

Exploring Spatial and Temporal Patterns Across Solar Cycles Focusing on Active Longitudes

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A dolgozat magyar nyelvű összefoglalója

A Nap számos jelenségnek ad otthont, melyek tanulmányozása a napfizikusok régóta tartó feladata. Ezen jelenségek vizsgálata kulcsfontosságú a Nap működésének mélyebb megértéséhez, különös tekintettel az esetlegesen veszélyes napkitörések – a nagyenergiájú flerek és a korona-anyagkidobódások (CME) – előrejelzéséhez, amelyek jelentős hatással lehetnek a modern emberi infrastruktúrára. A napkitörések és a Föld közötti kölcsönhatás egy összetett eseménysorozatból áll, amelyet összefoglaló néven űridőjárásnak nevezünk. A flerek és a CME-k jelentős károkat okozhatnak, akár néhány órán belül is, mind a Földön, mind az űrben működő technológiákban, ipari létesítményekben és szolgáltató hálózatokban. Néhány múltbéli példa: 1989-ben egy geomágneses vihar több órás áramkimaradást és több napos áramfennakadást okozott a kanadai Quebecben. 2012-ben az eddig ismert legveszélyesebb CME-kitörés mindössze 30 fokkal kerülte el a Földet. 2014 májusában egy hatalmas napkitörés érte el a Földet, amely több napig fennakadásokat okozott az északi országok elektromos ellátásában és a GPS-rendszerekben.

Az 11 éves napfoltciklus során ezek a nagyenergiájú napkitörések jellemzően "viharos időszakokhoz" köthetők. Ezek a "viharos időszakok" több héten keresztül tarthatnak, és hosszabb, hetekig vagy hónapokig tartó csendesebb időszakok választják el őket egymástól. Ezekben a "viharos időszakokban" történnek a legveszélyesebb űridőjárási események, amelyek fennakadásokat okozhatnak a modern földi technológiában.

A korábbi modellezési és előrejelzési erőfeszítések főként a néhány órától napokig terjedő időtávokra koncentráltak. A nemrég felfedezett aktív hosszúságok azonban lehetőséget nyújthatnak arra, hogy néhány héttel előre jelezzük a nagyenergiájú napkitöréseket. Az aktív hosszúságok olyan hosszúsági tartományok a Napon, amelyek körülbelül 60 fok szélesek, és akár 2-6 hónapig is fennmaradhatnak. Ezeken az aktív hosszúságokon a naptevékenység különféle jelenségei – például a napfoltok vagy napkitörések – jelentősen gyakrabban fordulnak elő, mint más hosszúságokon. Mindezenáltal további vizsgálatok szükségesek annak megállapításához, hogy az aktív hosszúságok valóban összefüggésben állhatnak-e a legerősebb napkitörésekkel. Ezek a nagyenergiájú napkitörések általában olyan "nagy, rossz és mérges" aktív régiókból származnak, amelyek mágneses terükben jelentősen összetettebbek az átlagos aktív régióknál. Egyes feltételezések szerint ezek az aktív régiók gyakran jelennek meg olyan hosszúsági tartományokban, amelyeket aktív hosszúságoknak nevezünk. Az aktív hosszúságok létezése azonban még nem került alapos vizsgálatok alá olyan adathalmazokon, amelyek több mint 100 évet ölelnek fel; eddig csak körülbelül 11 vagy 22 éves időszakokra vonatkozó adatok alapján tanulmányozták.

A fentieket figyelembe véve a jelen kutatásom fő célja, hogy azonosítsam és megerősítsem az aktív hosszúságok létezését, valamint megvizsgáljam, hogy ezek a "nagy, rossz és mérges" aktív régiók hajlamosak-e az aktív hosszúságokon megjelenni. Továbbá célom az aktív hosszúságok kapcsolatának feltárása a nagyenergiájú napkitörésekkel. Ennek érdekében egy több mint 140 évet (13 teljes napciklus) lefedő napfoltkatalógus alapos elemzésével vizsgáltam az aktív hosszúságok létezését. Emellett egy több mint 70 évre visszatekintő napkitörés-katalógus segítségével igyekeztem megállapítani, hogy fennáll-e kapcsolat az aktív hosszúságok és a "napvihar idősza-kok" között. Az aktív régiók és a flerek szinoptikus térképeken kerültek ábrázolásra, majd Kernel Density Estimation (KDE) módszer segítségével azonosítottam aktív

hosszúságokat a napfoltok és a flerek esetében minden Carrington-rotáció során.

A munkám során világossá vált, hogy a napfoltok és a flerek nem véletlenszerűen, hanem aktív hosszúságokon mentén jelennek meg, és ezek az aktív hosszúságok akár több héten vagy hónapon keresztül is fennmaradhatnak. Annak ellenőrzésére, hogy ezek az aktív hosszúságok valódiak-e, és nem véletlenszerű eloszlásból erednek, Kolmogorov-Smirnov tesztet alkalmaztam. Az eredmények megerősítették, hogy ezek az eloszlások nagy valószínűséggel valódiak: a teszt P-értéke $7 \cdot 10^{-5}$.

A vizsgálat során az is feltűnt, hogy ezek az aktív hosszúságok periodikusan jelennek meg az egyes napciklusok során. Ezért wavelet-analízis segítségével számos kvázi-kétéves oszcillációs (Quasi Biennial Oscillation - QBO) periódust azonosítottam a napfoltok és flerek aktív hosszúsági adataiban. A QBO-k oszcillációk egész családját jelentik, melyek egyik fontos főszereplői a mágneses Rossby-hullámok. Gyakori feltételezés, hogy ezek a mágneses Rossby-hullámok felelősek az aktív hosszúságok kialakulásáért. A QBO-k, kb. 0.6-4 éves periódusait már sok napjelenységben kimutatták, úgymint a besugárzás, a napfoltszám vagy a napszél időbeli változásai.

Jelen munkám során megerősítettem, hogy erős kapcsolat áll fenn az aktív hosszúság létezése, a "nagy, rossz és mérges" aktív régiók, valamint a nagyenergiájú napkitörések között, illetve a QBO-k családja között. Jövőbeni vizsgálatom célja annak feltárása, hogy az aktív hosszúságok periodikus megjelenése milyen lehetőségeket biztosíthat a várható "napvihar időszakok" előrejelzésére.

Abstract

Solar active regions have long been established to exhibit latitudinal time-dependence, as manifested in the butterfly diagram. Longitudinal time-dependence, however, has only been hypothesized for a long time, with several supporting studies presenting observational signs of this phenomenon. This longitudinally enhanced activity has been dubbed "active longitude," where certain regions may produce solar magnetic activity across several Carrington rotations on the solar surface.

The dataset used here to search for active longitudes naturally includes a catalog of sunspot groups, starting from as early as 1874. However, since higher magnitude flare activities are also clearly associated with active regions, we additionally employ flare data to affirm the findings with higher certainty. To accomplish this, flare data starting from 1938 will also be used in this study.

To search for the existence of active longitudes, we employ the methodology of Kernel Density Estimation to demonstrate that the appearance of these regions is not random: they tend to form clusters in longitudinal directions. This is evidenced by both the sunspot groups and the flares.

Furthermore, the active longitudes found show evidence of oscillating behaviour, as detailed by the associated wavelet analysis. The periods identified are consistent with the periodicities observed as quasi-biennial oscillations.

Our findings necessitate and highly encourage the development of new dynamo models that are able to account for not just the formation of such "active longitudes," but also address the oscillatory behaviour of these longitudinally localised active regions. Our results urge a paradigm shift towards developing a non-axisymmetric dynamo theory in three dimensions.

Introduction

Solar cycle - Spacial and temporal evolution of the solar activity

The global magnetic field plays a critical role in governing the behavior of the solar atmosphere and profoundly influences the dynamics of solar activity. Understanding these interactions is essential for advancing our knowledge of the Sun's influence on space weather. Space weather is of significant importance today, as large solar eruptions such as solar flares and coronal mass ejections (CMEs) can trigger strong space weather events, which in turn affect our modern society. One critical question that arises is how we can predict upcoming "solar storm seasons," and their potential impacts.

The first observations of the solar magnetic field were conducted by Hale (1908), who detected sunspots with magnetic fields as strong as 0.3 T—almost 100,000 times stronger than Earth's magnetic field. The Sun's magnetic field extends well beyond sunspots and is observed in the upper solar atmosphere, namely in the chromosphere and corona. While the magnetic field is rather strong in sunspots, reaching around 3000 G (0.3 T), outside of sunspots it may still be important, even though measuring only about 10 G. On the Sun, we typically refer to active regions, areas where groups of sunspots form, known as sunspot groups. These regions are sites of increased activity: larger flares, CMEs, and bright areas called plages often occur around them.

Sunspots and active regions are key indicators of the solar cycle and are central to understanding solar activity. Their number, distribution, and magnetic polarity change in a predictable pattern over the 11-year solar cycle and the 22-year magnetic cycle, known as the Hale cycle.

Active regions are often changing parts of the photosphere: larger ones could last many months, while smaller ones only last hours, before disappearing. The number of sunspots at a given time on the Sun gives us a quasi-periodic function of time, with the period being $\approx 11 \pm 2$ years. It is important to mention that the reason for the $\pm$ sign is not that the cycle's length is difficult to measure; it is that these cycles vary in length, hence the label, quasi-periodic. The most commonly given measure of this is the monthly smoothed sunspot number (monthly SSN). Plotting this in Figure 1, we can easily see that there are periods when there are no sunspots, which we call solar minima, and there are periods of varying intensity peaks, called solar maxima, with these alternating between one another (Hathaway, 2015).

Observations of many other solar phenomena show a strong correlation with this SSN, therefore it makes sense to demonstrate the Sun's 11-year activity using SSN. The solar minimum is considered to be the beginning of a given cycle. Typically, the time it takes for the cycle to increase to its maximum is considerably shorter than the

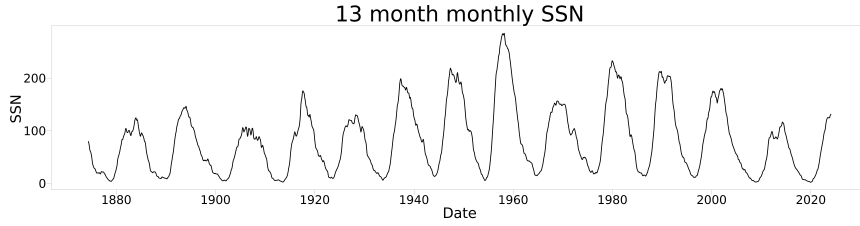


Figure 1: Plot of the 13-month average sunspot number, with data from [SILSO](#). The x -axis represents the year, and the y -axis represents the sunspot number.

time it takes for it to decrease back to the minimum, although there can be significant variances in this. Another important observation is that sunspots appear at lower and lower latitudes as the cycle progresses, resulting in the widely known butterfly diagram.

