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Games in partition function form with restricted cooperation

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Chapter 1

Introduction

In this paper we deal with strategic situations in which both externalities and restricted cooperation are important. There are lots of examples of situations like that in Politics and Economics. Here we present two of them.

First we consider a situation from Economics, in particular from Industrial Organization. It is well known that the profit of a given firm depends on the structure of the industry in which the firm competes. If a small firm faces large rivals, then it is less likely to perform well, while the large ones are more able to outperform their smaller rivals. Therefore if two firms merge, they will be better off. This fact provides an incentive to firms for mergers. However, while the profits of firms increase by mergers, the welfare of consumers may decrease. Thus the competition authorities do not approve all mergers.

It is clear that in this situation externalities between players are crucial: firms' profits depend on which market structure is formed by the other firms. However, firms cannot realize any market structure – by arbitrarily merging with each other – due to the competition authorities. So cooperation between the firms is restricted. One can ask the question: which market structure will be stable in this environment?

Our second example is from Politics, in particular from Russia in the late 1990-s. During the second presidency of Yeltsin, the prime minister Anatoly Chubais put an end to the corrupt "loans-for-shares" system in privatizations. Before that the oligarchs and the government cooperated in a way that the oligarchs provided loans for the government, and they cheaply acquired the large public companies in return. For further details see Hoffman (2001). When Chubais made this decision, he faced the strategic problem described below.

Before the change the oligarchs and the government cooperated, but by this decision

Chubais deviated from that situation. The oligarchs had to make the decision how to react to that step. If they would continue to cooperate with the government under the new rules, they were find themselves in an inferior position. Instead they could fight with the government. The outcome of the fight could depend on whether they join forces against the government, or stay in small groups and fight even with each other. The resulting outcome was influenced by what coalitions are formed by the agents, therefore externalities between players were crucial.

In addition, there were hierarchal relations within the government and even within the oligarchs. Due to these hierarchies some coalitions could not form. So as above, we again have a situation with externalities and restricted cooperation.

In the examples mentioned above externalities play an important role. The standard model of cooperative game theory, the game with transferable utility (TU game) is not able to express any externalities between players. However, the games in partition function form (PFF games) first introduced by Thrall and Lucas (1963) are able to do that. There are various solution concepts in the literature for PFF games, see for example Aumann and Peleg (1960); Myerson (1977b); Shenoy (1979); Huang and Sjöström (2003) and Kóczy (2007) among others. In contrast to the TU games, regarding the PFF games one of the important questions is what partition will be formed by the players. In other words, the PFF games often focus on the problem of *coalition formation*.

There is also an important non-cooperative approach in the study of the formation of coalitions in the presence of externalities, important contributions include Bloch (1996); Ray and Vohra (1999) and Yi (1997). Another part of the literature studies the relationship between the cooperative and non-cooperative solution concepts, for example Kóczy (2009) and Huang and Sjöström (2006).

Focusing on the cooperative approach we highlight the recursive core of Kóczy (2007) among the several solution concepts. It has a "central position" in the literature being a refinement between a somewhat weak and an extremely strong stability concepts, the α -core (Aumann and Peleg, 1960) and the ω -core (Shenoy, 1979). The stability concept of α -core assumes that residual players hurt deviators as much as they can, while the concept ω -core assumes that residuals will help deviators as much as possible. Since these concepts assume that residuals make efforts to hurting (or helping) the deviators even if they hurt themselves as well. This assumption is counterintuitive, and the concept of

recursive core assumes that residuals make only "credible" reactions to every possible deviation. Another attractive purpose of the recursive core, as Kóczy (2009) shows, is that its pessimistic (or large, according to the term we use) version of the recursive core coincides with the set order-independent sequential equilibrium coalition structures of Bloch (1996). In this paper we generalize the recursive core of Kóczy (2007) to the newly introduced PFF games with restricted cooperation.

As the above examples show, in practical situations it is often the case that some coalitions cannot form due to e.g. communication problems, hierarchical relations among the players, or institutional restrictions (for example the decisions of the competition authority regarding the mergers). For TU games, restricted cooperation is extensively discussed by the literature. Aumann and Dreze (1975) study the implications of restricted cooperation on various, widely used solution concepts of TU games. Other authors (Myerson, 1977a; Owen, 1986; Derks and Peters, 1993) propose different frameworks for restricted cooperation.

Owen (1986) and Myerson (1977a) use graph structures to define the set of permitted coalitions. They consider a graph where the nodes are the players, and coalition S is permitted if all players in S are connected in the graph. Derks and Peters (1993) use a different approach. They introduce a so-called restriction function, a monotonic projection of 2^N to 2^N which assigns a subcoalition to each coalition. There is another part of the literature which studies hierarchical permission structures where some agents are not free to cooperate without the presence of other agents; see Gilles, Owen, and Brink (1992); Brink (1997) among others.

A quite new direction of research regarding restricted cooperation assumes a certain structure of the feasible coalitions. This structure can be convex geometries (see e.g. Bilbao and Losada (1998)), antimatroids (see e.g. Algaba, Bilbao, Brink, and Losada (2004)), union stable systems, also called weakly union-closed systems (see e.g. Algaba, Bilbao, Borm, and López (2000); Faigle, Grabisch, and Heyne (2010); Faigle and Grabisch (2011)), matroids (see e.g. Bilbao, Driessen, Losada, and Lebrón (2001)), regular set systems (see e.g. Honda and Grabisch (2008) and Lange and Grabisch (2009)), distributive lattices (see e.g. Grabisch and Xie (2011); Grabisch (2011)), various ordered structures (see e.g. Grabisch (2009), and symmetric ordered structures (see e.g. Grabisch (2004)) among others.

Despite this extensive work on cooperative games with restricted cooperation, up to the best of our knowledge there is no paper about PFF games with restricted cooperation, however our examples above illustrates that there are political and economic situations in which both externalities and restricted cooperation play an important role.

In this paper we introduce the PFF games with restricted cooperation, and generalize the recursive core to this class of games. We also show that as in Kóczy (2007), the recursive core is a refinement of the α - and the ω -core pair. In addition, we relate the recursive cores of PFF games with and without restricted cooperation. At the end of the paper we present an example for a possible empirical application of our model using the merger simulation method of Nevo (2000a). Together with the merger simulation, our model can determine the stable industrial structures for real markets and predict possible mergers on those markets.

