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Design of a quadratic boost converter

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Contents

Abstract	IV
Kivonat	V
Introduction	VI
1 Theoretical background	1
1.1 The Boost converter	1
1.2 CCM and DCM	2
1.3 Other topologies	2
1.4 The Quadratic Boost converter	3
2 Modelling the converter	6
2.1 The ideal model	6
2.2 The transfer functions of the ideal model	7
2.3 The non-ideal model	11
3 Simulation and controller design	13
3.1 Simulation of the quadratic boost converter	13
3.2 Average current mode control	14
3.2.1 Design of the current loop	16
3.2.2 Design of the voltage loop	17
3.2.3 Discrete controller	18

3.2.4	Simulation of the controller	19
3.2.5	Simulation with fuel cell input	20
3.3	State Space Controller design	23
3.3.1	Extending the SVFB by an integrator	23
3.3.2	Linear Quadratic Regulator design	25
3.3.3	Simulation with fuel cell input	28
3.4	Comparison of the two control strategies	31
3.4.1	Dynamic response of the different controllers at rated load	33
3.4.2	Performance at different loads	33
3.4.3	Noise suppression	33
3.4.4	Other aspects and summary	36
3.5	State estimation	37
3.5.1	Advanced plant model	37
3.5.2	Kalman Filter	38
3.5.3	Summary and conclusion	39
4	Design of the circuit	42
4.1	Schematic	42
4.2	Element selection	46
4.3	Printed Circuit Board	48
5	Hardware	50
5.1	Building the converter	50
5.2	Measurements	51
5.3	Future plans	51
	Acknowledgement	54
	Bibliography	56

Appendices	57
F.1 Derivation of the ideal model	57
F.1.1 On state of the converter	57
F.1.2 Off state of the converter	58
F.1.3 The complete state space model	59
F.2 Derivation of the non-ideal model	62
F.2.1 The on state of the non-ideal model	63
F.2.2 The off state of the non-ideal model	64
F.2.3 The complete non-ideal state space model	65

Abstract

Nowadays DC-DC converters are increasingly gaining importance in smaller electronic devices and in high power applications as well. One of the most frequently used converters is the boost converter which is used to amplify (boost) the input voltage. The semiconductor switch in the converter is driven by pulse width modulation. Depending on the duty ratio theoretically even infinitely large gains could be reached. However in practice, due to the different non-ideal effects, the output voltage can be maximum 3-4 times higher than the input voltage. In such applications where the boost converter's gain is not enough, for example in case of fuel- or photovoltaic cells, the presented quadratic topology, in which the gain can be varied with the quadratic function of the duty ratio, proves to be an effective solution. The main advantage of the quadratic topology over several Boost converter's series connection is that only one semiconductor switch is needed.

The goal of our project is to simulate, design and build a laboratory model of a quadratic boost converter. During our progress with the topic besides the circuit analysis we put a high emphasis on the design and simulation of two output voltage control methods, the average current mode control and LQR (and LQG) control.

Our study covers the obtained differential equations of the system, derived from the circuit's structure. The piecewise linear-nonlinear converter's averaged state-space representation, with the inclusion of the effect of the circuit's non-ideal elements, is also derived from the equations. With the help of the state-space model the different transfer functions which are necessary for controller design are also acquired. The two control algorithms and the estimator were also examined both in discrete and continuous time. The simulations were carried out in Matlab Simulink environment. Besides the theoretical part the selection of the different circuit elements and the design of the converter's printed circuit board are also described in detail.

After the completion of the circuit the different control algorithms tested in simulation are going to be implemented on the hardware. The quality parameters of the different methods are going to be measured and compared to their simulated counterpart. The acquired results are going to be compared to the solutions found in the technical literature.

Kivonat

Napjainkban a DC-DC konverterek egyre nagyobb szerepet kapnak, legyen szó akár kisebb elektronikus eszközökről vagy akár nagy teljesítményű berendezésekről. Az egyik leggyakrabban alkalmazott DC-DC konverter a feszültségnövelő, angol elnevezéssel Boost konverter. A Boost konverter félvezető kapcsolóját impulzus szélesség modulációval vezérlik és a kitöltési tényező függvényében elméletileg akár végtelen nagy erősítés is elérhető. Ugyanakkor a különböző nemideális hatások következtében legfeljebb 3-4-szeres feszültségnövelés valósítható meg. Azokban az alkalmazásokban ahol ez az erősítési érték nem elegendő – ha a bemenő feszültség széles tartományon változik, mint például napelemek vagy üzemanyagcellák esetében – jól alkalmazható a dolgozatban bemutatott kvadratikus feszültségnövelő, angol elnevezéssel Quadratic boost konverter, amelyben a kitöltési tényező értékével négyzetesen változtatható az erősítés. A topológia előnye, a több egymás után kötött Boost fokozattal szemben, hogy csak egy félvezető kapcsolóelemre van szükség.

A dolgozatunk célja egy ilyen kvadratikus feszültségnövelő konverter szimulációja, tervezése és laboratóriumi modelljének megépítése. A munkánk során az áramkör elemzése mellett nagy hangsúlyt fektettünk két feszültségszabályozó eljárás, egy belső áramszabályzóval rendelkező kaszkád szabályzó, és egy LQR (valamint LQG) szabályzó tervezésére, szimulációjára.

Dolgozatunkban részletesen kitérünk az áramkör felépítésére, mely magában foglalja a rendszer differenciálegyenleteinek felírását. Az egyenletek segítségével levezetésre kerül a szakaszonként lineáris nemlineáris konverter átlagolt állapotter modellje a nemideális áramköri elemek hatásának figyelembevételével. Az állapotter modell segítségével a szabályzótervezéshez szükséges átviteli függvényeket előállítottuk, és a kettő szabályozási algoritmust folytonos és diszkrét időben is vizsgáltuk. A szimulációs vizsgálatokat Matlab Simulink környezetben végeztük. Az elméleti háttér mellett ismertetésre kerül az áramkörhöz szükséges elemek kiválasztása, és a hozzájuk tartozó nyomtatott áramkör tervezése.

Az áramkör elkészülte után gyakorlatban is implementálásra kerülnek a szimulációk segítségével vizsgált szabályzó algoritmusok, melyek minőségi paramétereit így mérésekkel is összehasonlíthatjuk. Az így kapott eredményeket a szakirodalomban fellelhető megoldásokkal is összevetjük.

Introduction

The aim of this study is to design and build a Quadratic Boost converter, which is a variation of the step-up DC-DC converters. The step-up or also called boost converters are widely used in different applications, for example in case of battery power systems using stacked cells. In an electric car usually a much higher output voltage is needed than the output of one cell. One solution is to stack huge number of cells in series to obtain higher voltage at the output. The disadvantage of connecting the cells in series is, that not only more cells are required, but the space occupied by them becomes huge, making it impossible to use this solution in applications where the space and weight are critical.

On the other hand, step-up DC-DC converters can increase the voltage, making it possible to use less cells. With this solution the application of the battery power systems can be extended, and used in equipments where the space is limited. In electric cars the dc voltage bus commonly ranges from 300 V to 600 V, which could only be produced with about 400–450 battery cells. The huge amount of cells would not only cause smaller space in the car, but represent higher weight. The number of the cells can be further reduced if Quadratic Boost converters are used instead of the basic boost topology, as they can achieve considerable higher voltage amplification.

Near batteries fuel cells are also being utilized in electric cars. Fuel cell operates in a similar way as a battery cell. It converts chemical energy into electrical energy. Similar to battery cells, multiple fuel cells are stacked in series to increase the voltage level as one fuel cell typically generates only about 0.7[V]. The main difference between batteries and fuel cells is that latter ones never need to be recharged as long as the supply of fuel and oxygen are available. As a matter of fact, the voltage generated by a fuel cell strongly depends on both the load power demand and the operating conditions. Besides, as a result of many design aspects, fuel cells are low voltage and high current electric generators. Therefore, electric loads, like the motor of the electric car, supplied by fuel cells are typically fed via a boost converter. The specifications of such a power converter are to be able to step up the input voltage with a high ratio and a good efficiency. A ratio of 5-10 is a classic situation. As it was mentioned the output voltage of a fuel cell stack depends on the loading condition. The voltage swing is typically about 50% with respects to open-circuit voltage resulting in a fluctuating input voltage for the boost converter

while the output voltage should be kept constant. As it will be shown later the Quadratic boost converter presented in this work, can be a feasible solution for such an operating condition as well. [1]

Quadratic boost converters can also be a good solution in the utilization of solar energy. Total power output of a solar PV module changes with the direction of sun, the level of solar irradiation and temperature. Each PV panel has a well-defined maximum power point (MPP) on the voltage vs. power curve, which is continuously moving depending on the operating conditions. Knowing that the efficiency of the solar PV module is low, while their cost is high, it is desirable to operate the module at the peak power point, so this way the maximum power can be harvested and delivered to the load under varying temperature and irradiation conditions, thus a MPP tracker (MPPT) is used for extracting the maximum power from the solar PV module.

In recent years the grid connected PV systems have become popular because they do not need battery back-ups to ensure MPPT. A typical grid connected PV system has two stages, the first is used to increase the PV array voltage with a boost DC-DC converter and track MPP and the second realize the conversion of the DC voltage to AC voltage. Nowadays, the range of PV panels output voltage is between 24 and 48[V] for remaining underneath security voltage. These values are low compared to the necessary voltage for the second stage. As a consequence the voltage gain of the boost converter stage must be high enough. Furthermore the input voltage continuously fluctuates due to the weather conditions and the required high amplification ratio can also varied over time. The quadratic boost converter can be a proper solution for such circumstances. [10]

In the first chapter, the theoretical background of the converters are discussed briefly. The continuous and discontinuous conduction mode are introduced, and different topologies are studied. The operation of the Quadratic Boost converter is described. In the second chapter the derivation of the state space model and corresponding transfer functions can be found. The equations were linearised in the small environment of the working point. In the third chapter various controller algorithms are designed and tested in simulations with Matlab/Simulink. At first an average current mode control is applied to the converter, then a Linear-Quadratic Regulator is presented. After this the two control strategies are compared to each other. At last state estimation possibilities are also discussed. In the fourth chapter the design of the printed circuit board, and selection of the elements are presented. The segments of the circuit are introduced in detail. In the last chapter the building of the converter, and open loop measurements can be found. The implementation of the controller algorithms to the microcontroller is also mentioned.

Chapter 1

Theoretical background

1.1 The Boost converter

DC-DC converters are nowadays widely used in various applications, such as low power equipments, and high power solutions. These converters' operation is based on PWM, which makes it easy to control the output voltage. In this study step-up (Boost and Quadratic Boost) converters are investigated, which – as the name implies – are able to convert the input voltage to a higher output voltage [12].

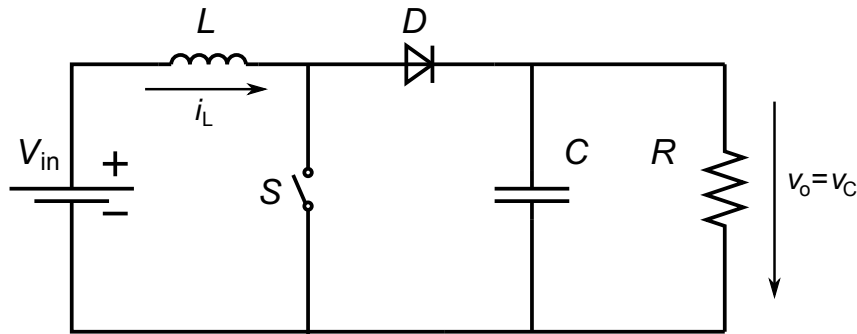


Figure 1.1: *Boost Converter.*

On Figure 1.1. a Boost converter can be seen [12]. The operation of this converter can be divided into two parts depending on the state – on or off – of the semiconductor switch S . During the *on* period, the diode is reverse biased, thus the input voltage charges the L inductor. When the switch is turned off, the load receives energy not only from the input, but also from the previously charged inductor. This process makes it possible to obtain a higher output voltage as the input. The ratio between the output and input voltage can be expressed as:

$$\frac{V_o}{V_{in}} = \frac{1}{1 - D}, \quad (1.1)$$

where D donates the duty ratio and V_o is the output, V_{in} is the input voltage.

1.2 CCM and DCM

When talking about DC-DC converters, the CCM – Continuous Conduction Mode and DCM – Discontinuous Conduction Mode need to be mentioned. These modes are related to the current of the inductor, and widely discussed in [8] and [12]. When the converter operates in CCM, the current of the inductor does not reach zero until the end of the *off* period of the semiconductor switch, thus conducting current in both *off* and *on* period. In contrast with CCM, at Discontinuous Conduction Mode the current of the inductor reaches zero at the *off* period, and remains zero until the switch is turned on. On Figure 1.2. the time diagrams for Continuous and Discontinuous Conduction Mode of the Boost converter can be seen, where T_s is the switching period, v_o is the output voltage and V_o is the average of the output voltage, V_d is the input voltage, v_L and i_L are the voltage and the current of the inductor respectively. The *on* and *off* state of the switch can be expressed as $t_{on} = DT_s$ and $t_{off} = (1 - D)T_s$.

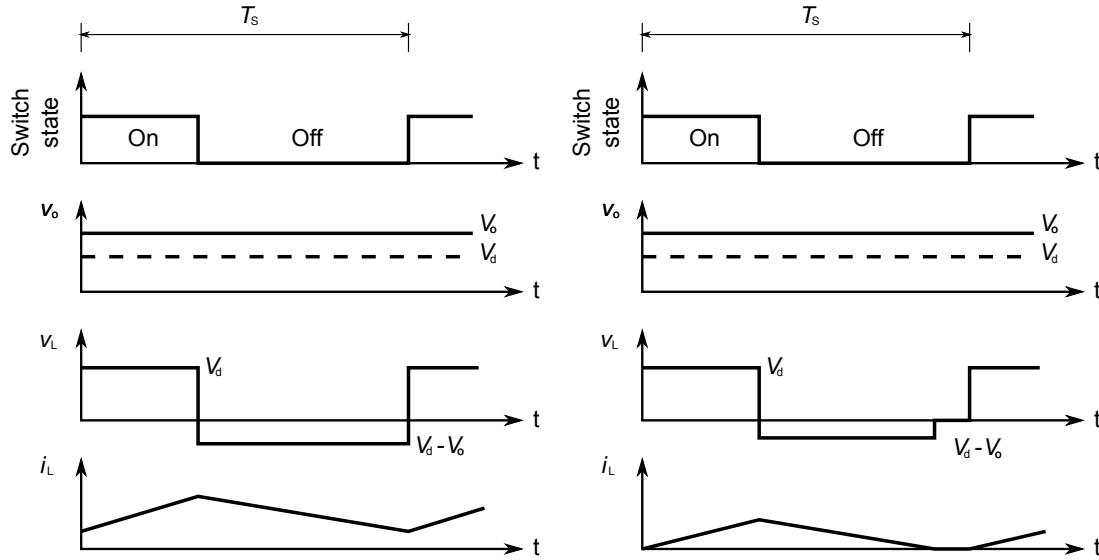


Figure 1.2: CCM and DCM of the Boost converter, based on [12].

It can be seen that at DCM, when the inductor current reaches zero, the voltage of the inductor will also be zero, thus the equations derived for the CCM will not be valid. It should be emphasized that the relation between the input and output voltage given by (1.1) is not valid. Later on when modelling the converters always Continuous Conduction Mode is assumed.

1.3 Other topologies

Due to the non-ideal circuit elements, in the applications where the input voltage varies in a wide range – e.g. fuel cells, photovoltaic cells or output voltage of the diode bridge rectifier – and the needed output voltage is much higher than the input a single stage boost converter

cannot be used, other topologies are required. At high Duty ratios, the amplification of the converter differs from the theoretical one due to the non-ideal elements. The real and theoretical characteristics of the amplification in function of the duty ratio can be seen in Figure 1.3.

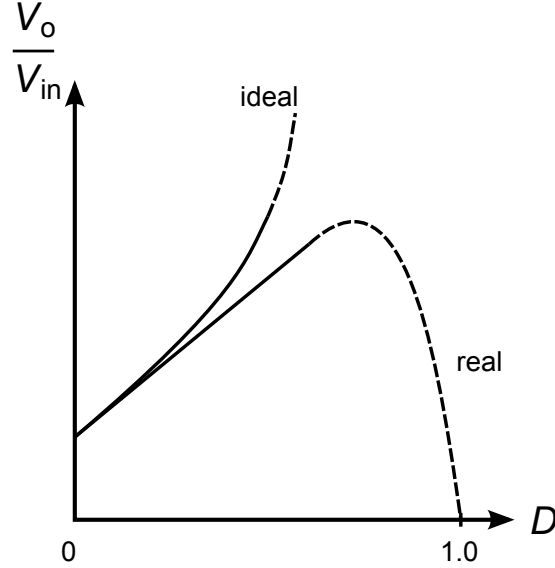


Figure 1.3: Amplification of the boost converter, based on [8].

At high conversion where very low or high duty ratio is needed, the minimum on and off time of the switch also prohibits the operation [6]. Solving this problem, various DC-DC converters with wide conversion ratio are discussed in [6]. Another solution is to connect multiple Boost converters in series, thus achieving higher amplification. Two Boost converters connected in series can be seen on Figure 1.4. In this case the output voltage and input voltage ratio can be expressed as $\frac{1}{1-D_1} \frac{1}{1-D_2}$, thus if $D_1 = D_2$, the amplification is proportional with the 2^{nd} power of the Duty ratio. With this topology higher amplification could be reached, but the losses however are more significant than that of a Boost converter due to the two semiconductor switches.

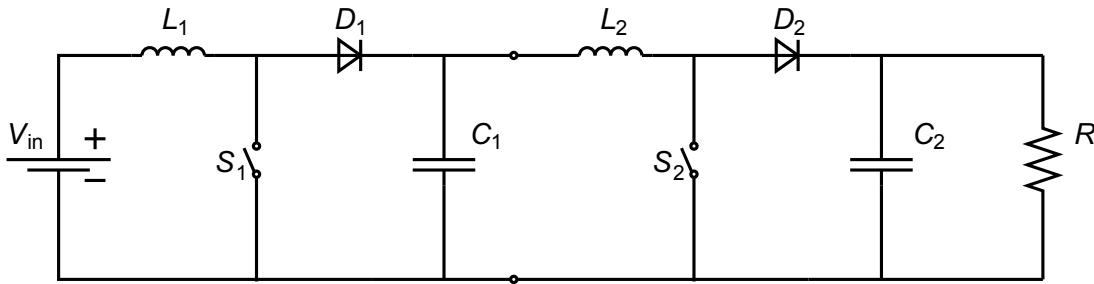


Figure 1.4: Two Boost converters connected in series.

1.4 The Quadratic Boost converter

The topology discussed in this study is the Quadratic Boost converter, which can be seen on Figure 1.5. The advantage of this converter is that only one semiconductor switch, thus only

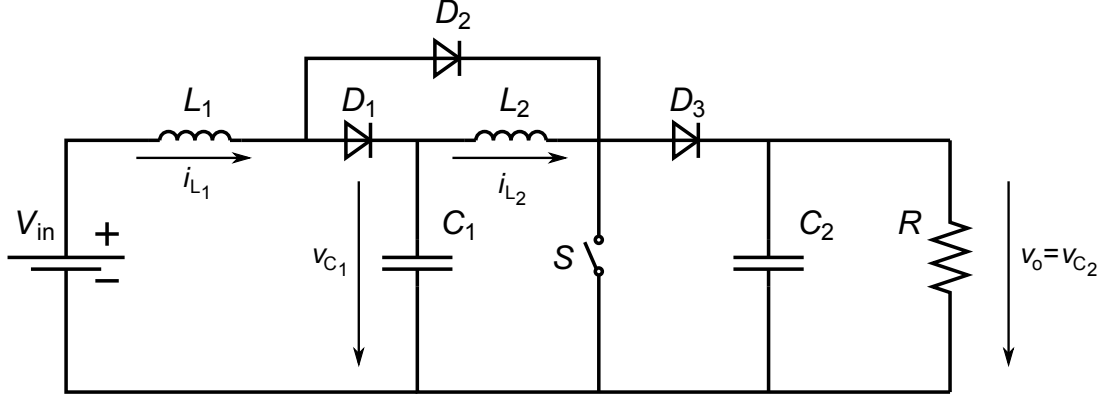


Figure 1.5: Quadratic Boost Converter.

one MOSFET driver is needed. This topology and the control of the converter are discussed in [3] and [5]. In this case it can be reached with only one switch, that the V_{in} input and V_o output voltage ratio is a quadratic function of the D duty ratio, which then can be expressed as

$$\frac{V_o}{V_{in}} = \frac{1}{(1 - D)^2}. \quad (1.2)$$

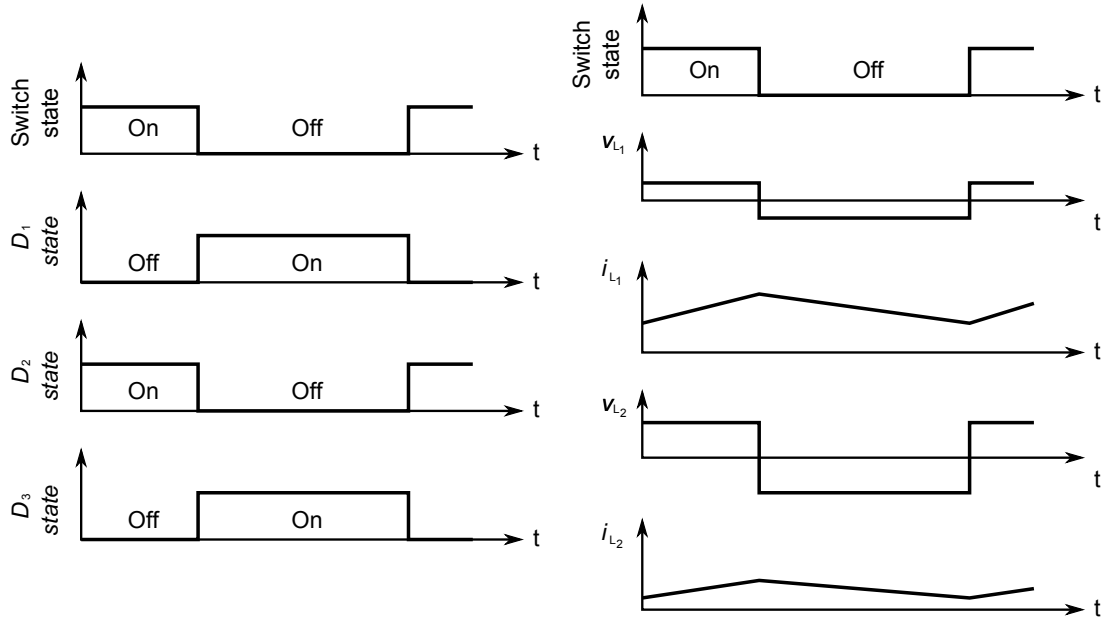


Figure 1.6: Operation of the Quadratic Boost converter.

On Figure 1.6. the time diagrams of the Quadratic Boost converter can be seen assuming ideal components. It can be seen that in the *off* period of the switch, the D_1 and D_3 diodes are conducting, while diode D_2 is reverse biased. In the *on* period this is reversed, D_2 is conducting, D_1 and D_3 are reverse biased. From the time diagrams of the voltages and currents of the

inductors it can be determined, that during the *on* state both of the inductors are charged by the input voltage. Correspondingly the voltage of the inductors are positive during the *on* cycle and negative during *off*.

Chapter 2

Modelling the converter

Before designing the controller algorithm, and setting its parameters we have to investigate the models of the converter, to obtain information of its behaviour. This investigation includes the derivation of the state space model, from which the corresponding transfer functions can be obtained. The elements of the converter – e.g. the diodes, inductors and the semiconductor switch – are assumed ideal. The basic steps of the derivation of the ideal converter are discussed in [3], and the resulting matrices are given also in [5]. Based on the basic steps of [3] the ideal state space model of the converter is derived, in a more detailed way.

For the controller design algorithms we also need to obtain the transfer functions from the state space models. These transfer functions are used for the visualization of the frequency- and time response diagrams.

To be able to simulate the steady state errors and losses of the circuit, the state space model assuming non-ideal elements was also derived.