The butterfly diagram provides a visual representation of the latitudinal distribution of sunspots over the course of a solar cycle, serving as a fundamental tool for studying solar activity. This diagram showcases a distinctive pattern, characterized by the emergence of sunspots at higher latitudes at the onset of a solar cycle, with subsequent appearances progressively shifting closer to the equator as the cycle progresses, see in Fig 2.

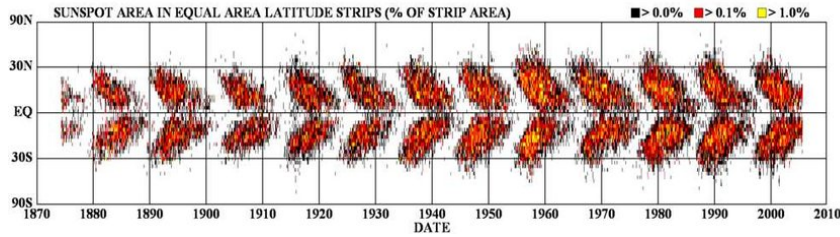


Figure 2: The butterfly diagram illustrates the distribution of sunspots across different latitudes over time. The darkest spots on the diagram represent areas with the largest sunspot coverage, averaged over individual solar rotations. Initially, sunspots appear at approximately 30 degrees latitude and subsequently migrate towards the solar equator. It is also evident that the number of sunspots increases with solar activity. Credit: [D. H. Hathaway](#)

Spatially, sunspot activity not only adheres to the latitudinal migration depicted in the butterfly diagram but also exhibits a tendency for active regions to cluster at preferential longitudes, referred to as active longitudes. This concept has been extensively studied since the early 20th century, with significant contributions from researchers such as (Losh, 1939; Bogart, 1982; Zhang et al., 2011), among others. Studies on active longitudes have consistently suggested that various solar activity indicators, including sunspots, the global magnetic field (Benevolenskaya et al., 1999; Bumba et al., 2000), the heliospheric magnetic field (Ruzmaikin et al., 2001; Mursula and Hiltula, 2004), and solar flares (Bai, 1988, 2003; Bumba and Howard, 1969; Gyenge et al., 2016), show a pronounced preference for specific longitudinal ranges, thereby reinforcing the idea of active longitudes. Furthermore, several studies have shown that active regions disobeying the hemispheric helicity rule appear to form in "activity nests" or active longitudes (Canfield and Pevtsov, 1998; Pevtsov and Canfield, 1999,

2000; Zhang and Bao, 1999; Tian et al., 2001; Pevtsov and Balasubramaniam, 2003). Helicity in active regions is likely related to dynamo processes, and thus, the presence of these helicity nests may suggest their subphotospheric origin.

Active longitudes are typically observed to be 20 to 60 degrees wide (Bai, 1988, 2003; Bumba and Howard, 1969; Gyenge et al., 2016) for about 10-15 carrington rotations (Elek et al., 2024). Gaizauskas et al. (1983) noted that complexes of solar activity within these active longitudes are sustained by the frequent emergence of new magnetic fluxes. Nevertheless, the underlying mechanisms driving these spatio-temporal patterns remain a subject of ongoing debate. Interestingly, some argue against the very existence of active longitudes. From a theoretical standpoint, current dynamo theories have yet to provide a model that can account for or predict the presence of active longitudes. Additionally, Korsós et al. (2024) provided observational evidence that active longitude could be important to predict the upcoming solar storm seasons. This result has motivated me to further explore the potential connection between active longitudes and large solar flare eruptions using more than 100 years of data.

Periods shorter than the solar cycle

In addition to the well-documented 11-year solar cycle (Schwabe, 1844), solar activity is characterized by several shorter periodic oscillations, which are manifestations of complex processes within the Sun's magnetic and dynamo systems. Investigating these shorter cycles offers valuable insights into the variability of solar phenomena and their potential impacts on space weather. Recent studies have identified a stronger correlation between active longitudes and "solar storm seasons," underscoring the profound implications for forecasting space weather events (Gurgenashvili et al., 2016; Dikpati et al., 2022). These shorter periods of intensified solar activity include:

- Short-term variations are dominated by the 27.26-day Carrington rotation periodicity, which is the most prominent among them.
- Mid-term oscillations, which range from 27.26 days to the 11-year solar cycle, include a variety of periodic phenomena such as quasi-biennial oscillations (QBOs), Rieger-type periods, the 1.3-year variation associated with the tachocline, and oscillations driven e.g. by magneto-Rossby waves. These intermediate timescales reflect complex processes within the Sun's magnetic field and dynamo mechanisms. Connections between magneto-Rossby waves and the observed features of the global solar magnetic activity on the short-to-medium term have also been established by Dikpati and McIntosh (2020), which, as mentioned before, have implications for the occurrences of solar phenomena, such as sunspots or flares.

Identifying periodicities beyond the 27.26-day and 11-year cycles in solar data has long been of interest, as mid-term patterns may reveal mechanisms driving solar variability. Studies have examined periodic variations over intermediate timescales, including:

- **Quasi-biennial oscillations (QBOs):** The QBOs are oscillations, as their name suggests, with periods of around two years. This approximately two years in practice means periods of 0.6 to 4 years. Oscillations with periods on the

order of 2 years are of particular interest, as they appear to be connected to the internal fine structure of the solar magnetic field. A key characteristic of these quasi-periodic solar variations is their instability; they occur intermittently, with periods fluctuating within the range of 0.6 to 4 years. In the literature, periodicities within this range are commonly referred to as QBOs. However, it is also common to see them described as intermediate- or mid-term quasi-periodicities (e.g., Valdés-Galicia et al., 2005; Chowdhury et al., 2009; Kudela et al., 2010; Kostyuchenko and Bruevich, 2021)

There is now consensus that QBOs are as fundamental a feature of solar activity as the 11-year cycle; however, the nature of QBOs remains unclear. The most likely physical mechanisms that may drive QBOs are reviewed in Bazilevskaya et al. (2014) and references therein.

- **~1.3-year variation:** In 1994, Richardson et al. (1994) discovered an oscillation in the speed of the solar wind. This study used data from 1 AU, as well as from the Voyager 2 spacecraft at the outer heliosphere. Both of these measurements were starting from 1987, and both showed a change in amplitude of about 100 km/s and a period of approximately 1.3 years. This suggested significant changes in the Sun with this period. Later studies by Gazis (1995, 1996) showed that these oscillations are present everywhere in the heliosphere, and they cannot come from the structural changes during the evolution of the wind. They concluded that this periodicity must originate from the Sun itself, eliminating other, external causes.

Howe et al. (2000) discovered that the 1.3-year periodicity at the base of the convection zone is most pronounced in the solar rotation rate at the equatorial region of the tachocline, and the ~150–160 days period (Rieger-type periods - see later) is seen to vary approximately in phase with the ~1.3 year variation.

- **Magneto-Rossby waves:** Rossby waves, originally discovered in meteorological studies (Rossby, 1939), are a type of inertial wave, occurring in a rotating shell of fluids, due to a gradient of pressure and that of the Coriolis force as a function of latitude. Later, it was realised that these kinds of motions can occur not just in planetary atmospheres, but in objects entirely made of fluids, such as the Sun. Rossby waves in our star were first found in 1969 by Gilman (1969a,b), who suggested that these waves were generated close to the base of the convection layer. In the Sun, we expect to also find magnetic Rossby waves, since fluids of plasma are highly influenced by magnetic fields. More recently, more and more studies have found signs of magneto-Rossby waves on the Sun (Dikpati and McIntosh, 2020; Korsós et al., 2023), while theoretical and numerical investigations help us gain more understanding about the properties and exact behaviour of this phenomenon (Dikpati et al., 2022). In light of such recent findings, the view is becoming more prevalent, that magneto-Rossby waves can have an influence on the solar magnetic field and thereby solar activity on timescales of ~0.8-2.4 years.
- **Rieger-type periods:** Rieger-type periods are typically in the range of 150-160 days, their effects have been identified in many different solar phenomena, such as X-ray flares and sunspot areas (Rieger et al., 1984). In 2010, it was hypothesized by Zaqarashvili et al. (2007) that these Rieger-type periodicities

may be the result of unstable magnetic Rossby waves. These are expected to reside in the tachocline and lead to quasi-periodic eruptions of magnetic flux to the surface, driven by differential rotation and the toroidal magnetic field. In other developments, it was demonstrated that there is an anti-correlation between the strength of a solar cycle and the period found within the cycle. In other words, the shorter the Rieger-type periodicities within a cycle, the higher the solar activity at that time (Gurgenashvili et al., 2016). From this, it thus follows that the observed length of these periods and the dispersion relation of magnetic Rossby waves could be used to give an estimate for the activity of the solar dynamo (Zaqarashvili and Gurgenashvili, 2018).

The applied datasets

Analysing active longitudes is essential for advancing our understanding and improving predictions of potentially hazardous space weather events weeks to months in advance. To achieve this, a long-term photospheric sunspot catalogue covering 13 solar cycles was used to detect active longitudes.

The applied sunspot catalogue

The data used in this study come from the database described in Mandal et al. (2020). The authors composed this catalogue using daily observations coming from a total of nine observatories, which are: RGO (Royal Greenwich Observatory), Kislovodsk, Pulkovo, Debrecen, SOON (Solar Optical Observing Network), Kodaikanal, Rome, Catania and Yunnan. The dataset covers the full period between 1874 and 2019 (the second half of SC 11 to the beginning of SC 25). The bulk of this period is covered by the RGO and its multiple observing stations around the planet. The data compiled here covers the period between 1874 and 1976. This fact, combined with this observatory having 99% coverage over its active years and the smallest reported spot area of $1 \mu\text{Hem}$ (this latter feat is shared with Kislovodsk, Pulkovo and Debrecen) makes the RGO the best choice for a reference catalogue.

For the remainder of the period (1976–2019), other catalogues were used, with four in total (including RGO) labelled "primary", meaning that these catalogues cover mostly the entire time period used, and at sufficiently high resolution. Any gaps in this primary time series were then filled with data by the remaining, "secondary" observatories.