The paper is organized as follows. In Chapter 2 we overview the TU games, the baseline cooperative model and the PFF games, which are able to express the externalities between the players. We take the restricted cooperation into account in Chapter 3 where we introduce the PFF games with restricted cooperation, define the $\Lambda(N)$ -restricted recursive core and discuss some of its properties. Chapter 4 presents the example for the empirical application. Chapter 5 concludes the paper.

Chapter 2

TU games and PFF games

In this section we overview the model of TU games, which is the baseline model of cooperative games. After that we consider the games in partition function form.

2.1 TU games

First of all we are going to show the simplest, and in previous works the most frequently used cooperative game theory model, the coalitional TU game (shortly TU game, see for instance Peleg and Sudhölter (2003)).

Definition 2.1.1. *Let N be a non-empty finite set of the players, the cooperative transferable utility game (henceforth TU game) is the function $v : 2^N \rightarrow \mathbb{R}$ such that $v(\emptyset) = 0$. Moreover, let $\mathcal{G}^N$ be the class of TU games with players set N .*

TU games are cooperative games played by players in N , and players form so-called *coalitions*: that is, coalitions are subsets of N , therefore 2^N denotes the class of the possible coalitions. A TU game can be given by the coalitional function (also called characteristic function) v , where v assigns a real number – the value of the coalition – to each coalition. The fact that we allow the formation of any possible coalitions must be emphasized (in other words, any elements of 2^N can arise as a coalition).

The most frequently used solution concept of TU games is the *core* (Gillies, 1959). The *core* is a set of so-called *imputations*, which are payoff vectors, such that all players get at least as they could achieve by themselves alone; and the sum of all player's payoff is equal to the value of the grand coalition. Formally:

Definition 2.1.2. Let $v \in \mathcal{G}^N$, then the set of imputations of v is defined as

$$I(v) = \{x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N) \text{ and } x_i \geq v(\{i\}), i \in N\}$$

The core is the set of imputations, where for all coalitions the members of the given coalition get at least as much as the value of the coalition.

Definition 2.1.3. Let $v \in \mathcal{G}^N$, then the core of v is defined as

$$C(v) = \{x \in I(v) : \sum_{i \in S} x_i \geq v(S), \text{ for all } S \subseteq N\}$$

2.2 Core and domination

In the literature the core is often defined as the set of undominated outcomes, however the two approaches are not exactly the same. Now we illustrate this difference with an example. First we define dominance relation.

Definition 2.2.1. Let $x \in I(v)$. The payoff vector $y \in I(v)$ dominates x via coalition S (in notation: $x <_S y$), if

1. $y_i > x_i$ for all $i \in S$, $S \subseteq N$,
2. $\sum_{i \in S} y_i \leq v(S)$.

Example 2.2.2. Consider the following TU game with four players:

$$v(S) = \begin{cases} 2 & \text{if } |S| \leq 2 \text{ and } S \neq \{1, 2\}, \\ 5 & \text{if } S = \{1, 2\}, \\ 6 & \text{if } |S| = 3, \\ 8 & \text{otherwise.} \end{cases}$$

Then $x = (2, 2, 2, 2) \notin C(v)$, because $v(\{1, 2\}) > x_1 + x_2$, however, $x \in I(v)$, and there is no $y \in I(v)$ and coalition $T \subseteq N$ such that $y >_T x$. (The symbol $>_T$ means that $y_i > x_i$ for all $i \in T$.) That is, x is not a core element, but it cannot be blocked by any imputation.

As the example above shows, there is a difference between the two approach. For the sake of consistency we will use the approaches in Definition 2.1.3 for defining the recursive core and the $\Lambda(N)$ -restricted recursive core.

2.3 PFF games and the recursive core

Games in partition function form differ from TU games in that they can model externalities between players. So in the case of PFF games it is possible, that a certain coalition can have different depending on the realized partition (in effect, depending on the coalitional structure formed by other players).

We adopt the definition of Myerson (1977b) which uses the set of embedded coalitions (*ECL*) to define the PFF game.

Definition 2.3.1. *Let ECL_N be defined as follows:*

$$ECL_N = \{(S, \pi) : S \in \pi \in \Pi(N)\}.$$

Then, PFF game with restricted cooperation V with players set N is a point in $\mathbb{R}^{ECL_N}$, that is, V is a function from ECL_N to $\mathbb{R}$.

Let Γ^N denote the class of partition function games with player set N .

Note, that this definition differs from the definition in Kóczy (2009). The discrete partition function in Kóczy (2009) exactly determines the payoff of each player in a given partition, while the definition above only determines the payoff of each coalition in a given partition. Let see how the PFF games are able to express the externalities in contrary to the TU games.

Example 2.3.2. Let $N = \{1, 2, 3\}$ be the set of players, and for simplicity consider only the partitions $\pi_1 = (\{1\}, \{2\}, \{3\})$ and $\pi_2 = (\{1\}, \{2, 3\})$.

Look first at TU game v , $v(\{1\}) = 2$, $v(\{2\}) = 3$, $v(\{3\}) = 4$, $v(\{2, 3\}) = 8$. Since v is defined on the coalitions only, $v(\{1\}) = 2$ regardless of whether the other two players are in one coalition or not. Therefore TU games cannot express external effects.

Consider now the PFF game V . Now the realization of π_1 yields $V(\{1\}, \pi_1) = 2$, $V(\{2\}, \pi_1) = 3$, $V(\{3\}, \pi_1) = 4$, and the realization of π_2 yields $V(\{1\}, \pi_2) = 1$, $V(\{2, 3\}, \pi_2) = 8$. So in the case of PFF game the payoff of players who create a one-player coalition depend on whether player 2 and player 3 coalesce or not; so the PFF game can model externalities.

Notice, that since in TU games only the payoffs are relevant, in PFF games besides the payoffs the realized partitions are also important. Therefore in the definition below both dimensions are expressed.

