

10.2 – Mathematica brachiation

Az acrobot mozgásegyenlete

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ω1 = D[q1[t], t];
ω2 = D[q1[t], t] + D[q2[t], t];
rA1B1 = {L1 Sin[q1[t]], -L1 Cos[q1[t]], 0};
rA1S1 = {xG1 Sin[q1[t]], -xG1 Cos[q1[t]], 0};
rA2S2 = {xG2 Sin[q1[t] + q2[t]], -xG2 Cos[q1[t] + q2[t]], 0};
vs1 = Cross[{0, 0, ω1}, rA1S1];
vs2 = Cross[{0, 0, ω1}, rA1B1] + Cross[{0, 0, ω2}, rA2S2] // Simplify;
T = 1 / 2 m1 vs1.vs1 + 1 / 2 (I1 - xG1^2 m1) ω1 ω1 + 1 / 2 m2 vs2.vs2 + 1 / 2 (I2 - xG2^2 m2) ω2 ω2;
U = -m1 g xG1 * Cos[q1[t]] - m2 g (L1 Cos[q1[t]] + xG2 Cos[q1[t] + q2[t]]);
T11 = D[T, D[q1[t], t]];
T12 = D[T11, t];
T13 = D[T, q1[t]];
U1 = D[U, q1[t]];
T21 = D[T, D[q2[t], t]];
T22 = D[T21, t];
T23 = D[T, q2[t]];
U2 = D[U, q2[t]];
eqHELP1 = T12 - T13 + U1 // Simplify;
eqHELP2 = T22 - T23 + U2 // Simplify;
m1,1 = Coefficient[eqHELP1, q1''[t]]
m1,2 = Coefficient[eqHELP1, q2''[t]]
m2,1 = Coefficient[eqHELP2, q1''[t]]
m2,2 = Coefficient[eqHELP2, q2''[t]]
c1 = eqHELP1 /. {q1''[t] → 0, q2''[t] → 0, g → 0} // Simplify
c2 = eqHELP2 /. {q1''[t] → 0, q2''[t] → 0, g → 0} // Simplify
φ1 = U1 // Simplify
φ2 = U2 // Simplify

I1 + I2 + L1^2 m2 + 2 L1 m2 xG2 Cos[q2[t]]

I2 + L1 m2 xG2 Cos[q2[t]]

I2 + L1 m2 xG2 Cos[q2[t]]

I2

-L1 m2 xG2 Sin[q2[t]] q2'[t] (2 q1'[t] + q2'[t])

L1 m2 xG2 Sin[q2[t]] q1'[t]^2

g ((L1 m2 + m1 xG1) Sin[q1[t]] + m2 xG2 Sin[q1[t] + q2[t]])

g m2 xG2 Sin[q1[t] + q2[t]]

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Pumping the swing of acrobat

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eq1 = m1,1 q1''[t] + m1,2 q2''[t] + c1 + φ1;
eq2 = m2,1 q1''[t] + m2,2 q2''[t] + c2 + φ2;
q2des = q20 + A Sin[ω t];
q2desdot = D[q20 + A Sin[ω t], t];
q2desddot = D[q20 + A Sin[ω t], {t, 2}];
eq1p = eq1 /. {q2[t] → q2des, q2'[t] → q2desdot, q2''[t] → q2desddot};
q1ddotP = q1''[t] /. Solve[eq1p == 0, q1''[t]][[1]];
tau0 = eq2 /. q1''[t] → q1ddotP /.
  {q2[t] → q2des, q2'[t] → q2desdot, q2''[t] → q2desddot} // Simplify;

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Runge Kutta

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RK4Step[f_, {tj_, xj_}, h_] :=
Block[{k1, k2, k3, k4},
  k1 = f[tj, xj];
  k2 = f[tj + h / 2, xj + h / 2 k1];
  k3 = f[tj + h / 2, xj + h / 2 k2];
  k4 = f[tj + h, xj + h k3];
  {tj + h, xj + h / 6 (k1 + 2 k2 + 2 k3 + k4)}]
RK4[f_, {t0_, x0_}, h_, n_] := NestList[RK4Step[f, #, h] &, {t0, x0}, n]

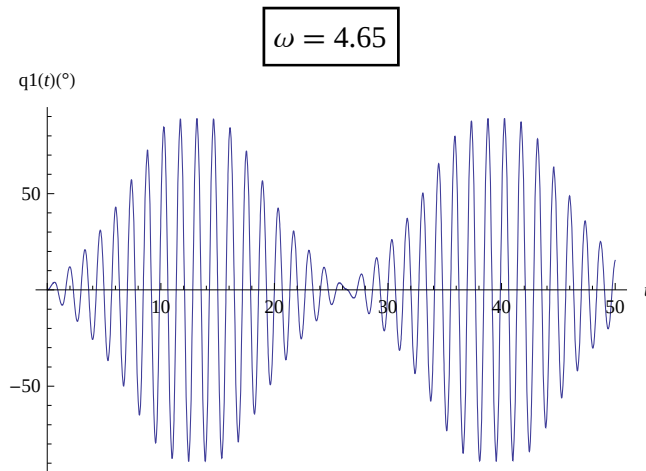
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Simulation

