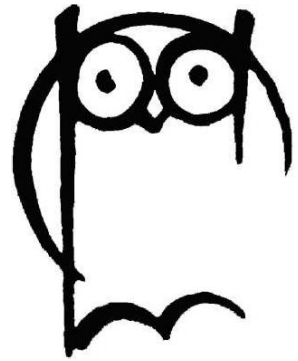


University of Debreceni
Faculty of Engineering
Department of Mechanical Engineering



Performance Analysis of a Drone Propeller

Author: Naveen Kaspapettai Kuppusamy, BSc Student.

Supervisor: Dr. Szodrai Ferenc, Associate Professor.

Head of Department: Dr. Mankovits Tamás, Associate Professor.

Debrecen

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1 Introduction

The objective of this study is to investigate the performance of a small quadcopter propeller in hovering condition, which is important for the drones used for photogrammetry within the automotive industry 4.0. As the industry adopts advanced technologies like digital twin, applications of drones are increasing to create digital replicate of the production environment and also for the regular inspection of the facility. To fly the drone within the indoor facility is complex due to the limited space and the presence of other objects and machinery. So smaller drones are preferred for the indoor flying, but the need to carry different payloads requires more research to optimize smaller propellers to produced maximum lift. This study focuses on evaluating straight blade propeller performance by calculating aerodynamic forces through analytical and numerical methods only in hovering condition and determining the propeller efficiency, contributing to the peak functionality of small size drones in production process monitoring and optimization.

2 Literature Review

2.1 Propeller

Aircraft's propulsion is achieved through one or more propeller. Propeller produces thrust (a forward force) which overcomes the drag of the wing and airframe during the cruise flight and provides additional force to balance the components of weight during climb. The propeller has two velocities through air, angular velocity Ω of the blade about the axis of the propeller and the flight upward speed V which is in the direction of the axis. Propeller produce thrust T along the axis and torque Q about its axis as a result of aerodynamic reaction when the blades move through the air. A propeller is a certain form of a more general category which is called airscrew. Airscrew is defined as any type of mechanical device with blades spans along the radius of the device that rotates about its axis during its motion relative to the air. The main types of airscrews are:

1. Propeller: Type of airscrew designed for propulsion through air and to give a high thrust power $V T$ for a given torque power ΩQ .
2. Windmill: An airscrew designed to absorb power from its axial motion relative to the air. Difference between propeller & windmill is that propeller has high axial velocity and drag is an issue, but windmill has low axial velocity and drag is not an issue.
3. Fan: An airscrew designed to create a current of air or a flow of air.
4. Anemometer: An airscrew used in meteorology to determine the wind speed through the blades rate of rotation.

There are propellers that do not move through the air in the direction of their axis. Instead, it uses a phenomenon called autogyro to move in a particular direction. This type of airscrew are used in helicopters[1]. Multi-copter drones use more than one propeller depends on the type of the configuration of the drone. In multi-copter drone propeller move along the axis during vertical flight. In other flights like forward, rear, roll and yaw the motion is achieved by changing the RPM of a certain propeller to create imbalance of force which drives the drone in the required direction. A propeller has finite number of identical blades placed with equal space between them. All the blades are maintained in a uniform rotation motion about its axis by the torque of the internal combustion or gas turbine or electric engines. Cross-section of a blade at any radial location has a unique shape called airfoil which is discussed in detail in the further topics. Each point on the blade experience lift and drag which then resolve into two components of force which is thrust and torque experienced by the propeller. The characteristic of a propeller is expressed in a non-dimensional form called advanced ratio. It gives the relation between axial velocity V , number of revolutions in unit time n and the diameter of the propeller D . The advanced ratio is defined as the ratio of forward velocity to the propeller tip velocity[1].

$$J = \frac{V}{n \cdot D} \quad [1]Eq-1$$

Where, J is the advance ratio, V is the forward flight velocity, n is the number of revolutions in unit time and D is the diameter of the propeller disk.

2.2 Types of propellers

Propellers convert mechanical energy into force which has two components thrust and torque used to propel the vehicle it is attached. It has two or more lifting surfaces called propeller blades. Propellers can be classified into different types based on several factors which are explained below.

Types of propellers based their placement on the aircraft

Tractor Propeller: the propeller is mounted to the nose of the aircraft. since the propeller is at the forefront of the aircraft the air flow is uninterrupted. During take-off and landing nose is always at high position from the ground, so less ground clearance is suitable. In winter season ice is formed on the fuselage, when the aircraft is at flight it is more likely to fly off the fuselage. Such incidences are called damage due to FOD (Foreign Object Debris). In the tractor configuration such accidents cannot be possible and increases the safety of the aircraft.

Also, the exhaust of the engine flows backwards and the propeller does not become hot. Obstruct the pilots view and it requires the back side of the blade to be painted black. The downwash of the propeller has a helix motion and it flows around the fuselage results in increased drag[4].



Figure 1 Tractor Propeller[4]

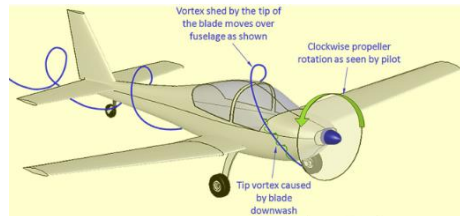


Figure 2 Downwash of the Propeller[4]

Pusher propellers: The propeller is mounted at the rear side or tail of the aircraft fuselage. This is mostly used in the small category aircrafts enabling the pilot to have obstacle free front view from cockpit. Since the noise is produced at the back of the aircraft and the craft is moving forward the cabin experiences less noise offers a pleasant flight for the passengers. In the moving flight the air flows through the propeller disk have a helical or swirl pattern. This swirl of air doesn't flow around the fuselage, so the aircraft feels less drag and more stability. It also has few disadvantages, during take-off and landing the tail of the aircraft encounters close experience with the ground. Since the propeller is mounted at the tail, the aircraft need high ground clearance. In winter seasons if the ice is deposited on the fuselage, it is possible that the ice fly back during flight and hit the propeller. So additional features need to be installed to prevent the propeller from ice. Hot air from the exhaust of the engine also flow towards the backside of the aircraft will make the propeller hot[4, p. 14].



Figure 3 Pusher Propeller[4]

Nacelle: Nacelles typically defined as a streamlined body that is completely attached outside the airframe by pylon that contains certain parts of the aircraft like engines. This type of propeller configuration is used in comparatively big aircrafts and has similar advantages of tractor type but is more efficient than tractor because there is no blockage in the air flow behind the propeller. The disadvantage of nacelle type mount is the structure support. The engine is located on the wing and forward of the wing it creates more stress to the wing. Thus, more structural support is required[4].



Figure 4 Nacelle propellers[4]

Types of propellers based their pitch

Pitch is defined as the angle between the rotation disk and the chord line of the propeller. Propellers can be classified based on how the pitch is changed. It is important to understand that changing the pitch of the propeller and change in pitch of the blade is two different things.

Changing the pitch of the propeller usually means collective pitch, that the entire blade is rotated about the axis (from blade root to blade tip) along the span of the blade. The rate of Change is equal at each radial location. This change in blade position can be change as per the requirement either on ground or in flight.

$$\tan\theta = \frac{P_D}{2 \cdot \pi \cdot r_{0.75}} \quad [4] \text{ Eq-2}$$

Where, θ Pitch angle, is the, P_D is the pitch Distance (Distance the propeller moves for one revolution) and $r_{0.75}$ is the radius at 75% of the propeller radius R .

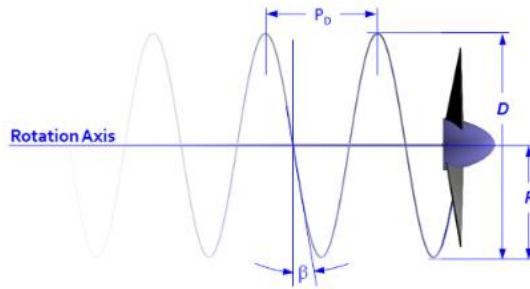


Figure 5 Some Propeller Properties[4]

Pitch distance is usually given by the manufacturer. It indicates for one revolution of the propeller, the distance it propels forward.

But change in pitch of blade usually means twist in cross-section of blade at each radial location. This is called blade *twist*; it is the difference between the pitch angle at root of the blade and the pitch angle relative to radial coordinates. This can never be changed one the propeller is manufactured. Blade twist angle can be calculated by

$$\theta = \theta_0 + (r \cdot \theta_{tw}) \quad [5] \text{ Eq-3}$$

Where, θ pitch angle, θ_0 pitch angle of wing section at blade root, r radial coordinate and θ_{tw} is the linear twist rate.

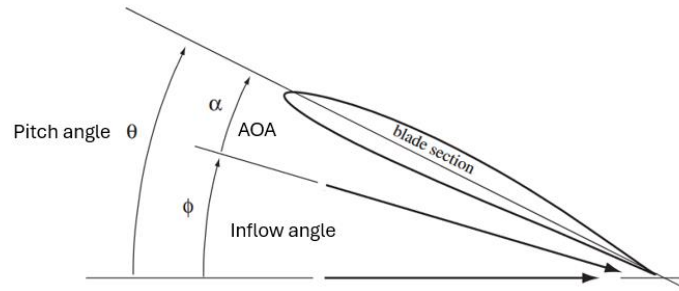


Figure 6 Angles of a Blade or Wing Section[5]

Twist angle is the change in pitch relative to the blade section at root. Blade angle is the change in pitch relative to rotation disk. If a twist is added to a propeller, then blade angle usually exists only at the root of the blade.

Below mentioned types only talks about the blade angle not the twist angle.

Fixed-pitch propellers: Blade pitch angle cannot be changed. It is less expensive, light weight and requires little maintenance and failsafe. But in case of engine failure, they rotate freely like a windmill thus increases drag and reduce glide ratio. Such propellers are designed for best efficiency at either climb (climb propeller) or cruise (cruise propeller) [4].

Limited-Pitch propellers: In this type of propellers the pitch angle can be adjusted using a simple tool. But it can only be adjusted to two specific angles. Either to a climb pitch angle or a cruise pitch angle. It can be adjusted on the ground or in flight using a mechanical system. This allows the pilot to change to climb pitch while taking-off and once cruise altitude is reached pilot can change to cruise pitch angle. This way flight stability and fuel consumption can be achieved [4].

Fully adjustable-Pitch propellers: this type of propellers comes with a hydraulic system on board, which is connected to the propeller through its hub. And the blade pitch angle can be adjusted for a range of angles as the pilot would require. This allows most efficient thrust output. Not only for the climb and cruise but also based on the weather conditions (Air density) it can be adjusted for better performance [4].

Constant-Speed Propellers: This type of propellers automatically adjust the pitch angle to maintain the current RPM of the engine. A part called governor in the engine is responsible for the automatic adjustment of the pitch angle [4], [6].

Feathering Propellers: This feature is only present in fully adjustable or constant speed propellers. This allows the pilot to adjust the blade perfectly colinear with the relative air flow. During engine failure this prevents the propeller from windmilling or rotating uncontrollably. So, the aircrafts glide ratio is not affected by the drag [4].

Reversing Propellers: It also exists only in the fully adjustable and constant speed propellers. The blades can be turned in way that the produced thrust is face not the forward but rear side of the aircraft. This position is used to reverse the aircraft on its own from the air terminal [4].

Below types are based on the blade twist.

Constant Pitch propeller: Pitch at every radial location is equal when measured from the rotation disk. The difference between the blade root pitch and the tip pitch is zero. The lift profile is linear across the span of the blade.

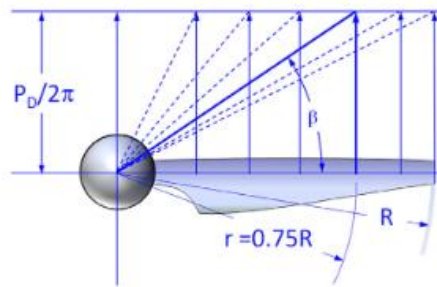


Figure 7 Constant Pitch Propeller Lift Profile[4]

β angle in the image is denoted with θ in this study which is pitch angle [4].

Variable Pitch propeller: The air speed across the blade depends on two components axial velocity and tangential velocity.

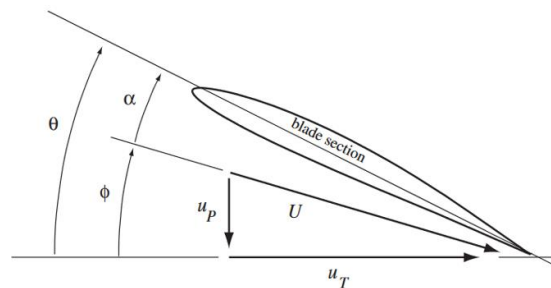


Figure 8 Velocity Components of the Blade or the wing section[5]

Where, U resultant velocity, U_P axial velocity and U_T tangential velocity.

Axial velocity is same across the radial coordinates but the tangential velocity changes with respect to radius. So, the blade section at the tip produces more lift than the blade section at the root. This will result in flapping of the blade; at maximum speed the blade is more likely to break. So, a twist is added to the propeller that the blade section at the tip has less pitch angle with the rotation disk than the blade section at the root. This rate of change in twist can be linear or cubical. The maximum lift exists at 75% of the span. Its lift profile can be seen below.