2.1 The ideal model

On Figure 2.1. the circuit of the converter can be seen, where the elements are assumed ideal.

Small letters were used to denote the time dependency of the current variable, and capital letters for the steady state values. The currents of the inductors L_1 and L_2 are denoted with i_{L_1} and i_{L_2} , the voltages of the capacitors C_1 and C_2 are v_{C_1} and v_{C_2} accordingly, V_{in} is the input voltage and D is the duty ratio nominal value, d is the duty ratio.

The state space model for the quadratic boost converter is given in (2.1), its derivation steps can be seen in F.1. As in this model $\mathbf{A}$ depends on the duty ratio d , and in the input matrix only the effect of the input voltage V_{in} is included, the model had to be linearised near the working

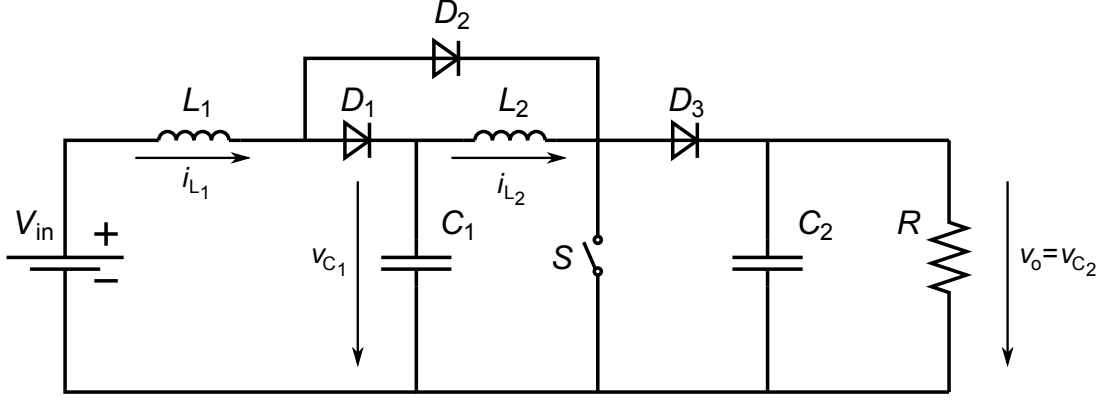


Figure 2.1: Quadratic Boost Converter.

point.

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_{C1} \\ \dot{v}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-d}{L1} & 0 \\ 0 & 0 & \frac{1}{L2} & -\frac{1-d}{L2} \\ \frac{1-d}{C1} & -\frac{1}{C1} & 0 & 0 \\ 0 & \frac{1-d}{C2} & 0 & -\frac{1}{RC2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L1} \\ 0 \\ 0 \\ 0 \end{bmatrix} v_{in}. \quad (2.1)$$

The linearised (small signal) state space model of the converter can be seen in (2.2). The derivation steps can be found also in F.1. In this model $\mathbf{A}$ depends only on the nominal value of the duty cycle D , and the small deviation of the duty ratio $\tilde{d}$ is included in the input vector.

$$\begin{bmatrix} \dot{\tilde{i}}_{L1} \\ \dot{\tilde{i}}_{L2} \\ \dot{\tilde{v}}_{C1} \\ \dot{\tilde{v}}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L1} & 0 \\ 0 & 0 & \frac{1}{L2} & -\frac{1-D}{L2} \\ \frac{1-D}{C1} & -\frac{1}{C1} & 0 & 0 \\ 0 & \frac{1-D}{C2} & 0 & -\frac{1}{RC2} \end{bmatrix} \begin{bmatrix} \tilde{i}_{L1} \\ \tilde{i}_{L2} \\ \tilde{v}_{C1} \\ \tilde{v}_{C2} \end{bmatrix} + \begin{bmatrix} \frac{V_{in}}{L1(1-D)} & \frac{1}{L1} \\ \frac{V_{in}}{L2(1-D)^2} & 0 \\ -\frac{V_{in}}{RC1(1-D)^4} & 0 \\ -\frac{V_{in}}{RC2(1-D)^3} & 0 \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}. \quad (2.2)$$

Substituting the nominal value of the duty cycle – obtained from the input voltage and the desired output voltage – into $\mathbf{A}$ and $\mathbf{B}$ the state space model can be linearised near the working point of the converter.

2.2 The transfer functions of the ideal model

As in (2.2) the state space model for small signals were obtained, we can derive and investigate the transfer functions of the converter. From the state space model we can get the transfer functions with the well known formula. It's derivation is not discussed in this study. The formula is given in [4]:

$$\frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D. \quad (2.3)$$

In this case the corresponding transfer functions are derived using the given values of the elements, with help of the Matlab software. The parameters of the elements can be seen in Table 2.1.

Table 2.1: Values of the elements in the converter.

$L_1[\mu H]$	100
$L_2[\mu H]$	560
$C_1[\mu F]$	22
$C_2[\mu F]$	100
$R[\Omega]$	100

As the derived transfer functions also depend on the duty ratio and the input voltage, we need to investigate them varying the parameters. For the later design of the control algorithms we need more transfer functions: the output voltage – duty ratio $\frac{v_{C_2}(s)}{\tilde{d}(s)}$, the first inductor current – duty ratio $\frac{i_{L_1}(s)}{\tilde{d}(s)}$, and the output voltage – first inductor current $\frac{v_{C_2}(s)}{i_{L_1}(s)}$. In the case of ideal elements the output voltage V_o equals with the second capacitor voltage v_{C_2} .

At first we need to investigate the $\frac{i_{L_1}(s)}{\tilde{d}(s)}$ transfer function, which were derived with the Matlab software and can be seen in equation (2.4). This transfer function is required during the design of the inner current loop for the current mode control algorithm, see later in 3.4.. The equation (2.4) is valid when $D = 0.5$ and $V_{in} = 12[V]$.

$$\frac{i_{L_1}(s)}{\tilde{d}(s)} = \frac{(2.4 \cdot 10^5) s^3 + (4.6 \cdot 10^8) s^2 + (4.0 \cdot 10^{13}) s + 7.8 \cdot 10^{15}}{s^4 + 100.0 s^3 + (2.0 \cdot 10^8) s^2 + (1.9 \cdot 10^{10}) s + 5.1 \cdot 10^{14}} \quad (2.4)$$

On Figure 2.2. the bode diagram of the $\frac{i_{L_1}(s)}{\tilde{d}(s)}$ transfer function can be seen. This transfer function has two resonant peaks, to which abrupt changes in the phase diagram belong. As the phase never reaches -180 degrees this transfer function is stable and minimum phase.

As it was mentioned, the transfer functions depend on the duty ratio, as well as on the input voltage, thus we need to investigate the effects of these variables. In Figure 2.3a. the effect of the duty ratio to the $\frac{i_{L_1}(s)}{\tilde{d}(s)}$ transfer function can be observed. If the duty ratio is increased, the two resonance peaks are moving to the lower frequencies, and their amplitude, as well as the DC gain of the transfer function increases. The points of the abrupt changes in the phase are moving correspondingly, however the change becomes smaller. This duty ratio dependent behaviour of the converter makes it difficult to design a controller algorithm, that suits for small and high duty ratios as well. During the design the desired application of the controller needs to be taken into account, to optimize the algorithm for the desired duty ratio.

On Figure 2.3b. it can be observed that by varying the input voltage, the frequency response of the plant varies only in smaller steps. The phase of the plant is always the same, at any input

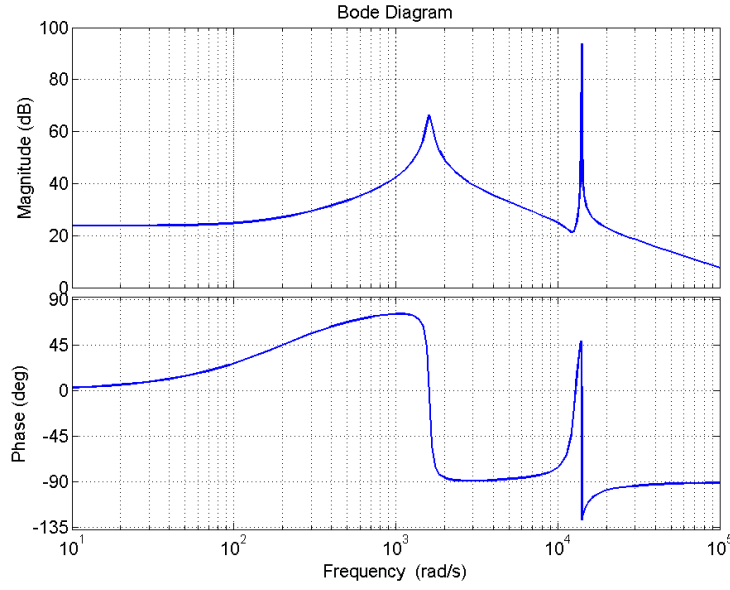
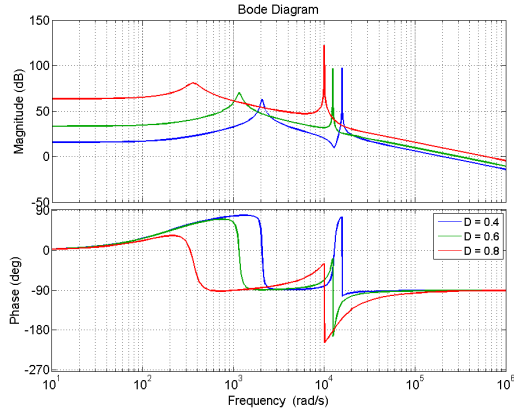
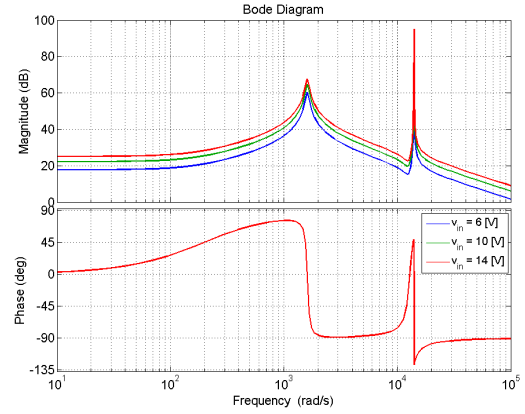


Figure 2.2: $\frac{i_{L1}(s)}{d(s)}$ transfer function, $D = 0.5$, $V_{in} = 12[V]$.



(a) $\frac{i_{L1}(s)}{d(s)}$ transfer function, $V_{in} = 12[V]$.

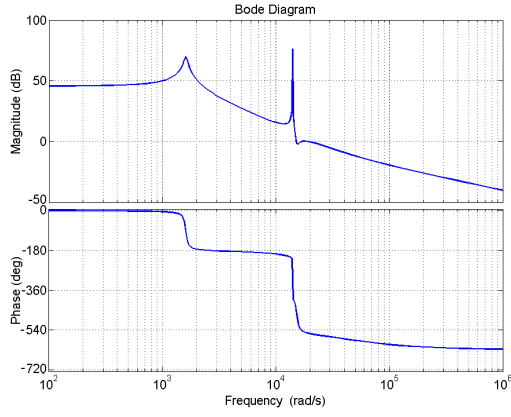


(b) $\frac{i_{L1}(s)}{d(s)}$ transfer function, $D = 0.5$.

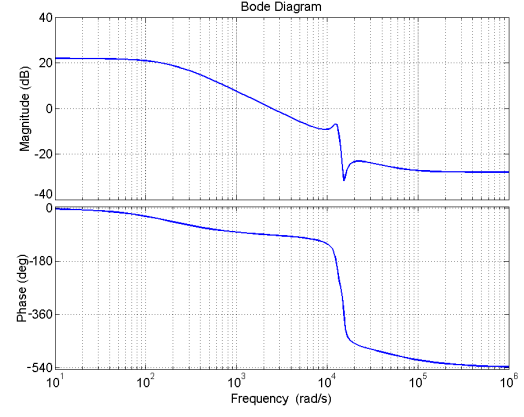
Figure 2.3: Bode plots

voltage values and the resonance peaks are also at the same frequency. This makes it possible to have a control algorithm that can provide the same output voltage when the input is fluctuating.

The next transfer function to be investigated is the $\frac{v_{C2}(s)}{d(s)}$ which is between the output voltage (which is equal to the second capacitor voltage at ideal model) and the duty ratio. Equation (2.5) shows the expression of the transfer function, while Figure 2.4a. shows the frequency response, when $D = 0.5$ and $V_{in} = 12[V]$. This transfer function has also two resonance peaks, its phase diagram however is different, the abrupt changes can also be observed.



(a) $\frac{v_{\tilde{C}_2}(s)}{\tilde{d}(s)}$ transfer function.

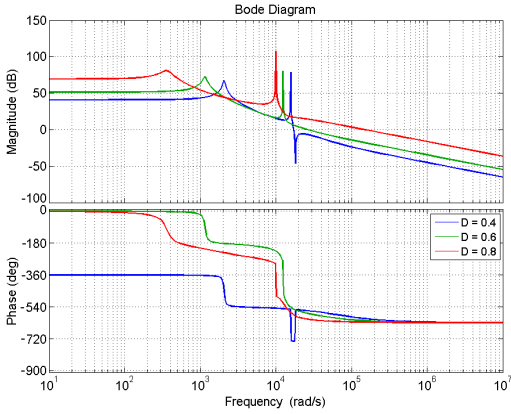


(b) $\frac{v_{\tilde{C}_2}(s)}{\tilde{i}_{L_1}(s)}$ transfer function.

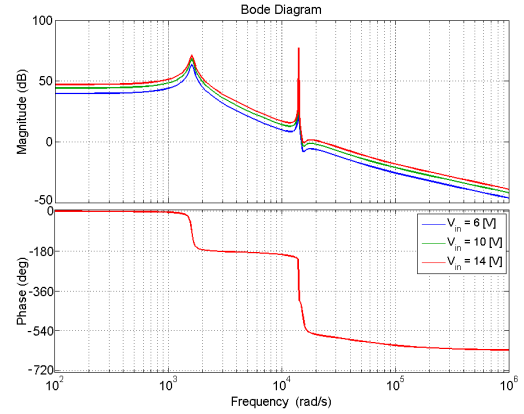
Figure 2.4: Bode plots for $D = 0.5$, $V_{in} = 12[V]$.

$$\frac{v_{\tilde{C}_2}(s)}{\tilde{d}(s)} = \frac{(-9.6 \cdot 10^3) s^3 + (4.3 \cdot 10^8) s^2 - (2.6 \cdot 10^{12}) s + 9.7 \cdot 10^{16}}{s^4 + 100.0 s^3 + (2.0 \cdot 10^8) s^2 + (1.9 \cdot 10^{10}) s + 5.1 \cdot 10^{14}} \quad (2.5)$$

On Figure 2.5. the deviations in the $\frac{v_{\tilde{C}_2}(s)}{\tilde{d}(s)}$ transfer function can be seen, when the duty ratio or the input voltage is varied. In this case the same can be observed as at the previous $\frac{\tilde{i}_{L_1}(s)}{\tilde{d}(s)}$ transfer function; there are greater differences when the duty ratio is varied, the changes in the input voltage do not have that large effects on the frequency response of the plant.



(a) $D = 0.4 - 0.8$, $V_{in} = 12[V]$



(b) $D = 0.5$, $V_{in} = 6 - 14[V]$

Figure 2.5: Bode plots for $\frac{v_{\tilde{C}_2}(s)}{\tilde{d}(s)}$ transfer function.

For the design of the outer – voltage loop we need to derive the $\frac{v_{\tilde{C}_2}(s)}{\tilde{i}_{L_1}(s)}$ transfer function. This can be obtained if we divide the $\frac{v_{\tilde{C}_2}(s)}{\tilde{d}(s)}$ transfer function with $\frac{\tilde{i}_{L_1}(s)}{\tilde{d}(s)}$. The obtained transfer function can be seen in equation (2.6), the corresponding bode diagram in Figure 2.4b. These are valid

for the previously discussed parameters.

$$\frac{\tilde{v}_{C_2}(s)}{\tilde{i}_{L_1}(s)} = \frac{-0.04 s^3 + 1786.0 s^2 - (1.104 \cdot 10^7) s + 4.058 \cdot 10^{11}}{s^3 + 1918.0 s^2 + (1.67 \cdot 10^8) s + 3.247 \cdot 10^{10}} \quad (2.6)$$

In this section the transfer functions needed for the controller design were obtained. The obtained bode diagrams show similar results to the ones in [5] and [3].

2.3 The non-ideal model

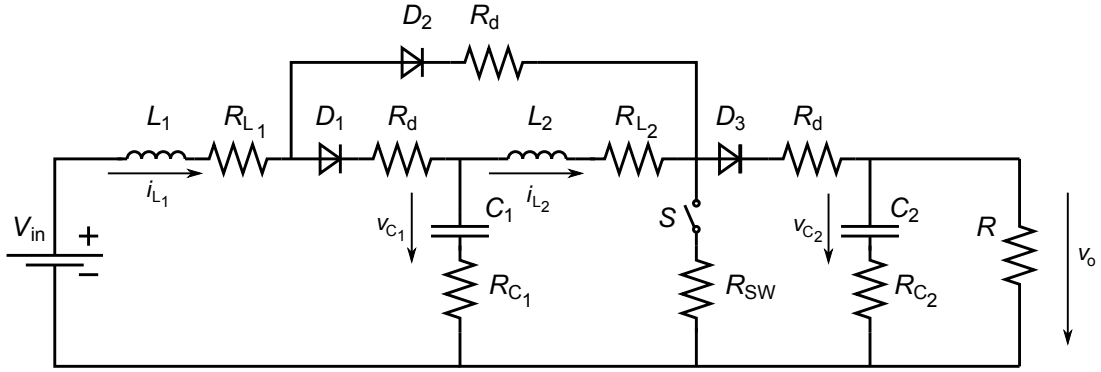


Figure 2.6: Non-ideal model of the converter.

The model of the converter with non-ideal, parasitic elements has to be investigated to be able to compute and simulate the steady state error in open loop, and the losses of the circuit. The model with the non-ideal elements can be seen in Figure 2.6.

The non-ideal elements are the resistance of the inductors – R_{L_1} and R_{L_2} , which include the shunt resistor as well, the equivalent series resistance (ESR) of the two capacitors – R_{C_1} and R_{C_2} , the resistance of the diodes – R_d , and the resistance of the semiconductor switch – R_{SW} . The forward voltage drop of the diodes, denoted by V_F , are assumed to be constant and taken into consideration in the input vector $\mathbf{B}_{adv}$. The non-linearised state space model of the converter with parasitic elements can be seen in (2.7), its derivation steps are detailed in F.2.

$$\dot{\mathbf{x}} = \mathbf{A}_{adv}\mathbf{x} + \mathbf{B}_{adv}v_{in} + \mathbf{B}_{adv_d} \quad (2.7a)$$

$$\mathbf{y} = \mathbf{C}_{adv}\mathbf{x} \quad (2.7b)$$

Where

$$\mathbf{A}_{adv} = \begin{bmatrix} \frac{-R_{C_1}(1-d)}{L_1} - \frac{R_{L_1}+R_d}{L_1} - \frac{R_{SW}d}{L_1} & \frac{R_{C_1}(1-d)}{L_1} - \frac{R_{SW}d}{L_1} & -\frac{1-d}{L_1} & 0 \\ \frac{R_{C_1}(1-d)}{L_2} - \frac{R_{SW}d}{L_2} & -\frac{\left(R_d + \frac{R R_{C_2}}{R+R_{C_2}}\right)(1-d)}{L_2} - \frac{R_{SW}d}{L_2} - \frac{R_{C_1}+R_{L_2}}{L_2} & \frac{1}{L_2} & -\frac{R(1-d)}{L_2(R+R_{C_2})} \\ \frac{1-d}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{R(1-d)}{C_2(R+R_{C_2})} & 0 & -\frac{1}{C_2(R+R_{C_2})} \end{bmatrix} \quad (2.8a)$$

$$\mathbf{B}_{adv}^T = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \end{bmatrix} \quad (2.8b)$$

$$\mathbf{B}_{adv_d}^T = \begin{bmatrix} -\frac{V_F}{L_1} & -\frac{V_F(1-d)}{L_2} & 0 & 0 \end{bmatrix} \quad (2.8c)$$

$$\mathbf{C}_{adv} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.8d)$$

Chapter 3

Simulation and controller design

In Chapter 2 the State Space model and the corresponding transfer functions of the converter were derived. These can be used for designing controller algorithms and setting their parameters. At first an average current mode control is designed and investigated. Current mode control algorithms for quadratic boost converters are discussed in more detail in [3] and [5].

After the classical approach a Linear-Quadratic Regulator is also designed to control the converter. After this a brief comparison of the two control methods is shown. As a last step state estimation possibilities are also discussed and an LQG controller is presented. The methods used in the state space controller design are briefly discussed in [11][9].

The control algorithms were designed for the ideal plant in continuous time, then they were implemented in discrete time and while applying non-ideal elements as well. As a last step the Matlab code for the whole control loop was written, and tested with the same plant.

3.1 Simulation of the quadratic boost converter

The simulation of the converter and the designed controllers were implemented in Matlab Simulink. For the converter the SimPowerSystems library of Simulink was used. Two models of the quadratic boost converter were created, one with ideal, and another with non-ideal elements. The non-ideal subsystem of the converter can be seen on Figure 3.1. where the resistance of the inductors, the equivalent series resistance (ESR) of the capacitors, the voltage drop of the diodes and the on state resistance of the semiconductor switching device are taken into consideration.

On Figure 3.1. it can be seen that for the input of the plants controlled voltage sources are used. The PWM signal is generated by comparing a repeating sequence signal to the duty ratio reference. The real model besides the parasitic parameters of the elements inherits the two shunt

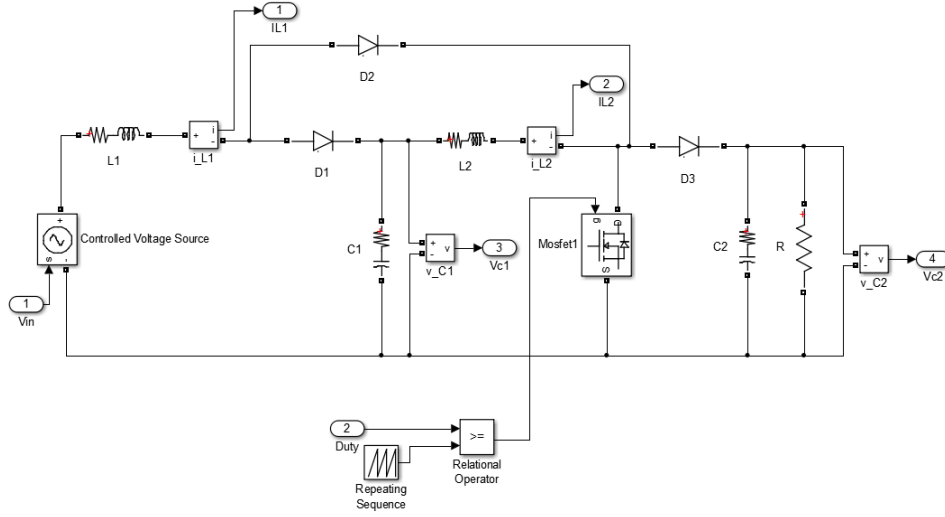


Figure 3.1: The real subsystems of the converter.

resistors in the value of the inductors' resistances. The values of the parasitic elements can be seen in Table 3.1. In opposite with the real model, in the ideal model the parasitic elements, such as the resistance of the inductors and capacitors are zero.

Table 3.1: Parasitic elements of the converter.

Element	Value
First inductor resistance	$0.047[\Omega]$
Second inductor resistance	$0.087[\Omega]$
First capacitor ESR	$0.5[\Omega]$
Second capacitor ESR	$0.2[\Omega]$
Diodes forward voltage drop	$0.7[V]$
Diodes resistance	$0.035[\Omega]$
Mosfet resistance	$0.12[\Omega]$

In Figure 3.2. an open loop simulation of the converter with parasitic elements can be seen. The simulation starts at an input voltage of $10[V]$ and a duty ratio of 0.4 . At 0.05 seconds the input voltage is increased stepwise to $12[V]$, while at 0.1 seconds the duty ratio is increased to 0.6 . From this simulation the open loop behaviour of the circuit can be observed. It can be seen that both the output voltage and first inductor current have an overshoot, however this phenomena is more significant in case of the current as the current overshoot can damage the circuit. The losses of the circuit can also be investigated in this simulation, as until 0.05 seconds the input voltage is $10[V]$ and duty ratio is 0.4 , thus theoretically the output voltage have to be $27.8[V]$, however it only reaches $24.4[V]$.

