



Simulation of Fluid Pound Condition

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Nomenclature

A	-	cross section of the rod [m^2]
c	-	damping coefficient [$1/\text{s}$]
CLF	-	cyclic load factor [-]
E	-	Young's modulus of elasticity for the rod material [Pa]
F_p	-	force acting on the plunger [N]
F_{fdown}	-	friction of the plunger during downstroke [N]
F_{fup}	-	friction of the plunger during upstroke [N]
F_o	-	fluid load [N]
h	-	parameter in the kinematics of the pumping unit
I_2	-	moment of inertia of the rod [m^4]
k_T	-	spring constant of the tubing [N/m]
l	-	length of the rod section under compression [m]
L_L	-	height of the liquid level above the standing valve [m]
L_{PL}	-	length of the plunger [m]
L_{SD}	-	setting depth [m]
L_{SR}	-	stretched length of the rod string [m]
L_{Total}	-	length of the rod string [m]
MPRL	-	minimum polished rod load [N]
P_{hydr}	-	required power at the pump [W]
PPRL	-	peak polished rod load [N]
PRHP	-	polished rod horsepower [W]
RP_L	-	relative position of the liquid level [m]
s	-	position of the polished rod [m]
SF	-	service factor [-]
t	-	time [s]
T_{CB}	-	counterbalance torque [Nm]
TF	-	torque factor [-]
T_{Net}	-	net torque [Nm]
T_{RL}	-	torque caused by the rod load [Nm]
u	-	displacement [m]
u_L	-	displacement of the plunger at the last time step [m]
u_{LD}	-	last displacement of the plunger during downstroke [m]

u_{LLD}	-	last displacement of the plunger during downstroke with fluid load pumping phase [m]
u_{LU}	-	last displacement of the plunger during upstroke [m]
u_P	-	displacement of the plunger [m]
v	-	dimensionless damping factor [-]
w	-	specific weight in air [kg/m]
W_{pump}	-	work done by the pump [J]
v_s	-	sound velocity in the rod material [m/s]
x	-	distance [m]
α	-	parameter in the wave equation
β	-	parameter in the wave equation and in the kinematics of the pumping unit(they have different meaning)
γ	-	parameter in the wave equation
η_{lift}	-	lifting efficiency
σ_b	-	minimum tensile strength of the rod [Pa]
σ_{max}	-	maximum rod stress [Pa]
σ_{min}	-	minimum tensile strength of the rod [Pa]
ϕ	-	parameter in the kinematics of the pumping unit
ψ	-	parameter in the kinematics of the pumping unit
ψ_b	-	parameter in the kinematics of the pumping unit
θ	-	crank angle [degree/radian]

1. Introduction

Sucker rod pumping is the first and the most widely used artificial lifting method in the world. The sucker rod pumping system uses a positive-displacement plunger pump. Among the many advantages of this kind of artificial lifting method, the most important is that it can be used until the end of a well's productive life, when the reservoir pressure is very low.

If the reservoir pressure is extremely low and the inflow is limited, the pump barrel doesn't completely fill with fluid. This pumping condition is the so called "fluid pound". Many sucker rod pumping systems producing in this condition all around the world in matured oil fields.

Usually these pumps operate intermittently. When the liquid fillage in the pump drops below a given level the pumps stops, and after a specified time it starts to operate again. There are some predictive calculation methods for the optimization of the pumping system, that operate under normal conditions, but there are none in case of intermittent pumping. The main reason for this is that there aren't any adequate mathematical models for it.

My aim with this paper is to give a detailed mathematical model for the calculation of operating parameters in case of fluid pumping condition. This includes the mathematical model, the numerical solution, the evaluation, and the effect on the operation conditions.

2. Introduction to Sucker Rod Pumping

When an oil reservoir starts to produce, the pressure of the reservoir is high enough for the oil to flow naturally to the surface through the wells. As the reservoir depletes, the pressure decreases at the well bottom, and after a given level it won't be sufficient to overcome the sum of pressure losses occurring along the flow path to the separator, and the well dies.

After a well dies there is a need for an artificial lifting system. As already mentioned the most popular method is sucker rod pumping. The main advantages of this method are: it is well-known, simple to operate, easy to design and it can operate under a wide variety of operational conditions, with acceptable efficiency.

2.1. Components of a Sucker Rod Pumping System

The main components of a sucker rod pumping system are shown on Figure 2.1. The components can be divided into two major group: surface and downhole equipment.

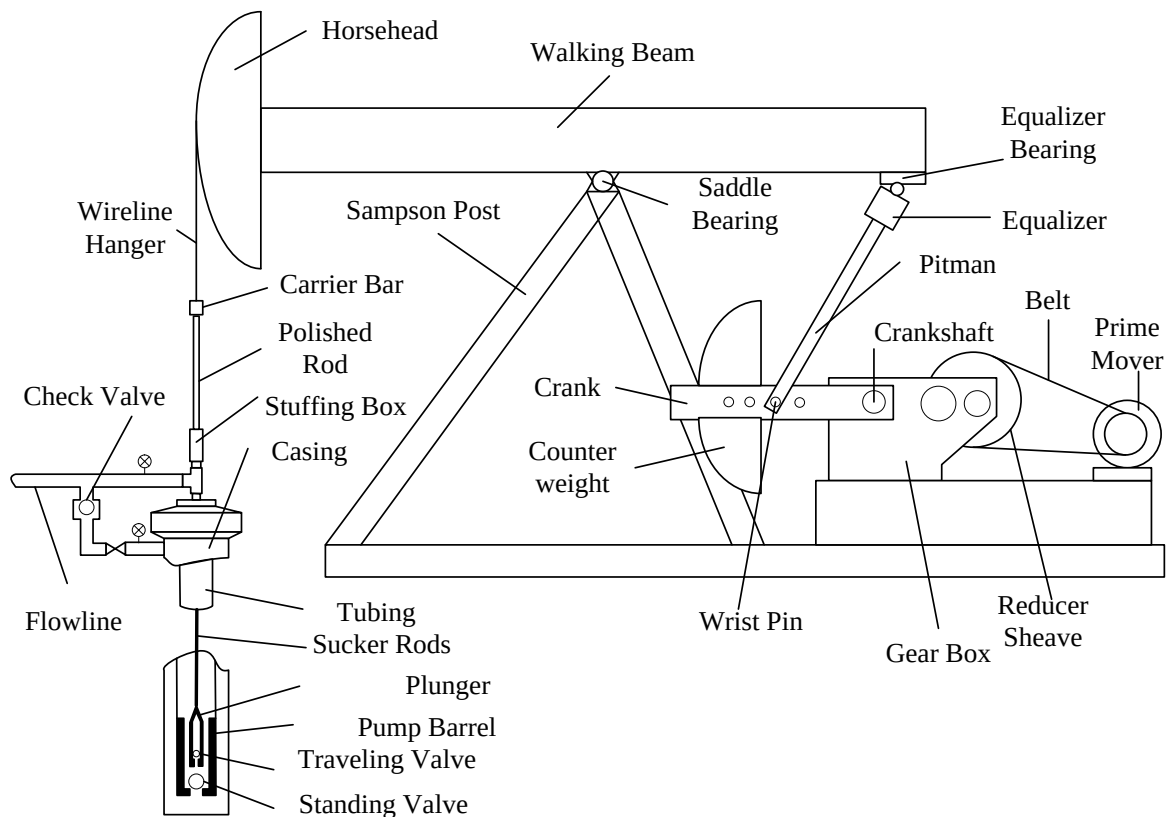


Figure 2.1-The Main Components of a Sucker Rod Pumping System

The most important surface components are: prime mover, gear reducer, pumping unit and polished rod. The prime mover provides the necessary driving power, while the other

parts basically convert the rotational motion into alternating motion with the required speed.

There are two essential downhole parts: the sucker rod string and the pump. The sucker rod string can be longer than 2,000 meters, and provides a mechanical linkage between the surface equipment and the pump. The pump is that part which transmits the energy to the transported fluid, so it plays a key role in the operation of the system.

The pump consists of four parts: plunger, barrel and two check valves. As we can see later, the two check valves also play a key role in the proper operation of the pump.

2.2. Important Operating Parameters

The most important operating parameters are the following:

- plunger stroke length,
- plunger displacement,
- polished rod loads,
- polished rod horsepower,
- peak net torque.

These parameters can be calculated with the help of the API RP 11L procedure [1] or the wave equation. The API RP 11L is a recommended practice, which is based on electrical analog experiments. It gives simple and accurate solution, but it only stands for normal pumping condition. The solution of the wave equation is much harder, but it provides a possibility to develop a procedure that works in case of fluid pound pumping condition.

In addition to operating parameters there are some factors, which reflects the operating efficiency and the quality of design. These factors are the following:

- safety factors of the rod,
- volumetric efficiency of the pump,
- overall efficiency of the system,
- cyclic load factor.

These factors can deteriorate when fluid pound occurs. Because of that if someone wants to design and optimize an intermittently operating sucker rod pumping system, one has to watch on these factors.

3. Fluid Pound Condition

Many pumping malfunctions can arise during the lifetime of a sucker rod pumping system, which can significantly decrease the pumping efficiency. Because of this it is important to detect and handle these problems.

The leaking check valve and pump is caused by pump wear. This problem can be solved only by the replacement of the pump or check valve.

Gas pound can arise, if the oil is volatile and the downhole separator is inefficient. Generally gas pound can be eliminated by the right design of the downhole separator.

Fluid pound is also a regular pumping malfunction that occurs when the reservoir pressure is extremely low. This problem cannot be simply eliminated or fixed, so the investigation of how this problem affects the pumping efficiency can be important.

3.1. Occurrence of Fluid Pound

The phenomena called fluid pound occurs when the pumping rate is significantly higher than the inflow rate. In that case the fluid level drops below the top of the pump, as can be seen on Figure 3.1. It means that the pump barrel isn't completely filled up with fluid, so when the downstroke starts the traveling valve doesn't open immediately, only when the plunger reaches the liquid level.

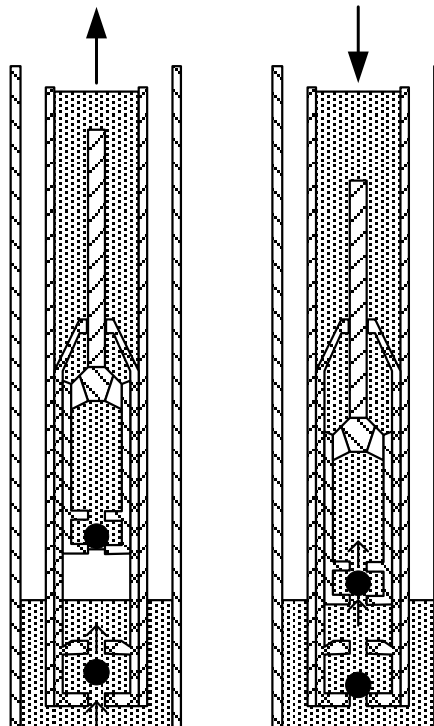


Figure 3.1-Sum of Forces Acting on a Rod Element

As already mentioned the fluid pound deteriorates many pumping parameters, which leads to inefficient pumping and to failures.

3.2. Effect on operation parameters

The upstroke proceeds just as in normal case without any irregularity. When the downstroke starts the traveling valve doesn't open immediately, only when the plunger hits the liquid level. This hitting action creates a sudden impact force, which is transmitted to the surface along the rod string. This shock load has many harmful effects on the pumping system.

This shock in rod string can cause buckling, which leads to rod breaks and rod-to-tubing wear. Coupling failure due to unscrewing also can be the result of shock loads. On the surface this shock load can damage the bearings and can be the source of harmful torques in the speed reducer, which exceeds the allowed levels.

In addition to the harmful effects, the fluid pound decreases the efficiency of pumping. First of all, it deteriorates the volumetric efficiency of the pump and decreases the pumping rate. As the torque changes the cyclic load factor also deteriorate, which is the measure of how good the counterbalance is.

4. Simulation of the Rod String's Behavior

It is already approved, that only the simulation of the rod string's behavior can provide the proper description of the pumping system. Other approximate methods don't have adequate accuracy, the necessary accuracy can only be provided by simulation. Other procedures use simplified models, that often lead, to unacceptable errors in the design or the analysis of a pumping installation.

The greatest difficulty in the calculation is caused by the elastic behavior of the rod, it makes it difficult to calculate the downhole conditions from the surface data or vice versa. All mechanical effects generated by the pumping unit's motion or the operation of the subsurface pump are transmitted along the rod in the form of stress waves. These stress waves travelling with the speed of sound along the rod string. The interferences and reflections of these waves have a significant effect on the displacements and loads that can be observed at different points along the string.

The most popular method, which is adequate for simulation is published by Gibbs [2], it is called the one-dimensional wave equation.

4.1. The Wave Equation

Figure 4.1 shows an infinitesimally short rod section. The coordinate axes x and u are directed downward, x represents axial distance and u represents displacement. The force balance must be written up on the infinitesimal rod section as the first step of the solution. The forces acting on this rod section can be seen in Figure 4.1, where:

- W = the buoyant weight of the rod,
- F_x and $F_{x+\Delta x}$ = tension forces, which represent upward and downward pull,
- F_d = damping force opposing the movement of the rod, which includes both viscous and mechanical friction.

As Newton's Second Law states, the sum of forces acting on the element must equal the mass of the element times its acceleration:

$$F_x - F_{x+\Delta x} + W - F_d = m \frac{\partial^2 u}{\partial t^2} \quad (4.1)$$

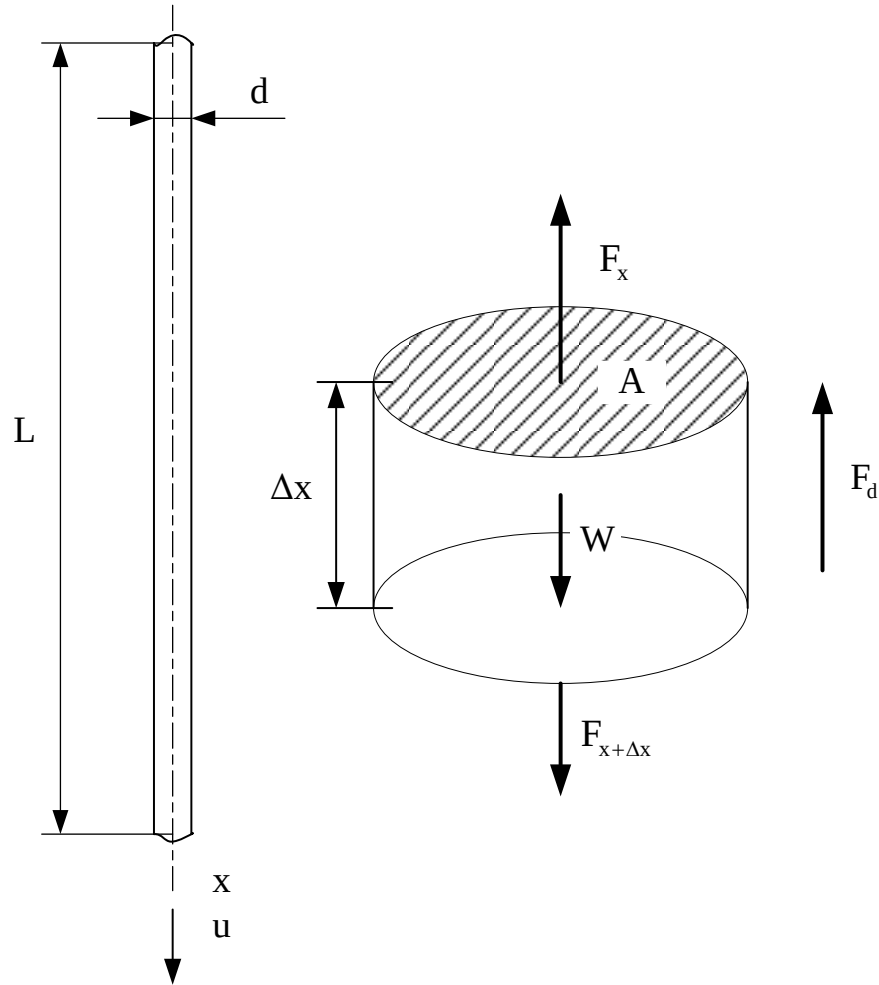


Figure 4.1-Sum of Forces Acting on a Rod Element

The buoyant weight of the rod element is constant during the whole pumping cycle, so it can be left out from the equation, and its effect can be simply added to the solution of the equation at the end. The tension forces can be expressed with the mechanical stresses present in the rod section and with the application of Hooke's Law. These stresses can be written up as the linear functions of deformation, which is the first derivative of the displacement. The damping forces responsible for the energy loss of the system. The damping forces can be divided into viscous and mechanical friction. The proper determination of these forces are almost impossible, so Gibbs approximated the sum of them with the following semi-empirical formula:

$$F_d = c \cdot \Delta x \cdot \frac{\rho \cdot A}{g} \cdot \frac{\partial u}{\partial t} \quad (4.2)$$

where:

$$c = \frac{\pi \cdot v_s \cdot \nu}{2 \cdot L} \quad - \quad \text{damping coefficient [1/s]} \quad (4.3)$$

$$\nu \quad - \quad \text{dimensionless damping factor [-]}$$

$$v_s = \sqrt{\frac{g \cdot E}{\rho}} \quad - \quad \text{sound velocity in the rod material [m/s]} \quad (4.4)$$

$$L \quad - \quad \text{total rod length [m]}$$

$$E \quad - \quad \text{Young's modulus of elasticity for the rod material [Pa]}$$

So both the viscous and mechanical friction approximated by viscous damping.

After substitution and some rearranging we get the following equation:

$$v_s^2 \frac{\partial^2 u}{\partial x^2} - c \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} \quad (4.5)$$

This is the most often used form of wave equation, which is valid if the cross section of the rod is constant. It is a linear, second-order hyperbolic partial differential equation, which can be solved with different methods.

4.2. Solution Possibilities

With the help of the wave equation the displacement of any rod element can be calculated in the function of the vertical distance and time. The solution of a linear, second-order hyperbolic differential equation require two initial and two boundary condition. Depending on how the boundary conditions are given, there are two different cases:

- Diagnostic case : the measured displacement and load on the polished rod gives the two boundary conditions.
- Predictive case : the kinematical boundary condition is given by the kinematics of the pumping unit and the dynamical boundary condition is given by operation of the sucker rod pump.

Gibbs solved the first case analytically[3], but it can be solved numerically as well[4]. The predictive case can be solved only numerically, because of the form of the dynamical boundary condition. The different pumping condition can be simulated by sufficient description of the dynamical boundary condition, which is given by the operation of the pump.

5. Numerical Solution in Case of Fluid Pound

The numerical solution methods, which are adequate to solve partial differential equation, can be classified into two groups: continuous and discrete. The continuous methods like the finite element method approximate the solution of the partial differential equation on the whole domain of the function., while the discrete methods approximate them only on specified, discrete points of the function domain.[5]

The finite difference method is the most simple, while provides the necessary accuracy. It can be classified to the group of discrete methods. The method doesn't require any restrictions on the function or on the initial and boundary conditions, which is favourable in our case, when the dynamical boundary condition isn't completely given initially.

