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To see the forest -
A study on the directed spanning arborescence framework used in systemic risk analysis based on financial networks

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I Introduction

Imagine a set of international banks and other financial institutions. In the globalized economy and financial sector, it is inevitable that the actors take part in certain processes simultaneously and they are interdependent on each other. This situation results in a network-like system of connections. It is important to know the properties of these networks, for instance its stability and the possible risky components. After the financial crisis of 2007-2008, the focus of research and regulation has shifted to the systemic nature of the institutions. Imagine, that any part of the network goes bankrupt and then, this event spreads across the rest of the network, similar to a contagion effect. The chances of shock propagation across the network need to be examined, as well as the routes that it may follow. Further questions are whether it is possible within the global financial system to separate or create independent sub-systems, and how this issue is connected to the concept of systemic risk and to the present analysis.

In this paper I compare the predictive power of the complete network and spanning arborescence variables on the performance of the financial institutions. In the analysis, after choosing the appropriate stock market data, I create a connectedness network of the banks using a pairwise Granger-causality method. Common centrality measures, such as node strength, closeness and betweenness are calculated on the weighted and directed complete network. Then, the concept of maximum spanning arborescence is used to filter out the redundant and less important connections, resulting in a smaller graph, that is more comprehensible both visually and numerically. The centrality measures of the arborescence include the same node strength, closeness and betweenness values, as well as an average edge weight variable. A regression framework is built to test the general connection between the performance of firms and the different variables if both the complete and the arborescence network.

Existing research suggests that for example minimum spanning tree or arborescence methods can be used to further deepen the analysis of network properties as systemic risk variables. The main research question of this paper is whether the centrality measures of the arborescence network generally have better explanatory power than the original network variables. Thus, my first hypothesis states that the arborescence network properties are suitable for the comparison, as they follow similar patterns than the variables of the

complete network, such as an increased overall strength of connections during turbulent times. My second hypothesis is that the arborescence structure will explain better the selected firm performance variable, which is drawn from the idea, that filtering the network will mostly reduce noise in the data but will not alter the explanatory power of the key edges and nodes. Further hypotheses, considering the early-warning indicator nature of the network properties are not addressed in this paper.

Results show that the arborescence graphs indeed provide a better general explanation on the performance level of firms. In the regression models, both those specifications that only include arborescence variables and those that contain a mixed set of variables outperform the base models that only use the complete network centrality measures. In the first set of regression models that are estimated on all three periods of the data, the adjusted R^2 values are calculated, and are significantly higher in models that use arborescence variables. For the second set of models, where the model is fitted on the first two periods of the data and the third section is used for prediction, the root mean squared error is also calculated, showing the stability of the fitted model on out of sample data, concluding that arborescence models are characterized by lower RMSE values.

The paper is structured in the following way: in chapter 2, I summarize the relevant literature and highlight the contexts in which I place the present analysis. In the next chapter, I introduce the methods and data that are used in this paper, for creating the networks and centrality measures, as well as the regression models. Chapter 4 shows the results of the empirical analysis along figures and summary statistics of the networks and the regression tables. The following chapter contains a short robustness analysis, where some of the parameters are changed to test for the stability of the results, while further possibilities are discussed. In chapter 6, I conclude the results and consequences of the analysis and the last chapter contains a discussion of further research ideas.

II Literature review

There is a wide literature on the measurement and analysis of systemic risk, so first, I summarize what is generally considered systemic risk in contemporary research. Then, I list all the contexts of the present analysis, listing the papers that it is built on, starting from the systemic risk literature, with the different approaches to measure systemic risk.

Then, moving closer to the topic of this paper, I present the literature that contains a network science approach to systemic risk, listing the available methods to create the networks. Following that, I summarize the research on the pairwise Granger causality approach that I used, along with its limitations and improvements. Finally, I show articles about the process of filtering the networks, some of which have been applied to the same systemic risk problem in the financial sector.

A systemic event in the economy is characterized by simultaneous losses across multiple firms, following an economic shock that starts from one or few institutions. One keyword here is contagion, the mechanisms of spreading losses throughout a set of institutions. Following this approach, the systemic risk literature has shifted from the concept of 'too big to fail' towards 'too interconnected to fail' (Markose et al. (2012)), hence the argument for approaching the problem by measuring connectedness, using concepts of networks for example. Another important notion is the general trust in the financial system: when a set of circumstances hold, so that the public confidence in the stability of the system is impaired, it is an indicator of systemic risk. Therefore, it can be decided for any bankruptcy whether it is a systemic event, which may result in counter-intuitive conclusions when only looking at firm sizes. Often, smaller, but more interconnected entities in the market cause more problems. Definitions and overview of systemic risk appear in ECB (2009), De Bandt and Hartmann (2000) and Kaufman and Scott (2003), as well as in the introductions of the empirical analyses, such as Billio et al. (2012).

After the attempt to define or describe the concept of systemic risk, it is important to overview the methods used to measure it. Different approaches exist based on what aspect of systemic risk they try to capture and what data sources they use. Multiple methods exist that focus on the losses, leverage or liquidity of the firms or the sector under a crisis, which is an earlier approach of describing systemic risk. More recently, however, the focus has shifted to the linkages and connections of the financial institutions as a determining factor of systemic risk. This division of approaches is well described in Billio et al. (2012). Some frequently used methods that are built on the loss distributions of the firms include the conditional value at risk (CoVaR) by Adrian and Brunnermeier (2011) or the marginal expected shortfall (MES) by Acharya et al. (2010). When providing a ranking of financial institutions based on several systemic risk variables, Nucera et al. (2016) highlight a few others as well, for example the leverage ratio and the $\beta * MV$ as defined in Engle et al.