Definition 2.3.3. Let $V \in \Gamma^N$ be PFF game, then the set of outcomes of PFF game V is as follows:

$$\Omega(V) = \left\{ (\pi, x) \in (\Pi(N), \mathbb{R}^N) : \forall S \in \pi : \sum_{i \in S} x_i = V(S, \pi) \right\}.$$

The definition of residual game helps to understand the recursive core. Here we define the situation of players who stay in the game.

Definition 2.3.4. Let $V \in \Gamma^N$ be a PFF game and consider a set of players $N \subset R$. Assume that $N \setminus R$ have committed to form partition $\pi_{N \setminus R} \in \Pi(N \setminus R)$. Then the residual game $V_{\pi_{N \setminus R}}$ is the PFF game over player set R , with partition function $V_{\pi_{N \setminus R}}$ such that for all $\pi_R \in \Pi(R)$, $S \in \pi_R$:

$$V_{\pi_{N \setminus R}}(S, \pi_R) = V(S, \pi_R \cup \pi_{N \setminus R}) .$$

In the following part we introduce the small (large) recursive core. Our definition differs from the one in Kóczy (2009) in several points. First, our model there are no fixed payoffs for the players in each given partition, since the payoffs are assigned to the coalitions. Second, in this model we use the same approach as in Definition 2.1.3, instead of defining the core by undominated outcomes. Finally we use the terms *small* and *large* instead of *optimistic* and *pessimistic* recursive core, because we think that these expressions would be misleading (the optimistic recursive core is smaller than the pessimistic one).

Definition 2.3.5. *Small (large) recursive core (RC).*

1. *Trivial game:* The recursive core of $V \in \Gamma^{\{i\}}$ is the only outcome with the trivial partition:

$$RC(V) = (\{i\}, V(\{i\}, \{\{i\}\})) .$$

2. *Induction assumption:* Assume that RC has been defined for all games with at most k players. Then for each PFF game $V \in \Gamma^R$, $|R| \leq k$, let

$$A(V) = \begin{cases} RC(V) & \text{if } RC(V) \neq \emptyset, \\ \Omega(V) & \text{otherwise,} \end{cases}$$

where $\Omega(V)$ is the set of all possible outcomes of V .

3. *The small (large) recursive core: The small (large) core of PFF game $V \in \Gamma^N$, $|N| = k + 1$ is as follows:*

$$RC_S(V) = \left\{ (\pi, x) \in \Omega(V) : \forall R \subseteq N, (\pi'_R, \pi'_{N \setminus R}) \in \Pi(N), (\pi'_{N \setminus R}, x'_{N \setminus R}) \in A(V_{\pi'_R}), \right. \\ \left. \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V(S, \pi'_R \cup \pi'_{N \setminus R}) \right\},$$

$$\left(RC_L(V) = \left\{ (\pi, x) \in \Omega(V) : \forall R \subseteq N, \exists \pi'_{N \setminus R} \in \Pi(N \setminus R), (\pi'_{N \setminus R}, x'_{N \setminus R}) \in A(V_{\pi'_R}), \right. \right. \\ \left. \left. \forall \pi'_R \in \Pi(R), \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V(S, \pi'_R \cup \pi'_{N \setminus R}) \right\} \right).$$

The idea behind the definition is that the recursive core consists of outcomes that are stable in some sense. An outcome is stable if there are not any set of players which can get higher payoffs by deviating from the outcome. (Note that in our model the payoffs are assigned to coalitions not to players, however if a coalition gets higher payoff, then its members can divide the surplus in a way that each of them gets more than before.) Since there are externalities between the players, when some players want to deviate from a given partition, they have to take into account the possible reaction of the others.

When reacting to the action of the deviating players the residuals face a residual game according to the Definition 2.3.4. In fact this residual game is also a PFF game with a smaller number of players. These residual players will form a stable partition in their own residual game. This stable outcome will be the recursive core of this residual game (or any outcome, if the residual core is empty). Here lies the recursion in the definition: to check whether an outcome is in the recursive core or not, we have to consider the possible deviations. For a given deviation we have to consider the reaction of the residual players which also will be an element of the recursive core of the residual game. To check whether an outcome of the residual game is stable, we have to check the possible deviations, and so on. Therefore the definition above starts with defining the recursive core of the *trivial game* (one player game), and builds from this.

Now we discuss the difference between the small and large recursive core. The small recursive core consists the outcomes that are extremely stable in a sense that for any deviations, *whatever credible outcome of the residual game* realizes (credible outcomes

are elements of $A(V)$), the deviators cannot be better off. Contrarily, the large recursive core is less restrictive. An outcome is an element of the large recursive core, if for any deviations there *exists a credible outcome of the residual game* for which the deviators cannot be better off.

Chapter 3

Restricted cooperation

In this section we introduce the notion of restricted cooperation: we relax the assumption that every subset of 2^N can form as a coalition. First we define our concept of PFF games with restricted cooperation then generalize the recursive core to this class of games and we discuss the properties of the newly defined $\Lambda(N)$ -restricted recursive core.

3.1 PFF games with restricted cooperation

As we mentioned in the introduction, the concept of restricted cooperation in TU games has a developed literature. However, we have to use a new concept of restricted cooperation for our purposes. For example, consider the situation when there are three players, and we allow all coalitions to form except the grand coalition. Myerson (1977a)'s graph structures cannot describe such a situation: each player should be connected by the graph since we permit all possible two-member coalitions, but if all players are connected by the graph then the grand coalition also must be permitted. In the case described above the restriction function of Derks and Peters (1993) is also inappropriate for our purposes since we cannot assign a definite subcoalition to the not permitted grand coalition.

In addition, our set of permitted coalitions must depend on the given partition. Let us give an IO example. Suppose there are four firms, $N = \{A, B, C, D\}$. The competition authority permits the merger of firms A and B if firms C and D do not merge, and also permits the merger of firms C and D if firms A and B do not merge. The two mergers jointly are prohibited. In this case the coalition $S = \{A, B\}$ is permitted if C and D are not in one coalition and is not permitted otherwise. It is clear that the different definitions

of restricted cooperation in TU games are not able to model these kind of permission structures; in TU games when we look at a given coalition then does not matter which coalitions the other players form.

