



Transient currents caused by sudden cooling in tokamak plasmas

Student Research Paper

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Abstract

The performance of fusion devices and thus their possible future applications depend greatly on the stability and transport properties of the plasma edge. Both experimental results and theoretical predictions indicate a clear connection between the plasma edge stability and the local plasma current density. Due to the nature of the tokamak geometry aside from the the electric field driven ohmic current, significant non-inductive currents appear in this region of which one of the most significant is the bootstrap current [1].

One of the greatest obstacle to fusion power are the edge localized modes (*ELM*), which are present in H-mode, driven by the large density and temperature gradients present in the pedestal region. These modes can cause a large amount of energy and material to leave the plasma and thus ultimately reach the divertor plates. This causes a high transient heat load which, in case of a reactor sized tokamak can cause serious damage to the device. The ELM-free region of the parameter space is very limited, and these instabilities also have beneficial effects on the plasma impurity composition, thus completely avoiding them is impractical. Instead, methods are being developed to mitigate their deleterious effects, one of which aims to increase the ELM frequency thus decreasing their amplitude. According to experimental results, this can be accomplished by the injection of a small projectile, a *pellet* [2].

The ablation of pellet material causes a sudden cooling of the plasma along with a sharp increase in density, both of which can cause a significant transient current. During the course of my work I investigated the magnitude and distribution of transient currents after pellet injection in ELM pace making scenarios. This was done by solving the diffusion equation for the electric field in a circular cylinder surrounded by vacuum, while including the non-inductive current sources. Further approximations were applied, by taking only flux surface averages, thus reducing the problem to 1×1 dimensions. Due to the large temperature and density gradients caused by the pellet, the bootstrap current was derived from a recent paper by P. J. Catto et al. [3]. Results for several ASDEX Upgrade scenarios imply that the transient currents decrease normalized current thus further mitigating the destabilizing effects of the increased normalized pressure gradient.

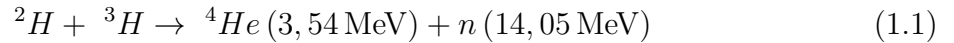
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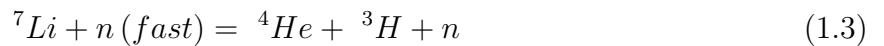
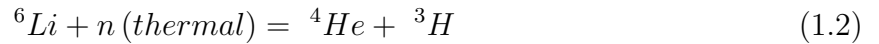
1 Introduction

One of the most promising future energy sources is the fusion of light nuclei. The condition for this process is that the distance between the interacting particles is small enough, so that the merging becomes possible due to quantum tunneling through the Coulomb-barrier. This requires very high energy, which in a macroscopic body means extremely high temperature (~ 100 million K). At such temperatures light atoms become completely ionized, creating a mixture of positive ions and electrons. This state of matter is referred to as plasma.

Currently the most viable fusion process for power generation is the fusion of deuterium (2H) and tritium (3H), since this requires the lowest temperature.



Let us assume that the net electric energy gain from the (1.1) reaction is 1 MeV! This is a very conservative estimation, still this would mean that a fusion power plant of 1 GW capacity would need only about 1.8 kg of deuterium on a daily basis, which in turn could be easily provided, since the D:H ratio in seawater is roughly $1,6 \cdot 10^{-4}$. Although the reaction also requires tritium, which is an extremely rare, medium fast decaying ($T_{1/2} \approx 12 \text{ years}$) isotope, fusion power is still viable through the breeding of tritium using the neutrons created by the fusion processes.



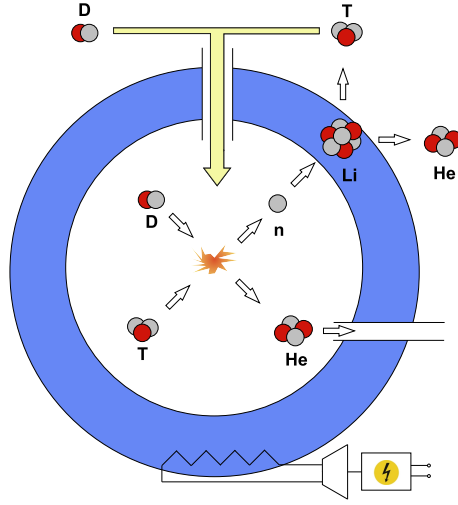


Figure 1.1: Fusion reactor conceptual design

Figure 1.1 shows a possible scheme for a fusion power plant, where the fuel for the (1.1) reaction would be supplied by the injection of small frozen projectiles, so called *pellets* into the plasma. The neutrons created during the fusion process would escape the plasma and react with the *Li* blanket of the device, while the α -particles would transfer most of their energy to the plasma, this is the so called α -heating. If the α -heating becomes large enough to balance the energy losses of the plasma, then outer energy sources are no longer required to sustain the fusion plasma, thus the plasma *ignites*.

1.1 Fusion devices

As it was mentioned in section 1, for the fusion of nuclei to occur, the particles must overcome the Coulomb-barrier, which in turn requires extreme temperature and pressure in the plasma. Fusion power generation also requires the plasma to be self sustaining, thus to be close to ignition. These can be combined into a condition for the intensive parameters of the plasma, which near the optimal temperature for the (1.1) reaction ($T \approx 25 \text{ keV}$) yield the Lawson criterion [1].

$$n\tau_E T \geq 10^{21} \frac{\text{keV s}}{\text{m}^3}, \quad (1.4)$$

where n denotes the plasma density, τ_E is the energy confinement time, T is the temperature (in $k_B T$ units). Since all known materials evaporate at temperatures of $\mathcal{O}(eV)$, the confinement of fusion plasma represents serious challenges. So far two confinement concepts have been investigated in detail, *inertial confinement* and *magnetic confinement*.

Inertial confinement means that the plasma itself is confined for only a very small amount of time. This is achieved by the compression of a small fuel pellet by lasers. In this case the compression force is in fact not provided by the laser itself, but by the evaporating surface of the fuel pellet. Of course to be viable for energy production dozens of such pellets would have to be “burnt” every second, which holds serious technical challenges.

Magnetic confinement makes use of the fact, that the particles are completely ionized in fusion plasmas, which allows the control of their motion through magnetic fields. In this case the ions and electrons follow helical orbits around the magnetic field lines (*Larmor-orbits*). In order to prevent the escape of plasma particles a closed magnetic topology is necessary (e.g. a torus Figure 1.2).

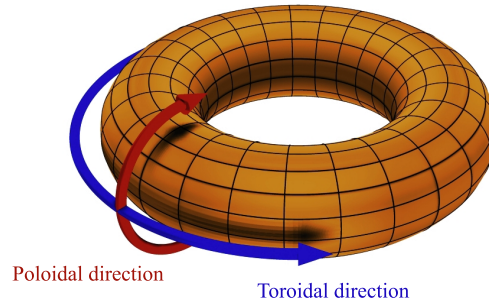


Figure 1.2: Torus and toroidal geometry

Due to the inhomogeneity and curvature of the electromagnetic field several drifts arise, which seriously degrade plasma confinement. Their effects can be mitigated by “twisting” the magnetic field lines, so that they take on a helical shape. Currently there are two widely used concepts to achieve such a magnetic configuration, the tokamak (Figure 1.3) and the stellarator.

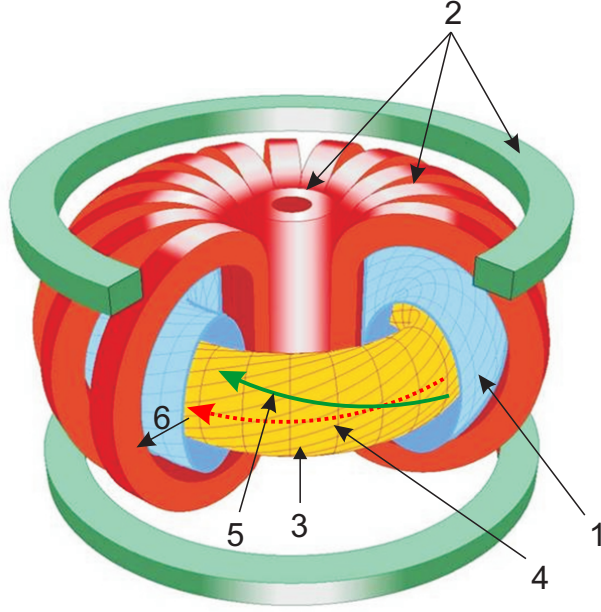


Figure 1.3: Sketch of a tokamak device. 1.-vacuum chamber wall, 2.-magnetic coils, 3.-plasma, 4.-plasma current, 5.-magnetic field lines, 6.-radial direction.

While the stellarator uses magnetic coils positioned along the vacuum chamber to create the desired magnetic configuration, the tokamak utilizes the poloidal magnetic field induced by a large toroidal plasma current. The latter method is also useful to heat the plasma (ohmic heating), which can also be achieved by electromagnetic waves and neutral beams.

The basic evolution of the plasma in these devices obeys the ideal magnetohydrodynamics equations [1], which state that:

$$\nabla p = \mathbf{j} \times \mathbf{B} \quad \Rightarrow \quad \nabla p \perp \mathbf{j} \quad \nabla p \perp \mathbf{B} \quad (1.5)$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{j}$ is the current density and p is the pressure in the plasma. This means that ∇p is perpendicular to $\mathbf{B}$ and $\mathbf{j}$, thus the magnetic field lines form isobar surfaces. These embedded surfaces are called *flux surfaces*. The innermost degenerate flux surface is called the magnetic axis. Since transport parallel to the magnetic field lines is unhindered by the magnetic field (unlike the perpendicular transport), the

intensive parameters of the plasma are assumed to be identical along flux surfaces on longer timescales. This motivates the introduction of flux coordinates [1] (Figure 1.4).

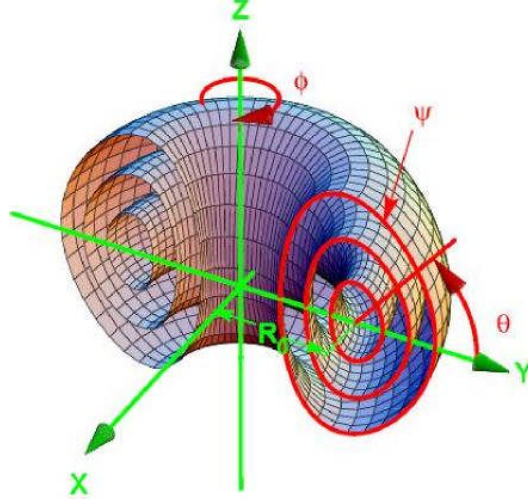


Figure 1.4: Flux coordinates

As shown on Figure 1.4, Φ is the toroidal angle, θ is the poloidal angle and Ψ is the normalized flux, which is quantity constant on flux surfaces that increases monotonically between the magnetic axis ($\Psi = 0$) and the last closed flux surface ($\Psi = 1$). It is customary to introduce the effective minor radius (r_{eff}), which equals to the radius of a circle with the same area as the poloidal projection of the given flux surface. From this it is possible to derive the normalized minor radius, $\rho \approx \frac{r_{eff}}{r_{LCFS}}$, which for elliptic flux surfaces is approximately $\rho = \sqrt{\Psi}$.

