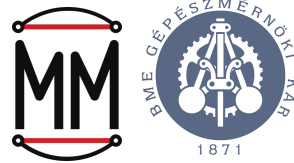




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Modelling of the Structural Dynamics of a Boring Bar Subjected to Bending and Torsion

by

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Abstract

This work covers the determination of the overall dynamics of long boring bars that are prone to torsional vibrations. The model consists of a parametric mechanical model of the bar, the passive absorber, and the measured machine dynamics. The possibility to include the machine tool frequency response functions (FRF) is ensured by using receptance coupling substructure analysis (RCSA). A vibration absorber is introduced into the mechanical system and the receptances at the corresponding cutting point are compared with the unattenuated mechanical system. A parameter optimization scheme is defined for choosing the optimal combination of parameters that would minimize the vibrations during the envisioned cutting process.

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Chapter 1

Introduction

One of the most common manufacturing processes for high valued parts is metal cutting (Shaw and Cookson 2005). The current attempts to reduce the carbon footprint of machined parts results in the design and production of thinner and lighter parts. However, this presents a new set of challenges. Thinning any part would negatively affect its stiffness and damping, and eventually lead to vibrations that can occur during the manufacturing of the part as well as the operation processes. Addressing this would require new sophisticated techniques in design and machining processes.

This work focuses on turning operation, in particular examines the boring bar system dynamics. The boring operation is used for increasing an existing hole diameter in a workpiece and provides the finishing of the part. As the design of such parts is limited by the length of the boring bar, increasing it is preferred. The boring bars are characterised by their length to diameter ratio, and once the ratio of around 4-5 is achieved, their static and dynamic stiffness drastically deteriorates (Rivin and Kang 1992). The reduction of the dynamic stiffness eventually leads to the so-called self-excited vibrations during the metal cutting procedure, also known as chatter (Tobias and Fishwick 1958, Altintas 2012). Typically, the vibrations are characterised by the bending modes and a number of suppression techniques are present in the research and industry fields, such as passive vibration absorbers (Rubio et al. 2013). However, this work poses a question of the torsional effects of the boring bars during the cutting process.

The main research question of this work is to determine to what extent the torsional effects are present in the boring operation of long boring bars. If the effects are found to be significant, what are the possible ways to attenuate these effects in order to achieve the high surface finish of the manufactured parts.

This work begins with an analytical modelling of a parametric boring bar system that

considers bending and torsional behaviour of the model. This mechanical model is used to analyse and compare the cutting tip receptances that originate from both bending and torsion. A well-known solution, that involves an introduction of a passive vibration absorber to the boring bar system, is implemented to improve the stability of the cutting process. Since the torsional behaviour is also considered in this analytical model, the effects of the introduced absorber on the torsional dynamics are analysed, where the absorber is tuned to primarily attenuate the bending vibrations. Next, a number of design parameter optimization methodologies are presented that focus on minimizing the vibrational effects and improving the stability of the boring bar system. Lastly, the measured machine dynamics, which can be applied during the Receptance Coupling Substructure Analysis (RCSA), are being utilized to improve the purely analytical model of the boring bar model (Astarloa et al. 2022).

In summary, in this work the boring bar system is described analytically, its kinetics characteristics are being analysed with an attempt to optimize the parametric mechanical model. Lastly, a measurement is performed to determine the machine dynamics and obtain a combined receptance.

Chapter 2

Analytical modelling

This chapter covers the analytical modelling of the boring bar machine structure. Firstly, the mechanical model is introduced. Next, a number of necessary definitions and methodologies are presented in order to derive the analytical response of the assembled boring bar structure. Additionally, the obtained analytical responses are verified with the numerical simulations performed on Ansys Mechanical software.

2.1 Mechanical Model

The parametric mechanical model of the boring bar consists of five separate segments, namely a machine, a solid segment, a hollow segment, a cutting head, and a passive absorber, see Figure 2.1.

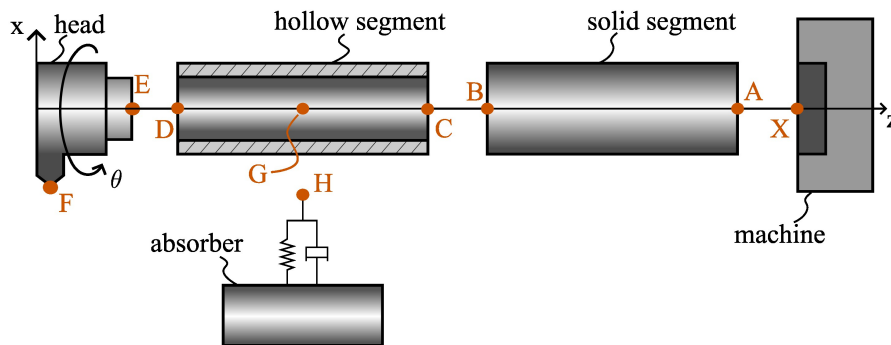


Figure 2.1: Mechanical model

In order to assemble the presented mechanical system, it is necessary to determine the dynamic properties of the separate segments. Once these are determined, the next step is to connect the separate elements. This is done according to the RCSA method. This methodology is based on the FRFs, or in other words, the responses in the frequency domain.

2.2 Frequency Response Function

The modelling in this work is mainly done in the frequency domain rather than the time domain. As a result the FRF must be introduced.

The spatial theoretical model can be described by the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t), \quad (2.1)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and the stiffness matrices respectively, $\mathbf{x}(t)$ is the displacement vector, and $\mathbf{F}(t)$ is the excitation vector.

It is necessary to transform the equation 2.1 from the time domain to the frequency domain, i.e. take a Laplace transform followed by a Fourier transform. The result is:

$$(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K})\mathbf{x}(s) = \mathbf{F}(s) \quad (2.2)$$

$$(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})\mathbf{x}(i\omega) = \mathbf{F}(i\omega). \quad (2.3)$$

In the following equations in this work, let $\mathbf{A}(\omega) \equiv \mathbf{A}(i\omega)$.

The frequency response function (FRF) is defined as the inverse of the stiffness function.

$$\mathbf{H}(\omega) = (-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})^{-1}. \quad (2.4)$$

Thus, the relationship between the response $\mathbf{X}(\omega)$, which is basically the surface quality of the workpiece, and the forcing $\mathbf{F}(\omega)$ is as follows:

$$\mathbf{X}(\omega) = \mathbf{H}(\omega)\mathbf{F}(\omega). \quad (2.5)$$

The analytical description of the boring bar is modelled in the three dimensional space, and the mechanical model's FRF is defined as:

$$\mathbf{H}_{i,j}(\omega) := \begin{bmatrix} H_{i,j}(\omega) & L_{i,j}(\omega) & P_{i,j}(\omega) \\ M_{i,j}(\omega) & N_{i,j}(\omega) & Q_{i,j}(\omega) \\ V_{i,j}(\omega) & W_{i,j}(\omega) & T_{i,j}(\omega) \end{bmatrix} \begin{pmatrix} \text{m/N} & \text{m/Nm} & \text{m/Nm} \\ \text{rad/N} & \text{rad/Nm} & \text{rad/Nm} \\ \text{rad/N} & \text{rad/Nm} & \text{rad/Nm} \end{pmatrix}, \quad (2.6)$$

where i stands for the sensing point, j stands for the excitation point, $H_{i,j}(\omega)$, $L_{i,j}(\omega)$, $M_{i,j}(\omega)$, and $N_{i,j}(\omega)$ describe the responses related to the bending modes, $T_{i,j}(\omega)$ describes the torsional mode response, and $P_{i,j}(\omega)$, $Q_{i,j}(\omega)$, $V_{i,j}(\omega)$, and $W_{i,j}(\omega)$ describe the coupling terms.

According to Banerjee and Williams (1992), there is no mode coupling effects between the bending and the torsional modes if the mass axis and the elastic band, i.e. the loci of the mass centre and the shear centre of the cross-section, are coincident. As this is the exact case for a beam model, the defined FRF in the equation 2.6 can be simplified to:

$$\mathbf{H}_{i,j}(\omega) := \begin{bmatrix} H_{i,j}(\omega) & L_{i,j}(\omega) & 0 \\ M_{i,j}(\omega) & N_{i,j}(\omega) & 0 \\ 0 & 0 & T_{i,j}(\omega) \end{bmatrix} \begin{pmatrix} \text{m/N} & \text{m/Nm} & \text{m/Nm} \\ \text{rad/N} & \text{rad/Nm} & \text{rad/Nm} \\ \text{rad/N} & \text{rad/Nm} & \text{rad/Nm} \end{pmatrix}. \quad (2.7)$$

The following sections focus on the algebraic manipulations and the derivations of the defined FRF above.

2.3 Receptance Coupling Substructure Analysis

Receptance coupling is a technique that can assemble FRFs of separate structures and is usually used to combine the measured responses with an analytically derived FRFs (Schmitz, Davies, and Kennedy 2001).

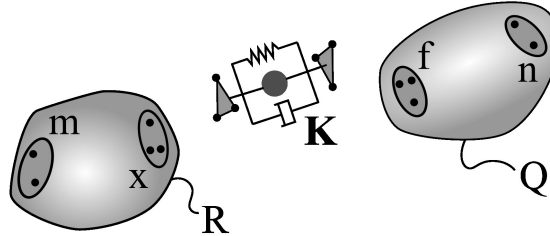


Figure 2.2: Structure R , structure Q , and connection $\mathbf{K}$

In this work the following notations for the receptances are taken:

- $\mathbf{R}_{i,j}$: analytical FRF matrix,
- $\mathbf{Q}_{i,j}$: analytical FRF matrix,
- $\mathbf{G}_{i,j}$: machine FRF matrix,
- $\mathbf{H}_{i,j}$: predicted FRF matrix,
- $\Phi_{i,j}$: measured FRF.

The RCSA is based on Newton's Laws, and according to Schmitz and Duncan (2005)

is described by the following set of equations:

$$\mathbf{H}_{n,n} = \mathbf{R}_{n,n} - \mathbf{R}_{n,f}(\mathbf{I} + \mathbf{K}, (\mathbf{Q}_{x,x} + \mathbf{R}_{f,f}))^{-1} \mathbf{K} \mathbf{R}_{f,n} \quad (2.8)$$

$$\mathbf{H}_{m,m} = \mathbf{Q}_{m,m} - \mathbf{Q}_{m,x}(\mathbf{I} + \mathbf{K}, (\mathbf{R}_{f,f} + \mathbf{Q}_{x,x}))^{-1} \mathbf{K} \mathbf{Q}_{x,m} \quad (2.9)$$

$$\mathbf{H}_{m,n} = \mathbf{Q}_{m,x}(\mathbf{I} + \mathbf{K}, (\mathbf{Q}_{x,x} + \mathbf{R}_{f,f}))^{-1} \mathbf{K} \mathbf{R}_{f,n}. \quad (2.10)$$

The presented equations describe the predicted direct and cross FRFs of an assembly of the two separate structures R and Q , see Figure 2.2. The set of structures is being characterised by the known receptances between the sets of points m and x for the structure R , and f and n for the structure Q . In addition, the contact properties are stored in the matrix $\mathbf{K}$.

If the connection properties are assumed to be perfectly rigid, the equations introduced above can be simplified to:

$$\mathbf{H}_{n,n} = \mathbf{R}_{n,n} - \mathbf{R}_{n,f}(\mathbf{Q}_{x,x} + \mathbf{R}_{f,f})^{-1} \mathbf{R}_{f,n}, \quad (2.11)$$

$$\mathbf{H}_{m,m} = \mathbf{Q}_{m,m} - \mathbf{Q}_{m,x}(\mathbf{R}_{f,f} + \mathbf{Q}_{x,x})^{-1} \mathbf{Q}_{x,m}, \quad (2.12)$$

$$\mathbf{H}_{m,n} = \mathbf{Q}_{m,x}(\mathbf{Q}_{x,x} + \mathbf{R}_{f,f})^{-1} \mathbf{R}_{f,n}. \quad (2.13)$$

These equations are used throughout the work to either add or subtract one structure to another. The equations presented in this section are used to make connection between points X and A, B and C, D and E, and finally G and H, see figure 2.1.

2.4 Bending Response

This section focuses on the analytical derivation of the beam element responses related to bending modes.

Timoshenko beam

The analytical model of the solid and hollow segments in this work is described by the well-known Timoshenko–Ehrenfest beam theory (Timoshenko 1921).

The Timoshenko–Ehrenfest beam dynamics, Figure 2.3a, with uniform cross-section

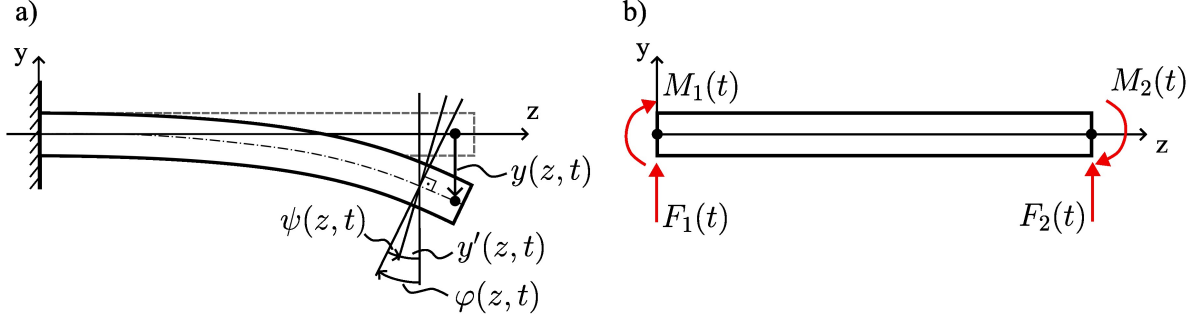


Figure 2.3: a) Timoshenko beam, b) Boundary conditions

are described by the following coupled system of two partial differential equations (PDE):

$$\begin{cases} \rho A \ddot{y} &= \kappa G A (y'' - \phi'), \\ \rho I_x \ddot{\phi} &= I_x E \phi'' + \kappa G A (y' - \phi). \end{cases} \quad (2.14)$$

The analytical model assumes the following: $y(z, t)$ is the transversal displacement, $\phi(z, t)$ is the angular displacement of the cross section (Euler-Bernoulli beam theory), $\psi(z, t)$ is the angular displacement due to shear force, and y' is the total local angular displacement. These parameters have a direct relationship with each other and both types of excitations, namely bending moment and shear force:

$$y'(z, t) = \phi(z, t) + \psi(z, t), \quad (2.15)$$

$$M_B(z, t) = -I_x E \phi'(z, t), \quad (2.16)$$

$$V(z, t) = \kappa A G \psi(z, t), \quad (2.17)$$

where I_x is the second moment of area along the x axis, E is the elastic modulus, κ is the so-called Timoshenko shear coefficient, Hutchinson (2000), A is the cross-sectional area, ρ is the density, and G is the shear modulus.