5.1. Basic Conception of Finite Difference Methods

The basic idea of the method is that the differential quotients can be approximated with difference quotients. These difference quotient approximations can be developed by the use of Taylor series. A given differential quotient can be approximated in different ways, here are some examples for $\partial u / \partial x$ with their notation[5]:

$$\frac{\partial u}{\partial x} \approx \frac{u(x + \Delta x, t) - u(x, t)}{\Delta x} + O(\Delta x) \quad - \quad \text{forward difference} \quad (5.1)$$

$$\frac{\partial u}{\partial x} \approx \frac{u(x, t) - u(x - \Delta x, t)}{\Delta x} + O(\Delta x) \quad - \quad \text{backward difference} \quad (5.2)$$

$$\frac{\partial u}{\partial x} \approx \frac{u(x + \Delta x, t) - u(x - \Delta x, t)}{2 \cdot \Delta x} + O(\Delta x^2) \quad - \quad \text{central difference} \quad (5.3)$$

The $O()$ describe the order of truncation error in the function of distance between two discrete points. Different approximations may have different order of truncation error. Different approximations leads to different solution schemes. If the approximation formula contains displacement of one unknown point, the problem can be solved explicitly. If it contains displacement of two or more unknown points, it leads to implicit solution scheme, so a linear equation system has to be solved in every time step. The implicit solution needs more memory and computation time, but it always gives a numerically stable solution procedure.

This problem can be solved explicitly only because of the dynamical boundary condition. The following subchapters contain construction and solution of the explicit numerical model for this problem.

5.2. Explicit Solution Scheme

The form of the wave equation, which is given by 4.5 formula, is only valid for a uniform rod section. If the wave equation has to be applied at the border of two rod sections the following form of the wave equation has to be used:

$$E \cdot A \cdot \left(\frac{\partial u}{\partial x} \Big|_{x+\Delta} - \frac{\partial u}{\partial x} \Big|_x \right) - c \cdot \frac{\Delta x \cdot E \cdot A}{v_s^2} \cdot \frac{\partial u}{\partial t} = \frac{\Delta x \cdot E \cdot A}{v_s^2} \cdot \frac{\partial^2 u}{\partial t^2} \quad (5.4)$$

The differential quotients are approximated with the following difference quotients in order to obtain an explicit scheme:

$$\frac{\partial u}{\partial x} \Big|_{x+\Delta x} \approx \frac{u(x+\Delta x, t) - u(x, t)}{\Delta x} \quad (5.5)$$

$$\frac{\partial u}{\partial x} \Big|_x \approx \frac{u(x, t) - u(x - \Delta x, t)}{\Delta x} \quad (5.6)$$

$$\frac{\partial u}{\partial t} \approx \frac{u(x, t + \Delta t) - u(x, t)}{\Delta t} \quad (5.7)$$

$$\frac{\partial^2 u}{\partial t^2} \approx \frac{u(x, t + \Delta t) - 2 \cdot u(x, t) + u(x, t - \Delta t)}{\Delta t^2} \quad (5.8)$$

For a simpler form some abbreviation will be established: i subscript will denote the position of the points on the x axis, j subscript will denote the position of the points on the axis, so $u(x+\Delta x, t)$ will be denoted as $u_{i+1, j}$. The constant parameters can be merged into groups for further simplification:

$$\alpha_i = \frac{\Delta x_i \cdot A_i \cdot E_i}{2 \cdot (v_{s(i)} \cdot \Delta t)^2} \quad (5.9)$$

$$\beta_i = \frac{\Delta x_i \cdot A_i \cdot E_i}{2 \cdot v_{s(i)} \cdot \Delta t} \cdot \frac{c}{v_{s(i)}} \quad (5.10)$$

$$\gamma_i = \frac{A_i \cdot E_i}{\Delta x_i} \quad (5.11)$$

After substitution of difference quotients and parameter groups, the wave equation converts into the following form:

$$\begin{aligned} & \gamma_{i+1} \cdot (u_{i+1, j} - u_{i, j}) - \gamma_i \cdot (u_{i, j} - u_{i-1, j}) - (\beta_i + \beta_{i+1}) \cdot (u_{i, j+1} - u_{i, j}) = \\ & = (\alpha_i + \alpha_{i+1}) \cdot (u_{i, j+1} - 2 \cdot u_{i, j} + u_{i, j-1}) \end{aligned} \quad (5.12)$$

The final equation, which can be used for the calculations, has to be expressed for the unknown $u_{i,j+1}$:

$$u_{i,j+1} = \frac{u_{i,j} \cdot [2 \cdot (\alpha_i + \alpha_{i+1}) + \beta_i + \beta_{i+1} - \gamma_i - \gamma_{i+1}] - u_{i,j-1} \cdot (\alpha_i + \alpha_{i+1})}{\alpha_i + \alpha_{i+1} + \beta_i + \beta_{i+1}} + \frac{u_{i+1,j} \cdot \gamma_{i+1} + u_{i-1,j} \cdot \gamma_i}{\alpha_i + \alpha_{i+1} + \beta_i + \beta_{i+1}} \quad (5.13)$$

Equation 5.13 is valid for tapered, composite sucker rod strings also. Figure 5.1 shows the mesh where the points involved in the 5.13 are illustrated by dots. Black dots represent the points whose displacement is known, while white dot represents the point whose displacement is the unknown in the equation.

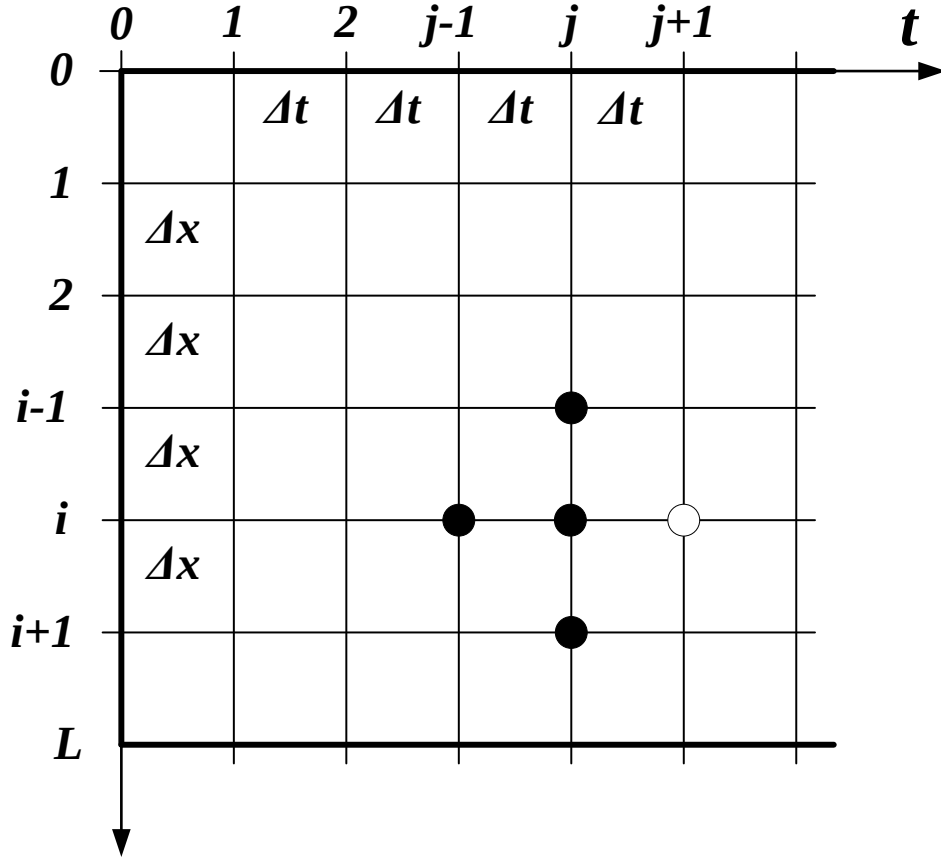


Figure 5.1-Points Involved in Solution Formula Illustrated on the Mesh

For the start of the calculation process the displacement of some points have to be known, These displacements can be determined from the initial and boundary conditions. The position of these points can be seen on Figure 5.2.

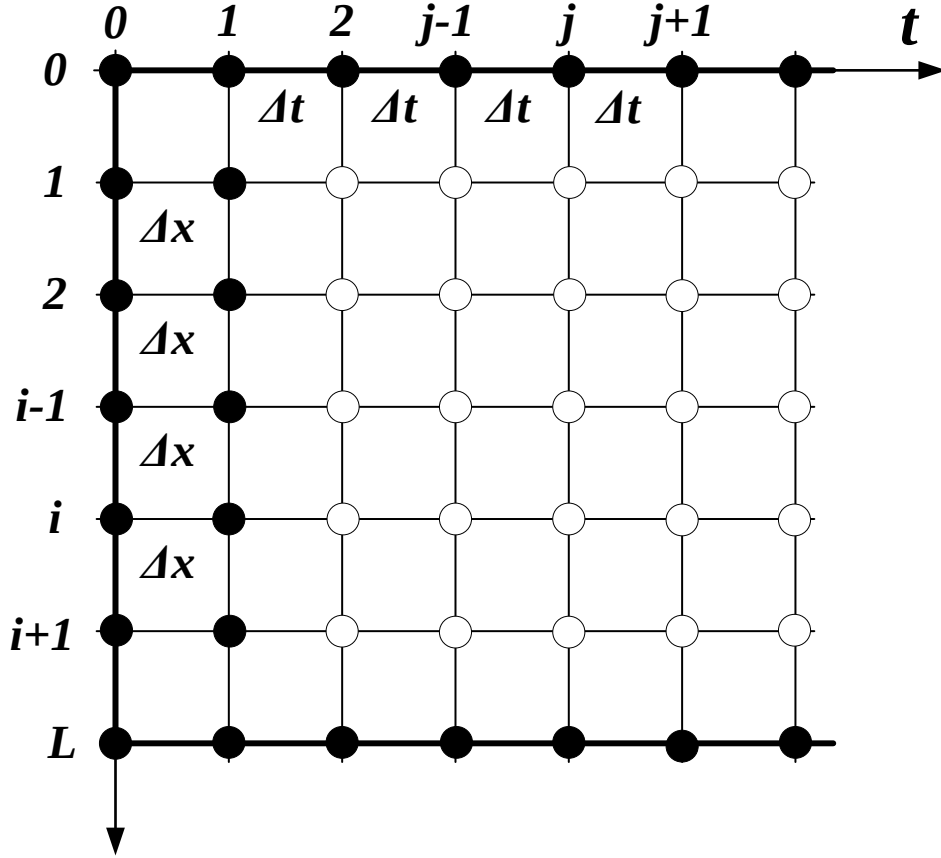


Figure 5.2-Position of Points Whose Displacement can be Determined from the Initial and Boundary Conditions

The points whose values are known from initial and boundary conditions, are represented by black dots. Other points, whose value can be calculated with equation 5.13, are represented with white dots. The initial and boundary conditions will be detailed later.

After the solution scheme is set up, the next step is the construction of the mesh of discrete points.

5.3. Discretization

In case of finite difference method the approximation involves the replacement of continuous domain by a network of discrete points, which is often called as "mesh". The displacement is only calculated at these points, this reduction provides much faster calculation. In the first step of the solution, the mesh has to be constructed.

If using any implicit scheme the solution procedure is always numerically stable, so we can freely determine the value of Δx and Δt , but an implicit scheme cannot be used because of the dynamic boundary condition. Numerical stability means that the procedure doesn't accumulate the errors. Explicit procedures can be numerically unstable, if the mesh

isn't adequate. The value of Δx and Δt must satisfy the stability criterion to avoid instability. This criterion is the following:

$$\Delta t \leq \frac{\Delta x}{v_s} \quad (5.14)$$

The physical meaning of this criterion is that any effect must propagate as fast in the numeric model as it propagates in real life. In the sucker rod string the stress waves propagate with the speed of sound. In our approximating model these stress waves' maximum propagation speed is $\Delta x/\Delta t$, which must be higher or equal to the speed of sound.

A sucker rod string is always made up of taper sections, of different cross-sections and lengths. Each taper must be approximated with integer number of points, so different Δx values must be used in every taper. A mesh of a sucker rod string having two tapers can be seen in Figure 5.3.

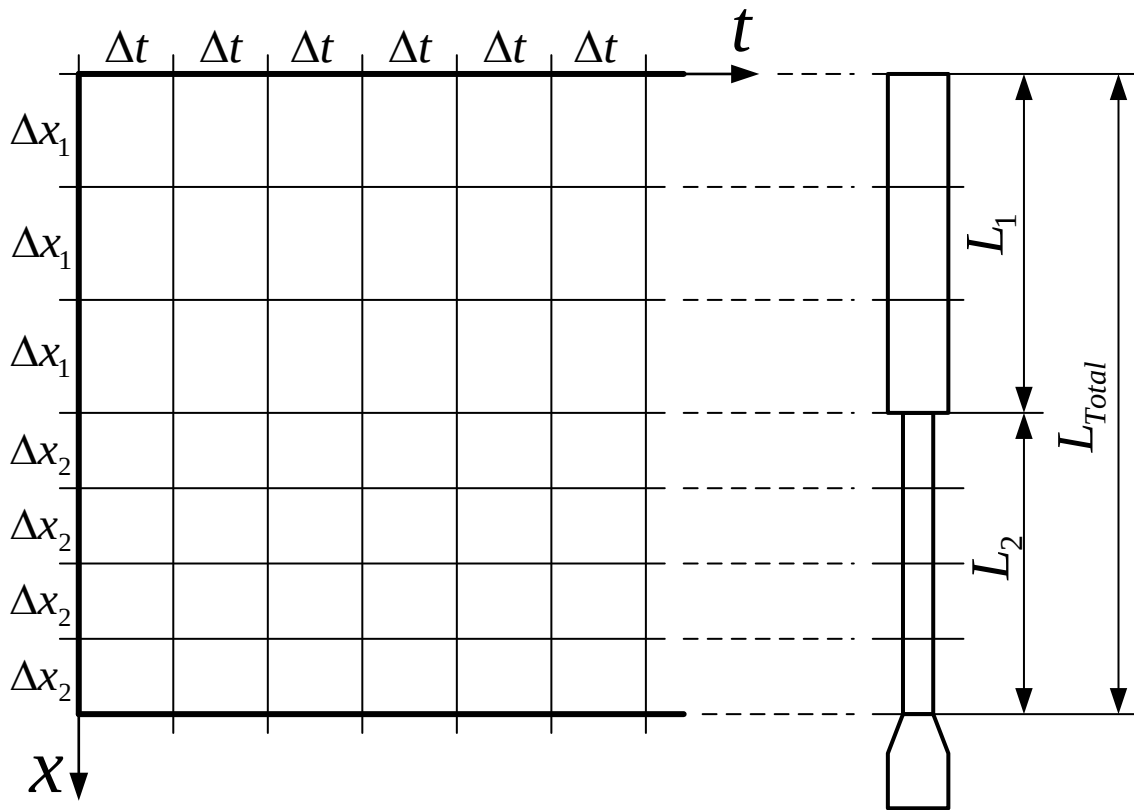


Figure 5.3-The Mesh of a Sucker Rod String with Two Tapers

The chosen value of Δt must satisfy the stability criterion in every taper section and it is generally constant on the whole time domain. In addition the duration of a pumping cycle must be dividable with the value of Δt .

After the values of Δt and Δx_i are determined, the mesh can be created and the next task is the calculation of initial and boundary value conditions.

5.4. Initial Condition

For the solution of a linear, second-order hyperbolic differential equation two initial conditions are needed. There aren't any information about them, so it is assumed, that the sucker rod pumping system starts to operate when our simulation is started. In this case the displacement and the velocity of every discrete point along the rod string is zero, which simply means that all of the displacements are zeros at the first two time step. This assumption will cause some transient effects that can be neglected after the simulation of 4-8 pumping cycles.

5.5. Kinematic Boundary Condition

In solid mechanics the boundary condition, which provides direct information about the displacement of points at the boundary of the object, is called kinematic boundary condition. In our case the displacement of the uppermost point of the sucker rod string is given by the kinematics of the pumping unit.

In modern sucker rod pumping there are four different kinds of pumping unit geometries, these are conventional, Mark II, air balanced and Torqmaster. The calculation of polished rod displacement in function of crank angle can be found in API Spec. 11E[6] and many books and papers[7,8,9]. The adequate equations valid for conventional geometry will be detailed only, others can be found in the cited references.

The position of the polished rod at any crank angle is found by:

$$s(\theta) = a \cdot (\psi_b - \psi) \quad (5.15)$$

where:

$$\psi_b = \arccos \frac{C^2 + K^2 - (P + R)^2}{2 \cdot C \cdot K} \quad (5.16)$$

$$\psi = \arcsin \frac{P \cdot \sin \beta}{h} - \arcsin \frac{R \cdot \sin(\theta - \phi)}{h} \quad (5.17)$$

$$\beta = \arccos \frac{C^2 + P^2 - K^2 - R^2 + 2 \cdot K \cdot R \cdot \cos(\theta - \phi)}{2 \cdot C \cdot P} \quad (5.18)$$

$$h = \sqrt{R^2 + K^2 - 2 \cdot R \cdot K \cdot \cos(\theta - \phi)} \quad (5.19)$$

$$\phi = \arcsin \frac{I}{K} \quad (5.20)$$

A, C, I, P, K, R - parameters of the pumping unit

The displacement of the uppermost point (polished rod) can be determined with equation 5.15 from the crank angle, which can be calculated from the elapsed time. The Figure 5.4 shows the position of the polished rod vs. crank angle for three different pumping unit.

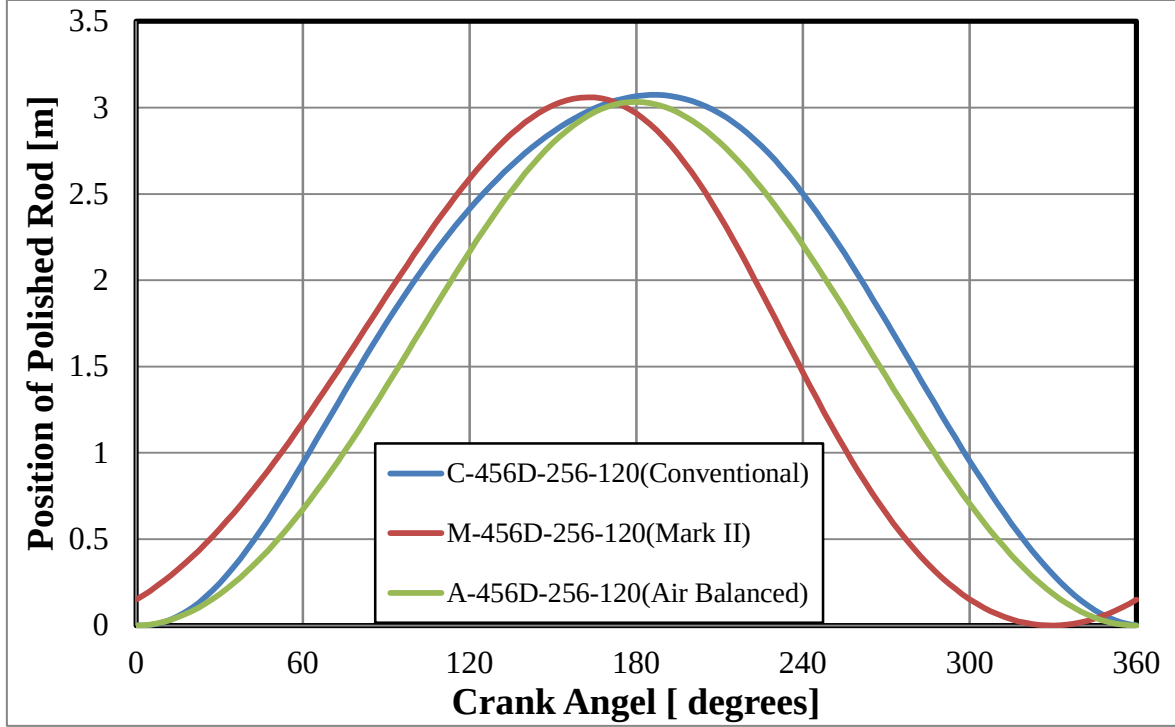


Figure 5.4-Position of Polished Rod vs. Crank Angle

The displacement of polished rod at zero crank angle isn't zero in all cases, this can cause a numerical error in form of a huge stress wave. To avoid this error, from the displacement of polished rod, which can be calculated by equation 5.15, the displacement at zero crank angle must be deducted.

