

TDK-dolgozat

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HYPERELASTIC MATERIAL MODEL STRUCTURE AND PARAMETER IDENTIFICATION WITH DEEP CONVOLUTIONAL NETWORKS

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Hyperelastic material model structure and parameter identification with deep convolutional networks

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This research presents an artificial intelligence (AI)-driven method to identify and estimate the structure and parameters of hyperelastic material models (like rubber), where a properly identified model is crucial in mechanical modeling and material science. The complex non-linear behavior of hyperelastic materials under stress requires precise models to accurately predict its performance. Traditional methods often require extensive manual adjustments. In response, this method employs a one-dimensional convolutional neural network (1D CNN) to automate classification for model structure selection and regression for model parameter estimation. The method categorizes data from six measurement types, including biaxial tension and planar compression, each aligned with specific material models such as Mooney-Rivlin, Neo-Hookean, and Ogden. Since we have six types of measurements, which are not always measured in each case, we propose to use a local model-based solution for identifying the model. It means, that we develop six neural network models for model structure selection and six neural networks for model parameter estimation, and the final result is predicted as an ensemble technique using weighted voting of the local models. This automated classification aligns the model closely with the behavior of the material, offering high-confidence results with minimal manual input. Beyond classification, the method improves parameter estimation by directly predicting model parameters, streamlining initialization, and minimizing iterative fitting. The results demonstrate high accuracy for the Mooney-Rivlin and Neo-Hookean models, with satisfactory performance for the first-order Ogden model. However, refinement is necessary for higher-order Ogden models due to their more intricate parametric interactions. Future work aims to expand training data and enhance network architecture, particularly to address these complexities in second- and third-order Ogden models.

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Nomenclature

1D CNN 1 dimensional Convolution Neural Network

AI Artificial Intelligent

CNN Convolution Neural Network

FEA Finite Element Analysis

GRNN General Regression Neural Network

MAE Mean Absolute Error

MSE Mean Squared Error

RMSE Root Mean Squared Error

1. Chapter

Introduction

Hyperelastic materials, such as rubber, exhibit a significantly nonlinear stress-strain relationship, posing considerable challenges in their precise modeling. Understanding the mechanical response of hyperelastic materials is crucial in engineering simulations and material science, where accurate predictions of material behavior under various conditions are imperative. These materials possess advantageous properties such as high elasticity, shock resistance, and wearability, which allows them to undergo large deformations, making them suitable for applications in fields such as biomedical engineering and aerospace engineering [1, 2, 3]. Common examples include rubbers, gels, and soft tissues (arteries, muscles, and skin), all of which demonstrate hyperelastic behavior.

Traditionally, determining an appropriate material model and its corresponding parameters has required extensive manual adjustments, a method that is labor intensive and susceptible to user bias. Common models for characterizing hyperelastic behavior include the Mooney-Rivlin, Neo-Hookean, and Ogden models. The Neo-Hookean and Mooney-Rivlin models are based on invariants of the left Cauchy-Green deformation tensor, while the Ogden model uses principal stretch ratios, offering more flexibility in capturing complex deformation behaviors at high strains [4]. The practical applications and theoretical background of these models have been widely explored in the context of finite element analysis, with numerous studies detailing their implementation and performance under a variety of deformation conditions [5].

To address these challenges, optimization-based approaches have been developed to streamline parameter estimation in hyperelastic modeling. For example, the COMPARE system, introduced by Gendy and Saleeb [6], uses a systematic optimization process to determine parameters by minimizing the

error between the experimental and predicted stress-strain data. Such methods provide structured frameworks for parameter fitting but can be computationally intensive and often require a high level of expertise to set up effectively. Similarly, Bedalov et al. [7] proposed an adaptive optimization method that iteratively adjusts model parameters to enhance predictive precision, further emphasizing the role of optimization techniques in improving hyperelastic material modeling.

Machine learning techniques are now increasingly applied in the field of material modeling, offering promising avenues for automating parameter estimation in complex material models. Various studies have explored AI-driven approaches to predict stress-strain responses in hyperelastic materials, with methods such as regression models that successfully characterize the mechanical properties of elastomeric materials [8]. Research such as Hou et al. [9] highlights the utility of machine learning to capture the complex, nonlinear properties of rubber-like materials, underscoring the relevance of AI techniques in this domain. These strategies help reduce the reliance on labor-intensive experimental adjustments and improve prediction consistency. Furthermore, neural networks have emerged as efficient alternatives to traditional constitutive models, capturing nonlinear behaviors effectively in finite element analysis (FEA) applications, as demonstrated in recent research [10]. Similarly, recent work on neural network-based constitutive modeling has shown potential to capture the nonlinear response of polymeric materials under complex loading conditions by leveraging molecular dynamics simulations, highlighting a data-driven approach to hyperelastic modeling [11]. Other studies, such as those of Li et al. [12], have further extended machine learning applications by optimizing hyperelastic parameter identification for specific biological tissues, demonstrating the flexibility and precision of AI-driven models in biomechanics.

Shen et al. [13] utilized a feedforward backpropagation neural network to approximate the strain energy function for rubber materials, bypassing the need for a predefined constitutive form. This data-driven approach, implemented in ABAQUS, demonstrated stable and reliable performance, providing an efficient way to model complex material behaviors without extensive manual parameter adjustments. Chen et al. [14] employed a General Regression Neural Net-

work (GRNN) to capture complex viscohyperelastic behaviors of rubber-like materials, facilitating the estimation of parameters in a way that effectively complements traditional optimization techniques. This approach highlights how neural networks can be integrated to streamline parameter identification in hyperelastic materials, addressing some of the limitations of conventional optimization.

In this context, our study uses a one-dimensional convolutional neural network (1D CNN) to automate model classification and parameter regression for hyperelastic materials, tailoring models to specific measurement types, such as biaxial tension, planar compression, and simple shear. Additionally, studies in related fields have demonstrated the effectiveness of ensemble learning approaches for increasing the accuracy and reliability of model predictions by combining outputs from multiple networks [15]. Inspired by these findings, our study incorporates a weighted ensemble of networks to refine the final predictions further, aiming to improve prediction consistency across varying test conditions.

Our proposed AI-driven methodology uses the 1D CNN architecture to classify hyperelastic models and perform parameter regression with high precision. By training individual networks for each measurement type, we ensure a customized approach to model selection, enhancing the robustness and accuracy of the results. Each measurement type receives a distinct model trained to classify the optimal hyperelastic material model and estimate its corresponding parameters. A weighted ensemble of networks further refines the final predictions, improving the consistency of the results under varying test conditions. Through this methodology, our aim was to reduce manual intervention, expedite the modeling process, and improve the accuracy of predictions for widely used models, such as the Mooney-Rivlin and Neo-Hookean models, while acknowledging the challenges associated with higher-order Ogden models.

The contributions of this study are as follows:

- Development of an AI-driven methodology to automate the selection and parameter estimation of hyperelastic material models.
- Utilization of a one-dimensional convolutional neural network (1D CNN) for both classification of hyperelastic models and parameter regression.

- Implementation of customized neural networks for each type of measurement, enhancing the reliability of model selection.
- Introduction of a weighted ensemble approach to improve the robustness and accuracy of the final predictions.
- Demonstration of high accuracy in predicting the parameters of the Mooney-Rivlin and Neo-Hookean models.
- Identification of challenges related to higher-order Ogden models, paving the way for future research improvements.

Further sections will thoroughly detail our method, the testing it underwent, and the significant findings that contribute to the field of hyperelastic material modeling. The insights gained in this research establish a foundation for future studies in this domain.

The structure of this document is organized as follows. Chapter 2 provides a detailed explanation of the AI-based methodology for hyperelastic model identification, including the architecture and training process of the 1D CNN. In Chapter 3, we present the experimental results, covering model performance, parameter estimation accuracy, and robustness analyses. Chapter 4 provides concluding remarks, summarizing the key findings of the study and outlining potential directions for future research. Finally, the bibliography lists the sources referenced throughout the study.

2. Chapter

Methodology

This methodology outlines an automated classification and parameter estimation process for hyperelastic material models, leveraging convolutional neural networks (CNNs) and a structured voting system. The process is designed to address the varying hyperelastic behaviors represented in the stress-strain data, thereby ensuring reliable model selection and accurate parameter estimation.

2.1 Classification and Voting System Formalization

The classification task aims to assign each hyperelastic material data sample (X) to one of three predefined material models: Neo-Hookean, Mooney-Rivlin, or Ogden. Each sample includes stress-strain measurements from different test types, and the CNN models analyze these measurements to identify the appropriate hyperelastic model.

Input Space, X : Each input X represents data from separate test types:

- n is the number of samples,
- m represents the features per sample (e.g., stress-strain values for biaxial tension, uniaxial compression).