Therefore, in our definition of restricted cooperation in PFF games we restrict the set of *partitions*, not the set of coalitions. It is a more general method to impose restrictions on the coalitional structure (for example, listing permitted partitions with coalitions that can be described with a graph structure discussed above, we exactly get the concept of Myerson (1977a)). Our definition is the following:

Definition 3.1.1. *Let $\Lambda(N) \subseteq \Pi(N)$ denote the set of all permitted partitions, where $\Lambda(N) \neq \emptyset$. We call $\Lambda(N)$ as a restriction of $\Pi(N)$.*

Given our concept of restricted cooperation, we generalize the Definition 2.3.1 and define the $\Lambda(N)$ -restricted PFF games.

Definition 3.1.2. *Let $ECL_{\Lambda(N)}$ be defined as follows:*

$$ECL_{\Lambda(N)} = \{(S, \lambda) : S \in \lambda \in \Lambda(N)\}.$$

Then, PFF game with restricted cooperation $V^{\Lambda}(N)$ with player set N and set of permitted partitions $\Lambda(N)$ is a point in $\mathbb{R}^{ECL_{\Lambda(N)}}$, that is, $V^{\Lambda}(N)$ is a function from $ECL_{\Lambda(N)}$ to $\mathbb{R}$.

Let $\Gamma^{\Lambda(N)}$ denote the class of partition function games with player set N and set of permitted partitions $\Lambda(N)$ (sometimes we also refer these games as $\Lambda(N)$ -restricted PFF games hereafter).

It is easy to see that if we choose $\Lambda(N) = \Pi(N)$, then we get exactly our definition of PFF games, so Definition 3.1.2 is indeed a generalization. In the rest of the paper if it does not make confusion we simply refer to $V^{\Lambda}(N)$ as V^{Λ} without mentioning the player set.

Similarly to the PFF games, the outcomes of the $\Lambda(N)$ -restricted PFF games are also pairs consisting of a partition and a payoff vector with feasible payoffs.

Definition 3.1.3. *Let $V^{\Lambda(N)} \in \Gamma^{\Lambda(N)}$ be PFF game with restricted cooperation, then the set of outcomes of PFF game with restricted cooperation $V^{\Lambda}(N)$ is as follows:*

$$\Omega(V^A(N)) = \left\{ (\lambda, x) \in (\Lambda(N), \mathbb{R}^N) : \forall S \in \lambda : \sum_{i \in S} x_i = V(S, \lambda) \right\}.$$

3.2 The $\Lambda(N)$ -restricted recursive core

In the previous section we introduced the $\Lambda(N)$ -restricted PFF games, and in this section we define the solution concept $\Lambda(N)$ -restricted recursive core for this class of games. As in the previous chapter, first we define the residual game which is crucial in the definition of the recursive core.

Definition 3.2.1. *Let $V^A \in \Gamma^{\Lambda(N)}$ be a PFF game with $\Lambda(N)$ -restricted cooperation and consider coalition $R \subset N$ such that there exists $\lambda \in \Lambda(N)$, $\lambda = \pi_R \cup \pi_{N \setminus R}$, where $\pi_R \in \Pi(R)$ and $\pi_{N \setminus R} \in \Pi(N \setminus R)$. Assume that $N \setminus R$ have committed to form partition $\pi_{N \setminus R} \in \Pi(N \setminus R)$ such that there exists $\lambda \in \Lambda(N)$, $\pi_{N \setminus R} \subseteq \lambda$. Then the residual game $V_{\pi_{N \setminus R}}^A(R)$ is the PFF game with $\Lambda(R)$ -restricted cooperation over player set R such that for all $\pi_R \in \Lambda(R)$, $S \in \pi_R$:*

$$V_{\pi_{N \setminus R}}^A(R)(S, \pi_R) = V^A(N)(S, \pi_R \cup \pi_{N \setminus R}),$$

where $\Lambda(R) = \{\pi \in \Pi(R) : \exists \lambda \in \Lambda(N), \pi_R \cup \pi_{N \setminus R} = \lambda\}$.

This definition is a modified version of Definition 2.3.4. It is important here that the set of the "leaving" players ($N \setminus R$) is not an arbitrary set of players, but there exists a permitted partition consisting of partitions of R and $N \setminus R$. In addition, the partition formed by the leaving players must be a subset of a partition of $\Lambda(N)$. Furthermore, the partition of the residual players (R) must be such a partition that is together with the partition of the leaving players constitute a permitted partition. We need these restrictions to ensure that all of the resulting coalition structures are in the set of permitted coalitions. Again, it is clear that choosing $\Pi(N)$ as $\Lambda(N)$ we get exactly Definition 2.3.4.

Now we are ready to define the small and the large $\Lambda(N)$ -restricted recursive cores. The setup of the following definition is similar to Definition 2.3.5, but in the case of restricted cooperation we cannot start from the one player games as trivial games, since it is possible that not all single-player coalitions are permitted. The definition of residual core (Definition 2.3.5) is based on the recursion on the number of the players, but in

the restricted cooperation case we cannot do so. The key of the definition below is that instead of the number of players, the recursion is on the number of permitted coalitions. Now even a game with three players can be a trivial game if there is only one permitted partition of the three players. It is important to emphasize that as in Definition 3.2.1, all partitions in the definition below are permitted partitions.

Definition 3.2.2. *The small (large) $\Lambda(N)$ -restricted recursive core (RC^Λ).*

1. *Trivial game: The $\Lambda(N)$ -restricted recursive core of $V_{\pi_{N \setminus R}}^\Lambda(R)$ such that $|\Lambda(R)| = 1$ is the only outcome with the trivial partition and with any x satisfying*

$$\sum_{i \in S} x_i = V(S, \lambda), \quad S \in \lambda \in \Lambda(R) .$$

2. *Induction assumption: Assume that RC^Λ has been defined for all games with number of allowed partitions at most k . Then for each PFF game $V_{\pi_{N \setminus R}}^\Lambda(R)$, $|\Lambda(R)| \leq k$, let*

$$A^\Lambda(V_{\pi_{N \setminus R}}^\Lambda(R)) = \begin{cases} RC^\Lambda(V_{\pi_{N \setminus R}}^\Lambda(R)), & \text{if } RC^\Lambda(V_{\pi_{N \setminus R}}^\Lambda(R)) \neq \emptyset \\ \Omega(V_{\pi_{N \setminus R}}^\Lambda(R)) & \text{otherwise} \end{cases} ,$$