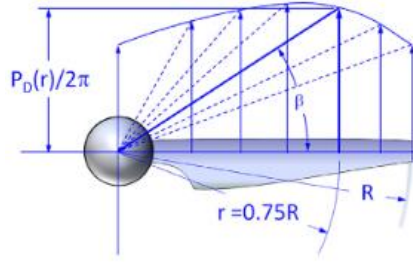


Figure 9 Variable Pitch Propeller Lift Profile[4]

β angle in the image is denoted with θ in this study which is pitch angle.

Some of the relationships of propeller rotation are given below. The speed of the propeller is generally mentioned in RPM. To convert the RPM to angular velocity (Ω) for calculation purpose:

$$\Omega = 2 \cdot \pi \cdot \left(\frac{RPM}{60} \right) \quad [4] \text{ Eq-4}$$

Angular velocity will be same regardless of position of radius. But the linear (tangential) velocity will change with respect to the radius.

$$U_T = \Omega \cdot r \quad [4], [5] \text{ Eq-5}$$

The axial velocity depends on two values. i.e., flight forward velocity V and inflow velocity v .

$$U_P = V + v \quad [5] \text{ Eq-6}$$

The resultant velocity magnitude and inflow angle are then given by:

$$U = \sqrt{U_T^2 + U_P^2} \quad [5] \text{ Eq-7}$$

$$\varphi = \tan^{-1} \left(\frac{U_P}{U_T} \right) \quad [5] \text{ Eq-8}$$

The aerodynamic angle of attack of the blade is $\alpha = \theta - \varphi$. All angles should be substituted either in rad or deg.

Types of propellers based the direction of rotation

propeller produces two resultant force thrust T and torque Q . Thrust helps the aircraft propel forward. But the torque acts on the aircraft and creates a moment on the airplane and make it roll; in helicopter it makes it yaw. So, a counter force is applied by the tail rotor in helicopter. In smaller aircrafts control surfaces are used to stabilize the plane. In multi-engine aircraft let's say two engine, one propeller rotates in CW and other propeller rotates in CCW direction. The opposite torque cancels each other so the moment is not created on the aircraft. The same happens in multi-copter drones. Quadcopter, Hexacopter, Octocopter or the copters with even number propellers are the most stable ones. The CCW propeller has blade turned 90° compared to the CW propellers. Both propellers have downwash in the same direction i.e., towards the tail of the aircraft or towards the ground for the helicopter or multi-copters.

2.3 Types of Drone Propeller Configurations

Drone doesn't have any control surface like an aircraft. Also, it doesn't have a tail rotor like helicopter to counter the torque. That's why a drone must have a minimum of two propellers. Propellers are used to achieve all the manoeuvres like climb, decent, roll, yaw, forward, backward, counter torque. So more the propellers more stable the drone is, also even number propellers are more stable than odd number of propellers. If a drone has odd number of propellers, then torque in one direction is more than the other, so opposite direction motor has to run at higher RPM to compensate. Based on the number of engines drone are classified into the following categories. One engine can have two propellers as well. One on the top of the engine and other at the bottom of the engine. These types of drones have specially designed electric motor, where the shaft of the motor extend in both sides. Both top and bottom propeller is either CW or CCW. Drone categories based on the number of motors are:

1. Bi-copter – 2 engines,
2. Tri-copter – 3 engines,
3. Quad-copter – 4 engines,
4. Hexa-copter – 6 engines,
5. Octo-copter – 8 engines.

These number of engines can be placed in different positions which determines the type of frame that drone requires. Most common drone frames are +, X, H, Y, V [11]. Number of engines and their placements can be seen in the following image.

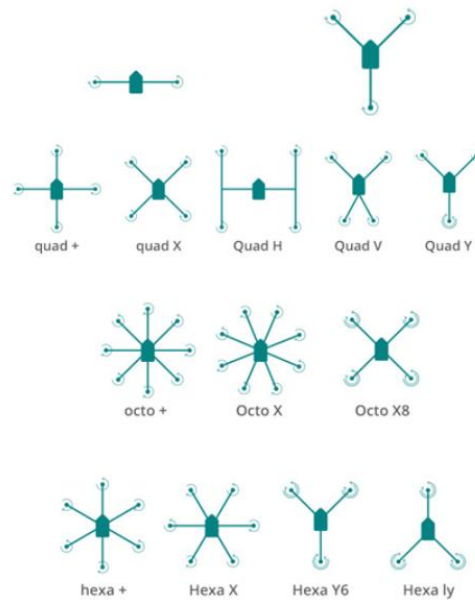


Figure 10 Drone Propeller Configuration[11]

2.4 Anatomy of a Propeller

2.4.1 Features of propeller

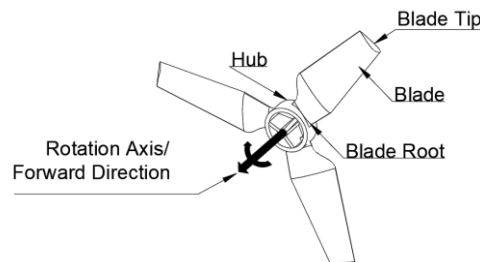


Figure 11 Anatomy of a Propeller

Every propeller has a hub which is a cylindrical object at the centre. It is used to connect the propeller to the engine/motor shaft. The engine power is transferred to the propeller blades through hub. Number of blades of a propeller varies depends on the application. If the propeller diameter is small but requires higher lift, then number of blades are increased. When a greater number of blades are used the drag also increases, leading to more fuel consumption. So, the number of blades always depends on the application. Blade tip experiences the max linear velocity. Thus, max lift produced at the tip, but this condition makes the propeller to flap. So, propeller shape is optimized so that the max lift exists at 75% of the span of the blade. Blade tip speed can be calculated by the following equation.

$$V_{tip} = \sqrt{V^2 + \left(\frac{\pi \cdot RPM \cdot D}{60}\right)^2} \quad [4] \text{ Eq-9}$$

Where, V_{tip} velocity at tip, V forward velocity (At hovering condition $V = 0$), D Diameter of propeller

Tip Mach number:

$$Ma_{tip} = \frac{V_{tip}}{c_s} \quad [4] \text{ Eq-10}$$

Where, Ma_{tip} blade tip speed in Mach Number, c_s = speed of sound.

The speed of sound refers to how quickly a small disruption or vibration moves through a medium like air, water or solids. This disturbance transfers energy between particles in the medium, allowing sound waves to propagate. The speed of sound can be calculated in different form.

Form 1:

Speed of sound in an ideal gas formula:

$$c_s = \sqrt{\frac{\gamma \cdot R_g \cdot T_k}{M}} \quad [12] \text{ Eq-11}$$

Where, γ = ratio of specific heat of the gas at constant pressure and volume, γ 1.4 for air [12], [13], R_g universal gas constant ($8.314 \text{ J} \cdot \text{mol}^{-1} \text{K}^{-1}$ [12]), T_k Temperature in Kelvin, M Molar mass of the gas ($28.95 \cdot 10^{-3} \text{ kg/mol}$ for air [12]).

Form 2:

Speed of sound in air formula:

$$c_s = \sqrt{\gamma \cdot R_g \cdot T_k} \text{ (m/s)} \quad [14], [15] \text{ Eq-12}$$

Where, γ , ratio of specific heat at constant pressure and volume, γ , 1.4 for air [12], [13], R_g , gas constant of air ($286 \text{ J kg}^{-1} \text{K}^{-1}$ [14]), T_k Temperature in Kelvin.

Form 3:

Using the Eq-12 the following graph for speed of sound in air medium can be plotted for every temperature.

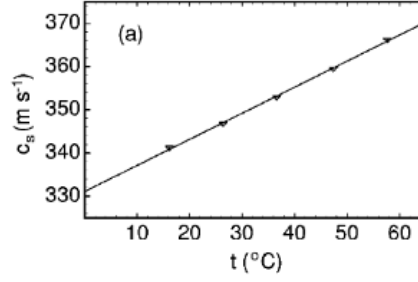


Figure 22 Temperature Vs Speed of Sound[12]

It is a linear function, and it is given by the following equation, this equation can be used to calculate speed of sound in air using temperature in degree Celsius.

$$c_s = c_{s0} + (0.61 \cdot T^o) \quad [12], [16] \text{ Eq-13}$$

Where, c_{s0} 331.5 m/s is the speed of sound at 0 °C and 1 atmospheric pressure [12], [17], 0.61 slope of the temp Vs c_s curve [12], [16], T^o Temperature in °C, Eq-13 can simply be written as $c_s = 331.5 + (0.61 \cdot T^o)$.

In above calculations T_k or T^o represents the temperature at the altitude (h) where the propeller is operating. In reality the temperature changes frequently. But for the calculation, standard earth atmospheric model can be used to find the temperature at the given altitude.

For $h > 25000$ m (Upper stratosphere)

$$T^o = -131.21 + 0.00299 \cdot h \quad [18] \text{ Eq-14}$$

For $11000 \text{ m} < h < 25000$ m (Lower stratosphere)

$$T^o = -56.46 \quad [18] \text{ Eq-15}$$

For $h < 11000$ m (Troposphere)

$$T^o = 15.04 - 0.00649 \cdot h \quad [18] \text{ Eq-16}$$

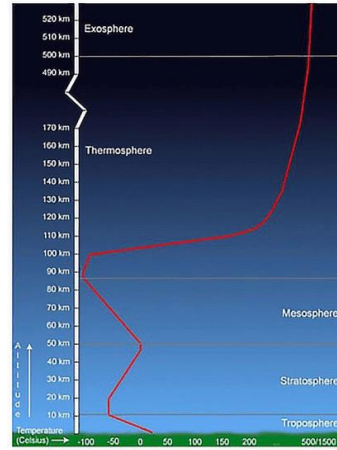


Figure 33 Average Temperature Profile of Earth atmosphere[19]

2.4.2 Propeller Disk

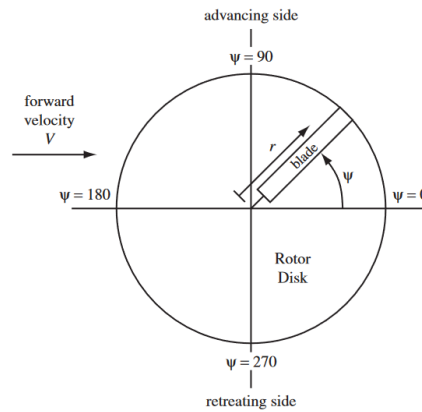


Figure 14 Propeller Top View[5]

When the propeller rotates it looks like a flat disk. The complete area the propeller covers during rotation is called *rotor disk*. Disk has 360° rotation angle it is called *azimuth angle* ψ of the blade, defined as zero in the downstream direction. The forward velocity in the figure 18 represent the inflow air direction during horizontal flight. In a helicopter scenario the propeller/helicopter will move in the opposite direction to V . when the leading edge of the blade moves away from the 0 angle that side is called advancing side of the propeller, experiences more drag because the blade moves against the air flow. When the leading edge moves towards the 0 angle that side is called retreating side of the propeller, experiences less drag because the blade moves along the air flow and produce less lift because forward velocity V is ineffective, and it can be seen in the Eq-6. Usually in American manufactured aircraft and helicopter the direction of propeller is CCW and French and Russian made are CW. Coordinated (r, ψ) is called the radial and azimuth position of the blade[5].

c blade chord, which for taper or other form of blade is a function of r , N , number of blades, A area of the rotor disk

$$A = \pi \cdot R^2 \quad [5] \text{ Eq-17}$$

Rotor solidity σ is the ratio of the total blade area to the rotor disk area. It is used to calculate other important parameters to estimate the propeller performance.

$$\sigma = \frac{\text{Total Blade Area}}{\text{Rotor Disk Area}} = \frac{NcR \text{ (for Constant Chord)}}{A} = \frac{N \cdot c}{\pi \cdot R} \quad [5] \text{ Eq-18}$$

2.5 Aerodynamics of Propeller

2.5.1 Forces on a Propeller

Forces produced by the propeller blades are the result of the aerodynamic reaction when the propeller moves through the air. Propeller blades produces two forces called *Lift* and *Drag*, both have two components which combines to form two resultant forces called *Thrust* and *Torque*. Lift and drag are dependent on AOA of the blade.

Lift: The force that causes an aeroplane to rise into the air is called lift, and it acts perpendicular to the direction of airflow. It is mostly produced by the wings/rotating wings (Propellers) because of their angle and shape (airfoil design), which results in lower air pressure on the top of the wing than at the bottom. To achieve and sustain flight, lift must balance the aircraft's weight.

Drag: Drag is caused by two factors at subsonic speed. One factor is the viscosity of the air, primarily due to the skin friction cause by the surface roughness of the aircraft. Other factor is due to the generation of lift[21]. Drag is of two types:

Parasite drag - cause by shape, size and roughness of the object.

Induced drag – caused by the generation of lift. Air resist the motion of objects through it. When propeller or aircraft move through air to produce life it also produce induced drag. For transonic aircraft i.e., for speed $Mach \sim 1$ and for supersonic and above speeds the drag also caused by the compressibility effect of the air.