5.6. Dynamic Boundary Condition

In solid mechanics the boundary condition, that gives information about the forces acting on the surface of the object, is called dynamic boundary condition. In this case forces acting on the plunger can be determined from its pump card.

5.6.1. General Equation

With the help of Hooke's Law the force acting on the plunger can be converted to the first derivative of displacement:

$$F_p = -E \cdot A \cdot \frac{\partial u}{\partial x} \quad (5.21)$$

where:

- F - the force acting on the plunger [N]
- A - the cross-section of the bottom rod taper [m²]
- E - Young's modulus of elasticity for the rod material [Pa]

For programming reasons in my model the u axes points upward and not downward as in the original, so there is a negative sign in equation 5.21 for this reason.

The required information from the boundary condition is the displacement of the lowest point of the rod string, so the first derivative must be approximated from the displacement of the lowest two or three points, using backward difference:

$$\left. \frac{\partial u}{\partial x} \right|_{L_{Total}} \approx \frac{u(L_{Total}, t) - u(L_{Total} - \Delta x, t)}{\Delta x} \quad (5.22)$$

For higher order of approximation $u(L, t)$ will be expressed by Taylor series about $u(L - \Delta x, t)$:

$$\begin{aligned} u(L_{Total}, t) &\approx u(L_{Total} - \Delta x, t) + \left. \frac{\partial u}{\partial x} \right|_{L_{Total} - \Delta x} \cdot \frac{\Delta x}{1!} + \left. \frac{\partial^2 u}{\partial x^2} \right|_{L_{Total} - \Delta x} \cdot \frac{\Delta x^2}{2!} \\ \left. \frac{\partial u}{\partial x} \right|_{L_{Total} - \Delta x} \cdot \frac{\Delta x}{1!} &\approx \frac{u(L_{Total}, t) - u(L_{Total} - \Delta x, t)}{\Delta x} \cdot \frac{\Delta x}{1} \\ \left. \frac{\partial^2 u}{\partial x^2} \right|_{L_{Total} - \Delta x} \cdot \frac{\Delta x^2}{2!} &\approx \frac{u(L_{Total}, t) - 2 \cdot u(L_{Total} - \Delta x, t) + u(L_{Total} - 2 \cdot \Delta x, t)}{\Delta x^2} \cdot \frac{\Delta x^2}{2!} \\ u(L_{Total}, t) &= 1.5 \cdot u(L_{Total}, t) - u(L_{Total} - \Delta x, t) + 0.5 \cdot u(L_{Total} - 2 \cdot \Delta x, t) \end{aligned} \quad (5.23)$$

If equation 5.23 is substituted into equation 5.22, it takes the following form:

$$\left. \frac{\partial u}{\partial x} \right|_{L_{Total}} \approx \frac{1.5 \cdot u(L_{Total}, t) - 2 \cdot 1.5 \cdot u(L_{Total} - \Delta x, t) + 1.5 \cdot u(L_{Total} - 2 \cdot \Delta x, t)}{\Delta x} \quad (5.24)$$

Let's abbreviate $u(L_{Total}, t)$ with u_p , $u(L_{Total} - \Delta x, t)$ with u_{p-1} , $u(L_{Total} - 2 \cdot \Delta x, t)$ with u_{p-2} and substitute equation 5.24 into equation 5.21:

$$F_p = -\frac{E \cdot A}{\Delta x} \cdot (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.25)$$

Equation 5.25 is in the adequate form to derive the required formulae for the different pumping phases.

5.6.2. Pumping Phases

Figure 5.5 shows a general pump card of a pump, which operates under fluid pound condition. Six distinguishable phases can be identified on the pump card, their differentiation is necessary, because each phase can be described properly by different equations, these phases are numbered in Figure 5.5. For each phases a sufficient equation must be derived for the calculation of the displacement of the lowest point. In addition conditions must be set up, that determine when the pumping phase changes to the next one.

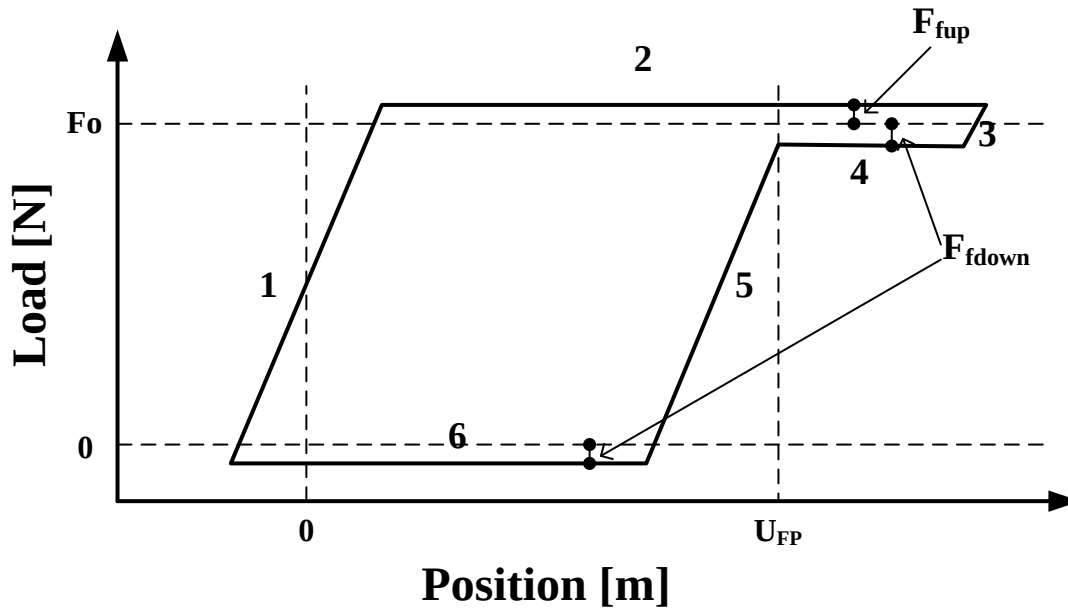


Figure 5.5-Pump Card of the Pump

Let's shortly overview these pumping phases:

1. Start of Upstroke: at the end of the downstroke the traveling valve immediately closes, and the pump picks up the fluid load. At the same time the standing valve opens, and the load on the tubing ceases, so the tubing returns to its unstretched position, which is the reason of the sloping shape of this phase on the pump card. When the value of the load reaches the fluid load plus friction the next phase starts.
2. Upstroke: the load on the plunger is equal to the fluid load plus the friction (F_{fup}) during the upstroke. It lasts until the plunger begin to move downward.
3. Start of Downstroke: when the plunger starts to move downward the traveling valve doesn't open as it should in normal case. The load on the plunger decreases until its value reaches the fluid load minus friction (F_{fdown}), then the next phase is coming.
4. Downstroke with Fluid Load: the traveling valve is still closed and the plunger holds the full fluid load. This phase lasts until the plunger reaches the liquid level and the travelling valve opens.
5. Fluid Pound: when the traveling valve opens the load on the plunger suddenly drops below zero (zero minus friction). At the same time the standing valve closes and the tubing carries the full fluid load, so it stretches, which is the reason for the

sloping shape of this phase on the Pump Card. When the value of the load reaches zero minus friction the next phases is coming.

6. Downstroke: the load on the plunger is constantly zero minus friction during the downstroke. This phase lasts until the plunger starts to move upward.

It is important that, when the simulation starts the displacement of the plunger is zero. In order to know, when the fluid pound phase starts, the relative position of the liquid level to the starting position of the pump must be known.

In the following subchapters the calculation of the relative position of liquid level and the above mentioned pumping phases will be detailed.

5.6.3. Determination of the Relative Position of the Liquid Level

At the start of the simulation all of the points have zero displacement including the lowest point, the plunger. The displacement of the lowest point at the start of the simulation is the reference point, so the position of the liquid level must be determined compared to this point.

The schematic of the problem can be seen in Figure 5.6. At the start of the simulation downstroke phase is supposed, so the rod is only loaded by its own weight which causes it to stretch. This stretch is significant, so the stretched length of the rod (L_{SR}) must be used in the calculations. At crank angle zero the position of the polished rod ($S_r(0)$) isn't zero, so it must be taken into consideration too. The other variables to be taken into consideration are: the length of the plunger (L_{PL}), the pump setting depth (L_{SD}) and the height of liquid level above the standing valve (L_L). It can be seen in Figure 5.6, that the relative position of the liquid level (RP_L) can be calculated from the following equation:

$$RP_L = (L_{SR} + L_{PL}) - (S_r(0) + L_{SD} - L_L) \quad (5.26)$$

The stretched length of the rod string can be calculated from the following formula:

$$L_{SR} = L_{Total} + \sum_{i=1}^N \left(\frac{L_i^2}{2} \cdot w_{r(i)} + \frac{\sum_{j=1}^{i-1} (L_j \cdot w_{r(j)}) - F_{b(i)}}{A_i \cdot E} \right) \quad (5.27)$$

where:

- w_r - specific weight of the rod [N/m]
- F_b - buoyancy force acting on the rod taper [N]

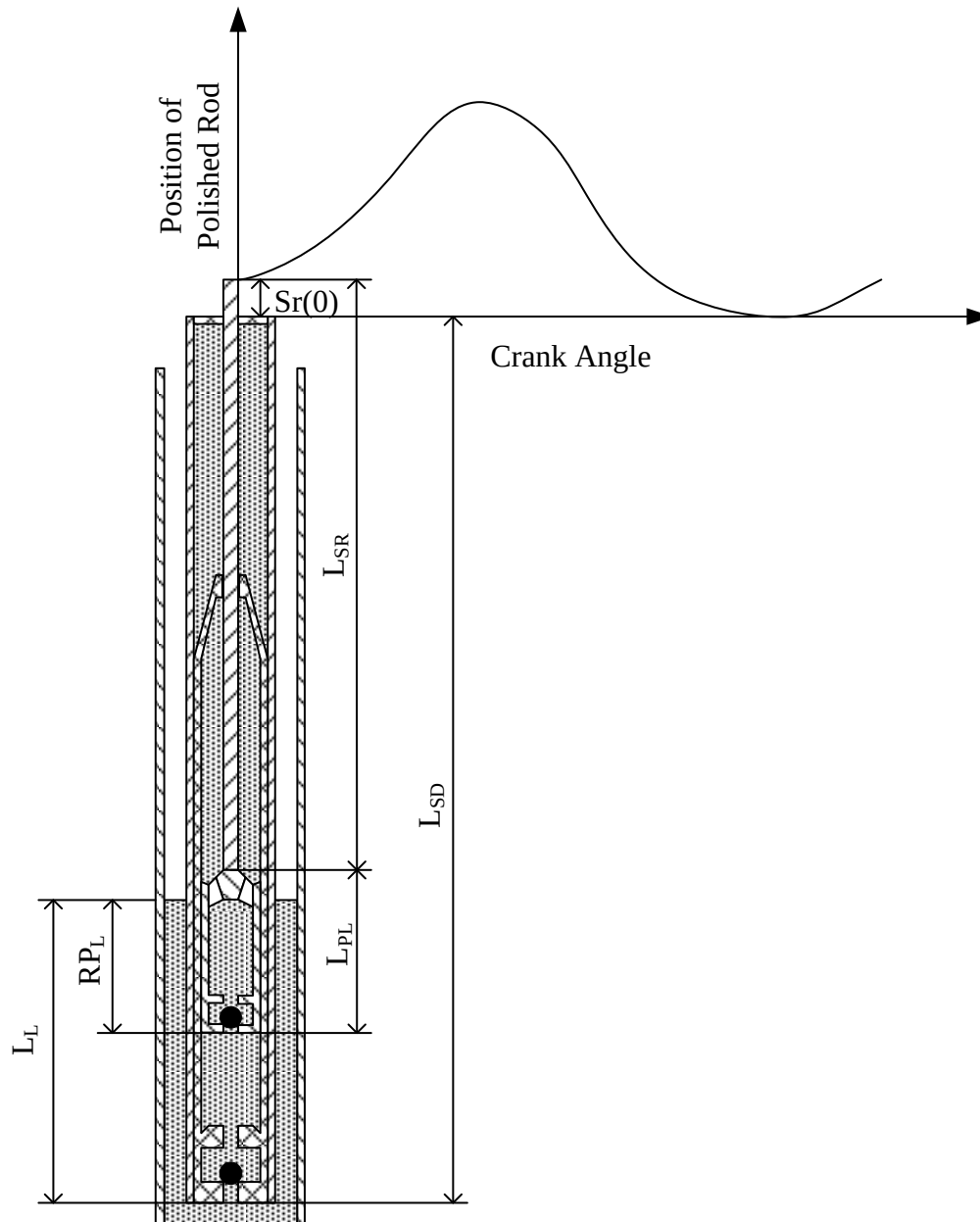


Figure 5.6-Schematic of the Liquid Level Problem

5.6.4. Start of Upstroke

There was already an introduction on what happens in each pumping phases. This subchapter will focus on the calculation of the displacement of the plunger and the phase change condition.

If the tubing is anchored, the position of the plunger doesn't change until the force in the rod doesn't reach the value of the liquid load. If the tubing isn't anchored, there is a relative movement between the plunger and the tubing while the rod string picks up the fluid load. The force acting on the plunger is linearly proportional with this relative movement, the proportional factor is the spring constant of the tubing. Taking into consideration that the

load wasn't zero, but equal to the friction during the downstroke, the following equation can be written:

$$F_P = (u_P - u_{LD}) \cdot k_T - F_{fdown} \quad (5.28)$$

where:

k_T - spring constant of the tubing [N/m]

u_{LD} - last displacement of the plunger during the downstroke [m]

If equation 5.28 is taken equal to equation 5.25, the following equation is received:

$$(u_P - u_{LD}) \cdot k_T - F_{fdown} = - \frac{A_N \cdot E_N}{\Delta x_N} \cdot (1.5 \cdot u_P - 2 \cdot u_{P-1} + 0.5 \cdot u_{P-2}) \quad (5.29)$$

where N subscript refers to the lowest rod taper.

After rearranging of equation 5.29 the following formula is reached for the calculation of the displacement of the plunger:

$$u_P = \frac{(u_{LD} \cdot k_T + F_{fdown}) \cdot \frac{\Delta x_N}{A_N \cdot E_N} + 2 \cdot u_{P-1} - 0.5 \cdot u_{P-2}}{\frac{k_T \cdot \Delta x_N}{A_N \cdot E_N} + 1.5} \quad (5.30)$$

The condition, which decides when to change to the next phase, is still to be determined. When the load on the plunger reaches the value of the fluid load plus friction the next phase is coming, which can be described by the following equation:

$$- \frac{E_N \cdot A_N}{\Delta x_N} \cdot (1.5 \cdot u_P - 2 \cdot u_{P-1} + 0.5 \cdot u_{P-2}) \geq F_o + F_{fup} \quad (5.31)$$

When this condition becomes true the next pumping phase is coming which is the upstroke.

5.6.5. Upstroke

The load on the plunger is equal to the fluid load plus friction during the upstroke, and its value is constant for the whole phase, so the following equation can be written for the load:

$$F_P = F_o + F_{fup} \quad (5.32)$$

When it is combined with the equation 5.25 the following equation is received:

$$F_o + F_{fup} = - \frac{A_N \cdot E_N}{\Delta x_N} \cdot (1.5 \cdot u_P - 2 \cdot u_{P-1} + 0.5 \cdot u_{P-2}) \quad (5.33)$$

after rearranging:

$$u_p = - \frac{(F_o + F_{fup}) \cdot \frac{\Delta x_N}{E_N \cdot A_N} - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}}{1.5} \quad (5.34)$$

The above equation is sufficient for the calculation of the displacement of the plunger during the upstroke. When it gives a lower value than at the last time, it means that the plunger has already started to move upward and the next pumping phase, the start of downstroke is coming. This condition expressed in mathematical form is really simple:

$$u_p \leq u_L \quad (5.35)$$

5.6.6. Start of Downstroke

The effects taking place here are similar to the ones at start of upstroke. The load is decreasing gradually until its value reaches the fluid load minus the friction. The load on the plunger is the following:

$$F_p = (F_o + F_{fup}) - (u_{LU} - u_p) \cdot k_T \quad (5.36)$$

where:

u_{LU} - last displacement of the plunger during upstroke [m]

The substitution of the right-hand side of equation 5.36 into the left-hand side of equation 5.25 gives the following equation:

$$(F_o + F_{fup}) - (u_{LU} - u_p) \cdot k_T = - \frac{E_N \cdot A_N}{\Delta x_N} (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.37)$$

after rearranging:

$$u_p = \frac{2 \cdot u_{p-1} - 0.5 u_{p-2} + (u_{LU} \cdot k_T - F_o - F_{fup}) \frac{\Delta x_N}{E_N \cdot A_N}}{\frac{\Delta x_N}{E_N \cdot A_N} \cdot k_T + 1.5} \quad (5.38)$$

This equation is adequate for the calculation of the plunger displacement during this pumping phase. Because the travelling valve hasn't opened when the plunger started to move downward, the plunger still carries the full fluid load, so the load decreases until its value reaches the fluid load minus friction. After that the next pumping phase is coming which is downstroke with fluid load. The mathematical form of this condition is:

$$F_o - F_{fdown} \geq - \frac{E_N \cdot A_N}{\Delta x_N} (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.39)$$

5.6.7. Downstroke with Fluid Load

During this pumping phase the plunger carries the full fluid load minus friction, because the traveling valve is still closed.

$$F_p = F_o - F_{fdown} \quad (5.40)$$

If the equation 5.40 is taken equal to equation 5.25, the following equation forms:

$$F_o - F_{fdown} = -\frac{E_N \cdot A_N}{\Delta x_N} (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.41)$$

If the desired displacement of the plunger is expressed from the above equation, it takes the following form :

$$u_p = \frac{(F_{fdown} - F_o) \cdot \frac{\Delta x_N}{E_N \cdot A_N} + 2 \cdot u_{p-1} - 0.5 \cdot u_{p-2}}{1.5} \quad (5.42)$$

The traveling valve stays closed until the plunger reaches the liquid level, then the phenomena called fluid pound occurs. The mathematical form of this phase change condition is:

$$u_p \leq RP_L \quad (5.43)$$

When this condition becomes true the next pumping phase follows.