Since the segments are modelled as straight, homogeneous and prismatic beams, the system of equations 2.14 can be decoupled. The result is presented below:

$$\rho A \ddot{y}(z, t) + I_x E y''''(z, t) - \rho I_x \left(1 + \frac{E}{\kappa G}\right) \ddot{y}'' + \frac{\rho^2 I_x}{\kappa G} \ddot{y} = 0. \quad (2.18)$$

For a mechanical system with no damping, the trial solution is assumed in the following way:

$$y(z, t) = Y(z) e^{i\omega t}. \quad (2.19)$$

By substituting the trial solution 2.19 back into the the equation of motion 2.18, the following ordinary differential equation (ODE) is obtained:

$$Y''''(z) + \omega^2 \left(\frac{1}{E} + \frac{1}{\kappa G} \right) Y''(z) + \left(\omega^4 \frac{\rho^2}{E\kappa G} - \omega^2 \frac{\rho A}{I_x E} \right) Y(z) = 0. \quad (2.20)$$

Let the trial solution for the ODE presented above be defined as $Y(z) = Ce^{\lambda z}$. After the substitution back into the equation 2.20, the following characteristic equation is obtained:

$$\lambda^4 + \omega^2 \left(\frac{1}{E} + \frac{1}{\kappa G} \right) \lambda^2 + \left(\omega^4 \frac{\rho^2}{E\kappa G} - \mu^4 \right) = 0, \quad (2.21)$$

where μ is a new parameter that originates from the Euler-Bernoulli beam model and is defined in the following way:

$$\mu := \sqrt[4]{\omega^2 \frac{\rho A}{I_x E}}. \quad (2.22)$$

The solution of the characteristic equation 2.21 yields the following roots and the new parameters δ and γ are being introduced:

$$\begin{aligned} \lambda_{1,2} &= \pm \frac{1}{\sqrt{2}} \sqrt{\beta - \alpha} := \pm \delta, \\ \lambda_{3,4} &= \pm \frac{1}{\sqrt{2}} \sqrt{-\beta - \alpha} := \pm i\gamma, \end{aligned} \quad (2.23)$$

where

$$\alpha = \frac{E + \kappa G}{\kappa G E} \rho \omega^2, \quad (2.24)$$

$$\beta = \frac{1}{\kappa G E} \sqrt{4(\kappa G E)^2 \mu^4 + (E - \kappa G)^2 \rho^2 \omega^4}. \quad (2.25)$$

Thus, the solution of the ODE 2.20, i.e the mode shape function, is defined in the following way:

$$Y(z) = C_1 e^{\delta z} + C_2 e^{-\delta z} + C_3 e^{i\gamma z} + C_4 e^{-i\gamma z}, \quad (2.26)$$

which can be rewritten as:

$$Y(z) = \tilde{C}_1 \cosh(\delta z) + \tilde{C}_2 \sinh(\delta z) + \tilde{C}_3 \cos(\gamma z) + \tilde{C}_4 \sin(\gamma z). \quad (2.27)$$

Boundary Conditions

The first step in the application of the free-free Boundary Conditions (BC) is to re-write the bending moment function 2.16 and the shear force function 2.17 in terms of the mode shape function $Y(z)$, see Figure 2.3b.

The equation 2.15, which described the relation between the spatial derivative of the transversal displacement of the beam with the angular displacements related to both shear force and bending moment, is differentiated spatially. The spatial derivative of $\phi(z, t)$ can be taken from the coupled system of equations 2.14. Next, the separation of variables should be applied to the angular displacement related to shear: $\psi(z, t) = \Psi(z)e^{i\omega t}$. Thus, the following relation is derived:

$$\Psi(z) = -\frac{\rho}{\kappa G}\omega^2 \int Y(\zeta) d\zeta \Big|_{\zeta=z} \quad (2.28)$$

The definition of the spatial derivative of the variable $\phi(z, t)$ in terms of the mode shape function can be taken from the initial coupled system of equations 2.14. After the separation of variables is applied, $\phi(z, t) = \Phi(z)e^{i\omega t}$, the following relation is obtained:

$$\Phi(z) = Y''(z) + \frac{\rho}{\kappa G}\omega^2 Y(z). \quad (2.29)$$

In order to combine the two definitions above with the excitation function, both the force and moment functions need to be transformed to the frequency domain:

$$\begin{aligned} V(0, t) &= \tilde{F}_1 e^{i\omega t}, \\ V(l, t) &= \tilde{F}_2 e^{i\omega t}, \\ M_B(0, t) &= \tilde{M}_1 e^{i\omega t}, \\ M_B(l, t) &= \tilde{M}_2 e^{i\omega t}. \end{aligned} \quad (2.30)$$

Finally, combining the equations 2.30 with the definitions of the angular displacements (2.28) and (2.29) yields the following set of equations:

$$\begin{aligned} \tilde{F}_1 &= -\kappa G A \alpha_T \left(\int Y(\zeta) d\zeta \right) \Big|_{\zeta=0}, \\ -\tilde{F} &= -\kappa G A \alpha_T \left(\int Y(\zeta) d\zeta \right) \Big|_{\zeta=l}, \\ -\tilde{M}_1 &= -I_x E (Y''(0) + \alpha_T Y(0)) (t), \\ \tilde{M}_2 &= -I_x E (Y''(l) + \alpha_T Y(l)) (t), \end{aligned} \quad (2.31)$$

where the new parameter α_T is defined in the following way:

$$\alpha_T := \frac{\omega^2 \rho}{\kappa G}. \quad (2.32)$$

Thus, the following matrix equation is derived by substituting the mode shape function

definition 2.27 into the equations 2.31 shown above:

$$\begin{aligned}
& \begin{bmatrix} 0 & 1/\delta & 0 & -1/\gamma \\ \sinh(\delta l)/\delta & \cosh(\delta l)/\delta & \sin(\gamma l)/\gamma & -\cos(\gamma l)/\gamma \\ \alpha_T + \delta^2 & 0 & \alpha_T - \gamma^2 & 0 \\ (\alpha_T + \delta^2) \cosh(\delta l) & (\alpha_T + \delta^2) \sinh(\delta l) & (\alpha_T - \gamma^2) \cos(\gamma l) & (\alpha_T - \gamma^2) \sin(\gamma l) \end{bmatrix} \begin{bmatrix} \tilde{C}_1 \\ \tilde{C}_2 \\ \tilde{C}_3 \\ \tilde{C}_4 \end{bmatrix} \\
& = \begin{bmatrix} -\tilde{F}_1/\kappa G A \alpha_T \\ \tilde{F}_2/\kappa G A \alpha_T \\ \tilde{M}_1/I_x E \\ -\tilde{M}_2/I_x E \end{bmatrix}.
\end{aligned} \tag{2.33}$$

Once solved, the analytical receptance of the Timoshenko–Ehrenfest beam can be assembled in the following way:

$$\mathbf{R}_{i,j}(\omega) = \begin{bmatrix} Y_i(\omega)/\tilde{F}_j(\omega) & Y_i(\omega)/\tilde{M}_j(\omega) & 0 \\ \Phi_i(\omega)/\tilde{F}_j(\omega) & \Phi_i(\omega)/\tilde{M}_j(\omega) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.34}$$

where the rotational response Φ is defined as:

$$\Phi(z) = Y'(z) + \alpha_T \int Y(z) \, dz, \tag{2.35}$$

considering $Y(z, \omega) := Y(z)$ and $\Phi(z, \omega) := \Phi(z)$ through $\tilde{C}_l(\omega) := \tilde{C}_l$.

Structural damping

The structural damping, which is described by the loss factor η (Dombóvári 2023), is introduced into the analytical beam model in the following way:

$$\rho A \ddot{y}(z, t) + (1 + \eta i) I_x E y''''(z, t) - \rho I_x \left(1 + \frac{(1 + \eta i) E}{\kappa G} \right) \ddot{y}'' + \frac{\rho^2 I_x}{\kappa G} \ddot{\ddot{y}} = 0. \tag{2.36}$$

2.5 Torsion Response

This section describes the torsional related response of a homogeneous prismatic beam.

Euler beam model

In this work, the torsional vibrations are described by the Euler-Bernoulli beam theory.

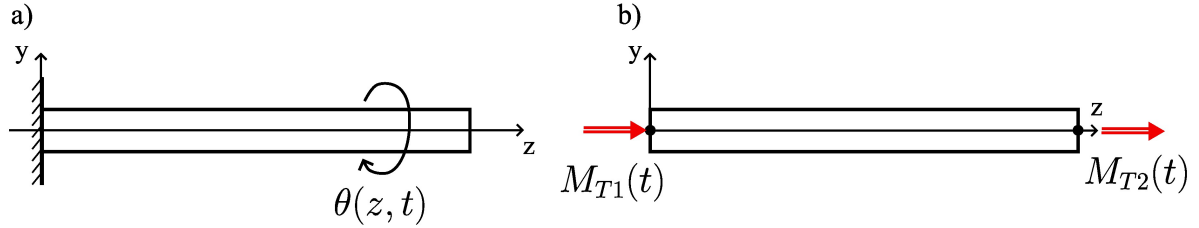


Figure 2.4: a) Euler-Bernoulli beam torsion, b) Boundary conditions

The dynamic model is described by the well-known equation:

$$\ddot{\theta}(z, t) - \frac{G}{\rho} \theta''(z, t) = 0, \quad (2.37)$$

where $\theta(z, t)$ is the angular displacement of the cross-section along the z axis, see Figure 2.4a. Trial solution of the equation 2.37 is given in the following form:

$$\theta(z, t) = \Theta(z) e^{i\omega t}. \quad (2.38)$$

Substitution of the trial solution back to the equation of motion leads to following ODE:

$$\Theta''(z) + \omega^2 \frac{\rho}{G} \Theta(z) = 0. \quad (2.39)$$

Assuming another trial solution, $\Theta(z) = K e^{\lambda z}$, leads to the following characteristic equation:

$$\lambda^2 + \omega^2 \frac{\rho}{G} = 0, \quad (2.40)$$

where the roots of the characteristic equation of the Euler-Bernoulli beam model are:

$$\lambda = \pm i\omega \sqrt{\frac{\rho}{G}} =: \pm i\beta_t. \quad (2.41)$$

Substituting the obtained roots back into the trial solution results in the following torsional mode shape function:

$$\Theta(z) = K_1 e^{i\beta_t z} + K_2 e^{-i\beta_t z}, \quad (2.42)$$

which can be re-written in the following way

$$\Theta(z) = \hat{K}_1 \cos(\beta_t z) + \hat{K}_2 \sin(\beta_t z). \quad (2.43)$$

Boundary Conditions

The methodology of the application of the BC on the beam model are similar to the bending beam model presented in the previous section. The relation between the torsional moment and the torsional angular displacement is as follows:

$$M'_T(z, t) = \rho I_p \ddot{\theta}(z, t). \quad (2.44)$$

The equation 2.44 can be re-written in terms of the torsional mode shape function 2.43:

$$M_T(z, t) = I_p G \theta'(z, t) = I_p G \Theta'(z) e^{i\omega t}. \quad (2.45)$$

The excitation function can also be transformed to the frequency domain in the similar manner to the shear force and bending moment functions from the previous section:

$$\begin{aligned} M_T(0, t) &= -\hat{M}_1 e^{i\omega t}, \\ M_T(l, t) &= \hat{M}_2 e^{i\omega t}. \end{aligned} \quad (2.46)$$

By combining the final equations related to the torsional moment, (2.45) and (2.46), with the derived mode shape function (2.43), the following set of equations is obtained:

$$\begin{aligned} -\hat{M}_1 &= I_p G \Theta'(0), \\ \hat{M}_2 &= I_p G \Theta'(l). \end{aligned} \quad (2.47)$$

By substituting the spatial derivative of the mode shape function $\Theta'(z)$ in the equations 2.47, the following matrix equation is obtained

$$\begin{bmatrix} 0 & \beta_t \\ -\beta_t \sin(\beta_t l) & -\beta_t \cos(\beta_t l) \end{bmatrix} \begin{bmatrix} \hat{K}_1 \\ \hat{K}_2 \end{bmatrix} = \frac{1}{I_p G} \begin{bmatrix} -\hat{M}_1 \\ \hat{M}_2 \end{bmatrix}. \quad (2.48)$$

Thus, after solving for the unknown coefficients $\hat{K}_1$ and $\hat{K}_2$, the torsional element can be added to the analytical FRF 2.34:

$$\mathbf{R}_{i,j}(\omega) = \begin{bmatrix} Y_i(\omega)/\tilde{F}_j(\omega) & Y_i(\omega)/\tilde{M}_j(\omega) & 0 \\ \Phi_i(\omega)/\tilde{F}_j(\omega) & \Phi_i(\omega)/\tilde{M}_j(\omega) & 0 \\ 0 & 0 & \Theta_i(\omega)/\hat{M}_j(\omega) \end{bmatrix} \quad (2.49)$$

Structural damping

The torsional damping characteristics are included in the similar manner to the bending model.

2.6 FEM Validation of the Beam Models

This section makes a validation of the analytical beam model described in the previous sections by comparing it to a Finite Element (FE) model. The section is divided into two subsections, namely bending related responses and torsion related responses. Firstly, the introduction of the structural damping is validated. Later, single solid cantilever beam responses are compared to the responses of an assembled structures of separate beam segments, where the assembled structure has the same length as a single cantilever beam. The receptances $H_{B,B}$, $H_{D,D}$, and $H_{F,F}$ correspond to one beam, two beams, and three beams bending related tip receptances respectively, see Figure 2.5. Torsion related FRFs are denoted by $T_{i,i}$.

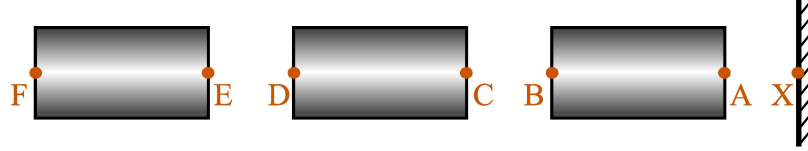


Figure 2.5: 3 segment cantilever beam

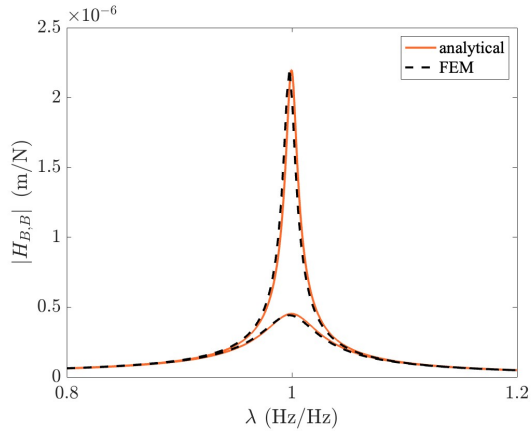
Bending

Firstly, the damping parameter η is varied in both analytical and numerical models and the results are compared. The simulations were performed with either 1% or 5% structural damping coefficient. As can be seen in Figure 2.6, for both damping values the receptances align almost perfectly. This difference should be related to a more sophisticated beam dynamics model used in Ansys Mechanical.

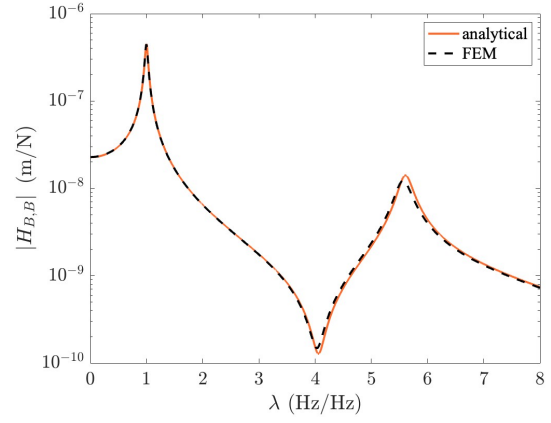
Now, that the single beam model was validated, the next validation step is related to the receptance coupling methodology. The single beam is cut in either two or three segments and RCSA is performed between these segments.

Figures 2.7 shows an exact match between the single beam receptance $H_{B,B}$ in comparison to either the two beam receptance $H_{D,D}$ or the three beam receptance $H_{F,F}$.

Thus, it can be said that the analytical model of the bending behaviour of the beam segment derived in this work is correct. Additionally, the RCSA proved to be implemented in the correct manner.