3.2 Average current mode control

For the average current mode control two controller loops are needed: one inner current control loop, and another outer voltage control loop. The inner loop needs to be faster then the outer,

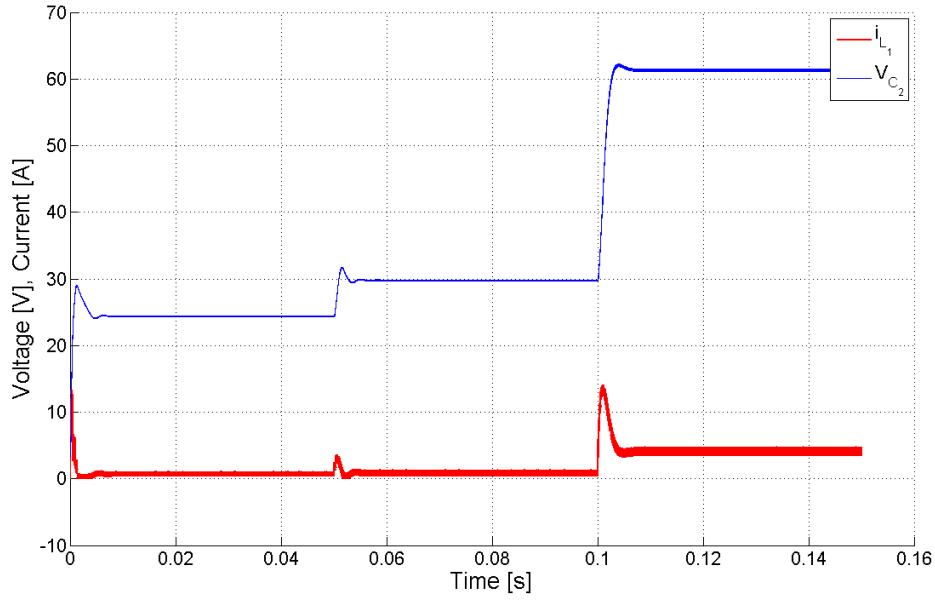


Figure 3.2: The open loop simulation of the converter.

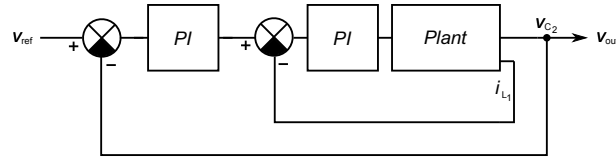


Figure 3.3: Block diagram of the control algorithm.

which has to be taken into account when designing the controller loops. The block diagram of the corresponding average current mode control for the quadratic boost converter the can be seen in Figure 3.4.

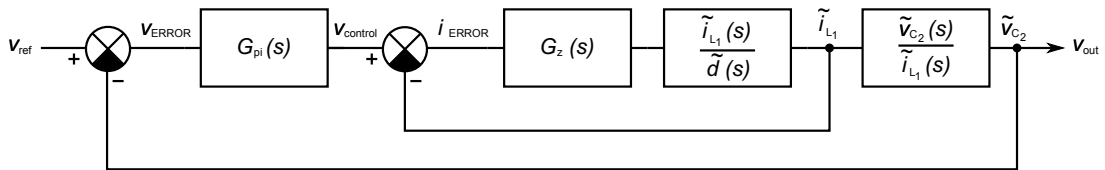


Figure 3.4: Block diagram of average current mode control.

The $\frac{v_{\tilde{C}_2}(s)}{i_{L_1}(s)}$ and $\frac{i_{L_1}(s)}{d(s)}$ are the previously derived transfer functions, see (2.6) and (2.4). The signals are the v_{ref} reference voltage, v_{ERROR} voltage error, $v_{control}$ control voltage – reference of the inner current loop, and i_{ERROR} current error. The controller blocks are $G_{pi}(s)$ and $G_z(s)$ which can be expressed with parameters as:

$$G_z(s) = \frac{K_p}{\omega_z} + K_p \frac{1}{s}, \quad (3.1)$$

$$G_{pi}(s) = P + I \frac{1}{s}. \quad (3.2)$$

From (3.1) and (3.2) it can be seen that at $G_{pi}(s)$ the proportional gain is P and the integrator gain is I , while in case of $G_z(s)$ the integrator gain is K_p and the proportional gain depends not only on K_p but on ω_z as well, and its value is $\frac{K_p}{\omega_z}$. These controller functions were chosen to suit the design steps in [5]. In our case the controller algorithms will be implemented in a microcontroller, however in [5] analog controller circuits are designed, thus not all of the design steps from it are used. The steps of the design from [5], and also the steps applied in our case are as follows:

1. Select the value of ω_z to be a decade below the half switching frequency.
2. Tune K_p such, that the $\frac{\tilde{i}_{L1}(s)}{v_{control}(s)}$ transfer function does not have resonant peaks.
3. Design a *PI* controller for the outer voltage loop, with the obtained closed inner loop transfer function.

The controllers were designed in case of a continuous system, and then the parameters were refined with the help of the simulations in discrete time. The advantage of the simulations is that they can show the effect of the switched mode operation of the system as well, which cannot be investigated with the linearised average state space model and transfer functions.

3.2.1 Design of the current loop

For the determination of the K_p and ω_z parameters the switching frequency of the semiconductor switch had to be defined. For this purpose $50[kHz]$ was chosen, which is the same as the sampling frequency of the control algorithm, which is high enough to avoid large ripples in the currents, and small enough to be able to implement it on a microcontroller. This way the value of ω_z was chosen to 2000. After this step the value of K_p could be determined, in order not to have resonant peaks on the $\frac{\tilde{i}_{L1}(s)}{v_{control}(s)}$ transfer function, which can be seen in Figure 3.5. Besides the resonant peaks it had to be taken into consideration, that the output of the G_z block is the duty ratio, thus it needs to be between 0 and 1. In this case the value of K_p could not be increased so high where the frequency response diagram would become completely smooth, but with $K_p = 300$ it could be achieved not to have regions with gain higher than $0[dB]$.

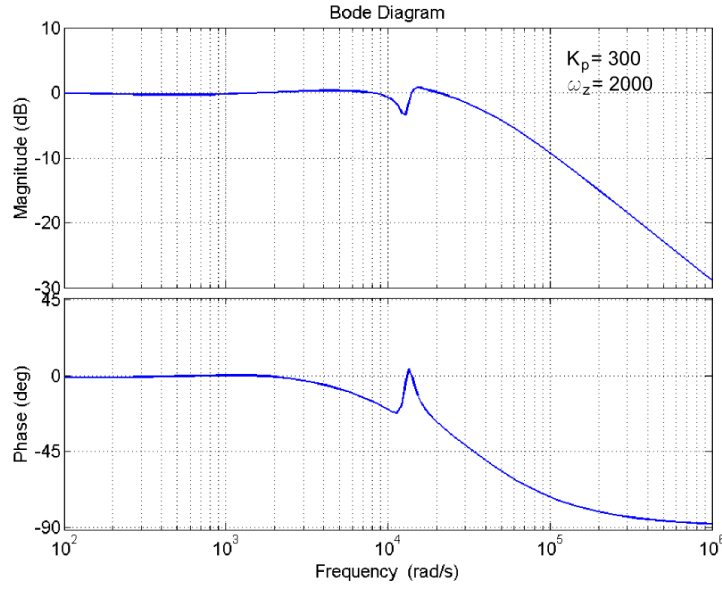
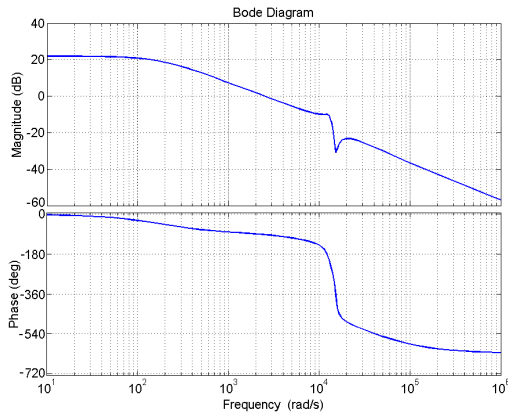


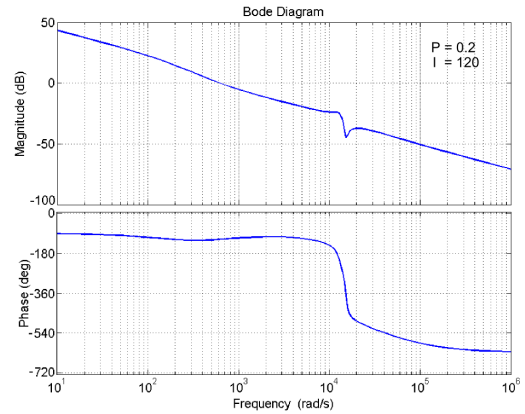
Figure 3.5: Bode diagram of the $\frac{\tilde{i}_{L1}(s)}{v_{control}(s)}$ closed current loop.

3.2.2 Design of the voltage loop

After the parameters of the inner current loop was obtained, the outer loop could be designed. According to Figure 3.4. the current loop transfer function needs to be multiplied with the $\frac{v_{\tilde{C}2}(s)}{\tilde{i}_{L1}(s)}$ transfer function to obtain the plant for which the outer *PI* controller needs to be designed. The bode diagram of the $\frac{v_{\tilde{C}2}(s)}{v_{control}(s)}$ transfer function can be seen on Figure 3.6a.



(a) Transfer function $\frac{v_{\tilde{C}2}(s)}{v_{control}(s)}$.



(b) Open loop system.

Figure 3.6: Bode plots

For the plant shown in Figure 3.6a. a *PI* controller needed to be designed. The integrator gain was selected in the range of K_p , its value after the refinement process was $K_p = 300$. The frequency response of the open loop system – the product of the previously mentioned plant and the G_{pi} controller can be seen on Figure 3.6b.

From Figure 3.6b. it can be seen, that the value of P can be selected in a wide range, for which the phase margin would be about 60 degrees. Due to this fact other considerations had to be taken into account: the outer loop has to be slower than the inner loop, and the current reference should not be higher than the current for which the board was designed. Being aware of these restrictions the value of P was chosen to 0.2.

3.2.3 Discrete controller

In the previous section the controller for the continuous system was designed. In the microcontroller the discrete version of the PI controllers are implemented with a sampling rate $f_s = 50[kHz]$. As the discretization has influence on the stability and performance it should be taken into consideration. The block diagram of the discrete system can be seen on Figure 3.7. The duty ratio is calculated by using the sampled signal of i_{L1} and v_o . The calculation takes less time than $T_s = \frac{1}{f_s}$, however, it is advisory to update the compare register of the PWM peripheral of the microcontroller only in the next switching period. It results in a fixed delay which can be expressed by using the first Padé approximation.

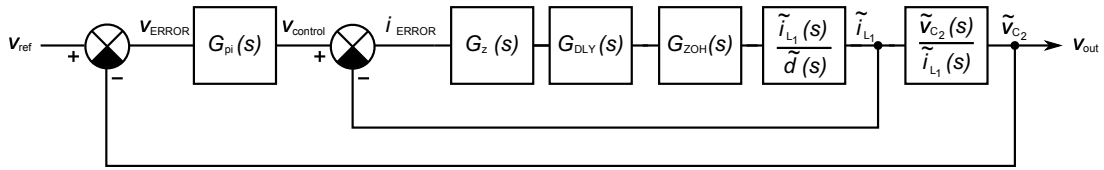


Figure 3.7: Block diagram of the discrete model.

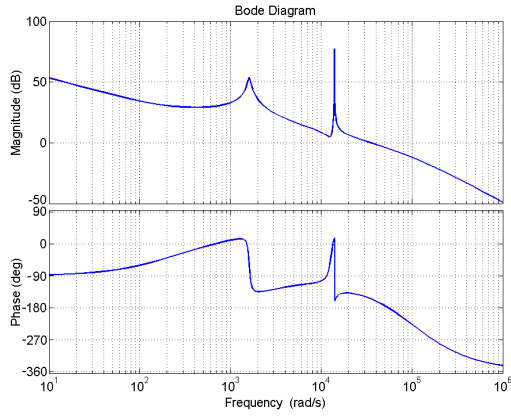
$$G_{DLY}(s) = \frac{-\frac{1}{2}T_s s + 1}{\frac{1}{2}T_s s + 1}, \quad (3.3)$$

where T_s is the sample time. The effect of the Zero order hold can be given by:

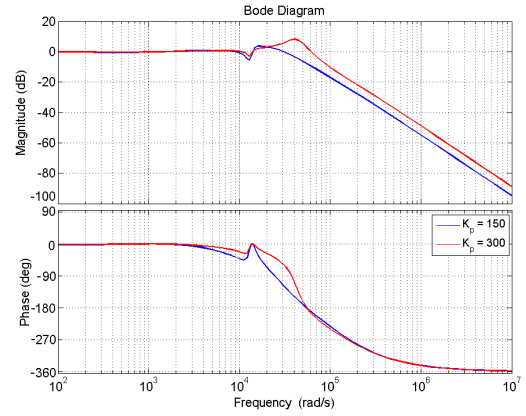
$$G_{DLY}(s) = \frac{1}{\frac{1}{2}T_s s + 1}. \quad (3.4)$$

In case of the discrete system, the previously designed parameters for the continuous system had to be refined. With $K_p = 300$ the $\frac{i_{L1}(s)}{v_{control}(s)}$ closed loop frequency response had again resonance peak. With help of the open loop bode diagram, the value of K_p was reduced to 150 with which the phase margin became 41.5 degrees, which can be seen on Figure 3.8a. The resonance peak was also reduced with this process, which can also be seen in Figure 3.8b.

As the inner current loop was refined, the outer voltage loop had to be verified. The open loop bode diagram of the discrete system can be seen on Figure 3.9., it can be observed that the phase margin became 61.7 degrees at the new value of K_p which ensures the stable operation.



(a) Open loop



(b) Closed loop

Figure 3.8: Bode plot of the $\frac{\tilde{i}_{L1}(s)}{v_{control}(s)}$ for discrete system.

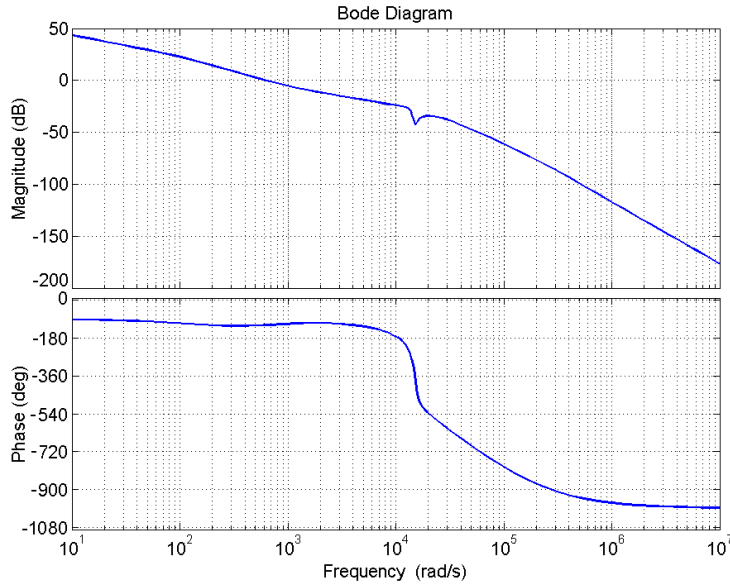


Figure 3.9: Bode diagram of the open loop discrete system.

3.2.4 Simulation of the controller

After the controller had been designed, the simulations could be carried out. At first the controller was simulated in continuous time with ideal parameters. When the results were obtained, the same controller was simulated again, but in discrete time, and the results were used for refining the controller. At last the discrete controller with non-ideal elements was investigated, whose block diagram can be seen on Figure 3.10. In the figure the 'real plant' block consists of the previously shown plant of the converter (Figure 3.1.). For protection of the circuit, an additional saturation was added at the current reference input of the inner current loop. The other saturation block at the duty output ensures that the switch cannot be always turned on. It

has to be mentioned that however saturation blocks are used the values of the current and duty ratio was not saturated otherwise anti-windup methods should be used.

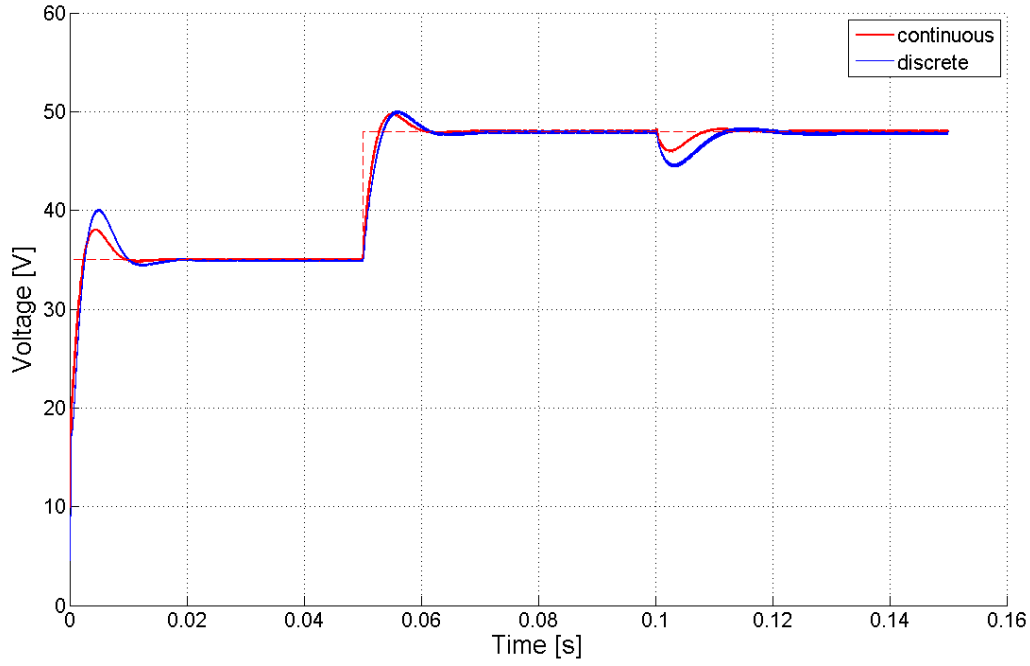
Figure 3.10: *The real plant with the discrete controller.*

The simulation results can be seen on Figures 3.11a. , 3.11b. and 3.11c. From these figures not only the behaviour of the controller, but also the difference between the continuous ideal and the real discrete model can be investigated. During the simulation the reference voltage increases stepwise from 35[V] to 48[V] at 0.05 seconds, while the input voltage decreases also stepwise from 12[V] to 8[V] at 0.1 seconds.

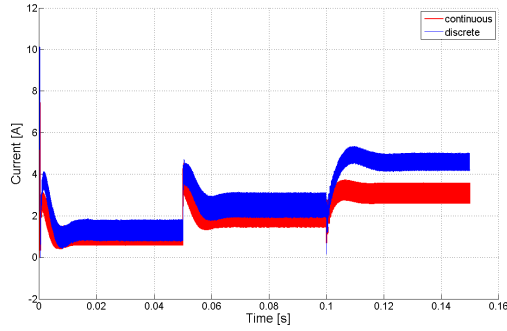
On Figure 3.11a. the output voltage of the converter can be seen. For the stepwise reference voltage changes it reaches the desired output at about 0.01 seconds with an overshoot. It should be mentioned, that the overshoot, as well as the voltage ripple is bigger in case of the discrete system.

On Figure 3.11b. the corresponding time functions of the first inductor current can be seen. As in case of the output voltage, the current responds also with an overshoot to the reference change. In the case of the discrete controller, for the same duty ratio higher current is needed, which can be the result of the parasitic elements modelled in the circuit.

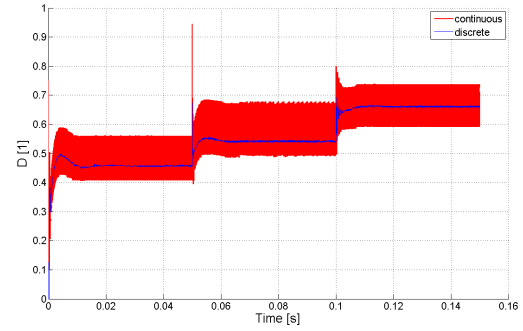
3.2.5 Simulation with fuel cell input



(a) Output voltage – v_{C_2} .



(b) First inductor current – i_{L_1} .



(c) Duty ratio (d)

Figure 3.11: Simulation results.

The parameters of the fuel cell were set based on EFOY PRO 2200. The voltage of the cell at $0[A]$ is $21[V]$, while at $1[A]$ is $16[V]$. The nominal operating point is $I_{nominal} = 7.5[A]$ and $V_{nominal} = 12[V]$. The maximum operating point is $I_{end} = 20[A]$ and $V_{end} = 9[V]$. The characteristic of the fuel cell can be seen on Figure 3.12.

During the simulation the reference voltage of the voltage loop increases stepwise from $35[V]$ to $48[V]$ at 0.05 seconds, while the load resistor is decreased from $100[\Omega]$ to $80[\Omega]$ at 0.1 seconds. It can be observed however, that the fuel cell voltage is depending on the changes of the load, thus it is not constant during the stepwise changes in the reference voltage, and it has a relative high ripple, the output voltage and first inductor current reaches te constant steady state value within 0.1 seconds. The output voltage also has smaller ripple than the voltage of the fuel cell.