Based on the frequency of collisions between particles three, so called *plasma transport regimes* exist [4]. In case of high collisionality a short mean-free-path closure of the fluid model is applicable to describe the plasma transport. This is the *Pfirsch-Schlüter regime* and it is characterized by the following condition:

$$\frac{\nu q R}{v_T} \gg 1 \quad (1.6)$$

where ν is the collision frequency, q is the safety factor and v_T is the thermal velocity of the chosen species. In case of low collisionality the short mean-free-path closure is no longer applicable, the trajectories of the individual particles are determined by the

toroidal geometry. This allows the emergence of trapped particles which are forced onto *banana orbits* by the magnetic mirroring effects. Thus this regime is called *banana regime* and it becomes relevant if:

$$\frac{\nu q R}{v_T} \ll \epsilon^{3/2}, \quad (1.7)$$

where ϵ is the inverse aspect ratio of the current radial location.

The Pfirsch-Schlüter and banana regimes are connected by the *plateau regime*, where the diffusion coefficient is independent of the collision frequency.

$$\epsilon^{3/2} \ll \frac{\nu q R}{v_T} \ll 1 \quad (1.8)$$

1.2 Bootstrap current

According to the MHD theory plasma confinement is achieved through the force balance of Equation (1.5). Using the definition of Ψ it can be shown that:

$$\frac{\mathbf{B} \times \nabla \Psi}{B^2} = \frac{I}{B^2} \mathbf{B} - R \hat{\varphi}. \quad (1.9)$$

where $I = RB_\varphi$ and $\hat{\varphi} = R\nabla\varphi$ is the unit vector in the toroidal direction [4]. Combining it with $\frac{\mathbf{B}}{B^2} \times (1.5)$ yields the diamagnetic current:

$$\mathbf{j}_\perp = \frac{\mathbf{B} \times \nabla p}{B^2} = \left(\frac{I\mathbf{B}}{B^2} - R\hat{\varphi} \right) \frac{dp}{d\Psi}. \quad (1.10)$$

If we take the divergence of (1.10) we find that while it is universally zero in a cylinder, it is usually finite in a torus:

$$\nabla \cdot \mathbf{j}_\perp = \nabla \cdot \left[\left(\frac{I\mathbf{B}}{B^2} - R\hat{\varphi} \right) \frac{dp}{d\Psi} \right] = \mathbf{B} \cdot \nabla \left(\frac{I}{B^2} \frac{dp}{d\Psi} \right) \neq 0, \quad (1.11)$$

where $\nabla \cdot (R\hat{\varphi} \frac{dp}{d\Psi}) = 0$ since zero since due to the axis symmetry of the system. We have also used the Maxwell equation $\nabla \cdot \mathbf{B} = 0$, which means that $\nabla \cdot (A\mathbf{B}) = \mathbf{B} \cdot \nabla A$ for any scalar A. Taking the quasi neutrality of the plasma into account, then the continuity equation for the charge density on scales larger than the Larmor-radius becomes:

$$0 = \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 + \nabla \cdot (\mathbf{j}_\parallel + \mathbf{j}_\perp) \quad (1.12)$$

Equations (1.11) and (1.12) together imply the existence of a $\mathbf{j}_{\parallel}$ current. If we substitute into the (1.12) continuity equation:

$$0 = \nabla \cdot \left(j_{\parallel} \frac{\mathbf{B}}{B} + \mathbf{j}_{\perp} \right) = \mathbf{B} \cdot \nabla \left(\frac{j_{\parallel}}{B} + \frac{I}{B^2} \frac{dp}{d\Psi} \right). \quad (1.13)$$

It follows that the expression in the brackets is constant along the flux surfaces, thus the parallel current becomes:

$$j_{\parallel} = -\frac{I}{B} \frac{dp}{d\Psi} + K(\Psi) B, \quad (1.14)$$

where $K(\Psi)$ is an arbitrary integration constant. Equation (1.14) can be viewed as the “return current” necessary to close the diamagnetic current. The fact that $\mathbf{j}_{\perp}$ is divergence free in a cylinder implies that the emergence of the non zero $j_{\parallel}$ is caused by purely geometrical effects. Taking the flux surface average of (1.14) yields:

$$K(\Psi) = \frac{I}{\langle B^2 \rangle} \frac{dp}{d\Psi} + \frac{\langle j_{\parallel} B \rangle}{\langle B^2 \rangle}. \quad (1.15)$$

Thus by substituting (1.15) into (1.14) the parallel current density becomes:

$$j_{\parallel} = \frac{B}{\langle B^2 \rangle} \langle j_{\parallel} B \rangle - \frac{I}{B} \frac{dp}{d\Psi} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) = (j_{Ohm} + j_{Bs}) + j_{PS}. \quad (1.16)$$

The first term can be resolved into an electric field driven term, the *ohmic current* (j_{Ohm}) and the remaining so called *bootstrap current* (j_{Bs}), of which the former dominates in case of collisional plasma. The last term is called the Pfirsch-Schlüter current (j_{PS}). It can be shown that the Pfirsch-Schlüter current has a vanishing flux surface average [4].

In the collisionless case the non-ohmic part of $j_{\parallel}$ becomes significant and may even dominate over j_{Ohm} . A heuristic explanation is that in the collisionless case a fraction of the particles become trapped in banana orbits because of the dependence of B on the major radius. Due to the presence of a non-zero pressure gradient, the number of trapped particles on outer banana orbits is smaller, this yields a net flow of particles (Figure 1.5).

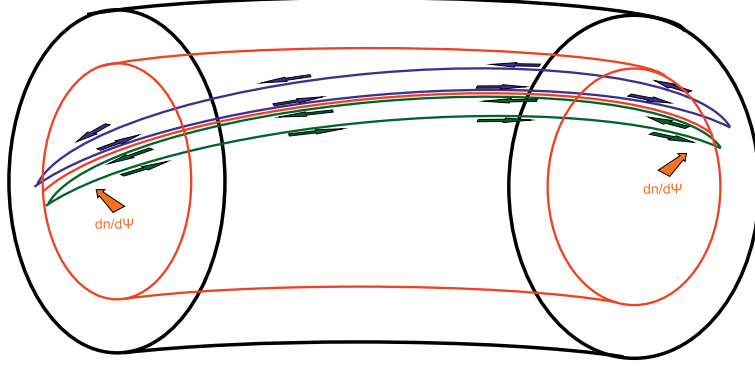


Figure 1.5: Schematic diagram of banana orbits of trapped particles. $\nabla n \neq 0$ implies a net current flow on the highlighted flux surface.

The bootstrap current can be calculated by subtracting the ohmic part of the parallel current, which according to [4] is:

$$j_{Bs} = j_{\parallel} - j_{Spitzer} = n_i Z e V_{i\parallel} - e \int f_{e1} v_{\parallel} d^3 v - j_{Spitzer} = -e \int (f_{e1} - f_{es}) v_{\parallel} d^3 v \quad (1.17)$$

where n_i is the ion density, V_i is the parallel flow of ions, f_e is the electron distribution function, f_{es} is the Spitzer function for electrons, n_i is the ion density, Z is the charge of ions and e is the elementary charge. Using the self-adjointness of the collision operators (C_e) [4], the flux surface average of Equation (1.17) yields:

$$\langle j_{Bs} \rangle = -\frac{T_e}{E_{\parallel}^A} \left\langle \int v_{\parallel} \frac{f_{es}}{f_{e0}} C_e (f_{e1} - f_{es}) d^3 v \right\rangle, \quad (1.18)$$

where $\mathbf{E}^A$ is the induced electric field, f_{e0} is the Maxwellian distribution function for the electrons while f_{e1} is the first order correction of the drift kinetic equation for electron distribution function for a linearized collision operator [4]. In the large aspect ratio limit ($\lim \epsilon \rightarrow 0$) the $C_e (f_{e1} - f_{es})$ collision term becomes localized in the trapped region, thus it can be approximated by the model operator [4] which in turn leads to the following expression:

$$\begin{aligned} \langle j_{Bs} \rangle = & -\frac{T_e}{E_{\parallel}^A} \left\langle \int v_{\parallel} \hat{f}_s C_e (f_{e1} - f_{es}) d^3 v \right\rangle = -f_t \frac{n_e I T_e^2}{m_e \Omega_e E_{\parallel}^A} (\nu_D^{ee} + \nu_D^{ei}) \hat{f}_s \\ & \times \left[\left(1 + \frac{T_i}{Z T_e} \right) \frac{d \ln n_e}{d \Psi} + \left(x^2 - \frac{3}{2} \right) \frac{d \ln T_e}{d \Psi} - \frac{0.173 d T_i}{Z T_e d \Psi} \right] \\ & - f_t \frac{n_e T_e^2}{m_e E_{\parallel}^A} (\nu_D^{ee} + \nu_D^{ei}) \hat{f}_{es}, \end{aligned} \quad (1.19)$$

where $\hat{f}_s = \frac{f_{es}}{f_{e0}}$, $x = \frac{m_e v^2}{2T_e}$, ν_D^{ei} is the electron-ion while ν_D^{ee} is the electron-electron deflection frequency and Ω_e is the electron Larmor-frequency, f_t is the fraction of trapped electrons.

In the large aspect ratio limit the perturbed distribution can be calculated, when the Spitzer function is known. Using a numerically obtained solution for f_{es} for singly charged main species we get the approximate formula for the bootstrap current [1].

$$j_{BS} = -\frac{\sqrt{\epsilon} n_e}{B_p} \left[2.44 \frac{T_e + T_i}{n_e} \frac{\partial n_e}{\partial r} + 0.69 \frac{\partial T_e}{\partial r} - 0.42 \frac{\partial T_i}{\partial r} \right] \quad (1.20)$$

1.2.1 Bootstrap current in the banana and plateau regimes

As it was previously pointed out, the bootstrap current is negligible in the Pfirsch-Schlüter regime. While the (1.20) formula is often a good approximation for the bootstrap current, it neglects the effects of the radial electric field and steep gradients which characterize the pedestal region in H-mode [5]. Of the many neoclassical corrections, the most recent are [3] and [6].