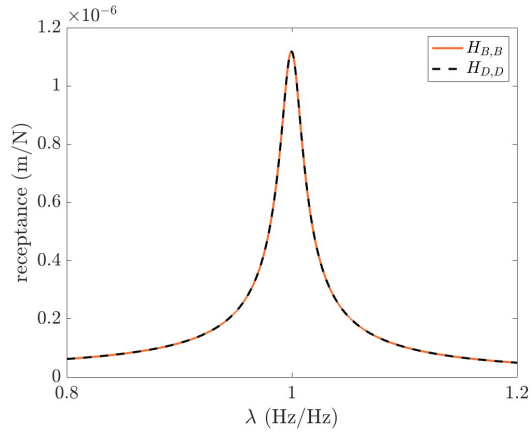


(a) Receptance in linear scale

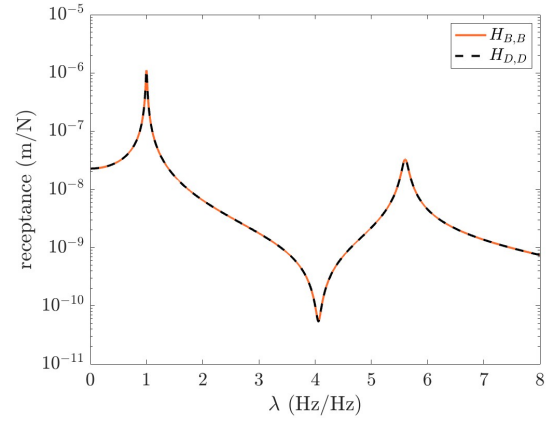


(b) Receptance in log scale

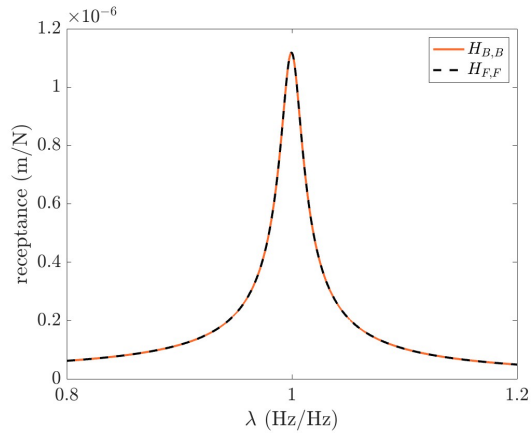
Figure 2.6: Bending receptance of a single beam



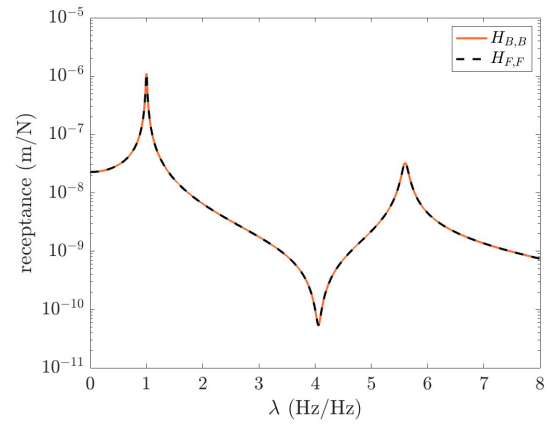
(a) 1 beam versus 2 beams linear scale



(b) 1 beam versus 2 beams log scale



(c) 1 beam versus 3 beams linear scale



(d) 1 beam versus 3 beams log scale

Figure 2.7: Bending receptance comparison

Torsion

The validation methodology for the torsional receptances is identical to the bending related validation presented in the previous subsection.

The comparison of the analytical and the numerical FRFs with varying damping coefficient produced an exact match between the two, see Figure 2.8. This fact showcases that the Ansys Mechanical uses Euler-Bernoulli torsional beam model to simulate torsion related responses.

As can be seen in Figure 2.9 both the two segment $T_{D,D}$ and the three segment $T_{F,F}$ receptances coincide with the single cantilever beam receptance $T_{B,B}$, validating the torsional related responses.

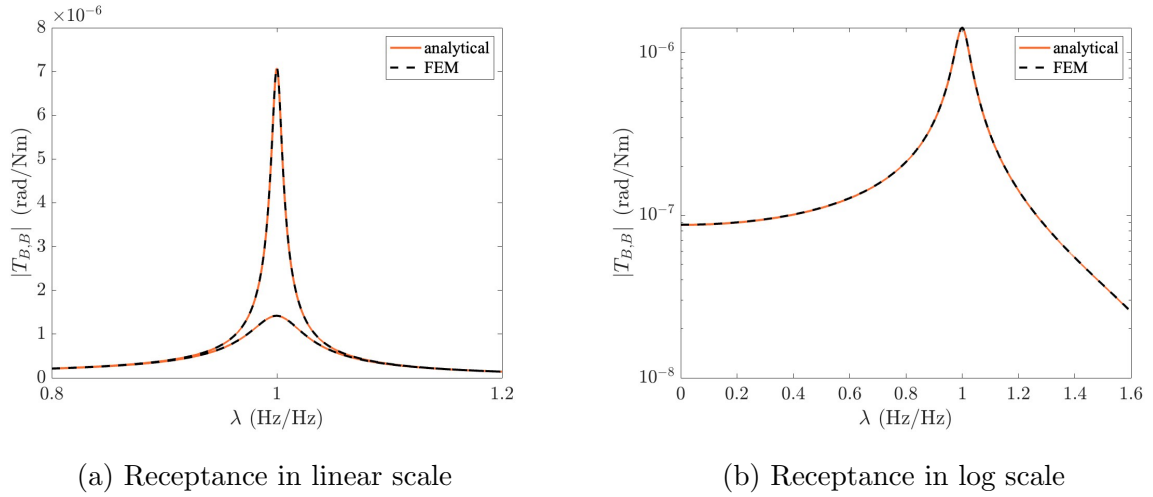
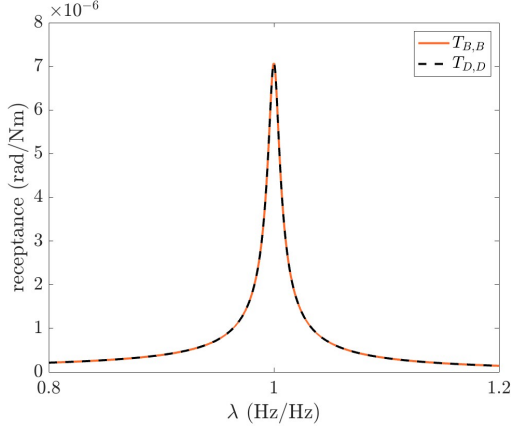
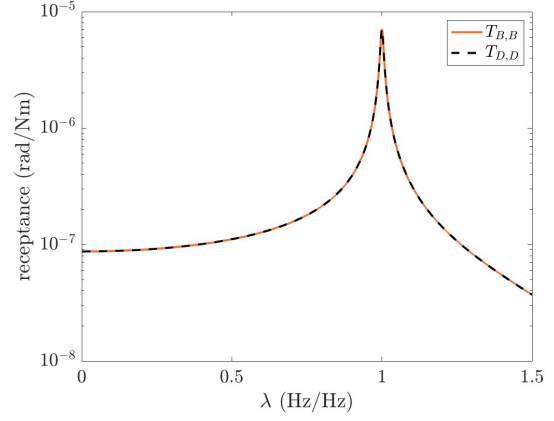


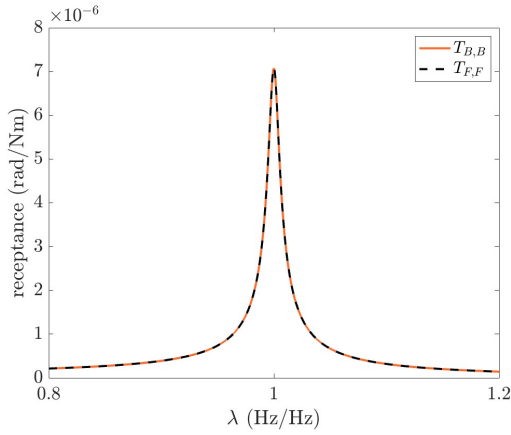
Figure 2.8: Torsion receptance of a single beam



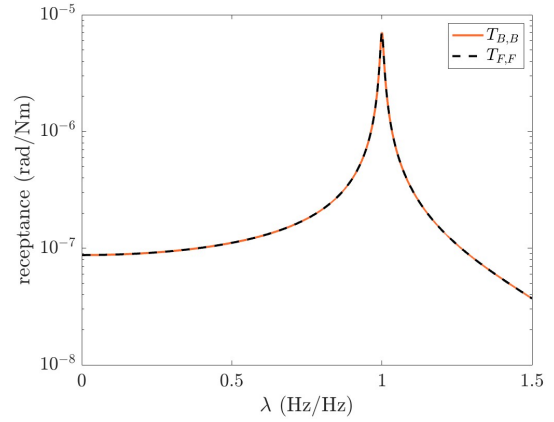
(a) 1 beam versus 2 beams linear scale



(b) 1 beam versus 2 beams log scale



(c) 1 beam versus 3 beams linear scale



(d) 1 beam versus 3 beams log scale

Figure 2.9: Torsion receptance comparison

2.7 Rigid Element

The final segment of the RCSA of the boring bar is the cutting head. The head possesses significant mass and inertia parameters and thus cannot be neglected during the assembly process. This section focuses on the derivation of an analytical FRF for a rigid element, see Figure 2.10a.

According to the basic law of dynamics, the following equations of motion are derived for the arbitrary rigid element shown in Figure 2.10a:

$$\begin{aligned} m_{\text{RE}} \ddot{y}(z, t) &= F_1(z, t) + F_2(z, t), \\ \Theta_{\text{CM}x} \ddot{\phi}(z, t) &= M_1(z, t) + F_1(z, t)z_1 + M_2(z, t) + F_2(z, t)z_2, \end{aligned} \quad (2.50)$$

where the rigid element is characterised by the mass m_{RE} , the moment of inertia at the centre of mass (CM) along x and z axes $\Theta_{\text{CM}x}$ and $\Theta_{\text{CM}z}$ respectively, and the distances

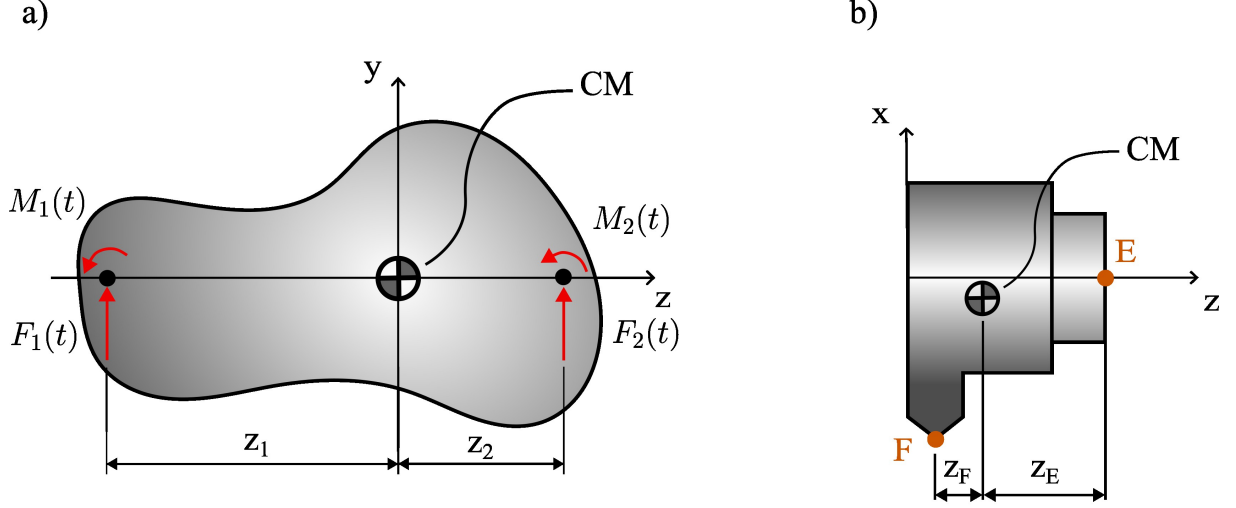


Figure 2.10: a) Arbitrary rigid element, b) Cutting head

z_1 and z_2 between the CM and the points of interest with a proper sign. In this particular model the distance z_1 is a negative value. Making a transformation to the frequency domain results in the following equations:

$$\begin{aligned} -\omega^2 m_{\text{RE}} Y(z) &= \tilde{F}_1 + \tilde{F}_2, \\ -\omega^2 \Theta_{\text{CM}x} \Phi(z) &= \tilde{M}_1 + \tilde{F}_1 z_1 + \tilde{M}_2 + \tilde{F}_2 z_2. \end{aligned} \quad (2.51)$$

The responses of both points of interest are defined as following:

$$\begin{aligned} y_i(z, t) &= y(z, t) + \phi(z, t) z_i, \\ \phi_i(z, t) &= \phi(z, t), \end{aligned} \quad (2.52)$$

which can also be transformed to the frequency domain:

$$\begin{aligned} Y_i(z) &= Y(z) + \Phi(z) z_i, \\ \Phi_i(z) &= \Phi(z). \end{aligned} \quad (2.53)$$

Taking into account the equation of motion 2.50 and the derived responses 2.53, it is possible to re-write the responses in terms of the excitation forces and moments:

$$\begin{aligned} Y_i &= -\frac{1}{\omega^2} \left(\frac{\tilde{F}_1 + \tilde{F}_2}{m_{\text{RE}}} + \frac{\tilde{M}_1 + \tilde{F}_1 z_1 + \tilde{M}_2 + \tilde{F}_2 z_2}{\Theta_{\text{CM}x}} z_i \right), \\ \Phi_i &= -\frac{1}{\omega^2} \left(\frac{\tilde{M}_1 + \tilde{F}_1 z_1 + \tilde{M}_2 + \tilde{F}_2 z_2}{\Theta_{\text{CM}x}} \right). \end{aligned} \quad (2.54)$$

In the similar manner, the response related to the rotation along z axis, i.e. torsion, can also be derived:

$$\Theta_i = -\frac{1}{\omega^2} \left(\frac{\hat{M}_1 + \hat{M}_2}{\Theta_{\text{CM}z}} \right). \quad (2.55)$$

Thus, the corresponding FRF matrix $\mathbf{R}_{i,j}(\omega)$ for the rigid element can be assembled in the following way:

$$\begin{aligned} \mathbf{R}_{i,j}(\omega) &= \begin{bmatrix} Y_i(\omega)/\tilde{F}_j(\omega) & Y_i(\omega)/\tilde{M}_j(\omega) & 0 \\ \Phi_i(\omega)/\tilde{F}_j(\omega) & \Phi_i(\omega)/\tilde{M}_j(\omega) & 0 \\ 0 & 0 & \Theta_i(\omega)/\hat{M}_j(\omega) \end{bmatrix} \\ &= -\frac{1}{\omega^2} \begin{bmatrix} \frac{1}{m_{R.E.}} + \frac{z_i z_j}{\Theta_{CMx}} & \frac{z_i}{\Theta_{CMx}} & 0 \\ \frac{z_j}{\Theta_{CMx}} & \frac{1}{\Theta_{CMx}} & 0 \\ 0 & 0 & \frac{1}{\Theta_z} \end{bmatrix}. \end{aligned} \quad (2.56)$$

The equation 2.56 is used to model the behaviour of the cutting head, which is essential during the receptance coupling, see Figure 2.10b. It should be mentioned that the rotational moment of inertia in the defined analytical FRF above is calculated with respect to the main axis of the bar.

2.8 FEM Validation of the Assembled Bar

This section performs a validation of the assembled boring bar without absorber, i.e. the rigid wall, the solid segment, the hollow segment, and the cutting head, Figure 2.11.

The FE simulations were performed for a bar of a total length of 2 m and a diameter of 20 cm. The variations in these simulations include the hollow and solid segments' length, see Table 2.1 for further details.

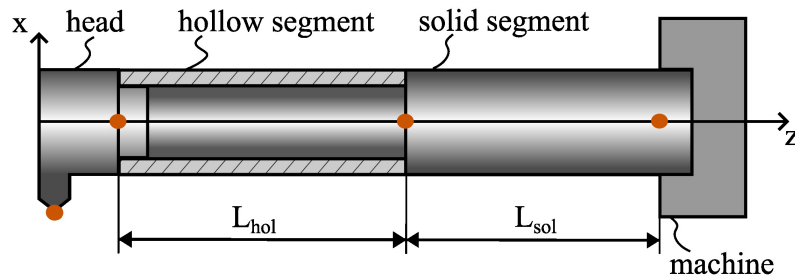


Figure 2.11: Assembled boring bar

Simulation number	L_{hol} (cm)	L_{sol} (cm)	D (cm)
1	100	100	20
2	50	150	20
3	140	60	20

Table 2.1: Simulation parameters

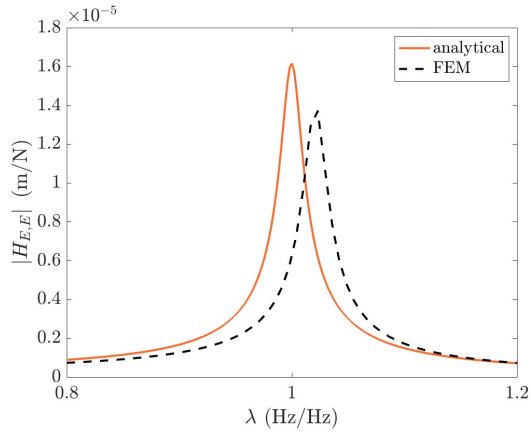
Simulation 1

The first simulation results are compared to the analytical calculations. Firstly, the bending related direct receptances are compared. Next, the torsion related FRFs are compared.