(2015), describing the risk of market capitalization of the firm. These previous methods are all centered around the losses or some other characteristic of the balance sheets of the firms. These methods are limited in their ability to be used as early warning indicators as some of them only show systemic risk once the shocks have struck the sector, with realized losses or higher volatility periods Billio et al. (2012).

In the paper of Billio et al. (2012), the introduction of a network-based model aims to overcome the limitations of the previous methods by describing the unconditional connections between institutions. Their new methods introduced first a principal component analysis (PCA) to define the connectedness of the whole system, as well as a pairwise Granger causality approach. They are the ones who introduce network centrality variables into the analysis, such as In and Out degrees, as well as closeness and eigenvector centrality. Their approach is not the only one to define a network of financial institutions: Diebold and Yılmaz (2014) use a generalized variance decomposition approach on a VAR model, creating the network edges from the portions of the error term variance explained by another institution. They introduce in detail the properties of the networks by discussing their topological features. The benefit of using networks to represent the interconnected financial sector is for example that the necessary data is easy to acquire, as it they are build from stock market data. Second, as Billio et al. (2012) highlighted, the unconditional connections between institutions do not depend on the simultaneous losses of multiple firms to show an increased connectedness in the system, and can account for the development of new connections as well.

The third context I place this paper in is the approach of constructing pairwise Granger causality networks. First, I show methods outside the Granger causality framework that can be used to estimate other types of pairwise connections. Also, I highlight the limitations and further improvements to the framework introduced by Billio et al. (2012). One basic model is the use of pairwise correlations between the time series. Correlation captures the general tendency between variables, and it would result in a weighted, but undirected network. Another, more recently used measure is the copula method that captures the tail dependency connection between time series (used in the analysis of Denkowska and Wanat (2020) and Yang et al. (2020)). That method turns to the assumption, that the connections between firms are more powerful in periods of economic downturn, at the lower tail of the returns distribution. Compared to the undirected but weighted options of

previous measures, the pairwise Granger causality approach, as introduced by Billio et al. (2012), at first resulted in an unweighted, but directed network. Ideally, the networks of financial institutions should be both directed and weighted, to account for the differences in the strength, as well as the direction of the connections. Direction matters because the differences in the strength of Out and In connections determine whether a firm contributes more to systemic risk or it is more exposed to it. Hué et al. (2019) improve the result of the Granger networks by introducing new variables based on the leave-one-out approach, to eliminate less important indirect connections from the system. Another improvement is the introduction to edge weights to the Granger networks using the F statistics from the Granger causality estimation to determine the strength of the connection, as it is defined and successfully used in the paper of Yao et al. (2016) and Gandica et al. (2020).

The final context I place the present analysis is the simplification of the network to filter out the redundant edges. Different network structures allow for different methods: an undirected, weighted graph may be filtered using a minimum/maximum spanning tree (MST), generated using Prim's algorithm (Prim (1957)). MSTs were applied on financial networks built from simple correlations and pairwise tail-dependency indices in, Denkowska and Wanat (2020), Yang et al. (2020) and Eryiğit and Eryiğit (2009). The directed nature of Granger networks requires a further step in using a filtering method. The minimum/maximum spanning arborescence is the equivalent of MSTs for directed graphs, showing probable transmission routes in the system. I use the spanning arborescence calculated by the Edmonds algorithm (developed independently by Edmonds (1967) and Chu (1965)). Spanning arborescence graphs have not yet been used to model systemic risk in the financial sector, but was used for the analysis of connectedness between industrial sectors by Yao et al. (2016).

III Methods

In this chapter I present the methods I used for the analysis of the networks and the regression design that helps to answer the research question. In section 1, before turning to the calculations, I summarize the main aspects of choosing variables and data sources. In the next section, I define more thoroughly the process of calculating the return and volatility data from stock prices, which form the basis of the network analysis. In section

3, I define the concept of pairwise Granger causality, which I use to estimate the connections between firms. In the next part, I introduce the network centrality measures that are used to estimate the systemic risk properties of firms using the complete network created by pairwise Granger causalities. In section 5, I show the different methods to simplify a network, which allows for analyzing an entirely different network structure than the complete graphs. The following section builds on the results of the simplified networks, where I calculate several other network centrality measures that serve as further characteristics of systemic risk. In section 7, I present the firm-level performance indicator variable and the regression analysis techniques I use to determine whether arborescence variables generally explain firm performance better than the original network properties, and whether they are good early warning indicators of losses in a systemic event. The chapter is concluded by the description of the data. For most calculations, I used the R software (R Core Team (2020)) and one Python package. I estimated the volatilities using the *fGARCH* package Wuertz et al. (2020), and I used the *igraph* R package (Csardi and Nepusz (2006)) and *networkx* Python package (Hagberg et al. (2008)) for the network concepts. The concepts of econometrics and time series analysis are taken from the popular textbook by Wooldridge (2016).

III.1 Networks in systemic risk analysis

The methodology of this paper is built on an approach that represents the connections of financial institutions as networks, on which the notion of systemic risk and the propagation of financial can be understood as the properties and dynamics of links. In such networks, the financial institutions represent the vertices and the connections of firms constitute the edges of the network. Therefore, the first task in the analysis of systemic risk through interconnected firms is to compose an appropriate network using available data that grasps the underlying links between institutions. The choice of data source and variables can impact the results of the estimation, however, the availability of data often determines this choice.

The intuitive approach to building financial networks would be representing the direct connections by data on interbank relations, for example existing investments, the ownership structures and interbank loans. However, most of the time, this approach is severely limited by the lack of public data of these variables, which would also be difficult to grasp

without using a large set of parameters and values. Consequently, a replacement choice of data source is necessary. The most often used choice for substitute is stock market data, which is publicly available for a large set of firms and for a wide period of time. Whether stock prices are appropriate for aggregating firm performance and prospects of risks accurately is a difficult question.