For each type of test, the input data subset $X_{\text{test_type}}$ is standardized and interpolated, resulting in:

$$X = \{X_{\text{biax_tension}}, X_{\text{planar_compression}}, \dots, X_{\text{simple_shear}}\}. \quad (2.1)$$

Output Space, Y : The output Y represents the class label for the chosen hyperelastic model:

- $Y = 0$: Neo-Hookean,
- $Y = 1$: Mooney-Rivlin,
- $Y = 2$: Ogden.

Each CNN, trained on data from a specific test type, outputs a prediction, and a voting system aggregates these predictions across test types to obtain the final classification:

$$Y = \text{Voting}(f_{\text{biax_tension}}(X_{\text{biax_tension}}), \dots, f_{\text{simple_shear}}(X_{\text{simple_shear}})). \quad (2.2)$$

The voting system integrates predictions from six CNNs, each tailored to a specific test type, to enhance the robustness of the final model classification. The system employs three strategies:

1. Majority Vote: In this strategy, each CNN prediction is treated as a vote. The class receiving the highest number of votes across all CNNs is selected as the final model:

$$Y = \arg \max_{y \in \{0,1,2\}} \sum_{i=1}^6 \delta(V_i = y), \quad (2.3)$$

where $\delta(\cdot)$ is 1 if $V_i = y$ (that is, the vote of CNN i -th matches the class y) and 0 otherwise. This approach prioritizes the majority decision without considering the confidence level of individual predictions.

2. Probability-Based Voting: Each CNN provides a probability distribution across the three classes. The final class Y is determined by averaging these probabilities across CNNs:

$$Y = \arg \max_{y \in \{0,1,2\}} \frac{1}{6} \sum_{i=1}^6 P_i(y), \quad (2.4)$$

where $P_i(y)$ represents the probability assigned by the i -th CNN to class y . This method incorporates the confidence level of each CNN in its predictions.

3. Weighted Probability Voting: Here, CNN predictions are weighted based on each accuracy of the network for its respective test type. Let A_i denote the accuracy of CNN i . The final class Y is obtained by weighting each

CNN's probabilities by its accuracy:

$$Y = \arg \max_{y \in \{0,1,2\}} \frac{\sum_{i=1}^6 A_i \cdot P_i(y)}{\sum_{i=1}^6 A_i}. \quad (2.5)$$

This approach gives more influence to CNNs with higher accuracy, enhancing reliability.

Figures 2.1, 2.2, and 2.3 illustrate the three voting structures, which collectively increase the robustness of the classification by balancing confidence and accuracy.

Majority Voting System

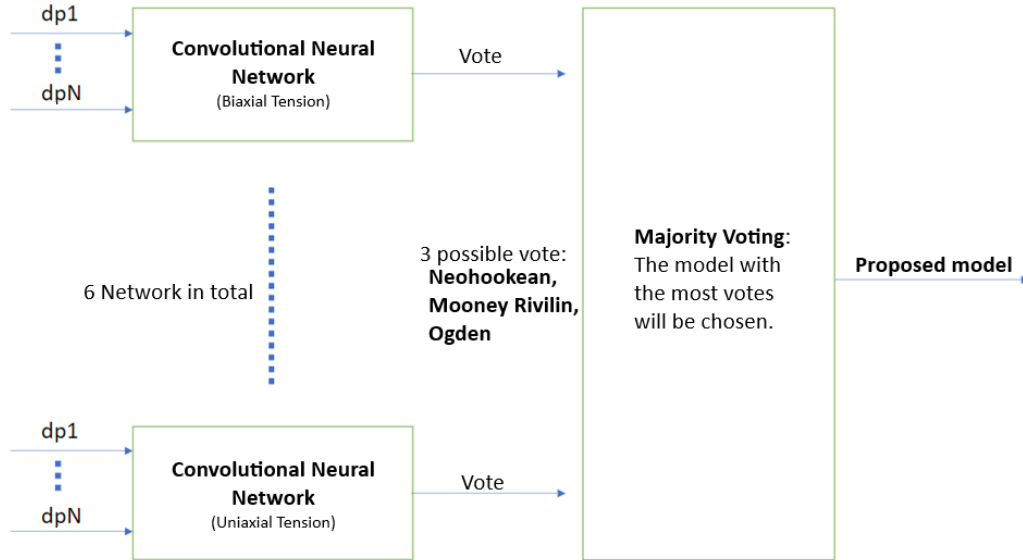


Figure 2.1: Majority Voting: Each CNN votes, and the model with the most votes is selected.

Probability-Based Voting System

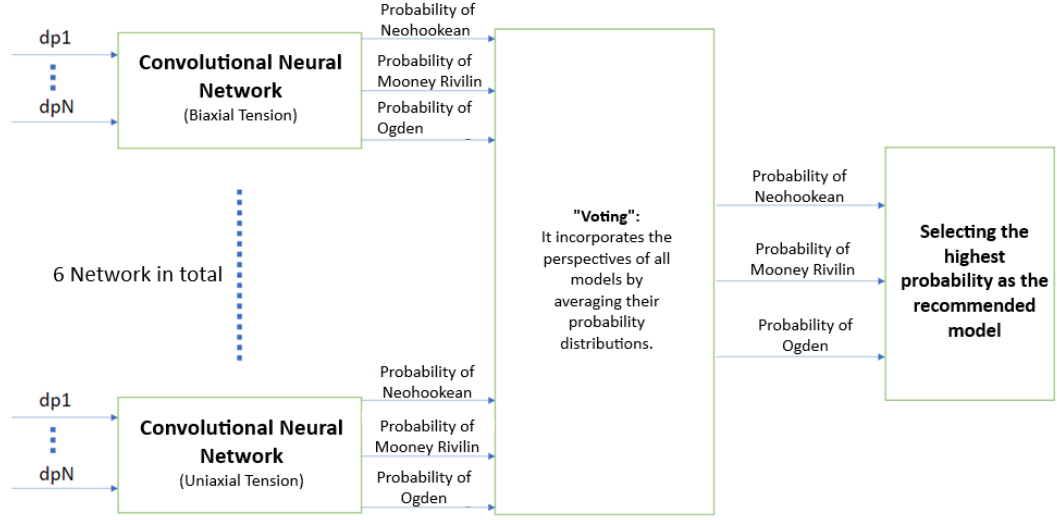


Figure 2.2: Probability-Based Voting: Each CNN assigns probabilities to models; the highest average probability determines the model.

Weighted Probability Voting System

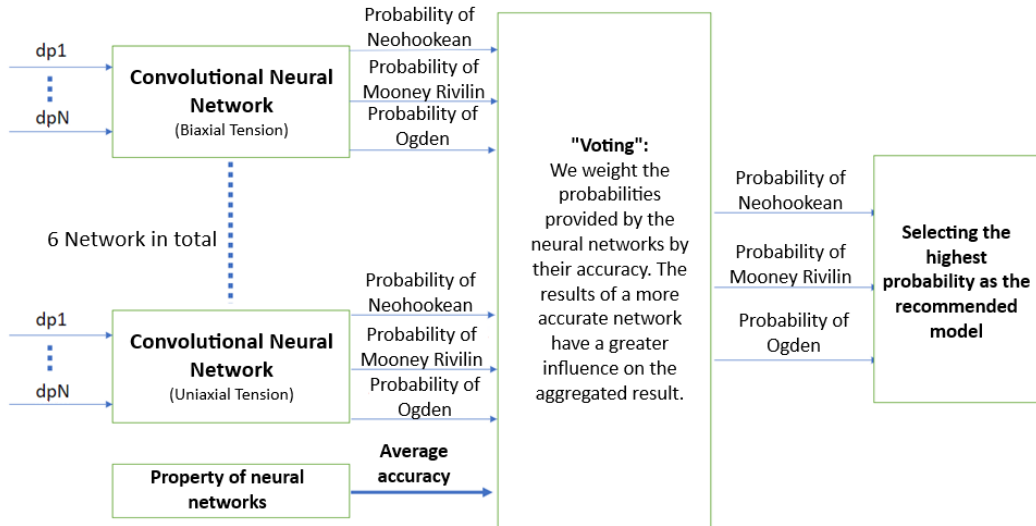


Figure 2.3: Weighted Probability Voting: CNN probabilities are weighted by accuracy, giving more influence to reliable models.

Each CNN utilizes a 1D convolutional layer, transforming the input $X \in$

$\mathbb{R}^{n \times m}$ through convolution filters that capture temporal dependencies across features. The output of the convolution layer Y for a kernel W of size k is given by:

$$Y_{i,j} = \sigma \left(\sum_{l=0}^{k-1} X_{i,j+l} \cdot W_l + b \right), \quad (2.6)$$

where b is the bias term, and σ represents an activation function such as ReLU. The model continues with additional convolution, pooling, and fully connected layers.