Total drag force of a wing in dimensionless form:

$$C_D = C_{Dp} + C_{Di} + \Delta C_{Dc} \quad [21] \text{ Eq-19}$$

C_D is called coefficient of drag. Total drag consists of three terms. First term C_{Dp} is the parasite drag. Second term consists of lift-dependent drag, it is related to lift coefficient C_L .

$$C_{Di} = \frac{C_L^2}{\pi \cdot (AR) \cdot e_{inviscid}} \quad [21] \text{ Eq-20}$$

AR is the aspect ratio of the wing or the propeller blade it is the ratio of wingspan to average chord.

$$AR = \frac{b}{c} \quad [22] \text{ Eq-21}$$

Skin friction may also affect the value of e (Oswald efficiency factor). Third term is the drag cause by compressibility effect. Third term exist only for speeds closer to Mach 1 and above. Mostly parasite drag C_{Dp} and e are obtained by experimental data.

Thrust: In airplane thrust can be interpreted as forward force which moves an aircraft forward. But in propeller point of view thrust is the vertical component of the produced lift. In airplane thrust must overcome horizontal component of the total drag to accelerate and maintain the speed. But in propeller/helicopter/multi-copter the thrust must overcome weight of the fuselage/frame and the vertical component of the drag.

Tangential force is a force applied in circular motion. In non-rotating wing the horizontal component of the lift and drag results in a linear motion, since the wing is not rotating around a pivot point. But in propellers the sum of horizontal component of lift and drag creates a circular motion result in tangential force. The tangential force multiplied with radius of the propeller gives the torque. Torque is created in the opposite direction to the direction of rotation of the propeller.

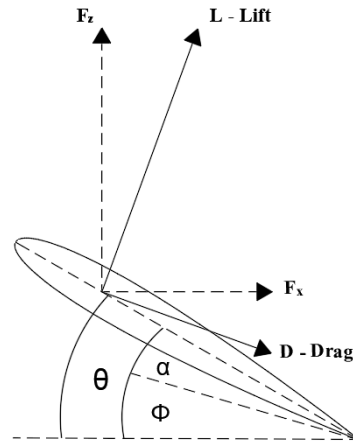


Figure 45 Forces on a Propeller

Where, L is lift, D is drag, F_z is elementary thrust and F_x is elementary tangential force.

The lift is always produced at an angle perpendicular to the inflowing air. And the drag is produced at an angle perpendicular to the lift force. So, the lift and drag can be resolved into two components as seen below.

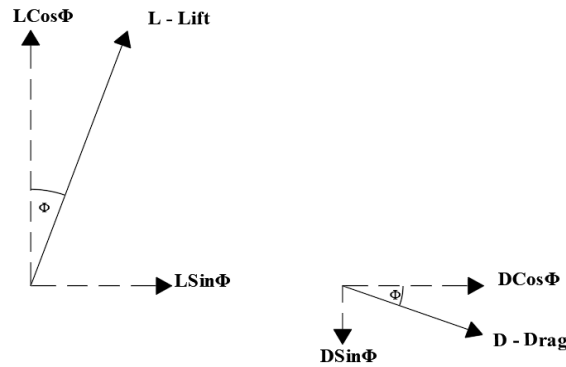


Figure 56 Resolved Lift & Drag Forces

$$F_z = L \cdot \cos\Phi + D \cdot \sin\Phi \quad [5] \text{ Eq-22}$$

$$F_x = L \cdot \sin\Phi + D \cdot \cos\Phi \quad [5] \text{ Eq-23}$$

2.5.2 Aerofoil

The surface on any type of propelled aircraft that produce the aerodynamics forces by the dynamic reaction on the air is called wing. It can be either fixed like in the airplanes or it can be rotating like the propeller (rotary wing), or it can be flapping like in the ornithopters. Regardless of the type of lifting surface, its aerodynamic performance is mainly affected by the geometry of the wing section, in other words airfoil/aerofoil[23]. The present-day shapes of airfoils are primarily developed by the “National advisory committee for aeronautics-NACA” present day NASA and is commonly referred as NACA airfoils. The development of

NACA airfoils began in 1929 with a scientific way of investigation in the “*Langley Variable-Density*” wind tunnel. The airfoils developed by NACA are classified into different series. Each series have varieties of airfoils shapes, and the name of individual airfoil begins with NACA followed by digits (NACA xxxx). Each series have similar thickness distribution, and the amount and type of camber is changed. The series of an airfoil can be identified by the number of digits present in the name. For example, NACA 2412 is an airfoil belong to 4-digit series. Similarly, there are 2, 5, 6, 7, 16 digit series and specialized airfoils are developed for certain purposes like SC(1)-0710 is an airfoil belong to super-critical airfoil series for helicopter rotor blades and airfoils for ornithopters etc[24].

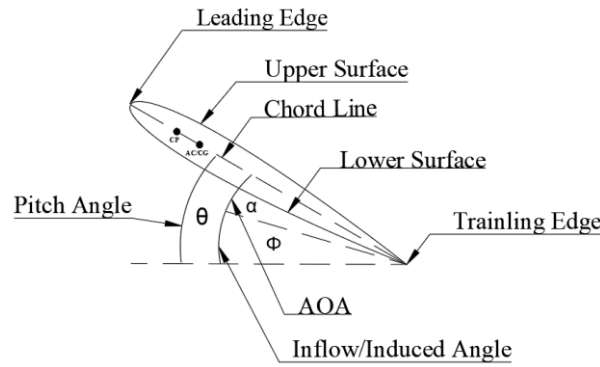


Figure 17 Anatomy of symmetry Aerofoil

In an airfoil there are several features like it is shown in the above image. Upper surface is the surface on the top of the chord line which experiences low pressure during air flow. Lower surface is the surface experiences high pressure during air flow. Chord line is a straight line that connects the leading edge (LE) and trailing edge (TE). The LE is a vital edge located at the very front of the airfoil. The shape of this edge is crucial which determines how the air flows around the airfoil and plays a significant role in generating lift. The LE is usually rounded in many airfoil shapes, as it reduces drag and helps in smooth transition of the airflow. In some high-speed airfoils like supersonic airfoils the LE is pointed. Because the effects (compressibility) at Mach > 1 is different from the effects at Mach < 1. Since LE is rounded, each airfoil has different radius, and the LE radius for 4-digit airfoil can be estimated with the following formula:

$$\frac{r_{LE}}{c} = 1.1019 \left(\frac{t}{c} \right)^2 \quad [25] \text{ Eq-24}$$

Where, r_{LE} is LE radius, c is chord length, t is max thickness of airfoil.

The LE centre can be found by drawing a line from the LE at an angle equal to the slope of mean line at LE and laying off a distance from the leading edge along this line equal to the LE radius. The slope at LE is theoretically infinite but for a theoretically finite load mean line the slope of the radius through the end of the chord is usually taken as the slope of the mean line at 0.5% chord since the slope increases to theoretically infinite values as x/c approaches zero. Below formula give the ordinate value at 0.5% chord length. Mean line is differentiated to find the slope values.

$$\frac{X}{c} = 0.005 \quad [24] \text{ Eq-25}$$

Where, X is ordinates along the chord of the airfoil and c is chord length.

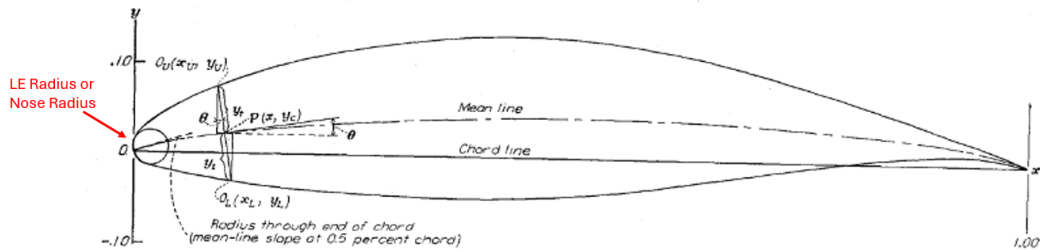


Figure 18 NACA 65,3-818 $a=1$ airfoil[24]

Mean line or Mean camber line is the line that divides the airfoil symmetrically from LE to TE. The max distance between the chord line and the mean line is called the *Maximum Camber* of the airfoil. Some airfoils have zero camber it is called *symmetric airfoil*. The mean line is formed by connecting two parabolic arcs which are tangent at the point of max camber[25]. For some airfoils a designation “ a ” is given. It represents the type of mean line used and it is given as $a=1$. The value of “ a ” ranges from 0-1. “ a ” multiplied by 100 gives the percentage of chord length from LE. There is a uniform chord wise loading from LE to “ a ” and after “ a ” the loading decreases linearly till the TE. If “ a ” is not mentioned, then it is usually taken as 1. Uniform loading type “ $a=1$ ” is commonly used in NACA 16-series airfoils[24]. The ordinate of “ a ” can be found by:

$$\frac{X}{c} = a \quad [24] \text{ Eq-26}$$

Where, X is Ordinates of airfoil, c is Chord length and a is type of mean line.

The trailing edge of an airfoil is the rear edge where upper surface and lower surface reconnect. It is the point where the airflow rejoins after moving past the airfoil. Trailing edge can be of different shape:

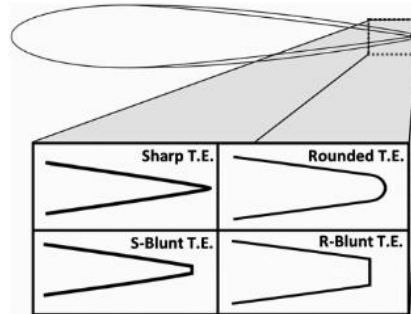


Figure 19 Types of Trailing Edge[26]

Sharp: In this type the upper and lower surface converge to a sharp point. The trailing edge has a finite angle.

Cusped: “Cusp” refers to curved lower surface near the TE of an airfoil. This type of trailing edge is used in supercritical airfoil for additional lift by forming a high-pressure region.



Figure 20 Cusped Trailing Edge[27]

Cusped feature can be designed with blunt or sharp trailing edge. At the sharp trailing edge, the airflow need to reach infinite velocity to satisfy the flow continuity and pressure conditions. However, this situation is physically impossible due to the viscosity of the fluid and other constraints. So, a special condition is applied for the sharp TE and this condition is only valid for steady incompressible potential flow, it is called *Kutta-Joukowski* condition. According to Kutta “There is no vorticity in the flow mass that at the trailing edge, the direction of the flow is tangent to the arcs forming the body”. According to Joukowski “The rear stagnation point always be at a fixed point regardless of the value of AOA”[28]. It is interpreted in many ways by different authors. As mentioned in reference (35) “Giesing” explains the velocity on the upper and lower surfaces at the TE should be equal in magnitude and opposite in tangential direction. And “whitehead” interprets as either velocity difference or the pressure difference at the TE is zero. But whitehead’s statement is only valid for sharp TE with finite incidence angle. For a cusped sharp TE, the gradient of the mean line becomes

infinite at TE, the incident angle of the TE approaches zero so both the flows leaving the TE is tangent to each other and has equal velocity. Thus, the velocity at the TE is finite and not zero[28].

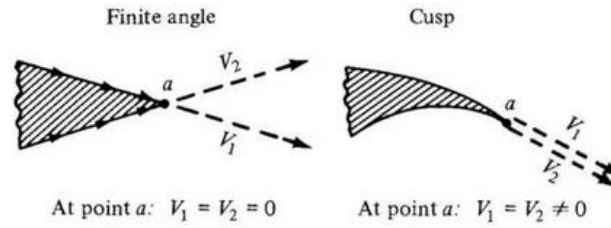


Figure 21 Types of Trailing Edge[29]

Flatback or Blunt or Square: There are two types of flatback TE. Type-A or S-blunt TE is created by building up thickness of the TE. Type-B or R-blunt TE is created by cutting off a part of the TE. Any type of flatback creates a pressure drag by forming a wake region at the behind. But this type of TE is ideally used for certain propellers and wind turbines.

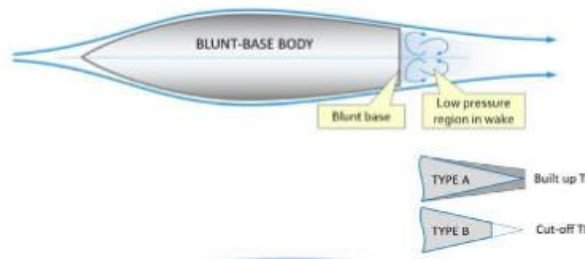


Figure 22 Blunt Trailing Edge[27]

Rounded: Rounded TE has thicker and round profile rather than sharp point. This type of TE is chosen for structural strength and easy manufacturability. It creates pressure drag due the wake region formed at the behind. But it has smaller impact on overall performance of low-speed airfoils.

Another notable feature of an airfoil is centre of pressure (CP) and aerodynamic centre (AC). In addition to linear resultant force, an airfoil immersed in fluid flow also produce a moment if it is fixed at a point along the chord. When the air flows its velocity changes at each point on the upper and lower surface of the airfoil, results in varying pressure at each point. When airfoil is fixed at a point along the chord the sum of pressure before this point is different from the sum of pressure after this point. Due to this imbalance in force, airfoil experiences a torque. If the torque has a lever arm, then it is called *moment or pitching*

moment. It usually given at AC The centre of pressure is defined as the point along the chord where the pitching moment is zero[27], [30].

$$CP = \frac{\int X p(X)dx}{\int p(X)dx} \quad [30] \text{ Eq-27}$$

Where, CP is the Centre of Pressure, X is the Ordinates of airfoil, p is the Pressure distribution.