3. *The small (large) recursive core: The small (large) $\Lambda(N)$ -restricted recursive core of game $V^{\Lambda(N)} \in \Gamma^{\Lambda(N)}$, $|\Lambda(N)| = k + 1$ is as follows:*

$$RC_S^\Lambda(V^\Lambda) = \left\{ (\lambda, x) \in \Omega(V^\Lambda) : \forall R \subseteq N, \forall (\pi'_R, \pi'_{N \setminus R}) \in \Lambda(N), \right. \\ \left. (\pi'_{N \setminus R}, x'_{N \setminus R}) \in A(V_{\pi'_R}^\Lambda(N \setminus R)), \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V^\Lambda(S, \pi'_R \cup \pi'_{N \setminus R}) \right\}, \quad (3.1)$$

$$\left(RC_L^\Lambda(V^\Lambda) = \left\{ (\lambda, x) \in \Omega(V^\Lambda) : \forall R \subseteq N, \exists \pi'_{N \setminus R} \in \Lambda(N \setminus R), \right. \right. \\ \left. \left. (\pi'_{N \setminus R}, x'_{N \setminus R}) \in A(V_{\pi'_R}^\Lambda(N \setminus R)), \forall \pi'_R \in \Pi(R), \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V^\Lambda(S, \pi'_R \cup \pi'_{N \setminus R}) \right\} \right). \quad (3.2)$$

Again, if we permit all partitions – $\Pi(N) = \Lambda(N)$ – we get Definition 2.3.5. If all partitions are permitted, then the games with $|\Lambda(R)| = 1$ are exactly the games with $|R| = 1$. The recursive structure and the concepts of small and large recursive core are the same as in Definition 2.3.5.

3.3 Properties of the $\Lambda(N)$ -restricted recursive core

In this subsection we discuss some of the properties of the $\Lambda(N)$ -restricted recursive core. First we show that the pair of the small and the large $\Lambda(N)$ -restricted recursive core is between the $\Lambda(N)$ -restricted α - and ω -cores. Then introducing the notion of the restriction of a PFF game, we relate the $\Lambda(N)$ -restricted recursive cores to the recursive cores.

One of the results of Kóczy (2007) is that the pair of the recursive cores is a refinement of the α - and the ω -core pair, i. e. $C_\omega(V) \subseteq RC_S(V) \subseteq RC_L(V) \subseteq C_\alpha(V)$. Here we argue that it is also true for the $\Lambda(N)$ -restricted recursive core. To show this, first we define the $\Lambda(N)$ -restricted equivalents of the α - and ω -cores. From now fix the set of permitted coalitions $\Lambda(N)$.

The α -core (Aumann and Peleg, 1960) is stable against the deviations where the residuals hurt the deviating players as much as they can. In other words, an outcome is an element of the α -core if for any set of deviating players, for any deviations, there exists a reaction of the residual players when the deviators are worse off. Observe that the existence of only one outcome like that is enough since the residuals will form the partition which hurts the deviators the most. In the $\Lambda(N)$ -restricted case it means the following:

Definition 3.3.1. *Let $V^A \in \Gamma^{\Lambda(N)}$ be a PFF game with restricted cooperation. The $\Lambda(N)$ -restricted α -core is as follows:*

$$C_\alpha^A = \left\{ (\lambda, x) \in \Omega(V^A) : \forall R \subseteq N \exists \pi'_{N \setminus R} \in \Lambda(N \setminus R), \right. \\ \left. (\pi'_{N \setminus R}, x'_{N \setminus R}) \in \Omega(V_{\pi'_R}^A(N \setminus R)), \forall \pi'_R \in \Pi(R), \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V^A(S, \pi'_R \cup \pi'_{N \setminus R}) \right\} \quad (3.3)$$

One can see that the difference between C_α^A and $RC_L^A(V^A)$ is that in the case of C_α^A the outcome realized by the residual players is an element of $\Omega(V_{\pi'_R}^A(N \setminus R))$, instead of $A(V_{\pi'_R}^A(N \setminus R))$ as it was in the case of the $RC_L^A(V^A)$.

In contrast to the α -core, the ω -core (Shenoy, 1979) contains outcomes that are stable against all possible deviations given that the residuals will help the deviating players as much as possible (that is, the ω -core contains extremely stable outcomes). We also give the definition of $\Lambda(N)$ -restricted ω -core:

Definition 3.3.2. Let $V^A \in \Gamma^{A(N)}$ be a PFF game with restricted cooperation. The $A(N)$ -restricted ω -core is as follows:

$$C_\omega^A = \left\{ (\lambda, x) \in \Omega(V^A) : \forall R \subseteq N, \forall (\pi'_R, \pi'_{N \setminus R}) \in A(N), \right. \\ \left. (\pi'_{N \setminus R}, x'_{N \setminus R}) \in \Omega(V_{\pi'_R}^A(N \setminus R)), \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V^A(S, \pi'_R \cup \pi'_{N \setminus R}) \right\} \quad (3.4)$$

The difference between C_ω^A and $RC_S^A(V^A)$ is the same as it was between C_α^A and $RC_L^A(V^A)$ above. From Definitions 3.3.1 and 3.3.2 can be seen that $C_\omega^A \subseteq C_\alpha^A$.

When we relate the $A(N)$ -restricted recursive cores to the $A(N)$ -restricted α - and ω -cores we also use the following corollary that immediately follows from Definition 3.2.2.

Corollary 3.3.3. $RC_S^A(V^A) \subseteq RC_L^A(V^A)$.

Now we can relate these solution concepts to each other. The result generalizes Corollary 7 of Kóczy (2007) to the class of PFF games with restricted cooperation.

Proposition 3.3.4. Let $V^A \in \Gamma^{A(N)}$ be a PFF game with restricted cooperation. Then we have

$$C_\omega^A(V^A) \subseteq RC_S^A(V^A) \subseteq RC_L^A(V^A) \subseteq C_\alpha^A(V^A).$$

Proof. We make the proof in two parts, first we prove that $C_\omega^A(V^A) \subseteq RC_S^A(V^A)$. Assume that outcome $(\lambda, x) \in \Omega(V^A)$ satisfies (3.4), hence $(\lambda, x) \in C_\omega^A(V^A)$. Since $A(V^A) \subseteq \Omega(V^A)$, then (λ, x) also satisfies (3.1), which means that $(\lambda, x) \in RC_S^A(V^A)$.