Each CNN is trained to minimize the mean squared error (MSE) between the predicted and true labels:

$$L(Y, \hat{Y}) = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2. \quad (2.7)$$

The optimal parameters θ^* of each CNN are obtained by minimizing this loss:

$$\theta^* = \arg \min_{\theta} \frac{1}{n} \sum_{i=1}^n L(Y_i, f(X_i; \theta)). \quad (2.8)$$

2.2 Regression Task for Parameter Estimation

Upon identifying the appropriate hyperelastic model, the next step is to accurately estimate the parameters of that model. Given the predicted model Y and the corresponding input X , the regression task maps X to a parameter vector $P \in \mathbb{R}^l$, where l represents the number of parameters specific to each model:

- $P_{\text{Neo-Hookean}} = \{\mu\},$
- $P_{\text{Mooney-Rivlin}} = \{C_{10}, C_{01}\},$
- $P_{\text{Ogden}} = \{\mu_1, \alpha_1, \mu_2, \alpha_2, \mu_3, \alpha_3\}.$

A dedicated 1D CNN architecture is used for parameter regression, and the model minimizes the MSE between the predicted parameters $\hat{P}$ and the true parameters P :

$$L(P, \hat{P}) = \frac{1}{n} \sum_{i=1}^n (P_i - \hat{P}_i)^2. \quad (2.9)$$

This architecture of the network closely follows that of CNNs of classification, but is adapted for regression.

Parameter estimation models are trained using the Adam optimizer with MSE loss. Techniques like early stop and batch normalization are applied to improve training stability and prevent overfitting. The precision of these models is evaluated using metrics such as the mean absolute error (MAE) and the R^2 score.

2.3 Model Evaluation and Validation

The performance of this methodology is evaluated by assessing both the classification accuracy and the regression precision. For classification, the accuracy, precision, recall, and F1 scores are calculated. For parameter estimation, metrics such as MAE and R^2 are used to measure the fit between the predicted and true parameter values.

To validate the robustness of the model, k-fold cross-validation is applied, dividing the data set into k subsets and cycling each as a validation set. This process provides an averaged assessment across all the folds, ensuring reliability.

Ensemble methods are also explored to further enhance robustness. By combining outputs from multiple models trained on different data subsets, this approach reduces variance and improves overall performance.

3. Chapter

Results

3.1 Experimental Data Overview

Before presenting the results, it is essential to understand the nature of the experimental data on which the model is trained. This section provides an overview of the strain-stress behavior for each test type across three hyperelastic models: Neo-Hookean, Mooney-Rivlin, and Ogden. By examining these strain-stress curves, we can observe distinct differences in material responses that serve as a basis for training and evaluating the models.

In the planar compression test, the Ogden model shows a significant decrease in stress with increasing strain, indicating a high sensitivity to compressive forces. The Mooney-Rivlin model exhibits a more moderate decline, while the Neo-Hookean model shows the least variation, suggesting it is less responsive to compression changes.

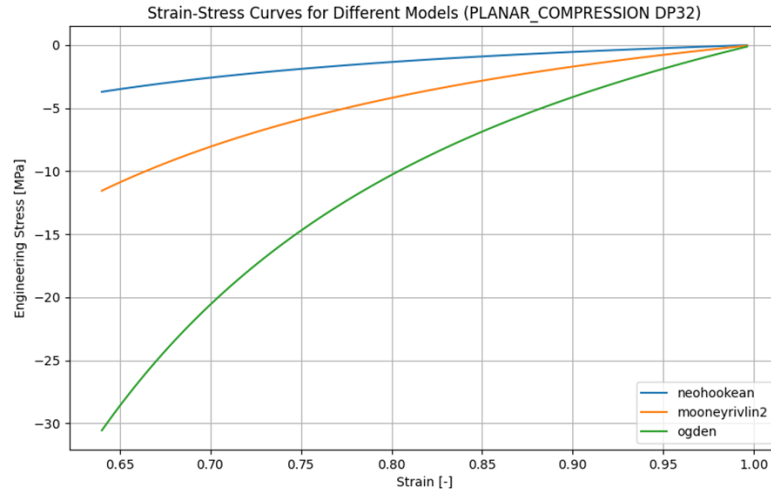


Figure 3.1: Strain-Stress Curves for Different Models (Planar Compression DP32).

Under biaxial tension, the stress of the Ogden model response increases sharply as strain increases, diverging significantly from the other models. Neo-Hookean and Mooney-Rivlin models exhibit more stable and moderate stress levels. This steep response from the Ogden model makes it highly distinguishable in biaxial tension conditions.

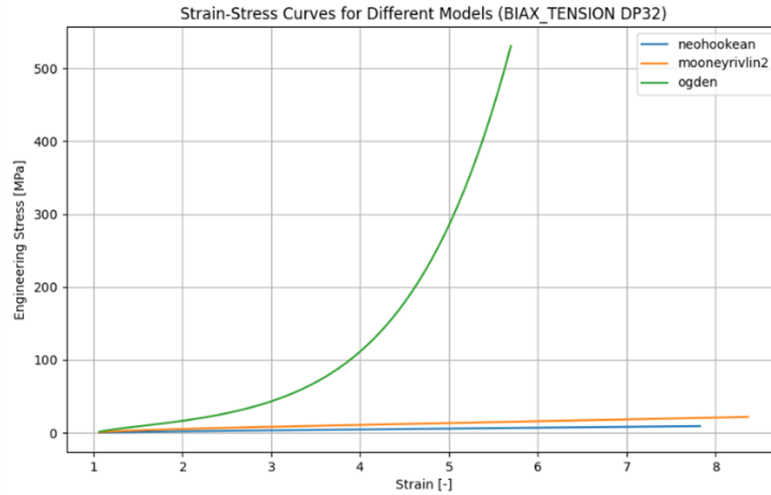


Figure 3.2: Strain-Stress Curves for Different Models (Biaxial Tension DP32).

In the planar tension test, the Ogden model again shows the highest stress response across the strain range, whereas the Neo-Hookean model exhibits the

lowest response. The Mooney-Rivlin model lies between the two, providing a moderate response to planar tension. This gradient in responses helps to differentiate models in tension scenarios.

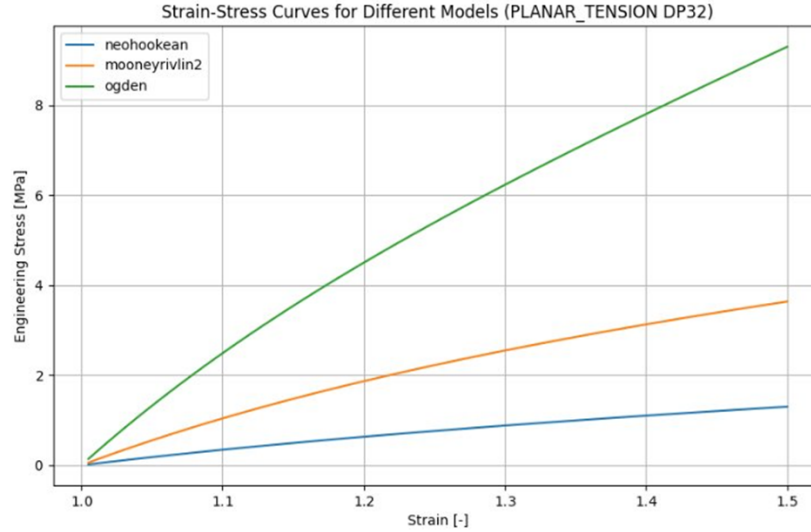


Figure 3.3: Strain-Stress Curves for Different Models (Planar Tension DP32).

For the simple shear test, each model shows a linear increase in stress as strain increases, but with different slopes. The Ogden model has the steepest slope, followed by the Mooney-Rivlin model, while the Neo-Hookean model has the lowest slope. These differences in shear response provide a clear distinction between the models in shear conditions.

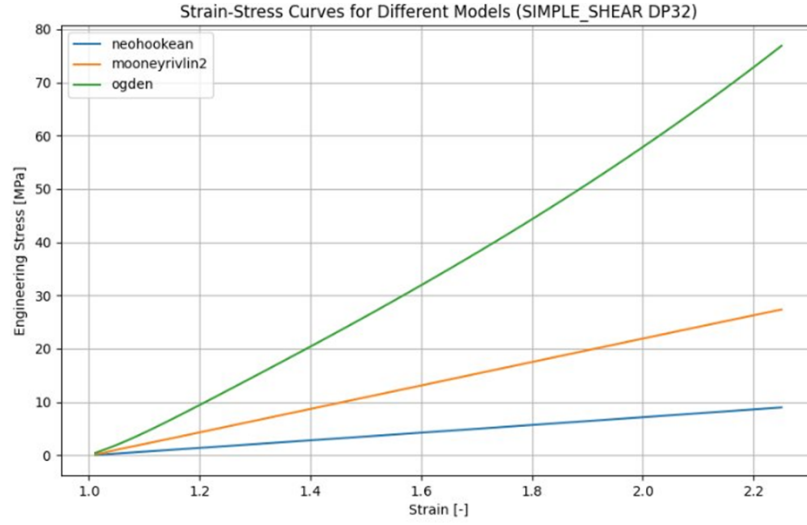


Figure 3.4: Strain-Stress Curves for Different Models (Simple Shear DP32).