5.6.8. Fluid Pound

After the plunger reaches the liquid level, the travelling valve opens and the plunger drops the load. At the same time the standing valve closes and the full fluid load burden the tubing. The sucker rod returns to its unstretched state, while the tubing stretches because of the fluid load. The stretch causes a relative movement between the sucker rod and the tubing, so this is the reason why the load decreases gradually. The load on the plunger in the function of its displacement is:

$$F_p = (F_o - F_{fdown}) - (u_{LLD} - u_p) \cdot k_T \quad (5.44)$$

where:

U_{LLD} - last displacement of the plunger during downstroke with fluid load pumping phase [m]

If this equation is combined with equation 5.25, the following equation is found:

$$(F_o - F_{fdown}) - (u_{LLD} - u_p) \cdot k_T = -\frac{E_N \cdot A_N}{\Delta x_N} \cdot (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.45)$$

After rearranging the equation, it takes its final form:

$$u_p = \frac{(F_{fdown} - F_o + u_{LLD} \cdot k_T) \cdot \frac{\Delta x_N}{E_N \cdot A_N} + 2 \cdot u_{p-1} - 0.5 \cdot u_{p-2}}{\frac{k_T \cdot \Delta x_N}{E_N \cdot A_N} + 1.5} \quad (5.46)$$

Equation 5.46 is adequate for the calculation of the displacement of the plunger during the fluid pound phase. The next pumping phase is coming, when the load decreases to zero minus friction, the mathematical form of this condition is the following:

$$-F_{fdown} \geq -\frac{\Delta x_N}{E_N \cdot A_N} \cdot (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.47)$$

5.6.9. Downstroke

The description of this pumping phase is really simple, because only friction is acting on the plunger during the downstroke. The mathematical expression of this is also simple:

$$F_p = -F_{fdown} \quad (5.48)$$

If it is substituted into equation 5.25, the following equation forms:

$$-F_{fdown} = -\frac{E_N \cdot A_N}{\Delta x_N} \cdot (1.5 \cdot u_p - 2 \cdot u_{p-1} + 0.5 \cdot u_{p-2}) \quad (5.49)$$

If this equation is expressed for the displacement of the plunger, the equation takes its final form:

$$u_p = \frac{F_{fdown} \cdot \frac{\Delta x_N}{E_N \cdot A_N} + 2 \cdot u_{p-1} - 0.5 \cdot u_{p-2}}{1.5} \quad (5.50)$$

This pumping phase is over, when equation 5.50 gives a higher value than in the last time step. Its mathematical form is really simple:

$$u_p \geq u_L \quad (5.51)$$

This was the last pumping phase during the pumping cycle, so the solution algorithm is next.

5.6.10. Solution Algorithm of the Dynamic Boundary Condition

After detailed descriptions of each pumping phase, now the solution algorithm is missing only. In addition some serious errors can occur which hasn't been mentioned yet.

In the program code it is expedient to make a separate subroutine for every pumping phase. There should be a phase identifier variable, which records the identification number of the current pumping phase. There is a need for another subroutine which is directly called by the main function and its task is to choose the proper subroutine.

The general schematic of the subroutines (exclude the phase choosing subroutine), which is valid for all pumping phases is given in Figure 5.7. Basically all subroutine calculate the displacement of the plunger, then check the phase change condition. If it is true, it calls the proper subroutine for the next phase which recalculates the displacement of the plunger, so it can be classified as a recursive function.

The algorithm can be realized several other ways, like function pointers and others, but this is a simple solution for the problem.

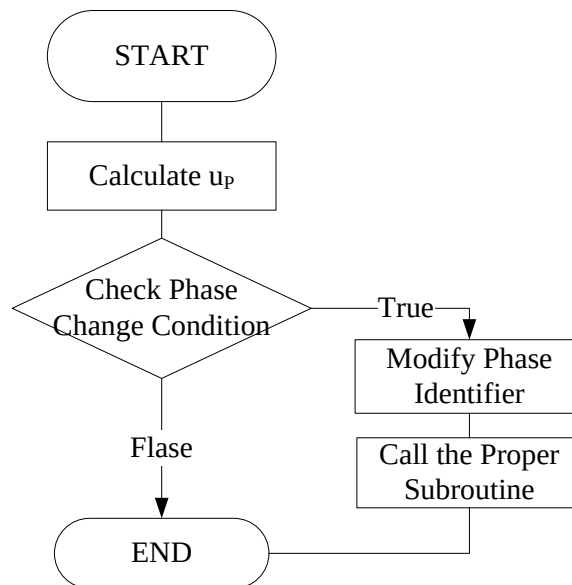


Figure 5.7-General Schematic of the Subroutines

This algorithm can be "cheated" by stress waves, when the plunger moves slowly at the end of upstroke or downstroke. A greater than average stress wave force the algorithm to think that the current pumping phases is over and the next is coming. Since this is not true, so the algorithm must recognize this problem and correct itself. This makes a small glitch on the pump card of the pump, but the result is still acceptable. For the sake of simplicity this won't be detailed.

5.7. Solution Algorithm

Each step of the solution was detailed in the previous subchapters. This subchapter will give an overview of the solution algorithm and the order of calculations. The main steps in the solution are the following:

1. Determination of Δx_i and Δt to satisfy the stability criterion., construction of the mesh and initializing vectors and matrices.
2. Determination of parameters: α_i , β_i and γ_i .
3. Calculation of the displacement of points coming from the initial condition.
4. Calculation of the displacement of points which coming from the kinematic boundary condition.
5. Calculation of the damping coefficient and the displacement of other points in the proper order as seen in Figure 5.8.
6. At the end of each column the displacement of the lowest point can be calculated from the dynamic boundary condition.
7. Point 5 and 6 must be repeated for every time step in the pumping cycle, and about 6 pumping cycles must be simulated to eliminate transient effects.

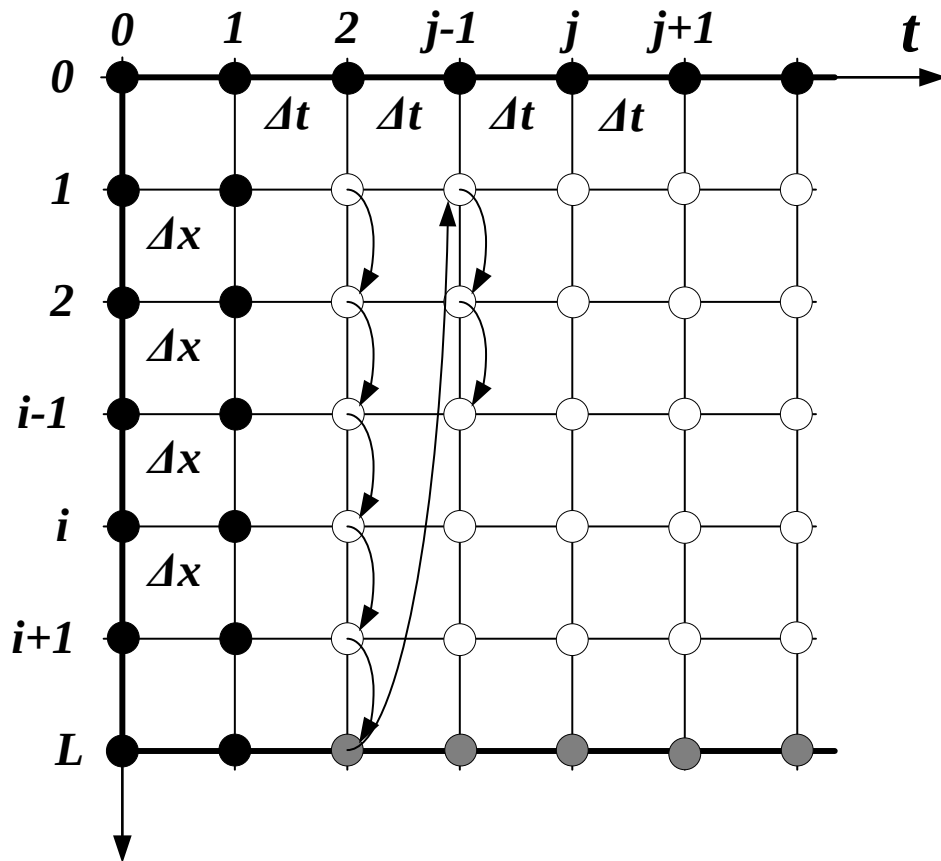


Figure 5.8-Order of Calculation on the Mesh

The flowchart of point 5, 6 and 7 can be seen in Figure 5.9, It doesn't contain the initialization (point 1-4) and the evaluation which will be described in the next chapter.

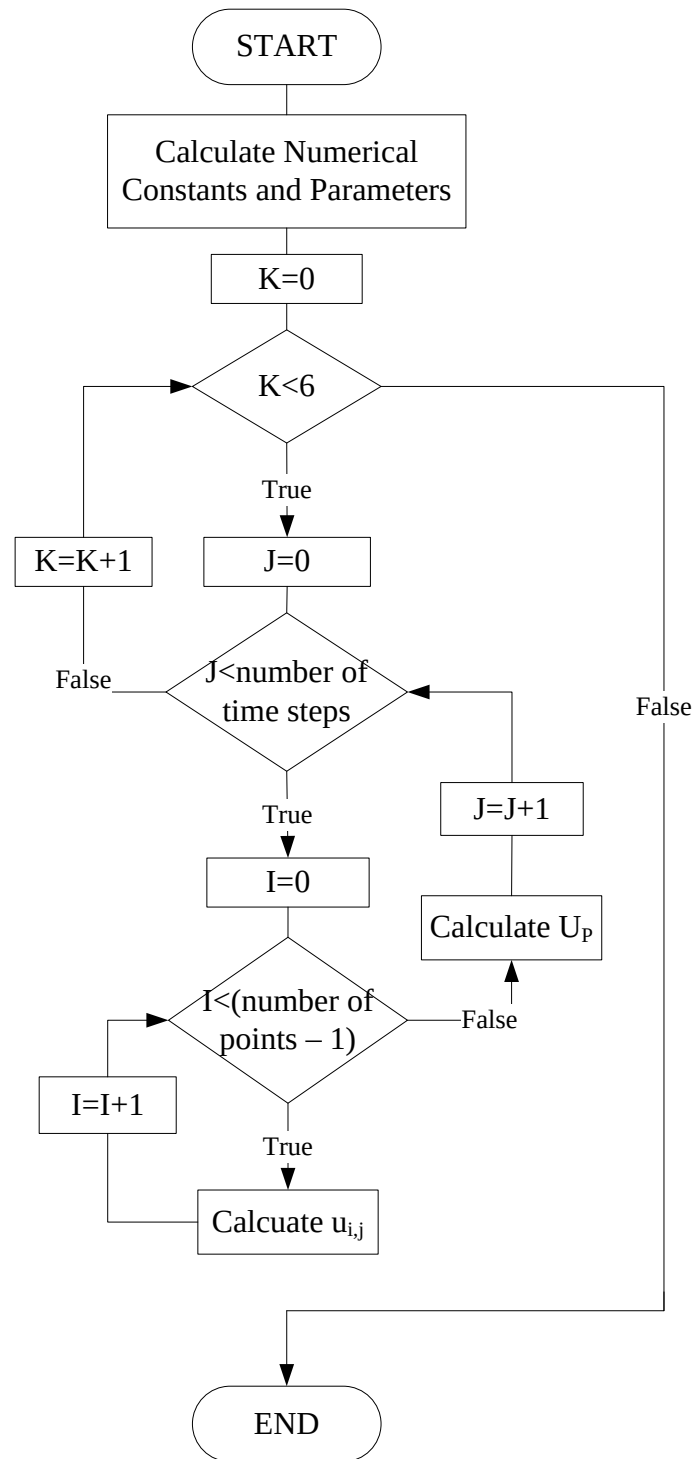


Figure 5.9-Flowchart of the Solution

6. Evaluation of the Results

The governing equations and the solution algorithm was already described in the last chapter. The next step is the evaluation of the data, which is obtained by the solution of the problem. The solution of the wave equation provides the displacement field in distance-time domain and that is not the information we are searching for. The loads, torques, safety factors and other parameters must be determined from the displacements.

The evaluation will be presented for a particular example, the basic data of this problem is the following:

Pumping Unit : C_456D_256_120	R (Crank) : 42"
Length of the Rod String : 1850 m	Rod Code : API 97
Length of the Tapers : 594.94 m(1"), 474.4 m(7/8") and 780.66 m(6/8")	
Stroke per Minute : 10 1/s	Diameter of the Plunger : 2"
Tubing : 2 7/8" (Spring Constant=1.68*10 ⁵ N/m)	
Setting Depth : 1853 m	Length of the Plunger : 1 m
Liquid Level Above the Standing Valve : 3 m	
Friction Up : 200 N	Friction Down : 400 N

6.1. Loads

The loads can be attained from the displacement by differentiation. There are many useful representation of the loads: in function of displacement, in function of crank angle or the distribution along the rod string. The following subchapters will cover the calculation and representation of these plots.

6.1.1. Calculation and Distribution of Loads Along the Sucker Rod String

The load is the first derivative of the displacement. Of course the differentiation must be done numerically, because the displacement is only known in some discrete points. Second order Taylor approximation should be used for better accuracy, which requires the displacement at three points. Figure 6.1 shows the two possible arrangement of the three points. In the first case the other two points involved in the derivation can be found below the given point, in the second case they are above. Most of the time the differentiation can be performed in both ways, but at the two ends of the rod and near connection of two tapers it can be accomplished in one way only.

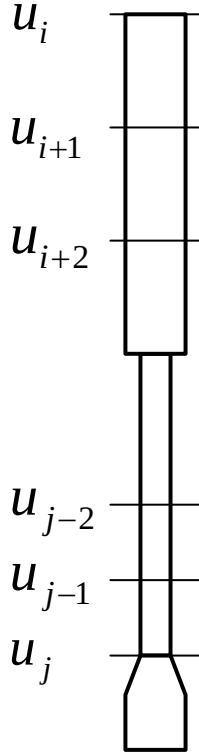


Figure 6.1-Possible Arrangements of the Discrete Points

The weight of the rod and the buoyancy also must be added to the equation, because they were not included in the model. The final equations are the following:

$$F_i = \frac{E \cdot A}{\Delta x} (1.5 \cdot u_i - 2 \cdot u_{i+1} + 0.5 \cdot u_{i+2}) + (1 - 0.128 \cdot \gamma) \cdot \sum_{k=i}^N w_k \cdot \Delta x \quad (6.1)$$

$$F_j = -\frac{E \cdot A}{\Delta x} (1.5 \cdot u_j - 2 \cdot u_{j-1} + 0.5 \cdot u_{j-2}) + (1 - 0.128 \cdot \gamma) \cdot \sum_{k=j}^N w_k \cdot \Delta x \quad (6.2)$$

The buoyancy can be accounted in two ways: buoyant weight or forces acting at the bottom of the taper. In equation 6.1 and 6.2 buoyancy is accounted as buoyant weight, which gives the so called effective load. If the buoyancy is accounted as a force coming from the hydrostatic pressure and acting on the bottom of the taper, it gives the so called true load. The true loads are the actual forces acting in the rod section, they are used for finding mechanical stresses in rod sections. For the investigation of the possible occurrence of buckling the effective loads can be used, because buoyancy cannot cause buckling [10] and the buoyant weight accounts it like loss of weight instead of a force. The true loads can be calculated with the following equations:

$$F_i = \frac{E \cdot A}{\Delta x} (1.5 \cdot u_i - 2 \cdot u_{i+1} + 0.5 \cdot u_{i+2}) + \sum_{k=i}^N w_k \cdot \Delta x - F_b \quad (6.3)$$

$$F_j = -\frac{E \cdot A}{\Delta x} (1.5 \cdot u_j - 2 \cdot u_{j-1} + 0.5 \cdot u_{j-2}) + \sum_{k=j}^N w_k \cdot \Delta x - F_b \quad (6.4)$$

The calculation of buoyancy won't be detailed here, it can be found in reference[10] or book[7].

Figure 6.2 shows the maximum and minimum forces along the sucker rod string calculated in both ways. The true loads will be used for the determination of service factors which indicate the safety against fatigue failure. Where the effective load is negative, buckling possibly occurs.

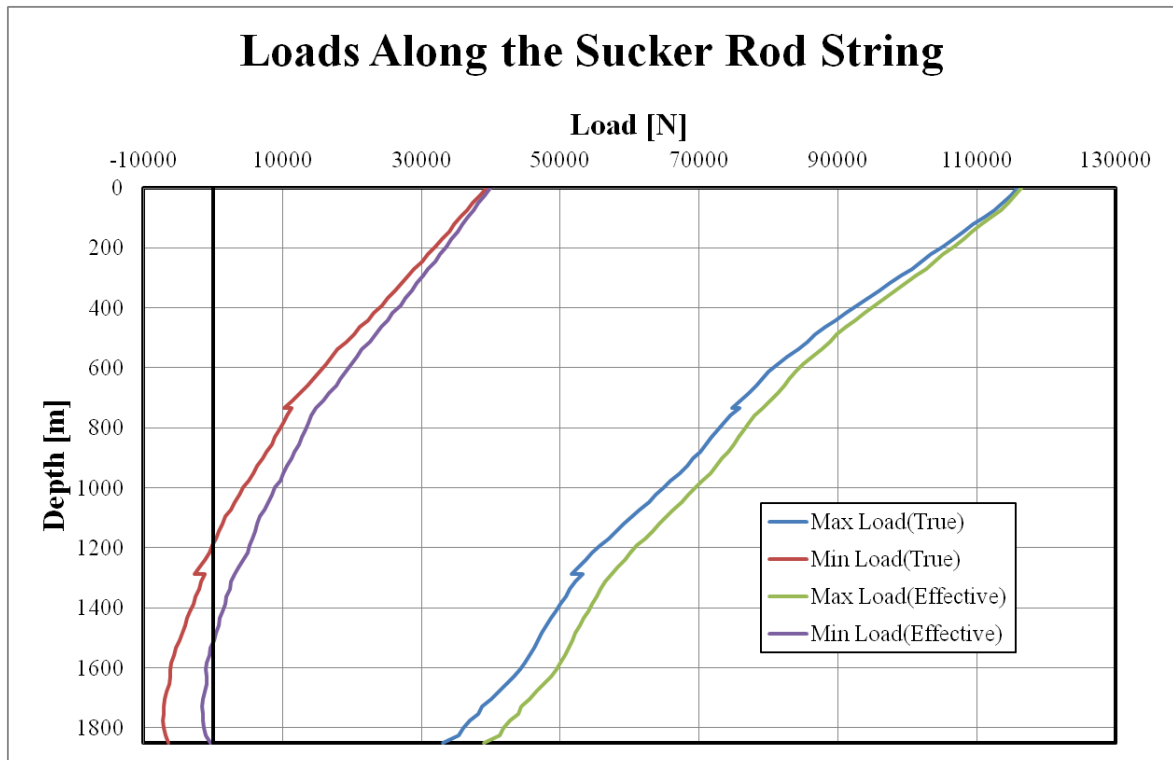


Figure 6.2-True and Effective Loads Along the Sucker Rod String

6.1.2. Dynamometer Card

The calculation procedure of loads has been shown already. Now another useful plot of the loads will be presented here. The diagram shown in Figure 6.3 is created, if loads are plotted versus displacement. This plot is called the dynamometer card. It can be used in the determination of the plunger stroke length, hydraulic power and polished rod horse power.