According to [6] retaining the above mentioned terms and calculating the bootstrap current in the **plateau regime** yields:

$$j_{BS} = -c_1 \frac{\epsilon B v_{Te}}{\nu_e R_0 \langle B^2 \rangle} \left[\frac{\partial(p_e + p_i)}{\partial r} + c_2 n_e \frac{\partial T_e}{\partial r} + \frac{J(U^2)}{2} n_e \frac{\partial T_i}{\partial r} \right], \quad (1.21)$$

where p_a , T_a are the pressure and temperature of species a , $\nu_e = \frac{\sqrt{2} n_e e^4 \ln \Lambda}{12 \pi^{3/2} \epsilon_0^2 m_e^{1/2} T_e^{3/2}}$ is the electron collision frequency, $v_{Te} = \sqrt{2T_e/m_e}$ is the electron thermal speed and the constants are $c_1 = \sqrt{\pi/2} \frac{\sqrt{2+4Z}}{Z(2+\sqrt{2}Z)}$ and $c_2 = \frac{\sqrt{2+13Z}}{2(\sqrt{2+4Z})}$, where Z is the charge of the main ion species. The constant multiplying the ion temperature gradient is

$$J(U^2) = \frac{1 + 4U^2 + 6U^4 + 12U^6}{1 + 2U^2 + 2U^4},$$

which is close to unity for small U . Here U denotes the normalized electrostatic field

$$U = \frac{B_0 v_{\mathbf{E} \times \mathbf{B}}}{v_{Ti} B_p} \simeq \frac{q}{v_{Ti} \epsilon B_0} \frac{\partial \phi}{\partial r},$$

where $v_{\mathbf{E} \times \mathbf{B}}$ is the $\mathbf{E} \times \mathbf{B}$ drift velocity, v_{Ti} is the ion thermal speed and B_p is the poloidal magnetic field.

According to [3] the bootstrap current takes the following form in the **banana regime**:

$$j_{BS} = -k_1 \frac{\sqrt{\epsilon} q B}{\langle B^2 \rangle} \left[\frac{\partial(p_e + p_i)}{\partial r} - k_2 n_e \frac{\partial T_e}{\partial r} - 7 \frac{A_b(U^2)}{6Z} n_e \frac{\partial T_i}{\partial r} \right], \quad (1.22)$$

where the constants are $k_1 = \frac{Z^2+2.21Z+0.75}{Z(Z+\sqrt{2})}$ and $k_2 = \frac{2.07Z+0.88}{Z^2+2.21Z+0.75}$. The constant multiplying the ion temperature gradient is:

$$A_b(U^2) = \frac{6}{7} \left[\frac{5}{2} + U^2 - \frac{\int_0^\infty e^{-y} (y + 2U^2)^{3/2} (y\nu_\perp + 2U^2\nu_\parallel)}{\int_0^\infty e^{-y} (y + 2U^2)^{1/2} (y\nu_\perp + 2U^2\nu_\parallel)} \right].$$

Here $\nu_\perp/\nu_i = 3\sqrt{2\pi}(\text{Erf}(x) - \Psi(x))/(2x^3)$ and $\nu_\parallel/\nu_i = 3\sqrt{2\pi}\Psi(x)/(2x^3)$ with $\Psi(x) = \text{Erf}(x) - x\text{Erf}'(x)$ and $x = \frac{v_{Te}}{v_{Ti}}$, where ν_i is the ion-ion collision frequency (by Braginskii's definition).

By looking at (1.22) and (1.21) it is apparent that the driving forces behind them are the same three gradients, however the coefficients are different. As $\frac{c_1}{k_1} = \mathcal{O}(1)$, the ratio of the main coefficients is $\mathcal{O}\left(\frac{\sqrt{\epsilon}v_{Te}}{qR_0\nu_e}\right)$ which is $\mathcal{O}(10)$ in the investigated scenarios, thus (1.21) yields much higher currents. It is also important to note, that while the electron temperature gradient is a driving force in (1.21) its sign changes in (1.22). This is important for our investigation as pellets interact with the plasma in a close to adiabatic manner which means that the pressure only changes slightly while the temperature gradients may change significantly.

1.3 Edge localized modes

In fusion plasmas the current density is known to drive instabilities like the edge localized modes (*ELMs*). These are a unique feature of the H-mode of tokamak operations. These scenarios arise once the heating power crosses a threshold after which confinement improves drastically due to the emergence of a transport barrier at the plasma edge [5]. This creates very steep edge gradients of temperature and density, forming the pedestal region. Unfortunately this leads to quasi-periodic bursts of energy and plasma material ejection, which in turn leads to not only degraded confinement, but extreme heat load on the tokamak's plasma limiting elements (limiter or divertor)[7]. These instabilities are called edge localized modes (*ELMs*). In case of a reactor sized device (e.g. ITER [8]) this heat load would be large enough to cause an unacceptable level of melting and erosion, that would seriously limit its lifetime. The presence of large ELMs also bear the danger of causing core MHD modes, that can lead to further confinement degradation [7]. Since

H-mode is the baseline operational scenario of ITER [9], it is essential that we understand the mechanism behind the ELMs and find a way to control them.

One would think that the most prudent method would be to completely avoid ELMs, but despite their negative effects, ELMs have a large role in removing impurities from the plasma edge and controlling plasma density. Nevertheless it was shown that high confinement without impurity buildup can be achieved without ELMs [10]. Unfortunately these regions in parameter space are rather small and hard to maintain.

1.3.1 Peeling-ballooning model

ELMs are believed to be caused by some form of MHD instability driven by the steep edge gradients, however their exact nature is not yet fully understood. The most widely accepted analytical model is the so called peeling-ballooning model. This model builds on the two instability sources near the plasma edge, the pressure gradient and the current density. By itself the pressure gradient drives the *ballooning* instabilities. This happens if the pressure gradient is so high, that it becomes possible for a perturbation to be localized to the area of destabilizing curvature, thus leading to localized modes, the ballooning modes [1]. Similarly the current density drives the *peeling* instabilities, which are in fact edge localized external kink modes [11]. The peeling-ballooning model of ELMs describes coupled peeling and ballooning modes, which are driven by both the current density and the pressure gradient [12]. In this picture the steep pressure gradient may lead to a large bootstrap current (chapter 1.2), which in turn could provide the free energy necessary to destabilize peeling and ballooning modes.

Thus the stability limits are set by these two types of instabilities as it is seen on Figure 1.6. The pure ballooning modes provide an upper limit for the normalized pressure gradient [7]:

$$\alpha = -\frac{2Rq^2\mu_0}{B^2}\nabla P < \alpha_{crit}. \quad (1.23)$$

Meanwhile the pure peeling instabilities have the following stability criterion [7]:

$$\alpha \left[\epsilon \left(1 - q_a^{-2} \right) - s\Delta_{sh} \right] \geq \frac{Rqs}{B}j_{\parallel}, \quad (1.24)$$

where ϵ is the inverse aspect ratio, q_a is the safety factor at the edge, s is the magnetic

shear, Δ_{Sh} is the Shafranov-shift and $j_{\parallel}$ is the total parallel current density.

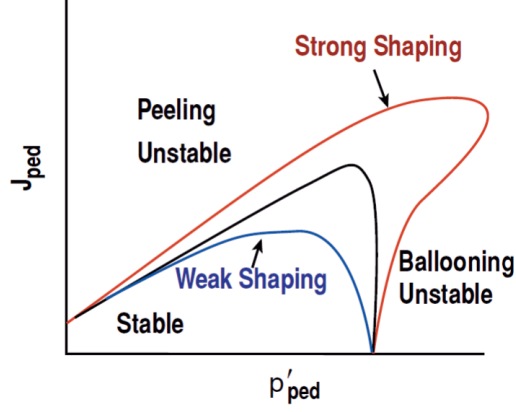


Figure 1.6: Schematic diagram of peeling–ballooning stability boundaries as a function of pedestal pressure gradient and current. Boundaries for weak and strong shaped discharges are characterized [12].

According to the heuristic picture behind the peeling-ballooning model the periodic cycle of the ELMs have 3 phases [7] (Figure 1.7):

1. The plasma pressure builds up up until it reaches the ballooning stability limit.
2. Since it is a “soft” limit, the plasma hovers there as the pressure gradient driven bootstrap current increases, until it reaches the peeling-ballooning boundary.
3. The ELM is triggered plunging the plasma into the unstable region.

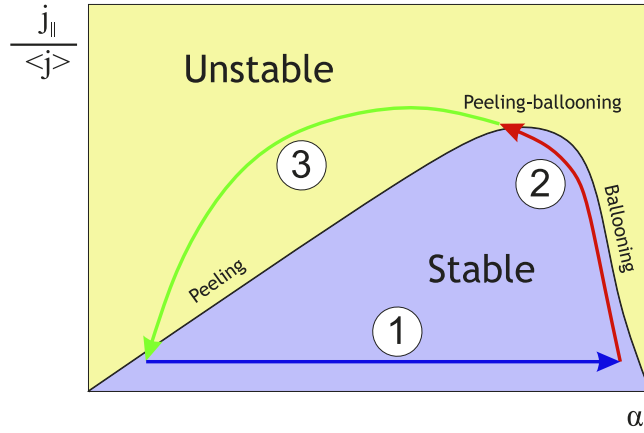


Figure 1.7: Phases of the ELM cycle on the peeling-ballooning stability diagram based on [7].

It should be noted that despite ELMs are believed to be caused by localized instabilities, they show dependence on the global (e.g. flux surface averaged) parameters [13].

1.3.2 ELM pace making

As it was mentioned before, although avoiding ELMs is possible, it is not practical. But since large amplitude ELMs (type I ELMs) pose a potential danger to the tokamak devices and to the fusion reactor concept, it is essential to find a way to mitigate their effects. Currently three ideas are being explored [7]:

- ELM control through magnetic perturbations means that a stochastic layer is created within the pedestal region, thus enhancing transport which limits the steepness of plasma gradients in the region. This is accomplished by using external coils.
- ELMs can be triggered by raising the edge current (see Figure 1.6). Raising the edge current is usually accomplished by applying a voltage perturbation to the poloidal field coils, which results in the up-down movement of the plasma.
- Experiments have found that injecting a small projectile (*pellet*) into the plasma can in fact trigger an ELM. By injecting pellets at the right frequency ELMs can be

triggered prematurely resulting in smaller amplitudes, which in turn greatly reduces the transient heat load on the plasma limiting elements [2]. This is often referred to as *pellet pace making*. Heuristically, as the pellet moves deeper into the plasma a cloud of ablated particles is created around it. This cold cloud creates a large increase in the local density and a decrease in temperature in the cooled region and this change could trigger an ELM. This picture, however, does not take the effect on the current density into account. Apart from these local effects the pellet injection might modify the global, flux surface averaged plasma parameters enough to contribute to the destabilization of the plasma. The aim of this paper was to determine if this is the case.