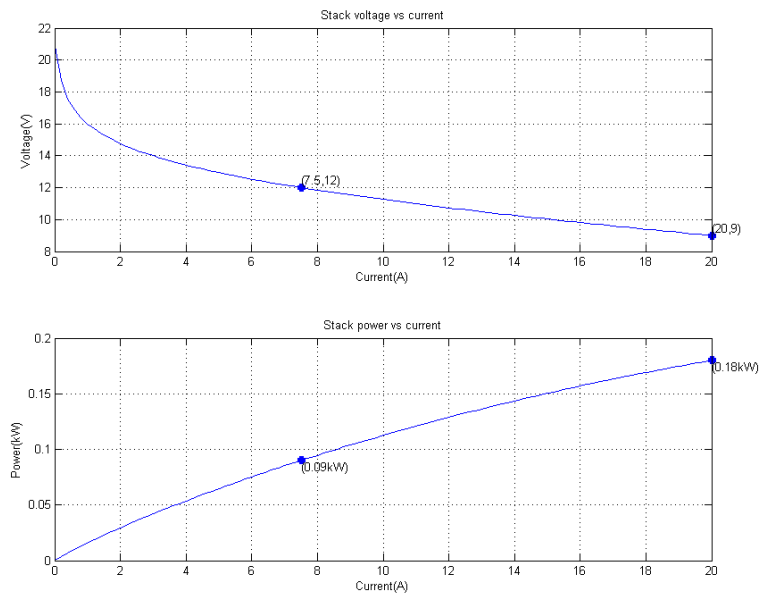


Figure 3.12: *Characteristic of the fuel cell.*

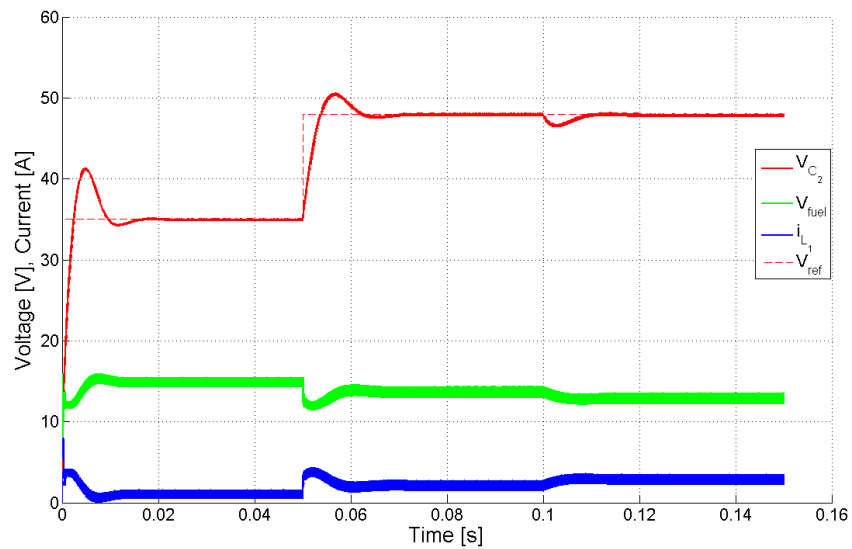


Figure 3.13: *Simulation result of the real controller with fuel cell input.*

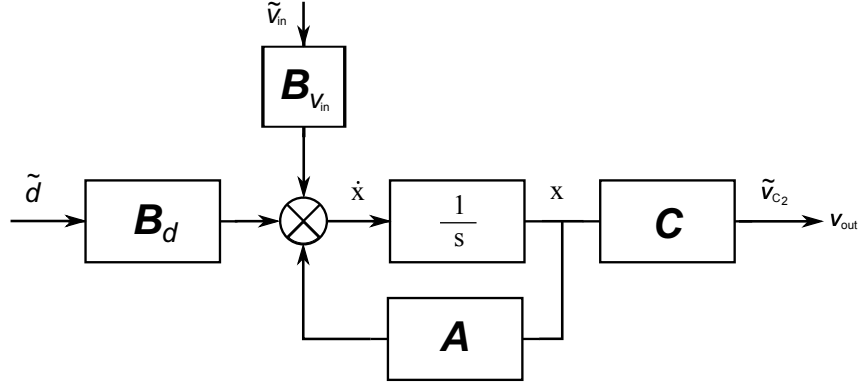


Figure 3.14: Block diagram of the separated state space equations

3.3 State Space Controller design

For state space controller design consider the small signal state space model of the converter described in (2.2). From the two inputs of the converter $\tilde{d}$ and $\tilde{v}_{in}$ only $\tilde{d}$ would function as an input the controller can set and the changes in $\tilde{v}_{in}$ will appear as disturbances in the system. Therefore it is advisory to separate the inputs in the state space equations to the form of $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_d\tilde{d} + \mathbf{B}_{v_{in}}\tilde{v}_{in}$:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_{C1} \\ \dot{v}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D}{L_2} \\ \frac{1-D}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-D}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} \frac{V_{in}}{L_1(1-D)} \\ \frac{V_{in}}{L_2(1-D)^2} \\ -\frac{V_{in}}{RC_1(1-D)^4} \\ -\frac{V_{in}}{RC_2(1-D)^3} \end{bmatrix} \tilde{d} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tilde{v}_{in}. \quad (3.5)$$

In order to alter the dynamics of a controllable system one can use State Variable Feedback (SVFB). In this method the state variables of the system are fed back to the inputs through a feedback gain matrix $\mathbf{K}$. As a result the new system matrix will be $(\mathbf{A} - \mathbf{BK})$. Moving the poles however changes the steady state gain therefore a gain compensator should be added if one wants to control the system using a reference signal. A more detailed description of SVFB and reference gain compensation can be found in [4].

3.3.1 Extending the SVFB by an integrator

The method presented above only creates open loop control. Even if the loop becomes closed the Quadratic Boost Converter with SVFB will be a type 0 system and thus it won't be able to eliminate the steady state error of the output signal. For this reason an integrator is added to the feedback loop. This new integrator yields an additional state variable x_i . The extended open

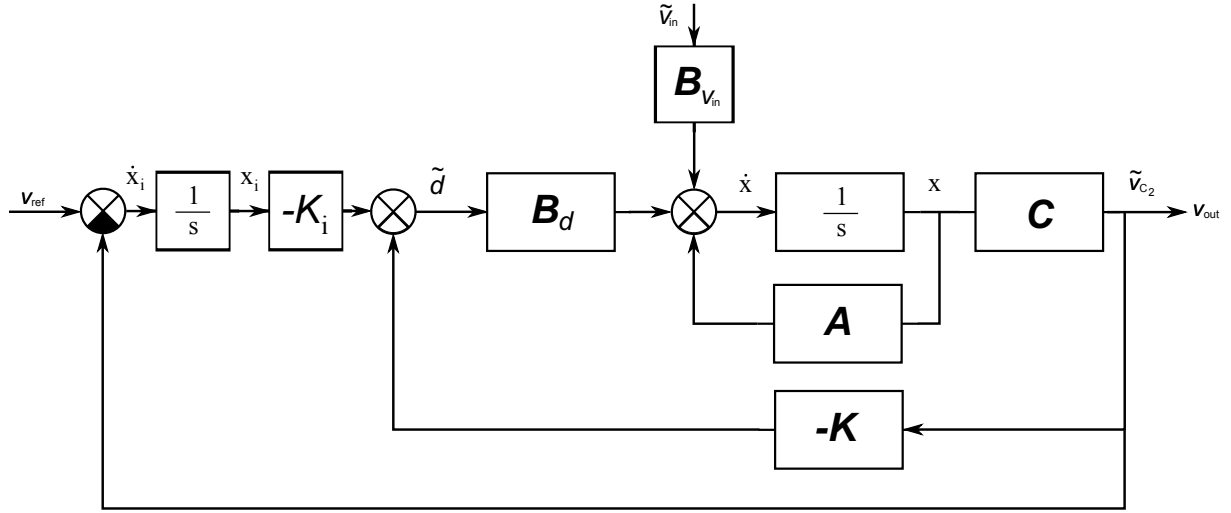


Figure 3.15: Block diagram of the converter with state variable feedback extended by integral action

loop state space model $\dot{\mathbf{x}}_{ext} = \mathbf{A}_{ext}\mathbf{x}_{ext} + \mathbf{B}_{d_{ext}}\tilde{d} + \mathbf{B}_{v_{in_{ext}}}v_{in}$ is :

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_{C1} \\ \dot{v}_{C2} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L_1} & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D}{L_2} & 0 \\ \frac{1-D}{C_1} & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1-D}{C_2} & 0 & -\frac{1}{RC_2} & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{i}_{L1} \\ \tilde{i}_{L2} \\ \tilde{v}_{C1} \\ \tilde{v}_{C2} \\ x_i \end{bmatrix} + \begin{bmatrix} \frac{V_{in}}{L_1(1-D)} \\ \frac{V_{in}}{L_2(1-D)^2} \\ -\frac{V_{in}}{RC_1(1-D)^4} \\ -\frac{V_{in}}{RC_2(1-D)^3} \\ 0 \end{bmatrix} \tilde{d} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} v_{in}. \quad (3.6)$$

The tuning of this controller is done by setting the feedback gain matrix $\mathbf{K}_{ext}$ and thus moving the poles of the system. The state space model of the closed loop system becomes:

$$\dot{\mathbf{x}}_{ext} = (\mathbf{A}_{ext} - \mathbf{B}_{d_{ext}}\mathbf{K}_{ext}) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_{ref} + \mathbf{B}_{v_{in_{ext}}}v_{in} \quad (3.7)$$

There are several methods for the calculation of $\mathbf{K}_{ext}$. If one knows the location of the system's desired poles he can use Ackermann's formula to calculate it. In most of the cases (as in our case as well) instead of the the desired poles certain performance criteria for the controller are known. In such cases a good approach is to use Optimal control theory. Optimal control problems define a cost functional J as a function of the state and control input variables. $\mathbf{K}_{ext}$ is then determined by solving for the minimum of J . Out of a number of try-outs the best performance was reached with the Linear-Quadratic Regulator (LQR) design.

3.3.2 Linear Quadratic Regulator design

The Infinite-horizon, continuous time LQR which was used for the controller design uses a cost function where the cost is described by a quadratic function [11]:

$$J = \int_0^{\infty} (\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt \quad (3.8)$$

For a system which has n state variables and m inputs $\mathbf{Q}$ matrix is an n by n positive semi-definite matrix which contains weights which penalize the deviations of the state variables from the desired state. The matrix $\mathbf{R}$ is an m by m matrix which contains weights that penalize the control effort. The $\mathbf{K}$ feedback gain matrix which provides the solution to the problem can be calculated by [11]:

$$\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \quad (3.9)$$

where $\mathbf{B}$ is the input matrix of the plant and $\mathbf{P}$ is the solution of the algebraic Riccati equation (ARE):

$$\mathbf{P} \mathbf{A} + \mathbf{A}^T \mathbf{P} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = \mathbf{0} \quad (3.10)$$

One of the main advantages of the LQR is that it guarantees at least a 60 degrees phase margin[11]. The LQ problem was solved for $\mathbf{K}_{ext}$ using the *lqr(sys,Q,R,N)* function in Matlab.

As the LQR method computes the feedback gain depending on the small signal state space model and the weighing matrices, the dynamic behaviour of the resulting closed loop system will be affected by D , the duty ratio around which the converter equations are linearised. Figure 3.16. shows the relation of the steady state state variables under different duty ratios at $V_{in} = 12V$. As it can be seen at $D = 0.5$ the characteristics of the states are rather flat. For this reason it was considered to design the controller for a greater nominal duty ratio. Using duty ratios higher than $d = 0.8$ however could not provide the expected voltage gains in open loop simulations. Due to this limitation nominal duty ratios (D 's) between 0.5 and 0.8 were investigated. The conclusion was that an overall better dynamic performance and a broader operating range could be achieved by linearising around $D = 0.65$.

The Quadratic Boost Converter extended by an integrator is a fifth order system so $\mathbf{Q}$ is a 5 by 5 matrix. As the state variables of the system $\tilde{i}_{L1}, \tilde{i}_{L2}, \tilde{v}_{C1}, \tilde{v}_{C2}$ have well defined physical meanings and x_i is the time-integral of the error signal $e(t)$, it is logical to penalize the transients of these individually by placing non-zero elements only in the diagonal of $\mathbf{Q}$. R 's dimension is 1 because the controller can only manipulate one input $\tilde{d}$.

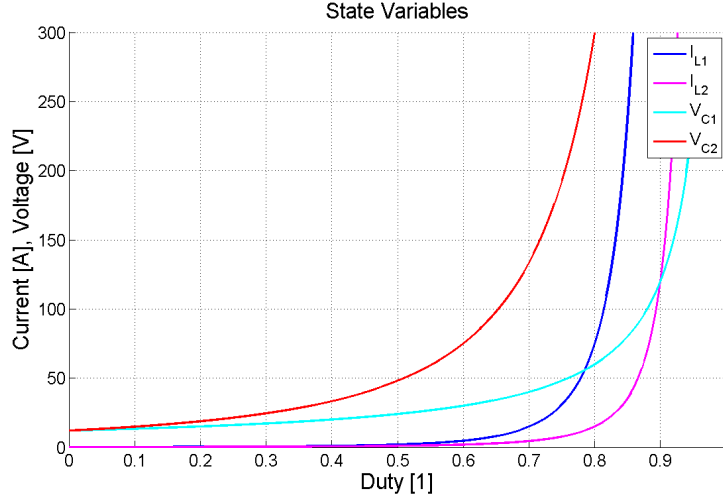


Figure 3.16: $I_{L1}, I_{L2}, V_{C1}, V_{C2}$ under different duty ratios at $V_{in} = 12V$

After numerous efforts the best solution turned out to be selecting the weighting factors with respect to energy considerations. Such an energy based controller design method for an Asymmetrical Interleaved DC-DC converter is described in [2]. In steady state the energies stored in the capacitors and inductors of the circuit are:

$$E_{L1} = \frac{1}{2} L_1 I_{L1}^2 \quad (3.11)$$

$$E_{L2} = \frac{1}{2} L_2 I_{L2}^2 \quad (3.12)$$

$$E_{C1} = \frac{1}{2} C_1 V_{C1}^2 \quad (3.13)$$

$$E_{C2} = \frac{1}{2} C_2 V_{C2}^2 \quad (3.14)$$

where $I_{L1}, I_{L2}, V_{C1}, V_{C2}$ are the steady state values of the respective inductor currents and capacitor voltages described in equations (F.1.16) – (F.1.19).

As the Q matrix is related to energy costs the weighting matrices were set to be:

$$\mathbf{Q} = \frac{1}{C_2 V_{C2}^2} \begin{bmatrix} L_1 I_{L1}^2 & 0 & 0 & 0 & 0 \\ 0 & L_2 I_{L2}^2 & 0 & 0 & 0 \\ 0 & 0 & C_1 V_{C1}^2 & 0 & 0 \\ 0 & 0 & 0 & C_2 V_{C2}^2 & 0 \\ 0 & 0 & 0 & 0 & C_2 \cdot 10^{10} \end{bmatrix} \quad (3.15)$$

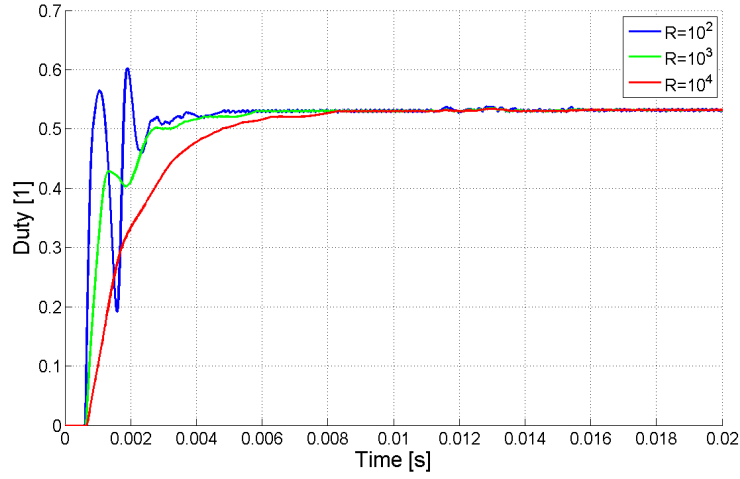


Figure 3.17: The duty ratio change during a 48V reference step response of the real plant with discrete controllers designed with different R weights

$$R = 10^3 \quad (3.16)$$

The coefficients of $i_{L1}, i_{L2}, v_{C1}, v_{C2}$ were set based on the average energies stored during steady state in the respective inductors or capacitors. In addition for design purposes the whole matrix was normalized in such a way that the coefficient of v_{C2} (the output) voltage is 1.

The coefficients of x_i and the input $\tilde{d}$ were set by systematic simulation and analytical calculations in such a way that the penalty on the control effort would cause the controller output to remain between 0 and 1 as it is necessary for properly controlling the switch. As it can be seen on Figure 3.17. by keeping $\mathbf{Q}$ the same one order of magnitude smaller R doesn't result in quicker response and also produces oscillation of the duty ratio. An order of magnitude higher R would result in a much slower system response. The penalty on x_i (the time-integral of the error) signal was set to be high enough to provide a fast dynamic response. Figure 3.18. shows the system's response to a 48V step reference signal with different x_i weights while keeping R the same.

During the design process as the plant is non-linear the designed controllers were always evaluated with simulation. After the controller has been designed in continuous time for the ideal plant it was changed to operate in discrete time with $T_s = 2 \cdot 10^{-5}$ sampling time. The plant in the simulations was also changed to the real plant with parasitic elements already described in the previous section. Some further optimization of the weighing matrices ($\mathbf{Q}, R$) was done to obtain the final controller parameters. The resultant feedback gain matrix $\mathbf{K}_{ext}$ of the optimal controller designed for $D = 0.65$ with the final parameters $R = 10^3$, and $\mathbf{Q}$ defined in (3.15) and is:

$$\mathbf{K}_{ext} = \begin{bmatrix} 0.01215 & 0.02445 & -0.00342 & 0.04211 & -32.28 \end{bmatrix} \quad (3.17)$$

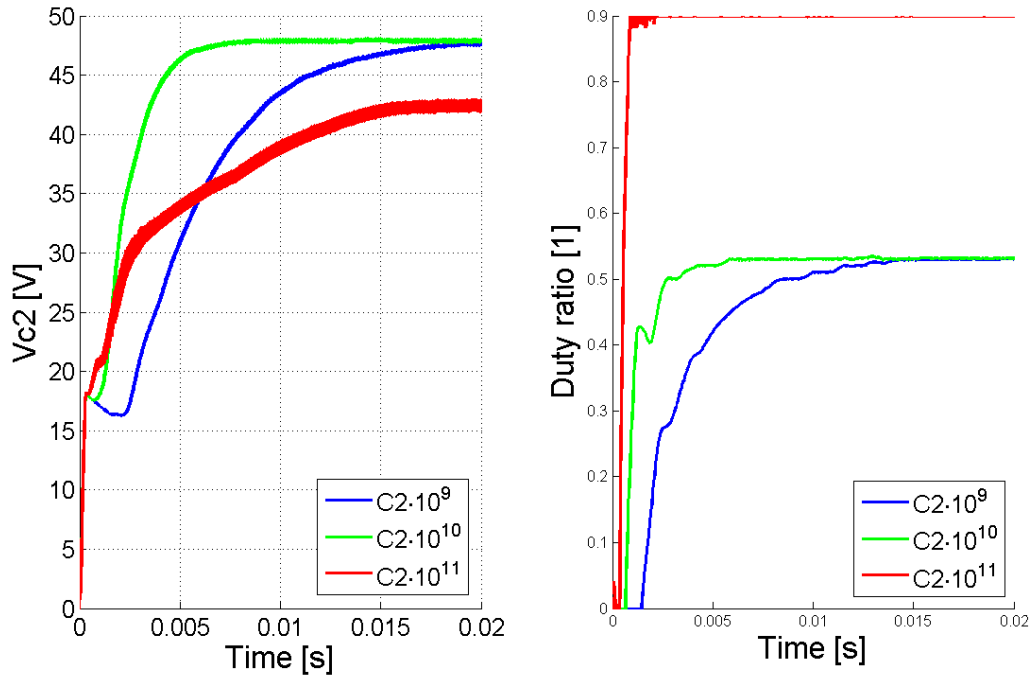


Figure 3.18: The real plant's response to a 48V reference step signal using discrete time controllers with different x_i weights

Figure 3.19. shows the bode plot of the resulting closed loop system. A final step in the LQ controller design was to set a saturation of the output with a lower limit of 0 and an upper limit of 0.8. This is necessary because if the MOSFET is turned on continuously for a longer period of time it can get damaged. The upper limit of 0.8 was determined based a set of simulations. From these it was concluded that for duty ratios higher than 0.8 the controller output did not reach the expected value therefore there was no point in allowing higher duty ratios on the controller output. This saturation helped to achieve a much broader operating range.

Figure 3.20. shows the Simulink block implementation of the discrete time regulator. The controller was also implemented in an embedded Matlab function block which provided the same simulation results as the block implementation. On Figures 3.21a., 3.21b., 3.21c. the simulation results of the final controller's performance can be seen.

3.3.3 Simulation with fuel cell input

The controller was also tested with the same fuel cell input as it has been used with using Average current mode control. The results can be seen on Figure 3.22.

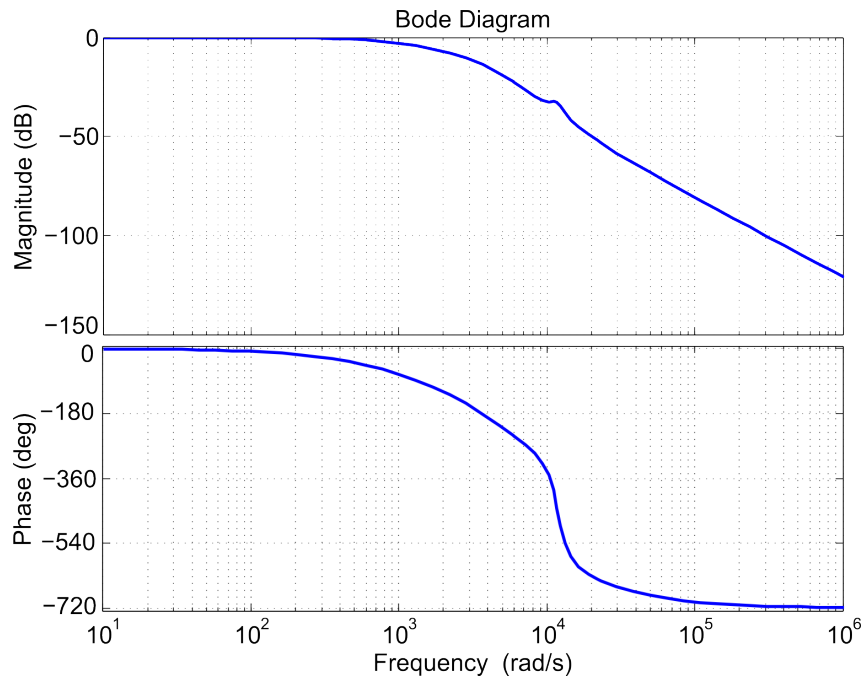


Figure 3.19: Bode plot of the closed loop system with LQ controller

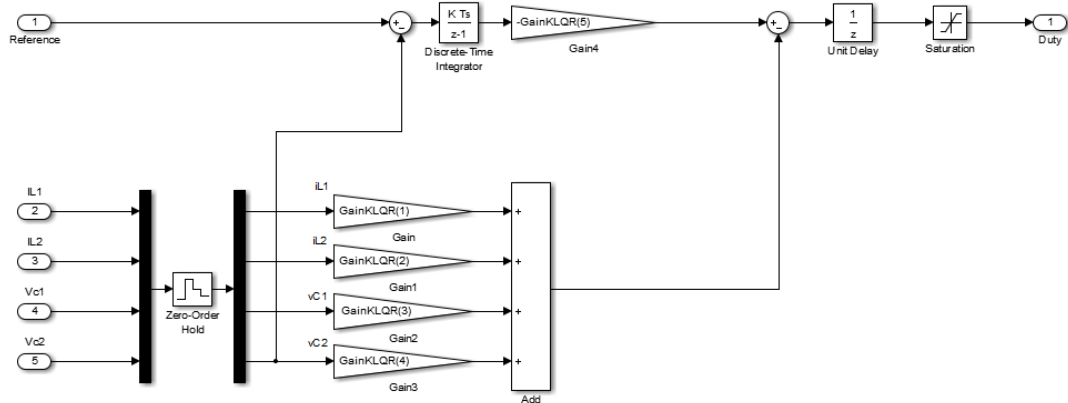
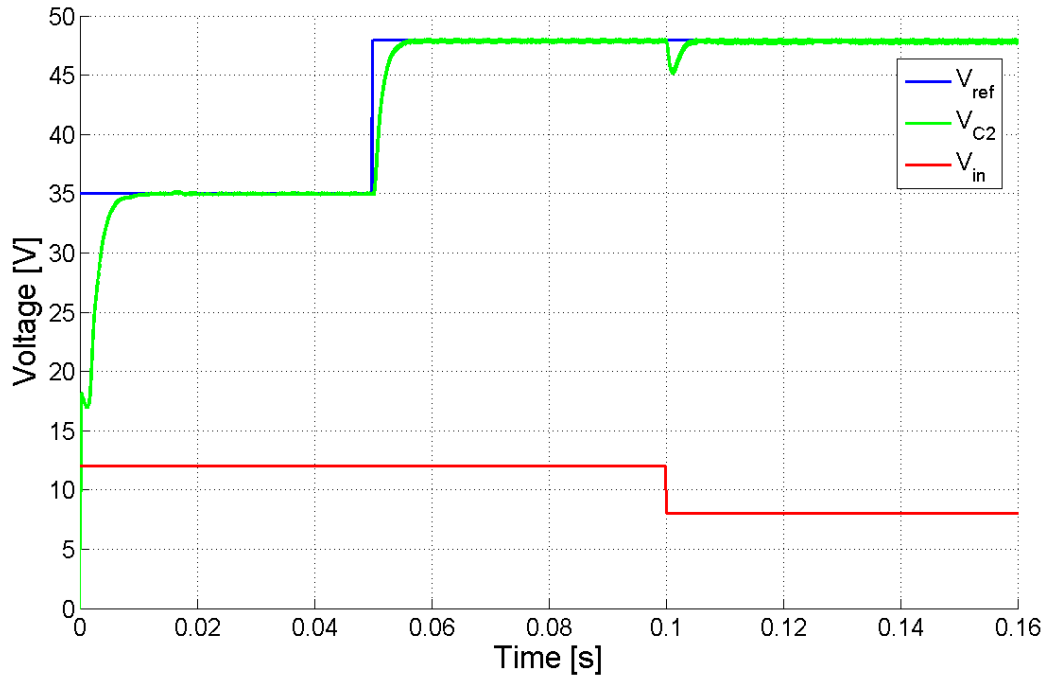
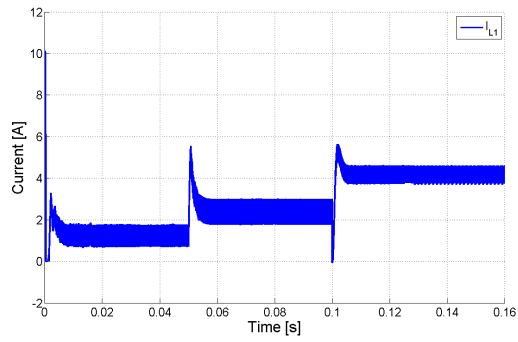