Now we show that $RC_L^A(V^A) \subseteq C_\alpha^A(V^A)$. Assume that outcome $(\lambda, x) \in \Omega(V^A)$ satisfies (3.2), hence $(\lambda, x) \in RC_L^A(V^A)$. Similarly as above, since $A(V^A) \subseteq \Omega(V^A)$, and we even had a $(\pi'_{N \setminus R}, x'_{N \setminus R}) \in A(V_{\pi'_R}^A(N \setminus R))$, then we also have such an outcome in $\Omega(V_{\pi'_R}^A(N \setminus R))$. From that we know that (λ, x) also satisfies (3.3), therefore $(\lambda, x) \in C_\alpha^A(V^A)$. $\square$

Now we introduce the notion of the restriction of a PFF game in order to make implications about a recursive core and the $A(N)$ -restricted recursive core of two somewhat related games. Consider a player set N . In a PFF game with the player set N the function V is defined on each embedded coalition and assigns a value each of them. In the $A(N)$ -restricted PFF game with player set N we have a coarser set of embedded coalitions on which the function V^A is defined. However, V is also defined on each embedded coalition on which V^A is defined. Taking this fact into account, we can define the *restriction* of a PFF game.

Definition 3.3.5. Let be N the player set, $\Lambda(N)$ is the set of permitted partitions, V is a PFF game and V^A is a $\Lambda(N)$ -restricted PFF game with players set N . Let also be $S \in \lambda\Lambda(N)$. We say that V^A is a restriction of V if

$$V^A(S, \lambda) = V(S, \lambda).$$

So if V^A is a restriction of V , then every outcome of V^A is also an outcome of V . Given this, it does make sense to compare the recursive core of V with the $\Lambda(N)$ -restricted recursive core of V^A . We formulate and prove the proposition below for the small recursive core (and small $\Lambda(N)$ -restricted recursive core) only, an analogous formulation and proof arises for the large objects.

Proposition 3.3.6. Let be N the player set, $\Lambda(N)$ is the set of permitted partitions, V is a PFF game and V^A is a restriction of V . Then, neither $RC_S(V) \subseteq RC_S^A(V^A)$ or $RC_S^A(V^A) \subseteq RC_S(V)$ is true.

Proof. We prove the proposition by counterexamples. Let the player set be $N = \{1, 2, 3\}$.

Denote the elements of $\Pi(N)$ as follows:

$$\begin{aligned}\pi_1 &= \left\{ \{1\}, \{2\}, \{3\} \right\}, \\ \pi_2 &= \left\{ \{1, 2\}, \{3\} \right\}, \\ \pi_3 &= \left\{ \{1, 3\}, \{2\} \right\}, \\ \pi_4 &= \left\{ \{2, 3\}, \{1\} \right\}, \\ \pi_5 &= \left\{ \{1, 2, 3\} \right\}.\end{aligned}$$

1.

Consider the following PFF game V_1 :

- $V_1(S, \pi_1) = 2$ for all $S \in \pi_1$,
- $V_1(S, \pi_2) = V_1(S, \pi_3) = V_1(S, \pi_4) = \begin{cases} 1 & \text{if } |S| = 1, \\ 6 & \text{otherwise.} \end{cases}$,
- $V_1(S, \pi_5) = 18$ for all $S \in \pi_5$.

Consider the outcome $(\pi_5, (6, 6, 6))$. No one-player coalition can improve its payoff by deviating, since deviation yields a payoff of 1 instead of the original 6 (the payoff 1 is because the remaining two players will form a coalition in their residual game ensuring

themselves a total payoff of 6 and they be better off with the 2–2 payoff in π_1). Neither any two-player coalition can improve its payoff by deviating, since deviation yields a payoff of 6 to the given coalition instead of the initial 12. So the small recursive core of V_1 is nonempty.

Take V_1^A , the restriction of V_1 with $\Lambda(N) = \{\pi_1, \pi_2, \pi_3, \pi_4\}$. The $\Lambda(N)$ -restricted small recursive core of V_1^A is empty. The only possible outcome with partition π_1 cannot be in $RC_S^A(V^A)$, since any two player coalition can improve its total payoff by forming a two-player coalition. Consider now the partitions π_2 , π_3 and π_4 . In any outcome with one of these partitions (let us say π_1), Player 3 has a payoff of 1. The other two players have 6 payoff, so at least one of them (let us say, Player 2) is getting $x \leq 3$. Then, this outcome is not in $RC_S^A(V_1^A)$, since Player 3 can form a coalition with Player 2 and change the partition of players to π_4 . Player 1 can offer a payoff of $3 + \varepsilon$ to Player 2, while she have $3 - \varepsilon$ which is still greater than her original 1 ($0 < \varepsilon < 1$). But given this outcome, now Player 1 can make the same offer to Player 3 and force her to deviate and so on. So the $\Lambda(N)$ -restricted small recursive core of V_1^A is indeed empty. We proved that $RC_S(V) \subseteq RC_S^A(V^A)$ is not true.

2.

Consider the following PFF game V_2 :

- $V_2(S, \pi_1) = 2$ for all $S \in \pi_1$,
- $V_2(S, \pi_2) = V_2(S, \pi_3) = V_2(S, \pi_4) = \begin{cases} 1 & \text{if } |S| = 1, \\ 6 & \text{otherwise.} \end{cases}$,
- $V_2(S, \pi_5) = 6.5$ for all $S \in \pi_5$.

Now $RC_S(V_2)$ is empty. Any outcome with partitions $\pi_1, \pi_2, \pi_3, \pi_4$ cannot be in $RC_S(V)$ because of the arguments stated above. Any outcome with partition π_5 also cannot be stable since either a single player deviates if her payoff is below 1, or a two-player coalition deviates if their payoff is below 6, and since $6 + 1 < 6.5$ at least one of these two cases will always happen, $RC_S(V_2)$ is empty.

