Thus, the model with the horseshoe prior estimates the network of time series better when the variance of time-varying coefficients is higher. Meanwhile, the inverse gamma is the better specification when the parameters are less volatile, i.e. when the parameter variance is closer to the constant theoretical mean of the inverse gamma prior’s variance¹⁴. The accuracy of both priors is inversely proportional to the variance level of the coefficients.

Since the forecast window of the DY-framework is set to 10, it means that the covariance matrix bears much larger weight in the computation of spillover measures. This fact can explain our main finding of this section, that when we have a structural break in our network, the accuracy-pattern and the behavior of different prior specifications in the estimation of connectedness is similar to those in the estimation of the time varying parameters themselves.

IV Concluding Remarks

In this study, we propose a framework to simultaneously incorporate the connectedness and structural breaks to model the dynamic nature of financial networks. We introduce a TVP-VAR-SV estimation method with two different prior specifications (horseshoe and inverse gamma priors)

¹³Defined in Equation 33

¹⁴See Section II.1.3

into the Diebold-Yilmaz spillover analysis, that is widely used to measure connectedness within and across different levels of markets. To assess the accuracy of the different prior specifications, we propose a three-part Monte Carlo simulation.

In the first part, we generate data from a univariate unobserved parameter model with stochastic volatility with independent abrupt breaks in both the unobserved parameter and the stochastic volatility process. Here we find that with a given level of variance in the coefficients and the stochastic volatility, it is the size of the break that determine the relative efficiency of the horseshoe and the inverse gamma prior. The larger the break, the more superior the horseshoe prior become because the inverse gamma prior captures the break more smoothly and more slowly as well. However, contrary to the findings of [Prüser \(2021\)](#), the horseshoe prior tends to be less accurate when there is no break in the system.

In the next part, we simulate a bivariate TVP-VAR-SV model with three different state-switching regimes in the system (no break, break in one coefficient, break in all coefficient). In this case, with a given state-switching regime, the horseshoe prior performs better when the time variation of the coefficients is higher, and seem to be more effective in the presence of structural breaks. The inverse gamma specification tends to perform better when the variance level is close to its theoretical prior variance. The results in this simulation also differ from those of [Prüser \(2021\)](#) because the advantage of the horseshoe prior is not that straightforward.

Finally, the estimation of connectedness with the DY-framework, shows very similar characteristics to the estimation of the underlying time series. The horseshoe prior performs better in the more volatile environments, while the inverse gamma prior in the less turbulent environments. The accuracy of both priors is also inversely proportional to the variance level of parameters, similarly to the previous part.

V Further Research

Both the topic and the results open numerous new directions for future research. The theoretical argument behind the finding that the horseshoe prior tends to be less accurate when there is no break in the system (Section [III.1](#)) is missing, so it would require extensive in-depth research in the Bayesian econometrics and probability theory to explain this property.

The results of Section [III.2](#) also brings up a question, but a rather empirical one. The fact that the inverse gamma prior specification with a constant mean of prior variance tends to lose a lot of efficiency when the variance level of the VAR coefficients are significantly higher than this prior mean implies the need of hyperparameter optimization with this specification.

The comparison of these two prior specifications with the approximate Kalman filtering method with forgetting factors ([Koop and Korobilis, 2013](#); [Antonakakis et al., 2020](#)) in network estimation would be an interesting extension and could make the results more robust. A similar comparison could be performed with the Bayesian estimation method of [Primiceri \(2005\)](#) too.

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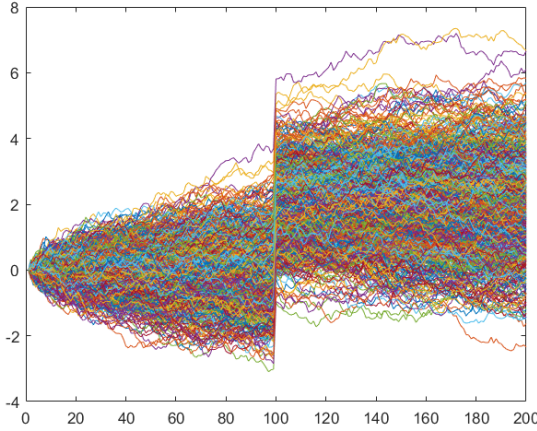
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Appendix