Stock prices are usually not able to represent the firm information accurately, however, it is still the best choice when aggregated data is needed. Also, perhaps the most important aspect of systemic risk, the market sentiment and the trust in a particular firm or sector can be seen from short-term price movements that follow rumors or news that do not necessarily reflect the true performance of the firm (for example Mullainathan and Thaler (2000)). Behavioral studies in economics have dealt with this phenomenon. In the following sections, I present the building blocks of the financial networks and the steps of subsequent analysis and calculation that aims to capture the predictive powers of systemic risk estimation on firm performance in periods of economic downturn. When defining the building blocks of the financial network, the analysis relies on the methods of both Billio et al. (2012) and Diebold and Yılmaz (2014), and any literature that follows the same steps, while the Granger causality estimation follows the improvements made by Hué et al. (2019) and Yao et al. (2016). The rarely used concept of the spanning arborescence is also built similarly to the approach of Yao et al. (2016). Any deviations from these steps in the methodology are clearly indicated.

III.2 Returns and volatility data

As presented in the previous section, the network is built using market data. Still, selecting the correct variable from all the available stock market data is not Among the most frequently used are stock prices, the returns and volatility. In this paper, I consider the latter two only because of the following stylized facts of the statistical properties and theoretical assumptions of these variables. One is that stock prices are usually neither stationary nor normally distributed, two major concerns and limitations for any financial time series data analysis. In contrast, returns, especially their natural logarithmic transformation tend to turn out as stationary more frequently. As for volatility, different measures exist for both the unconditional and conditional case. Volatility has an especially important role in risk analysis, as it is assumed that the connections between volatility

are better at estimating the risk spillover of financial institutions. As for the distribution of the above variables, stock prices are often considered to be log-normally distributed, meaning that their logarithmic transformation follows a normal distribution. In the next paragraphs I explore these concepts further in detail and highlight their relevance to the reliability and convenience of the present analysis. The differences in volatility and returns spillovers in the financial sector were discussed early in Diebold and Yilmaz (2008).

A stationary process in statistics is one for which the unconditional joint probability distribution does not change when shifted in time. In this paper, the time series are not properly tested for stationarity using a Dickey-Fuller test for example, where a hypothesis test determines whether a unit root exists in the data. Rather, it is assumed that stationarity will hold in most cases, for the following two reason. First, stock prices are usually not stationary, as there are increasing or decreasing trends. However, the two most common transformations of time series to make them stationary are differencing and the logarithmic transformation. When calculating log returns, these two steps are carried out on the price data, which is then most likely stationary. Similarly, as $\log(1+r) \sim r$ for small r, and after the logarithmic transformation, $1+r$ is assumed to be stationary, then the log returns are assumed to be stationary as well. Second, splitting the time interval into 3 parts can eliminate the perceived trend from the time series if the distinct time periods are different from each other, but are homogeneous within that time frame, for example the series of returns behave similarly throughout a crisis.

In this paper, I use log returns calculated from the daily closing prices of the stock shares of the banks, along with a daily estimation of conditional volatility based on a GARCH(1,1) model. Simple returns are equal to the percentage change in price between two consecutive days, whereas log returns are calculated as the difference in the natural logarithms of the prices:

$$R_{i,t}^{simple} = \frac{y_{i,t} - y_{i,t-1}}{y_{i,t-1}} \tag{1}$$

$$R_{i,t}^{log} = \log(y_{i,t}) - \log(y_{i,t-1}) \tag{2}$$

$$\log(y_{i,t}) - \log(y_{i,t-1}) = \log\frac{y_{i,t}}{y_{i,t-1}} \sim \frac{y_{i,t}}{y_{i,t-1}} - 1 = \frac{y_{i,t} - y_{i,t-1}}{y_{i,t-1}} \tag{3}$$

$$\textit{using the approximation: } \log(1+x) \sim x \tag{4}$$

Equation (4) shows the approximation of the logarithmic transformation, which means

that while the returns are relatively small, the simple and log returns are approximately the same. Based on the convenient assumption that stock prices follow a log normal distribution, the natural logarithm of prices, as well as the difference in log prices will be normally distributed:

Besides returns, I build the network of financial institutions using volatility data. Two distinct approaches exist for calculating volatility: one from the intra-day changes in prices, estimated from the open, close, minimum and maximum stock prices, and another from the changes of closing prices or returns over a period of time. In this paper, I use the latter approach on the daily returns. As volatility increases in a period, it means that stock returns take up more extreme values. The volatility of a period can also be interpreted multiple ways: unconditional, realized or conditional volatility. The unconditional volatility is the underlying variance of the data generating process of the returns, which is unknown but can be estimated using the assumptions of the data distribution. The variance calculated from the observed data points is the realized unconditional volatility. Finally, the method I use is the conditional volatility of a time series, which is calculated for each observation. In this case, the unobserved volatility is estimated based on additional information for each observation.

$$R_t^i = \mu_i + \sigma_{it}\epsilon_t^i \ , \quad \text{where } \epsilon_t^i \sim \text{WN}(0,1), \tag{5}$$

$$\sigma_{it}^2 = \omega_i + \alpha_i(R_{t-1}^i - \mu_i)^2 + \beta_i\sigma_{it-1}^2 \tag{6}$$

Here, I use the generalized autoregressive heteroskedasticity (GARCH) model to estimate the conditional volatility, which is conditioned on past values of itself and other error terms, which can be seen in equations (5) and (6). I use the GARCH(1,1) specification which means that only the first lags of both the conditional volatility and the error term are included in the autoregressive model. Billio et al. (2012) estimated the conditional volatility using the same GARCH(1,1) model, and they created an adjusted return values by dividing the them by the corresponding estimated volatility. My approach is partly different, as I build the networks using both the log returns and the estimated volatilities without any transformation. Following the literature on systemic risk variables, I expect that the volatility network will explain firm performance better as it contains more information on risks.