But the position of CP change with AOA and resultant aerodynamic force. At each phase of the flight wing has different AOA , so the wing experience different pitching moment. The centre of gravity (CG) of the aircraft or the centre of the propeller hub need to be aligned with one point of the wing, CP which changes at each phase of the flight is not suitable. So, engineers found a point theoretically and experimentally at which the pitching moment is almost constant at any AOA . The point at which the pitching moment is constant is called aerodynamics centre and this point of a wing, or a blade is aligned with the CG of the aircraft or hub in case of straight wing form. In case of different wing form like swept back the average AC is aligned with the CG . For a low-speed airfoils the AC is usually located at[31]:

$$AC = \frac{c}{4} \quad [31] \text{ Eq-28}$$

Where, AC is Aerodynamic Centre, c is chord length.



Figure 23 Position of Aerodynamic Centre

2.6 Theories of lift

Lift is produced because of the air flows around the curvature of the airfoil. But how the air follows the curvature of the airfoil rather than flowing in its original direction? Fluids have a natural behaviour to stick to any surface that is present in the trajectory of its flow. It is called *Coanda effect*. The Coanda effect explains the tendency of a fluid jet (such as air or water) to attach to a nearby surface and follow its curvature rather than travelling in a straight path. Due to these phenomena when air flows around the airfoil a pressure difference is created on the upper and lower surface of the airfoil which results in a generation of a force in perpendicular direction called lift force. But why is this pressure difference occurs? It can be explained in

three different ways and those are called *momentum theorem*, *Bernoulli theorem* and the *circulation or Vortex theorem*[27].

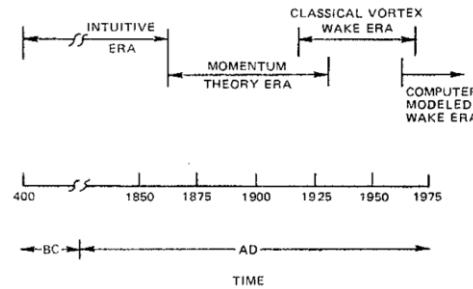


Figure 24 Propeller Theory Evolution Stages[3]

2.6.1 Simple Momentum Theorem

This theory relates rotor thrust and power to overall flow velocities through a global analysis. It was originally developed by W.J.M Rankine in 1865 and R.E. Froude in 1885 for marine propellers. Later it was adapted for airscrew propellers by including the rotation of the slipstream by A. Betz in 1920. According to momentum theorem lift for fixed wing & thrust for rotary wing is generated as a consequence of wing moving through the mass of air and gives the air a downwash motion. So, the vertical speed of air changes in a certain time. As the air moves downward a reaction force (Lift for wing & Thrust for rotary wing) is produced in the opposite direction according to *Newton's Third Law of Motion*. The magnitude of this reaction force can be estimated through the laws of conservation in fluid mechanics called conservation of momentum, energy and mass [5], [27].

Momentum Actuator Disk theorem:

It is also called as axial momentum theorem. In Momentum theory the rotor is modelled as actuator disk a rotor disk with infinite number of blades. This actuator disk is circular with diameter equal to the rotor diameter but with zero thickness. This disk can support the pressure difference, but not the velocity change thus accelerates the air when it flows through the disk. The blade loading varies in reality but in this theorem, it is assumed to be uniform. This theory helps to estimate the induced velocity and induced power for a given thrust and useful to find the ideal performance limit of propeller, but it is not enough to design the blade. Induced velocity can be found in three ways. *Momentum theory* uses conservation laws of fluid motion to solve this problem. *Vortex theory* uses Biot-savart law to find the induced velocity by the wake vorticity. *Potential theory* uses fluid dynamic equation for the velocity

potential or stream function to find the induced velocity. For similar model of the disk all three theories must give the same result for the induced velocity.

The downward motion of the air behind the propeller is called downwash and the air at a distance after the propeller is called *far wake region*. The velocity of air immediately after the rotor disk in downwash is called induced velocity and is denoted by v , the velocity in the far wake is wake induced velocity and is denoted by w [5].

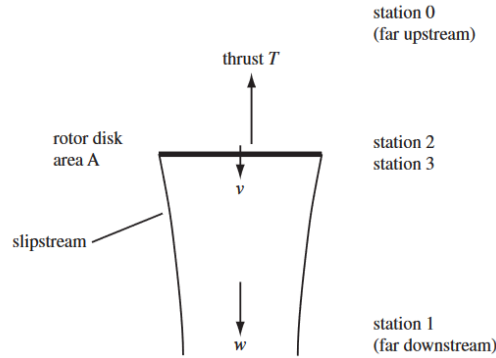


Figure 25 Flow model for Momentum Theory[5]

If the downwash velocities and the mass flow rate is known, then thrust force for propellers can be calculated using below method with the assumption that fluid is incompressible and inviscid.

$$\text{Momentum Conservation } T = \dot{m} \cdot v \text{ (For Hovering Rotor)} \quad [5], [27] \text{ Eq-29}$$

$$\text{Mass Conservation } \dot{m} = \rho \cdot A \cdot v \quad [5], [27] \text{ Eq-30}$$

$$\text{Energy Conservation } T \cdot v = \frac{1}{2} \cdot \dot{m} \cdot \omega^2 \quad [5] \text{ Eq-31}$$

$$\omega = 2 \cdot v \quad [5] \text{ Eq-32}$$

Thus, momentum theory relate rotor thrust and induced power for hover to induced velocity at the rotor disk:

$$T = \dot{m} \cdot v = 2 \cdot \rho \cdot A \cdot v^2 \quad [5] \text{ Eq-33}$$

$$P = T \cdot v \quad [5] \text{ Eq-34}$$

Where, T is thrust, $\dot{m}$ is Mass flow rate, v is induced velocity, ρ is air density, A is Disk Area, ω is velocity of air in downwash/far wake region, P Power.

There is another momentum theory called *extended or general momentum theory* which is a detailed analysis of rotor blades. It included the effects of swirl in the downwash which is not covered in this study.

2.6.2 Bernoulli Theorem

It was derived by a Swiss mathematician *Daniel Bernoulli* in 18th century. Generally, when explaining the generation of lift, Bernoulli theorem is used among aerodynamic professionals. Because of its simplicity to understand. It explains that the lift is produced as the result of pressure difference between upper and lower surface. Mathematically speaking the results are obtained by integrating the pressure on the surface of the airfoil. This theorem relates pressure and the velocity of the fluid at a point and along a streamline that goes through that point[5], [27]. Following equation is for incompressible flow:

If the swirl in the wake is neglected, the pressure at station 1 in the figure 30 is equal to the ambient pressure p_0 . Applying Bernoulli's equation between station 0 and station 2 the Thrust equation can be obtained similar to the actuator disk theorem.

$$p_0 = p_2 + \frac{1}{2} \cdot \rho \cdot v^2 \quad [5] \text{ Eq-35}$$

Applying the Bernoulli's equation between station 3 and 1 gives:

$$p_3 + \frac{1}{2} \cdot \rho \cdot v^2 = p_0 + \frac{1}{2} \cdot \rho \cdot v^2 \quad [5] \text{ Eq-36}$$

Combining Eq-35&36 gives:

$$\frac{T}{A} = p_3 - p_2 = \frac{1}{2} \cdot \rho \cdot \omega^2 \quad [5] \text{ Eq-37}$$

By substituting Eq-30 in Eq-37, we obtain Eq-31.

Where, p_2 is Pressure at station 2, p_3 is Pressure at station 3.

2.6.3 Circulation Theorem

A vortex is a rotating region of a moving fluid. Vortices are used in vortex theory to model the behaviour of flow around an airfoil or any object. The instantaneous rate of rotation of a fluid element in a fluid flow is called vorticity[32]. The function of the flow pattern with respect to the vortex radius is given by:

$$\psi_v = \frac{\Gamma}{2 \cdot \pi} \ln r_v + c'' \quad [23] \text{ Eq-38}$$

$$\omega_v = \nabla \cdot V_v \quad [32] \text{ Eq-39}$$

Where, ψ_v is the Flow Pattern of the Vortex, Γ is the strength of vortex/circulation, r_v is the radius of vortex, c'' is the integration constant, ω_v is the vorticity, V_v is the velocity of fluid around the airfoil.

Circulation is a key concept in describing the generation of lift and it is explained in detail under section 2.8.1. It represents the net spinning motion of the fluid around a closed path surrounding the airfoil. It is the strength of fluid rotation around the airfoil. Circulation is positive if the produced lift is upward. Circulation is given by:

$$\Gamma = - \oint_C \vec{V}_v \cdot d\vec{s} \quad [27] \text{ Eq-40}$$

Where, C is the Closed Curve around the airfoil

According to the kelvin's circulation theorem for an inviscid and incompressible fluid flow the circulation Γ is constant.

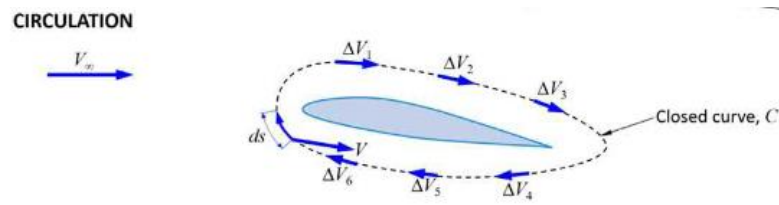


Figure 26 Circulation Around the Airfoil[27]

2.7 Theory of Infinite Wing

2.7.1 Kutta-Joukowski Theorem

KJ-Theorem is the first mathematical model of lift rather than an explanation. But it only applies to a particular family of airfoils and all the airfoils of this family have a cusped trailing edge. It calculates lift of a wing using Circulation theorem. Circulation around an airfoil is caused by an effect that occurs in the boundary layer of the flow around the airfoil. Boundary layer is the layer of a flowing fluid that is attached to the surface of the object. When solid and fluid interact, the layer of fluid that is directly attached to the surface of the object has zero velocity due to Van der Waals forces. Due to the viscosity in the fluid the zero velocity layer slows down the fluid layer right above it. The layers slowing down gradually decrease with increase in height. When the layer reaches the velocity equal to the free stream velocity the boundary layer ends[27].

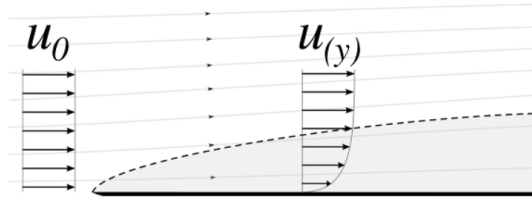


Figure 27 Boundary Layer on Airfoil[33]

Presence of circulation can be understood by the rotating cylinder. Consider a cylinder in a uniform flow field is rotating in a clockwise direction and the air is flowing from left to right. The upper portion of the cylinder moves towards right, since the surface is moving with the flow, there is no friction force in the opposite direction to the flow. So, the velocity of the flow on the upper part of cylinder is high. But in the lower surface of the cylinder moves towards left which is against the direction of the air flow causes the flow velocity to decrease. According to Bernoulli's principle if the velocity of a fluid is high then the pressure is low and vice versa. So, the upper part of the cylinder has low pressure compared to the lower part of the cylinder. This pressure difference generates the lift towards up[34].

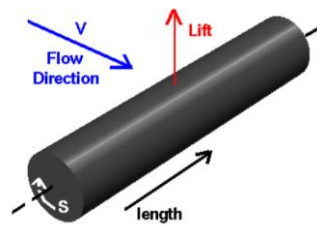


Figure 28 Lift of a Rotating Cylinder[34]

The same cylinder phenomena occurs around the airfoil as well. In the below image the first airfoil shows the air flows around it and the second airfoil shows the circulation present around it. On the upper surface of the airfoil both vectors add to have higher velocity and in the lower surface both vectors subtract to have lower velocity. It proves the Bernoulli's principle of pressure difference.



Figure 29 Resultant of Air Flow and Circulation[33]

KJ-Theorem relates the lift produced by a 2D object and the bound vortices (Circulation Γ). The Eq-41 is derived from KJ-Theorem, which give the lift of the wing.

$$L' = \rho \cdot U \cdot \Gamma \quad [27] \text{ Eq-41}$$

$$L = \rho \cdot U \cdot \Gamma \cdot b \quad [27] \text{ Eq-42}$$

If the c is the chord and C_L is the section lift coefficient the circulation can be written as:

$$\Gamma = \frac{1}{2} \cdot C_L \cdot U \cdot C \quad [35] \text{ Eq-43}$$

Helmholtz law states that whenever the value of Γ change along the span of the blade, a vortex filament (*Trailing Vortex*) of strength $d\Gamma = -(\partial\Gamma/\partial r) \cdot dr$ should leave the blade. By this law vortex theory determines the flows of the rotary wing[5], [35]. Biot-Savart Law is an important law to determine the flow field induced by a system of vortex filament.