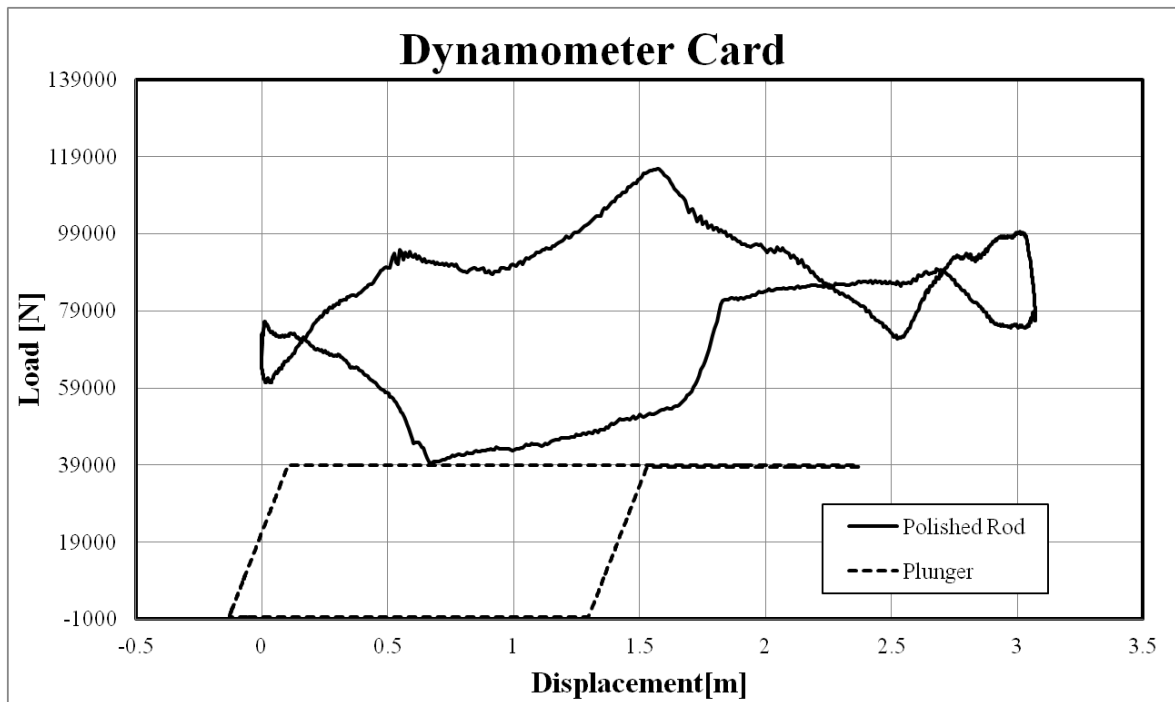


Figure 6.3-Dynamometer Card

6.1.3. Polished Rod Load

The torque calculations require the knowledge of polished rod load in function of the crank angle which can be seen in Figure 6.4.

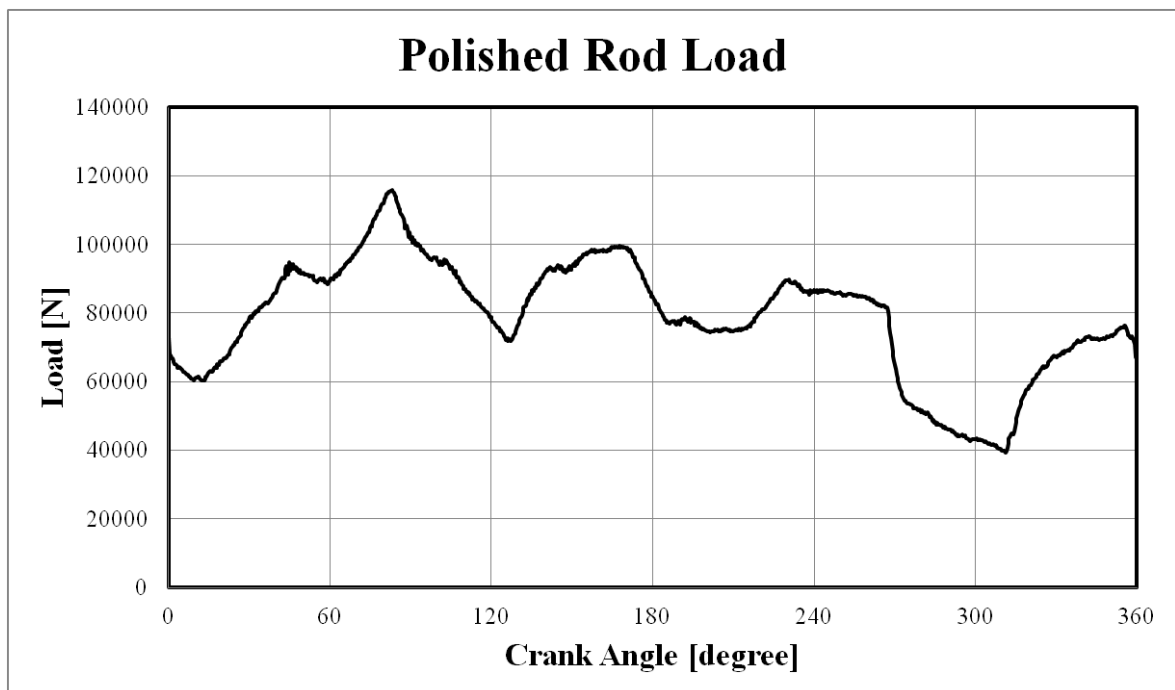


Figure 6.4-Polished Rod Load

6.2. Safety Factors

The most important operating parameter of the sucker rod string is the service factor that shows its endurance against fatigue failure. It can be calculated in the following way (for API materials) [11]:

$$SF = \frac{\sigma_{\max}}{\frac{\sigma_b}{4} + 0.5625 \cdot \sigma_{\min}} \quad (6.5)$$

where:

- $\sigma_{\max}$ - maximum rod stress [Pa]
- $\sigma_{\min}$ - minimum rod stress [Pa]
- σ_b - minimum tensile strength of the rod [Pa]

Minimum and maximum stresses must be determined from true forces.

A lower service factor denotes higher resistance against fatigue failure. If its value is higher than 1, the string surely breaks after a finite number of pumping cycles. The service factor increases upward along a sucker-rod taper, generally the most endangered part is the top of the sucker rod taper, where the service factor is the highest. The extreme stresses of the uppermost point of the tapers can be plotted on the non-dimensional modified Goodman diagram[11] as seen on Figure 6.5. This plot helps to visualize the problem.

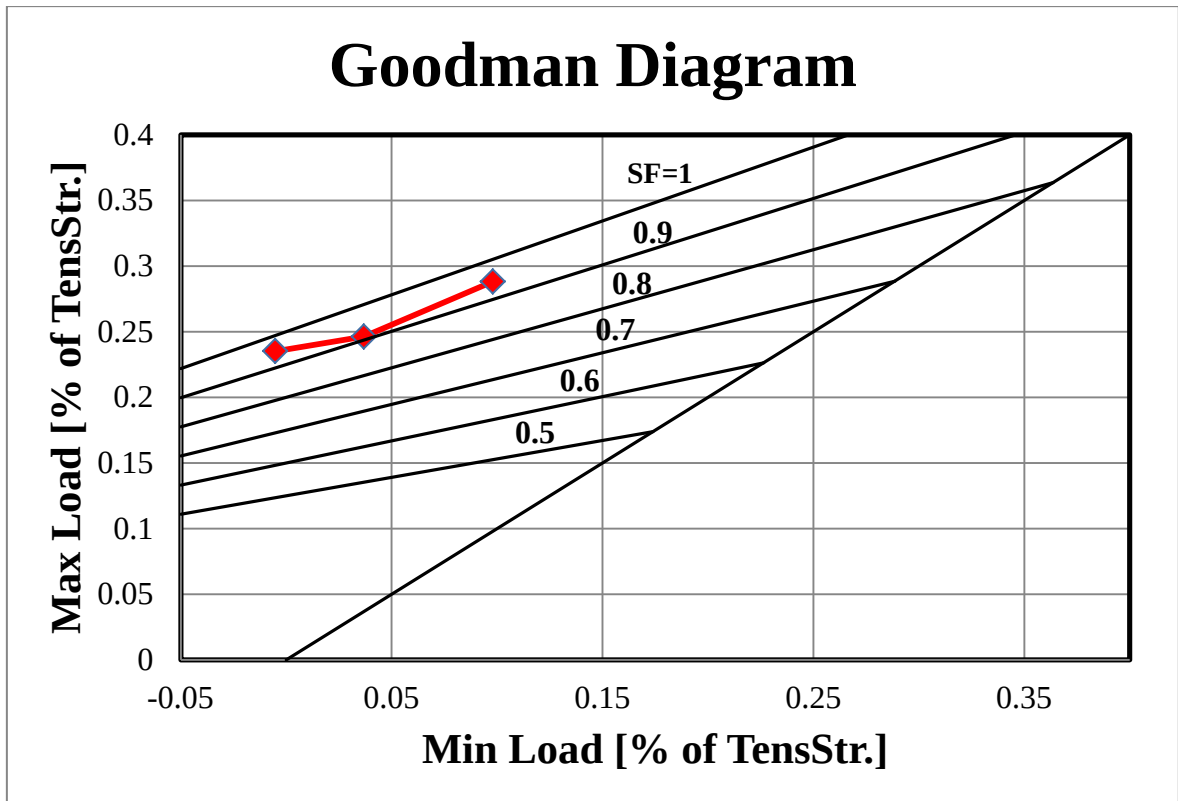


Figure 6.5-Goodman Diagram

6.3. Occurrence of Buckling

The sucker-rod string is a slender bar in the mechanical point of view. If a slender bar is under compression, it will buckle, and takes a sinusoidal or helix shape. The buckling causes additional bending stresses and mechanical friction, which seriously decrease the lifetime of a rod.

At the bottom of the sucker rod string there is always compression during the downstroke because of friction of the plunger etc. The critical force required to buckle the rod section, which is under compression, can be determined with the following equation after Euler:

$$F_{crit} = \frac{\pi^2 \cdot E \cdot I_2}{l^2} \quad (6.6)$$

where:

- I_2 - moment of inertia of the rod [m⁴]
- l - length of the rod section under compression [m]

The rod section, where the minimum effective loads are negative, has to be checked. If compressive forces are higher than the critical value, the rod buckles in that section.

In the lowest 320 meter of the sucker rod string the minimum effective load is negative, the critical force for this section is less than 1 N, so this section buckles in every downstroke.

6.4. Plunger Stroke Length

The surface stroke length easily can be determined from the kinematics of the pumping unit, but the plunger stroke length can considerably differ from that value. The plunger stroke length is defined as the travel of the pump plunger relative to the working barrel, because this relative movement causes the displacement of the produced fluids in the pump. It can be read from downhole dynamometer card as Figure 6.6 shows.

Its value in our case is 2.2679 meter which is significantly less than the 3.072 meter of the surface stroke length.

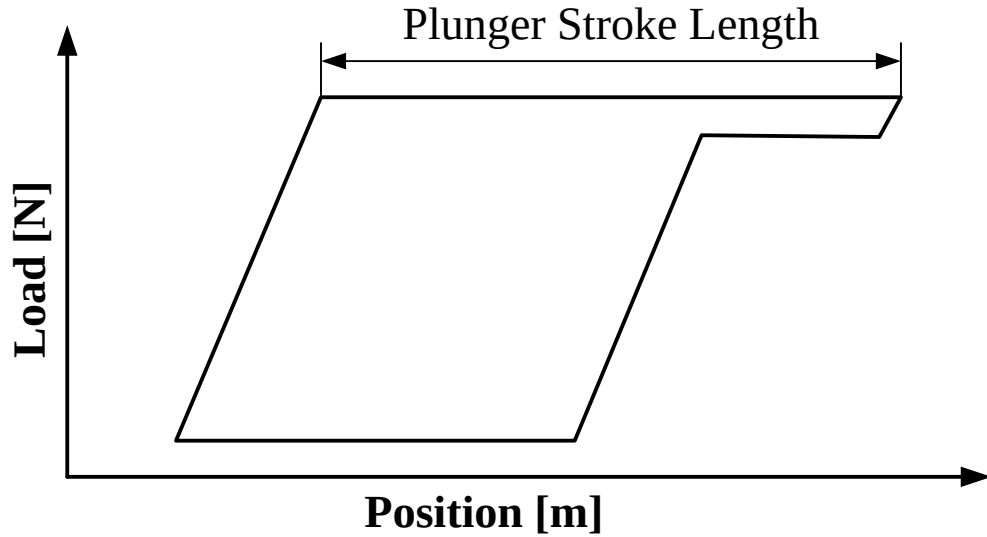


Figure 6.6-Determination of the Plunger Stroke Length

6.5. Hydraulic Power and Lifting Efficiency

The power requirement of pumping is an important technical and economical parameter of the pumping installation. The pumping system useful output work is done by the downhole pump. This work is generally described by the so-called hydraulic power (P_{hydr}), this is the power which required to move the fluid to the surface.

The work done by the pump during a pumping cycle equals to the area of the pump's dynamometer card (A_{pump}), which can be seen on Figure 6.7. It can be calculated with the following integral or its numerical approximation:

$$W_{\text{pump}} = \int_{1_pumping_cycle} F_{\text{pump}} du \approx \sum_{i=1}^N \left[\frac{F_{\text{pump}(i)} + F_{\text{pump}(i-1)}}{2} \cdot (u_{\text{pump}(i)} - u_{\text{pump}(i-1)}) \right] \quad (6.7)$$

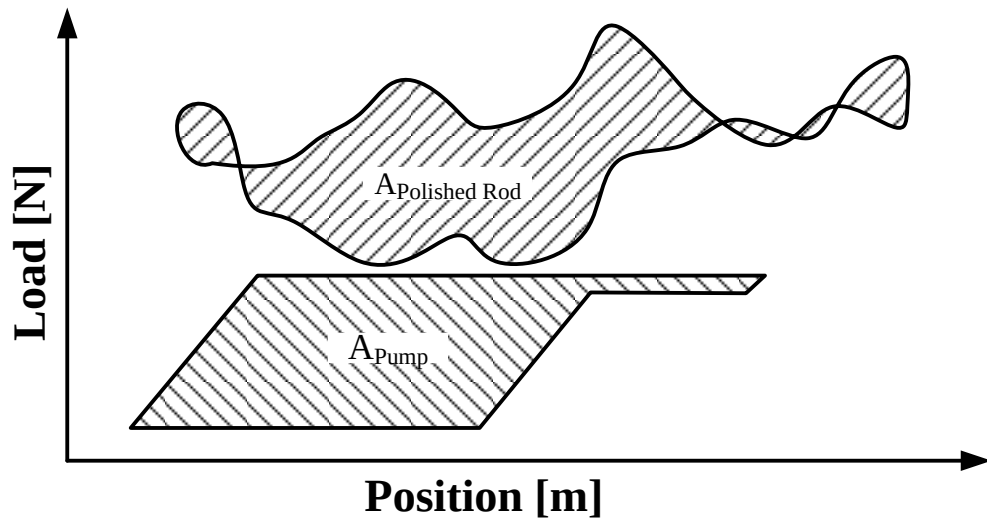


Figure 6.7-Work Done by the Pump and Polished Rod

From the work of the pump the required power is easily found:

$$P_{hydr} = \frac{W_{pump}}{(60/SPM)} \quad (6.8)$$

The power required at the polished rod contains in addition to the useful work the energy losses occurring in the well. The calculation of the work and power, which are required at the polished rod, is similar to pump's ones. It has to be mentioned, that the model doesn't account for leakage losses in the pump-barrel clearance.

The required power at the pump in case of the example is 9465.6W (12.70 HP). The same for the polished rod is 12818.1 W (16.19 HP).

In any system where energy loss takes place the power efficiency can be an informative number. Its calculation is very simple:

$$\eta_{lift} = \frac{P_{hydr}}{PRHP} \quad (6.9)$$

For this example its value is 0.738.

6.6. Torques and CLF

The polished rod load is transformed into torque at the crankshaft. In order to decrease the net torque arising on the crankshaft there are counterweights on the crank arms, which generate torque in the opposite direction. In the analysis of a pump installation the accurate prediction of the torques acting on the crankshaft is essential.

The torque caused by the rod load can be calculated by the following formula:

$$T_{RL}(\theta) = F(\theta) \cdot TF(\theta) \quad (6.10)$$

where F is the polished rod load and TF is the torque factor in the function of the crank angle. Torque factors represent imaginary lever arms which, when multiplied with the polished rod load valid at the same crank position, give the resultant crankshaft torque. The determination of the torque factor won't be detailed here.

The calculation of counter balance torque is much easier:

$$T_{CB}(\theta) = -M \cdot \sin(\theta) \quad (6.11)$$

where:

M - maximum moment of the counterweights, cranks, and crank pins [Nm]

The net torque, T_{Net} is the sum of the two kinds of torque. Figure 6.8 illustrates the different torques calculated for the example used for the evaluation. Red lines represent the standard peak torque capacity of the gearbox. Overload can be observed both upstroke and downstroke.

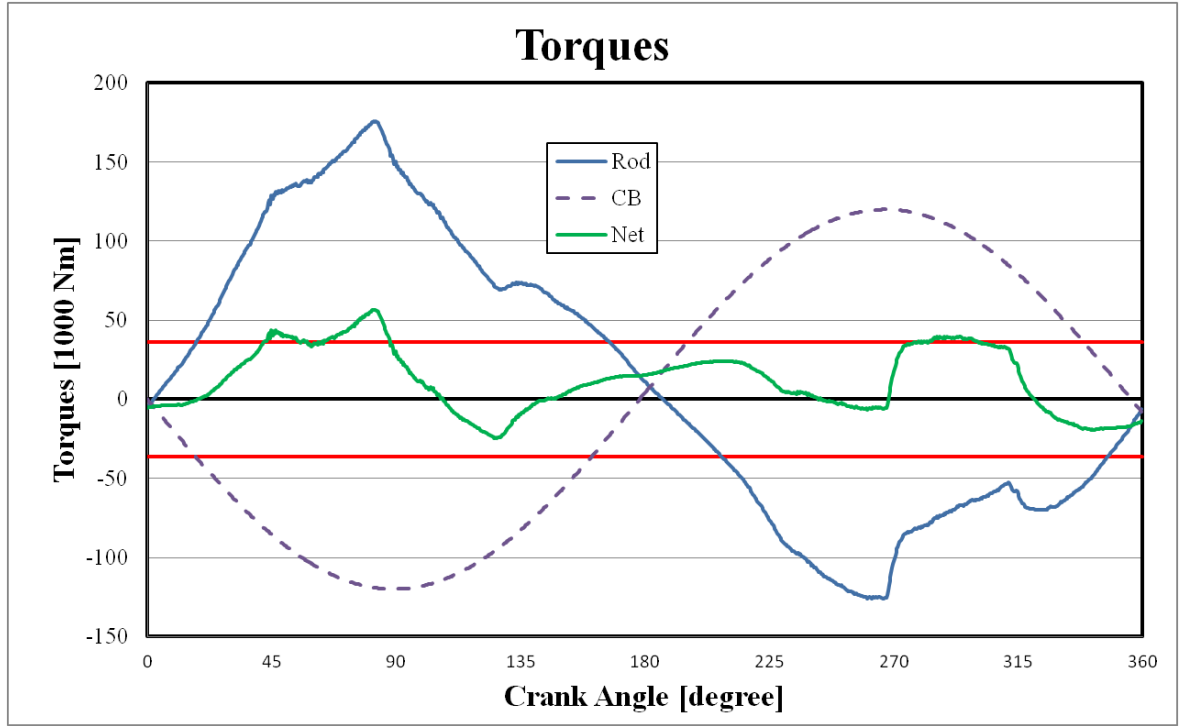


Figure 6.8-Different Kind of Torques

The mechanical cyclic load factor (CLF) used for the characterization of counterbalancing is similar to the electric cyclic load factor used for electric motors. The cyclic load factor characterizes the energy requirement of fluid lifting, so it is often used for the optimization of the counterbalancing. Since the required prime mover power is directly proportional to the cyclic load factor, the lower the cyclic load factor, the lower the energy consumption. The definition of cyclic load factor is[12]:

$$CLF = \frac{\int_{\theta=0}^{2\pi} \sqrt{[T_{Net}(\theta)]^2} d\theta}{\frac{\int_{\theta=0}^{2\pi} T_{Net}(\theta) d\theta}{2\pi}} \quad (6.12)$$

The value of cyclic load factor is 1.44 for the example.