III.3 Pairwise Granger causality

Once the appropriate stock market variables are identified and the return as well as the volatility time series are presented, the following task is to calculate the strength of connections between the firms. As I summarized in the literature review, there are several methods within two major approaches I listed. The Vector Autoregression techniques Diebold and Yılmaz (2014) that generate a network based on the analysis of the whole set of variables in one single model are not used in this paper. Instead, among the techniques of pairwise connections, I use the concept of Granger causality. It differs from the pairwise correlation and tail-dependency index in being able to account for both directed and weighted connections, whereas the other two cannot describe the directions. The pairwise approach has both its benefits and limitations. On one hand, it is more simple to implement and focuses more on the direct connections between each pair of institutions. The latter fact is also a limitation, as the position of one institution in the whole network is better captured in a VAR-like model, as a pair of connections is not viewed individually without context, but rather as a part of a system.

In technical terms, Granger causality, as introduced by Granger (1969) is a concept to measure the existence of a connection between two time series. This method does not detect true causality, as it is not possible, but it improves the results of simple correlation analyses. The concept itself is the following: given two time series, R_t^j is said to cause R_t^i if in the following autoregressive framework, past values of R_t^j also explain R_t^i besides its own past values. The notation in this paper follows that of Billio et al. (2012), except that in this paper I use the general description, allowing for a lag parameter $p > 1$ which means that observations from several periods before are included in the specification.

$$R_t^i = \sum_{k=1}^{p} a_k^i R_{t-k}^i + \sum_{k=1}^{p} b_k^{ij} R_{t-k}^j + e_t^i \tag{7}$$

$$R_t^j = \sum_{k=1}^{p} a_k^j R_{t-k}^j + \sum_{k=1}^{p} b_k^{ji} R_{t-k}^i + e_t^j \tag{8}$$

The existence of a Granger causality between two time series can be tested using a standard F-test, with a restricted model containing only the past values of the same time series, excluding the second one. The extended model is where the past values are both included as explanatory variables in the autoregressive equation. The null hypothesis for

the F-test is that the extended model does not explain the current values better than the restricted model: the corresponding b parameters are statistically not different from zero. We reject the null hypothesis if the F value is higher than a certain critical value derived from the F distribution depending on the number of observations and degrees of freedom.

In Billio et al. (2012), the authors calculated the results of the above hypothesis tests. Their network is unweighted, because there existed a directed edge between two firms if their corresponding time series had a statistically significant Granger causality relationship on a given significance level. The edges in the network were not weighted, therefore only took the values of zero or one. In this paper, I proceed with an alternative method, in which the resulting network edges are weighted by the F-statistics of the ranger causality hypothesis tests. In that case, no edges have zero weights. The major shortcoming of using unweighted networks is that it is not possible to distinguish connections based on their strength. The weighted network attempts to fix this limitation of the research of Billio et al. (2012). Greater values of F-stat represent stronger connections. This fact has two implications regarding the subsequent analysis: first, the edge weights must be inverted for some network centrality measures and second, the spanning arborescence algorithm needs the maximum edge weight specification.

In the last paragraphs of this section, I summarize the process of calculating Granger causalities and generating the network structure that is used for the rest of the analysis. The pairwise estimation method is implemented by iterating through the number of institutions, creating every possible pairing. For each pair, a Granger causality test is conducted and the resulting weight is saved. The structure that contains the network information is an edgelist table. The three columns of the table are the 'from', 'to' and 'weight' values. 'From' and 'to' are the tails and heads of the arrow representing a directed edge. The number of entries in the list is $n(n-1)$ where n is the number of institutions. The resulting network is a complete graph, as no edges are omitted from the list.

III.4 Network centrality

This part introduces the centrality measures of the complete network, discussing the methods to calculate them and their interpretation and use case in the analysis.

III.4.1 Degree and strength

In the following section I describe the degree and strength centrality measures of the complete network. To properly define these measures, a formal notation is necessary. Based on the previously explained edge list, any directed network edge $e_{ij} = F$ is represented by the two endpoints of the edge and its weight. The index j is the starting point of the edge, index i is the end, given by the values in the first two columns of the edge list respectively. The third column contains the value for the weight. Both the degree and node strength can be separated into In, Out and Total components, which describe the incoming and outgoing connections, as well as the sum of the two. In the present case, degree centrality is not well usable for a complete graph, as each vertex has the same amount of in and out connections.

On the other hand, node strength can be used to differentiate between the impact of vertices on others. Node strength takes advantage of the weighted network, it is the centrality measure that allows working with complete networks. As for the interpretation, higher node strength means a more central position in the network, as it is for most centrality measures. Higher In strength values represent higher exposure to systemic risk, while higher Out values represent higher contribution. The following summation formulas define the values of the In, Out and Total node strengths:

III.4.2 Closeness

Closeness centrality is another possible characteristic of network vertices. The basis of the concept of closeness is distance, in other terms, the shortest path between two vertices. A path in graph theory is a sequence of edges through which a certain node can be reached from a starting point. In a network with many edges, evidently, there are multiple possible paths between any pair of nodes. The length of a path is the number of traversed edges in an unweighted network and the sum of weights of traversed edges in a weighted network. The shortest path is minimum length element in the set of all paths between two nodes. Closeness centrality measures the average length of shortest paths from one node to all other connected nodes. The following formula defines the closeness centrality for vertex

v, based on Sabidussi (1966):

$$c_{Closeness}(v) = \frac{1}{\sum_{u \in V^*} d(v,u)} \tag{9}$$

where $d(u,v)$ is the shortest path between vertices u and v, V^* is the set of all nodes to which there exists a connection from v.