2.7.2 Thin Flat Plate Theorem



Figure 30 Flat Plate Airfoil[1]

In this theorem a symmetrical airfoil is modelled as flat plat with a finite thickness. One of the Joukowski airfoil solution is a flat plate at an angle of attack in a steady planar potential flow. In Joukowski airfoil solution the chord c of the plate is $c = 4R$. In all Joukowski airfoil solution it is valid that the flow could leave the sharp trailing edge smoothly. Mathematically to satisfy this condition a specific value of circulation and it mean that the lift coefficient depends on the shape of the airfoil. The relation obtained between the angle of incidence α_i and lift coefficient C_L is given by:

$$C_L = 2 \cdot \pi \cdot \alpha_i \quad [1] \text{ Eq-44}$$

The coefficient of moment at leading edge is calculated by:

$$C_M = \frac{1}{4} \cdot C_L = \frac{\pi}{2} \cdot \sin \alpha_i \quad [1] \text{ Eq-45}$$

The coefficient of drag is given by:

$$C_D = C_L \cdot \sin \alpha_i = 2\pi \cdot \sin^2 \alpha_i \quad [1] \text{ Eq-46}$$

Similarly, for cambered airfoil the coefficient of lift is given by Thin circular arc plate which is the basis of thin airfoil theorem.

2.7.3 Thin Airfoil Theorem

When analysing a 3D flow at high speed, two approaches can be used thin airfoil theory and vortex panel method. The vorticity creating circulation around an airfoil doesn't need to stem from a single vortex; it can be distributed throughout the region within or along the airfoil surface. However, the resulting circulation magnitude must satisfy the Kutta condition. A simplified approach is to concentrate the vorticity along the camber line, a concept foundational to the classic thin-airfoil theory developed by Munk and Glauert. This method is restricted to thin airfoils with small camber and positioned at small α_i [32].



Figure 31 Vorticity Concentrated on the Camber Line [32]

For an infinitely thin cambered plate at an angle of incidence, the lifting flow is modelled by a continuous line of infinitesimal vortices along the camber line. This line of variable vorticity replaces the camber line, with the total circulation around the chord being the sum of all vortex elements.

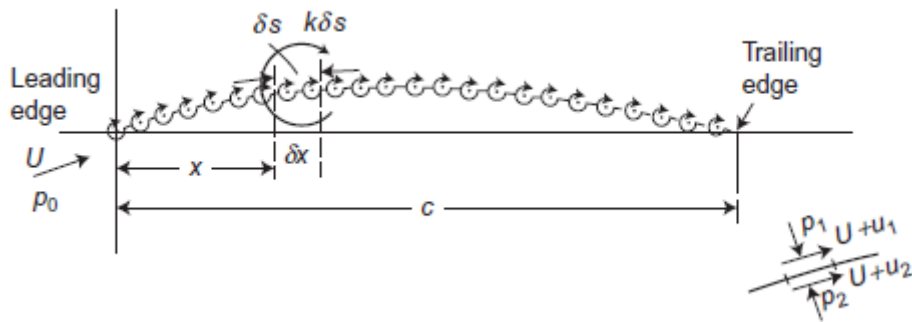


Figure 32 Vortex Sheet Model [32]

Then, the circulation is given by:

$$\Gamma = \int_0^c k \, ds \quad [32] \text{ Eq-47}$$

Where, k is the Vorticity of elementary vortex.

The elementary vorticity is given by:

$$k = 2 \cdot U \cdot \alpha \frac{(1 + \cos \theta)}{\sin \theta} \quad [32] \text{ Eq-48}$$

When Eq-47 is substituted in Eq-41 and by deriving equation for lift coefficient is given by:

$$C_L = \frac{L}{\frac{1}{2} \cdot \rho \cdot U^2 \cdot C} = 2 \cdot \pi \cdot \alpha \quad [32] \text{ Eq-49}$$

2.8 Theory of finite wing

2.8.1 Prandtl Lifting Line theorem

It is one of the numerical methods to analyse the aerodynamic characteristic of a wing. The invention of computers has revolutionized science and engineering, particularly in the field of aerodynamics, it allows to solve the *Navier-Stokes Equation* (NSE) for complex situation. But there are other numerical methods that are faster and inexpensive in terms of computer resources and also accurate like NSE. Those are lifting line theory and panel method (vortex lattice or doublet lattice). Lifting line theory can be used to calculate the performance of a straight wing[27]. The common method for calculating aerodynamic forces on a rotary wing is by LLT. In this theory a wing wake model is used to obtained loading on the blade section and a wake model is used to obtain the induced velocity. LLT allows to determine the 2D aerodynamics characteristic of a wing or a rotary wing from a 3D analysis[5]. In this theory wing is replaced by a straight line that is located through the aerodynamic centre of the wing. The circulation around the wing is replaced by vortex filament. This vortex filament lies along the span of the wing and the strength of the vortex is equal to the magnitude of the lift. The vortices lies on the wing are called *Bound Vortices* and the vortices extending in the downwash is called *Trailing Vortices*[23].

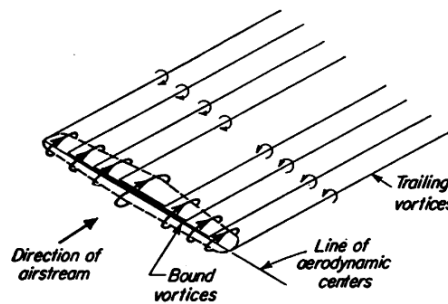


Figure 33 LLT model[23]

LLT solution was developed by Prandtl for an elliptical lift distribution and the relation between wing profile and the wing characteristics for elliptical lift distribution are given by:

$$C_D = C_d + \frac{C_L^2}{\pi \cdot A} \quad [23] \text{ Eq-50}$$

$$\alpha_i = \alpha_0 + \frac{C_L}{\pi A} \quad [23] \text{ Eq-51}$$

Glauert used Fourier analysis to develop a method to find solutions for a wing of any plan form and twist. Later *Anderson* applied *Glauert's* method to analyse the characteristics of wing with wide range of aspect ratio, straight taper and linear twist.

2.9 Theory of Propeller Blade

2.9.1 Vortex theorem

It is a circulation theory approach applied to a rotary wing. The vortex theory of the propeller models the rotating blades as generating trailing vortices that form helical vortex sheets in the slipstream. These vortices induce an "interference velocity" that alters the effective airflow around the blades, impacting lift and drag. Calculating this induced velocity is complex, so a common simplification assumes a high number of blades, creating a uniform velocity across each annular section of the propeller disc. With this assumption, the vortex theory aligns closely with blade element theory, where interference velocities are calculated using momentum theory and two-dimensional airfoil characteristics[1]. The Basic characteristics of vortices are determined by Biot-Savart law, Helmholtz law and Kelvin circulation law.

2.9.2 Blade Element Theory

Blade Element Theory (BET) is a method used to analyse the forces and performance of rotating blades, particularly for propellers, helicopter rotors, and wind turbine blades. The theory breaks down a blade into small elements along its length, allowing for a simplified, segment-by-segment analysis of lift and drag forces at each point on the blade. BET is a lifting line theory applied to a rotating wing. In this theory each blade section is considered as a 2D airfoil by integrating each sections aerodynamic forces along the propeller blade span entire blade characteristic can be analysed. This solution requires the wake induced velocity at the rotor disk, which is provided by the momentum theory, vortex theory or nonuniform inflow calculation. LLT is only for blade for high aspect ratio. The aspect ratio of the rotor blade is related to the solidity of the rotor disk which is discussed in section 2.5.2. BET is the ideal theory for rotor blade analyse because it deals with the detailed flow and loading of the blade. BET origin starts from the works of *William Froude* in 1878. But the major development was made by *Stefan Drzewiecki* between 1892 & 1920. For the derivation of force for a vertical flight of rotor blade low disk loading is assumed and neglecting stall and

compressibility effects[5]. In this study BET method is used to analyse the performance of the propeller.

3 Methodology

In this study the methodology is structured to provide a comprehensive analysis of the aerodynamic performance of a small racing quadcopter propeller through both analytical and Computer Numerical method. The primary aim of this approach is to create a framework for accurately predicting aerodynamic forces. For the analytical method though there are different theories and methods to evaluate the propeller performance, momentum actuator disk theorem and BET is used in this study due to its simplicity. The results that are obtained using other theories also give approximately similar results compared to the theories used in this study which is proved in the reference [36]. Due to the available computer resource ANSYS CFD CFX software is used to perform the numerical method with Reynolds-Average Navier-Stokes (RANS) study model.

For this study a simple racing quadcopter propeller (CW) with a straight blade (Constant Chord) planform and three blade configuration is considered due to the less requirement of the computational power. Some basic dimensions for a racing quadcopter propeller are taken from the DJI 5048S tri-blade propeller and DJI Snail 2305 racing motor series[37].



Figure 34 DJI Snail Motor & Propeller[38]

Data obtained from the reference propeller, motor and drone:

Radius of the propeller (R) = 63.5 mm = 0.0635 m

Hub diameter (D_H) = 20 mm = 0.02 m

Number of propellers per drone = 4

Number of blades per drone (N) = 3

Blade Span (b) = 53.5 mm = 0.0535 m

Chord $C = 20 \text{ mm} = 0.02 \text{ m}$

From Eq-17 the rotor disk area is given as:

$$A = \pi \cdot R^2 = 0.0127 \text{ m}^2$$

To assume a drone weight, a racing drone frame called *Martian-III Reptile* quadcopter frame which has 260 mm wheelbase is considered. Along with a 3S Li-po battery, flight controller and DJI snail ESC and other accessories, the weight is calculated to be around 500 gm including 4 motors and propellers.

Mass of the drone (m) = 500 gm = 0.5 kg

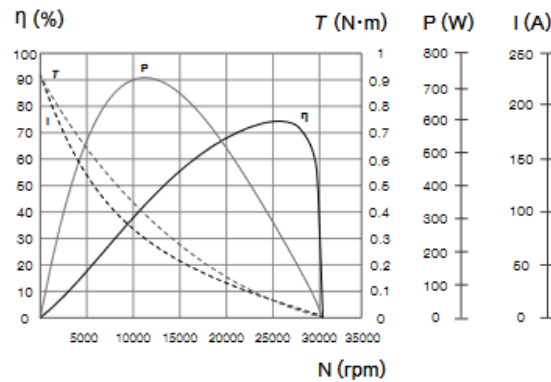
Based on the newton's second laws of motion the required thrust force for the drone of 0.5kg at hovering condition can be calculated as:

$$\text{Total required Thrust } (T) = m \cdot g = 0.5 \cdot 9.81 = 4.905 \text{ N}$$

$$\text{Thrust per propeller } (T) = \frac{\text{total require thrust}}{\text{number of propellers}} = 1.23 \text{ N}$$

$$\text{Thrust per blade } (T) = \frac{\text{thrust per propeller}}{\text{number of blade}} = 0.41 \text{ N}$$

2305 Racing Motor Performance



η- Efficiency, T- Torque, P- Output Power, I- Current, N- Rotational Speed

Figure 65 Motor Characteristic Curve of DJI 2305 DC racing motor[37]

The peak output power for the motor is at RPM approximately 11200.

Angular velocity of the propeller (Ω) = 1172.86 rad/s

Other required data

The drone is assumed to be hovering at the altitude of 100 m above the sea level.

$$h = 100 \text{ m}$$

To calculate the atmospheric air density below parameters are required. since the assumed altitude is below 11000m the atmospheric temperature at h can be calculated from Eq-16.

$$T^\circ = 14.39^\circ\text{C}$$

Since the assumed height is in troposphere the atmospheric pressure is given by:

$$p = 101.29 \cdot \left[\frac{T^\circ + 273.1}{288.08} \right]^{5.256} = 100.2 \text{ kPa} \quad [18] \text{ Eq-52}$$

Finally, the atmospheric air density at assumed altitude is given by:

$$\rho = \frac{p}{[0.2869 \cdot (T^\circ + 273.1)]} = 1.215 \text{ kg/m}^3 \quad [18] \text{ Eq-53}$$

0.2869 is the specific gas constant of air in kJ/kg K[39].

3.1 Modelling of Wing Profile

Among all the series of NACA airfoils, 4-digit symmetrical airfoils are commonly used for rotary wings applications like helicopter blades, aircraft propellers[40]. Propeller's required power is due to three factors drag, propulsive power and induced power. But airfoil drag has significantly less influence on the total required power. Thus, it is not required to invest more time in optimizing airfoils for propeller applications. The reference [41] proves that as long as the airfoil has a reasonable drag characteristic it is sufficient to use for the propeller applications. Since the propeller is for a quadcopter drone, its size is very small compared to airplane or helicopter propeller. So, the effects due to air flow like vortex, flow separation, are less important. An airfoil is only required to deviate the airflow from one side of the disk to the other side of the disk. A symmetrical airfoil is more than sufficient to do this job. A well preferred NACA symmetrical airfoil for electric small propeller application is NACA 0012 [41], [42], [43] which is used in this study.