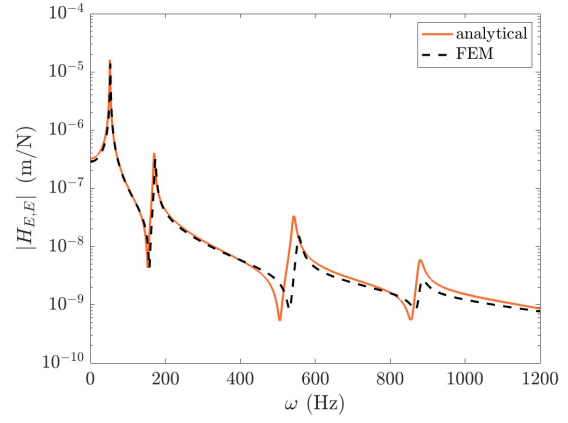
As can be seen in Figure 2.12b, the general trends of the analytical calculation are similar to the FE simulation results. It can be seen that the difference between the two plots is increasing steadily with each subsequent mode. Also, by looking at the Figure 2.12b, one can observe a slight deviation even in the first bending mode. The deviation is not that significant, the difference between the peaks is lower than 1 Hz.

As for the torsional receptances, a perfect match is achieved.

Since the analytical and FE simulation results coincide for the torsional related responses, the slight deviation in the bending receptances cannot be explained with the numerical errors which occur during both the analytical and FE calculations. As mentioned in the section 2.6, the bending related deviation in the receptances can be explained by a more advanced beam modelling scheme used in Ansys Mechanical compared to presented Timoshenko–Ehrenfest beam model.

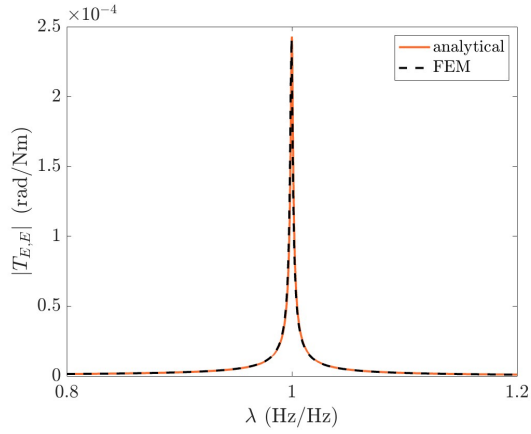


(a) First mode in linear scale

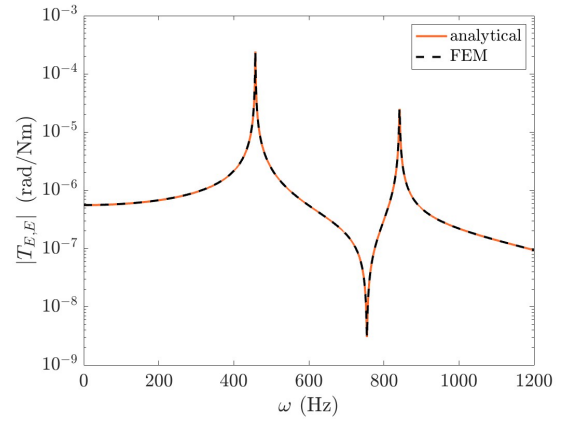


(b) Log scale

Figure 2.12: Bending receptance for simulation 1



(a) First mode in linear scale

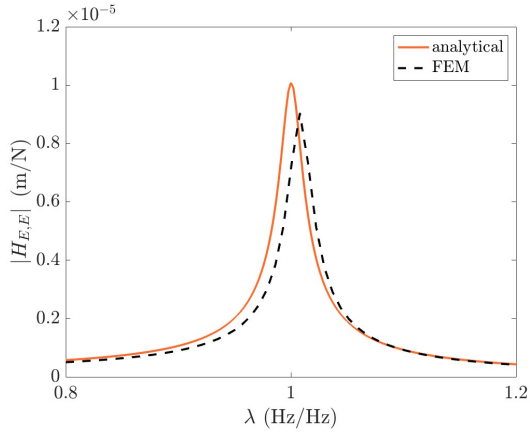


(b) Log scale

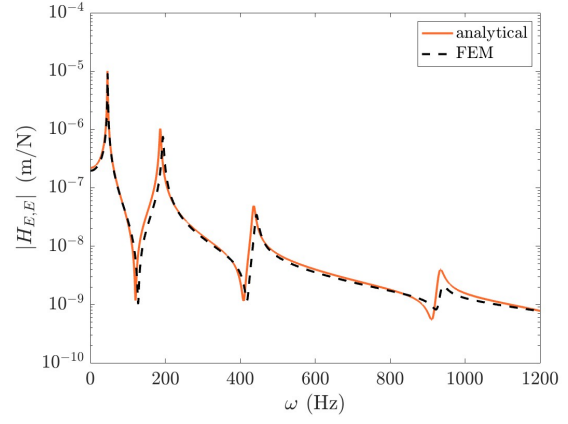
Figure 2.13: Torsion receptance for simulation 1

Simulation 2

Similar trends can be observed in the second simulation.

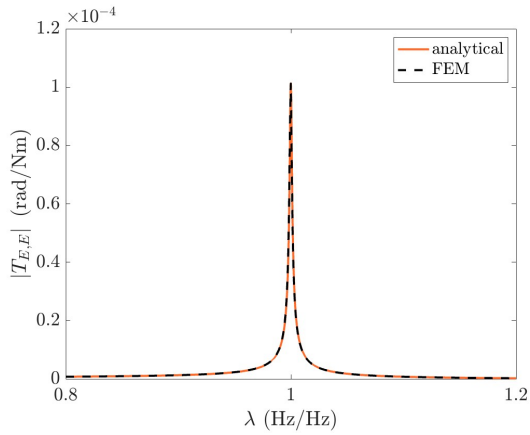


(a) First mode in linear scale

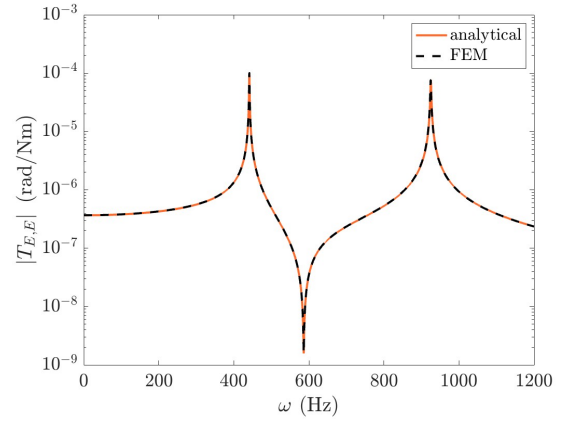


(b) Log scale

Figure 2.14: Bending receptance for simulation 2



(a) first mode



(b) Log scale

Figure 2.15: Torsion receptance for simulation 2

Simulation 3

The third simulation showcases similar trends as well.

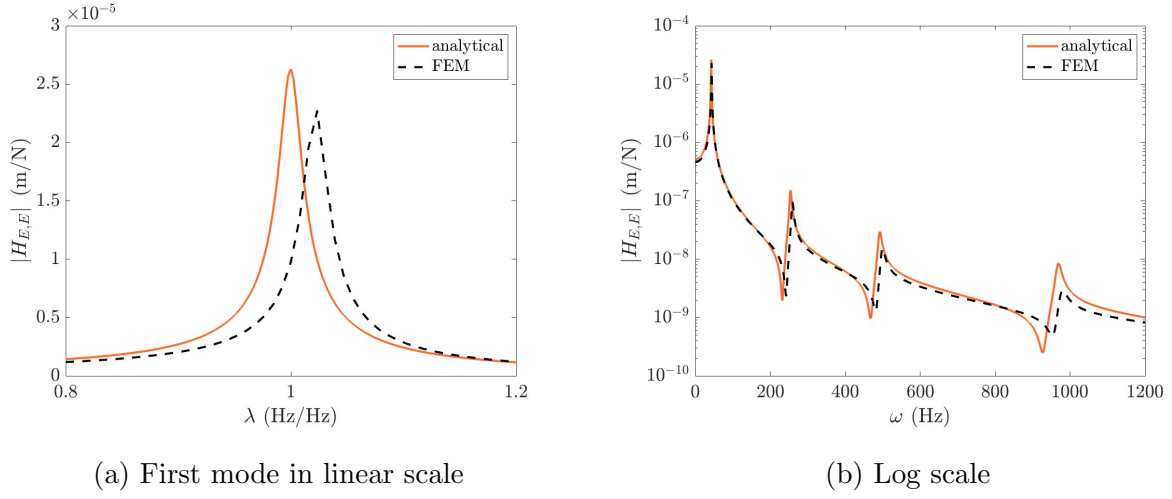


Figure 2.16: Bending receptance for simulation 3

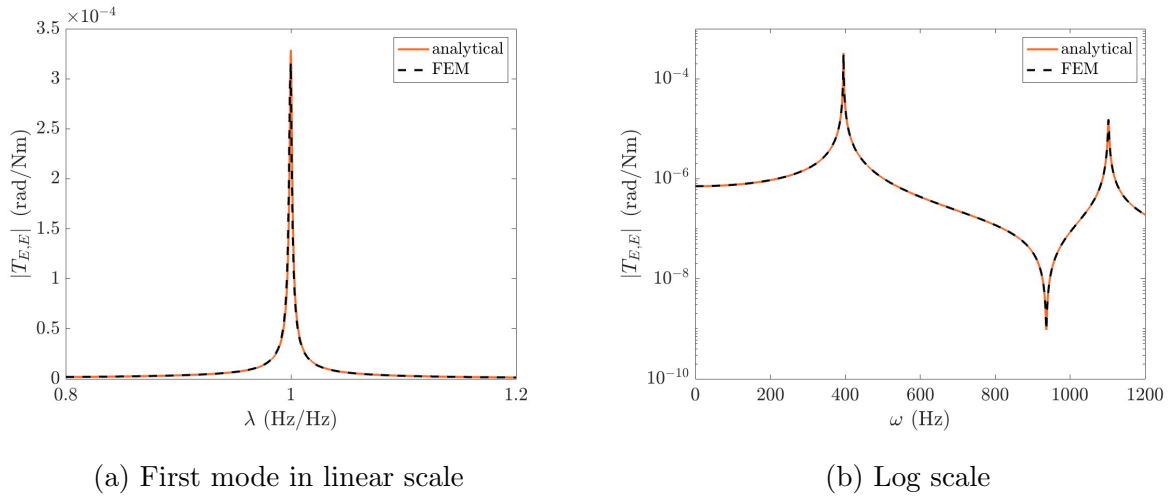


Figure 2.17: Torsion receptance for simulation 3

Thus, the implementation of the rigid element FRF 2.56 has been validated with the FE simulations.

2.9 Absorber

A dynamic vibration absorber was introduced by Herman Frahm back in 1909 and it is a common way to reduce vibrations of the cutting tool during the machining process (Munoa et al. 2016), see Figure 2.18.

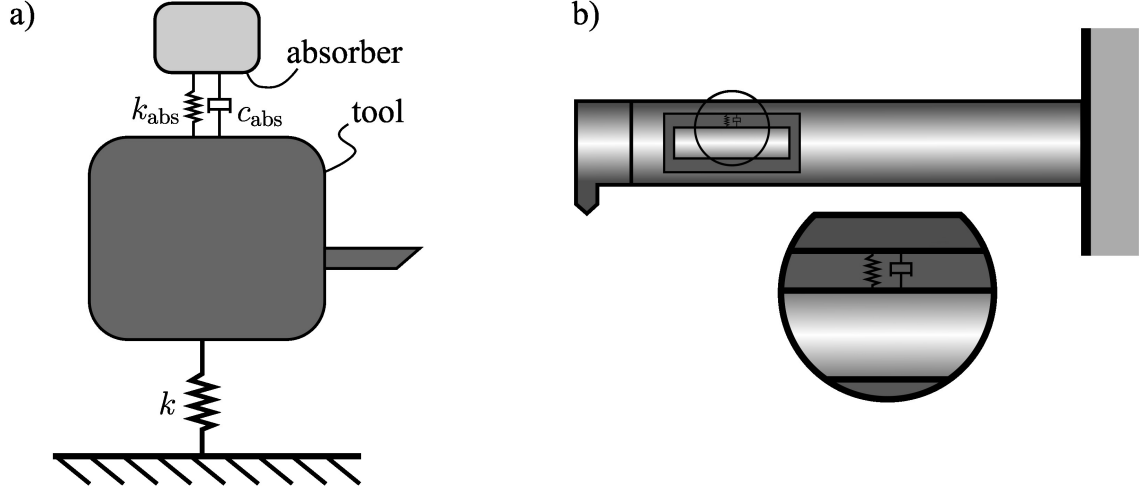


Figure 2.18: a) Tool and absorber system, b) Boring bar with introduced absorber

A tuning methodology for a dynamic vibration absorber with damping was famously introduced by Den Hartog (1985). The "equal-peak" methodology focuses on minimizing the amplitude behaviour of the main structure subjected to a harmonic excitation. However, the most dominant group of vibrations in metal cutting process is chatter. Regenerative chatter is closely related to the characteristics of the real part of the cutting tip FRFs. Accordingly, Sims (2007) introduced a tuning methodology that focuses on maximizing the negative real part of the FRF. This section develops a methodology that introduces a passive dynamic absorber in the boring bar machine structure.

Sims absorber

Firstly, a set of new variables and parameters must be introduced:

$$\begin{aligned}
 \text{main structure mass} \quad & m_M, \\
 \text{absorber mass} \quad & m_{\text{abs}}, \\
 \text{absorber moment of inertia for torsion} \quad & \Theta_{z,\text{abs}}, \\
 \text{mass ratio} \quad & \mu_{\text{abs}} = m_{\text{abs}}/m_M, \\
 \text{main structure natural frequency} \quad & \omega_M = \sqrt{k/m_M}, \\
 \text{absorber natural frequency} \quad & \omega_{\text{abs}} = \sqrt{k_{\text{abs}}/m_{\text{abs}}}, \\
 \text{frequency ratio} \quad & f, \\
 \text{damping ratio} \quad & \nu.
 \end{aligned}$$

The optimal frequency and damping ratios, which are functions of the mass ratio, are approximated by the following formulas

$$f_{\text{opt}}^2 = \frac{\mu_{\text{abs}} + 2 \pm \sqrt{2\mu_{\text{abs}} + \mu_{\text{abs}}^2}}{2(1 + \mu_{\text{abs}})^2}, \quad (2.57)$$

$$\nu_{\text{opt}}^2 = \frac{3\mu_{\text{abs}}}{8(1 + \mu_{\text{abs}})}. \quad (2.58)$$

Once the two optimal frequency and damping ratios are determined, it is possible to calculate the optimum stiffness and damping constants:

$$\omega_{\text{abs}} = f \omega_{\text{M}} \quad \Rightarrow k_{\text{abs}}, \quad (2.59)$$

$$\nu_{\text{opt}} = \frac{c_{\text{abs}}}{2 \omega_{\text{abs}} m_{\text{abs}}} \quad \Rightarrow c_{\text{abs}}. \quad (2.60)$$

Apparent Mass and Stiffness

The next step is to properly define the mass ratio μ_{abs} , where the mass of the absorber is quite obvious whilst the mass of the boring bar is questionable. The definitions of the apparent mass and stiffness, $m_{i,j,p,l}$ and $k_{i,j,p,l}$ respectively are utilized, where i is the sensing point, j is the excitation point, p is the sensing direction, and l is the excitation direction. The apparent/reflective/effective mass and the apparent/reflective/effective stiffness are concepts that quantify how stiff a point i in the given structure is if the load is applied at point j (Ewins 2009).