In the present case, where the edge weights are given by the F-statistic of Granger causality tests, some modification is necessary. The problem with the current weights is that they are understood as the strength of the links, meaning that higher values are interpreted as closer connections. Closeness assumes a distance metric, which is supposed to be lower for closer connections, similar to costs for example. Thus, the current edge weights need to be transformed into a distance measure, using a one-to-one correspondence function, so that the modified value can be reverted easily and the ordering of path lengths does not change. The process of transforming into a distance measure is often used for weighted networks, but the method is different when the original weight is constrained to a closed interval, as it is with correlation or tail-dependency measures (see Denkowska and Wanat (2020) or Yang et al. (2020)). For both the closeness and betweenness centrality, the ones that use shortest paths, the transformation of the edge weights is conducted as follows:

$$W^* = \frac{1}{W} = \frac{1}{F\ value} \tag{10}$$

where $W = F\ value$ is the original edge weight.

After the transformation, the formula for closeness centrality returns the correct values. This measure can be interpreted as the centrality that takes indirect connections into account as well. As the Granger network is complete, every node is directly connected to all other nodes, however, there are also multiple other paths traversing more nodes on the way. It is possible that the shortest path is not the direct connection, which implies that indirect links can be important as well. The interpretation of this centrality in the context of financial institutions is the following: higher closeness value for a node indicates closer links to other nodes in the network, considering both the direct and indirect connections. Higher closeness centrality of a financial institution implies greater contribution to systemic risk.

III.4.3 Betweenness

The concept of betweenness centrality also builds on shortest paths, so the edge weights are transformed in this part the same way as before. It aims to capture the position of a node relative to all other pairs of nodes and the paths between them. Betweenness measures the number of instances where a particular node is present in the shortest path between any other pair of vertices. The formula for betweenness is the following, based on Freeman (1977):

$$c_{Betweenness}(v) = \sum_{s \neq t \neq v \in V} \frac{\sigma(s,t|v)}{\sigma(s,t)} \tag{11}$$

where s and t are all other pairs of nodes, the numerator of the fraction is the number of shortest paths between s and t that run through the vertex v, while the denominator is the total number of shortest paths between s and t. The appropriate usage in R is described in Kolaczyk and Csárdi (2020).

I briefly highlight the differences in using betweenness centrality on unweighted and weighted networks in this paragraph. In the former case, when in addition the network is not complete, the above formula is the correct one, where because of the same edge weights, multiple shortest paths may exist for each pair of nodes. However, when using weighted networks, especially those where the range of weight values is wide, it is likely that no two shortest paths are the same. In that case, the formula is simplified, and the betweenness is the number of any shortest paths running through node v.

As for the interpretation in the systemic risk context, betweenness measures the importance of an institution as the degree to which it cannot be avoided when looking at a random chain of links through several institutions. The higher betweenness centrality implies that a node is less likely to stay out of the chain and more likely to actively take part in the propagation of risks.

III.5 Simplifying the network

The network representing the connections of financial institutions is by design a complete graph, that is, between each possible pair of nodes, there is an edge with a non-zero weight assigned to it. The first part of the analysis is conducted on the complete graph,

by measuring the node strength, closeness and the betweenness centrality. The reason for this is that even though several connections are not significant based on the Granger causality test, – the corresponding F statistic would be lower than the critical value for a given significance level – including the whole set of edges may still have some information on the nature of the set of systemic connections. This is one approach, where no links are omitted, in hope of finding the smallest one useful as well.

The other approach is a rather aggressive simplification of the network, involving the graph theory concepts of 'tree' and 'arborescence', which suggest a network structure where the minimum possible number of edges are considered. This can seem contradictory to the first approach, however, the two represent the two sides of a trade-off between possible detail in the connections and less noise in the data. Eliminating only the non-significant Granger causality links from the network – even half of them - does not significantly decrease the noise, while it does give up on detail. For this reason, the above two approaches are used in this paper, a complete graph and an extremely simplified one. In this chapter, I describe the methods used for simplifying the network, as well as the set of centrality measures that are calculated for the simplified graphs and later used to compare the two approaches.

III.5.1 Maximum Spanning Tree

The first method I describe is the concept of Minimum/Maximum Spanning Tree (MST or spanning tree from now on). Note, that both the Minimum and Maximum types are identical, they only differ in the interpretation of the edge weights: a maximum spanning tree is calculated when higher edge weights represent closer and stronger connections, while the minimum is considered when lower edge weights represent stronger links - as in the case of costs or distances. In the Granger network, higher F statistics represent stronger connections, so in this paper, for both the spanning tree and spanning arborescence explained in the following section, the maximum type is used. In graph theory, a tree is an acyclic graph where any two nodes are connected by exactly one link. Practically, in a graph with n vertices, this means that there are $n-1$ edges the network is still connected. A maximum spanning tree is an arrangement of $n-1$ edges so that the sum of edge weights across the whole tree are maximal.

The MST is not used as a tool of analysis in this paper, its short introduction only

serves the purpose of understanding how the spanning arborescence works, which is more complex, but is built on the concept of the MST. Spanning trees can only be used on undirected graphs, and as the present Granger network is directed, it cannot be applied in this case. In several research papers in this field, MST is often used on undirected networks built from pairwise correlation or tail-dependence metrics, calculated using Prim's algorithm (Prim (1957)).

III.5.2 Maximum Spanning Arborescence

A Spanning Arborescence is analogous to the spanning tree, except for the fact that it arranges directed edges. The same way, in the arborescence, there is exactly one path between any pair of nodes, but this has a particular direction. The maximum spanning arborescence (once again, using the maximum specification because higher weights represent strong connections in the Granger network), arranges the directed edges so that the sum of edge weights is maximal across the resulting graph. The construction of the arborescence is different from spanning trees, as in every case, there must be a designated root node, from which the chain of directed edges starts. It can be understood for example as the chain of spreading information or risk, where the starting point is crucial. Depending on the central node, the various arborescence graphs usually have a different arrangements of links.