7. The Effect of Fluid Pound on Operating Parameters

In the previous chapters a new numerical method was described that can treat the fluid pound condition. This method helps to investigate the changing of the operating parameters as the fluid level drops. In this chapter the fluid pound will be compared to the normal condition. The basic data of the pumping system is the same with the example in chapter 6. (excluding that friction of the pump is zero both in up- and downstroke). Four different fluid levels are taken to compare them with the normal condition, their heights above the standing valve are 2 , 2.5 , 3 and 3.5 meters.

7.1. Loads

Figure 7.1 compares the dynamometer cards of the normal condition and the fluid pound, when the liquid level is above the standing valve by 2.5 meters. The comparison of the other three cases can be found in Appendix A. The peak polished rod load (PPRL) is hardly changed by the fluid pound, but the minimum polished rod load (MPRL) is significantly increased. The rod load is high during most of the pumping cycle, so the stretch of the rod is increased. This additional stretch changes the displacement range of the pump, so it can hit down, which is another really harmful pumping malfunction.

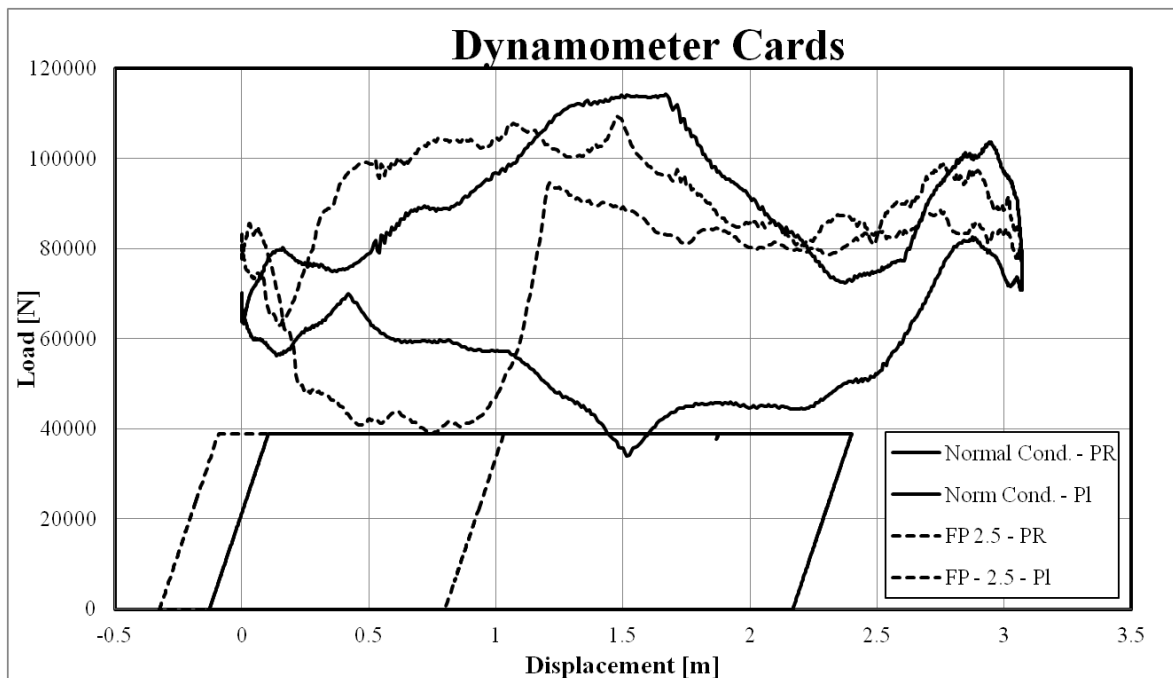


Figure 7.1-Comparison of Dynamometer Cards

The minimum and maximum load of normal condition and fluid pound are compared in **Error! Reference source not found..** The comparison of the other three cases can be found in Appendix B. Maximum load is decreased due to fluid pound, which is

favourable. The minimum load is much lower in the lowest section of the rod string, that greatly decrease the service factor and the endurance against fatigue failure. The minimum load in the uppermost section of the rod string is increased, that is also favourable.

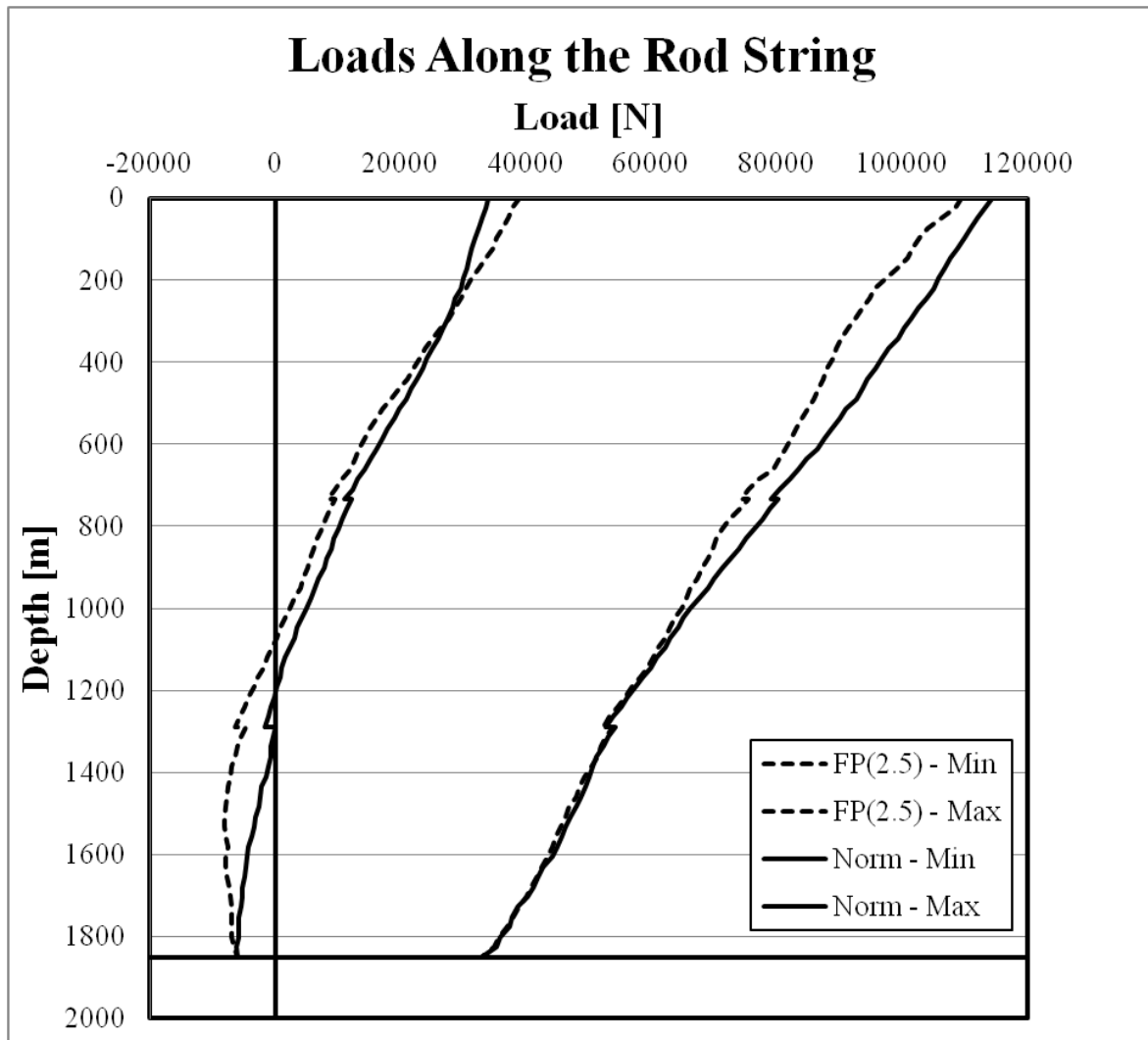


Figure 7.2-Loads Along the Rod String

7.2. Service Factors

Figure 7.3 shows the modified Goodman Diagram, where the stresses are plotted for the different cases. In most of the cases the fluid pound decreases the service factors, so it has a positive effect on it, excluding when the fluid level is in the middle of the displacement range of the pump (this is represented by the 2.5 m high liquid level). The possible reason of this is that the pump moves much faster in this region than in the two ends of its displacement range.

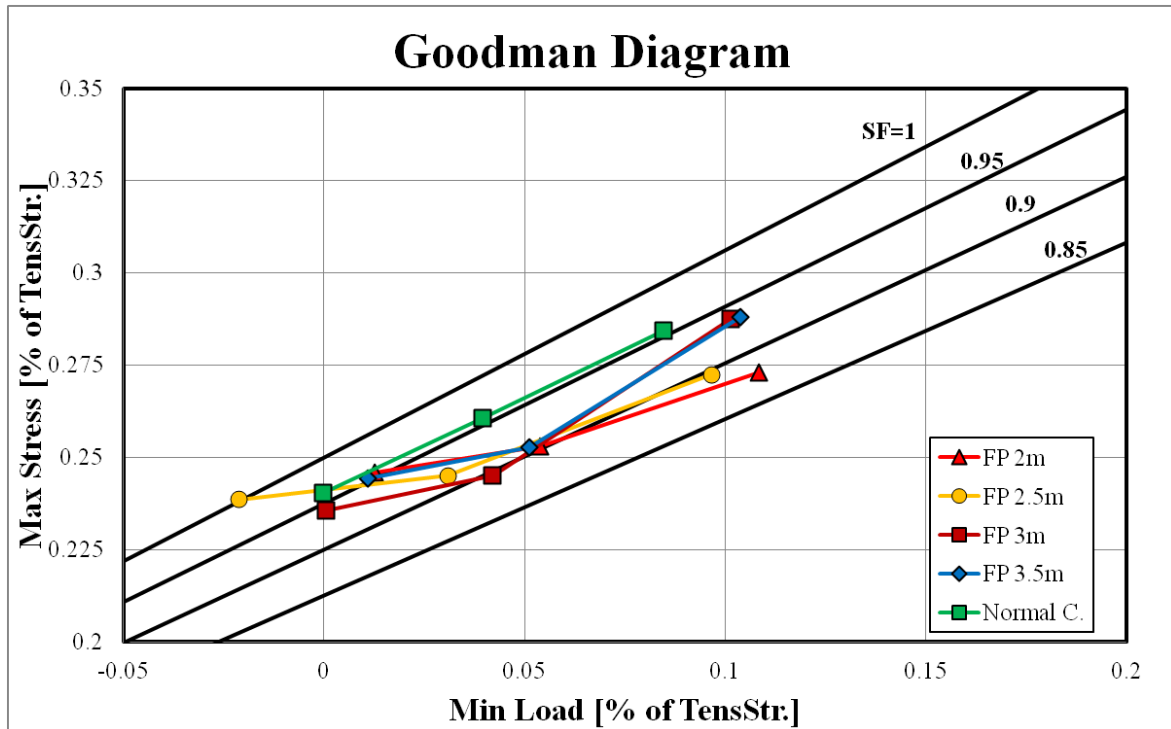


Figure 7.3-Comparison on Goodman Diagram

7.3. Buckling

Beside normal condition the lowest 115 m is exposed to buckling, this is caused by the fast pumping speed, at lower speed the occurrence of buckling is less likely. When the liquid level is 3.5 and 3 m above the standing valve the lowest 205 and 240 m buckles, which are twice longer sections than in normal operation. When the liquid level is 2.5 m above the standing valve the lowest 590 m buckles, this is an extremely high value. Surprisingly, when the liquid level is only 2 m above the standing valve there is no buckling at all, the reason can be that the pump is loaded with full fluid load for almost the whole pumping cycle and it is already slowed down, when it reaches the liquid level. The figures showing the effective loads along the string can be found in Appendix C.

7.4. Plunger Stroke Length

The plunger stroke length is 2.296 m for the normal condition. First the plunger stroke length decreases as the fluid level decreases (3.5m - 2.254 m), but after a given point it starts to increase (3m - 2.286 m) and finally its value becomes greater than in case of normal condition (2.5m - 2.417 m, 2m - 2.387 m).

7.5. Hydraulic Power and Lifting Efficiency

The following list shows how the parameters change as the liquid level drops:

- Normal Condition - $P_{Hydr} = 14.85$ kW, $PRHP = 18.34$ kW, $\eta = 80.97\%$

- Liquid Level 3.5 m above Standing Valve - $P_{Hydr} = 12.31$ kW, $PRHP = 15.71$ kW, $\eta = 78.35\%$
- LL. 3 m above SV - $P_{Hydr} = 9.27$ kW, $PRHP = 12.62$ kW, $\eta = 73.46\%$
- LL. 2.5 m above SV - $P_{Hydr} = 7.27$ kW, $PRHP = 10.76$ kW, $\eta = 67.53\%$
- LL. 2 m above SV - $P_{Hydr} = 3.59$ kW, $PRHP = 7.13$ kW, $\eta = 50.41\%$

As the liquid level drops and fluid pound occurs the hydraulic power and polished rod horse power decrease monotonically, and the lifting efficiency deteriorates as expected.

7.6. Torques and CLF

The moment of the counterweights was determined by minimizing the peak net torque[13] in order to avoid overloading the gearbox. Its value is 118 000 Nm and of course it doesn't change when the fluid pound occurs. The comparison of torques can be seen in Figure 7.4, the comparison in the other three cases can be found in Appendix D. The rod torque hardly changes during the upstroke, but it significantly increases during the downstroke and because of that the net torque also decreases in downstroke. The peak net torque doesn't have considerably higher value in any of the four investigated cases.

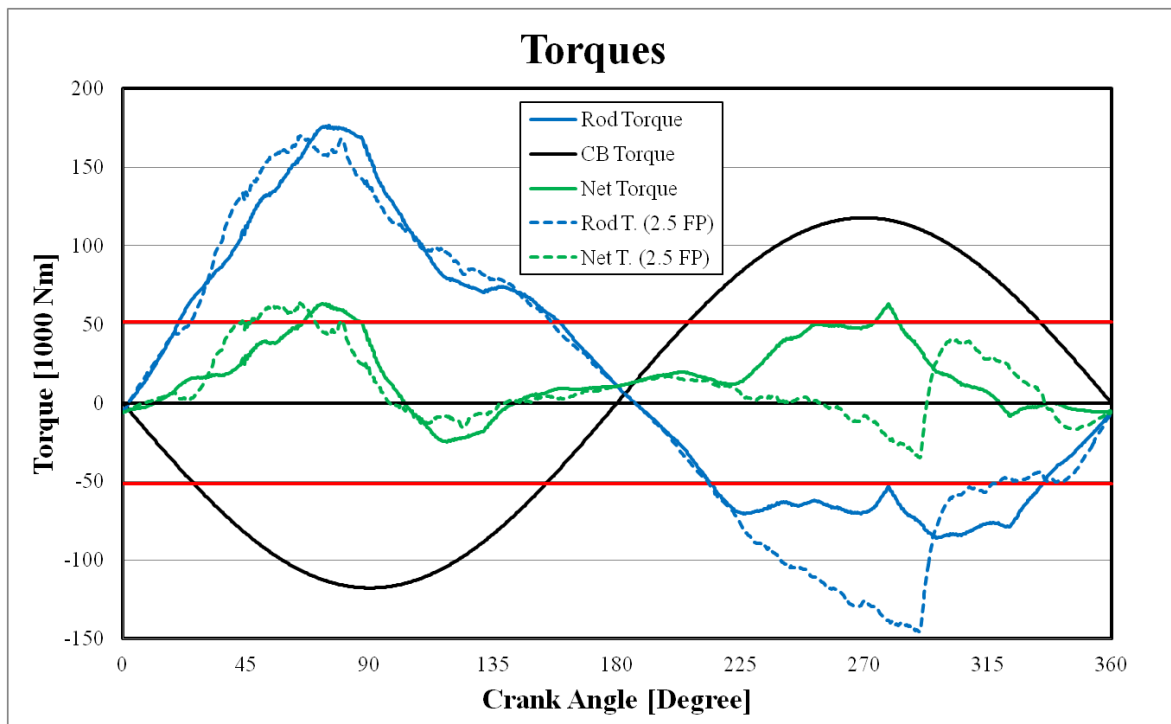


Figure 7.4-Comparison of Torques, Normal Condition vs. Fluid Pound

The cyclic load factor is 1.246 for the normal condition, its value is monotonously deteriorates as the fluid level drops (3.5m - 1.309, 3m - 1.490, 2.5m - 1.653, 2m - 2.486). As it can be seen from the numbers, this deterioration becomes more serious when the fluid level drops below the middle of the displacement range of the pump.

7.7. Conclusions

In the previous subchapter the effect of the fluid pound was investigated, sometimes it causes serious problems, sometimes only moderate ones. Because it is outside of the coverage of this paper, the effect of polished rod stroke length and stroke per minute on fluid pound were not studied. If the polished rod stroke length is shorter and the stroke per minute is lower, the fluid pound condition is less harmful. In these examples the polished rod stroke length and the stroke per minute was above the average. An average intermittent pumping installation generally operates with much lower stroke per minute and stroke length is also smaller most of the time, so the effect of fluid pound on safety factors and on buckling is less critical.

It can be stated generally that the pumping efficiency including both hydraulic and mechanical (CLF) deteriorates as the liquid level drops. Occurrence of buckling also increase significantly in most of the cases. The service factor increases only when the liquid level is in the middle third of the displacement range of the pump. The peak net torque was not increased considerably by fluid pound conditions.

The liquid level can be allowed to drop into the upper third of the displacement range of the pump, because it doesn't decrease the safety factors and doesn't cause serious efficiency loss. If the liquid level is in the middle third, it significantly increases the safety factors, which is dangerous. If the liquid level is in the lower third, it seriously deteriorate the pumping efficiency, so it has to be avoided too.