3.1.1 NACA 4-digit airfoil name interpretation:

Each airfoil that are developed so far around the world are named in a unique way. Similarly, airfoils that are developed by NACA/NASA are name with numbers. Depends on the number of digits present in the name, it can be understood each airfoil belongs to what series. Each number in that name has a meaning to it. Since a 4-digit airfoil is used in this study, the interpretation of those 4 digits are given below[24], [25], [40]:

A 4-digit airfoil name starts with “NACA” followed by 4 digits. For example, NACA 2412 airfoil is used below:

NACA 2412

The first number “2”, represent max camber of that airfoil. Camber is the distance between chord line and mean line. It is given in percentage of chord.

$$\text{Maximum Camber } M_c = 2\% = \frac{2}{100} \cdot C = 0.02 \cdot C \text{ (Units of chord)}$$

The second digit “4” represents the location of the maximum camber from the LE in tenth of the chord.

$$P_c = \frac{4}{10} \cdot C = 0.4 \cdot C \text{ (Units of chord)}$$

Third and fourth digit together “12” gives the max thickness of the airfoil in percent of chord.

$$\text{Max thickness } Y_{tmax} = \frac{12}{100} \cdot C = 0.12 \cdot C \text{ (Units of chord)}$$

Since NACA 0012 is a symmetrical airfoil, it doesn't have camber. So, the first two digits are zero. Only maximum thickness of 12%. And the assumed chord length is 20mm. The max thickness of the airfoil is:

$$\text{max thickness of NACA 0012} = \frac{12}{100} \cdot C = 0.12 \cdot 20 = 2.4 \text{ mm} = 0.0024 \text{ m}$$

3.1.2 NACA 4-digit airfoil construction analytical method

References for all the information in section 3.1.2 [24], [25], [40], [44].

To construct NACA 0012 airfoil first it is constructed for chord 10mm, at the end, the ordinate and abscissas of the upper and lower surface is multiplied by 2 to get the coordinates for 20mm chord.

1. First step is to evaluate the mean line coordinates (X, Y_C) and it is given by:

$$Y_c = \frac{M_c}{P_c^2} (2 \cdot P_c \cdot X - X^2) \text{ from } X = 0 \text{ to } X = P_c \quad \text{Eq-54}$$

$$Y_c = \frac{M_c}{(1 - P_c^2)} [(1 - 2 \cdot P_c) + 2 \cdot P_c \cdot X - X^2] \text{ from } X = P_c \text{ to } X = C \quad \text{Eq-55}$$

For NACA 0012 there is no value for P_C and Y_C since there is no camber. X coordinate for chord 10 mm is divided into:

Table 1 Ordinates of NACA 0012 airfoil

No.	X	No.	X
0	0	11	0.5
1	0.005	12	0.55
2	0.05	13	0.6
3	0.1	14	0.65
4	0.15	15	0.7
5	0.2	16	0.75
6	0.25	17	0.8
7	0.3	18	0.85
8	0.35	19	0.9
9	0.4	20	0.95
10	0.45	21	1

2. Second step is to evaluate the thickness distribution Y_t . It is positive for upper surface and negative for lower surface.

$$\pm Y_t = \frac{t}{0.2} (0.2969 \cdot X^{0.5} - 0.1260 \cdot X - 0.3516 \cdot X^2 + 0.2843 \cdot X^3 - 0.1015 \cdot X^4) \quad \text{Eq-56}$$

3. Determining the θ_t value. It is the $\tan^{-1}$ slope of the mean line curve.

$$\theta_t = \arctan\left(\frac{dY_c}{dX}\right) \quad \text{Eq-57}$$

4. Finally evaluate the coordinates of upper and lower surface. (X_U, Y_U) is the ordinate and abscissa of upper surface and (X_L, Y_L) is the ordinate and abscissa of lower surface.

$$X_U = X - Y_t \cdot \sin\theta \quad \text{Eq-58}$$

$$Y_U = Y_C + Y_t \cdot \cos\theta \quad \text{Eq-59}$$

$$X_L = X + Y_t \cdot \sin\theta \quad \text{Eq-60}$$

$$Y_L = Y_C - Y_t \cdot \cos\theta \quad \text{Eq-61}$$

Using the above equation the coordinates for the NACA 4-digit airfoil can be determined for chord 1 unit. For this study the initially the chord is assumed to be 10 mm and the below coordinates of upper and lower surface is for chord of 20 mm.

Table 2 Coordinates of upper and Lower Surface of NACA 0012-Chord 20mm

X_U	Y_U	X_L	Y_L
0	0	0	0
0.01	0.02443	0.01	-0.02443
0.1	0.07109	0.1	-0.07109
0.2	0.09366	0.2	-0.09366
0.3	0.10690	0.3	-0.10690
0.4	0.11475	0.4	-0.11475
0.5	0.11882	0.5	-0.11882
0.6	0.12003	0.6	-0.12003
0.7	0.11897	0.7	-0.11897
0.8	0.11606	0.8	-0.11606
0.9	0.11161	0.9	-0.11161
1	0.10588	1	-0.10588
1.1	0.09905	1.1	-0.09905
1.2	0.09127	1.2	-0.09127
1.3	0.08265	1.3	-0.08265
1.4	0.07328	1.4	-0.07328
1.5	0.06321	1.5	-0.06321
1.6	0.05246	1.6	-0.05246
1.7	0.04105	1.7	-0.04105
1.8	0.02895	1.8	-0.02895
1.9	0.01613	1.9	-0.01613
2	0.00252	2	-0.00252

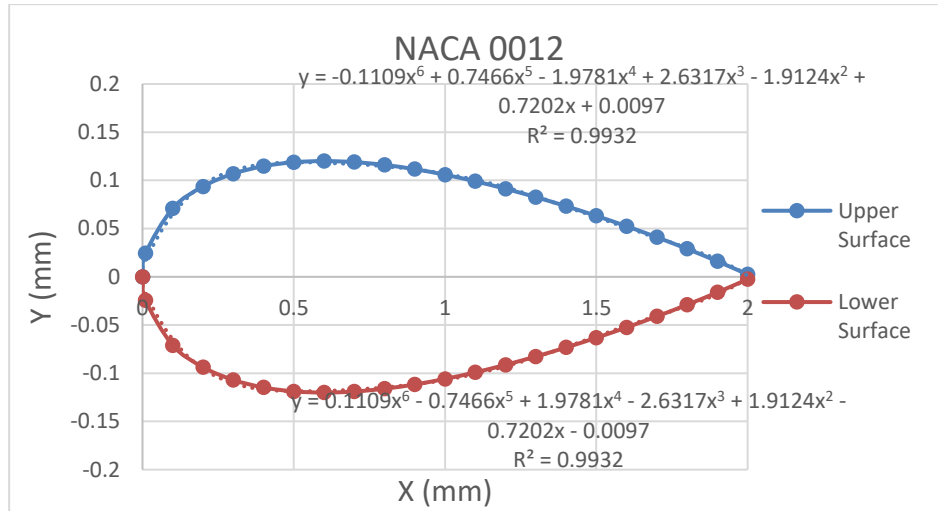


Figure 36 Constructed NACA 0012 Airfoil for Chord 20mm

In this method of construction, it gives an open trailing edge which is closed during 3D modelling with a finite angle trailing edge. The mathematically constructed curve is matched with the curve constructed using the coordinates from the NACA 0012 airfoil tool generator website which is the website of NASA. It can be seen in the below image. The “Validation” curve in the image is the curve by the coordinates from the NASA website. It align perfectly with the mathematically produced curve.

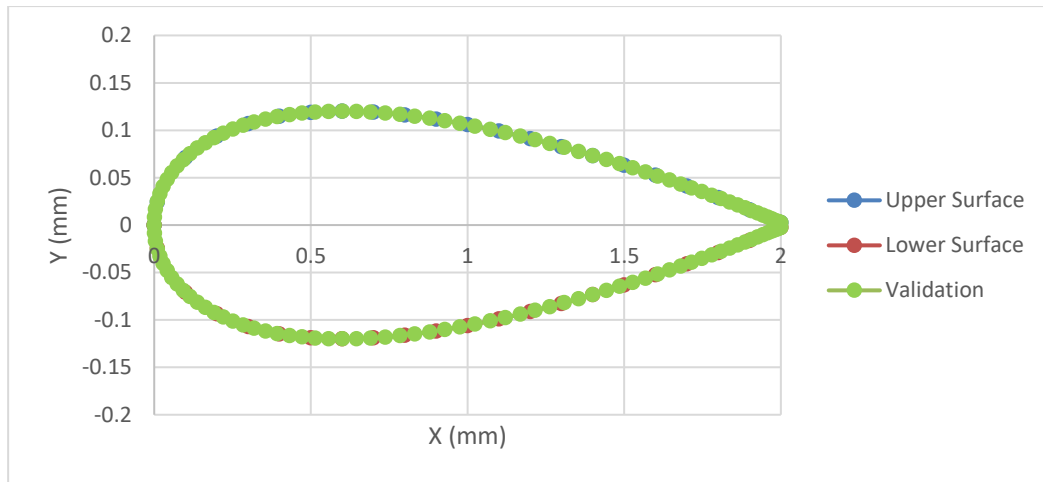


Figure 37 Validation of NACA 0012 airfoil surfaces

3.1.3 Lift and Drag coefficient Calculation

The coefficient of lift and drag can be obtained through the equations provided in the previous sections. But it gives the coefficients only for the inviscid flow. To find the coefficients for the viscous flow, it is difficult to calculate manually. So, software like XFOIL is used to find those values. To find the coefficients for viscous flow determining the

Reynold's number is mandatory. For the simplification of the study a uniform inflow inlet boundary condition is assumed. The coefficient of the airfoil at the blade root is applied throughout the blade in this study. So, the Reynold's number for the blade root is calculated and the coefficient of lift and drag of the airfoil at the blade root is found using XFOIL and applied throughout the blade span. For NACA 0012 the maximum pitch angle is 9°[45]. Reynolds number can be found using:

$$Re = \frac{\text{Inflow Velocity} \cdot \text{Chord Length}}{\text{Kinematic Viscosity}} = \frac{U \cdot C}{\gamma_k} \quad [46] \text{ Eq-62}$$

Inflow velocity has three components one is flight velocity V , rotational velocity U_T , cross-sectional induced velocity v . It is explained in the section 2.3 from Eq-5 to Eq-7. For hovering rotor $V=0$. Rotational velocity is equal to the motor rotational velocity. Since the motor's RPM is assumed to be 11200. The rotational velocity is given by the Eq-5.

$$U_T = \Omega \cdot r = 1172.86 \cdot r$$

The radial coordinate for the blades is divided into the following, it starts with the blade root. The hub is not considered in this coordinate.

Table 3 Radial Coordinate of a Blade

Radial Coordinate (m)
0.01
0.01535
0.0207
0.02605
0.0314
0.03675
0.0421
0.04745
0.0528
0.05815
0.0635

Induced velocity is calculated using the momentum theorem. From the Eq-33

$$v = \sqrt{\frac{T}{2 \cdot \rho \cdot A}} = \sqrt{\frac{0.41}{2 \cdot 1.215 \cdot 0.0127}} = 3.64 \text{ m/s}$$

Form Eq-6, the vertical component of the inflow velocity is:

$$U_p = V + v = 0 + 3.64 = 3.64 \text{ m/s}$$

Below table gives the rotational velocity along the radial distance and the inflow velocity U (Eq-7).

Table 4 Inflow velocity Across the Radial positions

r (m)	U _T (m/s)	U (m/s)
0.01	11.73	12.28
0.01535	18	18.37
0.0207	24.28	24.55
0.02605	30.55	30.77
0.0314	36.83	37.01
0.03675	43.10	43.26
0.0421	49.38	49.51
0.04745	55.65	55.77
0.0528	61.93	62.03
0.05815	68.20	68.30
0.0635	74.48	74.57

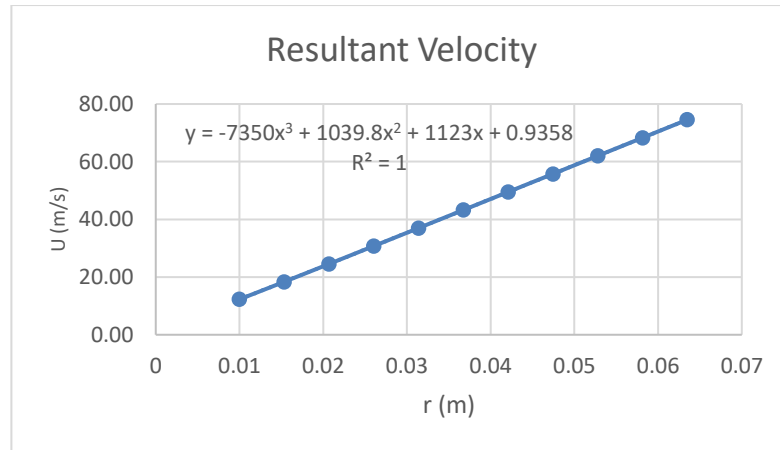


Figure 38 Inflow Velocity Across the Blade

The kinematic viscosity γ is given by:

$$\gamma_k = \frac{\text{dynamic viscosity}}{\text{air density}} = \frac{\mu}{\rho} \quad [46] \text{ Eq-63}$$

$$\text{dynamic viscosity coefficient } \mu = \left(\frac{T_k}{T_0}\right)^{1.5} \cdot \left(\frac{T_0 + S_\mu}{T_k + S_\mu}\right) \quad [46], [47] \text{ Eq-64}$$