The structure of the spanning arborescence is the following: the root node has no incoming edges, because it is the one that start the chain of links. Then, all other nodes have exactly one incoming edge, because by definition, in an arborescence, each of the vertices can be reached by only one path starting from the center. As for other node properties, there are no limitations: either the root, or any other vertex may have several outgoing connections, meaning that they are more central in the network. Examples figures are presented in the empirical results section.

In the following paragraphs, I present a short discussion of the algorithms, programming tools and techniques used to generate the spanning arborescence graphs. The Edmonds algorithm is used to create the spanning arborescence from the complete network of financial institutions Edmonds (1967). It is a recursive process, which begins by selecting a root node, then with each iteration, it removes edges from the initial network that have lower weights, until only a graph that fulfills the properties of the spanning arborescence

remains. My goal in this section is not to describe the process of the algorithm in detail but to highlight the key points in the implementation for the purpose of clarity and reproducibility.

I used the Python package, *networkx*, more specifically the *maximum_spanning_arborescence* command from the Branching section Hagberg et al. (2008). One important note regarding the implementation is that the above command in itself does not select a root node properly, but chooses one based on other parameters. Since choosing different root nodes is necessary to highlight the differences between network structures, and the properties of the different arborescence graphs are used to measure the importance of individual institutions, this limitation needed a solution. By definition, the first step of the Edmonds algorithm is to eliminate all incoming edges to the designated root node from the network, and this was the step that the above command seems to have missed. By removing the proper edges directed towards the root node before running the algorithm forces it to consider the chosen node as the root. This way, after running the script, an n-node network yields n different spanning arborescence graphs, where n is the number of institutions.

III.6 Centrality measures of the arborescence

For the analysis of arborescence network structures, I use the similar centrality measures as discussed in the complete network case. As the number of possible arborescence graphs is large, due to the choice of central nodes (the three time period specifications and the 45 possible central nodes add up to a total of 135 arborescence graphs), presenting all of them is not feasible. For selecting only a few arborescence graphs, the solution of Coelho et al. (2007) is implemented, which designated central nodes for the analysis of MSTs. Only those arborescence graphs are displayed or disclosed in detail, where the root node is the one that is the most central in the complete network according to one of the centrality measures. In this section, I provide a description of all the centrality measures and their role in the analysis. These variables will be part of the regression framework that estimates whether variables of the simplified network can better explain or predict the losses and systemic risk contribution of a firm than those variables that are only calculated from the complete network.

One of the measures is Out Degree, which is the number of outgoing connections of a

node. This measure was not used in the complete network, as any degree number would have been the same for each node. However, in case of the arborescence, it can be used because the Out Degree of nodes can differ greatly. The same is not true for the incoming edges, as it can be only one for each vertex, or zero for the central node.

Another measure is the strength of a node, the sum of weights in the edges connected to it. This is defined the same way as for the complete network. All three, the In, Out and Total strength might be used to determine the centrality of nodes. The same way, closeness and betweenness centrality can be calculated for each node, which is interpreted the same way as the complete network case. All of the above measures could be calculated for all possible arborescence structures. However, as stated in the previous section, only those will be included in the regression framework, that are based on an arborescence with a root that is an otherwise central node in the complete graph.

One last measure on the other hand is calculated from all arborescence graphs: that is the mean weight of all $n-1$ edges. Since there is an arborescence graph for each node, where it is the root, there is one mean edge weight value for each node as well. This measure can be interpreted as the overall strength of the connections assuming that the center of risk propagation is the chosen node. A higher value means that risk is likely to spread faster and further from that particular central node. The root node in this interpretation can be for example the starting point of a financial crisis. This measure aggregates the information from the different arborescence structures.

III.7 Dependent variable and regression design

Besides the variables derived from both the complete network and the arborescence, a dependent variable is needed in the regression framework to represent the firm-performance for each period. The firm-level performance indicator I use in this paper is the approach used by Hué et al. (2019). The variable they use is the average of downside returns. For each institution $k = 1, ..., 45$, it is given by

$$\text{Perf}_k = \frac{1}{m} \sum_{t=1}^{T_c} y_{k,t} Z_{k,t} \tag{12}$$

Where $y_{k,t}$ is the return at time t for firm k. The downside indicator $Z_{k,t}$ is one if the corresponding daily return value is lower than a threshold value and zero otherwise:

$$Z_{i,t} = \begin{cases} 1 & \text{if } y_{i,t} < \delta \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

The m parameter is the number of times the downside indicator takes the value 1.

The firm performance variable does not change in the regression analysis according to the network used, whether it is the return or volatility-based. In this section, I briefly introduce the methods within the regression framework I use, especially the measures and calculations to determine the predictive power of the regression or the stability of the estimation on the out of sample data. Also, I introduce the concept of dividing the data into an in sample and out of sample part for the second set of regression models.