In uniaxial compression, similar to planar compression, the Ogden model shows a pronounced stress decrease as the strain increases, indicating a higher sensitivity to compression. The Mooney-Rivlin and Neo-Hookean models exhibit less variation, with the Neo-Hookean model showing the smallest stress response.

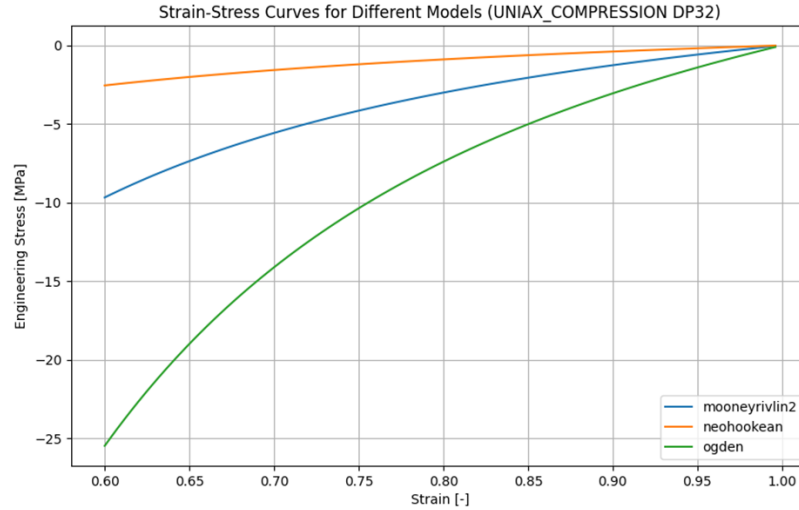


Figure 3.5: Strain-Stress Curves for Different Models (Uniaxial Compression DP32).

Under uniaxial tension, the Ogden model shows a rapid increase in stress at

higher strains, contrasting sharply with the Neo-Hookean and Mooney-Rivlin models, which show more stable and lower stress responses. This marked difference in tension response is especially pronounced in high-strain regions, aiding in model classification.

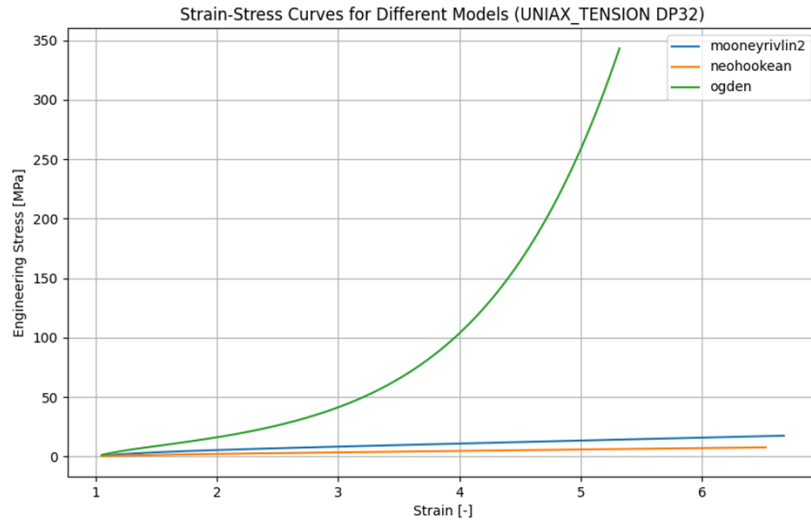


Figure 3.6: Strain-Stress Curves for Different Models (Uniaxial Tension DP32).

Having reviewed the experimental strain-stress data, we now turn our attention to the classification results. Table 3.1 summarizes the test accuracies for each model across different measurement types, highlighting the performance differences based on the specific test conditions.

Test type	Test accuracy, %
Biax tension	95.00
Planar compression	90.00
Planar tension	93.75
Simple shear	98.73
Uniax compression	91.25
Uniax tension	84.04

Table 3.1: Test accuracies of the models for different measurement types

The classification results show that the AI-driven methodology successfully identifies the correct hyperelastic models in the majority of test cases.

Results

The confusion matrices for each test type illustrate the classification performance in the Neo-Hookean, Mooney-Rivlin, and Ogden models. In general, most classifications are accurate, with distinct patterns of misclassification emerging in certain tests.

For most test types, the Ogden model shows strong classification accuracy, particularly evident in the Biaxial Tension, Planar Compression, and Uniaxial Tension tests, where it is consistently identified with minimal misclassifications. However, some confusion is observed between the Mooney-Rivlin and Neo-Hookean models, especially in simple shear and uniaxial compression tests, suggesting that these two models exhibit similar characteristics in these scenarios.

In tests such as Planar Tension and Biaxial Tension, a few misclassifications between Mooney-Rivlin and Ogden occur, highlighting challenges in distinguishing between these models under certain deformation types. However, the Neo-Hookean model is generally well distinguished from Ogden in most test types, as evidenced by the low misclassification rates between them.

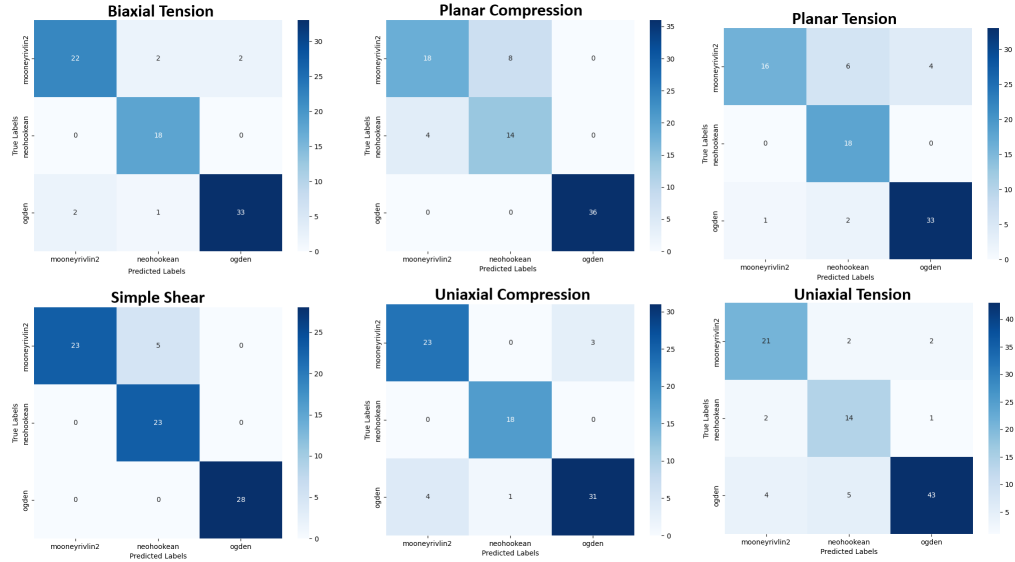


Figure 3.7: Confusion matrices for model classification across different test types, illustrating classification performance for the Mooney-Rivlin, Neo-Hookean, and Ogden models.

3.1.1 Classification Voting Systems and Their Accuracy

To increase the reliability of the prediction of the model, three different voting systems were used. These systems combine predictions from multiple models to make more accurate classification decisions about the material model corresponding to a given dataset. The voting systems and their accuracy are presented in Table 3.2.

Voting System	Accuracy (%)
Majority Voting	92.25
Probability-Based	97.00
Weighted Probability-Based	97.25

Table 3.2: Accuracy of different classification voting systems.

Each voting system is described as follows:

- **Majority Voting:** In this method, the class with the highest number of predictions among the models is selected. This simple approach achieves an accuracy of 92.25%.
- **Probability-Based Voting:** This approach uses the probability scores assigned by each model. The class with the highest combined probability is selected, resulting in an improved accuracy of 97%.
- **Weighted Probability-Based Voting:** In this system, models are weighted according to their individual accuracies, providing greater influence to more accurate models. This system yields the highest accuracy, achieving 97.25%.

These voting systems demonstrate the benefit of ensemble methods in improving model prediction accuracy, with the weighted probability-based voting system providing the most reliable results.

As shown in Table 3.1, each test type has a specific accuracy rate when evaluated individually, with values ranging from 84.04% to 98.73%. Although these accuracies indicate the performance of the model for specific types of measurement, the ensemble voting systems provide a more robust and generalized

prediction accuracy across all test types. By combining predictions through the voting systems, as shown in Table 3.2, the overall accuracy improves significantly. This approach not only improves individual prediction reliability but also ensures higher confidence in model classification when considering the combined data set of all test types.

3.2 Neo-Hookean Regression Model Results

The performance of the regression model for Neo-Hookean model was evaluated across multiple test types. In each test type, the predicted values are compared with the actual (real) values to assess the model's accuracy in parameter estimation.