2 Current diffusion model and realization

The aim of my work was to calculate the transient currents induced by a pellet injection. This is, however, a fairly complex problem, thus several simplifications and approximations had to be employed.

2.1 Model approximations

The evolution of the electric field and the current density are governed by the Maxwell equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.1)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}. \quad (2.2)$$

By taking $\nabla \times (2.1)$ and substituting Equation (2.2) into it yields:

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{j}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}. \quad (2.3)$$

Taking advantage of the quasi neutrality of the plasma yields $\rho = 0$, which in turn leads to $\nabla \cdot \mathbf{E} = 0$ thus $\nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \Delta \mathbf{E} = -\Delta \mathbf{E}$. The scales of the investigated process should also be examined. Based on experiments it is known that

it takes $\tau = \mathcal{O}(50 \mu\text{s})$ time to penetrate the pedestal and trigger an ELM, thus setting the characteristic time scale of the problem. The width of the pedestal is measured in centimeters, thus the characteristic distance is $\delta = 1 \text{ cm}$. Carrying out a scale analysis of Equation (2.3) yields:

$$\mathcal{O}\left(\frac{1}{\delta^2}\right) \mathbf{E} - \mathcal{O}\left(\frac{\mu_0}{\tau}\right) \mathbf{j} - \mathcal{O}\left(\frac{1}{c^2 \tau^2}\right) \mathbf{E} = 0, \quad (2.4)$$

$$\mathcal{O}(10^{-4}) \mathbf{E} - \mathcal{O}(8\pi 10^{-3}) \mathbf{j} \frac{\text{mVs}}{\text{C}} - \mathcal{O}(4 \cdot 10^{-9}) \mathbf{E} = 0. \quad (2.5)$$

Equation (2.5) shows that the third term in this case is negligible compared to the first one. Meanwhile Ohm's law states that:

$$\mathbf{j} = \mathbf{E}\underline{\underline{\sigma}} + \mathbf{k}, \quad (2.6)$$

where $\mathbf{k}$ is the non-inductive current density. Thus based on (2.5) and (2.6). Equation (2.3) can be simplified into a diffusion equation:

$$\Delta \mathbf{E} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} = \mu_0 \frac{\partial (\mathbf{E}\underline{\underline{\sigma}} + \mathbf{k})}{\partial t}. \quad (2.7)$$

What makes (2.7) different from a trivial inhomogeneous diffusion equation is that due to the pellet induced cooling and density increase the $\underline{\underline{\sigma}}$ conductivity tensor is not constant in time or space, making the search for an analytical solution in most cases futile. Considering that regarding ELM stability $j_{\parallel}$ is important, while assuming that $\underline{\underline{\sigma}}$ is diagonal, Equation (2.7) can be simplified into a scalar equation:

$$\Delta E_{\parallel} = \mu_0 \frac{\partial j_{\parallel}}{\partial t} = \mu_0 \frac{\partial (E\sigma + k)}{\partial t}. \quad (2.8)$$

From now on the $\parallel$ subscript from $j_{\parallel}$ and $E_{\parallel}$ will be dropped.

Solving Equation (2.8) is still rather complex in case of torus, thus further geometrical approximations were employed. First, toroidal symmetry was assumed, reducing the dimension of the problem from 3×1 to 2×1 . By assuming that all physical quantities are averaged along the flux surfaces this can be further reduced to 1×1 which also eliminates the Pfirsch-Schlüter currents. It is also helpful to neglect the geometrical effects of the toroidal topology, and assume a cylindrical topology, while keeping the bootstrap current. Together these approximations mean that the diffusion equation described in (2.8) is to be

solved in a circular cylinder with azimuthal and translational (axial direction) symmetries (Figure 2.1).

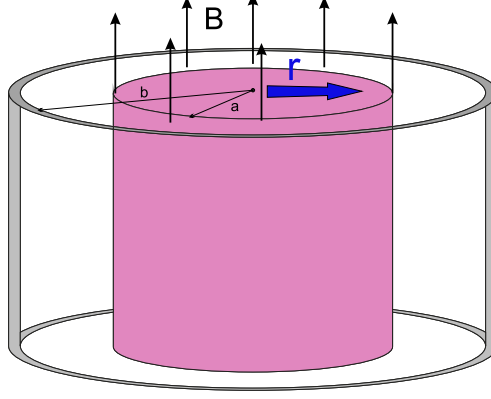


Figure 2.1: Cylindrical configuration used in the model.

Let us assume the the radius of the cylinder to be a and that it is surrounded by vacuum and an ideal metallic wall at $r = b > a$. In the vacuum region $j = 0$, thus according to (2.8) $\Delta E = 0$. Considering the assumed symmetries this yields:

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} = 0, \quad (2.9)$$

thus

$$E(r > a) = Cb \ln\left(\frac{b}{r}\right) + E_b \quad (2.10)$$

is the general form of the solution, where $E(b) = E_b$ is the boundary condition, that is prescribed by the applied loop voltage at the wall. Since there is no surface charge accumulation at the plasma-vacuum boundary thus the solution and its derivative has to be continuous: $E(r = a) = E_b + Cb \ln\left(\frac{b}{a}\right)$ and $E'(r = a) = -Cb \frac{1}{a}$. This means that the vacuum region can be replaced by a boundary condition at $r = a$:

$$E(a, t) + E'(a, t) a \ln\left(\frac{b}{a}\right) = E_b. \quad (2.11)$$

At the start of the simulation the plasma is assumed to be in steady state, which means that $E = E_0 = \text{const.}$, thus the starting current density is:

$$j(r, t = 0) = \sigma(r, t = 0)E_0 + j_{bs}(r, t = 0). \quad (2.12)$$

Taking advantage of the azimuthal symmetry and the diffusion equation yields another boundary condition at the center, since all physical quantities and their derivatives have to be continuous.

$$\left(\frac{\partial E}{\partial r}\right)_{r=0} = 0. \quad (2.13)$$

Although the calculation takes place in a cylindrical topology the used formulas assume a toroidal geometry (e.g. there would be no j_{bs} in a cylinder). Thus the conductivity (σ) is assumed to be a from of the Spitzer conductivity with neoclassical corrections [1]:

$$\sigma = \sigma_{Sp} \frac{(1 - \phi)(1 - C\phi)}{Z_{\text{eff}}} \frac{1 + 0.47(Z_{\text{eff}} - 1)}{1 + 0.27(Z_{\text{eff}} - 1)}, \quad (2.14)$$

where

$$C = \frac{0.56}{Z_{\text{eff}}} \left(\frac{3 - Z_{\text{eff}}}{3 + Z_{\text{eff}}} \right),$$

$$\sigma_{Sp} = 3\epsilon_0^2 (2\pi T_e)^{3/2} / (0.51 m_e^{1/2} e^2 \ln \Lambda),$$

$$\phi = f_T \left[1 + (0.58 + 0.2Z_{\text{eff}}) \frac{2Rq\epsilon^{-3/2}}{3\sqrt{2\pi c\tau}} \left(\frac{m_e c^2}{T_e} \right)^2 \right]^{-1},$$

where $\tau = 3(2\pi)^{3/2} \epsilon_0^2 \sqrt{m_e} T_e^{3/2} / (n_e e^4 \ln \Lambda)$ is the electron collision time, and the fraction of trapped electrons is approximately:

$$f_T = 1 - (1 - \epsilon)^2 / \left[(1 - \epsilon^2)^{1/2} (1 + 1.46\epsilon^{1/2}) \right].$$

It should be noted that the (1.21) and (1.22) bootstrap current formulas contain the $\frac{\partial \phi}{\partial r} = -E_r$ radial electric field. It is governed by the drift kinetic equations which describe the transport in the device [4]. Unfortunately it can not be solved analytically, and the low order solutions describe an ambipolar transport which means that the relation between the radial electric field and the plasma parameters is not known. However, in real devices the finite number of toroidal field coils breaks the perfect toroidal symmetry, leading to magnetic field ripple. In some cases this can become large enough so that its contribution to non-ambipolar transport dominates over the contribution of the higher order terms of the drift kinetic equation. Since radial electric field data is hard to come by, thus in this model the radial electric field was either assumed to be constant (ideal system) or the field ripple was assumed to be dominant thus:

$$E_r = \frac{T_i}{Z_i e} \left(\frac{\nabla n_i}{n_i} + 3.37 \frac{\nabla T_i}{T_i} \right), \quad (2.15)$$

where n_i is the ion density and Z_i is the effective charge of the ions. Using (2.15) can be motivated by the fact that recent measurements on Tore Supra [14] partially confirmed the validity of (2.15).

Also the (1.21) and (1.22) bootstrap current formulas are valid in the two distinct limits of $\frac{\nu q R}{v_T} \ll \epsilon^{3/2}$ and $\frac{\nu q R}{v_T} \gg \epsilon^{3/2}$, thus interpolation between them was required in case of $\frac{\nu q R}{v_T} = \mathcal{O}(\epsilon^{3/2})$. Thus the used bootstrap current was:

$$j_{interpol} = \frac{\xi^2 j_{banana} + \xi^{-2} j_{plateau}}{\xi^{-2} + \xi^{+2}}, \quad (2.16)$$

where $\xi = \frac{\epsilon^{3/2} v_T}{\nu q R}$ is the transport regime indicator function.

2.1.1 Limitations

The calculation of the diffusion of the electric field is depends on the plasma parameters whose temporal evolution is governed by the pellet injection. The detailed simulation of the pellet induced changes in the plasma parameters requires advanced pellet codes, and these are out of the scope of this work. Thus if no such data was available, then a quasi step-like change in plasma parameters was assumed as the pellet penetrates into the plasma. Since this is a rather rough approximation of the temporal evolution and it also neglects the coupling between the evolution of the current density and the plasma parameters, the results of these simulations are mostly just qualitatively correct. In case of fast pellets or small devices the timescale of the diffusion is larger than the timescale of the pellet cloud's temporal evolution, thus making this approximation viable.

Furthermore, the bootstrap current is only know in two distinct limits, while the simulated scenarios are bound to contain boundary regions between the plateau and the banana regimes. In this region an interpolation between the (1.21) and (1.22) bootstrap current formulas was employed. If it was applicable then the interpolation formula was partially fitted to measurement data (e.g. [15] for scenario in section 3.1). It should also be noted that in this model the bootstrap current is assumed to change on a timescale much shorter than the diffusion's time scale, thus making the bootstrap current's response to the perturbations prompt.

Nevertheless, the greatest limitation of the model, is that it can only simulate flux

surface averages, while neglecting local effects. Thus only the global contributions to ELM stability can be studied.