The Pulkovo and Kislovodsk institutions together cover the times from 1932 until 2018, both being located within the former Soviet Union and composing data from multiple locations. This and the relatively few gaps in the data in addition to the aforementioned high resolution means that these two observatories were labelled "primary".

The fourth and final observatory with the "primary" label is Debrecen. This site took up the official continuation of the RGO programme from 1977 onwards. The data primarily comes from the late Debrecen Heliographical Observatory and its Gyula observing station, however, gaps were filled with observations from all around the world.

The method used for cross-calibration in Mandal et al. (2020) is similar to the one described in Balmaceda et al. (2009). A short description of this is as follows:

1. the common days between any two observatories are identified;
2. scatter diagram of daily area values between any pair is plotted, and then a straight line is fitted with a forced pass through the origin;
3. points outside a 3σ threshold are labelled outliers and removed;
4. points close to the origin are also removed to eliminate bias to the slope;
5. a straight line is fitted again with the remaining points; and
6. a final slope is obtained after correcting for attenuation bias and applying the "bisector line" method.

These are then used as calibration factors together with their calculated errors.

The calibration factors between the "primary" observatories are all close to a value of one and include one within the margins of errors. None of the "secondary" data have calibration factors as close to a value of one as any of the "primaries". This confirms that the author's choice of "primary" observatories was a good one. Of note here is the very large factor for SOON of 1.48 ± 0.102 , likely due in part to the smallest resolved sunspot being $10 \mu\text{Hem}$, a worse number than that of the "primaries". On the other hand, this could be because of the rounding errors of the limb-correction overlay used on SOON drawings.

A big problem when dealing with calibration between two observatories is addressing the time-temporal evolution. This could be a result of ageing instruments or a change in measurement techniques. The authors corrected for this by determining the calibration factors for each pair of observatories with the datasets divided to each solar cycle. Their results showed that for the "primary" institutions, this time variation is small, within error bars, and therefore could be neglected. The cases, where these fluctuations are large, there always is a "secondary" source involved.

For the composite catalogue, RGO values were used where possible, and then the missing values were filled with Kislovsk data, due to the observatory's more stable history over Pulkovo, then Pulkovo and then Debrecen. The final catalogue includes values between 1874 and 2019, with 776 days missing, most of which are from a time where only RGO data were available, most of the rest are from after mid-2018, due to only Kislovsk data being available at the time of the publication of Mandal et al. (2020).

Since some studies use projected areas as input, the authors of the applied sunspot catalogue also performed the same analysis as before, except for the projected areas. These calculations were performed without the Pulkovo dataset as this does not provide projected areas or times of observation. The resulting calibration factors are both small and include the value one within their error bars. A direct comparison between each group within multiple datasets is a rather complicated task that was not done in the most exact way by Mandal et al. (2020), as it would require identifying the same group within multiple sources and also accounting for group evolution. The authors noted that this would be the topic of a separate study. So, for a simpler solution of this issue, the authors decided to only use the group area from the largest

group out of the datasets of the three observatories. Then, they proceeded with the same method as described before. The result is similar to earlier: small calibration factors with one included as a value within the error bars. Then, the time series was constructed. This shortcoming of the study, however, is not a problem for our analysis, as we are more concerned about the summed areas over some period of time, rather than the areas of individual groups. In addition, they provided the exact date of the observation with minute accuracy and the sunspotgroup areas, the data series also includes latitudes, longitudes (both in degrees) and relative distances from the centre of the disk for the sunspot groups (relative to the solar radius).

The applied flare catalogues

To identify the strong flare activity periods of most of the solar cycle, two types of catalogues were used. These two catalogues:

- Prior to the advent of space exploration, solar flares were monitored using $H\alpha$ filters, a practice dating back to 1938. This method relies on the red spectral line at a wavelength of 656.28 nm, emitted by hydrogen atoms, to classify flares based on their size in square degrees of heliocentric latitude, as detailed in Benz (2008). Flares are categorized as "S" for small, with additional classifications of 1 through 4 indicating increasing sizes. Specifically, classifications 3 and 4 denote flares with a visible area exceeding 12.5 square degrees. For this research, covering the period from 1938 to 1974, the focus is exclusively on flares classified as 3 and 4 in size, utilizing data from the USAF Solar Observing Optical Network (SOON¹) catalogue.
- The catalogue of the Geostationary Operational Environmental Satellite (GOES²) spans from 1975 to today and includes classifications for solar flares based on the maximum X-ray flux measured near Earth, ranging from wavelengths of 0.1 to 0.8 nm. These classifications categorise flares into five intensity levels: A, B, C, M, and X, with M and X being the most intense. Consequently, the analysis in this study focuses solely on M and X flares to examine periods of significant solar flare activity within each solar cycle.

The above two classification schemes could be matched with each other in general, as suggested in Table 1, based on Bhatnagar and Livingston (2005).

Analytical Techniques

In this study, patterns in solar activity across the last 13 solar cycles were examined using tools such as wavelet analyses and kernel density estimation.

¹<https://www.ngdc.noaa.gov/stp/space-weather/solar-data/solar-features/solar-flares/h-alpha/reports2/>

²https://hesperia.gsfc.nasa.gov/goes/goes_event_listings/

$H\alpha$ scheme		Soft X-ray scheme	
Class	Area	Class	Peak flux
	(10^{-6} solar disk)		(1-8 Å W/m ²)
S	200	A	10^{-8} - 10^{-7}
1	200-500	B	10^{-7} - 10^{-6}
2	500-1200	C	10^{-6} - 10^{-5}
3	1200-2400	M	10^{-5} - 10^{-4}
4	2400-	X	10^{-4} -

Table 1: This table shows how each of the $H\alpha$ -flare classes could be matched with the corresponding X-ray flare class, based on Table 5.4 on pg. 254 of Bhatnagar and Livingston (2005).

Wavelet analysis

To identify periodicity in the data, the wavelet analysis method constructed by Torrence and Compo (1998) was employed first, because the windowed Fourier transform is not preferred as it is both inaccurate and inefficient. It is inaccurate as it misses high and low-frequency components outside the window, and inefficient due to the necessity of analysing numerous frequencies at each separate time step. However, the wavelet analysis is a powerful tool for detecting non-stationary power at various frequencies within a data series. It operates by convolving the time series with a specific wavelet function, which must have an average value of zero and be localized in both time and frequency space. Typically, this convolution is performed in Fourier space to expedite calculations, resulting in the wavelet transform. The wavelet power spectrum, defined as the square of the absolute value of the wavelet transform, is crucial for this analysis. Similar to a Fourier spectrum, the higher the power in the wavelet power spectrum, the more dominant a given frequency or period is. This spectrum is then plotted over time and period space.

An essential feature is the global wavelet spectrum, which averages the wavelet power spectrum across the time axis to the period axis. The most commonly used wavelet function is the Morlet wavelet, represented by:

$$\Psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\eta^2/2}$$

where Ψ denotes the wavelet function, ω_0 is the nondimensional frequency, and η is the nondimensional "time" parameter.

To detect potential periodic signals, the wavelet power spectra with the default Morlet wavelet profile was used, utilizing the Python version of the Torrence and Compo (1998) wavelet script, available on their website³. The significance level of the wavelet power spectra is set at a 95% confidence level, established via a white noise model and the standard deviation of the input signal.

Kernel density estimation

Next, the method of kernel density estimation (KDE) was employed. This non-parametric statistical method estimates the probability density function of a continuous random variable based on observed data (Rosenblatt, 1956; Parzen, 1962).

³<https://atoc.colorado.edu/research/wavelets/>

Unlike histograms, which approximate data distributions discretely through binning, KDE offers a smooth and continuous view. This is particularly beneficial for visualising and analysing data in N-dimensional spaces, where it highlights regions of varying probability densities by evaluating the spatial distribution of data points.

In KDE, each data point contributes a kernel function —typically a Gaussian distribution— centred on the data point itself. The complete KDE is then the aggregate of these individual kernels, yielding a smooth probability density curve. The bandwidth of the kernel is pivotal, as it sets each kernel’s width and thus influences the smoothness of the resulting density estimate. A narrower bandwidth leads to a more detailed estimate closely following the data, whereas a broader bandwidth results in a smoother, more generalized view.

Comprehensive Analysis of Latitude and Longitude Distributions

To study the evolution of the solar activity between SC 11–24, synoptic maps were constructed from the applied sunspot catalogue, see Section . In order to make the synoptic maps for each Carrington Rotation (CR for short) covered by the dataset, we need to know their starting dates. To identify the exact time interval of each CR, we utilized a publicly available database for that purpose, which is accessible [here](#). This dataset includes dates for Carrington rotations from rotation number -10 at the start of 1853 to rotation 2300 in mid-2025. The start dates of these are given here in year-month-day, in years with 5 digits and the Julian date.

First, the data from Mandal et al. (2020) was separated by CR. Since the CR data table provides dates by days to the 4th digit, the full extent of the dates of the sunspot area catalogue was used with minute accuracy, if provided by the observatory. To do this separation, the dates were first converted into years by adding up all the days within a year up to the point of observation, then dividing by the number of days of the year. This was done for both the CR start date database and for the sunspot group database so that an exact determination could be made regarding which CR each group belongs to. Leap years were also taken into account, while remembering that 1900 wasn’t a leap year, but 2000 was. After organising the data into separate CRs, a synoptic map from the sunspot data catalogue was created using a 10-degree window centred on the solar disk, see e.g. Figure 3. This method provides a coherent view of sunspot distribution, making it possible to track solar activity and identify large-scale patterns over time.

Additionally, synoptic maps were created from the solar flare catalogues, as the occurrence time and location are also provided in the flare catalogues.

Distribution analysis of active regions and flares over 13 solar cycles.

The purpose of this analysis is to determine whether there is any clustering of sunspots and large energetic flares longitudinally and latitudinally. In analysing both coordinates, the following steps were carried out:

- Extracted the coordinates of the sunspot groups and flares from the synoptic maps, like e.g. Figure 3.