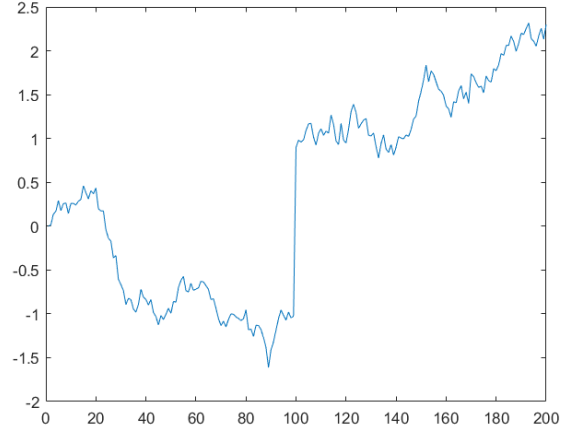
A Realizations of τ

As highlighted in Section III, the setting of σ_τ is different in Prüser (2021), than the setting that would produce their realizations (see Figure 1 in Prüser (2021)). In this section, I compare the realizations with Prüser (2021)'s setting and with mine, too.

First, let us see my parameter setting with $\sigma_\tau = 0.1$ on Figure A1. If one looks up the paper of Prüser (2021), Figure A1b has very similar characteristics than the four realizations presented on their Figure 1. Namely, the τ process follows a random walk around zero with quite notable variance, then the structural break is introduced, and after the process follows the random walk with the very same characteristics around 2.



(a) 1000 realizations of the DGP

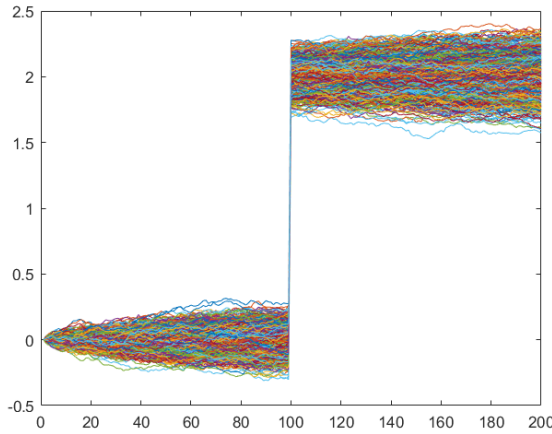


(b) A randomly chosen realization

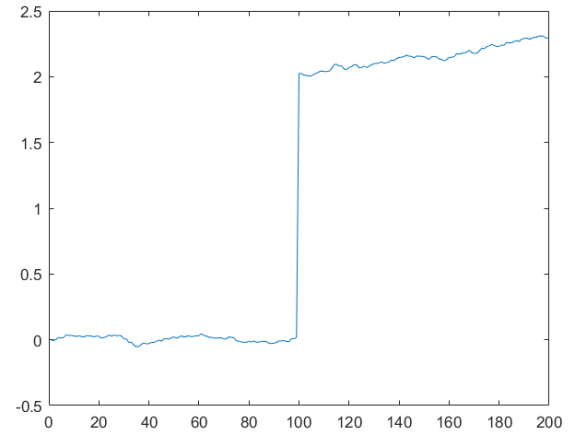
Figure A1: Realizations of the τ unobserved component of the DGP presented in Equation 26 and Equation 27, with the following parameter setting: $c = 2$, $t_b = 100$, $T = 200$, $\sigma_y = 1$ and $\sigma_\tau = 0.1$, i.e. my setting.

Second, we analyze the data generating process with $\sigma_\tau = 0.01$, that is presented on Figure A2. By comparing Figure A2b with Figure A1b and with Figure 1 of Prüser (2021), one might remark very different characteristics. In this case, the variation around the means before and after the structural break is hardly notable compared both to the structural break and to the variance caused by setting $\sigma_\tau = 0.1$.

If we compare Figure A1a with Figure A2a, probably the biggest difference is the density of trajectories on the vertical axis. With standard deviation 0.1, the trajectories seem to be most densely grouped somewhere between 0 and 4. However, with the 0.01 value, the trajectories before the break are the densest between -0.5 and 0.5 , and after the break between 1.5 and 2.5 . Also important to note that in Figure A2a not once out of a thousand realizations does the trajectories leave neither the $[-0.5, 0.5]$ interval nor the $[1.5, 2.5]$ interval, which is definitely not the case in any of Prüser (2021)’s τ -realizations.



(a) 1000 realizations of the DGP



(b) A randomly chosen realization

Figure A2: Realizations of the τ unobserved component of the DGP presented in Equation 26 and Equation 27, with the following parameter setting: $c = 2$, $t_b = 100$, $T = 200$, $\sigma_y = 1$ and $\sigma_\tau = 0.01$, i.e. Prüser (2021)’s setting.