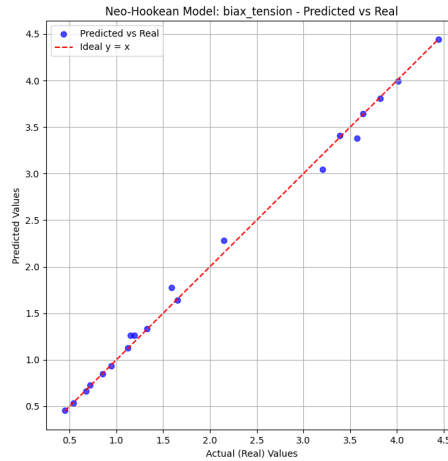


Figure 3.8: Neo-Hookean Model: Real vs. Predicted values for Biaxial Tension.

The results of the biaxial tension test show a close alignment between the real and predicted values, demonstrating the model's ability to accurately estimate parameters under this test type. Minor discrepancies observed in a few instances suggest areas for potential refinement, but the overall performance is satisfactory.

Results

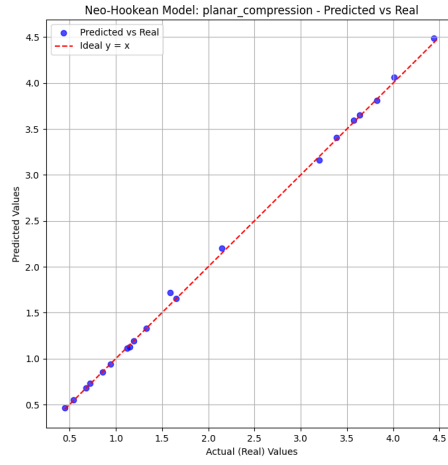


Figure 3.9: Neo-Hookean Model: Real vs. Predicted values for Planar Compression.

The planar compression test results indicate a similar level of precision, with slight deviations in certain regions. These deviations could reflect complex behavior under compression, which the model may not fully capture, suggesting room for further optimization.

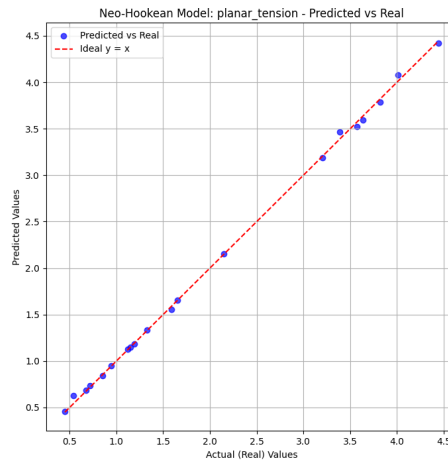


Figure 3.10: Neo-Hookean Model: Real vs. Predicted values for Planar Tension.

For the planar tension test, the Neo-Hookean model demonstrates good

Results

alignment between the real and predicted values. Although slight deviations occur in areas with rapid fluctuations, the model accurately captures the general trend, supporting its reliability under tension.

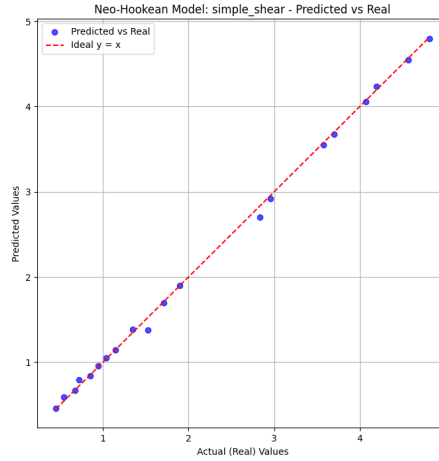


Figure 3.11: Neo-Hookean Model: Real vs. Predicted values for Simple Shear.

The simple shear test yields a strong match between the predicted and real values, showcasing the capacity of the model for accurate parameter estimation in shear conditions. The Neo-Hookean model performs particularly well under this type, with minimal discrepancies.

Results

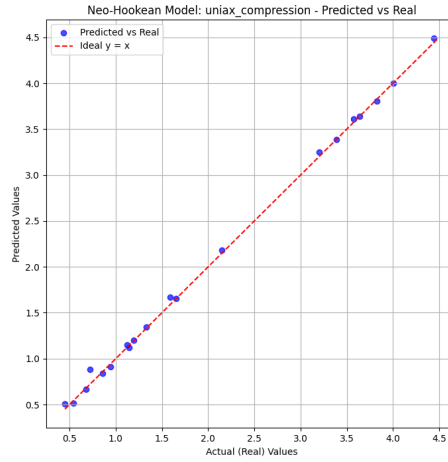


Figure 3.12: Neo-Hookean Model: Real vs. Predicted values for Uniaxial Compression.

The results of the uniaxial compression test demonstrate a close alignment between the real and predicted values. However, minor variances suggest areas where the model could benefit from further calibration, particularly in high-stress regions.

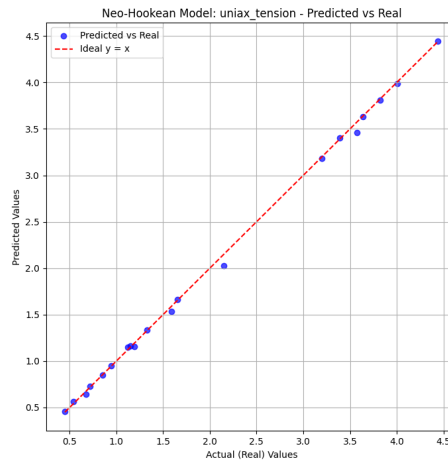


Figure 3.13: Neo-Hookean Model: Real vs. Predicted values for Uniaxial Tension.

For the uniaxial tension test, the model accurately tracks most real values,

though some underestimation occurs in high-stress points. Despite this, the overall alignment is robust, validating the efficacy of the Neo-Hookean model under tensile conditions.

Overall, the Neo-Hookean model performs consistently across different test types. Although some minor discrepancies are observed in high-stress regions and in tests with complex material behavior, the model demonstrates high accuracy in parameter estimation, supporting its suitability for various stress conditions.

3.3 Mooney-Rivlin Regression Model Results

The regression model for Mooney-Rivlin demonstrated strong classification accuracy, particularly in distinguishing between different test types. In each type of test, the parameters C_{10} and C_{01} were predicted and compared to their real values, highlighting the model's parameter estimation capabilities. The results for each type of test are discussed in the following subsections.

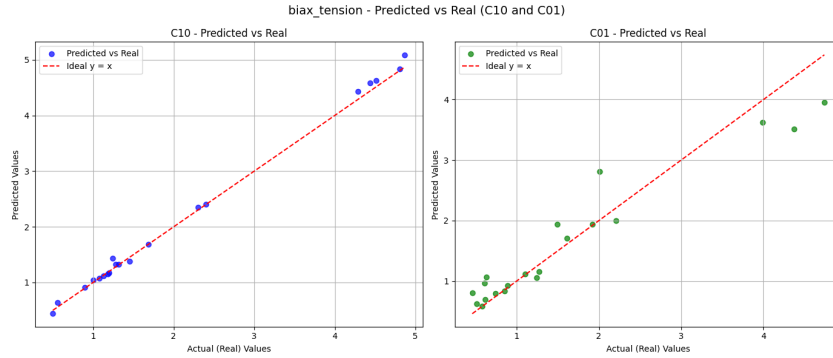


Figure 3.14: Mooney-Rivlin Model: Real vs. Predicted values for Biaxial Tension.

In the biaxial tension test, the Mooney-Rivlin model shows a close alignment between the real and predicted values for both C_{10} and C_{01} . The model effectively captures the parameter fluctuations, with only minor deviations observed in certain high-stress regions, suggesting that the model predictions are reliable under biaxial tension conditions.

Results

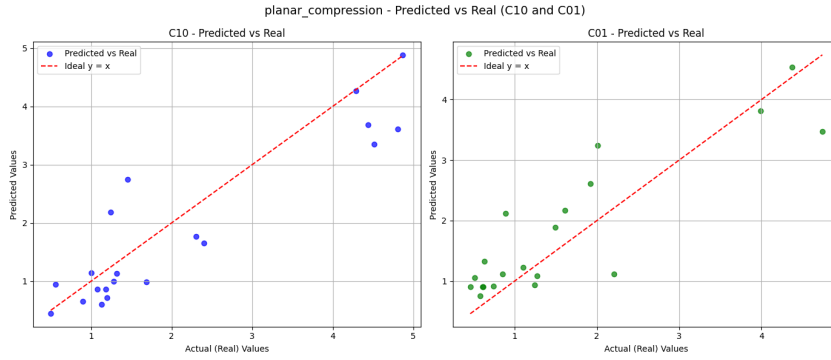


Figure 3.15: Mooney-Rivlin Model: Real vs. Predicted values for Planar Compression.

The planar compression test results demonstrate a strong correspondence between real and predicted values for both parameters. However, slight discrepancies are observed during parameter peaks, which may be attributed to complex material responses under compression. Despite this, the model maintains high accuracy, indicating its robustness in planar compression scenarios.