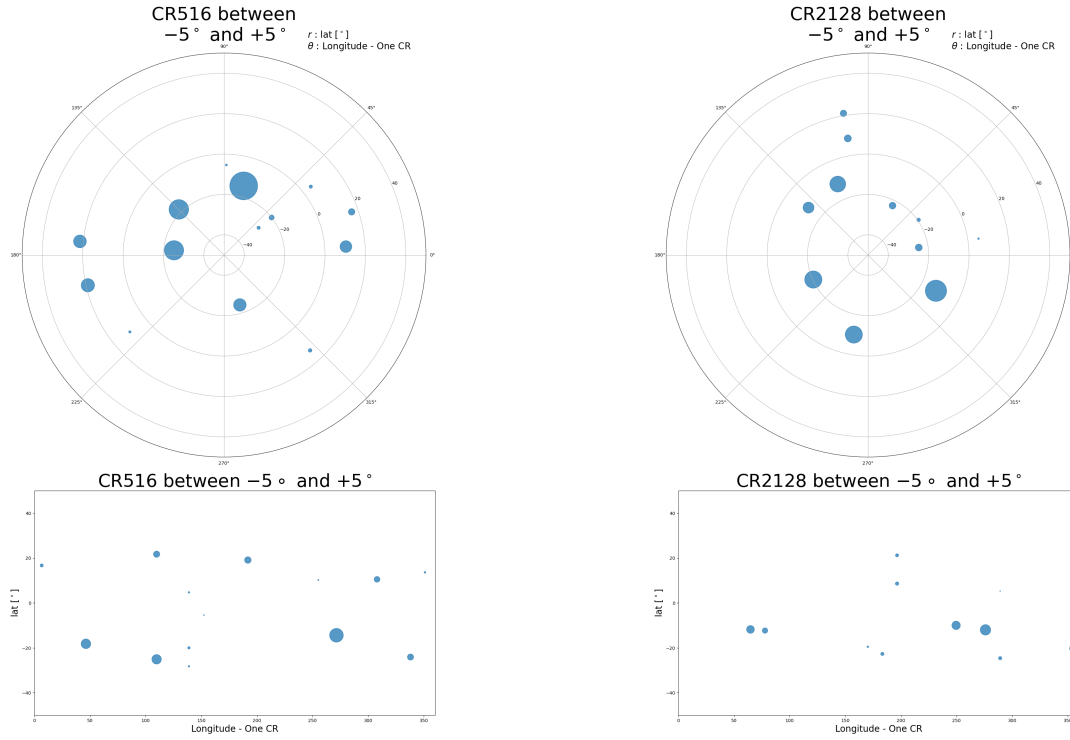


Figure 3: Examples of synoptic maps from the analysis. These synoptic maps correspond to the period: 1982.04.25–1892.05.23 and 2012.09.11–2012.10.08

- Performed Kernel Density Estimation (KDE) on the longitudinal and latitudinal coordinates of the sunspots and flares, to identify the most populated longitudinal and latitudinal coordinates at each CR. As an example, see Fig. 4, where the KDE was applied on sunspot longitude data.
- Identified the largest peaks of the KDE at each CR periods, as it can see via the red dots in Fig. 4.
- Used these peaks for further wavelet analysis.

Latitudinal direction analyses

First, let's focus on the evolution of the most populated latitudinal locations (the largest peaks identified in the KDE) of sunspots in each Carrington Rotation. For better visualization of this evolution, 2D KDEs were used to highlight the most populated latitudinal locations over each solar cycle. The expected result of these latitudinal investigations is the butterfly diagrams, which are indeed illustrated in Figures 5 – 7. However, what is more intriguing is the apparent structures that these diagrams reveal.

If we focus on Figure 5, sunspots will appear at higher latitudes in the early parts of the cycle and slowly drift to lower latitudes by the end of the cycle. From

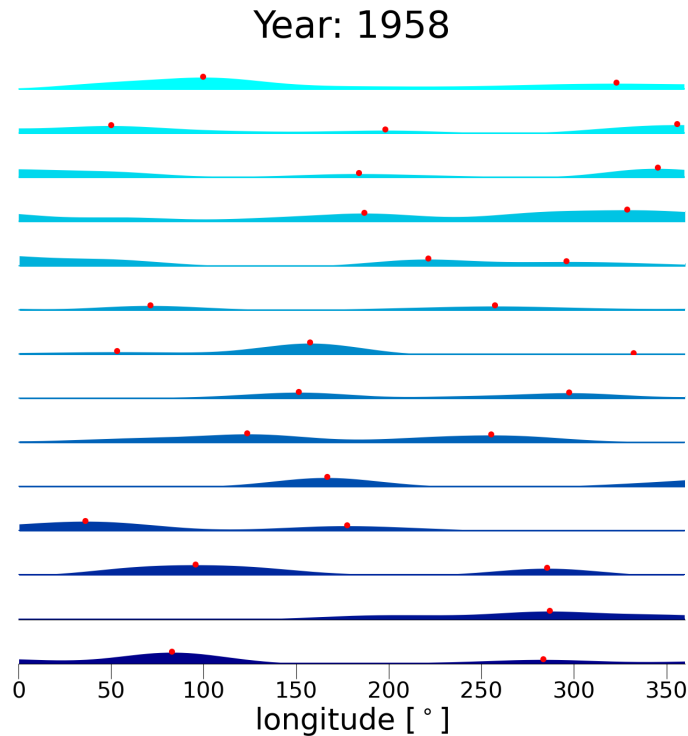


Figure 4: A ridge plot of KDEs within the year 1958, with each line denoting a Carrington Rotation within that year. Note, they do not necessarily begin or end in that year. The KDE was made with all the sunspot group longitudes in a single data series and later selected to only show one year. The peaks of this KDE are marked with red dots, these were used for later analysis. The y -axes of the individual subplots show the value of the KDE, but this does not have too much relevance, as the main point is the relative strength of these peaks.

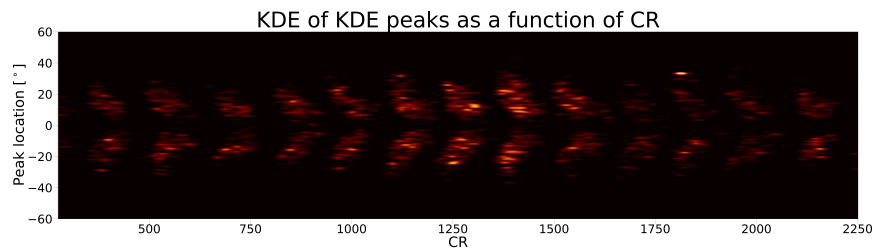


Figure 5: 2D KDE of the full sunspot group latitude data series.

the brightness of these patches, it is also clear that the solar cycle activity increased from Solar Cycle 12 to Solar Cycle 19 and began to decline after Solar Cycle 19. This is a trend that also appears in the sunspot number plot in Fig. 1 as a function of time. However, this plot also reveals that the appearance of sunspots varies by hemisphere across solar cycles. From Solar Cycle 12 to Cycle 14, the southern hemisphere was more active (brighter areas in Fig. 5). Starting in Solar Cycle 15, the northern hemisphere began to produce more active regions. During Solar Cycles 15 to 20, both hemispheres were active, but in Solar Cycle 20, the southern hemisphere was less

active compared to the north. In Solar Cycles 21 and 23, activity was low in both hemispheres, as seen by the faintness of the active belts compared to other cycles. Meanwhile, in Solar Cycles 22 and 24, the southern hemisphere was more active, as shown by its higher brightness.

To study the activity for each specific cycle, these KDE plots were analysed separately. For example, in the case of Solar Cycle 19, which produced the largest number of active regions, we observe in Figure 6 that both the northern and southern hemispheres became active almost simultaneously. However, the southern hemisphere initially took the lead, remaining more active for about 20 Carrington Rotations. After this period, both hemispheres showed significant activity together for another 20 Carrington Rotations. Around CR 1405, the northern hemisphere became rather active while the southern hemisphere diminished to relatively calmness.

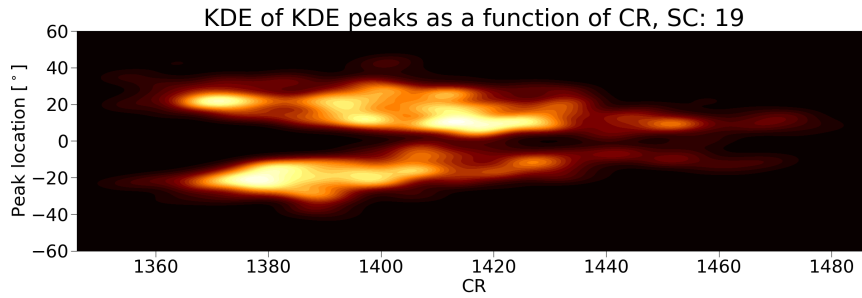


Figure 6: 2D KDE for sunspot group latitudes within Cycle 19 reveals brighter areas where active regions appeared more frequently.

In contrast, Solar Cycle 23, visible in Figure 7, produced a little more than half the number of sunspots compared to Solar Cycle 19. In this cycle, the activity levels between the two hemispheres were more synchronized over time. The northern and southern hemispheres were initially more simultaneously active between CR 1925–1960 and CR 1975–2000. There was a period of lesser, but similar activity, following this, with a larger peak in the southern hemisphere around CR 2010, lasting until about CR 2020. After CR 2030, the southern hemisphere became more active, continuing until the end of the solar cycle.

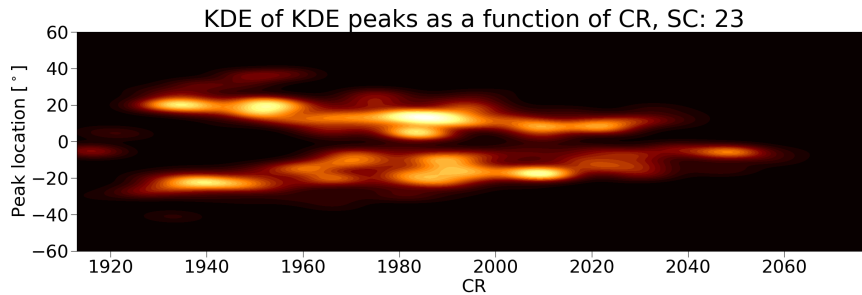


Figure 7: 2D KDE for sunspot group latitudes within Solar Cycle 23 reveals brighter areas where active regions appeared more frequently.