As the mass absorber is to be connected to the point G, see figure 2.1, it is necessary to determine the apparent mass and stiffness at that point, $m_{\text{G,G},y,y} \equiv m_{\text{G,G}}$ and $k_{\text{G,G},y,y} \equiv k_{\text{G,G}}$. This is done by analysing the real and imaginary parts of the direct receptance $H_{\text{G,G}}$ separately, see Figure 2.19.

The spectral properties, such as the natural frequency and the damping ratio of the model for the k^{th} mode, can be extracted from the real part of the receptance in the following way:

$$\omega_{n,k} = \sqrt{\frac{\omega_{\text{p},k}^2 + \omega_{\text{v},k}^2}{2}}, \quad (2.61)$$

$$\zeta_k = \frac{\omega_{\text{v},k}^2 - \omega_{\text{p},k}^2}{2(\omega_{\text{p},k}^2 + \omega_{\text{v},k}^2)}, \quad (2.62)$$

where $\omega_{\text{p},k}$ and $\omega_{\text{v},k}$ are the peak and valley frequencies for the k^{th} mode.

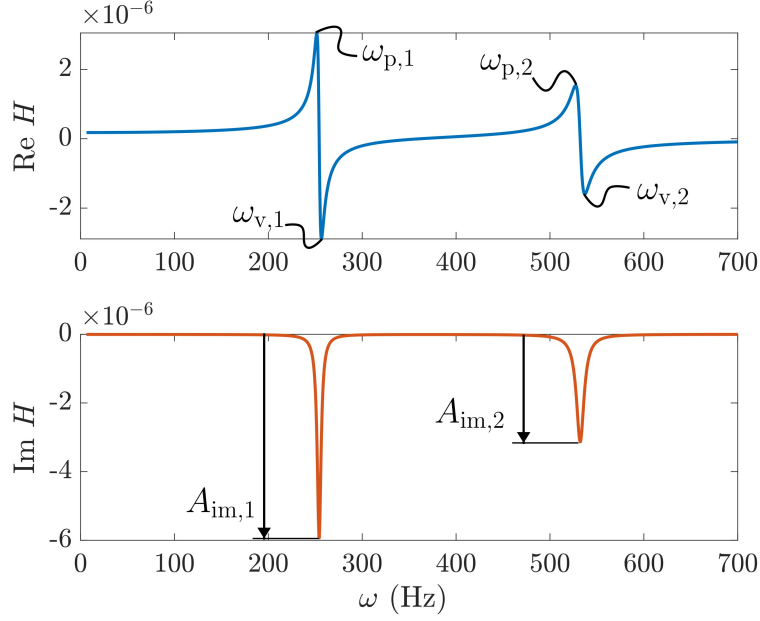


Figure 2.19: Real and Imaginary parts of FRF

The amplitude properties $A_{im,k}$ can be used to determine the necessary apparent mass and stiffness.

$$k_{i,j,k} = -\frac{1}{2\zeta A_{im,k}} \quad (2.63)$$

$$m_{i,j,k} = \frac{k_{i,j,k}}{\omega_{n,k}^2} \quad (2.64)$$

Thus, these two parameters can be used to tune the mass absorber according to the methodology developed by Sims.

Similar procedure can also be applied to the torsion related FRFs at the connection point G, $T_{G,G}$, see Figure 2.19.. This would ultimately result in the apparent moment of inertia $\Theta_{G,G}$ and the apparent torsional stiffness $k_{G,G,tor}$. This parameters can also be used for absorber tuning.

Stiffness Matrix

Another important parameter for the introduction of the mass absorber into the system is the connection properties. For the mechanical system described in this work, the $\mathbf{K}$ matrix is defined as:

$$\mathbf{K}(\omega) = \begin{bmatrix} k_{abs,rad} + i\omega c_{abs,rad} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k_{abs,tor} + i\omega c_{abs,tor} \end{bmatrix}. \quad (2.65)$$

Results

The results of the direct bending response for the cutting tip $H_{F,F}$, before and after the introduction of the passive absorber into the mechanical model are presented in Figure 2.20. The absorber is introduced at the point G, see Figure 2.1 for reference. Original system stands for the response of the boring bar model that consists of the rigid wall, the solid segment, the hollow segment and the cutting head. Sims system includes the tuned passive absorber into the mechanical model.

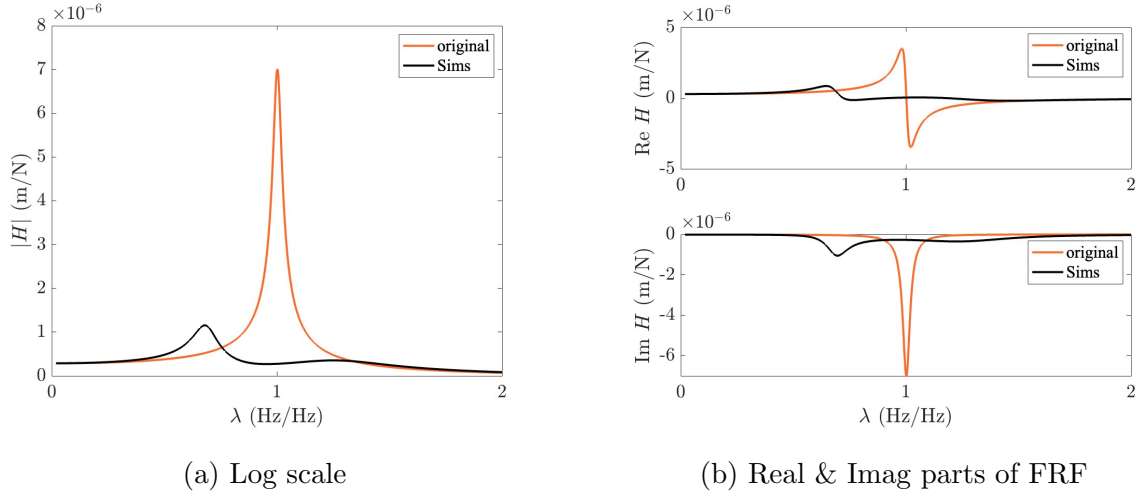


Figure 2.20: Tip FRF with introduced absorber

The torsional response with the introduced absorber is discussed and presented in the following chapter.

Chapter 3

Optimization

This chapter focuses on possible optimization methodologies with a goal to reduce the vibrational behaviour of the boring bar. It consists of three sections, namely the absorber size optimization, length ratio optimization, and finally the rubber bushings optimization.

3.1 Absorber Size Optimization

The mass ratio μ_{abs} is an open parameter, and therefore, there should be an optimal value for it. The limiting factor in this optimization cycle is the actual hollow volume of the boring bar, as the absorber has a limited space to be placed in. In this section the results of the analytical calculations with varying mass ratio μ_{abs} are presented.

Description of the Scheme

The mass ratio μ_{abs} depends on the apparent mass $m_{G,G}$ and the absorber mass m_{abs} . The only controlled parameter here is the absorber mass, however it is limited by the hollow volume inside the boring bar. Thus, a new parameter μ_{vol} is introduced and is defined as the volumetric ratio of the absorber and the hollow volume. The upper limit of the parameter is obviously one, however, rubber bushings and oil have to be introduced into the hollow segment as well. The simulation below varies the volumetric ratio μ_{vol} between the values of 0.3 and 0.95.

Results

The results of the scheme are presented in Figure 3.1 where the absorber mass ratio μ_{abs} and the maximum value of the absolute magnitude of the tip receptance $|H_{F,F}|$ are plotted as functions of the previously defined absorber volumetric ratio μ_{vol} .

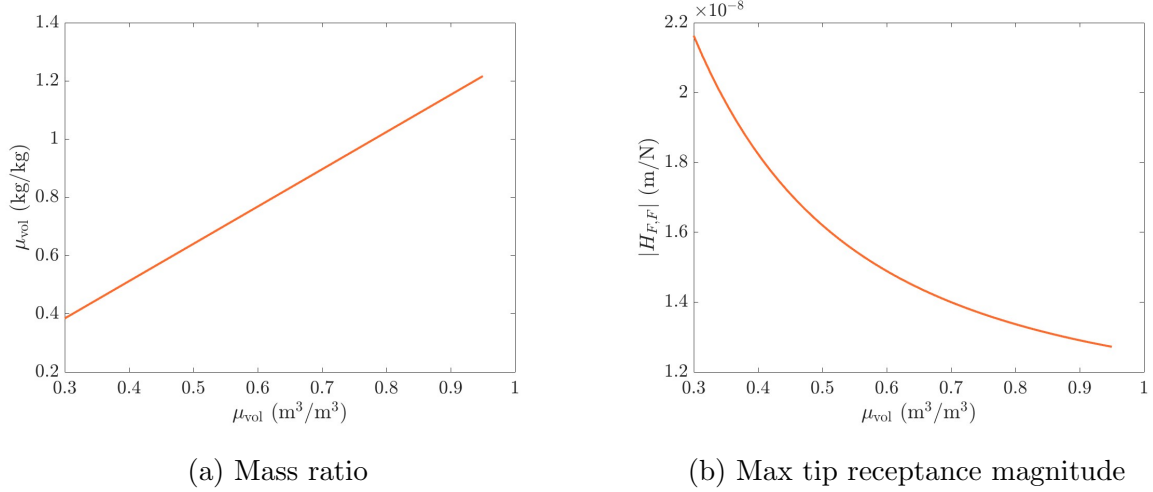


Figure 3.1: Absorber size variation

The mass ratio function possesses a linear trend, which was expected, however the more crucial observation is that the value of the mass ratio exceeds the value of one at around 80% of the volumetric ratio. This means that the absorber mass m_{abs} becomes larger than the apparent mass of the assembled boring bar $m_{\text{G,G}}$ after some point, whereas the traditional "equal-peaks" tuning methodology was introduced with examples of around 10-20% mass ratio.

The Figure 3.1b depicts the maximum value of the tip receptance as a function of the absorber volumetric ratio. An exponential decaying trend can be clearly seen, and therefore, it can be confidently stated that the largest possible mass absorber should be selected during the design process in order to minimize the vibration amplitude of the cutting tip.

3.2 Length Ratio Optimization

Now that it has been shown that the largest possible passive absorber should be selected, it is imperative to find the optimal length of the hollow segment. This can be described by a non-dimensional parameter σ which is defined as the ratio between the hollow segment length to the total length of the bar, see Figure 2.11.

$$\sigma := \frac{L_{\text{hol}}}{L_{\text{hol}} + L_{\text{sol}}}. \quad (3.1)$$

This section presents the results of the analytical calculations with varying the length ratio parameter σ between the values of 10% and 90%. The calculations were performed

with the following parameters:

$$\begin{aligned}
&\text{length to diameter ratio} & L/D &= 7 \text{ m/m}, \\
&\text{absorber volumetric ratio} & \mu_{\text{vol}} &= 0.8 \text{ m}^3/\text{m}^3, \\
&\text{wall thickness for the hollow segment} & w &= 5 \text{ mm}, \\
&\text{total length, shorter bar} & L^s &= 56 \text{ cm}, \\
&\text{diameter, shorter bar} & D^s &= 8 \text{ cm}, \\
&\text{total length, longer bar} & L^l &= 140 \text{ cm}, \\
&\text{diameter, longer bar} & D^l &= 20 \text{ cm}.
\end{aligned}$$

Results

The main results of this section are presented in Figure 3.2.

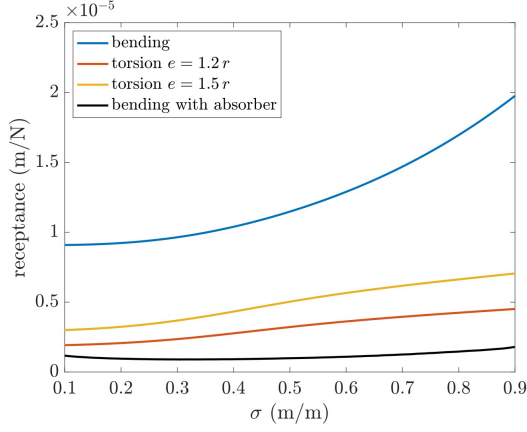
The first sub-figure, Figure 3.2a, presents the variation of the maximum absolute receptance of the cutting tip $H_{F,F}$ for the shorter bar. The purple line is related to the bending vibrations of the assembled bar without the passive absorber. Once the passive absorber is introduced into the mechanical model, the bending vibrations are greatly suppressed. This is shown with the black curve.

In addition, the torsional receptance maximum magnitude is also presented. In order to quantitatively compare the bending and torsion responses it is necessary to convert the torsional response $T_{F,F}$, which is measured in (rad/Nm), to the form of the bending response $H_{F,F}$, which is measured in (m/N). As the cutting tip is located at a certain eccentricity e away from the main axis of the bar, the relationship between the two types of receptances is defined in the following way:

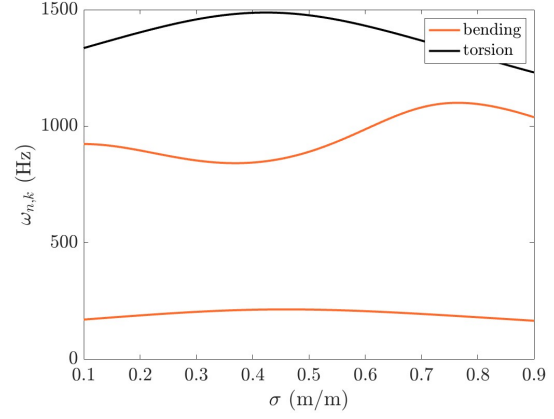
$$H_{F,F}^{\text{tor}} = e^2 T_{F,F}, \quad (3.2)$$

where $H_{F,F}^{\text{tor}}$ represents the displacement of the cutting tip along the y axis due to torsion.

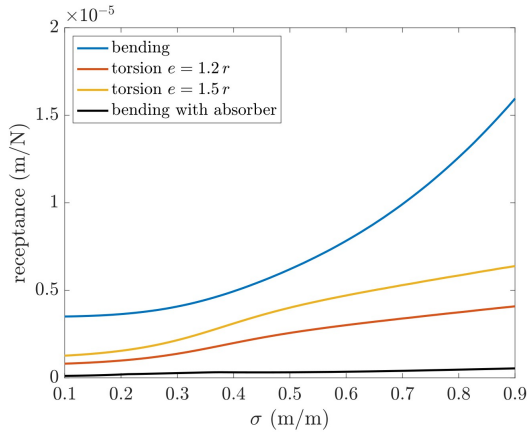
The last two curves on the Figure 3.2a are related to the transformed torsional receptance magnitude. The typical eccentricity values for boring bar cutting tips range between 120% and 150% of the bar's radius length. The two curves, namely orange and yellow, predict the cutting tip receptance magnitude due to torsion for these two eccentricity limiting cases. It is essential to highlight the fact that the torsion related



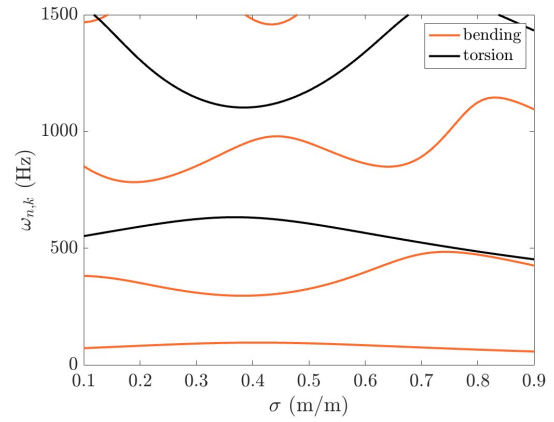
(a) Max tip receptance magnitude, shorter bar



(b) Natural frequency distribution, shorter bar



(c) Max tip receptance magnitude, longer bar



(d) Natural frequency distribution, longer bar

Figure 3.2: Receptance magnitude and frequency distribution along the length ratio variation

receptances possess larger values in absolute magnitude in comparison to the bending receptances once the passive absorber has been introduced.