2.1.2 Numerical realization

The numerical solution was calculated by a program I had written in Wolfram Mathematica (license provided by Chalmers University of Technology). This code utilizes the NDSolve numerical differential equation solver function of Mathematica that implements a finite differences method to calculate the derivatives. To increase precision while maintaining the computation time at an acceptable level, the grid spacing is not constant, the number of points is increased between the edge and the pellet penetration limit. Also the diffusion equation is solved for the electric field (E), from which an interpolating function is created for the current density (j).

2.2 Code verification

Before proceeding with the simulations the numerical diffusion equation solver had to be benchmarked. To remain as relevant to the problem as possible, the boundary conditions of equations (2.11) and (2.13) were kept. The test scenario had to be chosen in a way that a simple, nonsingular analytic solution could be derived, thus the plasma parameters were assumed to be constant in space and time. By searching for the solution of (2.8) in a form where the space and time variables are separated yields the analytical solution in the following form:

$$j(r, t) = R(r) \Theta(t) = C_1 e^{-\frac{k^2 t}{\mu_0 \sigma}} J_0(kr) + C_2, \quad (2.17)$$

where k , C_1 , C_2 are constants and J_0 is the Bessel function of the first kind. This form also fulfills the requirements of (2.13) thus we only need to fit it to (2.11). This sets C_2 as E_b and restricts the possible values of k from $\Re$ to the solutions of

$$ka \ln \left(\frac{b}{a} \right) J_1(ak) = J_0(ak). \quad (2.18)$$

The initial condition was set as $j_0 = 10^6 \frac{A}{m^2} J_0(ak)$ by choosing an appropriate k that satisfies (2.18). The analytical solution is shown on Figure 2.2.

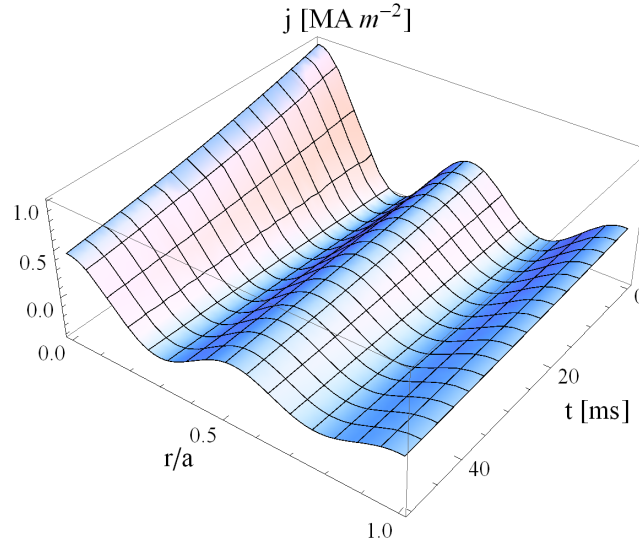


Figure 2.2: The analytical solution in test scenario according to (2.17).

Using the differential equation solver the numerical solution was also calculated using a grid size comparable to the ones used later. Figure 2.3 shows that the difference between the numerical and analytical solutions is within $2 \cdot 10^{-3}$ relative error, thus it is safe to say that the numerical diffusion equation solver is giving correct results.

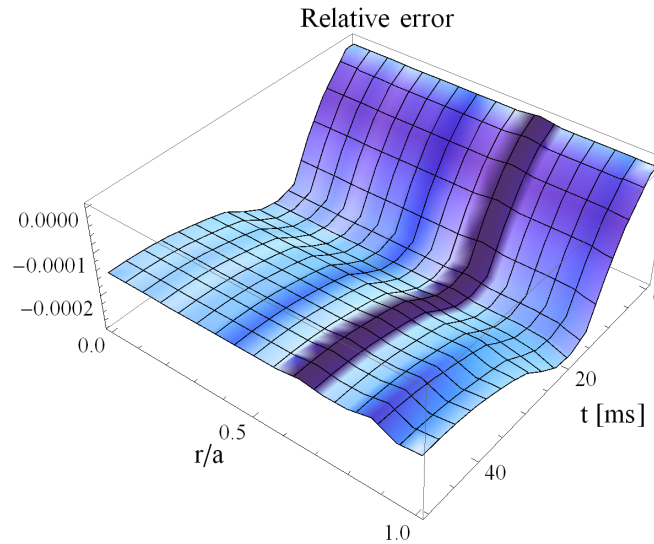


Figure 2.3: Relative difference between the numerical and analytical solutions of the test scenario.

2.3 Analytical solution

The scenario used for code verification was the case of ordinary diffusion with a constant σ parameter. An analytical solution of (2.8) for a more relevant, time dependent scenario can be derived if the transient event is space-independent while the effects of the cylindrical geometry are insignificant. In this case we can assume $\partial/\partial r \gg 1/a$ and

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \simeq \frac{\partial^2}{\partial r^2}.$$

If the conductivity (σ and the non-inductive current source ($k(r, t)$) are constants in space and change suddenly at $t = 0$ as

$$\sigma(t) = \sigma_0 + (\sigma_1 - \sigma_0)H(t), \quad (2.19)$$

$$k(t) = k_0 + (k_1 - k_0)H(t), \quad (2.20)$$

where H is the Heaviside function. Using (2.8) yields:

$$\mu_0 \frac{\partial j}{\partial t} = \frac{\partial^2}{\partial r^2} \left(\frac{j - k}{\sigma} \right).$$

For $t < 0$ the trivial solution is $j = \sigma_0 E_b + k_0 \equiv j_0$ and for $t > 0$ the current is given by the initial value problem using (2.8) and (2.11):

$$\frac{\partial j}{\partial t} = \frac{1}{\mu_0 \sigma(t)} \frac{\partial^2 j}{\partial r^2}, \quad (2.21)$$

$$a \ln \left(\frac{b}{a} \right) j'_a + j_a = \sigma_1 E_b + k_1 \equiv j_1, \quad (2.22)$$

$$j(r, 0) = j_0. \quad (2.23)$$

The solution is

$$j(r, t) = j_1 - (j_1 - j_0) \operatorname{erf} \left(\frac{a - r}{\sqrt{4Dt}} \right) + \frac{C}{\sqrt{4\pi Dt}} e^{-\frac{(r-a)^2}{4Dt}}, \quad (2.24)$$

where the constant C is chosen so that the boundary condition (2.22) is satisfied. This implies that

$$(j_1 - j_0) \frac{2}{\sqrt{4\pi Dt}} a \ln \left(\frac{b}{a} \right) + j_1 + \frac{C}{\sqrt{4\pi Dt}} = j_1$$

and this leads to

$$C = -2(j_1 - j_0) a \ln \left(\frac{b}{a} \right).$$

This solution has a singularity at $(r = a; t = 0)$, which is caused by the boundary condition (2.11), since the sudden change in local conductivity creates a similar change in the electric field, thus (2.11) forces the creation of a current sheet. This also means that a direct comparison with numerical results is somewhat problematic, since numerical solutions of differential equations have a finite resolution. Fortunately the singular part of the solution is transient, thus the numeric model should be able to find the nonsingular solution in the $t \rightarrow \infty$ limit. Since $r = a$ is the singular point the largest difference between the analytical and numerical results are expected at this position.

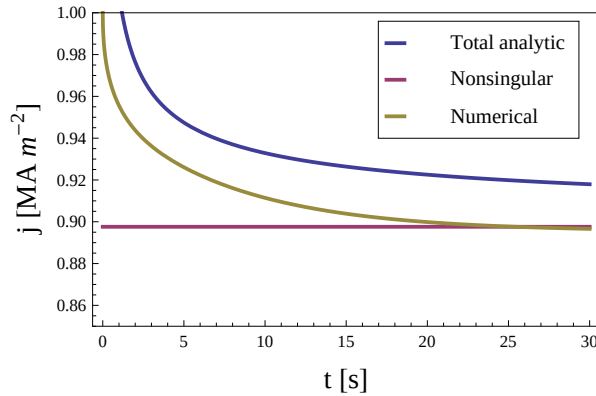


Figure 2.4: The numerical and analytical solutions several seconds after the change in plasma parameters at $r = a$.

Figure 2.4 shows clearly that the numerical result is identical to the analytical in the $t \rightarrow \infty$ limit while the transient behavior is somewhere between the behavior of the total and the nonsingular terms of (2.24). Since the simulated scenarios only contain continuous functions, it is safe to assume that the numerical method will yield correct results.

3 Results and interpretation

The aim of this paper is to show whether the global effects of the transient currents can influence ELM stability, thus the evolution of the global transient current has been

simulated for several ELM pace making scenarios. For the modeling of the temporal evolution of plasma parameters the results of pellet simulation codes were used.

3.1 KSTAR ELM pace making scenario results

The first tested scenario based on an ELM pace making simulation done by Kim et al. [16], which details the results of a transport code for a deuterium pellet injection into a KSTAR-like plasma. Unfortunately [16] only contains profiles only for a plasma without a real pedestal, thus this first scenario is just a demonstration of the model. The initial density ($n_e^{initial}$) and temperature ($T_e^{initial}$) profiles were fitted analytically (Figure 3.2):

$$T_e^{initial} = T_0 \left(1 - 0.5(r/a)^2\right)^{4.6}, \quad (3.1)$$

$$n_e^{initial} = n_0 \left(1 - 0.575(r/a)^2\right)^{1/2}, \quad (3.2)$$

where $a = 0.5$ m is the minor radius, $T_0 = 4.45$ keV, $n_0 = 5.9 \cdot 10^{19} \text{ m}^{-3}$ while E_b for the boundary condition was obtained from setting the total current to 1 MA (Figure 3.1). The effective charge of impurities was assumed to be a constant $Z_{eff} = 1.35$. Other parameters were from either [15] or [16].

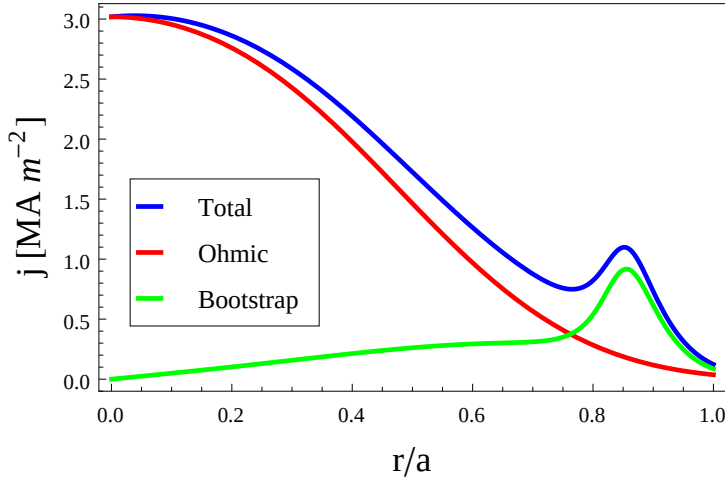


Figure 3.1: Components of the total current density in the steady state

Since [16] only contains data about the change of density, the temperature profile

after the pellet injection was calculated by the following approximation:

$$T_e^{after} = \frac{p_e^{init} - \Delta n_e (E_d + E_i)}{n_e^{init} + \Delta n_e}, \quad (3.3)$$

$$T_a^{after} = \frac{p_a^{init}}{n_a^{init} + \Delta n_a}, \quad (3.4)$$

where n_e , T_e , p_e are the electron density, temperature and pressure while n_a , T_a , p_a are the density, temperature and pressure of other plasma species. $E_d = 4.48$ eV and $E_i = 13.6$ eV are respectively the dissociation and ionization energy of the deuterium. Thus the (3.3) formula assumes that the cooled particles are co-moving with the ablated particles and that a thermal equilibrium is not reached between the different plasma species on the timescale of the calculation. The pellet induced cooling and density increase is shown on Figure 3.2.