In the case of latitude distribution of large energetic flares, the correlation between flare activity and sunspot locations along longitude is not very strong, as we compare Figs. 5 and 8. However, we still observe some correlation, such as during the peak of SC 19 (see Fig. 9) and the latter half of SC 23 (see Fig. 10). For Solar Cycle 19,

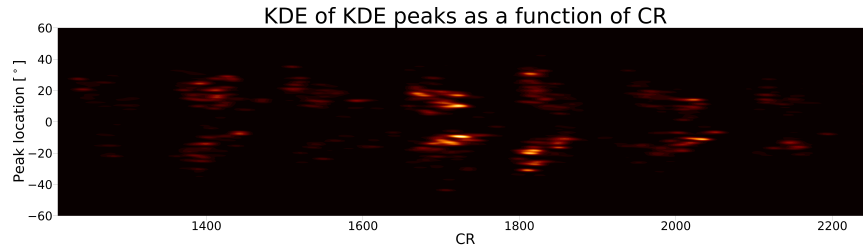


Figure 8: 2D KDE for the full flare latitude data series.

at the beginning, the northern hemisphere was more productive in flare activity, while sunspot activity was stronger in the south. As the cycle approached its maximum, the northern hemisphere began producing larger flares, and sunspot production also intensified in the northern hemisphere during this peak period. However, towards the end of the cycle, no clear connection between flare and sunspot locations is observed, when comparing Fig. 6 and Fig. 9.

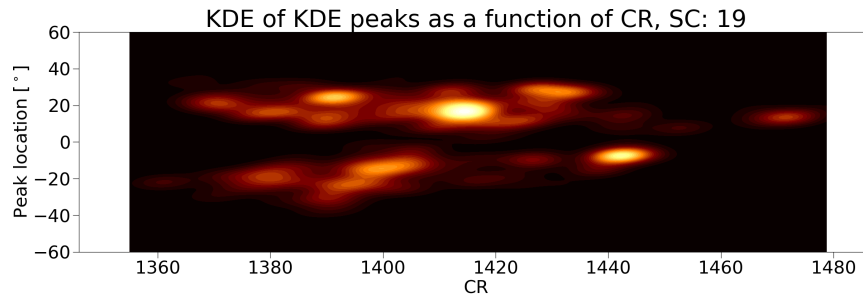


Figure 9: 2D KDE for flare latitudes within cycle 19.

Solar Cycle 23 exhibited slightly more flare activity in the northern hemisphere, similar to the sunspot emergence as shown in Fig. 7 and Fig. 10. Toward the end of this cycle, it becomes more evident that the southern hemisphere produced a greater number of flares, as depicted in Fig. 10.

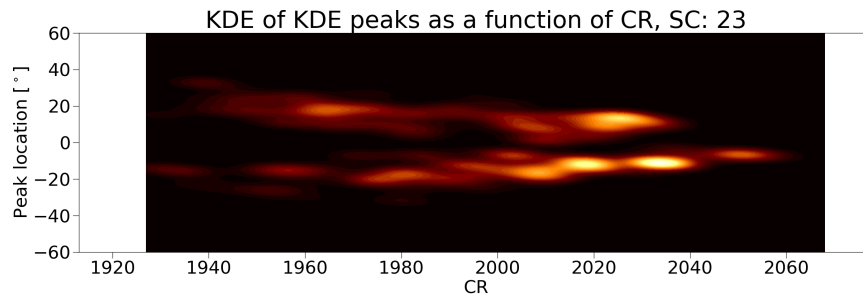


Figure 10: 2D KDE for flare latitudes within cycle 23.

Longitudinal direction analysis

To further pinpoint the potential locations of magnetically complex active regions that produce large solar eruptions, the longitudinal distribution is also analysed using KDE. Studying the development of heightened activity periods in each hemisphere

could help us better identify where the most magnetically complex active regions emerge, as shown in Fig. 11.

Longitudinal analysis showed some more surprising trends, in that there seemed to be a noticeable time dependence of which longitudes preferred for a longer time period for production active regions (Fig. 11) and large solar flares (Fig. 14).

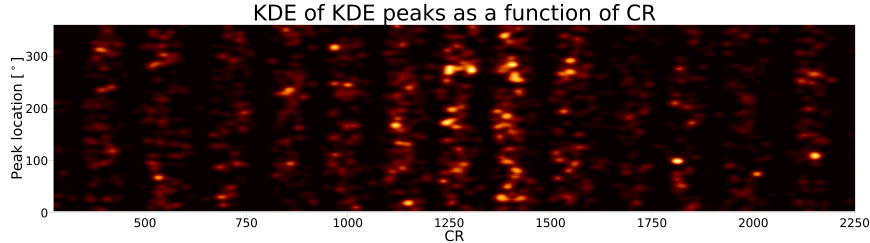


Figure 11: 2D KDE analysis of the full sunspot group longitude data series.

Similar to the latitudinal distribution, the most populated longitudinal locations (the largest peaks identified in the KDE) of sunspots are analysed for each Carrington Rotation. To better visualize this evolution, 2D KDEs were used to highlight the most populated longitudinal locations from Solar Cycles 11 to 24, as shown in Figure 11. In this figure, we observe the longitudinal distribution, with 30–50 degree-wide longitude bands indicating regions of higher sunspot activity (brighter areas) in each cycle. These regions are referred to as active longitudes, as previously mentioned. Notably, in Solar Cycles 18 and 19 (between CRs 1210 and 1487), which were the strongest cycles, more distinct active longitudes are observed.

The longitudinal distribution of each solar cycle was studied separately. For example, in the cases of Solar Cycles 19 and 23 (Figures 12 and 13.), we see multiple active longitudes, but each has a dominant one (the brightest). For Solar Cycle 19, the strongest ~ 50 -degree-wide active longitude occurs around CR 1415, centred at ~ 280 degrees. Meanwhile, for Solar Cycle 23, the primary ~ 30 -degree-wide active longitude is located around CR 2010, centred at ~ 80 degrees.

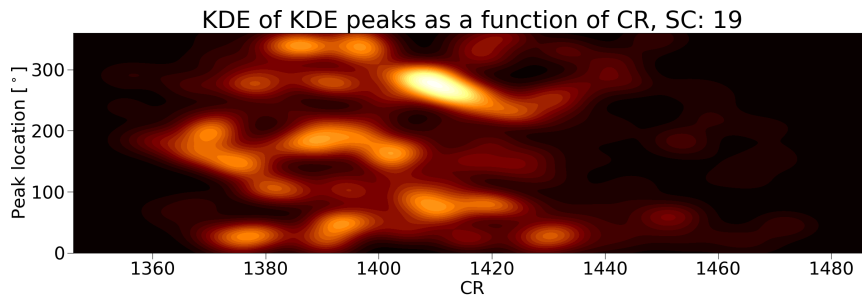


Figure 12: 2D KDE for sunspot group longitudes within cycle 19.

Similar to sunspots, the most populated longitudes for energetic flares were identified between Solar Cycles 17 and 24 as well as including Cycle 25 up to the summer of 2024. Here, as seen in Figure 14, it is observed that energetic flares tend to occur at preferred longitudes, similar to sunspots. However, it is important to note that the data sample for flares is much smaller compared to that for sunspots.

The longitudinal distribution of flares in each solar cycle was also investigated. For example, in Solar Cycle 19, as seen in Figure 15, the KDE identified the most

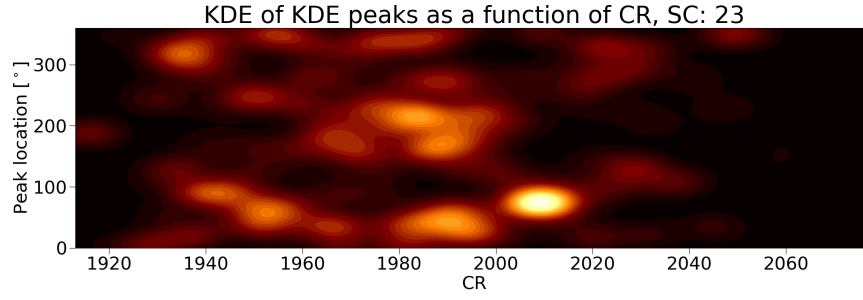


Figure 13: 2D KDE for sunspot group longitudes within cycle 23.

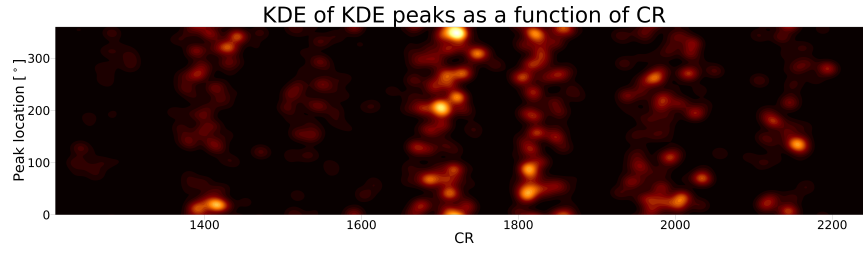


Figure 14: 2D KDE analysis of the full flare longitude data series.

favourable longitudes around 0 degrees and 280 degrees, which remained consistent throughout the cycle. The 280-degree longitude was also identified as a favourable longitude for sunspots in this cycle.

In the case of Solar Cycle 23 (Figure 16), the most dominant longitude for flares was around 80 degrees, matching the preferred location for sunspots. However, an additional prominent longitude was identified at around 250 degrees, approximately 180 degrees away. This alignment is consistent with the hypothesis that active longitudes often have a counterpart located 180 degrees away (Elek et al., 2024).

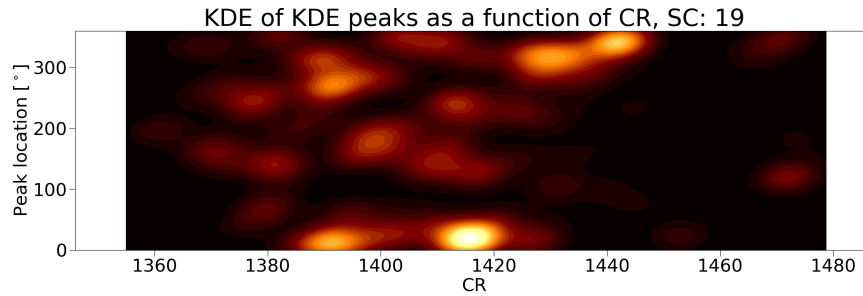


Figure 15: 2D KDE for flare longitudes within cycle 19.

Examining the KDE distribution of longitudinal locations of sunspots and flares for each solar cycle, particularly in Solar Cycles 19 and 23, some oscillatory behaviour can be observed between the populated regions within each cycle. To further investigate this pattern, wavelet analysis was used.