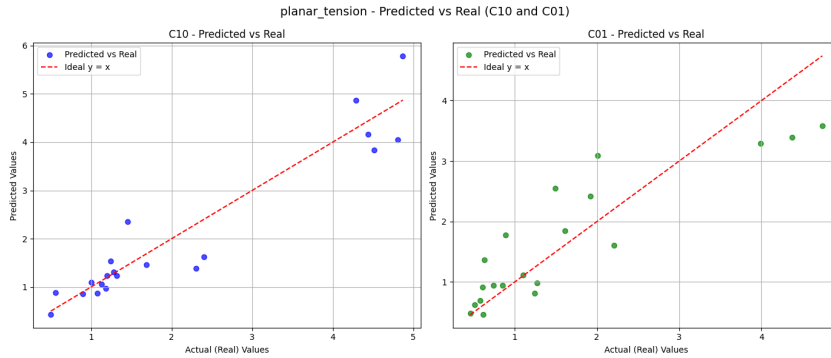


Figure 3.16: Mooney-Rivlin Model: Real vs. Predicted values for Planar Tension.

In the planar tension test, the Mooney-Rivlin model accurately tracks both the parameters C_{10} and C_{01} . Minor misalignments are observed at certain points, but do not significantly impact the overall trend. This high level of agreement reinforces the model's suitability for capturing tension-related behaviors.

Results

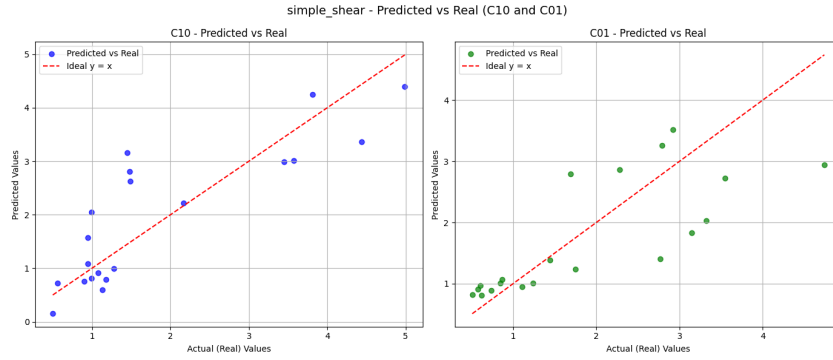


Figure 3.17: Mooney-Rivlin Model: Real vs. Predicted values for Simple Shear.

The simple shear test yields closely aligned predictions, with both parameters C_{10} and C_{01} showing minimal deviation from the real values. This result highlights the Mooney-Rivlin model's accuracy in predicting parameter values under shear conditions, with only slight variations that do not affect the model's reliability.

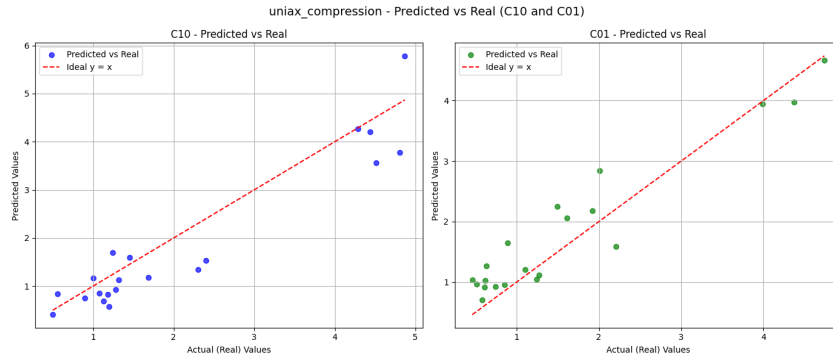


Figure 3.18: Mooney-Rivlin Model: Real vs. Predicted values for Uniaxial Compression.

For uniaxial compression, the model provides accurate estimates for both parameters. Although small discrepancies appear at higher values, the predicted curves maintain close alignment with the real data, confirming the efficacy of the model in compressive scenarios.

Results

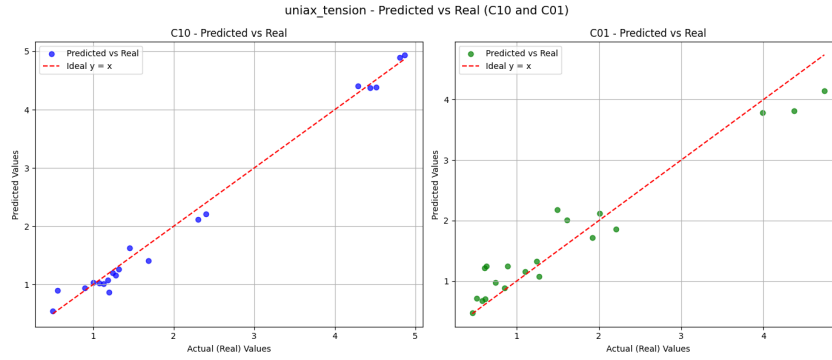


Figure 3.19: Mooney-Rivlin Model: Real vs. Predicted values for Uniaxial Tension.

In the uniaxial tension test, the Mooney-Rivlin model accurately follows the trends of the real values for both parameters. Slight underestimations occur in certain high-stress regions, but overall, the predictions align well with the actual values, supporting the model's applicability under tensile conditions.

Across the various test types, the Mooney-Rivlin model consistently demonstrates accurate parameter predictions, particularly for C_{10} and C_{01} . Although some minor discrepancies are observed, especially in high-stress regions, the general alignment of the model with real values underscores its reliability in representing complex material behaviors across diverse loading conditions.

3.4 Ogden Regression Model Results

The Ogden model, particularly in its higher-order forms, presents challenges for parameter estimation due to its complexity. However, results for the first-order Ogden model show reasonable accuracy, while higher-order models highlight areas for improvement.

3.4.1 First-Order Ogden Regression Model

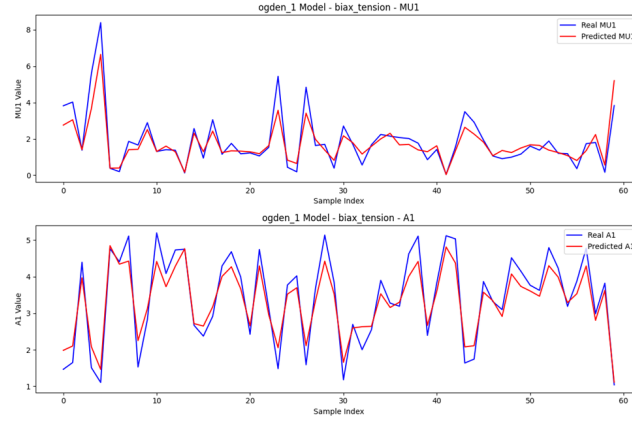


Figure 3.20: Ogden Model (First-Order): Real vs. Predicted values for μ_1 and A_1 under Biaxial Tension.

The first-order Ogden regression model shows a strong ability to capture the trend in the biaxial tension test, particularly for parameter μ_1 . However, some deviations in A_1 indicate areas where the model could be fine-tuned to improve accuracy.

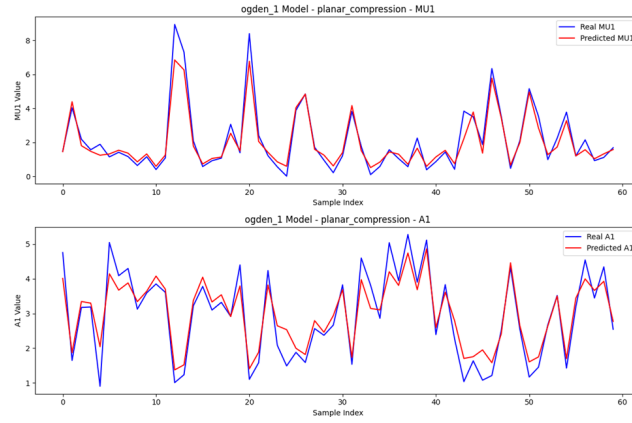


Figure 3.21: Ogden Model (First-Order): Real vs. Predicted values for μ_1 and A_1 under Planar Compression.

In the planar compression test, the first-order Ogden model maintains a close alignment between real and predicted values for both μ_1 and A_1 , though slight fluctuations suggest possible improvements in capturing finer variations in material response.