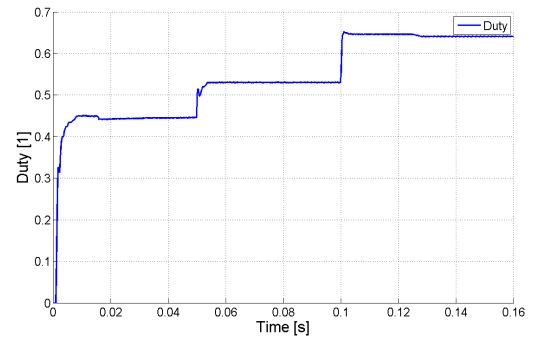
Figure 3.20: Simulink implementation of the closed loop LQR



(a) The reference, input and output voltages



(b) Inductor current (i_{L1})



(c) Duty ratio (d)

Figure 3.21: Simulation results for the discrete real LQR controller

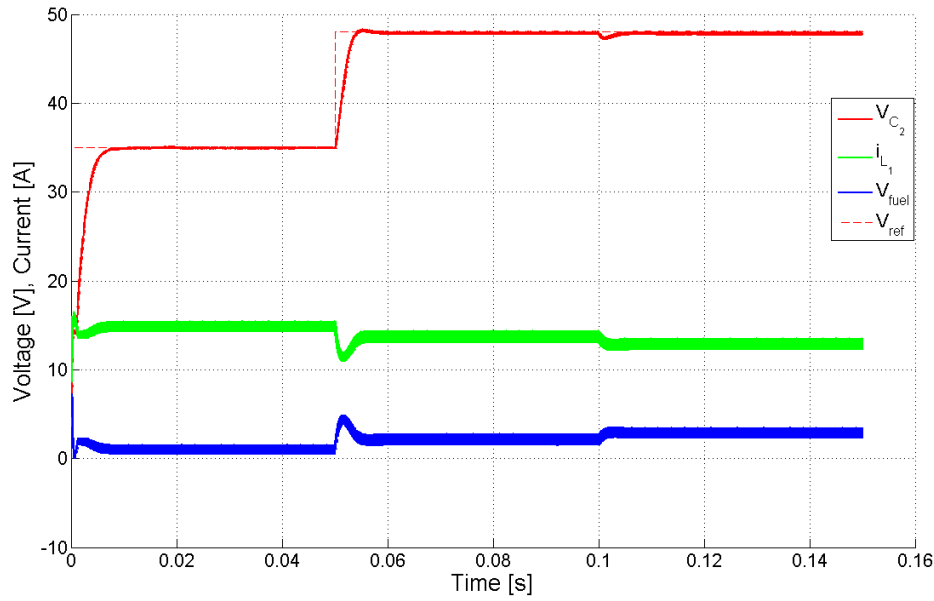


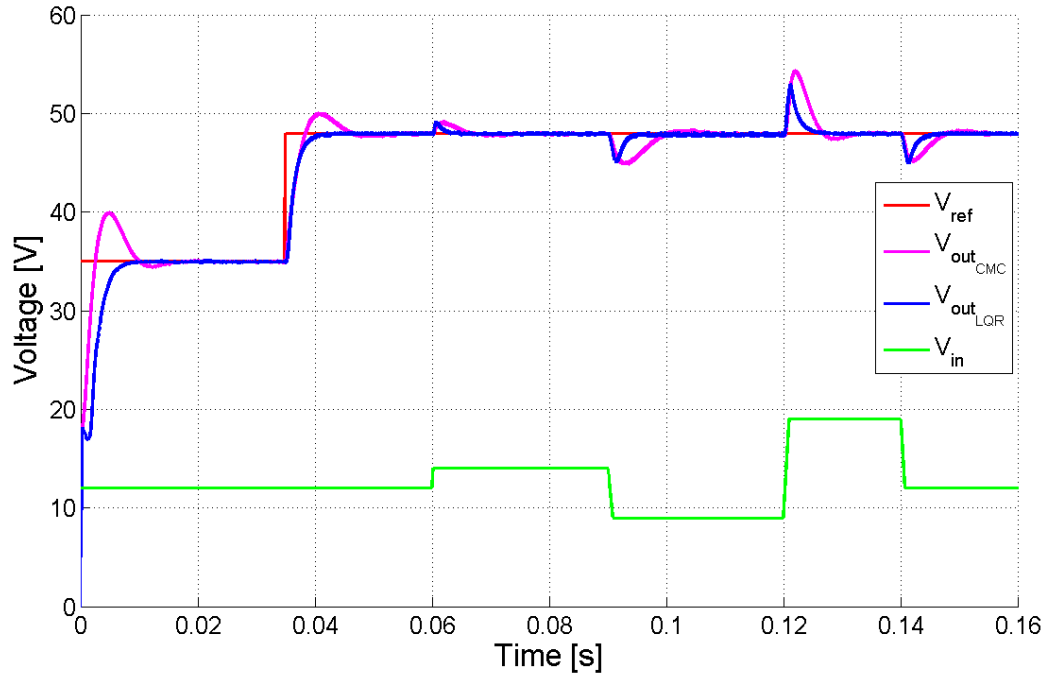
Figure 3.22: Simulation results of the discrete time LQ controller with fuel cell input

3.4 Comparison of the two control strategies

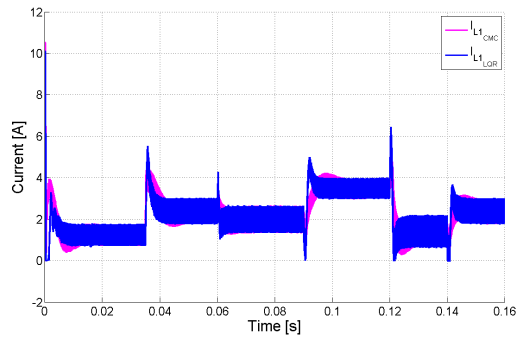
This section is aimed to give a deep comparison of the implemented control strategies and provide a conclusion of the simulation experiences by emphasising the advantages and disadvantages of the two different approaches. Numerous simulations were carried out on the real plant model with the parasitic elements using the two different discrete time controllers with identical sampling times both implemented in embedded Matlab functions. During these tests both systems were exposed to the same reference signals and disturbances.

The main aspects considered during the comparison of the two methods were:

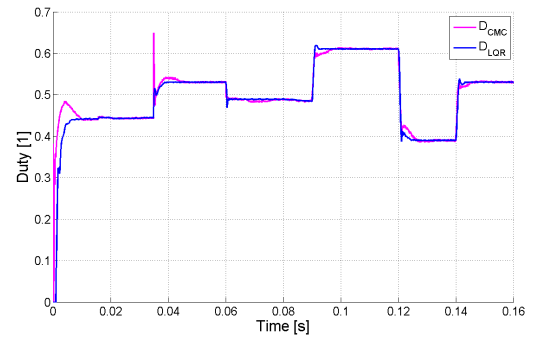
- Dynamic response for step/sudden reference voltage change
- Dynamic response for step/sudden input voltage change
- Performance at different loads (other than $R = 100\Omega$)
- Peak first inductor current during transients
- Stable operating range
- Noise suppression
- Parameter sensitivity
- Feasibility and costs



(a) The reference, input and output voltages



(b) Inductor current (i_{L1})



(c) Duty ratio (d)

Figure 3.23: Dynamic response of the system using the two different controller types

3.4.1 Dynamic response of the different controllers at rated load

Figure 3.23a. shows the response of the two closed loop systems under the following circumstances. V_{in} starts from $12V$ and V_{ref} from $35V$. First the reference voltage is raised to $48V$ to observe the dynamic responses for step reference voltage change. After this the input voltage is varied to test the dynamic responses for step/sudden input voltage changes.

It can be seen that the LQ controller has a much quicker response to both the reference and input voltage changes. The LQR controlled system's response has no overshoot to step reference voltage changes and has generally a smaller peak deviation after changes in the input voltage. Figure 3.23c. shows that during steady state operation the duty ratio has a slightly smaller oscillation using the LQ controller. On the other hand the Average current mode controller (CMC) produces smaller peak inductor currents (Figure 3.23b.).

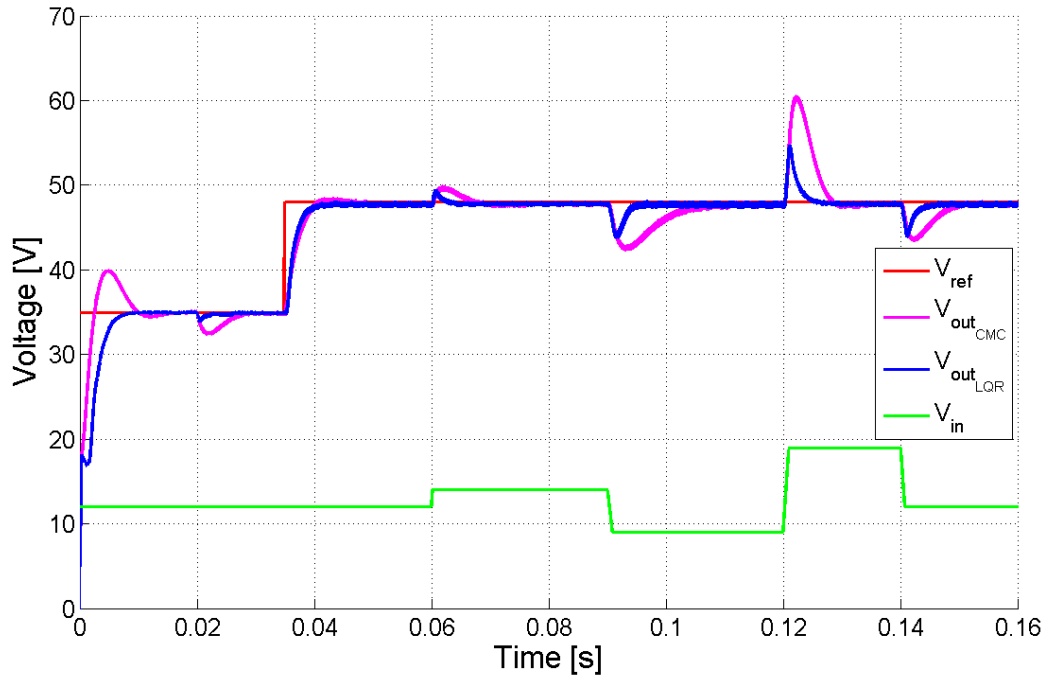
3.4.2 Performance at different loads

Figures 3.24a., 3.24b., 3.24c. show the responses of the system with the two different controllers being subjected to the same reference and input voltage changes as in the previous section. The difference is that at $t = 0.02s$ the load's resistance drops from $R = 100\Omega$ to $R = 50\Omega$. The LQ controller's response to the sudden load change itself is much quicker than that of the Average current mode controller's and the LQR also provides faster dynamic responses with smaller overshoot for the following reference and input voltage changes. The overall better performance in this case comes with the same disadvantage as before: while the CMC's peak inductor current is about $5.5A$ in the LQ controller's case it tops at $6.5 - 7A$.

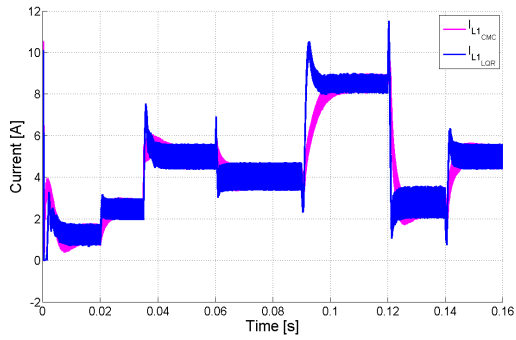
If the load resistance is changed in the other direction (R is increased to 200Ω at $t = 0.02s$) the LQ controller also proves to be better. While the Average CMC starts to produce smaller overshoots the LQR still provides a faster transient response and a better response to the load change itself (Figure 3.25a.). Furthermore if the load resistance is increased further and the controller starts to operate in Discontinuous Conduction Mode (DCM) the current mode controller will start to lose stability while the LQ regulator provides only a slightly worse performance as it does in the Continuous Conduction Mode (CCM) (Figure 3.25b.).

3.4.3 Noise suppression

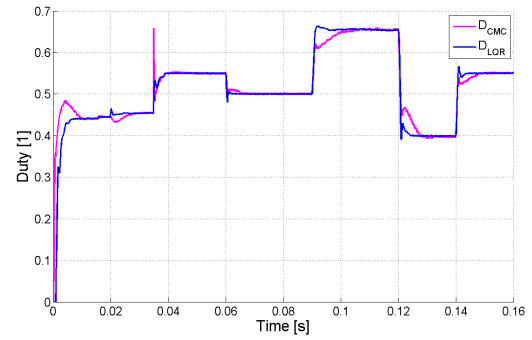
The modelling of measurement noises was done by superposing Gaussian noises with different covariances to the state measurements. In the simulations both controllers seemed to withstand smaller amount of noises without problems. Figure 3.26a. shows an extraordinary amount of



(a) The reference, input and output voltages

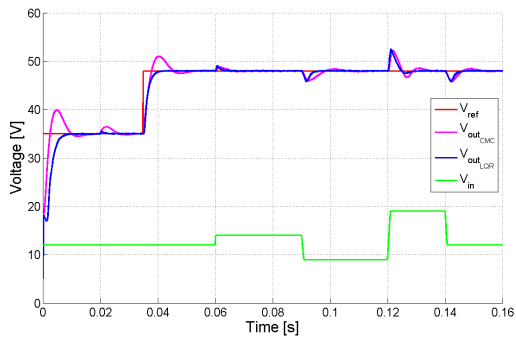


(b) Inductor current (i_{L1})

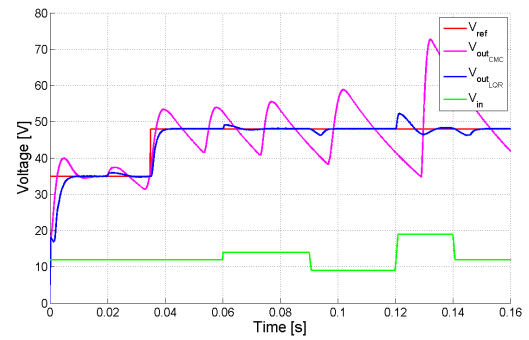


(c) Duty ratio (d)

Figure 3.24: Dynamic response of the systems with load change to $R = 50\Omega$ at $t/0.02s$

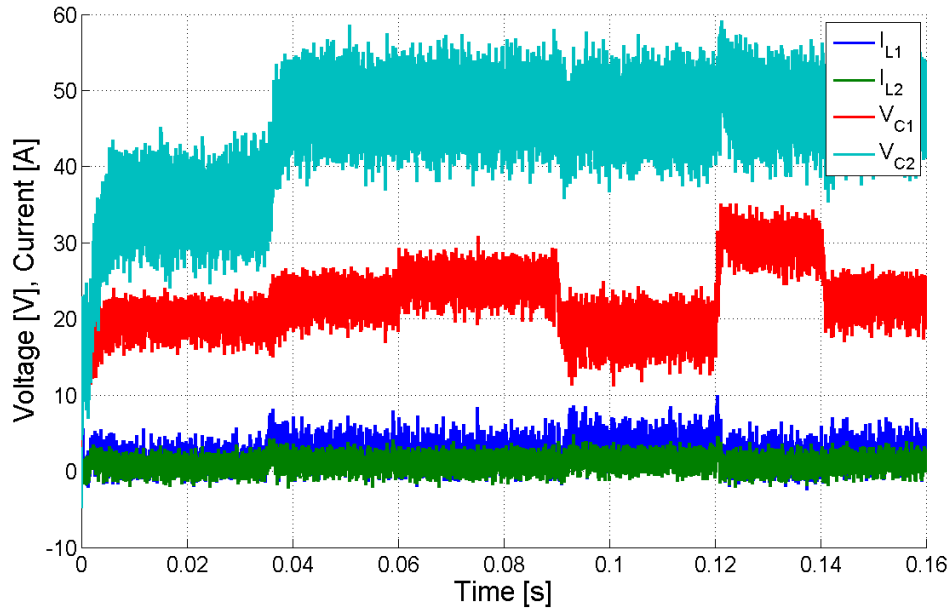


(a) To $R = 200\Omega$ at $t/0.02s$

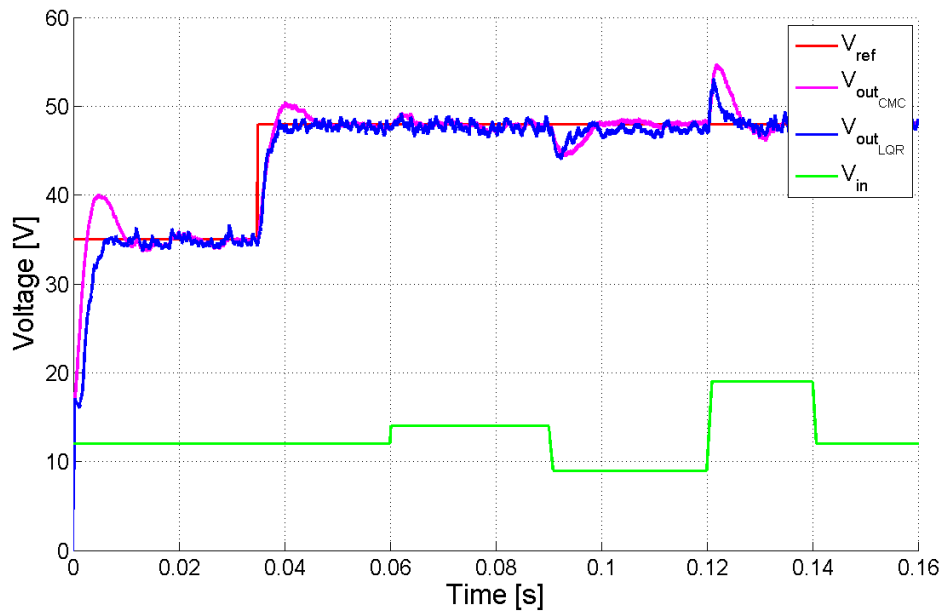


(b) To $R = 200\Omega$ at $t/0.02s$

Figure 3.25: Dynamic response of the systems with load change



(a) *The noisy measurements*



(b) *The reference, output and input voltages*

Figure 3.26: *Dynamic performance of the controllers with noisy measurements*

noise on the different measurements. The performance of the two controllers under these circumstances can be seen on (Figure 3.26b.). Both from output voltage and inductor current point of view the LQ controller seems to be more susceptible to the excessive amount of measurement noise.

3.4.4 Other aspects and summary

Based on the simulation results neither controllers are particularly sensible to $\pm 10\%$ variations in the converter parameters. At the rated load $R = 100\Omega$ the LQ regulator has a wider operating range. The maximum reachable and sustainable output voltage with $V_{in} = 12V$ is around $90V$ which for the current mode controller is about $75V$.

A brief summary of the comparison results based on our simulations can be found in Table 3.2. below:

Table 3.2: Comparison of the two controllers

	Average CMC	LQR
Dynamic performance		
35V $\rightarrow$ 48V step reference change ($V_{in} = 12V$)		
Settling time	$\sim 11ms$	$\sim 6ms$
Overshoot	15%	0%
Peak I_{L1}	4.6A	5.8A
12V $\rightarrow$ 8V step input change ($V_{ref} = 48V$)		
Settling time	$\sim 18ms$	$\sim 4ms$
Max. deviation from V_{ref}	3.5V	3V
Peak I_{L1}	5.3A	6.2A
Operating conditions ($R = 100\Omega$, $V_{in} = 12V$)		
Operating range	15 – 75V	13 – 90V
Good performance at loads	$R < 250\Omega$	$R < 1000\Omega$
Performance in DCM	Very poor	Good
Noise suppression	Slightly better	Good
Parameter sensitivity	Not sensitive	Not sensitive
Current limitation possible	Yes	No
Number of measurements	2	4

It can be concluded that the LQ regulator has a much faster dynamic response and is much more robust than the Average current mode controller. It's main disadvantage is that 2 voltage and 2 current measurements are necessary for proper operation. Both algorithms need about the same amount of basic arithmetical operations per sampling period but the LQ control needs 4 instead of 2 ADC conversions to finish during each cycle. The application of the Average current mode control is advised if the costs of the additional measurements is not affordable or if a maximum inductor current saturation is necessary. In other cases the LQR seems to be a much better choice overall.

3.5 State estimation

The circuit design of the Quadratic Boost Converter (Chapter 4) supports the measurement of all state variables. Even though the sensors required are rather cheap there could be applications where the measurement of all state variables is not advantageous (due to the costs for example). For this reason and for the demonstration of a complete LQG compensator design state estimation possibilities were also investigated. The goal was to create an estimator which uses the input and output voltage (v_{in}, v_{C2}) measurements and which can run at the same sampling frequency as the LQR controller ($50kHz$).

3.5.1 Advanced plant model

If a system is observable all state variables can be estimated from the measured outputs. The observer itself is a mathematical model of the plant. This means that for creating a good and accurate observer a good mathematical model is necessary. Using the averaged state space equations presented in (2.1) and substituting the actual duty ratio $d(t)$ values did not provide acceptable results. Due to the need for a more precise model the averaged state space equations for the converter considering the non-ideal, parasitic elements was used for observer design. The derivation of it can be found in Appendix F.2. The state equations of the advanced model are described in (2.7) and (2.8). To implement them in discrete time forward euler method was used:

$$\frac{\mathbf{x}(k+1) - \mathbf{x}(k)}{T_s} = \dot{\mathbf{x}}(k) = \mathbf{A}_{adv}(k)\mathbf{x}(k) + \mathbf{B}_{adv}v_{in}(k) + \mathbf{B}_{adv_d} \quad (3.18a)$$

$$\mathbf{x}(k+1) = \mathbf{x}(k) + T_s(\mathbf{A}_{adv}(k)\mathbf{x}(k) + \mathbf{B}_{adv}v_{in}(k) + \mathbf{B}_{adv_d}) \quad (3.18b)$$

From (3.18) it can be seen that the resultant discrete system matrix is $(\mathbf{I} + T_s\mathbf{A}_{adv}(k))$ where $\mathbf{I}$ is the identity matrix. To obtain the state observer's equations the error of the model's output $\mathbf{y}_{err} = \mathbf{y}_{measured} - \mathbf{y}_{estimated}$ has to be fed back through a gain matrix $\mathbf{L}$ (with negative feedback) to the state equation [4]:

$$\hat{\mathbf{x}}(k+1) = \left(\mathbf{I} + T_s\mathbf{A}_{adv}(k) - T_s\mathbf{C}_{adv}\mathbf{L} \right) \hat{\mathbf{x}}(k) + T_s\mathbf{B}_{adv}v_{in}(k) + T_s\mathbf{B}_{adv_d} + T_s\mathbf{L}v_{C2_{meas}}(k) \quad (3.19a)$$

$$\hat{\mathbf{y}}(k) = \mathbf{C}_{adv}\hat{\mathbf{x}}(k) \quad (3.19b)$$

Where $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ denote that they are estimated values and $\mathbf{A}_{adv}(k)$ denotes that this matrix is a function of the duty ratio $d(k)$ and thus its time-variant.

In the observer-based controller implementation the duty ratio $d(k-1)$ which was the controller output in the previous sampling time is used to re-calculate the system matrix $\mathbf{A}_{adv}(k)$ during each cycle.