Investigating the oscillatory behaviours

Wavelet analysis was performed by taking the latitude and longitude of the highest amplitude KDE peak within each Carrington Rotation, and applying it to the analysis

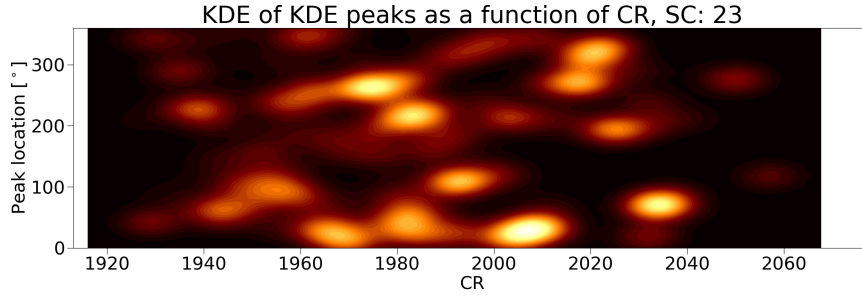


Figure 16: 2D KDE for flare longitudes within cycle 23.

of sunspots and flares. To avoid interference from the 11-year Schwabe cycle in the spectrum, these wavelets were computed separately for each solar cycle.

The steps taken in this part were the following:

- Construct a time series of the relevant KDE peak locations, in other words, the most populated latitude and longitude locations.
- Interpolate linearly for the missing data points.
- Smooth the curve using a moving average of two points before and after the central point.
- Provide this time series to wavelet analyses code, where the significance parameter set to 2σ .
- Generate plots showing the time series, the wavelet power spectrum, and the global spectrum with the significance threshold marked.

It should be noted that the smoothing process "pulls" the time series towards the middle value, thus the time series plots are intended to be qualitative in showing periods.

Latitudinal direction analysis

In this section, the oscillatory behaviour of the most populated latitudinal positions of sunspots and flares during CRs was measured during each solar cycle. The results for SC 19 and SC 23 are illustrated in Figs. 17–20, as an example.

Let's first focus on the sunspots data analyses. In the case of Solar Cycle 19 in Figure 17, both the wavelet power spectrum and the global wavelet analysis reveal a strong ~ 38 CR period throughout the cycle, which corresponds to approximately 2.8 years. For Solar Cycle 23, seen in Figure 18, an 8 CR and 27 CR period (about 0.6 and 2 years) is observed in the early part of the cycle, transitioning to a dominant 44 CR period that persists throughout the cycle, corresponding to approximately 3.3 years. All solar cycles were analysed in a similar manner, and the measured periods are summarized in Table 2. These identified periods closely resemble the quasi-biennial oscillations, as described in section .

In the case of flares, the periodicities are not as pronounced as those observed for sunspot groups. However, for Solar Cycles 19 and 23 (shown in Figures 19 and

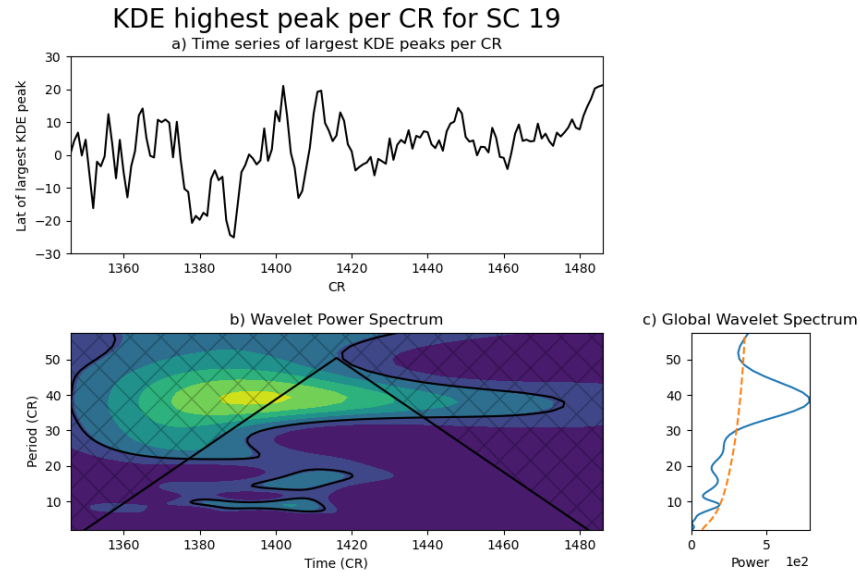


Figure 17: Wavelet analysis of the highest spot KDE peaks in Solar Cycle 19 in the latitudinal direction. The upper panel shows the time series of the most populated latitude of sunspots at each Carrington Rotation. At the bottom, the wavelet power spectrum is on the left, and the global wavelet analysis of the time series is on the right.

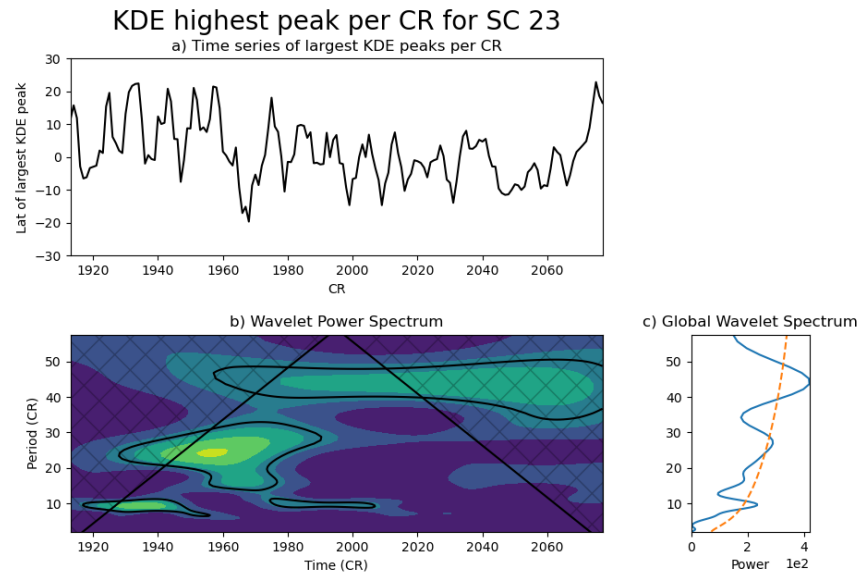


Figure 18: It is similar to Fig. 17, but this is the wavelet analysis of the highest spot KDE peaks in Solar Cycle 23, in the latitudinal direction.

20, respectively), a ~ 20 Carrington Rotation period can be seen, corresponding to about 1.5 years. The identified 20-CR period found here was not observed in the case of sunspots. However, this 20-CR period could represent a harmonic of the 38-CR period in Solar Cycle 19. In the case of Solar Cycle 23, a 20 CR period was measured at the first part of the cycle.

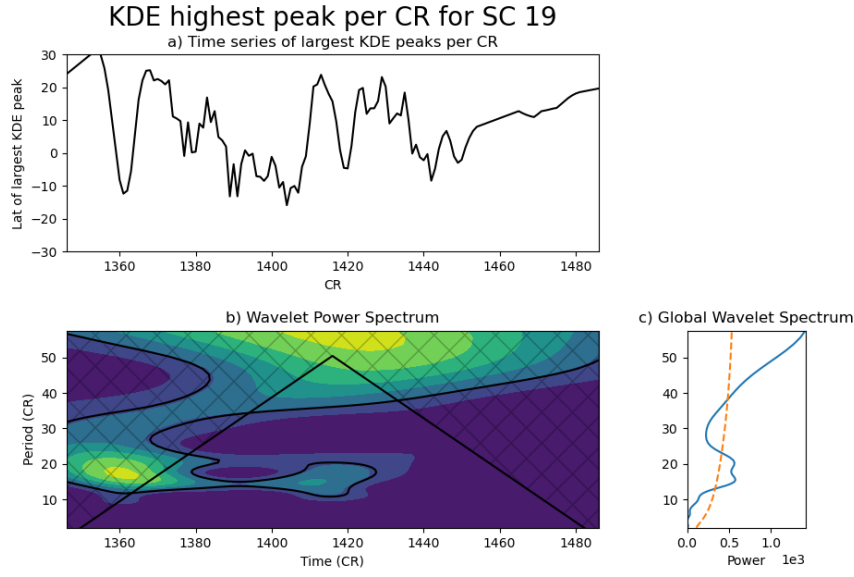


Figure 19: It is similar to Fig. 17, but this figure shows the wavelet analysis of the highest flare KDE peaks in Solar Cycle 19, in the latitudinal direction.

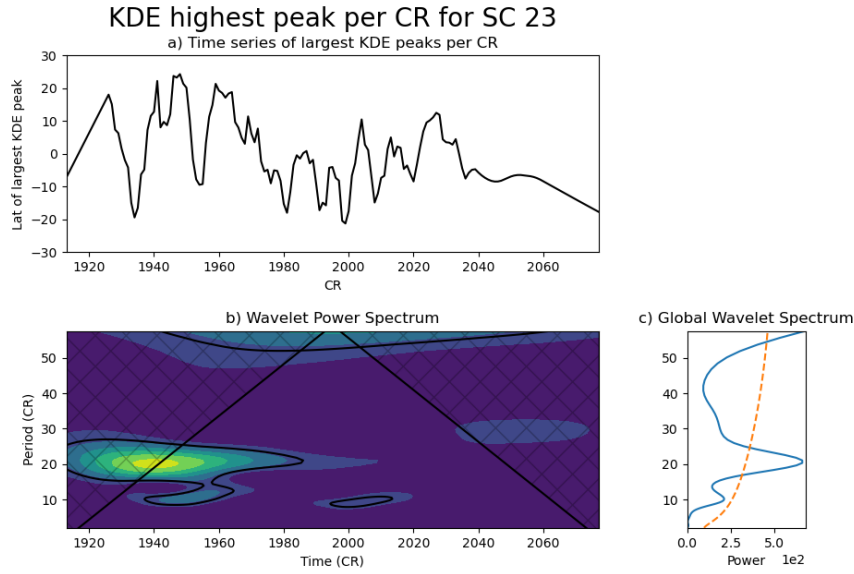


Figure 20: It is similar to Fig. 19, but for Solar Cycle 23.

Longitudinal direction analysis

The most populated longitude values exhibited more distinct periodic trends. Quasi-periodicities are apparent, as shown in Figures 21 to 24, even when examining the time series directly. These figures highlight recurring patterns, suggesting a structured rhythm in solar activity across the studied Carrington Rotations, with clear implications for the timing and distribution of sunspots and flares.