Take now V_2^A , the restriction of V_2 with $\Lambda(N) = \{\pi_1\}$. Now the game is a trivial one, and its $\Lambda(N)$ -recursive core contains the outcome $(\pi_1, (2, 2, 2))$. In this case we see that $RC_S^A(V^A) \subseteq RC_S(V)$ is not true.

Putting together the two statements above we proved the proposition. $\square$

Chapter 4

Guidelines for an empirical application

In this section we show a possible empirical application of our model introduced above. The area of application is Industrial Organizations, particularly the prediction of possible mergers. The method is presented here relies on the merger simulation method of Nevo (2000a). The consideration of other methods of merger simulation is out of this paper's focus, refer to Budzinski and Ruhmer (2009) for a survey of the alternative models.

Using this method presented below, with the proper data, one can calculate the profits of the firms before and after any mergers. These profits are the payoffs of the coalitions in question. Using the results of the estimated econometric model and the observations about the past merger control decisions in the relevant economy, one can obtain the set of permitted partitions. The permitted partitions are firm ownership structures that can be achieved by approved mergers. Given the set of permitted partitions and the payoff for each embedded coalitions, one can give the $\Lambda(N)$ -restricted recursive core of the resulting PFF game with restricted cooperation. In this particular case the partition realized in the outcomes of the recursive core are interpreted as stable industry structures. Knowing what structures are stable, if we observe that the actual industry structure is unstable we can predict the future mergers in the given industry.

In Section 4.1 we show how to determine the payoffs for the embedded coalitions. Section 4.2 is about how to construct the set of permitted partitions. In Section 4.3 we provide an example of the search for the $\Lambda(N)$ -restricted recursive core in a given game.

4.1 Determining the payoffs of coalitions

Now we introduce the merger simulation method described in Nevo (2000a). We give here an overview of the paper, and do not go much into the details, the interested reader should consult with Nevo (2000a). The framework assumes a realistic market structure with relatively high concentration and differentiated products.

The first task during a merger simulation is to estimate the demands produced by the competing firms. Nevo (2000a) uses a random coefficient logit specification for the demand estimations. This framework is very flexible since it allows to assume a heterogeneity in consumer preferences for the different product characteristics and also heterogeneous sensitivity to price. The firms compete according to the Bertrand-Nash model. For further details about the model he uses, one can also see Berry et al (1995) and Nevo (2000b).

We have $t = 1, \dots, T$ markets, $i = 1, \dots, I$ consumers and $j = 1, \dots, J$ products. The indirect utility of consumer i from product j at market t is given by the following equation:

$$u_{ijt} = x_{jt}\beta_i^* + \alpha_i^*p_{jt} + \xi_{jt} + \varepsilon_{ijt} \doteq V_{ijt} + \varepsilon_{ijt}, \quad (4.1)$$

where x_{jt} is a vector of observed product characteristics, p_{jt} is the price of good j at market t . The coefficients β_i^* and α_i^* are the consumers' taste parameters for the product characteristics and the price. These coefficients are conditional of the observed demographic characteristics of the consumers. ξ_{jt} is a market-specific product characteristic which is observed by the consumers but not observed by the econometrician, and ε_{ijt} is the error term. The demands for a given product are calculated using a discrete-choice model based on the utility function given by (4.1), and using the demands we can calculate the market share (s_j) for each good.

On the demand side there are F firms producing the set of products $\mathcal{F}_f$. The firms' profit maximization problem is given by

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) I s_j(p) - C_f, \quad (4.2)$$

where mc_j is the constant marginal cost of product j and C_f is firm f 's fixed cost of production. Now assuming Bertrand-Nash equilibrium and strictly positive prices, the price-cost markups can be calculated as follows:

$$p - mc = \Psi^{pre}(p)^{-1} s(p). \quad (4.3)$$

The estimated marginal costs can be recovered using (4.3):

$$mc = p - \Psi^{pre}(p)^{-1}s(p). \quad (4.4)$$

$\Psi^{pre}(p)$ refers to the pre-merger ownership structure and given by

$$\Psi_{jr}^{pre}(p) = \begin{cases} -\partial s_j(p)/\partial p_r, & \text{if } \exists f : \{r, j\} \subset \mathcal{F}_f, \\ 0 & \text{otherwise.} \end{cases} \quad (4.5)$$

Now if we take the matrix Ψ^{post} defined in the same way as (4.5) but given by the post-merger ownership structure, we can obtain the post-merger price p^* which satisfies

$$p^* = mc + \Psi^{post}(p^*)^{-1}s(p^*). \quad (4.6)$$

There are three implicit assumptions in (4.6): the Bertrand-Nash model of firm conduct; the same pre- and post-merger marginal costs and that Ψ^{pre} and Ψ^{post} use the same demand estimates with the different ownership structures. Nevo (2000a) discusses the validity of these assumptions.

With the proper Ψ^{post} we can simulate the post-merger prices for every possible ownership structure, and if we have the prices from (4.6), then using the estimated marginal costs from (4.4), we can substitute these values into (4.2) and can calculate the profits for the firms for all ownership structure. In the terms of PFF games, the (possibly merged) firms are the coalitions and the ownership structures are the partitions, and the profits are the values of the coalitions. There are obviously externalities between the players (one firm's payoff depend on whether the other firms are merged or not), so the PFF games are coherent models of this situations.

4.2 Determining the set of permitted partitions

We showed how to calculate the profits for the firms for all possible ownership structure, however obviously it is not the case that all possible ownership structure can realize due to the rulings of the competition authorities. In game theoretic terms, the players cannot form all possible partitions, so we have a game (in addition, a PFF game as we seen above) with restricted cooperation.

Competition authorities do not allow mergers when it is possible that the post-merger market will be significantly worse for the consumers than the pre-merger market was.

The structural merger simulation framework of Nevo (2000a) described above (using the estimated demand functions) is also able to analyze the change in the consumer welfare in each simulated post-merger equilibrium. The compensating variation¹ for consumer i is given by

$$CV_i = \frac{\ln \left[\sum_{j=0}^J V_{ij}^{post} \right] - \ln \left[\sum_{j=0}^J V_{ij}^{pre} \right]}{\alpha_i^*}, \quad (4.7)$$

where V_{ij}^{pre} and V_{ij}^{post} are the pre- and post-merger utilities for consumer i given by (4.1). Summing up the compensating variations we can obtain the total consumer welfare change.