Results

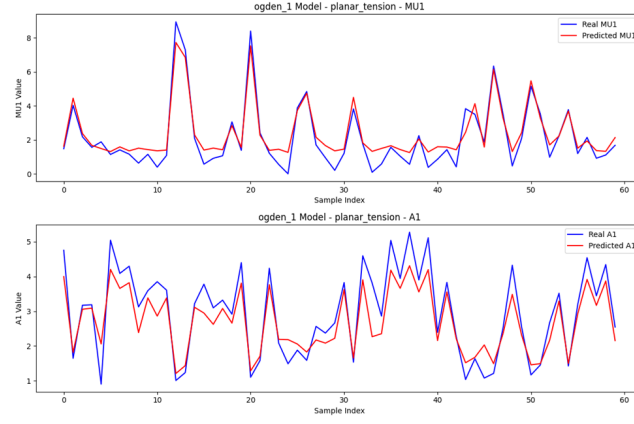


Figure 3.22: Ogden Model (First-Order): Real vs. Predicted values for μ_1 and A_1 under Planar Tension.

For planar tension, the predicted values closely match the actual measurements, indicating that the first-order Ogden model performs well under this test type. However, some minor misalignments are present, particularly at higher values of μ_1 .

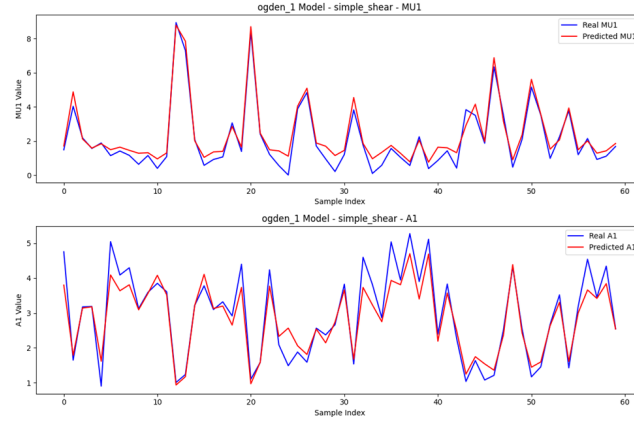


Figure 3.23: Ogden Model (First-Order): Real vs. Predicted values for μ_1 and A_1 under Simple Shear.

In the simple shear test, the first-order Ogden model achieves good accuracy in predicting both parameters, with only minor deviations, indicating reliable performance under this type of deformation.

Results

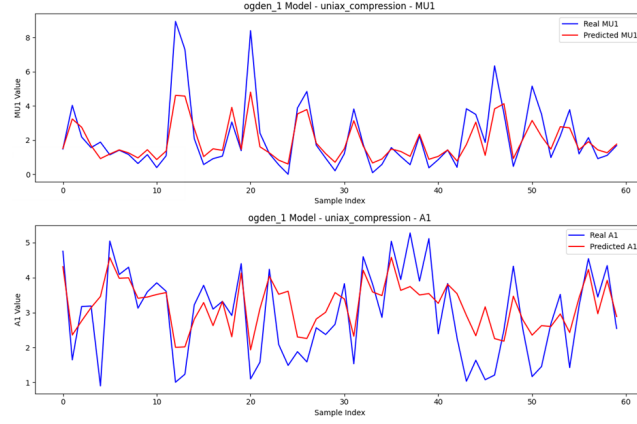


Figure 3.24: Ogden Model (First-Order): Real vs. Predicted values for μ_1 and A_1 under Uniaxial Compression.

The results of the uniaxial compression test reveal a close match between the predicted and actual values, with only minor discrepancies in the parameters A_1 . This indicates that the model is well-suited to capture the behavior in compression scenarios.

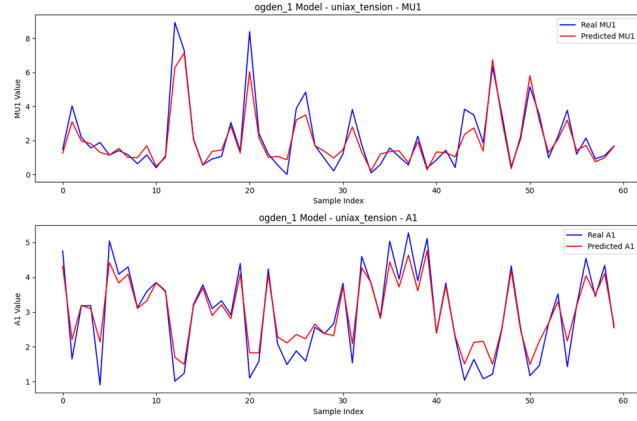


Figure 3.25: Ogden Model (First-Order): Real vs. Predicted values for μ_1 and A_1 under Uniaxial Tension.

For uniaxial tension, the first-order Ogden model provides satisfactory predictions for both μ_1 and A_1 , although some discrepancies arise in specific ranges. These results suggest potential areas for refinement to further improve accuracy.

The first-order Ogden model provides reasonable accuracy across multiple

test types, capturing general trends in the data for key parameters such as μ_1 and A_1 . The model performs particularly well in tests like simple shear and uniaxial compression, where the predicted values align closely with the real measurements. However, certain discrepancies are observed in tests with higher deformation complexities, such as biaxial and planar tension, where slight deviations highlight areas for potential improvement.

Overall, the first-order Ogden model demonstrates a strong capability for parameter prediction in simpler test cases, but further refinement and adjustments may be necessary to enhance accuracy in more complex deformations.

3.4.2 Second-Order Ogden Regression Model

The second-order Ogden regression model exhibited similar prediction challenges as observed in higher-order models. In this case, the predicted parameters struggled to align with the real values, as demonstrated by the results of the biaxial tension test shown in Figure 3.26. This trend of misalignment was consistent across other test types, indicating limitations in the ability of model to learn from the provided dataset.

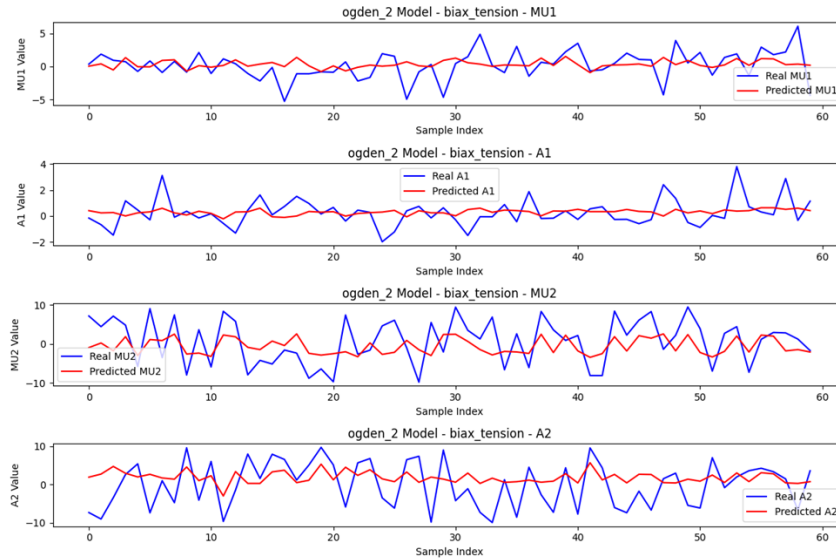


Figure 3.26: Second-Order Ogden Model: Real vs. Predicted parameter values for Biaxial Tension.

3.5 Third-Order Ogden Regression Model Results

The third-order Ogden regression model presents significant challenges in parameter estimation. The complexity and high degree of non-linearity associated with this model make it difficult to accurately capture the behavior of the material with the given data set. The estimated parameters show considerable deviations from the actual values, indicating that the model struggles to generalize effectively under current data constraints.

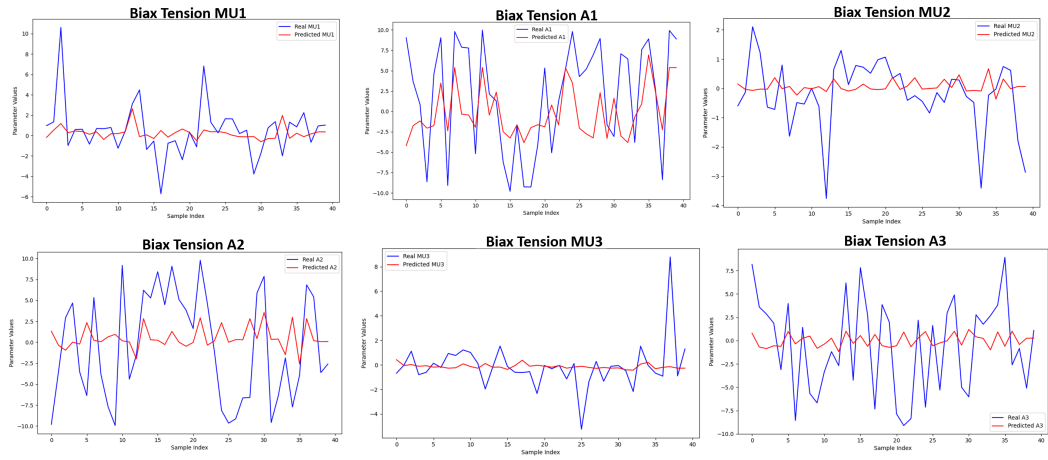


Figure 3.27: Third-Order Ogden Model: Real vs. Predicted values for parameters μ_1 , a_1 , μ_2 , a_2 , μ_3 , and a_3 under Biaxial Tension. In each plot, the blue lines represent the real values, while the red lines indicate the predicted values for each parameter.