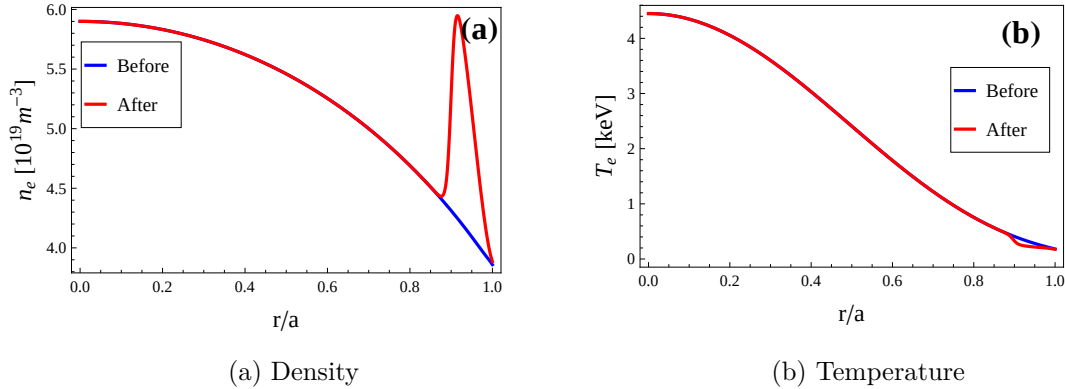


Figure 3.2: Plasma profiles before and after pellet injection for KSTAR ELM pace making scenario

The major radius is $R_0 = 1.8$ m, minor radius is $a = 0.5$ m and the metallic wall is at $b = 0.75$ m. The pellet velocity is assumed to be $v_{pellet} = 200$ m/s, thus the transition time (τ) between the two states shown on Figure 3.2 is assumed to be $\Delta\tau = d_p/v_{pellet}$, where d_p is the penetration depth of the pellet (Figure 3.3).

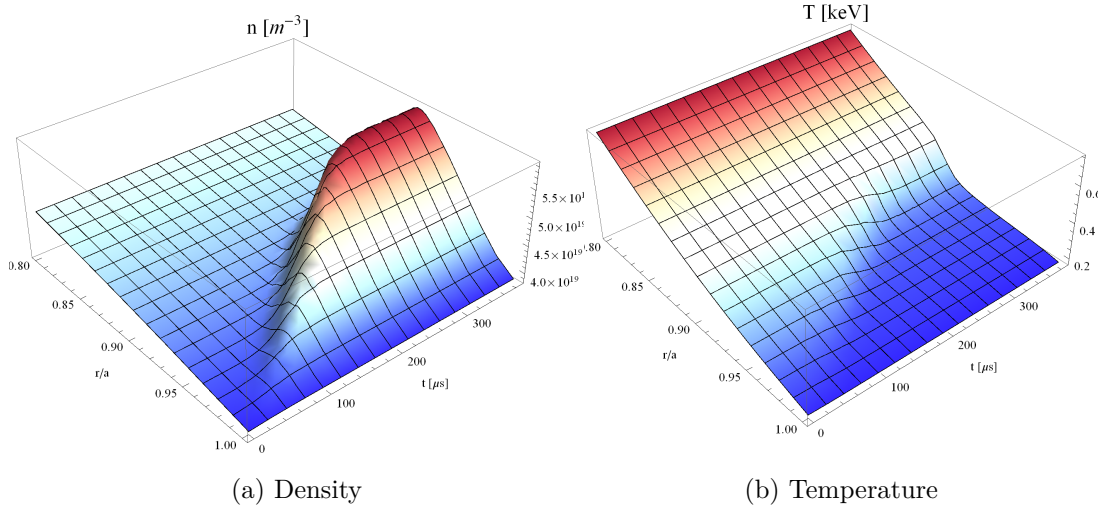


Figure 3.3: Plasma profile evolution in the KSTAR scenario

Based on (2.8) the current density was calculated, first without taking the bootstrap current into account, yielding Figure 3.6.

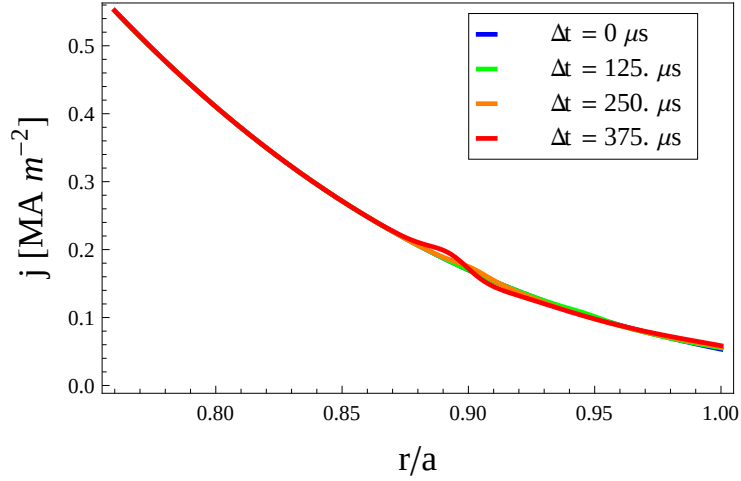


Figure 3.4: The change of current density after pellet injection (without j_{bs})

As it is shown on Figure 3.4, despite being a relatively large pellet, it causes little to no change in the current density. The same calculation was carried out with taking the bootstrap current into account while assuming the radial electric field to be a constant $E_r = -10^4$ V/m, which is in the typical magnitude of radial electric fields in tokamak

pedestals. Figure 3.5 clearly shows that the bootstrap current significantly increases at plasma edge, which in turn leads to a spike in the current density at $r/a \approx 0.85$. Naturally it remains unaffected in the core region ($r/a < 0.8$).

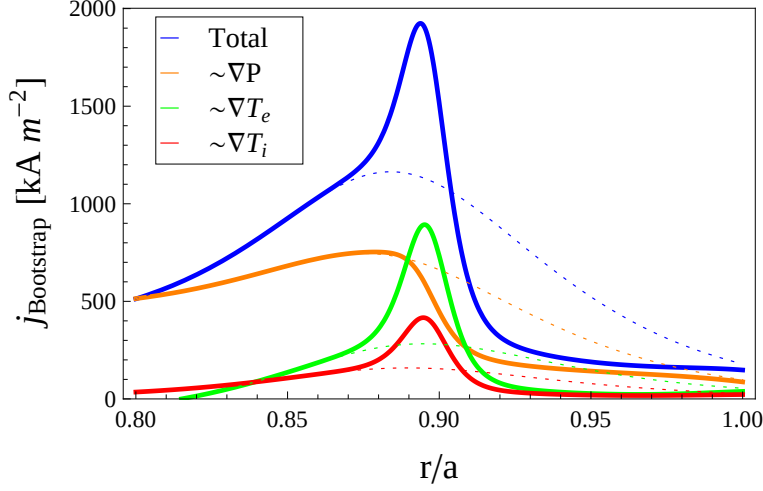


Figure 3.5: Components of the bootstrap current before and after pellet injection for KSTAR. The dotted lines denote the currents before while the straight lines the currents after the pellet injection.

Figure 3.5 also shows that the ∇P component dominates the bootstrap current and changes only slightly, as the effects of the pellet on the plasma is nearly adiabatic according to (3.3). Meanwhile the ∇T_e and ∇T_i components change significantly as the density increase cools the plasma, thus creating very steep gradients at the border of the cooled region, making these terms the same order as the ∇P component, thus creating a current spike.

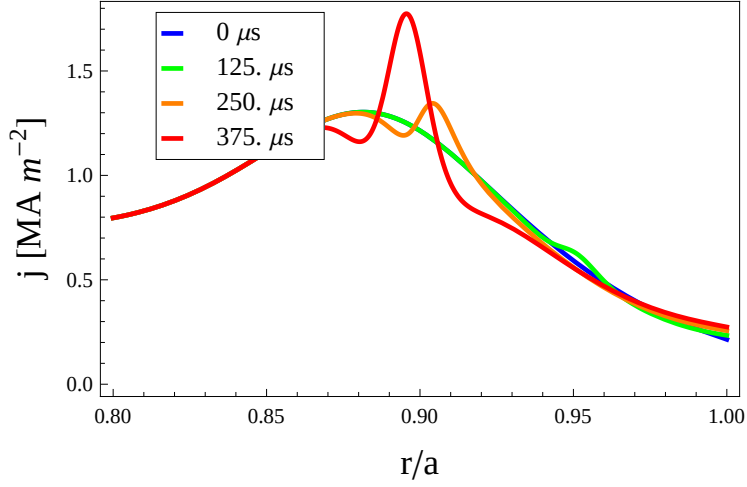


Figure 3.6: The change of current density after pellet injection for KSTAR scenario

As it is shown on Figure 3.6, compared to the case of only ohmic currents, this spike in the bootstrap current drastically changes the evolution of the total current. The result is a spike at the border of the cooled region which slowly diffuses on a larger timescale $\mathcal{O}(10ms)$. Based on these results it becomes obvious that the bootstrap current plays a significant role in the creation of transient currents. It should also be noted that Figure 3.6 shows a change in the plasma edge current density, which according to section 1.3.1 (Figure 1.6) has a role in ELM stability.