8. Summary

After a brief overview of the basics of sucker rod pumping and the theory of fluid pound, a novel mathematical approach was introduced to describe the fluid pound condition. Every step of the numerical solution was detailed including the solution scheme, discretization, initial condition, boundary conditions and the solution algorithm. The dynamic boundary condition was described in more detail, because it is far the most complicated part of the solution

The numerical simulation yields the displacement field on the time-depth domain, which requires evaluation to provide the necessary information about the operation of the pumping system. First of all the loads along the sucker rod string have to be determined from the displacement. From it the safety factors can be calculated directly and the occurrence of buckling is easily detectable on the effective load vs. depth plot. The hydraulic power and the polished rod horsepower can be calculated from the dynamometer cards. The torques and the cyclic load factor can be determined from the polished rod loads.

After the detailed description of the evaluation procedure the effect of the fluid pound on the pumping system was investigated including all the important operation parameters. Generally it can be stated that the fluid level can be allowed to drop into the upper third of the displacement range of the pump, whereas fluid level in the lower two third it are not recommended.

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10. Appendices

Appendix A

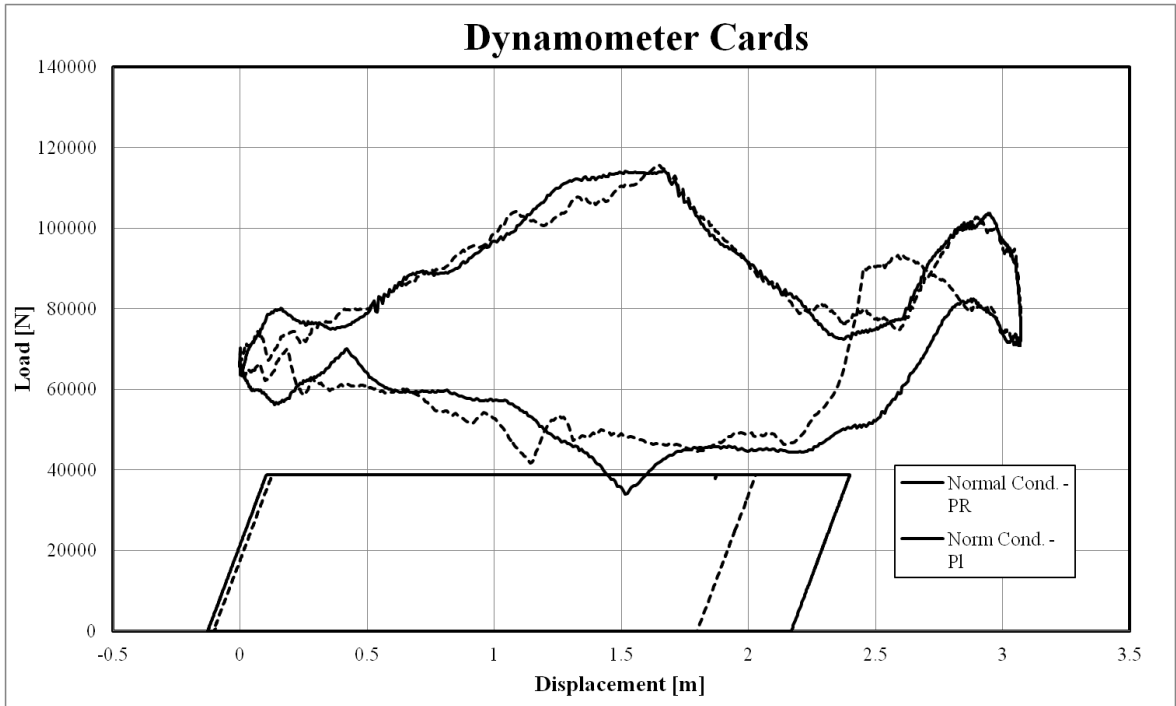


Figure A.1- Comparison of Dynamometer Cards (3.5 m)

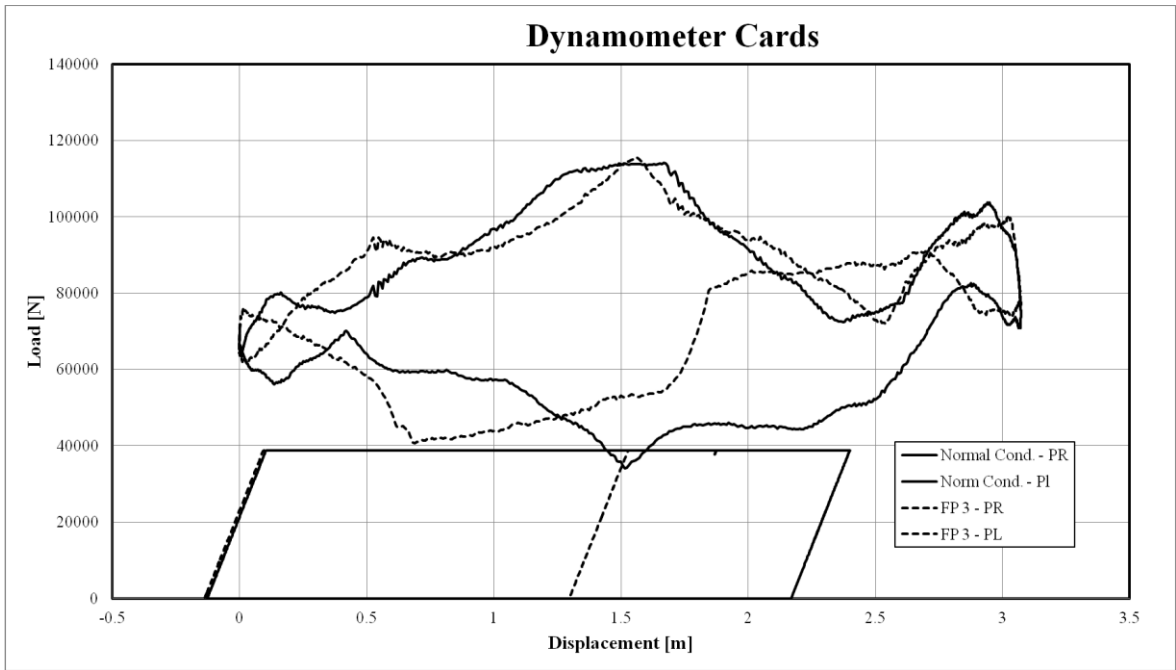


Figure A.2- Comparison of Dynamometer Cards (3 m)

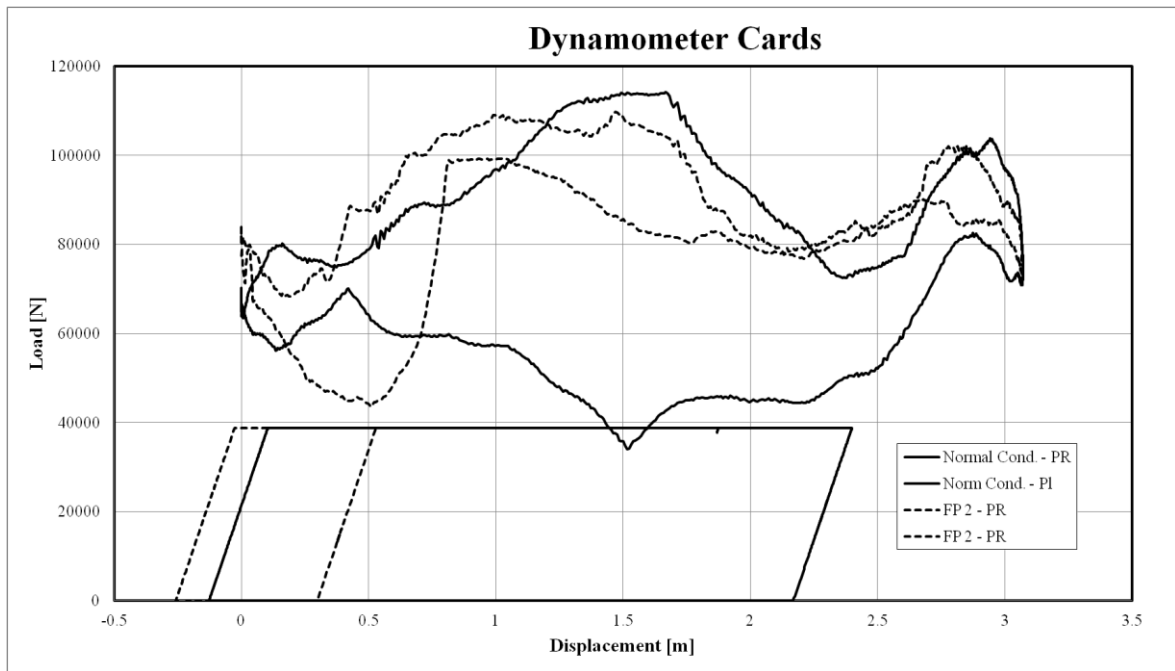


Figure A.3- Comparison of Dynamometer Cards (2 m)

Appendix B

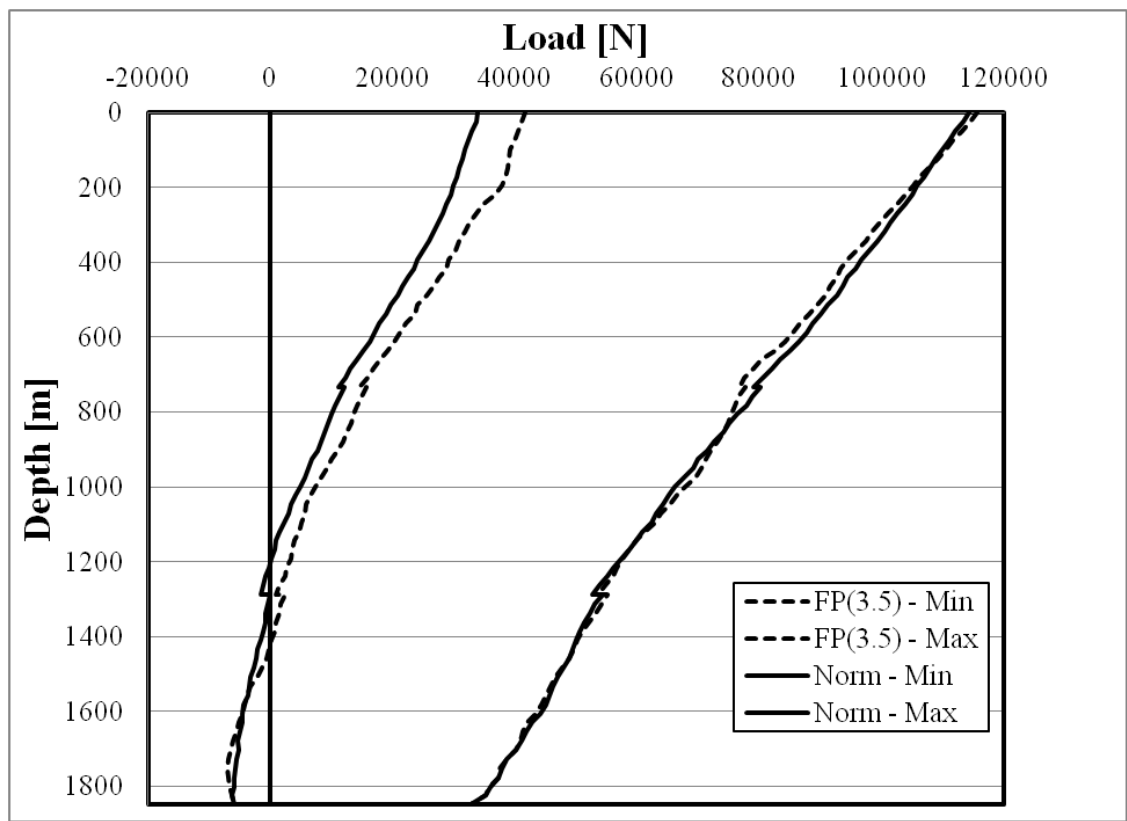


Figure B.1-Comarison of True Loads Along the Rod String(3.5 m)

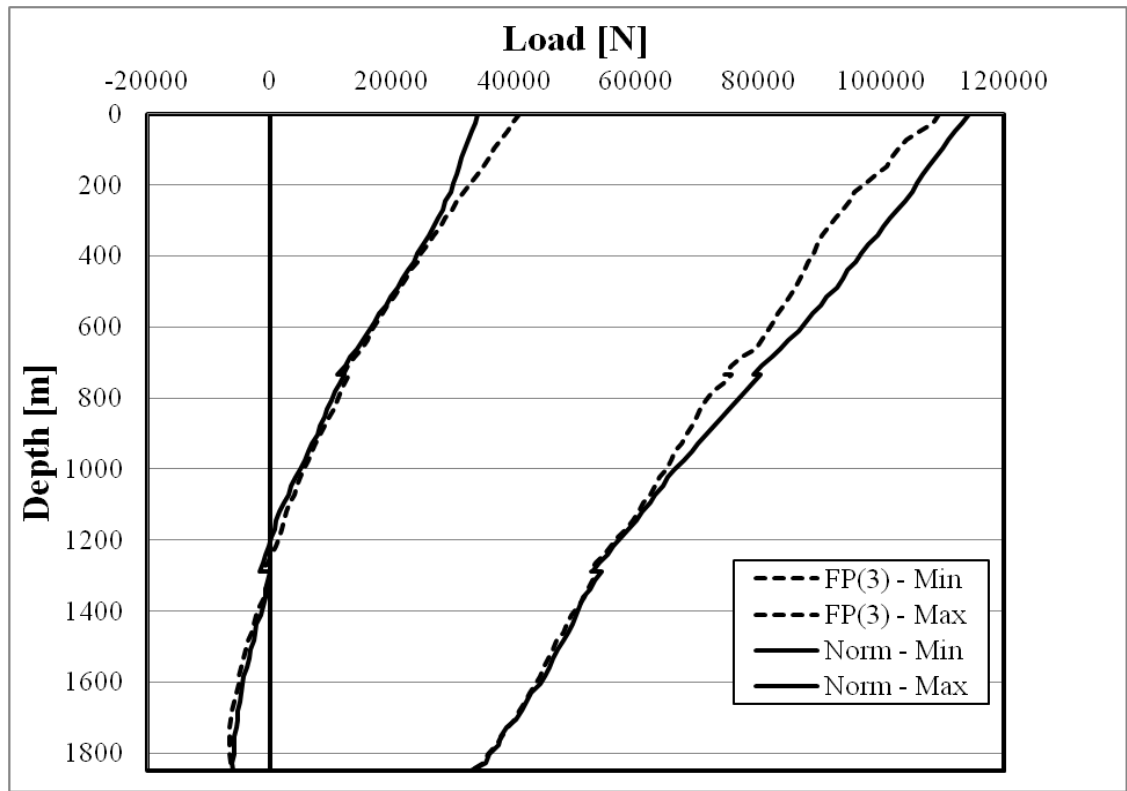


Figure B.2- Comparison of True Loads Along the Rod String(3 m)

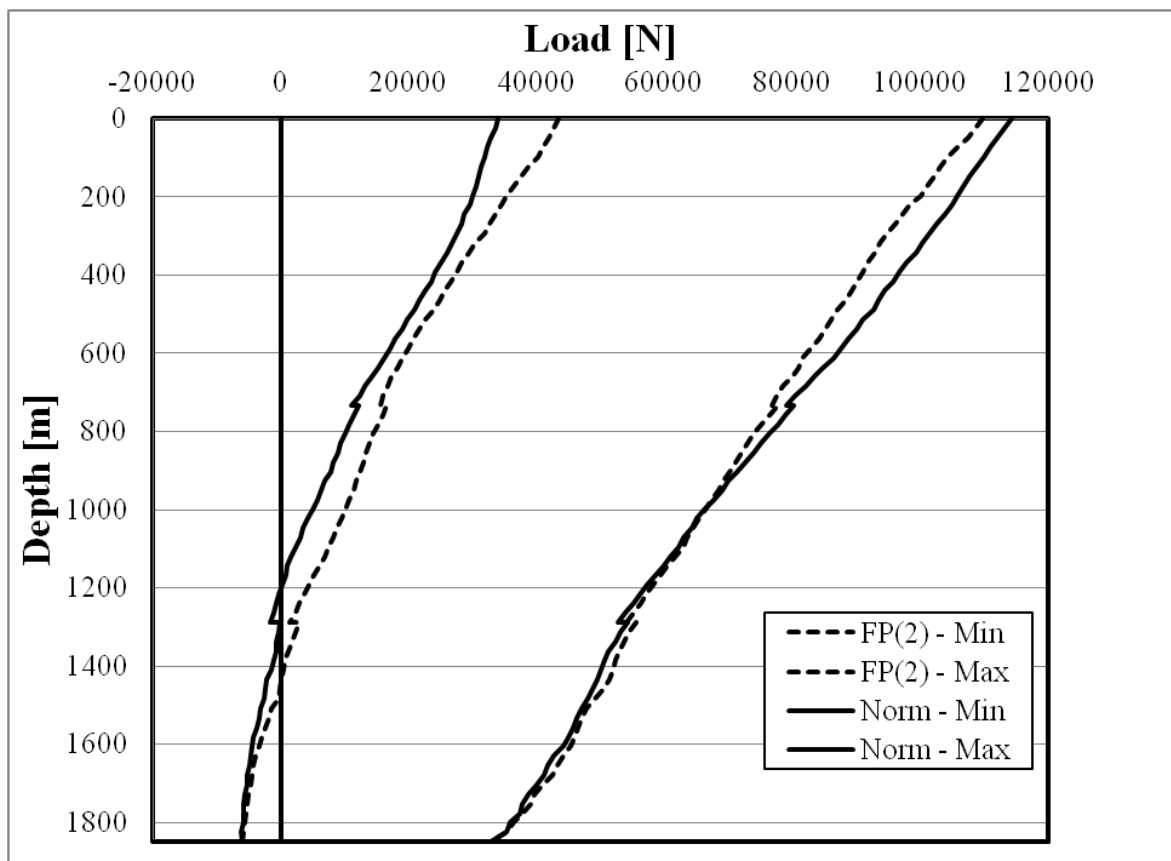


Figure B.3- Comparison of True Loads Along the Rod String(2 m)