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param = {g → 9.81, L1 → 0.28, L2 → 0.28, m1 → 1.129, m2 → 0.762, I1 → 0.042857,
  I2 → 0.0406244, xG1 → 0.140526, xG2 → 0.198104, q20 → 0, A → Pi/5, ω → 4.65};
q1ddotP /. {q1[t] → q1, q1'[t] → q1dot};
eswing[t_, y_] := Block[{q1 = y[[1]], q1dot = y[[2]]},
  {q1dot, (-g L1 m2 Sin[q1] - g m1 xG1 Sin[q1] + A I2 ω2 Sin[t ω] + A L1 m2 xG2 ω2
    Cos[q20 + A Sin[t ω]] Sin[t ω] + 2 A L1 m2 q1dot xG2 ω Cos[t ω] Sin[q20 + A Sin[t ω]] +
    A2 L1 m2 xG2 ω2 Cos[t ω]2 Sin[q20 + A Sin[t ω]] - g m2 xG2 Sin[q1 + q20 + A Sin[t ω]]) /
    (I1 + I2 + L12 m2 + 2 L1 m2 xG2 Cos[q20 + A Sin[t ω]])} /. param];
eswing[t, {q1, q1'}];
eswing[0, {1, 1}];
RK4Step[eswing, {0, {0, 0}}, 0.01];
RK4[eswing, {0, {0, 1}}, 0.01, 5];
tend = 50;
step = 0.01;
stepnum = Round[tend/step];
sim1 = RK4[eswing, {0, {0, Pi/90}}, step, stepnum];
rk4plt1 = ListLinePlot[MapThread[List, {sim1[[All, 1]], 180/Pi sim1[[All, 2]][[All, 1]]}],
  PlotRange → All, AxesLabel → {t, q1[t] [°]}, PlotLabel → Style[Framed[ω = 4.65], 14, Black]]
tau0 /. {q1[t] → q1, q2[t] → q2, q1'[t] → q1dot, q2'[t] → q2dot} /. param // Simplify // CForm

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-0.5519156816215348*Sin(4.65*t) +
  0.042267469440000004*Power(q1dot,2)*Sin((Pi*Sin(4.65*t))/5.) +
  ((0.0406244 + 0.042267469440000004*Cos((Pi*Sin(4.65*t))/5.))*
    (-3.6494559077400006*Sin(q1) +
      (0.5519156816215348 + 0.5742381230589251*Cos((Pi*Sin(4.65*t))/5.))*
        Sin(4.65*t) + 0.24698413895007534*q1dot*Cos(4.65*t)*
          Sin((Pi*Sin(4.65*t))/5.) +
            0.36080445376262216*Power(Cos(4.65*t),2)*Sin((Pi*Sin(4.65*t))/5.) -
              1.4808709828800002*Sin(q1 + (Pi*Sin(4.65*t))/5.)))/
    (0.14322220000000002 + 0.08453493888000002*Cos((Pi*Sin(4.65*t))/5.)) +
    1.48087098288*Sin(q1 + (Pi*Sin(4.65*t))/5.)

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Partial Feedback Linearization

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q1ddotPFL1 = Solve[eq1 == 0, q1''[t]][[1]];
tauPFL = eq2 /. q1ddotPFL1 /. q2''[t] → u // Simplify;
tauPFL /. {q1[t] → q1, q2[t] → q2, q1'[t] → q1dot, q2'[t] → q2dot} // CForm
eq5 = M11 * q1ddot + M12 * q2ddot + c1 + Q1;
sol = Solve[eq5 == 0, q1ddot][[1]];
M21 * q1ddot + M22 * q2ddot + c2 + Q2 /. sol // Simplify

c2 + Q2 + M22 q2ddot -  $\frac{M21 (c1 + Q1 + M12 q2ddot)}{M11}$ 

I2*u + L1*m2*Power(q1dot,2)*xG2*Sin(q2) + g*m2*xG2*Sin(q1 + q2) -
((I2 + L1*m2*xG2*Cos(q2))*(I2*u + L1*m2*u*xG2*Cos(q2) + g*L1*m2*Sin(q1) +
g*m1*xG1*Sin(q1) - 2*L1*m2*q1dot*q2dot*xG2*Sin(q2) -
L1*m2*Power(q2dot,2)*xG2*Sin(q2) + g*m2*xG2*Sin(q1 + q2)))/
(I1 + I2 + Power(L1,2)*m2 + 2*L1*m2*xG2*Cos(q2))

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