Here, let's focus on the sunspots first. In the wavelet spectrum analysis and the global wavelet, we can observe that the peak values show a periodicity of approximately 13 Carrington Rotations, which is about 0.9 years, during the increasing phase

of Solar Cycle 19 (Figure 21). When the cycle reaches its maximum, a weaker 30 CR periodicity (around 2.4 years) can be measured. This 30 CR period is actually around the first harmonic of the 13 CR periodicity. In the case of Solar Cycle 23 (Figure 22), a strong 14 and 22 CR periodicity, of about 1.1 and 1.6 years can be measured throughout the cycle. These values are similar to the so-called quasi-biennial oscillation and are also close to the periods of magneto-Rossby waves.

KDE highest peak per CR for SC 19

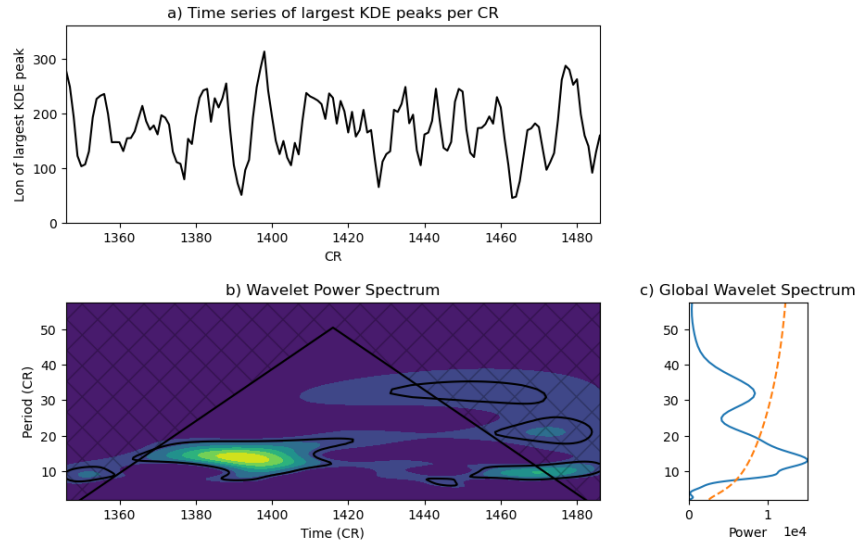


Figure 21: It is similar to Fig. 17, but this is the wavelet analysis of the highest spot KDE peaks in Solar Cycle 19, in the longitudinal direction.

KDE highest peak per CR for SC 23

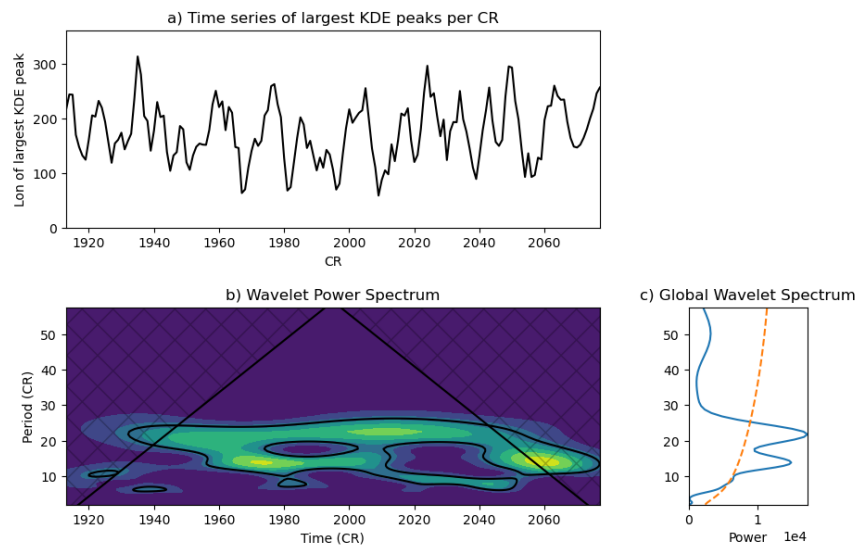


Figure 22: It is similar to Fig. 21, but in the case of Solar Cycle 23.

Flares also had similar periodicity ranges as found for sunspot group longitudes. In Solar Cycle 19, we can identify a strong 16 and 32 CR periods (about 1.2 and 2.4 years) during the cycle, as shown in Figure 23. In the case of sunspots, a 14 CR periodicity was also observed. During the increasing phase of solar Cycle 23, an 18 CR period (approximately 1.3 years) was detected, and from just before the first maximum, a 30 CR period (about 2.2 years) became evident, as visible in Figure 24. For sunspots in this cycle, a 14 and 22 CR periods were measured throughout. As with the sunspot groups, these periods also are similar to quasi-biennial and magneto-Rossby wave periods.

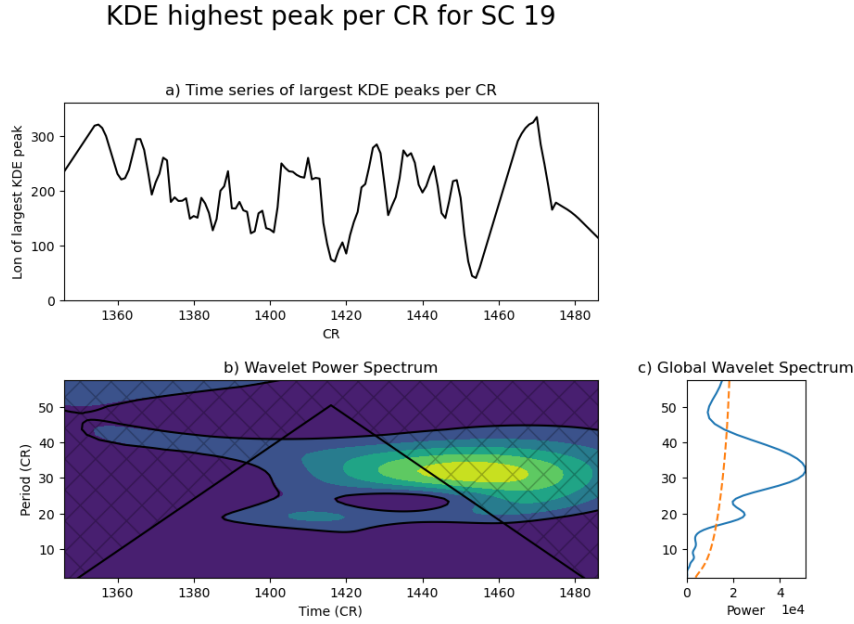


Figure 23: It is similar to Fig. 21, but here we can see the wavelet analysis of the highest flare KDE peaks in Solar Cycle 19, in the longitudinal direction.

Comparison with a randomized dataset

The question of whether these structures could be explained by the prevalent theory that sunspot groups appear on a uniform distribution of longitudes naturally arises. In this section, I explore this topic by simulating sunspot groups with randomized parameters, focusing primarily on longitude and size. I then apply the same analytical steps as detailed earlier in this chapter to identify any differences or similarities. Subsequently, I use the number of KDE peaks found per CR to argue that the observed longitudes of sunspot groups are inconsistent with a random distribution.

The random generator utilizes the synoptic maps previously created as a reference for estimating the number of CRs, sunspots, and their sizes. I employed Python's "random" module **I think it has to discuss what type of random generator and even give a reference** to generate appropriate distributions for each parameter. Gaussian distributions were chosen for simulating the length of the solar cycle and the sunspot content, while a triangular distribution was applied as a simplification to model the number of sunspot groups per CR. The distribution of sunspot sizes was modeled using an exponentially decreasing function, which aligns with the trends

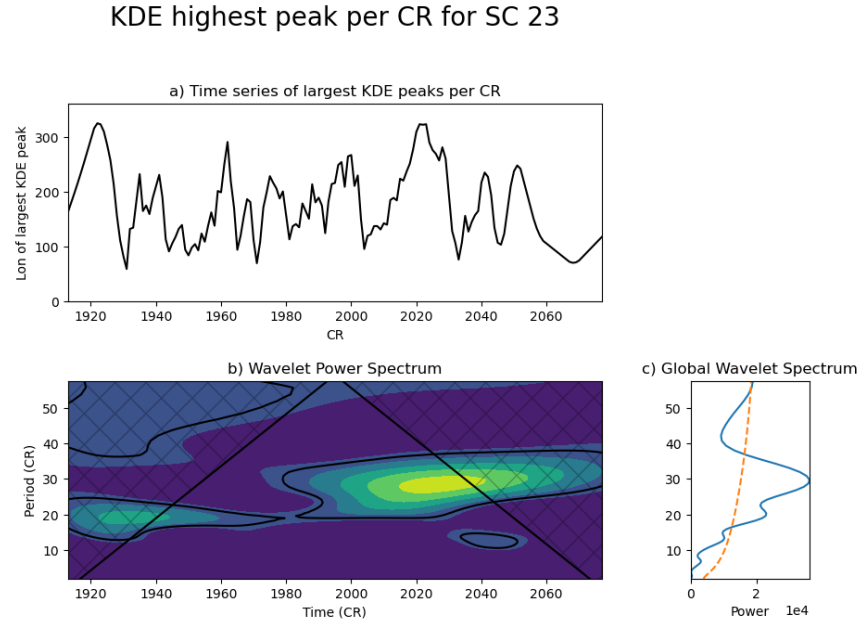


Figure 24: It is similar to Fig. 23, here the analyses correspond to Solar Cycle 23.

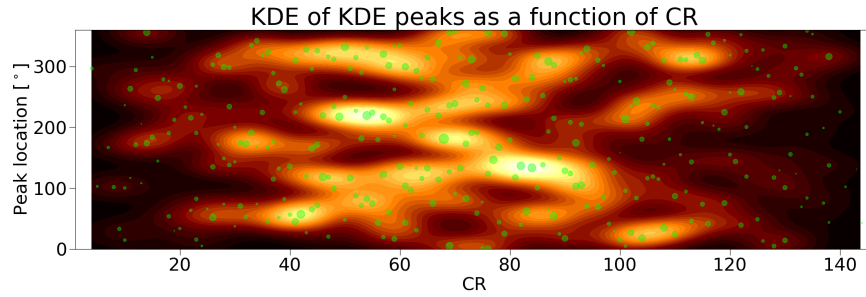


Figure 25: An artificially generated solar cycle, after KDE analysis. We can observe a different structure than those in Figures 12 or 13. There are more simultaneous bright areas here, as well as these "active longitudes" seem significantly narrower, than in the observed data. Furthermore, the lack of larger, empty regions is also noticeable.

observed in a histogram of the actual data. Lastly, each sunspot group was assigned a random longitude using a uniform distribution.