Appendix C

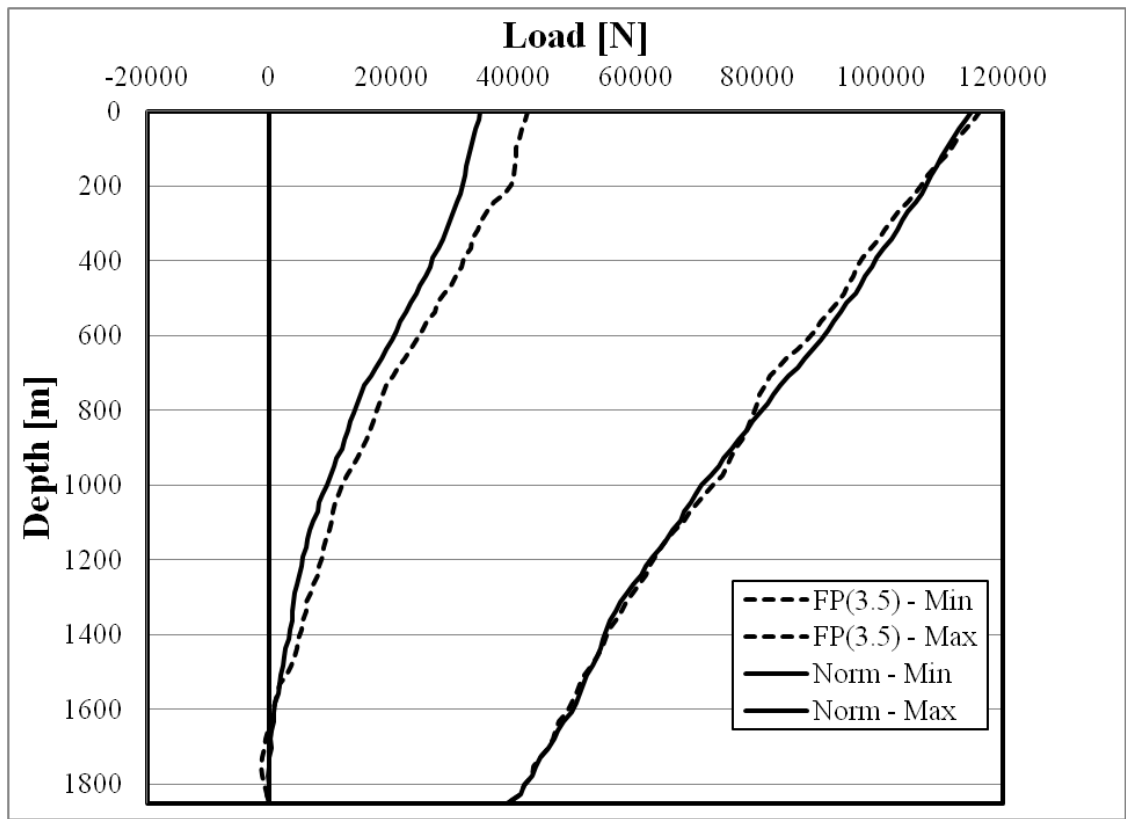


Figure C.1- Comparison of Effective Loads Along the Rod String(3.5 m)

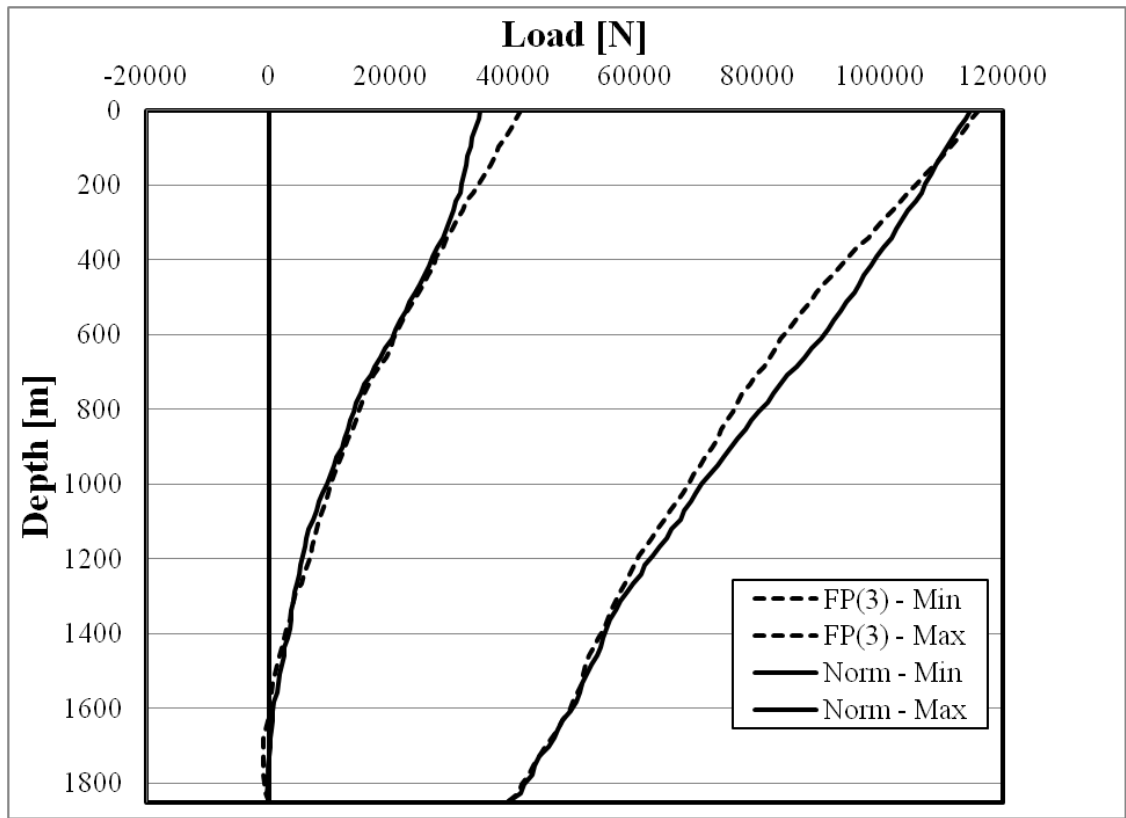


Figure C.2- Comparison of Effective Loads Along the Rod String(3 m)

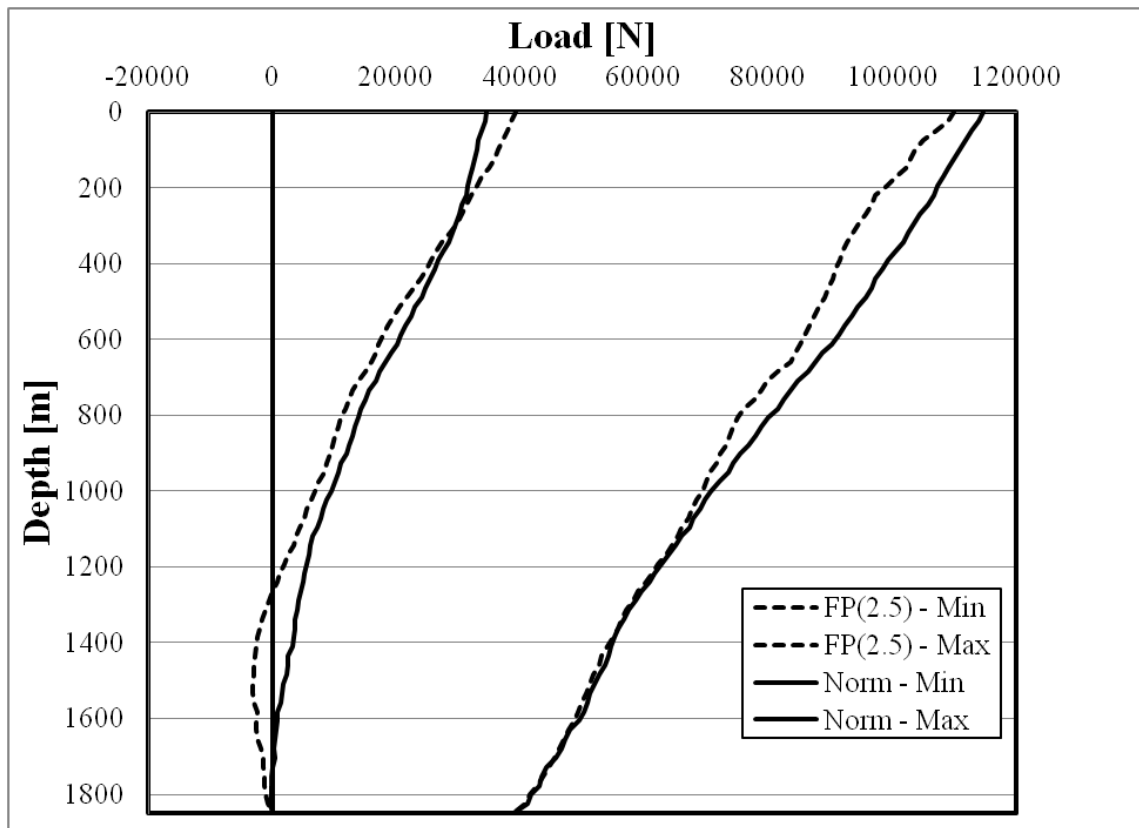


Figure C.3- Comparison of Effective Loads Along the Rod String(2.5 m)

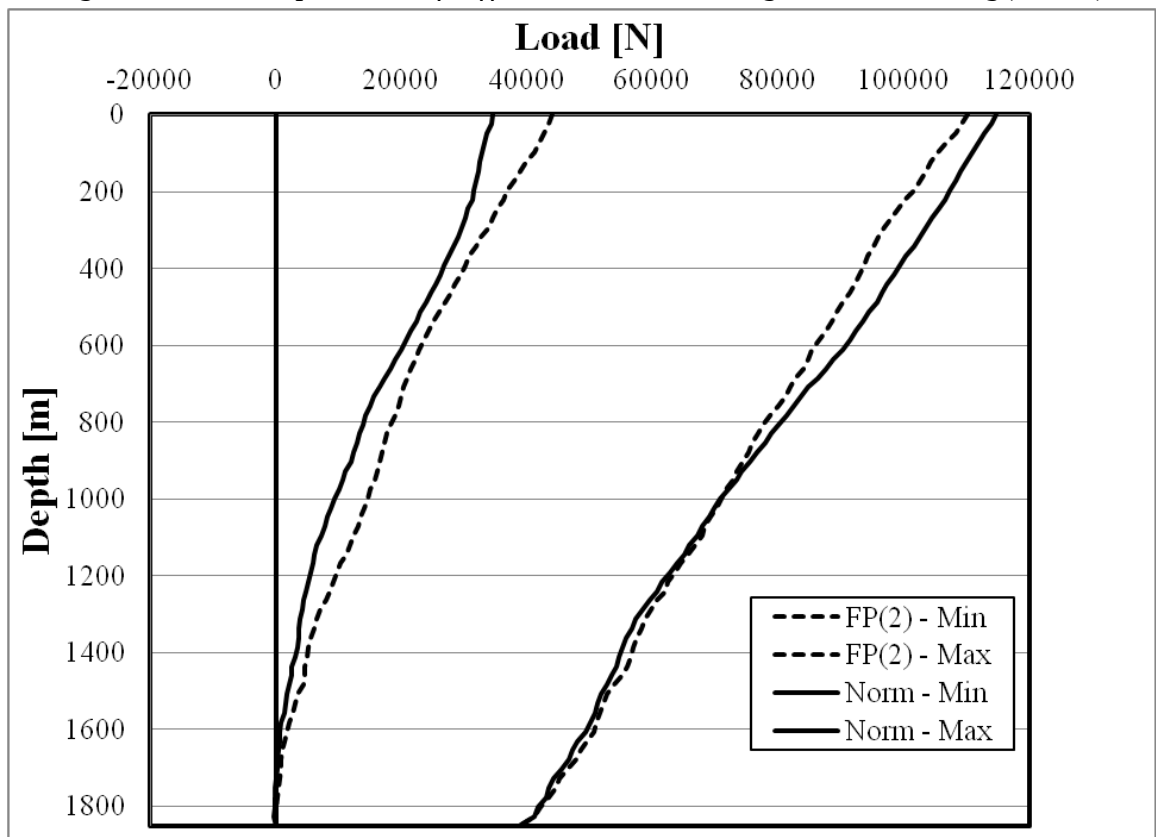


Figure C.4- Comparison of Effective Loads Along the Rod String(2 m)

Appendix D

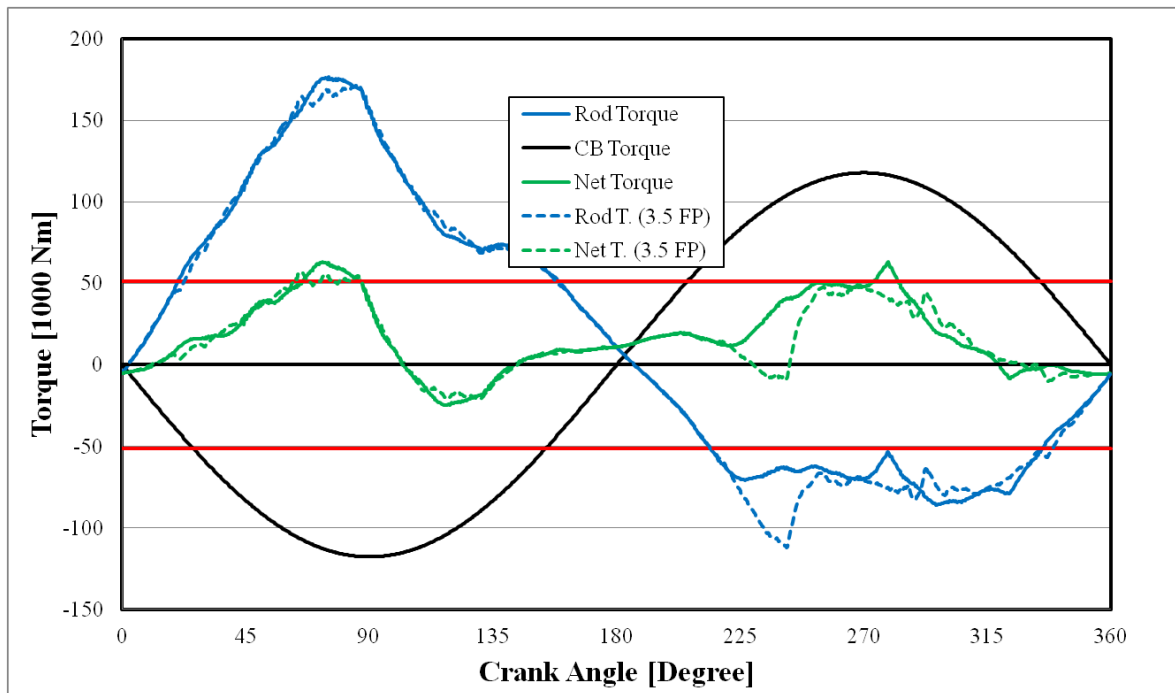


Figure D.1-Comparison of Torques (3.5m)

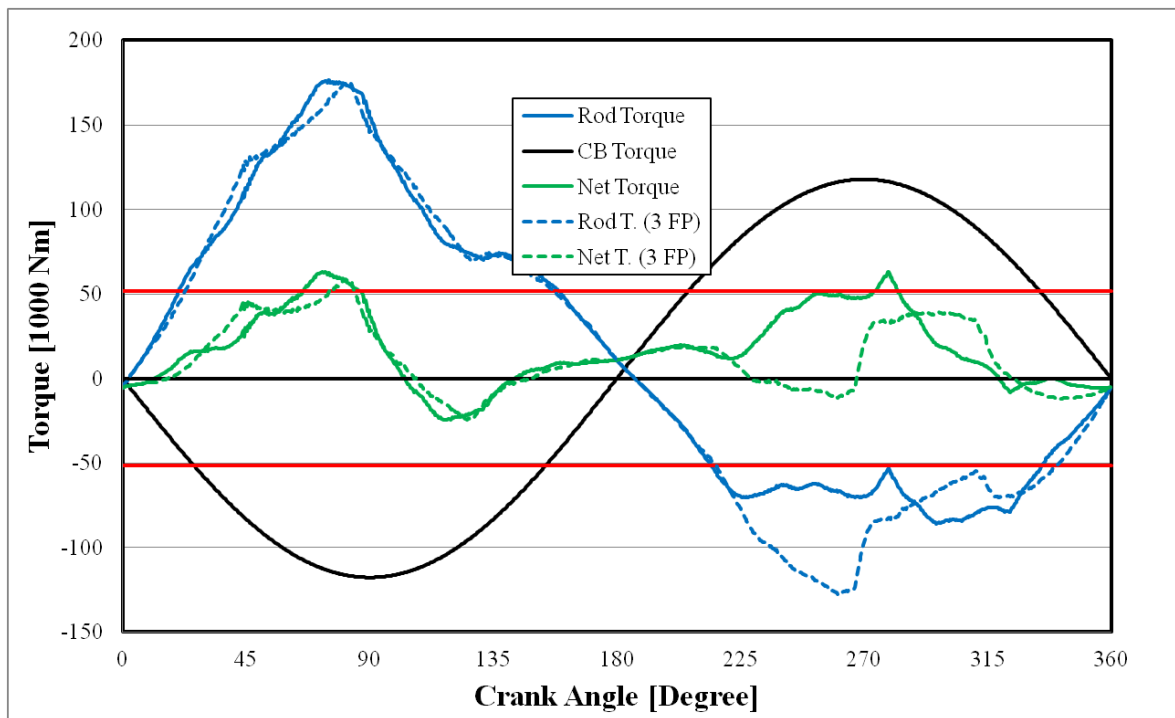


Figure D.2- Comparison of Torques (3 m)

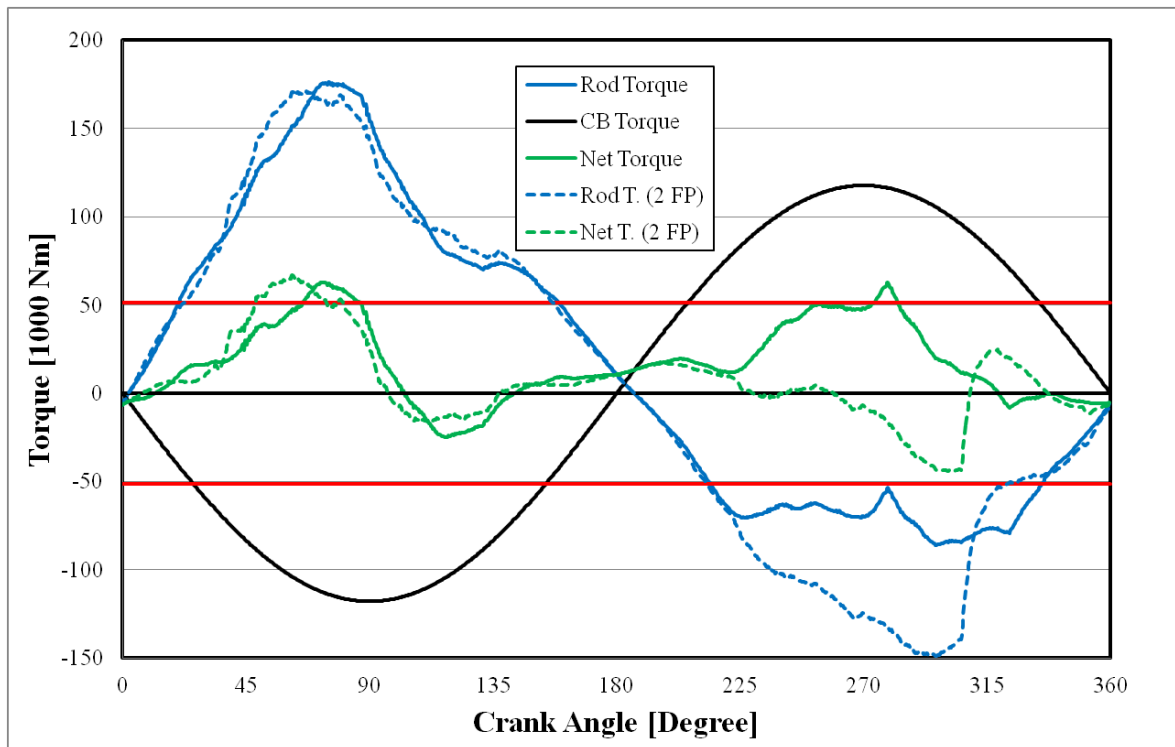


Figure D.3-Comparison of Torques (2 m)