3.5.2 Kalman Filter

Kalman Filters are used for both filtering and state estimation purposes. For our state estimation problem due to feasibility aspects a steady-state Kalman Filter is used - which uses a constant gain matrix $\mathbf{L}$ - as this solution requires much less computational cost than the time-varying one, which updates the gain in every sampling period [7]. Consider the state space model with process ω and measurement noises v included to the system description.[9] With these the equations are:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \omega \quad (3.20a)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + v \quad (3.20b)$$

For the optimal design of the observer gain matrix $\mathbf{L}$, the covariance matrix of the process noises $\mathbf{Q}_o$ and the covariance matrix of the measurement noises $\mathbf{R}_o$ have to be set. $\mathbf{L}$ then is determined using a cost functional similar to that in the case of the LQR. The resultant Kalman Filter is also called as a Linear-Quadratic Estimator (LQE). The LQR and LQE problems can be solved independently and if they are combined the resultant controller is also called as the Linear-Quadratic Gaussian (LQG) compensator. The continuous-time solution to the optimal observer problem (similarly to the LQR) is: [11]

$$\mathbf{L} = \mathbf{P}_o \mathbf{C}^T \mathbf{R}_o^{-1} \quad (3.21)$$

Where $\mathbf{P}_o$ is the solution to the algebraic Riccati equation:

$$\mathbf{A}\mathbf{P}_o + \mathbf{P}_o\mathbf{A}^T - \mathbf{P}_o\mathbf{C}^T\mathbf{R}_o^{-1}\mathbf{C}\mathbf{P}_o + \mathbf{Q}_o = \mathbf{0} \quad (3.22)$$

This calculation can be and was executed in Matlab using the *kalman(sys,Qn,Rn,Nn)* function. The function returns the discrete observer gain matrix if the system *sys* is in discrete time. The LQE problem was solved for $\mathbf{L}$ using the resultant discrete system matrix of the advanced equations $(\mathbf{I} + T_s\mathbf{A}_{adv})$ with $D = 0.65$ nominal duty ratio and the output matrix $\mathbf{C}_{adv}$. $\mathbf{Q}_o$, $\mathbf{R}_o$ were determined by estimating the process noise from the results of simulations and the possible measurement noises based on the circuit design (Chapter 4). The final $\mathbf{Q}_o$, $\mathbf{R}_o$ matrices are:

$$\mathbf{Q}_o = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \quad (3.23a)$$

$$R_o = 1 \quad (3.23b)$$

$R_o = 1$ means for example that the voltage measurements will contain less or equal to $\sqrt{1} = 1V$

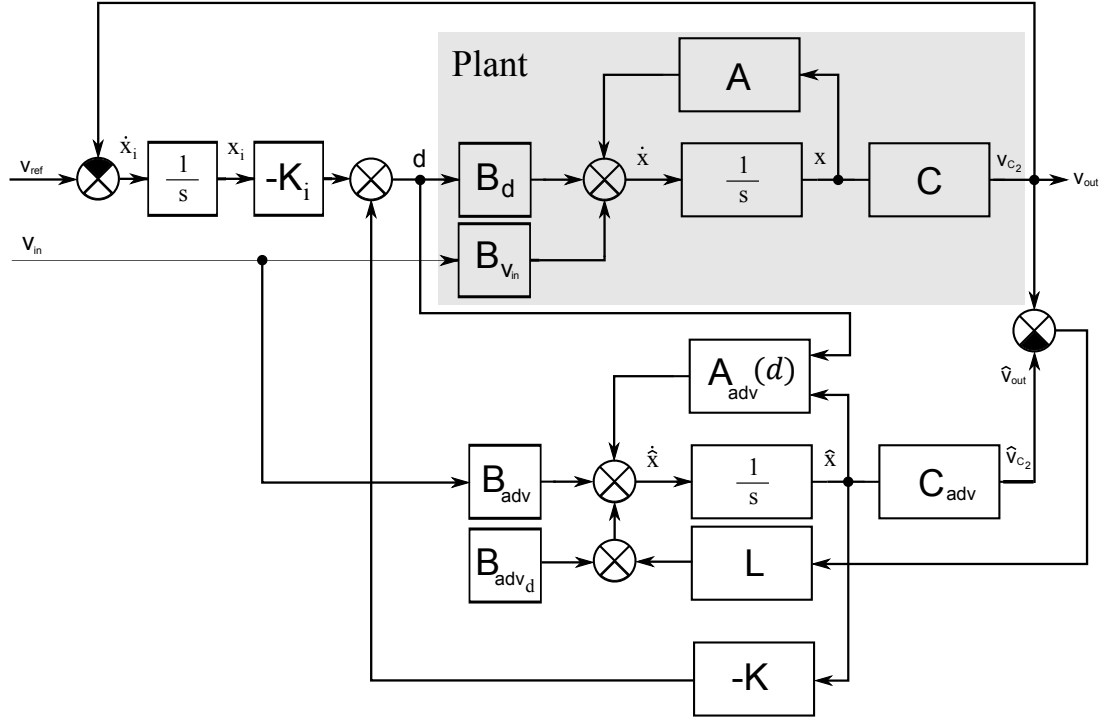


Figure 3.27: The block diagram of the LQG controlled converter

measurement noise with $\sim 68\%$ probability. The resulting gain matrix is:

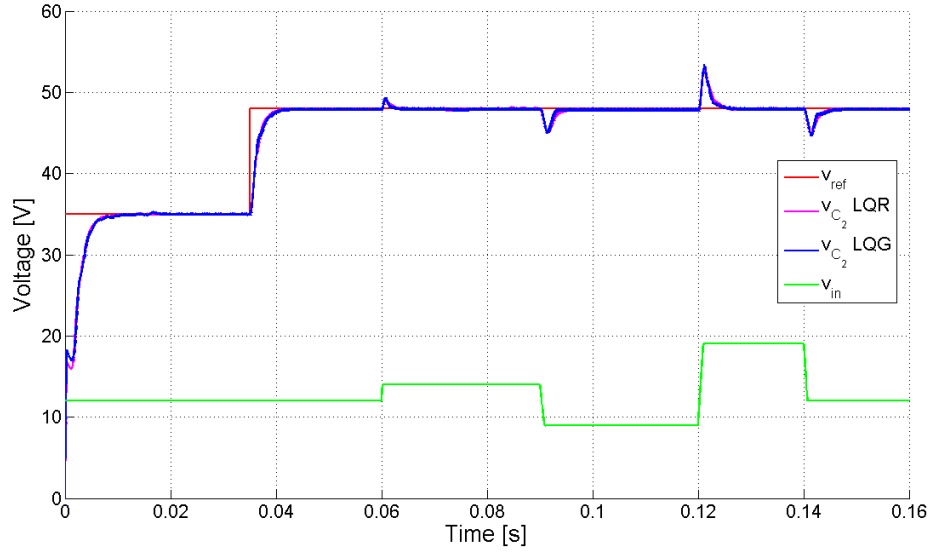
$$\mathbf{L}^T = \begin{bmatrix} -0.28 & 0.75 & -2.67 & 0.62 \end{bmatrix} \quad (3.24)$$

The block diagram of the complete LQG controller which shows the operation in continuous time can be seen on Figure 3.27. (The discretization of the controller has been excluded from the diagram for the sake of simplicity). Figure 3.28a. shows the response of the designed LQG controller compared to the LQR with four measurements and Figure 3.28b. shows the state variables and their estimates.

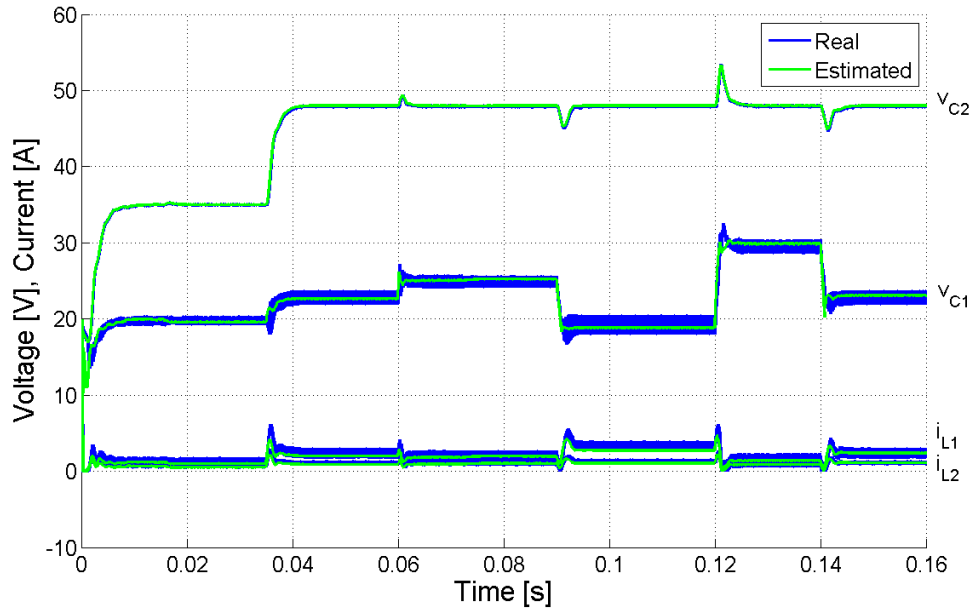
3.5.3 Summary and conclusion

In this section state estimation possibilities of the Quadratic Boost Converter were discussed. The main results of this section are:

- Using the ideal averaged state-space model (2.1) of the Converter is not sufficient for accurate state estimation. A more detailed (advanced) model has to be used (2.7).
- The state estimation's accuracy weakens if the load of the converter R differs from the nominal value. The resulting LQG still functions for a broader range of loads than the



(a) Dynamic performance of the LQR (without estimator) and LQG (with estimator) compensators



(b) The state variables and their estimates

Figure 3.28: Dynamic performance of the controllers with noisy measurements

current mode controller due to the robustness of the LQR, but the state estimations will contain errors.

- At rated load the LQG compensator's performance is almost identical to the LQR's.
- The LQG requires around 2-3 times more computation effort, but also requires 2 A/D conversions per sampling period instead of 4.
- If the input voltage is not expected to change more than $\pm 10\%$ the input voltage measurement can be substituted by putting the nominal input voltage into the observer equations (3.19). In this special case only the output voltage v_{C2} has to be measured.

The conclusion is that the use of an LQG compensator for controlling the Quadratic Boost converter is possible but it is not advised as in most of the cases the inclusion of the two current and two voltage measurements is relatively easy to implement. Other drawbacks are the extra modelling and controller design time required and the weaker performance at non-rated loads ($R \neq 100\Omega$). Its advantage besides the less amount of measurements required is that the measurement-noise suppression of the LQG can be enhanced by increasing R_o during the design process.

Chapter 4

Design of the circuit

The converter had to be designed and built in order to be able to implement the control algorithms designed before. At first, the elements of the converter had to be chosen. From the several simulations done before, the values of the voltages and currents as minimum requirements for the elements could be used. After the parts were chosen, the other elements of the circuit had to be designed, which were not involved in the simulation, such as the current and voltage measurement circuits, or the over-voltage protection. After the schematic the Printed Circuit Board needed to be designed with respect to the real converter parameters.

4.1 Schematic

To build the circuit in real, at first the schematic was designed for the theoretical circuit shown in 2.1. Besides the mentioned elements the design and selection of other smaller circuit components was also required. As this converter is designed for measurement and development purposes, such a circuit was designed, that is suitable for several control algorithms, and can be connected to any kind of microcontroller. To achieve this all of the state variables need to be measured. These values are transferred to the controller through a standard ribbon cable. To be able to measure the different signals with oscilloscope, measurement pins were placed on the PCB, thus in the schematic as well. The complete schematic of the converter can be seen on Figure 4.1. , however the smaller parts of it are discussed in details.

At first the measurement of the two inductors' current had to be designed. Current shunt monitors from Texas Instrument were chosen for current measurement, as using shunt resistors is a common way of measuring current. The Texas Instruments ICs are needed because of the floating voltages of the two shunt resistors. These ICs consist of an operational amplifier and four resistors which determine the gain of the IC. The connection of the ICs to the shunt resistors can be seen in Figure 4.2. The reference voltage of the ICs are connected to the digital ground

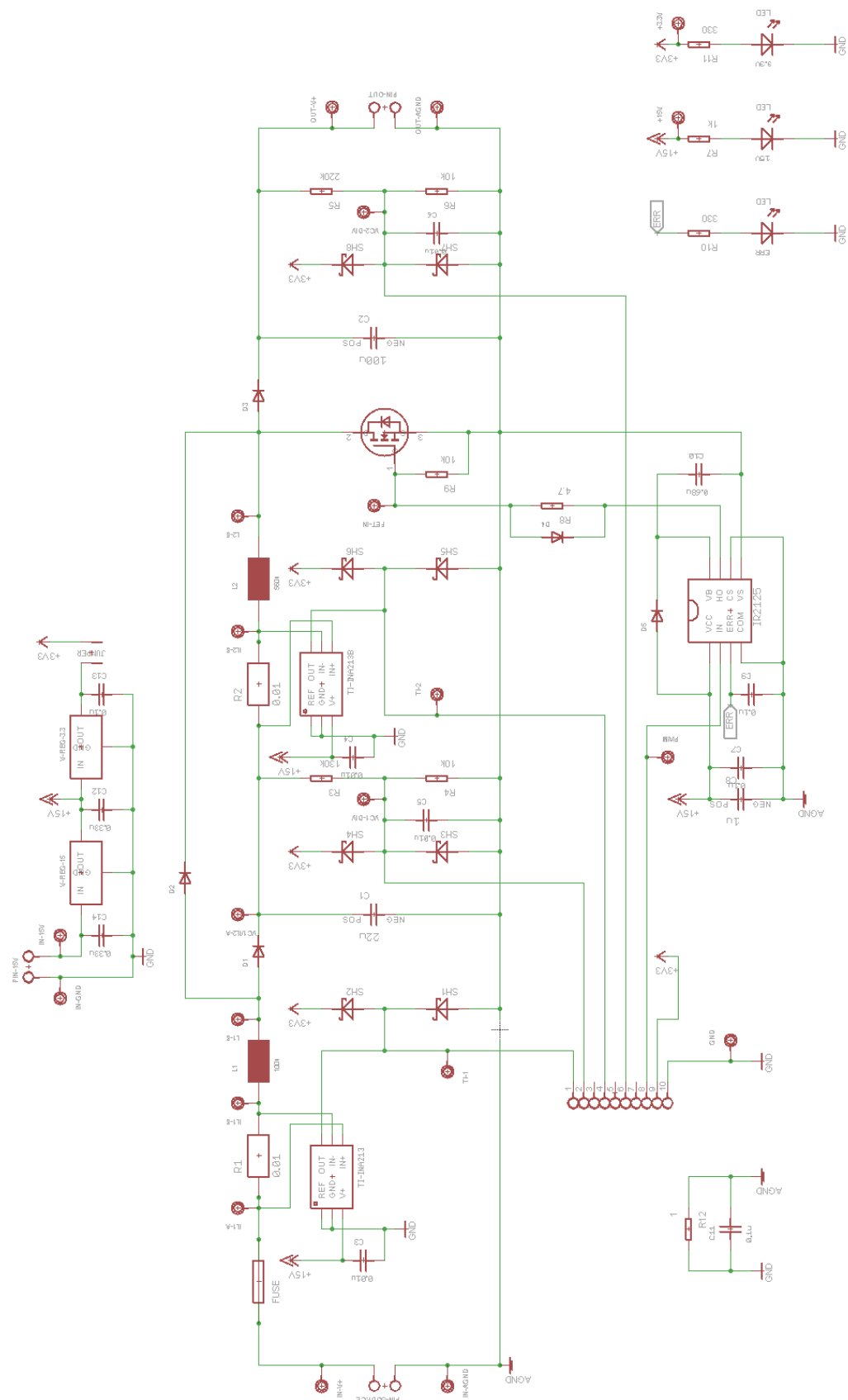


Figure 4.1: Schematic of the converter.

(GND), between the power supply and the ground a $0.01[\mu F]$ bypass capacitor is needed for stability according to the data sheet. On the figure the $IL1 - A$ and $IL1 - B$ are the corresponding measurement pins.

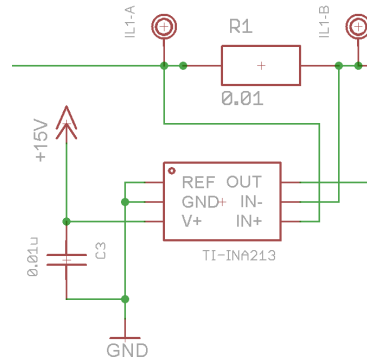


Figure 4.2: Connection of the current shunt monitor IC.

As a second step the power supply of the elements – such as the current shunt monitors – had to be designed. For this two voltage regulators were chosen and connected in series, one 15 [V] and one 3.3 [V] type. The connection of the voltage regulators can be seen on Figure 4.3. The 15 [V] output voltage regulator is needed for the supply of the current shunt monitors and the mosfet driver. As it was mentioned before, the board can be used with any type of microcontroller, the use of the 3.3 [V] voltage regulator is optional, it is connected with the ribbon cable and with the elements only if the jumper on the pins is applied. If the jumper is not applied one can use the voltage of the controller, or if it is applied the PCBs power can be used to supply the controller. As in the case of the current shunt monitors, different values of bypass capacitors are needed for stability. The input of the voltage regulators are separated from the high power input of the converter.

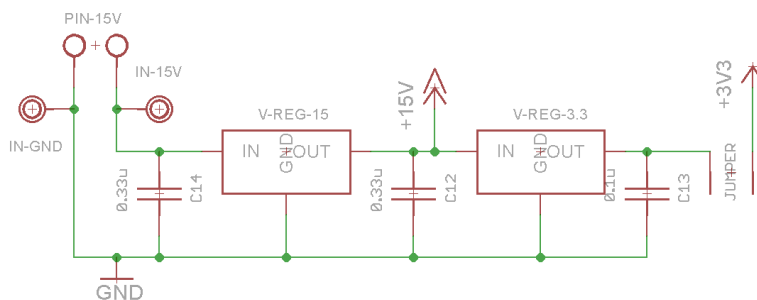


Figure 4.3: Power supply sub-circuit.

Over-voltage protection circuits are designed for every measured signal – at the outputs of the current shunt monitors and at the measured voltages. The protection consists of two Schottky diodes with 0.2 [V] opening voltage, which can be seen on Figure 4.4a. One Schottky diode is connected to the measured signal and to the 3.3 [V] (which can be other value, if the jumper is not applied), thus if the signal is higher than 3.3 [V] the diode becomes forward biased therefore

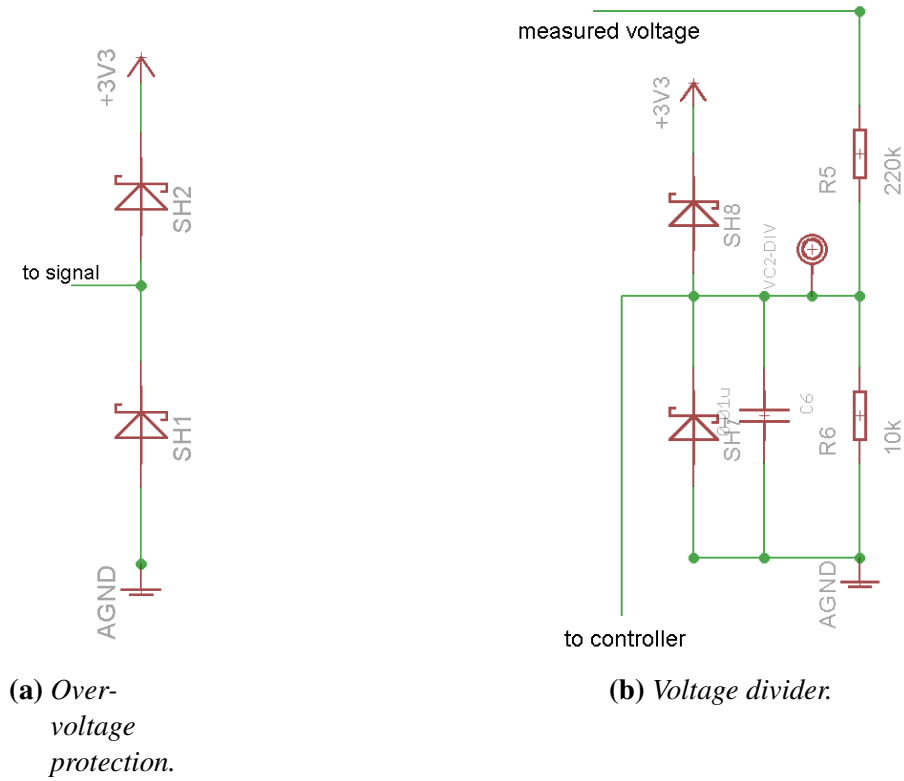


Figure 4.4: Circuit elements

the maximum voltage transferred to the controller is 3.3 [V]. The other diode is connected to the ground and to the measured signal, thus the minimum voltage cannot be smaller than the ground signal. The response of this circuit needs to be fast enough to protect the controller, therefore fast Schottky diodes are used.

The measurement of the capacitor voltages are implemented with simple voltage dividers, which can be seen on Figure 4.4b. To avoid the noises coming from the measurement, a capacitor is applied in parallel with the resistor on which the the voltage drop is measured. As this output is connected directly to the ribbon cable, thus to the microcontroller, the same over-voltage protection with Schottky diodes are applied.

To be able to utilize the PWM signal of the microcontroller, a MOSFET driver circuit is needed. For this purpose the IR2125 driver circuit was chosen, whose connection can be seen on Figure 4.5b. The PWM signal from the controller is connected to the *IN* terminal, the *HO* terminal is connected to the MOSFET through a series resistor. With respect to the data sheet of the IR2125 IC, the *VCC* and *VB* Terminals are connected with a diode. Between the *VB* and *ERR* terminals and the ground bypass capacitors are applied. In the case of the *VCC* supply voltage input terminal, two bypass capacitors are applied – one electrolyte and one ceramic.

To visualize some signals, 3 leds are placed on the PCB. Two of them can indicate if the 15 [V] supply or the 3.3 [V] supply is turned on and operating, the third one is connected to the IR2125 MOSFET driver's Error terminal. These are however not essential part of the circuit, but can

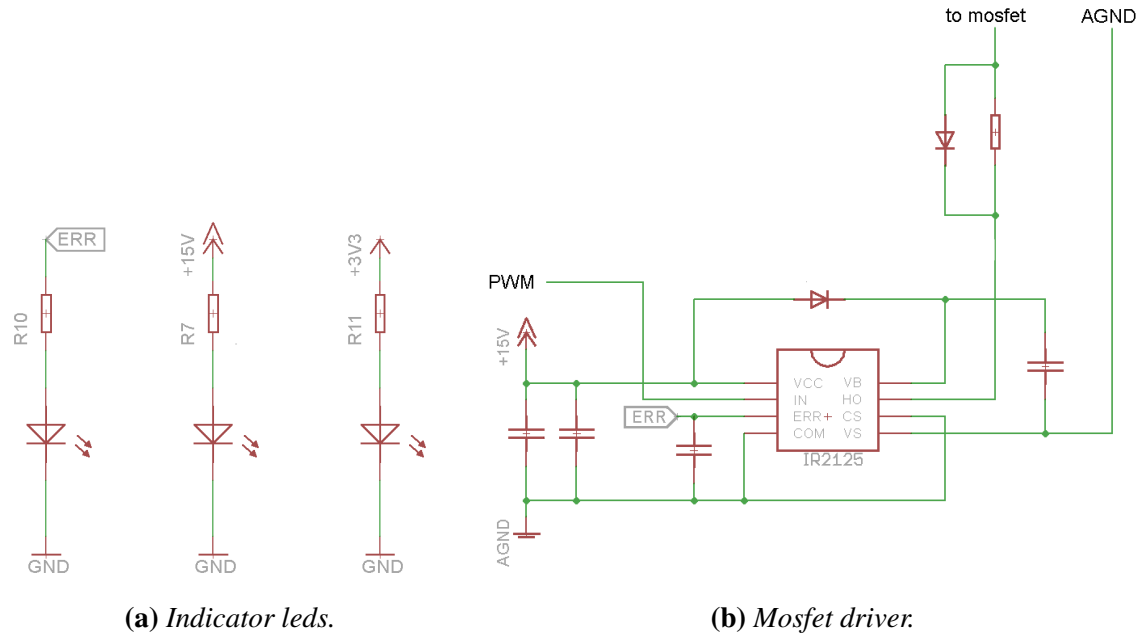


Figure 4.5: Circuit elements

be helpful diagnosing the voltages and errors. The corresponding circuit can be seen on Figure 4.5a., where the leds are connected to the signals through a series resistor.