Where, T_0 is the temperature at sea level, given by the Eq-16. T_k is the Temperature at reference altitude and S_μ is the Sutherland's Constant (111K)

$$\text{From Eq - 64, } \mu = 1.714 \cdot 10^{-5} \frac{Ns}{m^2}$$

$$\text{From Eq - 63, } \gamma = 1.411 \cdot 10^{-5} \frac{m^2}{s}$$

Substituting inflow velocity at blade root $U=12.28\text{m/s}$ from table 4, $C=0.02\text{m}$, γ_k in Eq-62,

$$Re = 17400$$

For this Reynolds number the coefficient of lift and drag is calculated using the XFOIL software to be:

$$C_L = 0.3 \text{ \& } C_D = 0.1$$

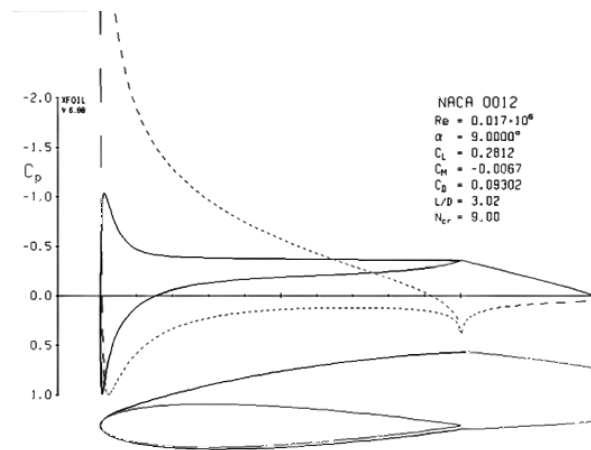


Figure 39 Coefficient of Lift & Drag Calculation using XFOIL at $Re=17400$

3.2 Aerodynamic Forces Calculation By Analytical Method

3.2.1 Lift Force calculation

The lift of the airfoil can be calculated by rearranging the Eq-49.

$$C_L = \frac{L}{\frac{1}{2} \cdot \rho \cdot U^2 \cdot C} \rightarrow L = \frac{1}{2} \cdot \rho \cdot U^2 \cdot C \cdot C_L$$

Since the inflow velocity change with respect to the radial coordinate. The lift produced by a blade also changes. Below table give the lift with respect to the radial coordinate.

Table 5 Lift Force of a Blade with respect to the r

r (m) - radial coordinate	L (N) - (Cross-Sectional Lift)
0.01	0.550
0.01535	1.230
0.0207	2.197
0.02605	3.451
0.0314	4.992
0.03675	6.820
0.0421	8.935
0.04745	11.338
0.0528	14.027
0.05815	17.003
0.0635	20.266

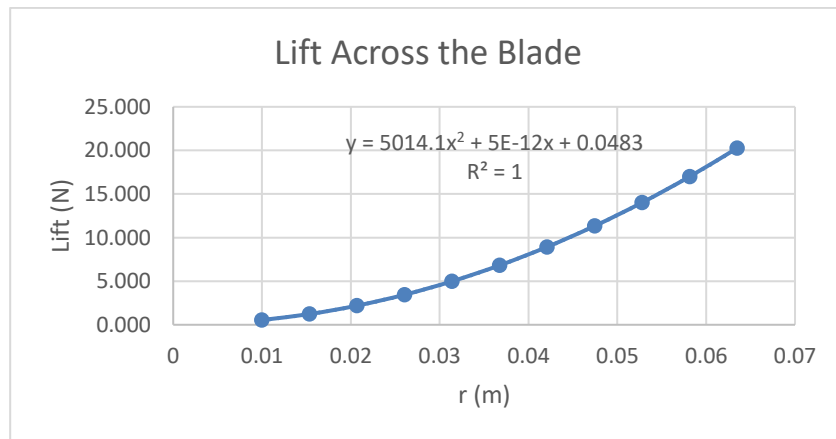
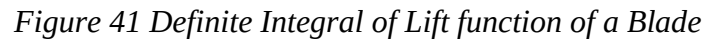


Figure 40 The lift function with respect radial coordinate of a Blade

Above graph shows the lift distribution across the radius of the blade. By interpolation the quadratic equation of the lift curve is also shown in the figure. To find the total lift force of the blade, definite integration is performed:

$$\int_{0.01}^{0.0635} \frac{dy}{dx} = \int_{0.01}^{0.0635} 5014.1x^2 + 5E - 12x + 0.0483$$



Total lift produced by a propeller = $0.4289 \cdot 3 = 1.2867\text{N}$.

The drag force of the airfoil can be found by the below equation:

Since the lift changes along the span of the blade the drag also changes along the span of the blade, and it is given in the following table.

r (m) - radial coordinate	D (N) - (Cross-Sectional Drag)
0.01	0.183
0.01535	0.410
0.0207	0.732
0.02605	1.150
0.0314	1.664
0.03675	2.273
0.0421	2.978

0.04745	3.779
0.0528	4.676
0.05815	5.668
0.0635	6.755

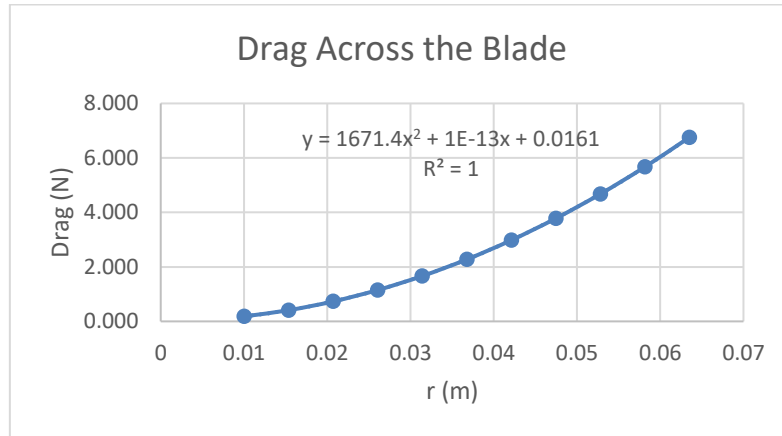


Figure 42 The Drag function with respect radial coordinate of a Blade

Above graph shows the Drag distribution across the radius of the blade. By interpolation the quadratic equation of the Drag curve is also shown in the figure. To find the total drag force of the blade, definite integration is performed:

$$\int_{0.01}^{0.0635} \frac{dy}{dx} = \int_{0.01}^{0.0635} 1671.4x^2 + 1 \cdot E - 13x + 0.0161$$

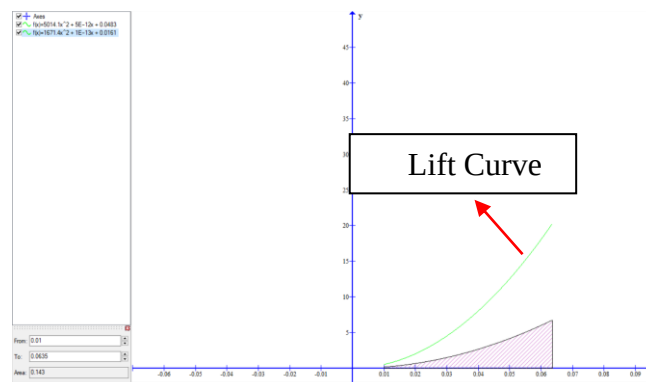


Figure 43 Definite Integral of Drag function of a Blade

Area under the curve = Total drag force of the blade = 0.143N

Since the number of blades are 3.

Total drag produced by a propeller = $0.143 \cdot 3 = 0.429\text{N}$.

3.2.3 Thrust Force Calculation

Form the figure 21 it can be understood that in a propeller, thrust is the sum of vertical force of lift and drag. Vertical force of lift and drag acts in opposite direction, so the vertical component of drag force need to be subtracted from the vertical force of the lift. The vertical components of the lift and drag can be found if the inflow angle Φ is known and it can be calculated using the Eq-8. The inflow angle as a function of r is given below in the table.

Table 7 Inflow Angle with Respect to the Radial Coordinates

r (m)	ϕ -inflow angle
0.01	0.30
0.01535	0.20
0.0207	0.15
0.02605	0.12
0.0314	0.10
0.03675	0.08
0.0421	0.07
0.04745	0.07
0.0528	0.06
0.05815	0.05
0.0635	0.05

Eq-22 gives the elementary thrust force and the thrust force across the span of the blade is in the below table.

Table 8 Thrust force with Respect to the Radial Coordinates

r (m) - radial coordinate	Thrust (N)
0.01	0.47
0.01535	1.12
0.0207	2.06
0.02605	3.29

0.0314	4.80
0.03675	6.60
0.0421	8.69
0.04745	11.07
0.0528	13.73
0.05815	16.68
0.0635	19.91

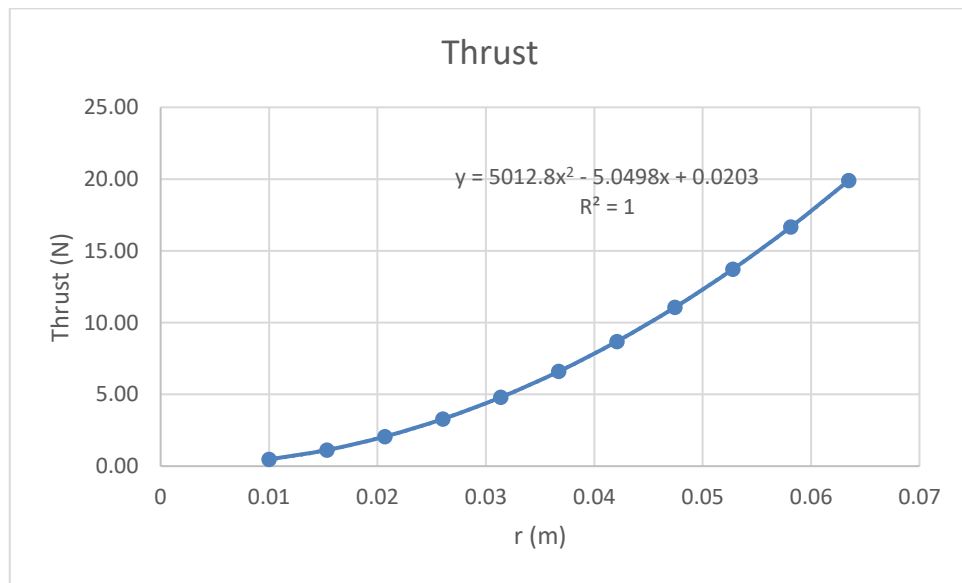


Figure 44 Function of Thrust with Respect to r

Above graph shows the Thrust distribution across the radius of the blade. By interpolation the quadratic equation of the thrust curve is also shown in the figure. To find the total thrust force of the blade, definite integration is performed:

$$\int_{0.01}^{0.0635} \frac{dy}{dx} = \int_{0.01}^{0.0635} 5012.8x^2 - 5.0498x + 0.0203$$

0.0528	5.49
0.05815	6.57
0.0635	7.74

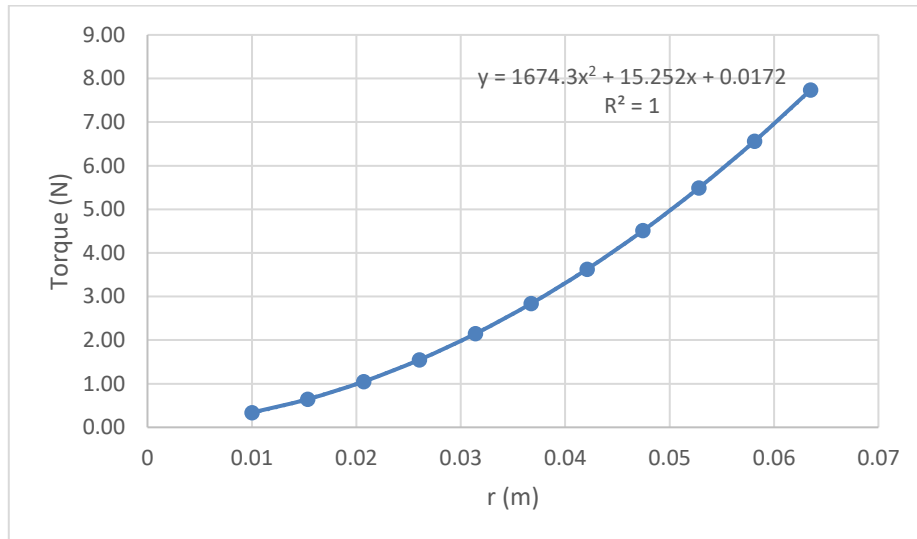


Figure 46 Function of Tangential Force with Respect to r

Above graph shows the tangential force distribution across the radius of the blade. By interpolation the quadratic equation of the torque curve is also shown in the figure. To find the total tangential force of the blade, definite integration is performed:

$$\int_{0.01}^{0.0635} \frac{dy}{dx} = \int_{0.01}^{0.0635} 1674.3x^2 + 15.252x + 0.0172$$

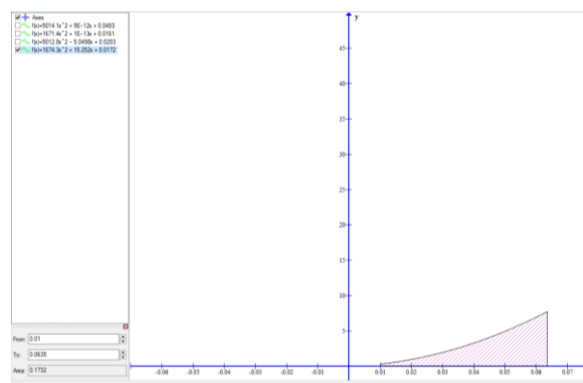


Figure 47 Definite Integral of Torque function of a Blade

Area under the curve = Total tangential force of the blade = 0.1732N

Since the number of blades are 3.