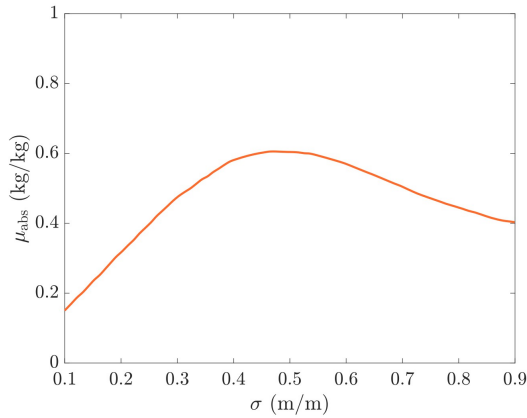
The Figure 3.2c depicts the calculated magnitude results for the longer case. Generally, similar trends can be seen. However, it is important to highlight that for the longer bar case there exists a sudden increase in the torsional receptance magnitude, which starts at around 40% length ratio. This factor must be considered in the optimal length ratio selection if present.

If the two cases are to be compared, the shorter bar possesses considerably higher stiffness characteristics for the undamped bending mode. This is evident in the difference of the two receptance magnitude values in relation to the length ratio σ , e.g. the value of the receptance magnitude of the shorter bar is approximately twice as much as of the

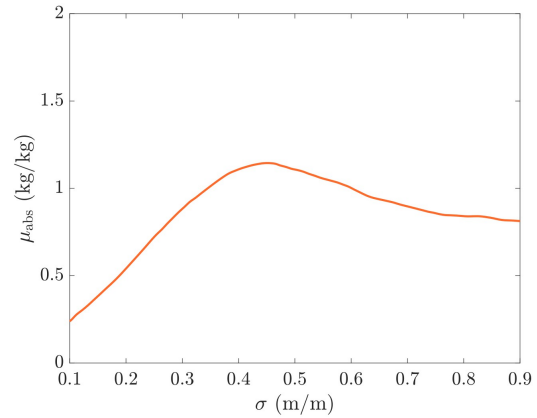
longer bar at 10% length ratio. This is related to the fact that the wall thickness w of the hollow segment is the same for both cases, meaning a larger, and relatively thinner, ring cross-section appears to be stiffer. However, as the dynamic behaviour of the ring cross-section is difficult to predict, a definitive statement regarding the reason of such findings cannot be made.

The second and the forth sub-figures, Figure 3.2b and d, depict distribution of the natural frequencies $\omega_{n,k}$ along the length ratio σ variation. It should be emphasized that first torsional natural frequency is significantly higher than the first, or dominant, bending natural frequency.

Another finding that should be highlighted is the obtained absorber mass ratio μ_{abs} as a function of the length ratio σ . It was shown in the previous section that the highest possible mass ratio μ_{abs} is preferred, as it tends to decrease the final bending receptance amplitude. Therefore, the determined relationship between the mass ratio and the length ratio, Figure 3.3, can be used for the optimal length ratio selection. For both cases, namely the shorter and the longer bars, the optimum value for the length ratio σ is located around 50%. Besides, it should be noted that the absolute value of the absorber mass ratio is larger for the longer bar case, which can be explained by the actual free volume space difference between the two cases where the absorber is placed.



(a) Absorber mass ratio, shorter bar



(b) Absorber mass ratio, longer bar

Figure 3.3: Absorber mass ratio

3.3 Rubber Bushings Optimization

The connection properties, such as stiffness and damping, between the boring bar and the passive absorber are provided by rubber bushings and oil. Based on the previous section, it is clearly evident that the analytical model of the boring bar possesses significant torsion related effects on the response of the tip, which is directly related to the final surface quality of the workpiece and thus should be minimized. This section analyses the relation between the radial $k_{\text{abs,rad}}$ and torsional stiffnesses of the rubber bushings and tries to find an optimal design parameters to minimize the torsional vibrations whilst being initially tuned to absorb the bending vibrations.

Model description

The rubber bushing is characterised by the length L , the outer radius b , the inner radius a , and the elastic shear modulus G_r , see Figure 3.4.

According to Horton and Tupholme (2006), the radial stiffness of rubber bushings can be accurately approximated in the following way:

$$k_{\text{rad}} = 4\pi \left(\frac{S^2 + 2.45}{S^2 + 1.45} \ln \left(\frac{b}{a} \right) - \frac{b^2 - a^2}{b^2 + a^2} \right)^{-1} G_r, \quad (3.3)$$

where S is the so-called shape factor, which is defined as the ratio of the loaded bonded area to the force-free lateral surface area:

$$S = \frac{2L(b^3 - a^3)}{3(b^2 - a^2)^2}. \quad (3.4)$$

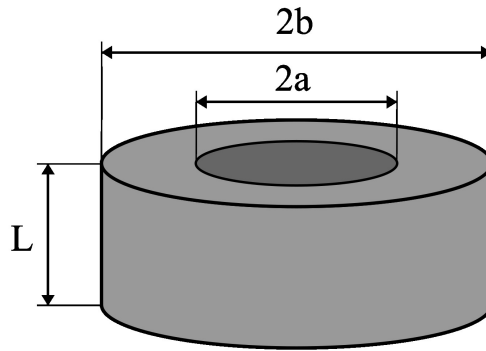


Figure 3.4: Rubber bushings

According to Adkins and Gent (1954), the torsional stiffness is defined as:

$$k_{\text{tor}} = 4\pi \frac{a^2 b^2 L}{b^2 - a^2} G_r. \quad (3.5)$$

Optimization methodology

The design and optimization procedure of the rubber rings is initiated with the given optimal radial stiffness, that is obtained from the Sims tuning methodology. The radial stiffness is a function of all dimension parameters, $k_{\text{rad}} = f(a, b, L)$. Let the outer radius b be a constant value which is equal to the inner radius of the hollow section of the boring bar. Consequently, the radial stiffness is simplified to be a function of only one variable, $k_{\text{rad}} = f(a)$ or $k_{\text{rad}} = f(L)$, as the remaining parameters a and L are coupled. As the length of the rubber bushings does not have a limiting characteristics, it is more effective to vary the inner radius a instead. To properly describe the design optimization process, a new non-dimensional parameter χ , a rubber bushing radii ratio, is introduced:

$$\chi := a/b, \quad (3.6)$$

where the ratio cannot exceed a value of one.

As it is impossible to derive a closed form equation that describes the length of the rubber bushing in terms of the inner and outer radii, and the radial stiffness, the value of L is determined via the Newton-Raphson scheme. The analytical formula that it used in the Newton-Raphson scheme is rather long and therefore is not presented in this work.

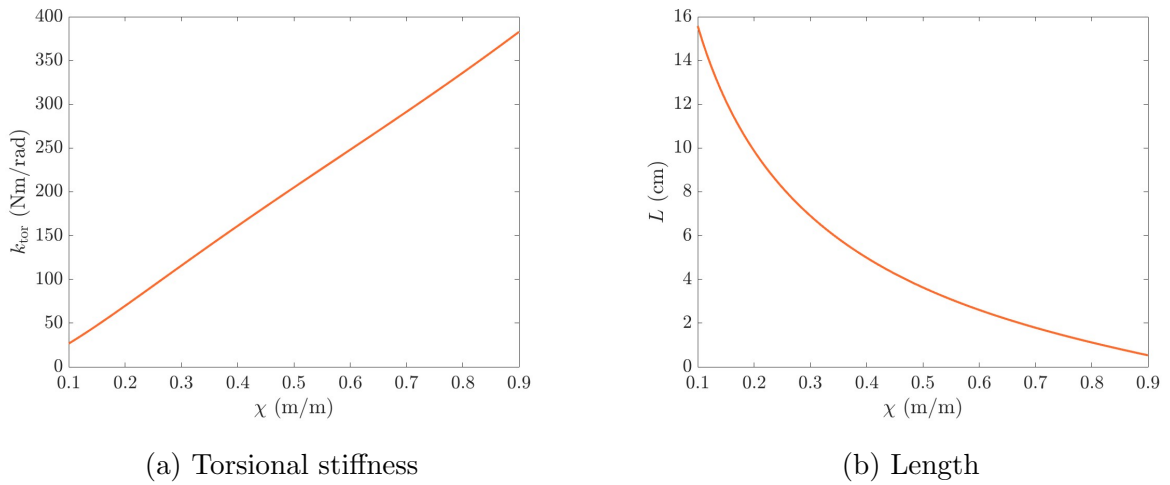


Figure 3.5: Radii ratio variation plots

The Figure 3.5 depicts the length L and torsional stiffness k_{tor} with relation to the radii ratio χ .

A reasonable question to ask at this stage is how to properly select the radii ratio in order to minimize the torsional vibrations. If the tuning methodology presented in the previous chapter is applied to the torsional FRFs, the obtained optimal torsional stiffness is far greater than the maximum possible value presented in the Figure 3.5a. Thus, it can be stated that the rubber bushings design methodology should focus on maximizing the torsional stiffness k_{tor} . However, this causes a number of problems with the real life application, as it is necessary to reduce the thickness, i.e. radii difference, of the rubber bushings as much as possible. This would ultimately lead to the reduction of the length of the rubber rings as well.

Next, a variation of the outer radius b is also considered. The goal of this scheme is to determine the behaviour of the torsional stiffness and the length of the rubber bushings and find the possible optimal value for the outer radius. The results are depicted in Figure 3.6. It can be clearly seen that the maximum values of the outer radius b is preferred.

Thus, the optimal design of the rubber bushings involves the maximization of the both inner and outer radii, a and b respectively, until it is reasonable.

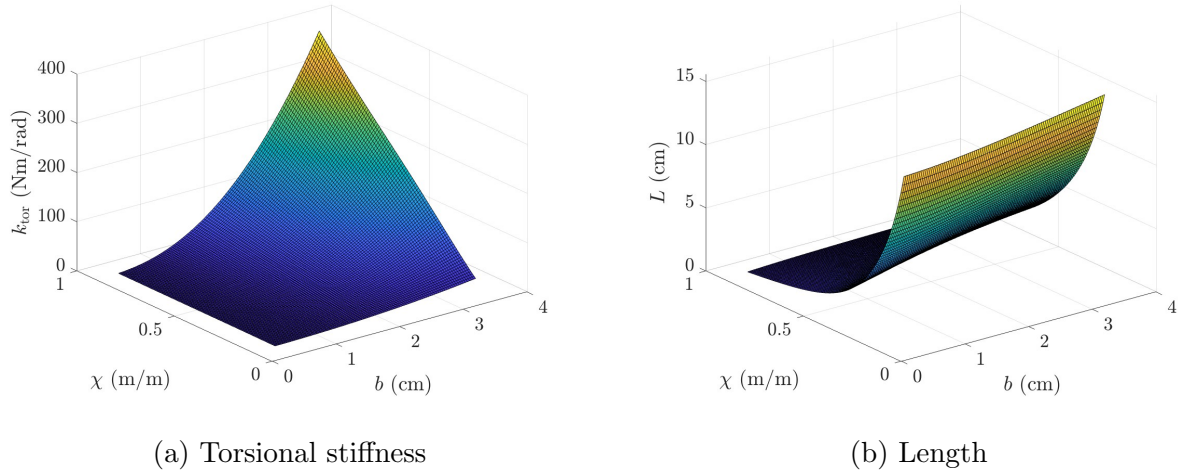


Figure 3.6: Radii ratio variation surfaces

Analytical Simulation

In this section an analytical calculation of the cutting tip receptance is performed, where the radial stiffness k_{rad} is based on the tuning methodology whilst the torsional stiffness k_{tor} is obtained based on the tuned radial stiffness k_{rad} .

Based on the statements made in the previous sections, the following parameters are assumed:

$$\begin{aligned} \text{length to diameter ratio} \quad L/D &= 7 \text{ m/m}, \\ \text{bar length} \quad L &= 140 \text{ cm}, \\ \text{bar diameter} \quad L &= 20 \text{ cm}, \\ \text{length ratio} \quad \sigma &= 0.45 \text{ m/m}, \\ \text{eccentricity} \quad e &= 1.2 D/2 \text{ cm}, \\ \text{inner hollow segment radius} \quad r_{\text{hol}} &= 9.5 \text{ cm}, \\ \text{rubber bushing outer radius} \quad b &= r_{\text{hol}} \text{ cm}, \\ \text{rubber bushing radii ratio} \quad \chi &= 0.8 \text{ m/m}, \\ \text{absorber volumetric ratio} \quad \mu_{\text{vol}} &= 0.9 \text{ m}^3/\text{m}^3. \end{aligned}$$

It is also assumed that the optimal radial damping constant can also be achieved. The following question arises: "What should the torsional damping constant be?" In the following simulation it is assumed that the ratio between the radial and torsional damping constants is proportional to the corresponding stiffness constants ratio.

$$\frac{1}{n} \frac{c_{\text{rad}}}{c_{\text{tor}}} = \frac{k_{\text{rad}}}{k_{\text{tor}}}. \quad (3.7)$$

Taking the assumption presented above, the calculated cutting tip receptances related to both bending and torsion are presented in Figure 3.7, where the the parameter n is varied between the values of 1, 10, and 50. The reason for the parameter n variation is the uncertainty of the radial and torsional damping properties for the boring bar and passive absorber structure.

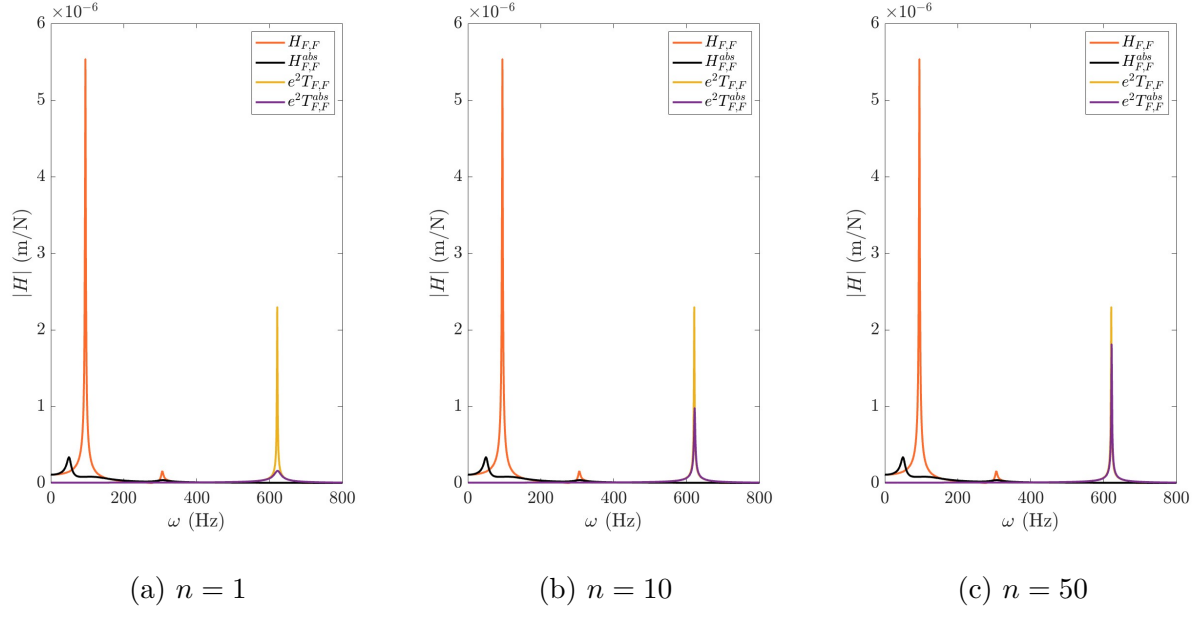


Figure 3.7: Cutting tip FRFs with introduced absorber

It should be brought to attention that the transformed torsional receptance $e^2 T_{F,F}^{abs}$, depicted with the purple curve in Figure 3.7, cannot be sufficiently suppressed with the previously presented absorber design and tuning.

Chapter 4

Measurement

The following chapter introduces the measurement set-up and the results. The aim of the measurement is to determine the machine behaviour.

4.1 Set-up

The measurement set-up consists of the following items:

- machine tool
- short bar
- long bar
- bar holder
- modal impact hammer
- plastic hammer tip
- metal hammer tip
- uniaxial acceleration sensor
- signal processing unit
- computer

Firstly, the bars were secured inside the bar holder. The insertion length for both bars was 50 mm. Once the bars were secured inside the bar holder, the bar holder itself was clamped to the CNC machine tool. The following step was to introduce the uniaxial acceleration sensor to the bars and begin the measurement.

The time signal of the measurement was selected to be 2 seconds long. This was done to achieve a half frequency resolution.