3.2 ASDEX Upgrade ELM pace making scenario results

The same calculations were carried out for several ASDEX Upgrade scenarios (shots #20041, #20043 with different pellet sizes and velocities), using the results of a detailed ELM pace making simulation [17]. This means that the temporal evolution of plasma parameters is known from the simulation (Figure 3.7), unlike in the KSTAR scenario in section 3.1. In this section mainly the results for shot #20043 will be detailed. The total plasma current for this shot was $I_{tot} = 1\text{ MA}$ while the ohmic current was just $I_{Ohm} = 30\text{ kA}$ and the effective charge of ions was assumed to be a constant $Z_{eff} = 1.5$. The major radius of the device is $R_0 = 1.65\text{ m}$, the minor radius is $a = 0.45\text{ m}$ and the metal wall was put at $b = 0.7\text{ m}$. The pellet's radius was 0.66 mm while its speed

was 600 m/s. As the characteristic time for a pellet to trigger an ELM is its time of flight through the pedestal, which in this case is $\mathcal{O}(100\ \mu\text{s})$, while the time of density homogenization is of the order of the ion collision frequency which in this case is $\mathcal{O}(300\ \mu\text{s})$. This means that using this timescale the global density is to be considered constant in time. Also, based on a paper regarding ASDEX Upgrade H-mode operations [18] the radial electric field was set at $E_r = -5000\ \text{V/m}$. In this model all non-inductive currents, except for the bootstrap current, are neglected and only taken into account when calculating $\langle j \rangle$ in section 3.4.

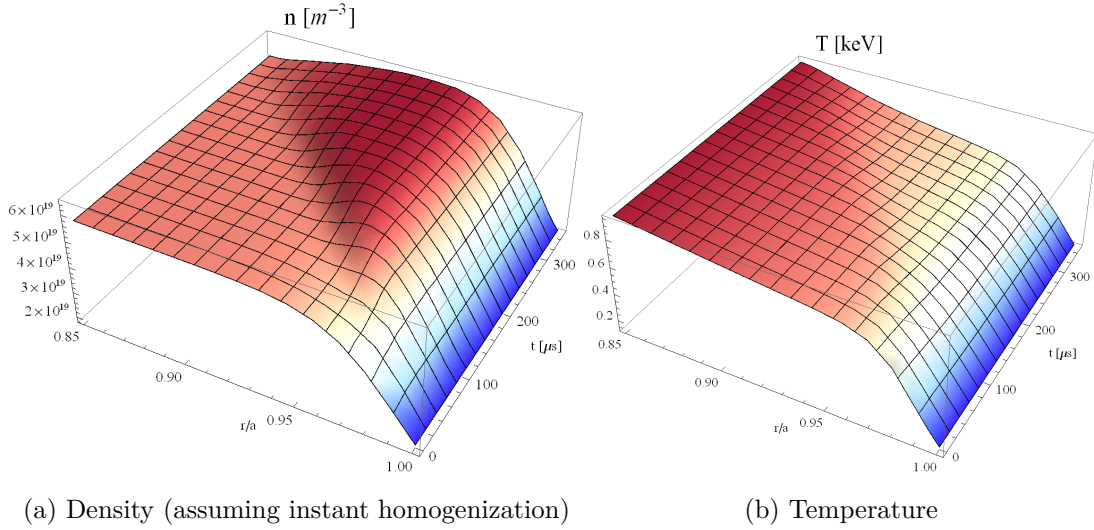


Figure 3.7: Plasma profile evolution after pellet injection for ASDEX Upgrade ELM pace making scenario (#20043)

As shown on Figure 3.7 this is an H-mode scenario with a very steep pedestal, thus ideal for the investigation of ELM triggering. It is also apparent that the quasi step-like approximation used in section 3.1 gives similar results (Figure 3.3). Although the pellet used for ELM pace making in ASDEX Upgrade is much smaller than the one used in the KSTAR simulation, it has been experimentally proven that it is sufficient to trigger an ELM [2].

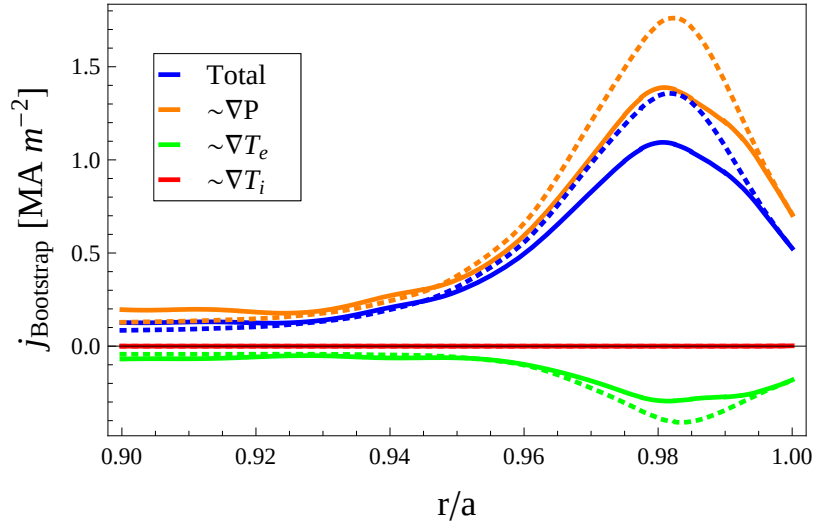


Figure 3.8: Components of the bootstrap current before and after pellet injection for ASDEX Upgrade scenario (#20043). The dotted lines denote the currents before while the straight lines the currents after the pellet injection.

Figure 3.8. shows that the bootstrap current is dominant in the pedestal, thus it governs the evolution of the local current density. Indeed, Figure 3.9 shows that main effect of the pellet can be interpreted as the shifting of the peak of the bootstrap current, thus of the total current.

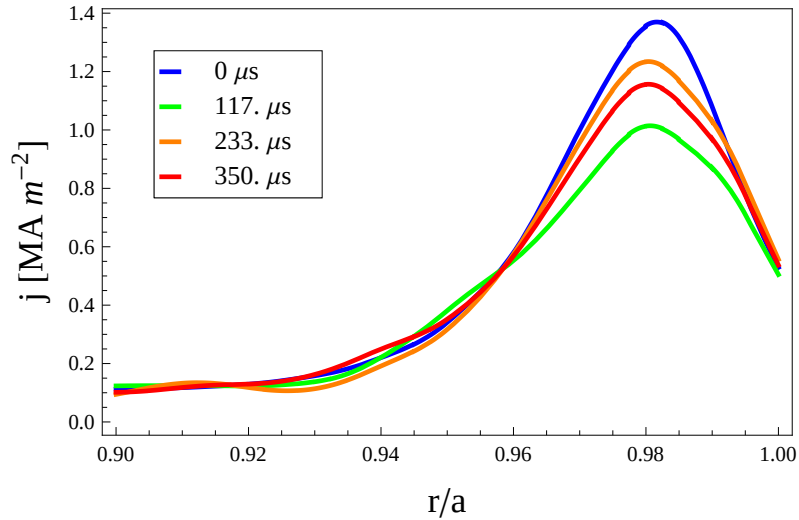


Figure 3.9: The change of current density after pellet injection for the ASDEX Upgrade scenario (#20043)

3.3 Role of radial electric field model

Previously, all systems were assumed to be ideally axisymmetric, thus the radial electric field was set as a constant. These calculations were repeated with different, constant radial fields, yielding qualitatively similar results, only the ∇T_i component of the bootstrap current changing. The same calculations have also been done by assuming large magnetic field ripple thus the radial electric field is set as Equation (2.15). By comparing Figure 3.9 and Figure 3.8 with Figure 3.10 we can see that the results in the ASDEX Upgrade scenarios remained qualitatively the same, only the bootstrap contribution increased

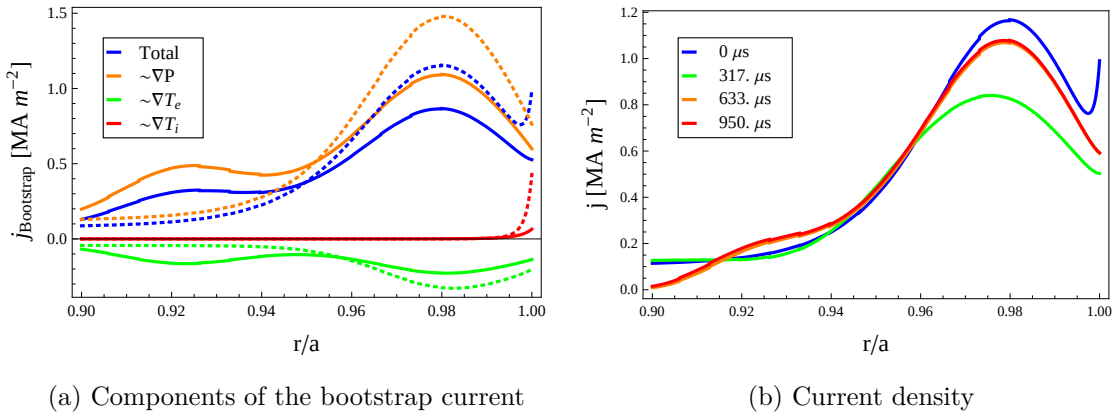


Figure 3.10: Results for the ASDEX Upgrade #20043 scenario in the magnetic field ripple dominated case using (2.15) formula

Thus we can conclude that the difference is quantitative and rather small so the choice of the radial field calculation bears no significant consequence on the results.

3.4 Implications on stability

As a well defined pedestal was lacking (Fig. 3.2a) for the KSTAR scenario, analysis of the results regarding ELM stability was only possible for the ASDEX Upgrade scenarios. In this case the temporal evolution of plasma parameters was also known from detailed simulation [17].

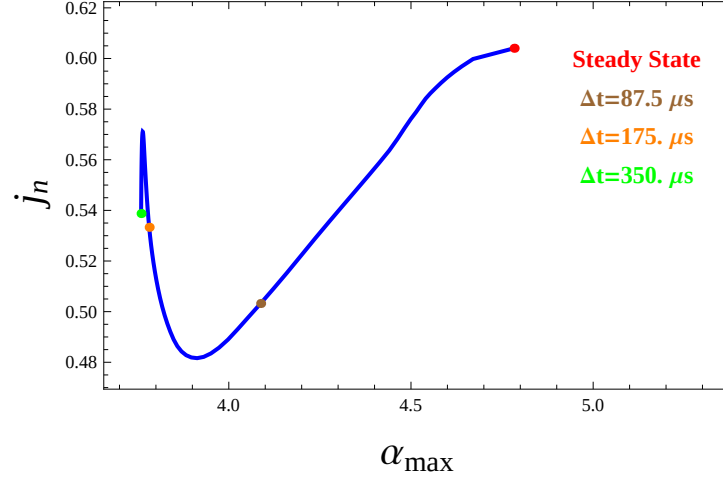


Figure 3.11: Trajectory of the system in the stability diagram for ASDEX Upgrade pellet pace making scenario (#20043).