Generally, the regular R^2 is used first to determine the explanatory power of a regression estimation. It shows the part of variance of the dependent variable that is explained by the variance of the independent variables. Later in the empirical analysis, however, I compare in tables the adjusted R^2 values of the models, that account for the number of variables in the model and counter the increase in explanatory power that is only the result of chance. Equations (14) and (15) show the formulas for both R^2 values:

$$R^2 = 1 - \frac{RSS}{TSS} \tag{14}$$

$$R^2_{adj} = 1 - \left[\frac{(1 - R^2)(n-1)}{n-k-1} \right] \tag{15}$$

The first set of estimations is only carried out on the full sample, spanning all three periods. In that case, only the adjusted R^2 is used to compare the different specifications. However, for the second set of estimations, that include the pre-crisis in sample estimation and the out of sample test phase, I use another measure to compare models. The goodness of fit of the predicted values on the realized firm performance variable is given by the root-mean-square error (RMSE). Lower RMSE values for a certain model indicate that the estimated connection between the network variables and firm performance is more stable and fits better on the realized data. The calculation of RMSE is displayed in equation

(16):

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \hat{x}_i)^2}{N}} \tag{16}$$

III.8 Data

In this paper, I chose to include only banks out of the few possible branches of financial institutions. This limitation is addressed in the limitations and further research section. For the present research question, that is, whether a spanning arborescence provides better explanation of firm performance can be tested on a specific and limited set of institutions as well. A total of 45 banks are chosen, from both North America (United States and Canada) and Europe, which at least partly addresses the question of diversity of geographical location. Also, besides the largest banks in both regions, some smaller institutions are included as well. Table (9) in Appendix 1 lists all institutions by abbreviation, full name and country.

Up to 50 banks were considered when acquiring the data, but some smaller institutions were removed from the sample, because the stock price data was either missing for a certain period, or it was reported in a way that resulted in 0 log returns over a significant period. The two core papers on which I built my analysis, Billio et al. (2012) and Hué et al. (2019) use more institutions, the first differentiates four sectors on 100 institutions of the US market while the second includes 90 banks from around the world. Other papers build networks on for example 38 European insurance firms (Denkowska and Wanat (2020)), 46 US financial institutions (Gandica et al. (2020)) or 53 equity market indices (Coelho et al. (2007)). This means that generally, the sample size used in this paper is in line with recent research.

The full sample spans the period between 2002/01/01 and 2009/06/30 For most of the analysis, the data is divided into three sections, following the approaches of both Billio et al. (2012) and Hué et al. (2019) . The period of the crisis is taken from Hué et al. (2019), and it lasts from 2007/07/01 to 2009/06/30, a 2-year long interval. The remaining 5 and a half years is divided into two parts, where the first half is based on an interval determined by Billio et al. (2012), spanning from 2002/01/01 to 2004/12/31, constituting the first period out of the 3. The last, middle interval is the remaining 2 and a half years

from the data. The most tranquil interval is the second one, while the most volatile is financial crisis in the last one. The number of observations in the return or volatility data is not the same in each period: the time series for each firm contain 784, 650 and 523 observations.

IV Empirical analysis

In this chapter, I introduce the results of the empirical analysis, based on the data and methods described in previous chapters. In the first section, I introduce the research design, which contains the set of parameters and the variables used for the regression analysis. Section 2 contains the summary statistics of the original data, the returns time series of the firms. Descriptive statistics provide a way to show the basic properties of the data and their change over the examined time frames. In the next sections, I present the characteristics of the generated networks, their edge weight distribution, and several centrality measures. Section 4 follows by a description of the arborescence networks and their corresponding centrality measures, by summary statistics and figures of the graph. Finally, Section 5 shows the properties of the dependent variable used in the regressions, as well as the regression analysis results. Finally, I close this chapter by discussing the limitations of the present regression framework.

IV.1 Research design

This section summarizes briefly the parameters and variables used throughout the empirical analysis. For the pairwise Granger causality test, I use a maximum lag length of 3, which means that the estimated lag order may be between 1 and 3. Similar to Billio et al. (2012), the lag order is determined by the Bayesian Information Criteria (BIC), that is often used in model selection. Following the pairwise Granger causality estimation, centrality measures on the complete network are calculated. The In, Out and Total strength, as well as closeness and betweenness centralities are scaled to the [0,1] interval in the visualization process, but for the regression analysis, the original values are used. The arborescence graphs are generated using the Edmonds algorithm and the process is repeated for each node in each time period. The arborescence centrality measures are

calculated for a selected root node for each period, which is chosen based on the node centralities of the complete network.

The dependent variable of the regression models follows the approach of Hué et al. (2019), calculating a mean losses in periods of downturn, where the daily returns fall below a threshold value δ. In the present analysis, for the base version, I chose $\delta = 3\%$. The parameters may be changed during a robustness analysis, where the stability of the estimations is tested. At the end of this paper, I present a short robustness analysis, discussing the results when the dependent variable is calculated with different parameters.

IV.2 Returns table

Table IV.2 shows the summary statistics of the log returns, the firms pooled for each of the three periods of time, as well as the full sample. This means that each row represents all firms for the given period, where every daily return value is considered part of a single set of observations. Properties of the distribution are addressed by the different percentile values, as well as the mean and standard deviation, which represent the main tendencies and the volatility of the returns. Note that instead of displaying the minimum and maximum values in the distribution, I included the 1st and 99th percentiles, due to extreme outliers, that deviate too much from the general tendency. The range and interquartile range of the distribution can be calculated by the differences in the 99th and 1st percentiles and the 75th and 25th percentiles respectively.

	P1 (%)	P25 (%)	P50 (%)	P75 (%)	P99 (%)	Mean (%)	St.Dev.
Full sample	−8.51	−0.95	0	0.93	8.57	−0.02	3.03
First period	−5.71	−0.88	0	0.95	6.17	0.04	2.06
Second period	−3.83	−0.59	0	0.71	3.88	0.06	1.35
Third period	−14.52	−2.04	−0.08	1.57	14.26	−0.21	5.06

Table 1: Summary statistics of the daily log returns. Each row shows the different sections of the data, starting with the full sample, through the three distinct periods. Columns indicate different summary statistics: the first five are the 1st, 25th, 50th, 75th and 99th percentiles, while the last two are the mean and standard deviation. These seven values describe the distribution of the returns for each period. First period: January 2002 to December 2004, Second period: January 2005 to June 2007, Third period (the crisis): July 2007 to June 2009.