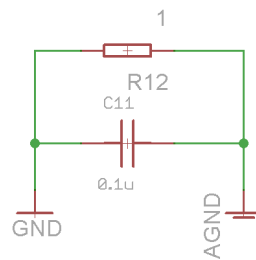


Figure 4.6: Connecting the two grounds.

As it was mentioned, the circuit has two separate grounds, one for the high power analogue elements – *AGND*, and one for the digital parts – *GND*. These two grounds are connected through a $1[\Omega]$ resistance and a parallel $0.1[\mu F]$ capacitor, as it can be seen in Figure 4.6.

4.2 Element selection

At first the range of the input and output voltage had to be determined. It was decided, that the converter should be designed to be able to convert 12 [V] input voltage to about 48-50 [V]. For this application several simulations were made, from which the corresponding minimum requirements for the elements were obtained. The values of the currents and voltages that we used for the determination of the elements can be found in Table 4.1.

Table 4.1: *Parameters for element selection*

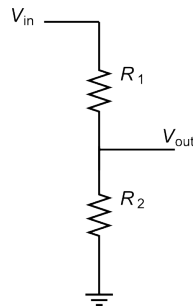
Description	Steady state value
1 st Inductor current	3 [A]
2 nd Inductor current	1.2 [A]
1 st Capacitor voltage	23 [V]
2 nd Capacitor voltage	48 [V]
Mosfet voltage	48 [V]
Max. Diode current	3 [A]

The gains for the measurement of the state variables also needed to be defined. The maximum voltage of the ADC of the used micro-controller is $3.3[V]$, thus the measured values need to be amplified into this range. The values of the currents are $2 - 3[A]$ at steady state, but as the transient also needs to be measured, the gain of the Texas Instruments ICs were chosen to 50, while the values of the shunt resistors to $0.01[\Omega]$. Thus the maximum measurable current is:

$$i_L = \frac{V_{ADC}}{TI_{gain}} \frac{1}{R_{shunt}} = \frac{3.3}{50} \frac{1}{0.01} = 6.6[A], \quad (4.1)$$

where V_{ADC} is the maximum voltage of the ADC, TI_{gain} is the gain of the current shunt monitor and R_{shunt} is the shunt resistor. As the maximum allowable current of the inductors are $7[A]$ the maximum measurable $6.6[A]$ suits our application.

The resistor values of the voltage divider circuits were also obtained. The voltage of the first capacitor – v_{C_1} is about the half of the second capacitor voltage – v_{C_2} , thus the resistor values are chosen differently in the two cases.

**Figure 4.7:** *Voltage divider.*

In table 4.2. the selected values for the voltage dividers can be seen with the naming conventions of Figure 4.7. The maximum measurable voltages are computed with equation (4.2).

$$V_{in} = \frac{R_1 + R_2}{R_2} V_{out}, \quad (4.2)$$

Table 4.2: *Selected values for the voltage dividers*

V_{in}	R_1	R_2	V_{inmax}
1 st Capacitor voltage	130[k Ω]	10[k Ω]	46.2[V]
2 nd Capacitor voltage	220[k Ω]	10[k Ω]	75.9[V]

where $V_{out} = 3.3[V]$ maximum output voltage, V_{in} measured voltage.

The selected resistance values are high enough to only use 0.125[W] resistors. The maximum measurable voltage covers the operation range for which the circuit was designed.

4.3 Printed Circuit Board

After the schematic was designed, and the parts were selected, the board layout had to be developed. From the data sheets of the elements the footprint of every part could be designed. For the design of the physical layout some considerations had to be taken into account, which are from [13]:

- Enough copper area should be placed to decrease thermal stress.
- If vias are conducting high current, sufficient vias are needed to minimize impedance.
- Power ground and small signal ground should be separated and just in one point connected.
- Small signal traces should be connected on other layer.
- At current sensing the bypass capacitor and the IC should be as close to the pins as possible.
- Small signal nets should be 10-15 mil thick.
- High power elements should be as close to each other as possible.
- The output capacitor should be close to the load.

Being aware of these suggestions, the printed circuit board was designed which can be seen on Figure 4.8. The board is double sided, its dimensions are 20x10 [cm] in Eagle. The high power elements were placed on the top of the board close to each other, the connecting nets are at the solder side. A bigger copper island is placed between Diodes D_2 and D_3 , the second inductor and the MOSFET for heat dissipation. The other nets connecting the high power elements are 160 mil thick, which enables high current conduction.

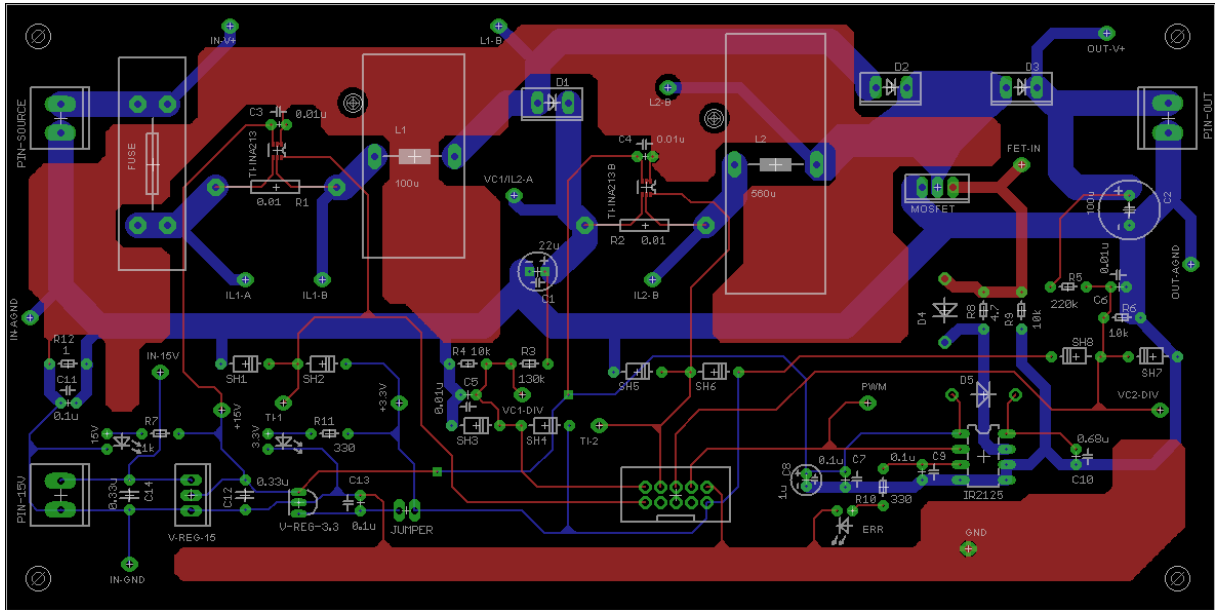


Figure 4.8: The design of the board. Component side – red, solder side – blue

The current shunt monitor ICs are placed close to the shunt resistors, with equal trace length. The bypass capacitor is placed near the pins of the IC. The output capacitor is placed near the output pins, at the right side of the board.

The small signal nets are placed on the component side to avoid the noises coming from the high power elements. Two ground islands are also placed on the component side for noise reduction. The measurement pins are placed near the components, leaving enough space for the probe of the oscilloscope.

Chapter 5

Hardware

5.1 Building the converter

When the printed circuit board was designed, the Gerber files could be exported, and sent to the manufacturer. The finished board can be seen on Figure 5.1. with the two SMD current shunt monitor ICs, which were implanted at Elekterv Kft.

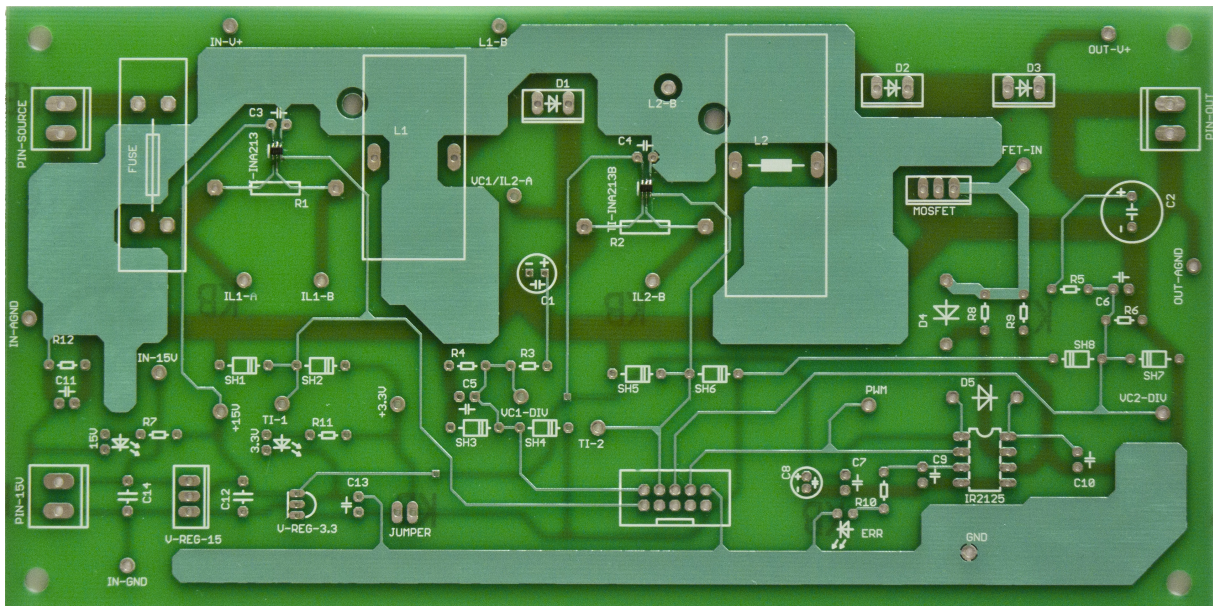


Figure 5.1: *Component side of the board.*

Except for the current shunt monitors, all elements are through hole type, thus they could be soldered into the PCB by hand, which can be seen on Figure 5.2. It can be seen, that a $6[A]$ fuse is used at the input for protection. As it was mentioned before, the connector where the micro-controller can be connected with a standard ribbon cable is located at the middle of the board. The colour of the supply voltage indicator LEDs are green, while the error signal indicator of the switch driver is red.

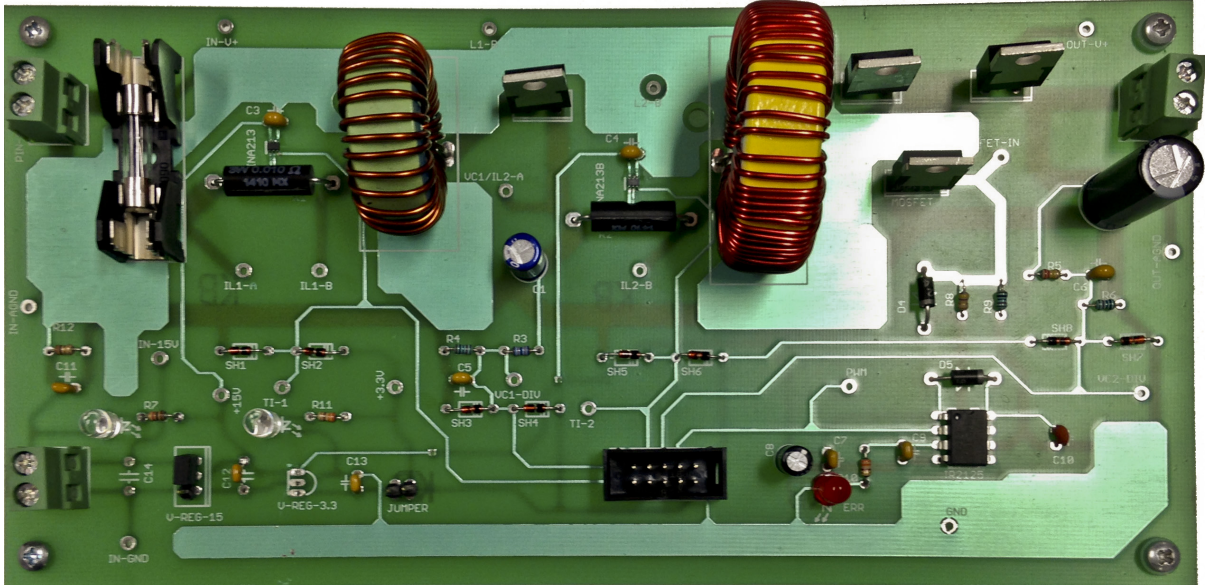


Figure 5.2: *The converter with the soldered elements.*

5.2 Measurements

As at this stage of the development there are no heat sinks, thus the measurements were carried out at a lower power level. An open loop measurement result can be seen on Figure 5.3., the input voltage is $3[V]$, the duty ratio is 0.5, the switching frequency is $\sim 50[kHz]$. From the figure it can be observed, that the output voltage of the converter is $7.88[V]$, which differs from the theoretical $12[V]$ because of the losses of the circuit. It also has to be mentioned, that however the microcontroller's PWM frequency is set to $50[kHz]$, the measured value is only $47.69[kHz]$ as the internal clock signal is not precise. Later on by using an external crystal the switching frequency can be set accurately to $50[kHz]$. The high power input voltage of the converter was supplied by a switching mode power supply, thus its output voltage was not constant, causing noises in the measured values of the converter.

5.3 Future plans

We are working currently on the implementation of the controller algorithms which were mentioned in the study. The algorithms will be implemented on a *STM32F401RE* microprocessor, which has a $84[MHz]$ floating point core. The *STM nucleo board* can be seen on Figure 5.4.

On Figure 5.5. the flowchart of the microcontroller's program can be seen. The main function initializes the ADC, PWM and system clock, and enables the PWM interrupt. At the beginning of each PWM period a PWM interrupt handler function is called. At the first step the AD conversion is initiated. After the measurement of the state variables ADC interrupt is called. The ADC interrupt handler is used to store the measurement results. After all measurement

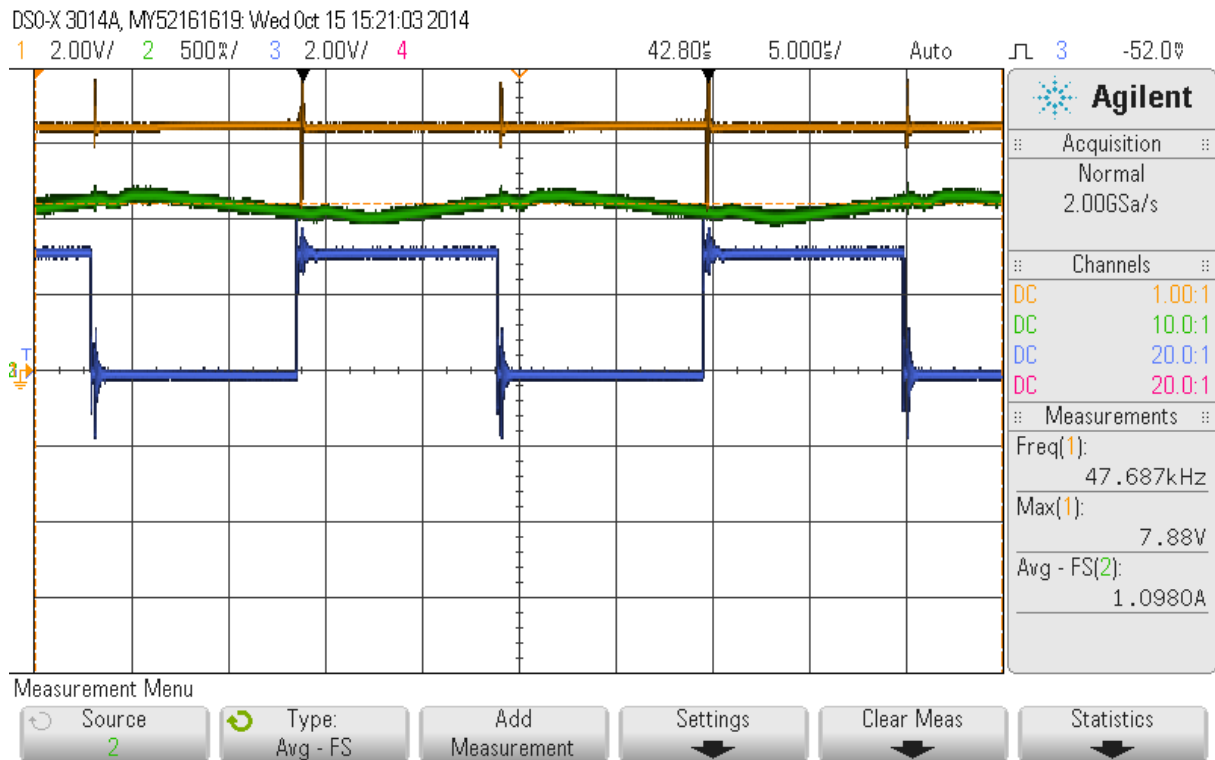


Figure 5.3: Measurement of the output voltage (yellow), first inductor current (green), and duty ratio (blue).

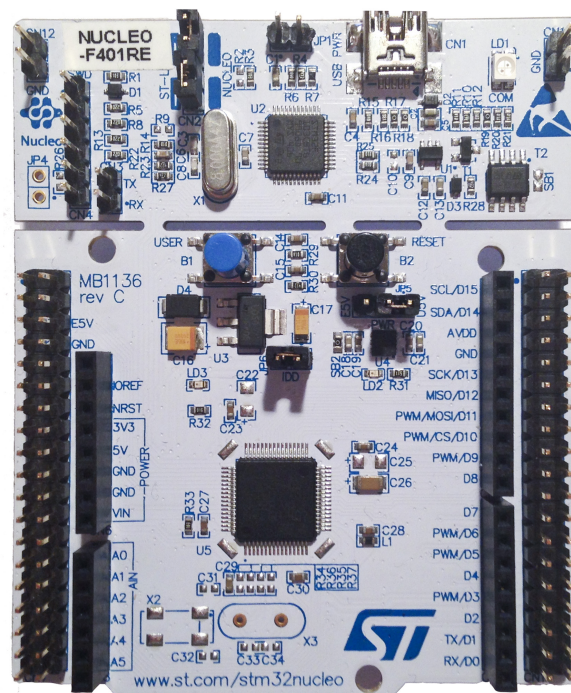


Figure 5.4: STM Nucleo board with STM32F401RE microprocessor.

results have been stored the control algorithm is performed and the duty ratio for the next PWM cycle is set using the controller output. After deleting the ADC interrupt flag, the program returns to the PWM interrupt handler which returns to the main function. Here the program

idles until the next sampling period begins.

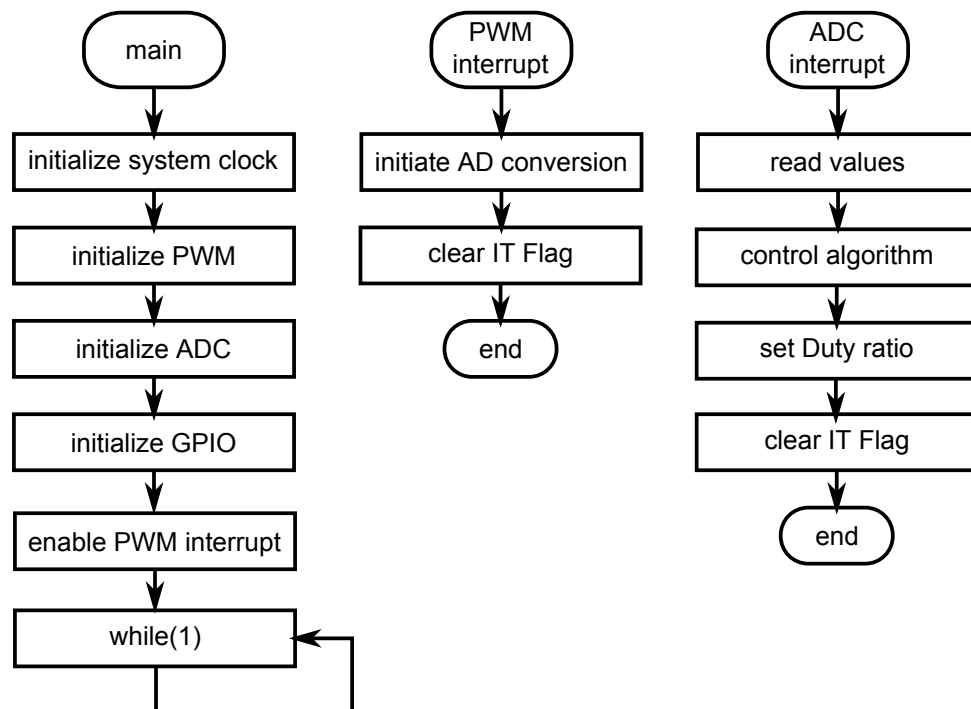


Figure 5.5: Flowchart of the microcontroller's program.

After the implementation our plan is to design controller methods for PFC applications, as well as for maximum power point tracking in case of photovoltaic cell inputs. We also would like to linearise the state space model with non-ideal, parasitic elements, and compare the obtained transfer functions with the ideal ones. Another plan is to further investigate and improve the estimator-based solutions.

Acknowledgement

We would like to thank Dr. Peter Stumpf for the continuous support that he provided in the simulation, measurements, model derivation and all of the topics presented in the paper.

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Appendices

F.1 Derivation of the ideal model

The small signal state space model was mentioned in (2.2). It's derivation is described in detail in this chapter.

In order to obtain the state space model, we have to investigate it in two separate states: when the switch is turned on and when it is turned off. These two states have two different circuit alignments, thus two different models. These models can be derived from the previously mentioned time diagrams of Figure 1.6.

F.1.1 On state of the converter

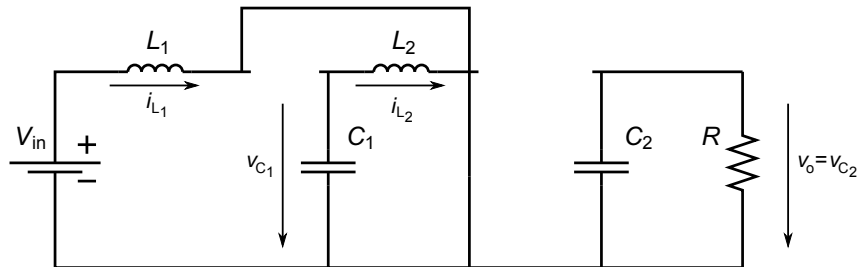


Figure F.1.1: On state of the converter.

The equivalent circuit of the *On* state of the converter can be seen on Figure F.1.1. In this state, when the semiconductor switch is on, the D_2 is conducting, while the D_1 and D_3 diodes are reverse biased. From this circuit, we can obtain the structure graph of the *On* state of the converter, which can be seen on Figure F.1.2.

From the structure graph of the model, the differential equations can be obtained. The loop equations are:

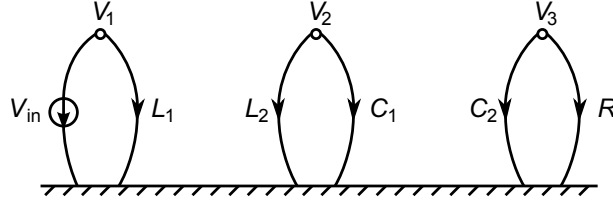


Figure F.1.2: Structure graph model of the converter at On state.

$$V_{in} = L_1 \frac{di_{L_1}}{dt}, \quad (\text{F.1.1})$$

$$L_2 \frac{di_{L_2}}{dt} = v_{C_1}, \quad (\text{F.1.2})$$

and

$$v_{C_2} = -RC_2 \frac{dv_{C_2}}{dt}. \quad (\text{F.1.3})$$

In order to construct the state space model, we need one more – in this case node – equation:

$$C_1 \frac{dv_{C_1}}{dt} = -i_{L_2}. \quad (\text{F.1.4})$$

From the equations from (F.1.1) to (F.1.4) the state space model can be obtained. As these equations are derived from the *On* state of the model, we have to multiply the elements of the matrices with the duty ratio – d .

$$\begin{bmatrix} \dot{i}_{L_1} \\ \dot{i}_{L_2} \\ \dot{v}_{C_1} \\ \dot{v}_{C_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_2}d & 0 \\ 0 & -\frac{1}{C_1}d & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{RC_2}d \end{bmatrix} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ v_{C_1} \\ v_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1}d \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in} \quad (\text{F.1.5})$$

F.1.2 Off state of the converter

In order to get the state space model, that describes the behaviour of the converter at all states, the state space model of the *Off* state also has to be constructed. The circuit of the model, when the semiconductor switch is off, can be seen on Figure F.1.3. In this case the diodes D_1 and D_3 are conducting, while D_2 is reverse biased. From this circuit, the structure graph can also be constructed, which can be seen on Figure F.1.4.