The tolerated level of the negative welfare change can vary across countries, see for example Bergman et al (2011) for a comparison of merger control decisions in the European Union and the United States. Determining the cut-off value for the approved mergers is out of the focus of this paper, but in an empirical application one can figure out what mergers would be approved by the given competition authority and what mergers would not. The ownership structures reached via approved mergers constitute the set of permitted partitions in the analyzed game.

4.3 Solution of the game

Now we show an example how to find the $\Lambda(N)$ -recursive core of a PFF game with restricted cooperation. Assume that we analyzed a market with 4 major firms (A, B, C, D) with the method described above, and we found out that the permitted ownership structures are the following:

$$\begin{aligned} \lambda_1 &= \left\{ \{A\}, \{B\}, \{C\}, \{D\} \right\}, \\ \lambda_2 &= \left\{ \{A, D\}, \{B\}, \{C\} \right\}, \\ \lambda_3 &= \left\{ \{B, D\}, \{A\}, \{C\} \right\}, \\ \lambda_4 &= \left\{ \{C, D\}, \{A\}, \{B\} \right\}, \\ \lambda_5 &= \left\{ \{B, C, D\}, \{A\} \right\}. \end{aligned}$$

¹The compensating variation is a welfare evaluation tool for economic changes, see for example Mas-Colell et al (1995). It is the amount that must be paid to the consumer after the price change to granting her original utility level.

The values of the different coalitions conditional to the realized partitions are given by the table below.

	λ_1	λ_2	λ_3	λ_4	λ_5
$\{A\}$	8	—	7	7	5
$\{B\}$	4	3	—	3	—
$\{C\}$	3	2	2	—	—
$\{D\}$	1	—	—	—	—
$\{A,D\}$	—	11	—	—	—
$\{B,D\}$	—	—	6	—	—
$\{C,D\}$	—	—	—	5	—
$\{B,C,D\}$	—	—	—	—	9.5

To find the recursive core, we will check each permitted partition whether it satisfies the conditions of Definition 3.2.2 or not. In this example there will be no difference between the small and large recursive cores since $A^A(V_{\pi_{N \setminus R}}^A(R))$ will always have only one element.

Consider λ_1 first. In this case A and D forming $\{A, D\}$ can achieve a total payoff of 10 instead of their original $1+8=9$, (B and C face a trivial residual game as the reaction to this deviation), so λ_1 is not in the recursive core.

For λ_3 , players B , C and D forming $\{B, C, D\}$ can earn a total payoff of 9.5 instead of their original $6+2=8$ (A face a trivial game as reaction to this deviation), so λ_3 is not in the recursive core.

For λ_4 , players B , C and D forming $\{B, C, D\}$ can earn a total payoff of 9.5 instead of their original $5+3=8$ (A face a trivial game as reaction to this deviation), so λ_4 is not in the recursive core.

Consider now λ_5 . B must have at least a payoff of 3 and C must get at least 2 from the total 9.5 of the coalition of B , C , and D , because if they don't, then they will deviate forming one-player coalitions. In this case they can ensure themselves the payoffs 3 and 2 respectively, whatever is the reaction of the other players. So the maximum payoff that D can get from the total 9.5 of the coalition of B , C , and D is 4.5. Given this, players A and D forming $\{A, D\}$ can achieve a total payoff of 10 instead of their original $5+4.5=9.5$, (B and C face a trivial residual game as the reaction), so λ_5 is not in the recursive core.

Finally consider λ_2 . Players B and C cannot deviate from this partition (neither jointly or one-by-one). The only possible deviation of A and D is to form one-player coalitions. If

D form an one-player coalition, then she will get a payoff of 1 and player A will get a payoff of 8 (the residuals face a trivial game). So it is not worth to D forming the one-player coalition since she and A can get more. If player A forms a one-player coalition, then the residual players form $\{B, C, D\}$ (because they are better off than they can be with any other partition, see the reasons above), and A will get a payoff of 5 and D will get at most a payoff of 4.5. So first, forming one-player coalitions are not worth for any players of the coalition $\{A, D\}$. Second, as we seen above, if player D decides that she deviates together with B and C , then A will get only a payoff of 5. Knowing this, A is willing to offer any y to D from the total 10 payoff of their coalition as long as $y < 5$. All remaining possible deviations involve D (A do not want to deviate in itself, B and C cannot deviate), but no possible deviation can give her more payoff than 4.5. If she commits to form coalitions $\{B, D\}$, $\{C, D\}$ or $\{B, C, D\}$ given that B and C will always demand at least 3 and 2 respectively, the maximum payoff D can achieve is 3 (for $\{B, D\}$ and $\{C, D\}$) or 4.5 (for $\{B, C, D\}$). So if the payoff she receives from the total 10 of $\{A, D\}$ is greater than 4.5, D will never deviate neither alone, neither with other players. We can conclude that the partition λ_2 is stable with the proper payoff vectors.

The $A(N)$ recursive core of the game described above is (x, λ_2) , where $x = (5.5 - \varepsilon, 3, 2, 4.5 + \varepsilon)$ such that $0 < \varepsilon < 0.5$.

Chapter 5

Conclusion

In our paper we introduced the concept of games in partition function form with restricted cooperation. After defining the recursive core using Myerson (1977b)'s definition of PFF games, we developed a framework to deal with both externalities and restricted cooperation. Since in the PFF environment the well-being of one group of players may depend on what actions the other group of the players take, instead of restricting the set of possible coalitions (as the literature for TU games does), we restricted the set of partitions.

We generalized the concept of recursive core to this new class of games, and defining the $\Lambda(N)$ -restricted versions of α - and ω -core, we proved (for fixed restriction) the same relationship between these solution concepts (i. e. α -, ω -, and recursive core) as Kóczy (2007) did. We also showed that if we consider a restriction of a PFF game there is no containing relation in neither directions between the recursive cores of the original and the restricted PFF game.

Regarding the suggestions for further research, up to the best of our knowledge, the phenomenon of restricted cooperation to noncooperative coalition formation games is still not analyzed yet, together with the links between the cooperative and noncooperative approach in this environment. Another interesting open question is that what are the conditions for the nonemptiness of the $\Lambda(N)$ -restricted recursive core.

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