The same analysis steps were then applied to this simulated data as were used with the actual data. The resulting 2D kernel density estimations (KDEs) displayed a markedly different structure, characterized by more frequent bright areas and significantly fewer "voids" compared to the observed data. For instance, as illustrated in Figure 25, there are noticeably more bright areas and fewer dark spaces than in the KDEs of the observed data, shown in Figures 12 and 13. This discrepancy provides a possible, albeit challenging to quantify, indication that the observed longitudes of sunspot groups may not be random.

In terms of wavelet analysis, while periods have been identified in the simulated data, they lack the consistency observed in the original data. Although this discrepancy does not provide a robust qualitative indicator of difference, it is certainly noticeable and can be clearly observed in a histogram.

SC	Sunspot group latitudes period CR/yr	Flare latitudes period CR/yr	Sunspot group longitudes period CR/yr	Flare longitudes period CR/yr
11	8.86 CR / 0.66 yr 14.90 CR / 1.11 yr		21.07 CR / 1.57 yr	
12	9.83 CR / 0.74 yr 14.90 CR / 1.11 yr 23.38 CR / 1.75 yr		15.97 CR / 1.20 yr 50.01 CR / 3.73 yr	
13	12.10 CR / 0.91 yr 22.13 CR / 1.65 yr		22.58 CR / 1.69 yr 42.13 CR / 3.15 yr	
14	27.80 CR / 2.08 yr		14.90 CR / 1.11 yr 31.93 CR / 2.39 yr 43.62 CR / 3.26 yr	
15	26.85 CR / 2.01 yr		10.53 CR / 0.79 yr 17.72 CR / 1.33 yr 30.84 CR / 2.31 yr	
16	29.79 CR / 2.23 yr		19.66 CR / 1.47 yr 34.22 CR / 2.56 yr	
17	42.37 CR / 3.16 yr		13.43 CR / 1.01 yr 21.07 CR / 1.57 yr	23.38 CR / 1.75 yr
18	11.29 CR / 0.85 yr 51.87 CR / 3.87 yr	35.43 CR / 2.65 yr	14.39 CR / 1.08 yr 28.78 CR / 2.15 yr	30.84 CR / 2.31 yr
19	9.51 CR / 0.71 yr 37.97 CR / 2.84 yr	20.35 CR / 1.52 yr	12.97 CR / 0.97 yr	19.66 CR / 1.47 yr 31.93 CR / 2.39 yr
20	34.22 CR / 2.56 yr	51.87 CR / 3.87 yr	11.69 CR / 0.87 yr 18.34 CR / 1.37 yr	13.43 CR / 1.01 yr 23.38 CR / 1.75 yr
21	8.26 CR / 0.62 yr 15.42 CR / 1.15 yr	15.97 CR / 1.20 yr 31.93 CR / 2.39 yr	35.43 CR / 2.65 yr	19.66 CR / 1.47 yr 33.06 CR / 2.47 yr
22	8.26 CR / 0.62 yr 15.42 CR / 1.15 yr 43.62 CR / 3.26 yr	16.53 CR / 1.24 yr 29.79 CR / 2.23 yr	19.66 CR / 1.47 yr	14.39 CR / 1.08 yr 27.80 CR / 2.08 yr
23	9.83 CR / 0.74 yr 26.85 CR / 2.01 yr 43.62 CR / 3.26 yr	20.35 CR / 1.52 yr	13.90 CR / 1.04 yr 21.81 CR / 1.63 yr	20.35 CR / 1.52 yr 29.79 CR / 2.23 yr
24	10.53 CR / 0.79 yr 13.90 CR / 1.04 yr	20.35 CR / 1.52 yr	37.97 CR / 2.84 yr	17.72 CR / 1.33 yr 30.84 CR / 2.31 yr 46.75 CR / 3.49 yr

Table 2: All the periods found in the time series of largest KDE peaks per Carrington Rotation. Data for flares are only available starting Solar Cycle 17, hence the gaps. Since data at the beginning and starting cycles of the catalogues were incomplete, periods found there should not be given too much consideration and are merely there for display purposes. All periods have units of a Carrington Rotation and also in year.

As a potential indicator, I chose to use the number of KDE peaks per Carrington Rotation (CR) in a solar cycle, essentially calculating the average number of KDE peaks per CR for each cycle. I conducted simulations of 1000 random solar cycles and computed this "Peaks Per CR (PPC)" for all of them. Subsequently, a histogram was created, which also includes the PPC for all 13 complete cycles from the sunspot group data, as shown in Figure 26. Intuitively, it appears unlikely that the distribution represented by the 13 consecutive cycles could have originated from the distribution generated by a random uniform distribution of sunspot longitudes. To quantify this intuition, I performed a Kolmogorov-Smirnov test. The resulting p-value was approximately 7.72×10^{-5} , indicating that we can dismiss the hypothesis of randomness in the results with a confidence level of about 3.97σ .

Recent studies have demonstrated that the evolving global configuration of ac-

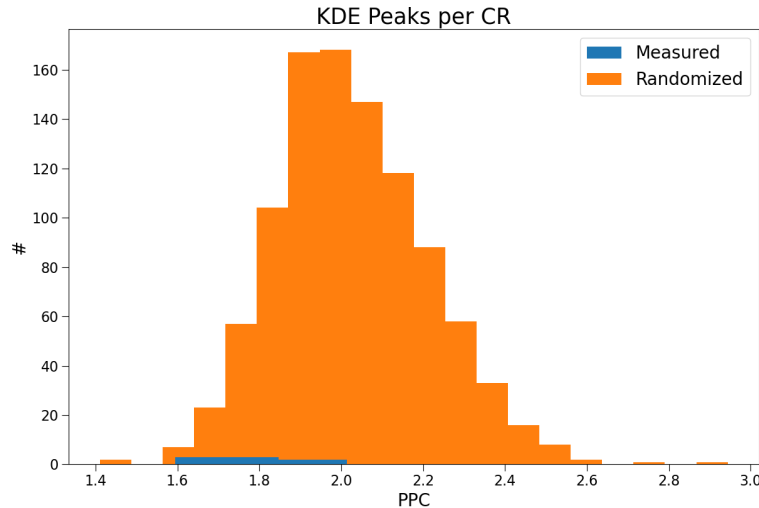


Figure 26: The number of cycles with a given PPC for both the randomized dataset (orange) and the measured data (blue). We can infer that the chances of 13 consecutive cycles having the seen distribution, while obeying the randomized distribution is not large.

tive regions throughout the solar cycle, particularly their distribution across latitudes and longitudes, provides valuable insights into pre-storm conditions.

Discussion

In this study, spanning Solar Cycles 11 to 24, the aim was to determine whether sunspots and large energetic flares exhibit clustering along longitudinal and latitudinal coordinates. This approach involved extracting the coordinates of sunspot groups and flares from Carrington maps and applying Kernel Density Estimation (KDE) to highlight areas of concentrated activity. The largest KDE peaks at each Carrington Rotation were identified and were used in a wavelet analysis to detect periodic trends.

During this study, it became clear that sunspots and flares tend to cluster in terms of both latitude and longitude, as shown in the Carrington Rotation KDEs, see e.g. in Figures 5–13. This suggests that unknown factors may be influencing their distribution. Additionally, the discovery of periodicities, despite their uncertainty and inconsistency, further indicates that sunspot groups do not appear at random longitudes.

Some latitudinal analysis suggests additional structures within the butterfly diagram, as shown in the sunspot Figures 5–7. In these figures, we observe small, slightly separated clusters at very similar Carrington Rotations, mainly at the start and end of cycles. There is noticeable asymmetry between hemispheres, which is well-known but still evident in the details, such as in the southern hemisphere’s flare activity during Cycle 19 (see Fig. 7). The full Fig. 5 and Fig. 8 also reveal patterns, such as the clustering of spots in Cycle 20 after CR 1500 and the distinct behaviour of flares in almost every cycle, particularly around CR 2000 in cycle 23.

The longitudinal Figs. 11–16 clearly show regions of high activity (brighter) and low activity (darker). While the areas populated by flares and sunspot groups do not

always align, noticeable features appear in both datasets. For instance, in Cycle 23, both flares and spots display a bright region around 1980 at 200° , and hints of an "arc-like" shape extending from 0° to 100° are visible in both figures.

To check whether these findings were the result of sunspot groups appearing randomly in terms of longitude, the number of KDE peaks per CR in a solar cycle ("PPC") was used. The distribution of this "PPC" was different for the real data and the data generated with randomized sunspot group longitudes. A Kolmogorov-Smirnov test was performed to confirm that these distributions are in fact not the same, with a certainty of 3.97σ .

Finally, wavelet analysis was conducted to determine whether the most populated latitudes and longitudes for flares and sunspots exhibit periodic behaviour or if their appearances are random. The results are summarized in Table 2, which expresses the measured periods in years for sunspot group latitudes, flare latitudes, sunspot group longitudes, and flare longitudes across various solar cycles.

From Table 2, it can be seen that the wavelet analysis revealed a wide range of quasi-biennial oscillation periods, often with harmonics, particularly pronounced in longitude data for sunspots and flares. These periods are less clear in latitude data, but we can notice that the identified periods for sunspots and flares in latitude might be harmonics of each other. These findings show that it is challenging to pinpoint active nests, but they do reinforce the presence of active longitudes with periodic changes driven by internal solar processes. This behaviour might be linked to magneto-Rossby waves, which show similar periodic patterns to the quasi-biennial oscillations, highlighting their potential role in influencing solar activity cycles. Furthermore, this suggests the solar interior-atmosphere behaves somewhat like a perfect resonator, where each period $P_n = P_1/n$ is divided by an integer number such as 2, 3, and so on. Small differences in these periods can help us understand the complex nature of Sun as a three-dimensional oscillator.

These findings suggest the need for a more advanced solar dynamo model, one that is three-dimensional and accounts for varying longitudinal and latitudinal oscillations. This model should explain the differences in the harmonics, particularly why they are not whole-number multipliers of each other. Such a model would likely place the dynamo deeper beneath the solar surface, implying that the waves travel a greater distance to exhibit these harmonics. This contrasts with some recent theories that propose a dynamo operating just beneath the surface.

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