Total tangential force produced by a propeller = $0.1732 \cdot 3 = 0.5196\text{N}$.

Since the drone has four propellers, two propellers will rotate in clockwise and other two will rotate in counterclockwise direction. So, this torque will be cancelled out.

3.3 Propeller coefficients Calculation by Analytical method

The performance of a propeller is estimated using the thrust and torque coefficients of a propeller which are given below.

$$\text{Thrust Coefficient } C_T = \frac{T}{\rho \cdot A \cdot (\Omega \cdot R^2)} = 0.015 \quad [5] \text{ Eq-66}$$

$$\text{Torque Coefficient } C_Q = \frac{Q}{\rho \cdot A \cdot (\Omega \cdot R^2) \cdot R} = 0.096 \quad [5] \text{ Eq-67}$$

Since $C_P = C_Q$ [5]

$$P = C_P \cdot \rho \cdot A \cdot (\Omega \cdot R)^3 = 612\text{W} \quad [5] \text{ Eq-68}$$

At 11200RPM (1172.86 rad/s), the motor output power is around 710 W, which can be seen in the motor characteristic curve of figure 41.

3.4 Design Procedure of Drone Propeller

Hub of 20 mm diameter is created at the centre. The coordinates obtained in table 2 is plotted in the SolidWorks and a 2D curve is drawn with the centre point of the chord align with the central axis of the hub. The airfoil is positioned at 9° . The hub resembles the hub of DJI 5048S tri-blade propeller without the detailed features.

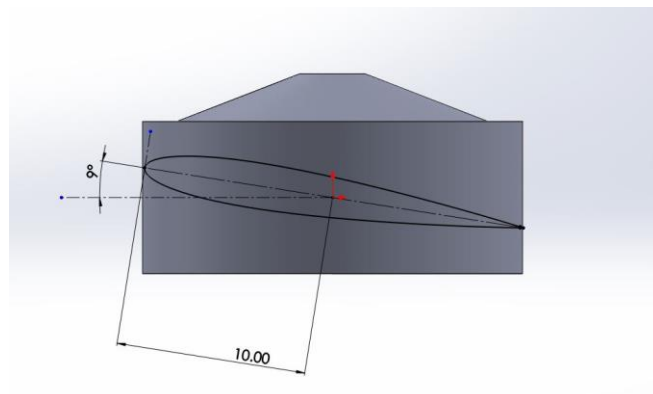


Figure 48 Hub & Airfoil Designed in SolidWorks

The trailing edge of the airfoil is closed with a finite angle method as shown below.

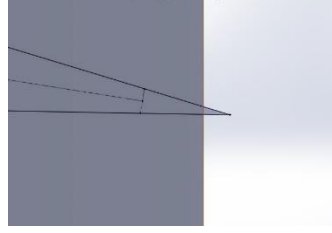


Figure 49 Airfoil Trailing Edge

The blade of span 53.5 mm is extruded as a solid body and using circular pattern tri-blade propeller is created.

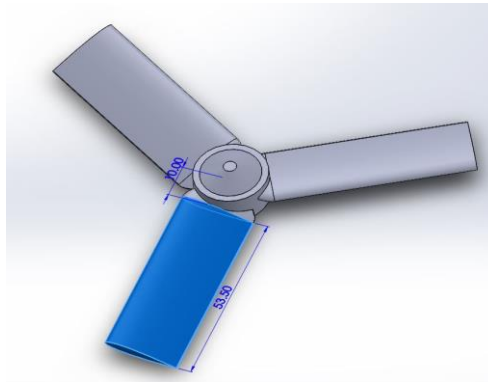


Figure 50 Designed Tri-Blade propeller with NACA 0012 Wing Section

3.5 Simulation Setup

ANSYS “Fluid Flow - CFX” RANS study method is used for fluid simulation. The designed propeller’s 3D model is uploaded as a geometry of the study. To reduce the mesh element count only the air near the rotor was modelled in this study. This proved to be the right choice in hindsight. The mesh comprised of 1 896 343 cells, with a base element size of 2 mm. To delineate the near wall zone, 10 layers of prismatic elements with a thickness of 0.5 mm were incorporated.

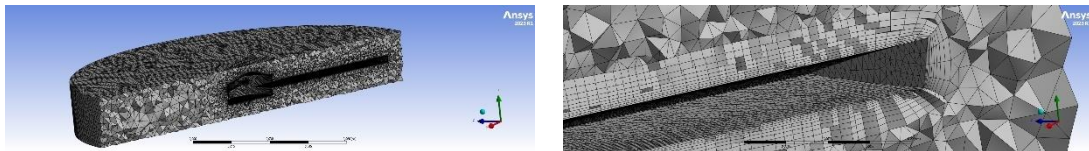


Figure 71 Mesh Model of the Propeller

In the setup incompressible ideal air at 25 °C at 1atm pressure was used due to the subsonic flow regime operation. In order to establish the conditions, incompressible ideal gas at 25 °C and one atmosphere of pressure was utilized, since the flow regime was subsonic. The turbulence model was SST k- ω . The boundary condition for the side and top of the air domain, pressure outlet were employed.

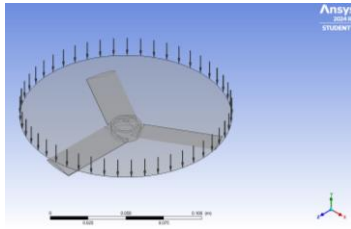


Figure 82 Inlet Boundary Condition

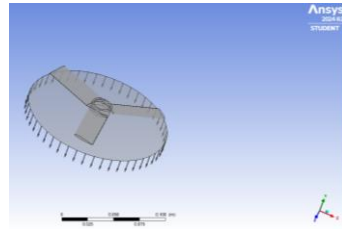


Figure 93 Outlet Boundary Condition

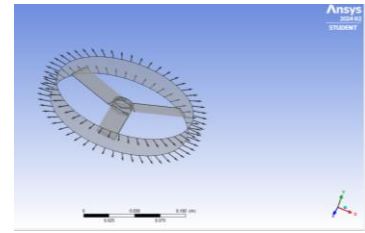


Figure 104 Side Boundary Condition

In the case of the rotor surface, a no-slip boundary condition was employed. In regard to the inlet boundary condition, the calculated value of 3.64 m/s was selected. The simulation comprised a minimum of 300 iterations, with the objective of achieving a residual value of 10^{-5} .

4 Results

From the Numerical simulation total vertical force of the propeller i.e., total force in Y direction is obtained. A parameter set analysis is done to analyse the output performance based on the different element sizes. Some of the input parameters are given in the table below.

Table 10 Parameter Set Analysis Results

	Mesh Element Size (mm)	Mesh Elements	Vertical Force (N)
1	8	0.8 M	1.432
2	4	1.0 M	1.540
3	2	1.8 M	1.360
4	4	3.5 M	1.174

Since the inlet boundary condition is set to be uniform flow. The flow velocity along the radius from the centre can be seen below.

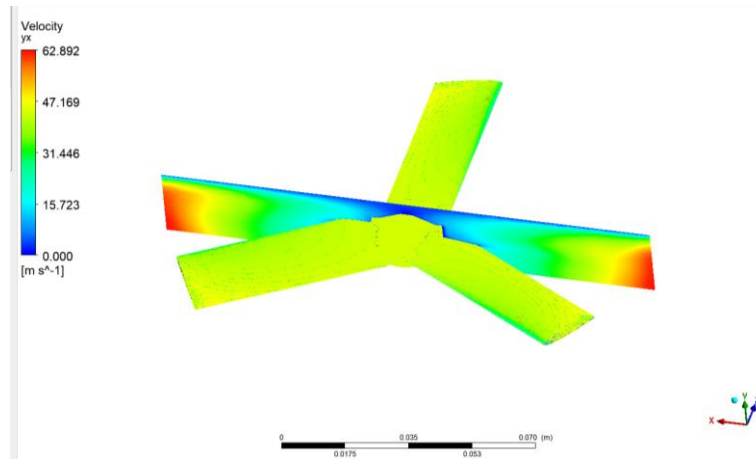


Figure 115 Inflow Velocity across the Radius

At higher velocity stagnation point occurs at the LE of the airfoil. Since the airfoil is positioned at an angle of 9° , the stagnation point moves to the LE of the lower surface. This is a phenomenon due to the circulation around the airfoil, which is explained by the lift of a rotating cylinder. In the below image the LE closer to the blade tip shows higher pressure due to stagnation because the rotational velocity of the blade increases with increase in radius.

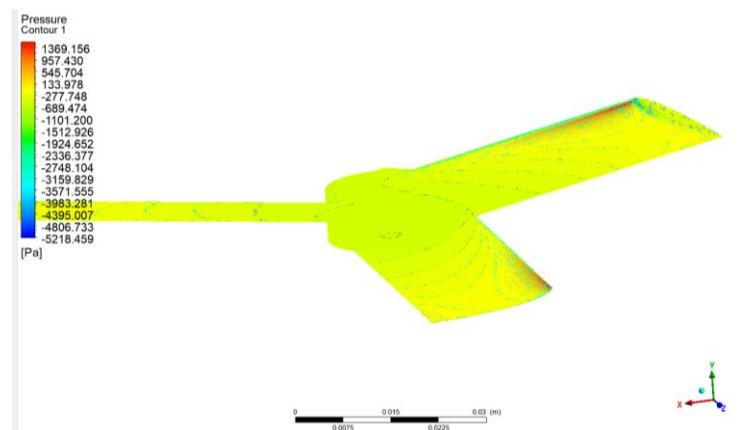


Figure 512 Stagnation Near Blade Tip

Air across the propeller flows in a helical motion. Since the simulation is set to the air near the propeller, the airflow within the rotor disk can be seen below, which has a helical motion.

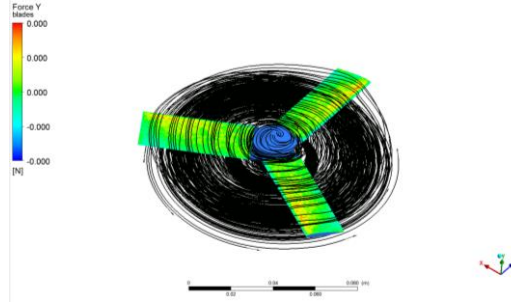


Figure 57 Airflow Across Propeller Disk

Flow around the wing section can be seen blow.

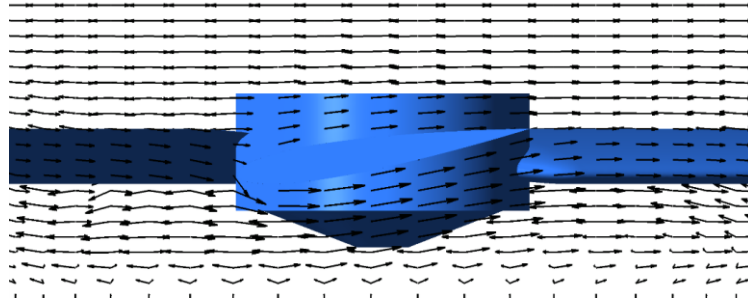


Figure 58 Flow Around the Airfoil

5 Discussions

In the study of drone propeller performance, predicting the thrust is critical. This research presents a comprehensive analysis of the thrust force calculated using both analytical and numerical simulation, shows the degree of accuracy and reliability of each method. In the analytical method thrust and torque is the vertical and horizontal forces of a propeller. In simulation the force in Y direction is the thrust and force in X direction is the torque. The analytical method yields a thrust per propeller value of 1.25 N, calculated through established theoretical models that consider various aerodynamic parameters. In the numerical simulation CFD analysis was conducted. The most accurate result was achieved using an element size of 2 mm, resulting in 1.9 million elements in the mesh. This setup produced a vertical force of 1.36 N, showing a strong correlation with the analytical result with a significantly less error.

$$\begin{aligned} \text{Percentage of error} &= \left(\frac{\text{Numerical value} - \text{Analytical Value}}{\text{Analytical Value}} \right) \cdot 100\% \\ &= 8.67\% \text{ error} \end{aligned}$$

This error could occur due to the multiple reasons, mainly due to the accuracy of the results from both methods. Accuracy of the analytical value may be affected by the rounding of digits and the accuracy of the parameter obtained from the software like XFOIL. Each

software has a margin of error in its results. Accuracy of the numerical value may be affected by the study model (RANS) used. And due to the difference in the boundary condition definition. Anyway, these errors can be rectified by iterative calculation of both analytical and numerical method.

The close relation of the results from both methods supports the use of analytical method for a quick, initial assessments of propeller performance, while numerical simulation offer more detailed insight for design optimization. This approach can be utilized to optimize propeller in small drones, particularly those used in precision application like photogrammetry in industrial processes.

This research work has also been submitted as a graduation thesis for BSc Mechanical Engineering, Automotive production Process Specialization.

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