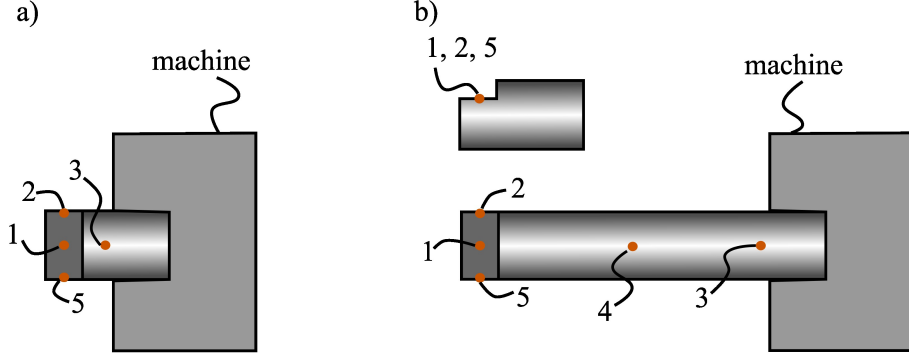


Figure 4.1: Measurement points for a) short bar, b) long bar



(a) Short bar measurement set-up



(b) Long bar measurement set-up

Figure 4.2: Measurement set-up

The Figure 4.1 presents the sets of sensing and excitation points for both the short and long bars. It can be seen that the tip side of each bar has an additional cut, which was done in order to sense and excite the system at a certain distance from the main axis, i.e. to measure the torsional vibration modes.

The assembled set-ups for both shorter and longer bars can be seen in the Figure 4.2.

4.2 Measurement Data

This section introduces the measurement results for both measurement set-ups, namely a short bar and a long bar. In this section the superscript "s" stands for the short bar, $\mathbf{H}_{i,j}^s$, whilst "l" stands for the long bar $\mathbf{H}_{i,j}^l$.

The impact type measurements were firstly taken with the plastic hammer tip. In this

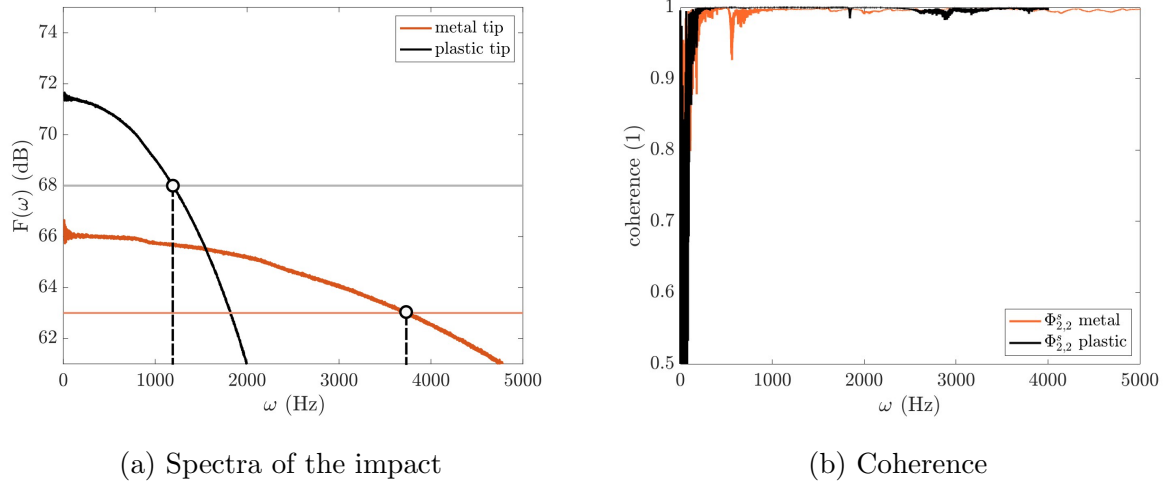


Figure 4.3: Measurement evaluation metrics

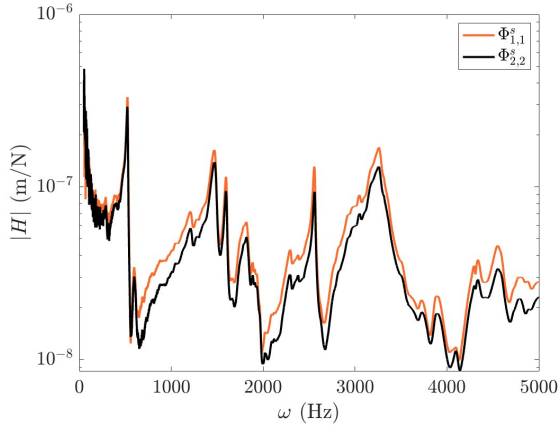
case there were not any problems with the multiple hits, even at the tip of the long bar the single hits were steadily achieved. However, the excitation bandwidth for the plastic tip was not sufficient to excite the first torsional mode of the long bar. The analytical torsional mode for the long bar occurs at around 3000 Hz, see Figure 4.5, whilst the acceptable frequency bandwidth for the plastic tip spans until around 1200 Hz, see Figure 4.3a.

Later, once all the permutations of sensing and excitation points were measured with the plastic tip, the metal tip was introduced to the modal impact hammer. As can be seen in the Figure 4.3a, the measured FRFs with the metal tip can be accepted until almost 4000 Hz (Ewins 2009).

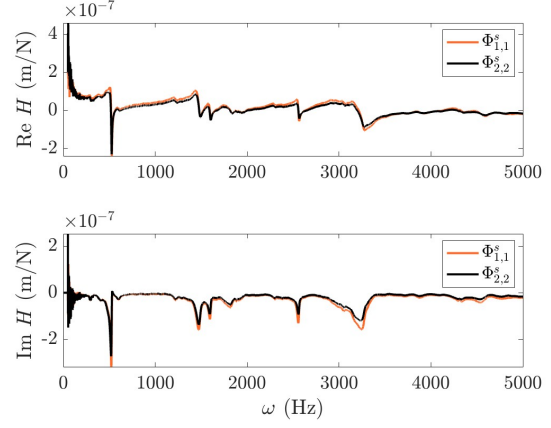
Each permutation of the sensing and excitation points were averaged based on three separate measurements. The Figure 4.3b depicts the calculated coherence for the direct off-centre measurement of the shorter bar with both plastic and metal hammer tips. Generally, throughout the whole measurement similar trends in the coherence curves could be observed. The receptance data contains a significant amount of noise up to 150 Hz and remains accurate for the rest of the frequency range.

The Figure 4.4 presents the directly measured FRFs of the short bar, $\Phi_{1,1}^s$ and $\Phi_{2,2}^s$, both in a logarithmic scale and linear scale. It can be clearly seen that the direct off-centre receptance has a lower absolute magnitude in comparison to the direct measurement $\Phi_{1,1}^s$. This could be explained by the additional static stiffness characteristics of the torsional vibrational mode.

The measured direct FRFs of the long bar are presented in the Figure 4.5 alongside

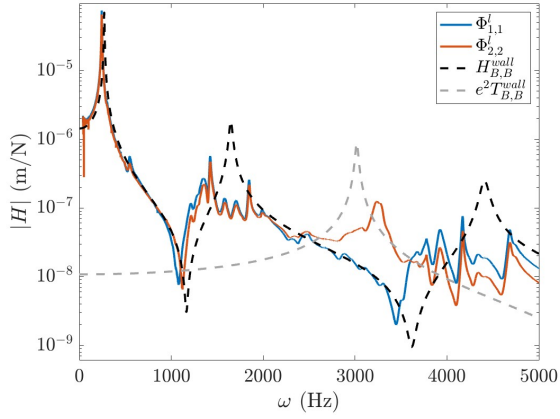


(a) Magnitude in logarithmic scale

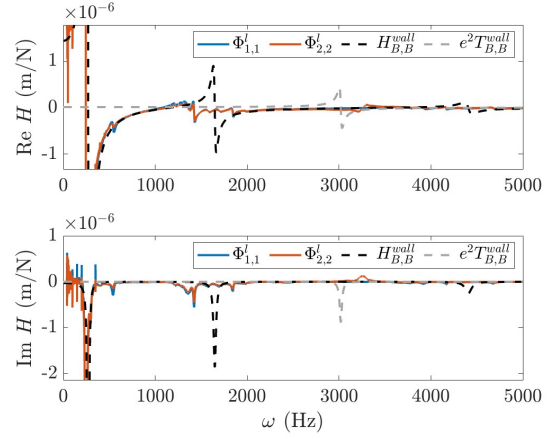


(b) Real and Imaginary parts in linear scale

Figure 4.4: Short bar measured FRFs



(a) Magnitude in logarithmic scale



(b) Real and Imaginary parts in linear scale

Figure 4.5: Long bar measured FRFs

with the analytical receptances of a cantilever beam with similar physical properties. The analytical FRFs are denoted with $H_{1,1}^{wall}$ and $T_{1,1}^{wall}$. The presence of the torsional mode in the direct receptance at the point 2 is distinctly noticeable. The first bending mode has a perfect match. The second bending mode is scattered due to the contact imperfections, which are magnified due to the second mode shape. The third bending mode is also suppressed, however, it should be noted that the natural frequency in this case is larger than the acceptable excitation frequency bandwidth.

4.3 Inverse Receptance Coupling

This section describes the inverse RCSA methodology used in this work.

Determination of the machine dynamics

The first bending and torsional vibration modes of the short bar occur at rather high frequencies, and thus, cannot be excited with the impact testing. Therefore, the short bar measurement are utilized in order to determine the unknown machine dynamics.

The unknown machine dynamics matrix is defined in the following way:

$$\mathbf{G}_{X,X}(\omega) := \begin{bmatrix} H_{X,X}(\omega) & L_{X,X}(\omega) & 0 \\ M_{X,X}(\omega) & N_{X,X}(\omega) & 0 \\ 0 & 0 & T_{X,X}(\omega) \end{bmatrix}. \quad (4.1)$$

Park, Altintas, and Movahhedy (2003) performed the inverse RCSA for two unknowns, $L_{X,X}$ and $N_{X,X}$. The suggested approach is also applied in the current work, however, it is extended from two to five unknowns which is presented in the equation 4.1. It can be argued that the off-diagonal elements are in fact equal to each other, however, this stands true only for a linear system. The machine tool dynamics cannot be assumed to be perfectly linear and thus, the assumption of the equivalent off-diagonal elements cannot be made.

The RCSA equations that were presented in the second chapter are used to derive the responses at all possible permutations of the sensing and the excitation points. As the data is measured in (m/N), only the first element of the predicted FRF matrices can be utilized. The equation are presented below.

$$\Phi_{1,1}^s = \mathbf{H}_{1,1}^s(1, 1) \quad (\text{m/N}) \quad (4.2)$$

$$\Phi_{1,3}^s = \mathbf{H}_{1,3}^s(1, 1) \quad (\text{m/N}) \quad (4.3)$$

$$\Phi_{3,1}^s = \mathbf{H}_{3,1}^s(1, 1) \quad (\text{m/N}) \quad (4.4)$$

$$\Phi_{3,3}^s = \mathbf{H}_{3,3}^s(1, 1) \quad (\text{m/N}) \quad (4.5)$$

$$\Phi_2^s = \mathbf{H}_{1,1}^s(3, 3) \quad (\text{rad/Nm}) \quad (4.6)$$

The equations 4.2 to 4.5 are rather straightforward, whereas, the torsion related equation 4.6 needs to be elaborated. The parameter Φ_2^s is defined in the following way:

$$\Phi_2^s := (\Phi_{1,1}^s - \Phi_{2,2}^s) e^{-2} \quad (\text{rad/Nm}). \quad (4.7)$$

The idea is that the difference between the direct measurements $\Phi_{1,1}^s$ and $\Phi_{2,2}^s$, see Figure 4.4, must be related to the torsional behaviour of the combined mechanical system, the bar and the machine.

The calculated torsion related parameter Φ_2^s can be seen in the Figure 4.6.

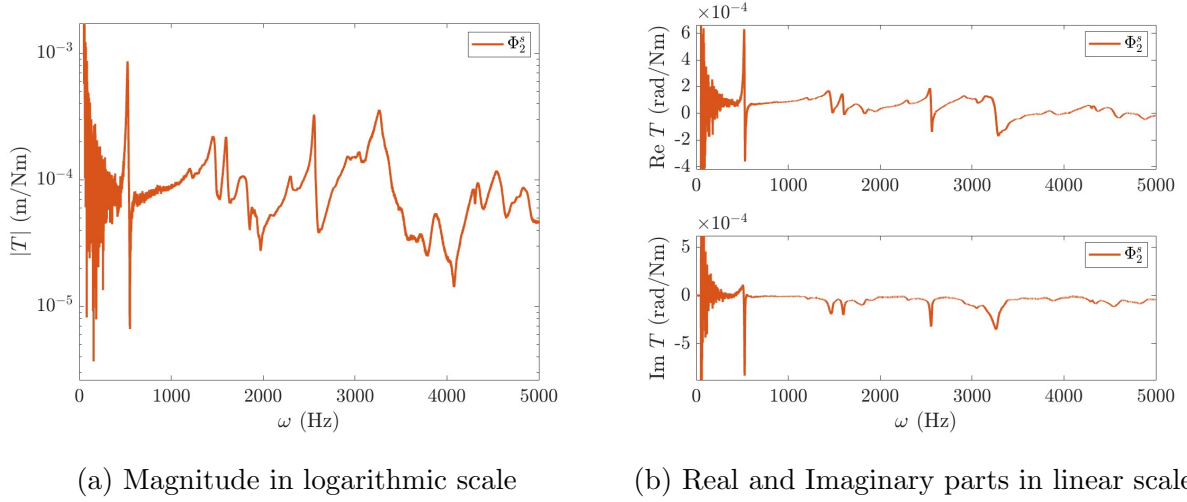


Figure 4.6: Torsion related receptance difference

The equations 4.2 to 4.6 create a determined linear system of 5 equations and 5 unknowns. Once rearranged, it is possible to derive a closed form solution for each unknown, $H_{X,X}(\omega)$, $L_{X,X}(\omega)$, $M_{X,X}(\omega)$, $N_{X,X}(\omega)$, and $T_{X,X}(\omega)$. These formulas are rather long and therefore are not presented in this work.

Thus, by performing the inverse RCSA, i.e. subtracting the short bar receptances $\mathbf{H}_{i,j}^s(k, k)$, from the the short bar measurements $\Phi_{i,j}^s$, the unknown machine dynamics matrix is determined.

The Figure 4.7 depicts all elements of the machine dynamics $\mathbf{G}_{X,X}(\omega)$. The sensing and the excitation points during the measurement could not possibly stay perfectly consistent throughout the whole measurement procedure. This could lead to the deviation of the determined machine dynamics results, which is especially true for the fifth element of the matrix, $N_{X,X}(\omega)$. The calculations were performed for the varying length of the short bar, where the total difference between the lowest and highest values is a couple of millimetres. The light grey curves depict the obtained machine dynamics for each value of the changing

bar length. The orange curve was selected as the most stable and least noisy machine FRFs. Additionally, the shaded region marks the acceptable excitation frequency bandwidth based on the fast Fourier transform of the impact and the coherence, see Figure 4.3.