Figure 3.11 shows the trajectory in the ASDEX Upgrade scenario case. The normalization is carried out the same way as in [13], where α_{max} and j_{max} are the maximums of the normalized pressure gradient (defined as Equation (1.23)) and current density in the pedestal. The heuristic picture behind pellet pace making puts the time of the ELM triggering around the time it takes for the pellet to traverse the pedestal, which in this case is $\mathcal{O}(100 \mu s)$. This means that only the falling part of the trajectory is relevant to ELM stability. It should be noted that while the total plasma current is taken into account when calculating $\langle j \rangle$, but due to the lack of information about the evolution of currents other than the ohmic and bootstrap currents, the axis of Figure 3.11 can be shifted with an unknown constant, thus Figure 3.11 only shows a relative trajectory.

Numerical stability calculation performed in [13] for ASDEX Upgrade shot #20116 resulted in the stability diagram shown on Figure 3.12.

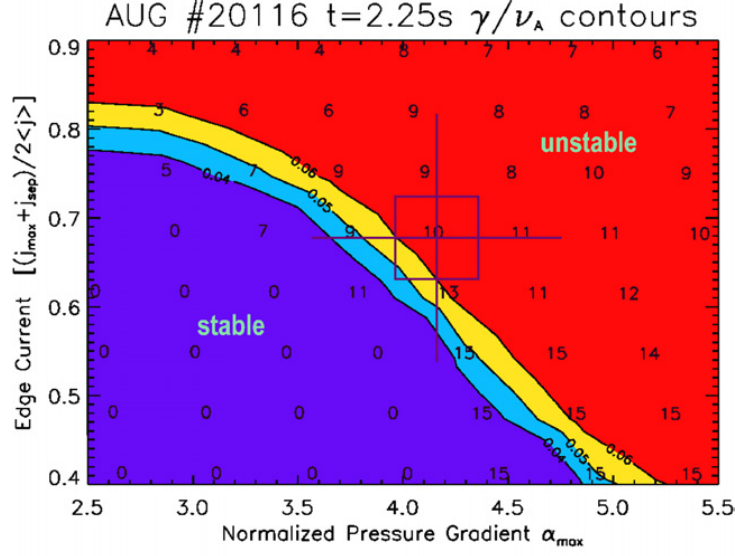


Figure 3.12: Stability diagram for ASDEX Upgrade based on shot #20116 from Maggi et al. [13], also showing the mode number of the triggered MHD instabilities.

Although Figure 3.12 shows numerical results for shot #20116 while the data used to get Figure 3.11 is for shot #20043, the two scenarios are similar enough to warrant a qualitative comparison. Based on Figure 3.11 we can see that first the α normalized pressure gradient decreases as the change of global density was neglected. Meanwhile the normalized current density also decreases, moving the system further away from the peeling-ballooning stability boundary. Plotting the trajectory on Figure 3.11 yields Figure 3.13 which shows the trajectory of the system. Although only a qualitative comparison is possible, Fig. 3.11 clearly shows that the effect of the global pressure gradient is more significant than the effect of global transient currents.

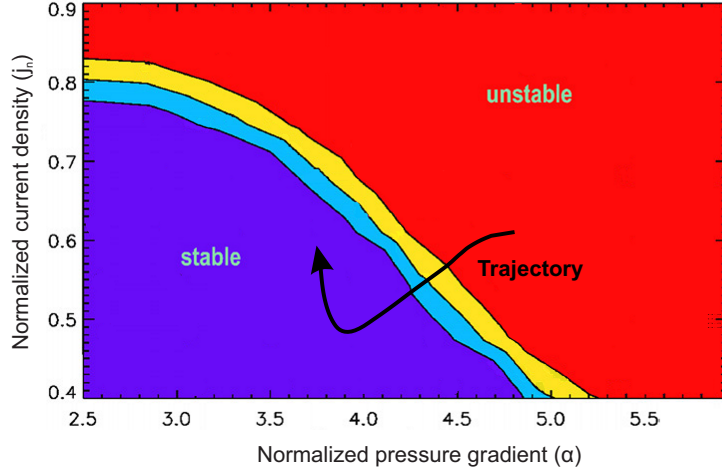


Figure 3.13: Trajectory of simulated system for shot #20041 plotted on stability diagram for shot #20116 from Maggi et al. [13]. Due to the different discharges used for the stability and trajectory calculation, only qualitative comparison is possible.

3.4.1 Sensitivity to pellet size and speed

As mentioned before, the results of several different pace making pellet scenarios were used. As expected, it was found that the size and speed of the pellet has significant effect on the results, as the larger and slower a pellet is, the greater the change in density and temperature on the outer flux surfaces.

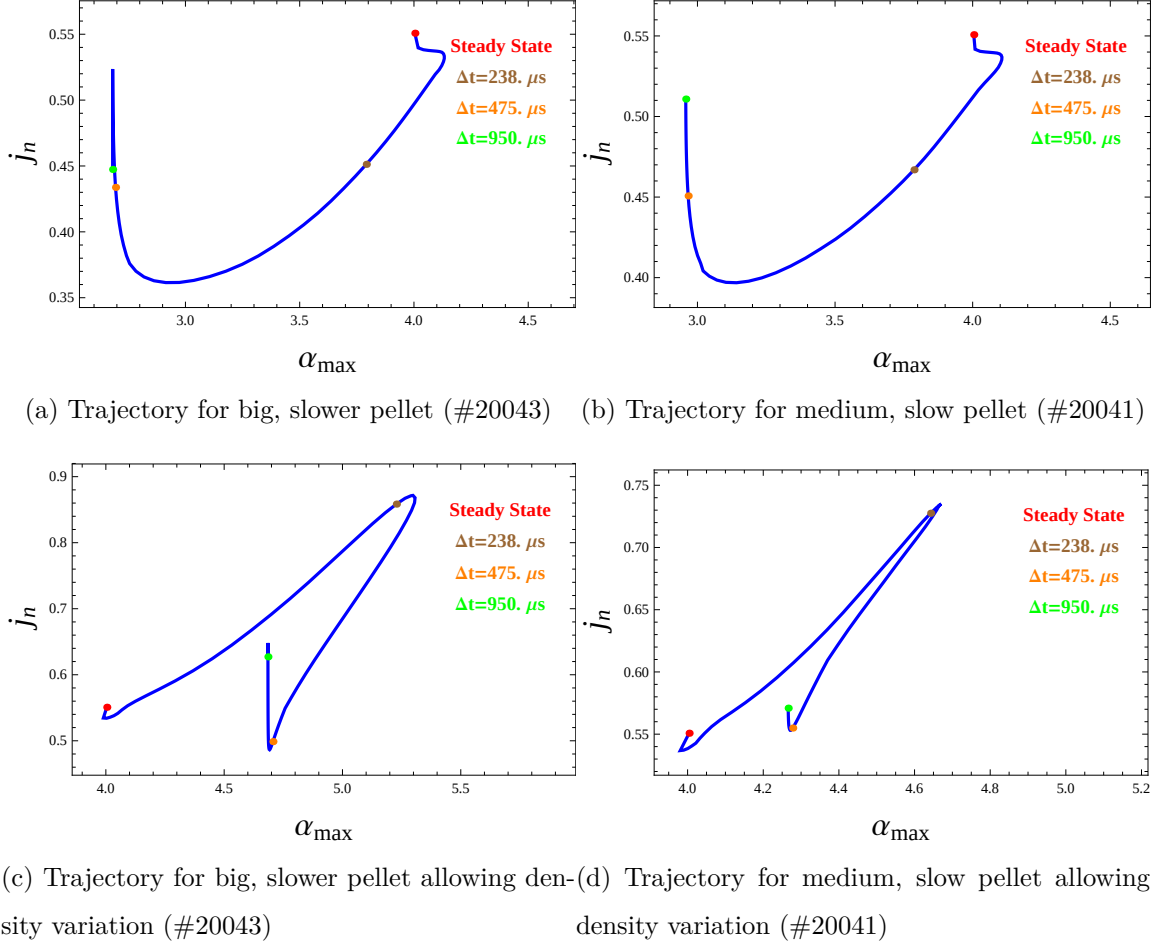


Figure 3.14: Results for different ELM pace making scenarios for ASDEX Upgrade.

Figure 3.14a shows the results for the ASDEX Upgrade #20043 scenario for a larger ($r = 0.91$ mm) and slower ($v = 240$ m/s) pellet, meanwhile Figure 3.14b shows the results for a medium sized ($r = 0.76$ mm) pellet with the same speed. Fig. 3.14c and Fig. 3.14d show the trajectories for the aforementioned scenarios if density homogenization is assumed which is made possible by the much lower pellet speed. By comparing Figure 3.11 with the above figures one can see that allowing the density homogenization changed the sign of both $\Delta\alpha$ and Δj_{norm} , and also increased their absolute value significantly. It is also apparent that the pellet size had no significant effect between the medium and large pellet scenarios without density homogenization. Thus the significance of j_{norm} regarding ELM stability scales with a function of the size and velocity of the pellet. Nevertheless, it should also be noted that the change in pellet velocity only influences the timescale of the

global diffusion, thus not necessarily the timescale of the ELM triggering as well. This means that an ELM could be triggered much sooner than the pellet leaves the pedestal, negating the scaling effects on the global contribution to the ELM stability.

All in all the global transient currents seem to have a stabilizing effect, while experiments showed that pellets do trigger ELMS, thus local effects must play a dominant role in the triggering process.

4 Conclusion and future plans

Despite being one of the most critical phenomenon of fusion plasmas, the nature of edge localized modes (ELMs) is still not fully understood. Since they represent a real danger to the plasma facing components of a reactor sized device, it is vital that their effects be somehow mitigated. One of the currently used mitigation methods is the pellet pace making which utilizes pellet injection to trigger ELMs prematurely, thus reducing the amplitude of the transient heat load. These pellets can cause transient currents, and the aim of this paper was to investigate the implications of global transient currents on ELM stability using the results of a pellet transport code for several ELM pace making scenarios as input.

During the course of this work I developed a model to describe the diffusion of the electric field during the pellet injection. First, several geometrical and physical approximations were applied thus reducing the problem to a cylindrical plasma surrounded by vacuum and a metallic wall, while calculating the bootstrap current and the conductivity with toroidal formulas. One KSTAR and several ASDEX Upgrade ELM pace making scenarios were investigated. The model successfully calculated the temporal evolution of the current density during and after the pellet injection. It was found that if density homogenization is neglected, than the global transient currents might have a stabilizing effect on the peeling-ballooning stability in the experimentally relevant cases, although local effects probably play the major role in the destabilization of the instability [19], as pellets do trigger ELMs. It should be noted that during these calculations the response of the bootstrap current to the pellet induced plasma parameter change was assumed to be

prompt thus the results depicted in this paper are an upper estimate of the contribution of the global transient currents.

During the course of this work a diffusion equation solver was developed that can model the spatial and temporal evolution of the current density not only for pellet injection scenarios but for arbitrary plasma parameter changes. This implies further possible applications of this model in calculations regarding runaway electron generation and MHD stability calculations.

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