From the structure graph we can derive the differential equations. We have to use 2 loop and 2

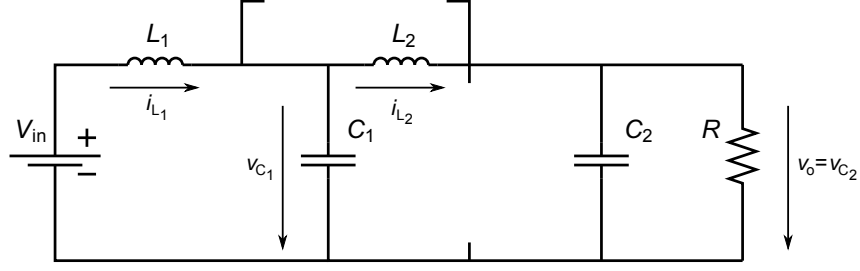


Figure F.1.3: Off state of the converter.

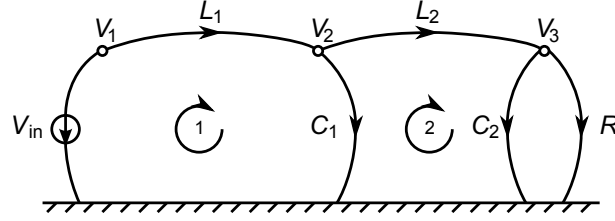


Figure F.1.4: Structure graph model of the converter at Off state.

node equations in this case. The loop equations are:

$$V_{in} = L_1 \frac{di_{L1}}{dt} + v_{C1}, \quad (\text{F.1.6})$$

$$v_{C1} = L_2 \frac{di_{L2}}{dt} + v_{C2}. \quad (\text{F.1.7})$$

The node equations:

$$i_{L1} = C_1 \frac{dv_{C1}}{dt} + i_{L2}, \quad (\text{F.1.8})$$

$$i_{L2} = C_2 \frac{dv_{C2}}{dt} + \frac{v_{C2}}{R}. \quad (\text{F.1.9})$$

From the equations (F.1.6) – (F.1.9) the state space model can be derived. As this model describes the *Off* state of the converter, we have to multiply the elements of the matrices with $(1 - d)$. Thus the model becomes:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_{C1} \\ \dot{v}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1}(1-d) & 0 \\ 0 & 0 & \frac{1}{L_2}(1-d) & -\frac{1}{L_2}(1-d) \\ \frac{1}{C_1}(1-d) & -\frac{1}{C_1}(1-d) & 0 & 0 \\ 0 & \frac{1}{C_2}(1-d) & 0 & -\frac{1}{RC_2}(1-d) \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1}(1-d) \\ 0 \\ 0 \\ 0 \end{bmatrix} v_{in}. \quad (\text{F.1.10})$$

F.1.3 The complete state space model

To obtain the State Space model valid for the whole system (assuming CCM – Continuous Conduction Mode), we have to use the superposition principle and add the two matrices obtained

from the two different states. Adding equation (F.1.5) and (F.1.10) results in:

$$\begin{bmatrix} \dot{i}_{L_1} \\ \dot{i}_{L_2} \\ \dot{v}_{C_1} \\ \dot{v}_{C_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-d}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-d}{L_2} \\ \frac{1-d}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-d}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ v_{C_1} \\ v_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} v_{in}. \quad (\text{F.1.11})$$

In equation (F.1.11) the state space model of the full converter is obtained, where matrix **A** depends on the current value of the duty ratio. The following the state space model is linearised around the nominal working point to obtain the transfer functions to design the controller parameters. To obtain this linearised model, first we have to express the steady state values of the state variables from the equations of (F.1.11). At steady state the first derivatives of the state variables are equal zero, thus:

$$-V_{C_1} \frac{1-D}{L_1} + \frac{1}{L_1} V_{in} = 0, \quad (\text{F.1.12})$$

$$V_{C_1} \frac{1}{L_2} - \frac{1-D}{L_2} V_{C_2} = 0, \quad (\text{F.1.13})$$

$$I_{L_1} \frac{1-D}{C_1} - \frac{1}{C_1} I_{L_2} = 0, \quad (\text{F.1.14})$$

$$I_{L_2} \frac{1-D}{C_2} - \frac{1}{RC_2} V_{C_2} = 0. \quad (\text{F.1.15})$$

From equations (F.1.12) to (F.1.15) the steady state values of the variables can be obtained as a function of the input voltage, duty ratio and the load resistance:

$$V_{C_1} = \frac{V_{in}}{1-D}, \quad (\text{F.1.16})$$

$$V_{C_2} = \frac{V_{in}}{(1-D)^2}, \quad (\text{F.1.17})$$

$$I_{L_1} = \frac{V_{in}}{R(1-D)^4}, \quad (\text{F.1.18})$$

$$I_{L_2} = \frac{V_{in}}{R(1-D)^3}. \quad (\text{F.1.19})$$

Now the variables can be substituted with the sum of their steady state and small signal values. The small signal values are denoted with a tilde. For example the input voltage $v_{in} = V_{in} + \tilde{v}_{in}$ or duty ratio $d = D + \tilde{d}$. As the first derivative of any steady state variable equals zero, these terms are eliminated from the equations. Substituting all variables with these expression results

in:

$$\begin{bmatrix} \dot{\tilde{i}}_{L_1} \\ \dot{\tilde{i}}_{L_2} \\ \dot{\tilde{v}}_{C_1} \\ \dot{\tilde{v}}_{C_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-(D+\tilde{d})}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-(D+\tilde{d})}{L_2} \\ \frac{1-(D+\tilde{d})}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-(D+\tilde{d})}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} I_{L_1} + \tilde{i}_{L_1} \\ I_{L_2} + \tilde{i}_{L_2} \\ V_{C_1} + \tilde{v}_{C_1} \\ V_{C_2} + \tilde{v}_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} (V_{in} + \tilde{v}_{in}). \quad (\text{F.1.20})$$

We can now substitute the equations from (F.1.16) to (F.1.19), thus we obtain:

$$\begin{bmatrix} \dot{\tilde{i}}_{L_1} \\ \dot{\tilde{i}}_{L_2} \\ \dot{\tilde{v}}_{C_1} \\ \dot{\tilde{v}}_{C_2} \end{bmatrix} = \mathbf{A}_t \begin{bmatrix} \frac{V_{in}}{R(1-D)^4} \\ \frac{V_{in}}{R(1-D)^3} \\ \frac{V_{in}}{1-D} \\ \frac{V_{in}}{(1-D)^2} \end{bmatrix} + \mathbf{A}_t \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} V_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \tilde{v}_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (\text{F.1.21})$$

where $\mathbf{A}_t$ denotes

$$\mathbf{A}_t = \begin{bmatrix} 0 & 0 & -\frac{1-D-\tilde{d}}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D-\tilde{d}}{L_2} \\ \frac{1-D-\tilde{d}}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-D-\tilde{d}}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}. \quad (\text{F.1.22})$$

As the equation (F.1.21) consists of two matrix-vector multiplications and two sums, we can investigate these operations separately. The first operation is:

$$\begin{aligned} \mathbf{A}_t \begin{bmatrix} \frac{V_{in}}{R(1-D)^4} \\ \frac{V_{in}}{R(1-D)^3} \\ \frac{V_{in}}{1-D} \\ \frac{V_{in}}{(1-D)^2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} V_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} -\frac{1-D-\tilde{d}}{L_1} \frac{V_{in}}{1-D} \\ \frac{1}{L_2} \frac{V_{in}}{1-D} - \frac{1-D-\tilde{d}}{L_2} \frac{V_{in}}{(1-D)^2} \\ \frac{1-D-\tilde{d}}{C_1} \frac{V_{in}}{R(1-D)^4} - \frac{1}{C_1} \frac{V_{in}}{R(1-D)^3} \\ \frac{1-D-\tilde{d}}{C_2} \frac{V_{in}}{R(1-D)^3} - \frac{1}{RC_2} \frac{V_{in}}{(1-D)^2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} V_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \\ &= \begin{bmatrix} \left(-\frac{1-D}{L_1} + \frac{\tilde{d}}{L_1}\right) \frac{V_{in}}{1-D} + \frac{1}{L_1} V_{in} \\ \frac{1}{L_2} \frac{V_{in}}{1-D} - \left(\frac{1-D}{L_2} - \frac{\tilde{d}}{L_2}\right) \frac{V_{in}}{(1-D)^2} \\ \left(\frac{1-D}{C_1} - \frac{\tilde{d}}{C_1}\right) \frac{V_{in}}{R(1-D)^4} - \frac{1}{C_1} \frac{V_{in}}{R(1-D)^3} \\ \left(\frac{1-D}{C_2} - \frac{\tilde{d}}{C_2}\right) \frac{V_{in}}{R(1-D)^3} - \frac{1}{RC_2} \frac{V_{in}}{(1-D)^2} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{L_1(1-D)} \\ \frac{V_{in}}{L_2(1-D)^2} \\ -\frac{V_{in}}{RC_1(1-D)^4} \\ -\frac{V_{in}}{RC_2(1-D)^3} \end{bmatrix} \tilde{d}. \quad (\text{F.1.23}) \end{aligned}$$

For the second operation we have to split matrix $\mathbf{A}_t$ into two parts, the first includes the steady state variables while the second the small signal ones:

$$\mathbf{A}_t = \mathbf{A}_{t,steady} + \mathbf{A}_{t,small}, \quad (\text{F.1.24})$$

where

$$\mathbf{A}_{t_{steady}} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D}{L_2} \\ \frac{1-D}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-D}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}, \mathbf{A}_{t_{small}} = \begin{bmatrix} 0 & 0 & \frac{\tilde{d}}{L_1} & 0 \\ 0 & 0 & 0 & \frac{\tilde{d}}{L_2} \\ -\frac{\tilde{d}}{C_1} & 0 & 0 & 0 \\ 0 & -\frac{\tilde{d}}{C_2} & 0 & 0 \end{bmatrix}. \quad (\text{F.1.25})$$

Thus, the second matrix multiplication from equation (F.1.21) can be expressed as:

$$\mathbf{A}_t \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix} = \mathbf{A}_{t_{steady}} \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix} + \mathbf{A}_{t_{small}} \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix}. \quad (\text{F.1.26})$$

In equation (F.1.26) there are also two matrix multiplications, from which at the second $\mathbf{A}_{t_{small}}$ is multiplied with the small signal state space vector. As the multiplication of two small signal components becomes even smaller, it can be neglected, thus equation (F.1.26) becomes:

$$\mathbf{A}_t \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix} = \mathbf{A}_{t_{steady}} \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D}{L_2} \\ \frac{1-D}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-D}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix}. \quad (\text{F.1.27})$$

If we substitute the obtained (F.1.23) and (F.1.27) expressions into equation (F.1.21), we can get the small signal state space model for the full converter:

$$\begin{bmatrix} \dot{\tilde{i}}_{L_1} \\ \dot{\tilde{i}}_{L_2} \\ \dot{\tilde{v}}_{C_1} \\ \dot{\tilde{v}}_{C_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D}{L_2} \\ \frac{1-D}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-D}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} \tilde{i}_{L_1} \\ \tilde{i}_{L_2} \\ \tilde{v}_{C_1} \\ \tilde{v}_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{V_{in}}{L_1(1-D)} & \frac{1}{L_1} \\ \frac{V_{in}}{L_2(1-D)^2} & 0 \\ -\frac{V_{in}}{RC_1(1-D)^4} & 0 \\ -\frac{V_{in}}{RC_2(1-D)^3} & 0 \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}. \quad (\text{F.1.28})$$

F.2 Derivation of the non-ideal model

The derivation of the non-ideal state space model which was mentioned on (2.7) is described in detail in this chapter. As in case of the ideal model, this also needs to be separated into two parts. The basic steps of the derivation of the equations are similar to the ideal ones.

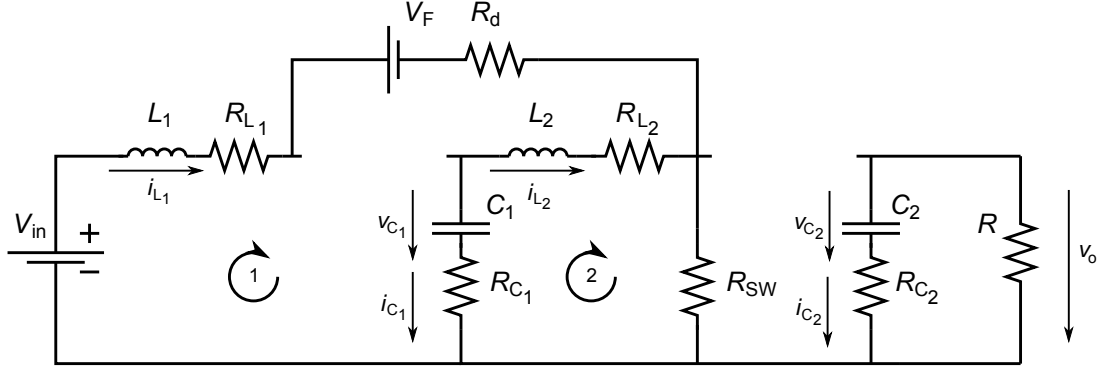


Figure F.2.1: Equivalent circuit of the on state of the non-ideal model.

F.2.1 The on state of the non-ideal model

The equivalent circuit of the non-ideal model when the semiconductor switch is turned on can be seen in Figure F.2.1. The voltage drop of the diodes are denoted with V_F . Three loop equations can be expressed, equations (F.2.1) and (F.2.2) are the first and second loop denoted in the figure, and equations (F.2.3) is the loop equation of the output.

$$v_{in} = i_{L1}R_{L1} + L_1 \frac{di_{L1}}{dt} + V_F + i_{L1}R_d - i_{L2}(R_{L2} + R_{C1}) - L_2 \frac{di_{L2}}{dt} + v_{C1} \quad (F.2.1)$$

$$v_{C1} - i_{L2}(R_{L2} + R_{C1}) - L_2 \frac{di_{L2}}{dt} = (i_{L1} + i_{L2})R_{SW}, \quad (F.2.2)$$

$$-v_{C2} = C_2 \frac{dv_{C2}}{dt} (R_{C2} + R). \quad (F.2.3)$$

As the value of the second inductor current $-i_{L2}$ equals with the value of the first capacitor current but in opposite direction, it can be written:

$$\frac{dv_{C1}}{dt} = -\frac{1}{C_1} i_{L2} \quad (F.2.4)$$

From equation (F.2.2) $\frac{di_{L2}}{dt}$ can be expressed:

$$\frac{di_{L2}}{dt} = -\frac{R_{SW}}{L_2} i_{L1} - \frac{R_{L2} + R_{C1} + R_{SW}}{L_2} i_{L2} + \frac{1}{L_2} v_{C1} \quad (F.2.5)$$

Substituting (F.2.5) into (F.2.1) results in:

$$\frac{di_{L1}}{dt} = \frac{1}{L_1} V_{in} - \frac{R_{L1} + R_d + R_{SW}}{L_1} i_{L1} - \frac{R_{SW}}{L_1} i_{L2} - \frac{1}{L_1} V_F \quad (F.2.6)$$

From equations (F.2.3), (F.2.4), (F.2.5) and (F.2.6) the state space model for the on state equiv-

alent circuit can be obtained, where the elements are multiplied with the duty cycle d , as this model is only valid when the PWM signal is high:

$$\begin{bmatrix} \dot{i}_{L_1} \\ \dot{i}_{L_2} \\ \dot{v}_{C_1} \\ \dot{v}_{C_2} \end{bmatrix} = \begin{bmatrix} -\frac{R_{L_1}+R_d+R_{SW}}{L_1}d & -\frac{R_{SW}}{L_1}d & 0 & 0 \\ -\frac{R_{SW}}{L_2}d & -\frac{R_{L_2}+R_{C_1}+R_{SW}}{L_2}d & \frac{1}{L_2}d & 0 \\ 0 & -\frac{1}{C_1}d & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_2(R+R_{C_2})}d \end{bmatrix} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ v_{C_1} \\ v_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1}d & -\frac{1}{L_1}d \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{in} \\ V_F \end{bmatrix} \quad (\text{F.2.7})$$

F.2.2 The off state of the non-ideal model

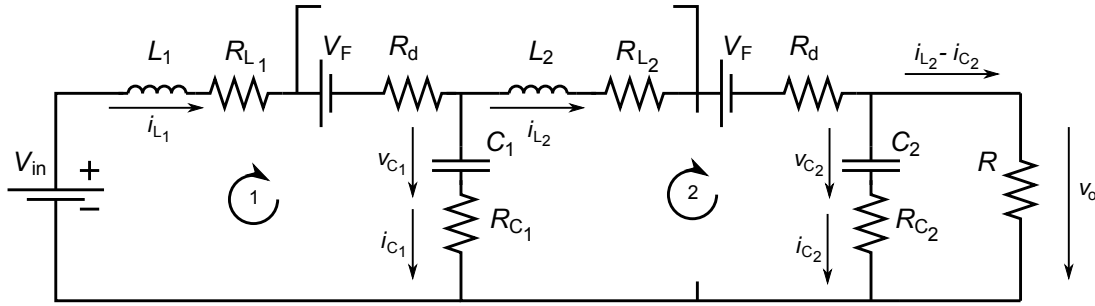


Figure F.2.2: Equivalent circuit of the off state of the non-ideal model.

The equivalent circuit of the non-ideal model when the semiconductor switch is turned on can be seen in Figure F.2.2. In this case again three loop equations can be expressed, (F.2.10) and (F.2.11) are 1. and 2. the loops shown in the figure, while (F.2.12) is the loop equation of the output. For the equations the relationship between the inductor currents and the first capacitor current is used, which:

$$i_{C_1} = i_{L_1} - i_{L_2}, \quad (\text{F.2.8})$$

thus

$$\frac{dv_{C_1}}{dt} = \frac{1}{C_1}i_{L_1} - \frac{1}{C_1}i_{L_2}. \quad (\text{F.2.9})$$

$$V_{in} = R_{L_1}i_{L_1} + L_1\frac{di_{L_1}}{dt} + V_F + R_d i_{L_1} + R_{C_1}(i_{L_1} - i_{L_2}) + v_{C_1}, \quad (\text{F.2.10})$$

$$v_{C_1} + R_{C_1}(i_{L_1} - i_{L_2}) = R_{L_2}i_{L_2} + L_2\frac{di_{L_2}}{dt} + V_F + R_d i_{L_2} + R_{C_2}C_2\frac{dv_{C_2}}{dt} + v_{C_2}, \quad (\text{F.2.11})$$

$$v_{C_2} + R_{C_2}C_2\frac{dv_{C_2}}{dt} = (i_{L_2} - C_2\frac{dv_{C_2}}{dt})R. \quad (\text{F.2.12})$$

From equation (F.2.12) $\frac{dv_{C_2}}{dt}$ can be expressed:

$$\frac{dv_{C_2}}{dt} = \frac{R}{(R + R_{C_2})C_2}i_{L_2} - \frac{1}{(R + R_{C_2})C_2}v_{C_2}. \quad (\text{F.2.13})$$

Substituting (F.2.13) into (F.2.11) results in:

$$\begin{aligned} \frac{di_{L_2}}{dt} = \frac{R_{C_1}}{L_2}i_{L_1} + \frac{1}{L_2}v_{C_1} - \frac{R_{L_2} + R_d + R_{C_1} + \frac{R_{C_2}R}{R_{C_2}+R}}{L_2}i_{L_2} - \frac{1}{L_2}V_F \\ - \frac{R}{L_2(R + R_{C_2})}v_{C_2}. \end{aligned} \quad (\text{F.2.14})$$

From (F.2.10) $\frac{di_{L_1}}{dt}$ can be expressed:

$$\frac{di_{L_1}}{dt} = \frac{1}{L_1}V_{in} - \frac{R_{L_1} + R_d + R_{C_1}}{L_1}i_{L_1} - \frac{1}{L_1}V_F + \frac{R_{C_1}}{L_1}i_{L_2} - \frac{1}{L_1}v_{C_1} \quad (\text{F.2.15})$$

From equations (F.2.9), (F.2.13), (F.2.14) and (F.2.15) the state space model for the off state can be obtained, the elements are multiplied with $(1 - d)$, as this model is only valid when the PWM signal is low:

$$\begin{aligned} \begin{bmatrix} \dot{i}_{L_1} \\ \dot{i}_{L_2} \\ \dot{v}_{C_1} \\ \dot{v}_{C_2} \end{bmatrix} = \begin{bmatrix} -\frac{(R_{L_1}+R_d+R_{C_1})(1-d)}{L_1} & \frac{R_{C_1}(1-d)}{L_1} & -\frac{1-d}{L_1} & 0 \\ \frac{R_{C_1}(1-d)}{L_2} & -\frac{(R_{L_2}+R_d+R_{C_1}+\frac{R_{C_2}R}{R_{C_2}+R})(1-d)}{L_2} & \frac{1-d}{L_2} & -\frac{R(1-d)}{L_2(R+R_{C_2})} \\ \frac{1-d}{C_1} & -\frac{1-d}{C_1} & 0 & 0 \\ 0 & \frac{R(1-d)}{C_2(R+R_{C_2})} & 0 & -\frac{1-d}{C_2(R+R_{C_2})} \end{bmatrix} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ v_{C_1} \\ v_{C_2} \end{bmatrix} \\ + \begin{bmatrix} \frac{1-d}{L_1} & -\frac{1-d}{L_1} \\ 0 & -\frac{1-d}{L_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{in} \\ V_F \end{bmatrix} \end{aligned} \quad (\text{F.2.16})$$

F.2.3 The complete non-ideal state space model

As the state space models for both the on and off state of the semiconductor switch were derived, using superposition the state space model for the complete non-ideal model can be obtained, if

(F.2.7) and (F.2.16) are added:

$$\dot{\mathbf{x}} = \mathbf{A}_{adv}\mathbf{x} + \mathbf{B}_{adv}v_{in} + \mathbf{B}_{adv_d} \quad (\text{F.2.17a})$$

$$\mathbf{y} = \mathbf{C}_{adv}\mathbf{x} \quad (\text{F.2.17b})$$

Where

$$\mathbf{A}_{adv} = \begin{bmatrix} \frac{-R_{C_1}(1-d)}{L_1} - \frac{R_{L_1}+R_d}{L_1} - \frac{R_{SW}d}{L_1} & \frac{R_{C_1}(1-d)}{L_1} - \frac{R_{SW}d}{L_1} & -\frac{1-d}{L_1} & 0 \\ \frac{R_{C_1}(1-d)}{L_2} - \frac{R_{SW}d}{L_2} & -\frac{\left(R_d + \frac{R R_{C_2}}{R+R_{C_2}}\right)(1-d)}{L_2} - \frac{R_{SW}d}{L_2} - \frac{R_{C_1}+R_{L_2}}{L_2} & \frac{1}{L_2} & -\frac{R(1-d)}{L_2(R+R_{C_2})} \\ \frac{1-d}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{R(1-d)}{C_2(R+R_{C_2})} & 0 & -\frac{1}{C_2(R+R_{C_2})} \end{bmatrix} \quad (\text{F.2.18a})$$

$$\mathbf{B}_{adv}^T = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \end{bmatrix} \quad (\text{F.2.18b})$$

$$\mathbf{B}_{adv_d}^T = \begin{bmatrix} -\frac{V_F}{L_1} & -\frac{V_F(1-d)}{L_2} & 0 & 0 \end{bmatrix} \quad (\text{F.2.18c})$$

$$\mathbf{C}_{adv} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{F.2.18d})$$