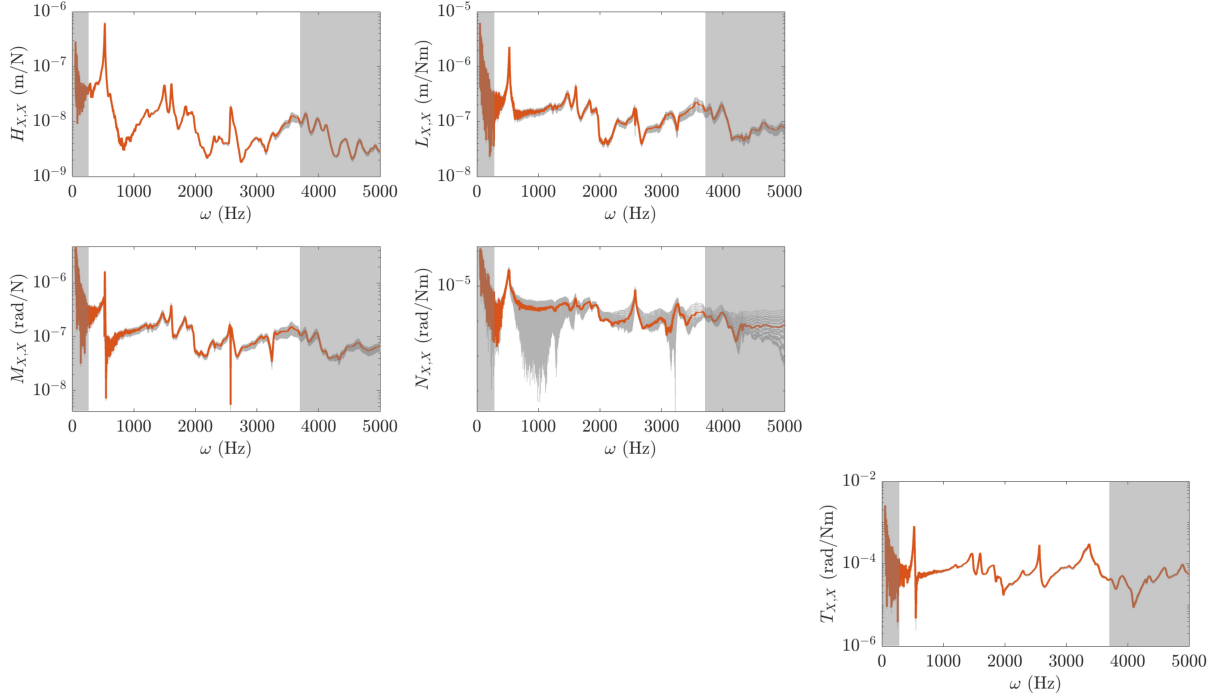


Figure 4.7: Machine dynamics (metal tip)

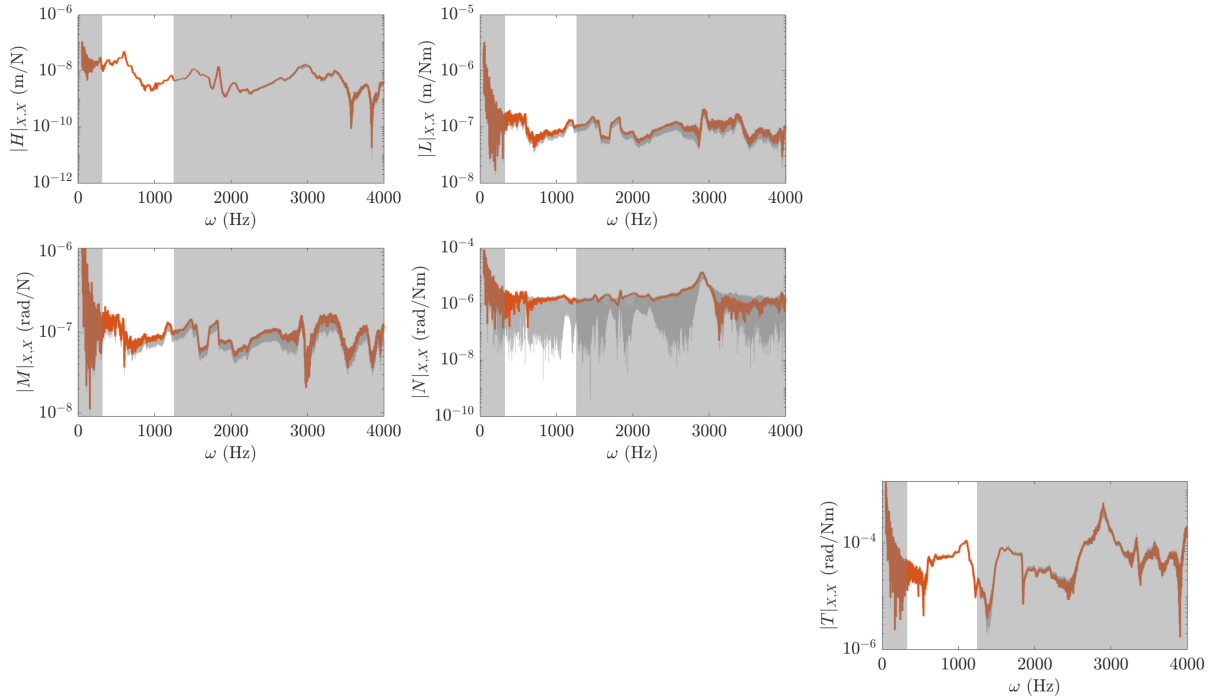


Figure 4.8: Machine dynamics (plastic tip)

The machine dynamics were also determined based on the plastic tip measurement data. The results can be seen in the Figure 4.8.

Validation of the machine dynamics

The modal analysis theory is based on linear systems (Dombóvári 2023), and thus, in theory, adding a short bar back to the obtained machine dynamics should produce the exact measurements results. The RCSA results between the machine dynamics and the short bar are presented below.

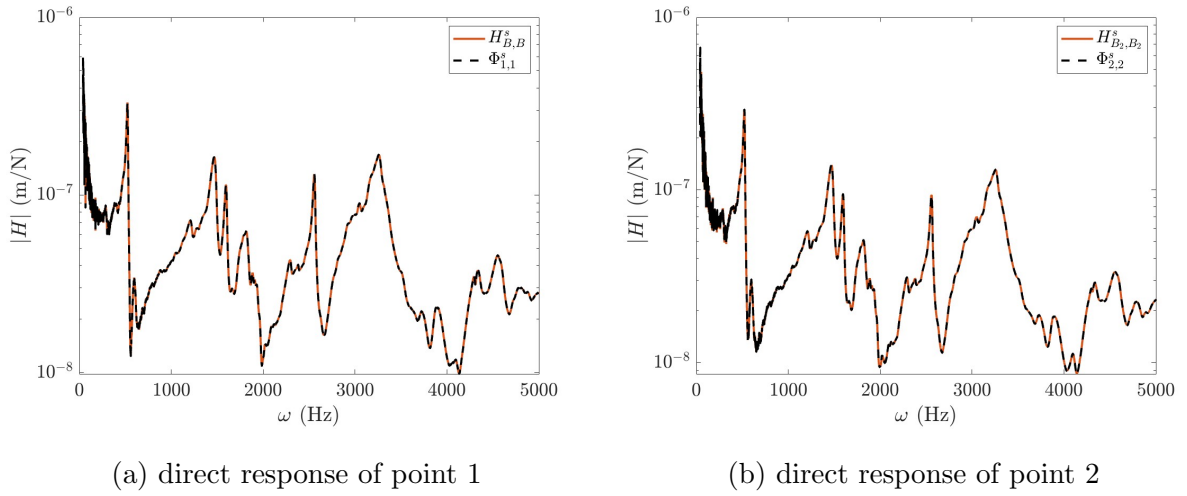


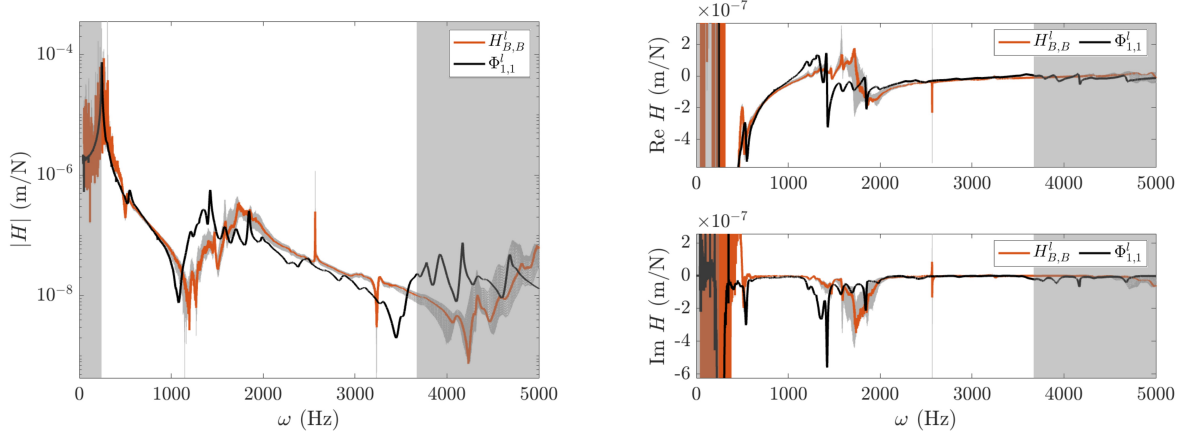
Figure 4.9: Short bar predicted and measured receptances

As can be seen in Figure 4.9, the predicted $H_{i,i}^s$ and the measured $\Phi_{i,i}^s$ receptances of the short bar coincide. Thus, the derived inverse RCSA scheme is validated.

4.4 Receptance Coupling with Machine Dynamics

In this section the results of the RCSA between the obtained machine dynamics and the long bar are presented and compared to the measurement data.

The obtained direct FRFs for the point 1 are plotted in comparison with the measurement data in Figure 4.10, where $H_{B,B}^l$ stands for the predicted receptance of the mechanical system that consists of the determined machine dynamics and a single beam segment, see Figure 2.5 for reference.



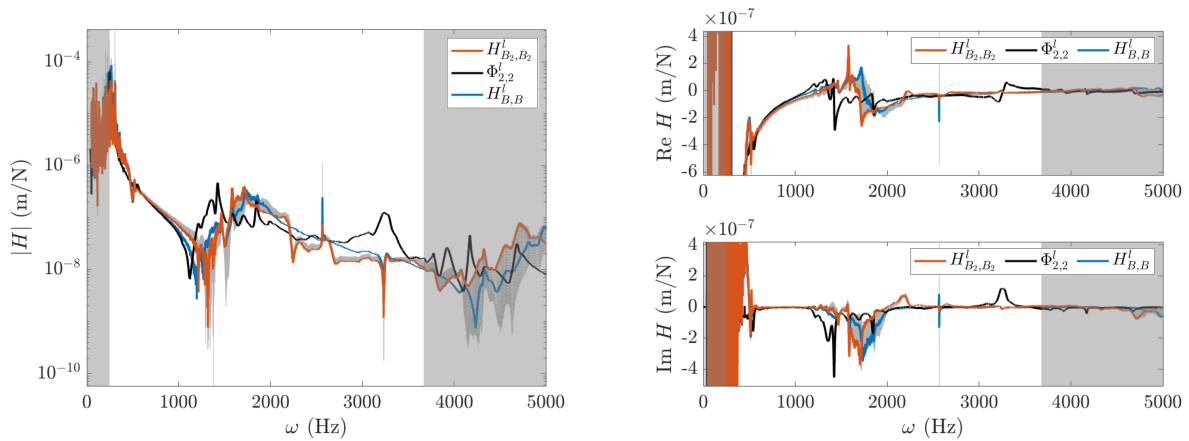
(a) Magnitude in logarithmic scale

(b) Real and Imaginary parts in linear scale

Figure 4.10: Long bar direct centre predicted and measured receptances

Generally, the predicted $H_{B,B}^l$ is similar to the measured data $\Phi_{1,1}^l$. As in the Figures 4.7 and 4.8, the light grey curves showcase the FRFs for a series of the calculated machine dynamics. Consequently, it was possible to avoid the sharp anti-resonances that can be seen at around 1100 and 3200 Hz with a proper selection of the accepted long bar FRFs $H_{B,B}^l$. Anti-resonances of such behaviour are the consequences of the numerical errors.

The predicted and measured FRFs for the off-centre sensing and excitation are plotted in the Figure 4.11. Since the predicted response H_{B_2,B_2}^l is in fact a variation of the direct centre FRF $H_{B,B}^l$, where their relationship is shown in the equation 4.7, the response $H_{B,B}^l$ has also been plotted.



(a) Magnitude in logarithmic scale

(b) Real and Imaginary parts in linear scale

Figure 4.11: Long bar direct off-centre predicted and measured receptances

At the first glance, the predicted response H_{B_2, B_2}^l is not so much different from the measured $\Phi_{2,2}^l$ response. However, no clear torsional mode can be seen. It must be pointed out that the predicted off-centre and centre responses H_{B_2, B_2}^l and $H_{B, B}^l$ differentiate from one another throughout the shown frequency range except at certain regions, such as the first and the second bending modes, as well as exactly at the measured torsional mode. This fact should be further analysed in order to identify the cause of the absence of the torsional mode in the predicted FRF. Additionally, it should be noted that the flipped torsional mode in the measured data $\Phi_{2,2}^l$ was expected, as the excitation point was on the other side of the bar. In other words, $\Phi_{2,2}^l \equiv \Phi_{2,5}^l$ in this work. This measured FRF is not used for any calculations, and therefore, exciting on the opposite side of the bar was preferred due to convenience.

Even though the machine dynamics produce similar predicted responses to the measured data, particularly for the bending modes, the results are far away from being accurate, and therefore, the model should be developed further.

Chapter 5

Conclusion

The purpose of this work was to study the dynamic properties of the presented boring bar mechanical system. The bending related issues with the long boring bars has already been studied in the research and industry fields (Rubio et al. 2013). However, the question of the torsional behaviour of the bar is still relevant to this day.

This work started the development of the boring bar dynamics in an analytical way. The bending and torsional behaviour of the bar was described based on the well-known Timoshenko and Euler beam models. A passive mass damper was introduced into the model in order to attenuate the dominant bending mode. Afterwards, the work focused on the parametric design optimization methodologies for the assembled boring bar model. Lastly, the measurement was performed with the aim of determining the machine dynamics.

The Analytical Modelling chapter introduced the continuous beam models that describe the bending and torsional vibrations. Timoshenko–Ehrenfest beam model was used for describing the bending behaviour of the boring bar, whilst Euler-Bernoulli beam model represented the torsional related characteristics. Both models were solved in the frequency domain as a means to determine the free-free analytical FRFs of the separate bar segments. A simplified receptance derivation for an arbitrary rigid element was also shown in this work. This model was used for describing the inertial properties of the cutting head for the subsequent receptance coupling. The analytical FRFs of an assembled boring bar, which consisted of the cutting head, a hollow segment, a solid segment, and a rigid wall, were validated with the help of FEM numerical simulations. Lastly, a passive vibration absorber was introduced to the hollow segment of the bar. The tuning was performed based on the tuning methodology presented by Sims (2007), which focuses on the regenerative vibrations.

Thereupon a number of possible design optimization methodologies for the assembled

boring bar system were developed. Firstly, it was shown that the absorber size should be maximized as much as physically possible. Secondly, the analytical receptance calculations for the cutting tip were performed for a continuously varying length ratio of the hollow segment to the total length of the bar. Parameters such as the magnitude of the receptances related to both bending and torsion, the natural frequency distribution, and the absorber mass ratio were plotted against the hollow segment length ratio. An optimal sizing parameter was suggested to be based on the maximum of the absorber mass ratio for the shorter bar calculation. As for the longer bar, the optimal sizing parameter was proposed to be selected based on the sudden increase in the torsional receptance magnitude. Lastly, the rubber bushings design scheme was presented. The methodology begins from the tuned radial stiffness value, which is based in the Sims tuning. The dimensions of the rubber are varied, whilst keeping the previously determined radial stiffness characteristics constant. It was discovered that the thinnest physically reasonable rubber rings are preferred, as they possess the highest torsional stiffness constants.

Lastly, the measurement of two solid bars was performed. The objective of the measurement was to determine the machine dynamics and to observe the torsional behaviour of the complete machine tool system. The inverse RCSA methodology was extended based on Park, Altintas, and Movahhedy (2003) previous work and was used for subtracting the short bar receptances from the measurement data as a means to isolate the machine dynamics. The determined machine dynamics were used in the subsequent RCSA with the analytical free-free responses of the longer bar, and the results were compared with the measured data.

In summary, in the analytical boring bar model it was shown that the torsional effects are indeed present and significant in comparison to the attenuated bending vibrations. Therefore, this work tried to create an optimization methodology for the boring bar design in a way to minimize the torsional vibrations whilst keeping the tuned bending related parameters constant.

As for the future work, the torsional behaviour of the bar should be studied further. In addition, the passive damper design that would be simultaneously optimal for the bending and torsional vibrations is still an open question. Besides, a more sophisticated measurement procedure should be performed with the aim of obtaining a more precise machine dynamics characteristics that would accurately represent the torsional behaviour.

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