The results of the biaxial tension test illustrate the difficulty in fitting the third-order Ogden model to the data. Although some general trends are followed, predictions often diverge significantly from the actual values, particularly in high-variance regions. This discrepancy suggests that the model fails to accurately capture the characteristics of the underlying material in the various parameters.

Overall, the results of the third-order Ogden model indicate substantial room for improvement. The observed performance issues may arise from both the complexity of the model and the limitations in the data set. Future work may involve exploring alternative modeling approaches or simplifying the model to achieve better predictive accuracy.

3.6 Parameter Estimation Results

In this section, we analyze the accuracy and reliability of parameter estimation for various hyperelastic material models using the evaluation metrics summarized in Tables 3.3, 3.4, and 3.5. The metrics include the mean squared error (MSE), root mean squared error (RMSE), mean absolute error (MAE), and R^2 score, each providing insight into the performance and suitability of each model for capturing the behavior of the materials.

As shown in Table 3.3, the Neo-Hookean and Mooney-Rivlin models demonstrate high accuracy in parameter estimation, particularly in terms of the R^2 score and low error values:

Metric	Neo-Hookean	Mooney-Rivlin	3rd Order Ogden
Mean Squared Error (MSE)	5.7483e-05	0.0521	19.5008
Root Mean Squared Error (RMSE)	0.0076	0.2242	3.9062
Mean Absolute Error (MAE)	0.0053	0.1480	3.1625
R^2 Score	0.999967	0.9741	-0.1969

Table 3.3: Model Evaluation Metrics for Different Hyperelastic Models

- Neo-Hookean Model:** The Neo-Hookean model achieves near-perfect predictions with an MSE of 5.7483e-05, an RMSE of 0.0076, and an R^2 score of 0.999967. These results suggest that the model is highly effective for simpler material responses, where fewer parameters are needed to capture the deformation behavior accurately.
- Mooney-Rivlin Model:** The Mooney-Rivlin model, though slightly more complex than the Neo-Hookean, also performs well, with an MSE of 0.0521, an RMSE of 0.2242, and an R^2 score of 0.9741. This model offers a balance between complexity and accuracy, making it suitable for materials requiring more detailed parameterization.

Table 3.4 presents the performance of the first-order Ogden model across different test types. The results indicate that the model is generally effective, but its performance varies depending on the test type:

Results

Test Type	Test Loss (MSE)	Mean Absolute Error (MAE)	R ² Score
Biaxial Tension	0.0790	0.1835	0.8916
Planar Compression	0.1791	0.3017	0.8181
Planar Tension	0.1023	0.2431	0.8976
Simple Shear	0.1306	0.2890	0.8688
Uniaxial Compression	0.4149	0.4642	0.5861
Uniaxial Tension	0.0989	0.2358	0.9032

Table 3.4: Model Evaluation Metrics for 1st Order Ogden Model Across Different Test Types

- The highest R^2 score (0.9032) is observed in the uniaxial tension test, where the model captures the material behavior with high accuracy. This is also reflected in the low MAE (0.2358) and MSE (0.0989), highlighting the model's suitability for this test type.
- In contrast, the uniaxial compression test exhibits the lowest R^2 score (0.5861) and the highest MSE (0.4149), indicating challenges in capturing the response of the material in compressive states.
- Overall, the first-order Ogden model demonstrates strong performance in tension-related tests (biaxial and uniaxial tension) with moderate success in compression-related tests. These results suggest that while the first-order Ogden model is a viable choice for parameter estimation, further tuning may be necessary for accurate predictions in complex deformation scenarios.

The second-order Ogden model results, shown in Table 3.5, reveal significant limitations in parameter estimation accuracy. The model consistently displays low or negative R^2 scores across all test types, indicating poor fit:

Results

Test Type	Test Loss (MSE)	Mean Absolute Error (MAE)	R ² Score
Biaxial Tension	0.9245	0.7710	0.0069
Planar Compression	0.9769	0.7759	-0.0439
Planar Tension	0.9617	0.7728	-0.0402
Simple Shear	0.9770	0.7841	-0.0947
Uniaxial Compression	0.9338	0.7365	-0.0155
Uniaxial Tension	1.0215	0.7874	-0.0067

Table 3.5: Model Evaluation Metrics for 2nd Order Ogden Model Across Different Test Types

- The model performs poorly in terms of both the MSE and the R^2 score in all types of tests, with particularly low R^2 values in the planar compression and simple shear tests. The negative R^2 scores suggest that the model's predictions deviate significantly from actual material behavior.
- The high MAE values, averaging around 0.77 across test types, indicate that the second-order Ogden model struggles to generalize effectively from the training data to unseen test data.
- This poor performance underscores the complexity and limitations of the second-order Ogden model, suggesting that its parameterization may be overly complex for the dataset used in this study, leading to overfitting or instability in parameter estimation.

As indicated in Table 3.3, the third-order Ogden model performs worse than the Neo-Hookean and Mooney-Rivlin models, with an MSE of 19.5008 and an R^2 score of -0.1969. These results highlight the model's struggles with parameter estimation:

- The high error values (MSE and RMSE) and negative R^2 score suggest that the model fails to capture the underlying material response accurately. The increased complexity associated with higher-order parameters appears to hinder its performance.
- The third-order Ogden model's inability to generalize effectively from the data may be attributed to the limited sample size or the model's sensitivity to parameter interactions, which complicates the estimation process.

4. Chapter

Conclusions

This study introduces an extensive AI-driven methodology aimed at automating the identification and parameter estimation of hyperelastic material models, with particular emphasis on the Mooney-Rivlin, Neo-Hookean, and Ogden models. Using a one-dimensional convolutional neural network (1D CNN), the study achieves an effective classification of hyperelastic materials and precise parameter prediction for more elementary models.

The methodology demonstrated high accuracy in identifying the appropriate material models; the classification aspect of the 1D CNN was successful across all models tested. For the Neo-Hookean and Mooney-Rivlin models, parameter estimation also showed strong performance, with predicted parameters closely matching actual values, validating the robustness of the 1D CNN for simpler hyperelastic models.

Challenges specifically arise in the parameter estimation of the higher-order Ogden models, particularly the second- and third-order variants. These models introduce complexities due to the phenomenon of parameter ambiguity, in which varying parameter sets yield comparable stress-strain responses. This phenomenon is depicted in Figure 4.1, which illustrates how the values of various parameters can result in stress-stretch curves almost indistinguishable under various loading scenarios, such as uniaxial tension. This ambiguity complicates the estimation process, as the model may converge on multiple sets of parameters that are functionally equivalent within the observed range. This convergence to different yet equally plausible parameter sets presents a substantial challenge for precise parameter estimation, as it becomes arduous to discern a unique parameter set that accurately characterizes the behavior of the material.

Conclusions

The findings of this study suggest that while the proposed AI-driven methodology is effective for classification across all hyperelastic models tested, additional research is required to improve the accuracy and reliability of parameter estimation for more complex models like Ogden-2 and Ogden-3. Future work will focus on optimizing the neural network architecture, expanding the diversity of the training dataset, and exploring methods to address parameter identifiability in complex hyperelastic models. This could involve introducing additional constraints or using a wider range of experimental data to better differentiate between parameter sets.

In general, this study represents a significant advancement in automating the modeling of hyperelastic materials, reducing the need for manual intervention, and providing a powerful tool for engineering simulations and material science applications. By continuing to refine and expand upon this approach, more complex materials and behaviors can be accurately modeled, further advancing the field of hyperelastic material modeling.

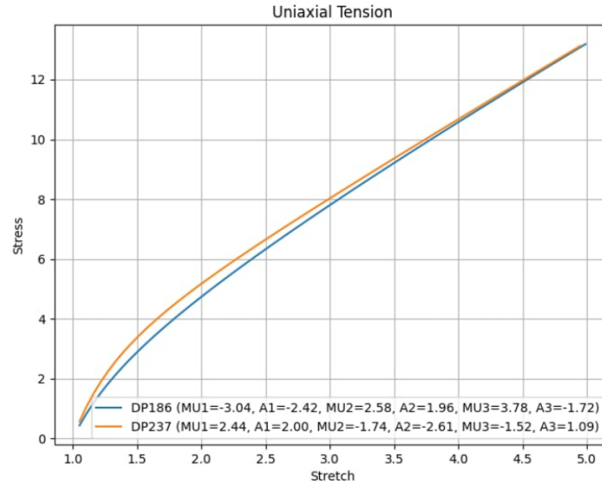


Figure 4.1: Comparison of third-order Ogden model stress-stretch curves for uniaxial tension with different parameter sets. The blue curve (DP186) and orange curve (DP237) have distinct parameter values but produce similar stress-stretch behavior, highlighting the parameter identifiability issue in higher-order Ogden models.

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