Results show that the third period that spans the 2007-2008 financial crisis is characterized

by significantly lower mean returns, as well as higher standard deviation, which is an indicator of higher risk, as it corresponds to the volatility of the time series. The differences in the distribution can be seen in the percentiles of the data as well. Higher range of values indicates more extreme returns, thus they represent a more volatile environment. Overall, it is evident, that the distribution of returns in the crisis is both shifted to lower values and it is more volatile. The distributions are close to being symmetric, but for example in the third period, the mean is lower than the median value, which indicates a left-skewed distribution. These results correspond to the expectations drawn from previous research, such as Diebold and Yilmaz (2008), that returns and volatilities change during a crisis,

IV.3 Granger causality network

Tables IV.3 and IV.3 contain the properties of the edge weights in the networks generated by the pairwise Granger causality estimation, for both the returns and volatility case. The network for each time period contains 1980 edges, representing a 45-node complete graph. For each network, summary statistics are calculated from the list of links. Mean edge weight provides information on the average strength of connections, while the different percentiles and the standard deviation give insight about the distribution of the edge weights.

	P1	P25	P50	P75	P99	Mean	St.Dev.
Full sample	0.001	0.97	5.51	16.40	131.83	15.96	27.45
First period	0.001	0.35	1.78	6.56	57.64	6.72	12.12
Second period	0.0003	0.24	1.14	3.85	24.22	3.19	5.05
Third period	0.001	0.44	2.30	6.80	43.16	5.96	9.36

Table 2: Summary statistics for the edge weights of the returns network. Each row shows the different sections of the data, starting with the full sample, through the three distinct periods. Columns indicate different summary statistics: the first five are the 1st, 25th, 50th, 75th and 99th percentiles, while the last two are the mean and standard deviation. The edge weights are the F-test values of the pairwise Granger causality tests on the returns time series. First period: January 2002 to December 2004, Second period: January 2005 to June 2007, Third period (the crisis): July 2007 to June 2009.

Results indicate that under market distress, the mean strength of connections increases, because in a period characterized by generally lower returns, the connections and correlations between individual time series tends to increase. A graphical representation of

any of the complete networks is not useful in any way, since the noise resulting from the amount of edges makes it difficult to draw any general conclusions. These results were expected based on Billio et al. (2012), where the authors conclude that the strength of Granger causality links increase during a period of crisis.

Note, that the comparison between periods is not obvious: the nature of Granger-causality tests is such, that higher sample size can lead to statistically more significant results, higher F-test values. That is why comparisons between a single period and the full sample is not possible, since the difference in sample size is too large. Although each period has a different length, since they were drawn from previous research, the differences are smaller. The reason for higher F-statistics during the first period is partly the difference in sample length, but also the fact that the first years of the decade were volatile as well.

	P1	P25	P50	P75	P99	Mean	St.Dev.
Full sample	0.06	5.01	11.35	20.61	73.03	16.68	46.28
First period	0.002	0.87	4.88	10.58	35.21	7.27	10.41
Second period	0.001	0.18	0.84	2.61	14.52	2.15	3.53
Third period	0.01	2.11	4.92	9.40	27.64	6.89	12.96

Table 3: Summary statistics for the edge weights of the volatility network. Each row shows the different sections of the data, starting with the full sample, through the three distinct periods. Columns indicate different summary statistics: the first five are the 1st, 25th, 50th, 75th and 99th percentiles, while the last two are the mean and standard deviation. The edge weights are the F-test values of the pairwise Granger causality tests on the volatility time series. First period: January 2002 to December 2004, Second period: January 2005 to June 2007, Third period (the crisis): July 2007 to June 2009.

IV.4 Centrality measures of the complete network

The following section presents the centrality measures and their characteristics that are generated for each iteration of the complete network. These are the In, Out and Total node strength, closeness and betweenness centralities. Figure (1) shows the five centrality values for each network next to each other on a bar plot. While plotting every variable on the same figure can be overwhelming and difficult to see through, in this case, it serves as a preliminary analysis of how the variables tend to move together as well. Since the interpretation of every centrality measure for these firms is nearly identical - as in higher centrality scores represent a more important node in the system, which corresponds

to higher influence on other institutions. While some variables correlate well, it is also interesting to see how the incoming connection variable relates to the others, as it is one that represents a different property, the exposure to systemic risk.

Results show that most centrality values move together to some degree, but there are significant differences between the In and Out strength of nodes as they capture different characteristics of institutions, as expected based on Gandica et al. (2020) for example. Since different centrality measures are on different intervals, I scaled all of them to the [0,1] interval, for the purpose of illustrating them. The maximum value of a centrality measure takes up 1 on the new scale, while the minimum value takes up 0, and it is important that the variables retain the rankings. One important measure among these is the Out strength of the nodes, as I used that to select the root node for the arborescence graphs: out of the 45 different arborescence networks that are generated for each period, I chose the one where the root node is the one that has the highest Out Strength in the complete network.

One main purpose of showing the centrality values for each institution is the ability to rank them. Systemic importance of firms can be ranked by any of the variables, which allows for many possibilities. Research that aims to identify systemically important institutions may instead use more sophisticated methods to aggregate the information of different rankings. This paper does not attempt to identify these firms accurately on the present set of data, and I only show examples of different rankings based on the individual centrality measures. Table (4) shows the top three institutions for each period according to each centrality measure.

IV.5 Spanning arborescence structure

As described in the methods chapter, the maximum spanning arborescence graphs are calculated for each period. A total of 45*3 are generated, one for every central node choice for every period. The number of possible graphs is too high and needs filtering. Based on the results of the previous section, only those arborescence graphs are shown here where the central node is the highest Out Strength centrality value for the corresponding period. The central node is not the same through the whole sample, but changes according to the most central node in each period.

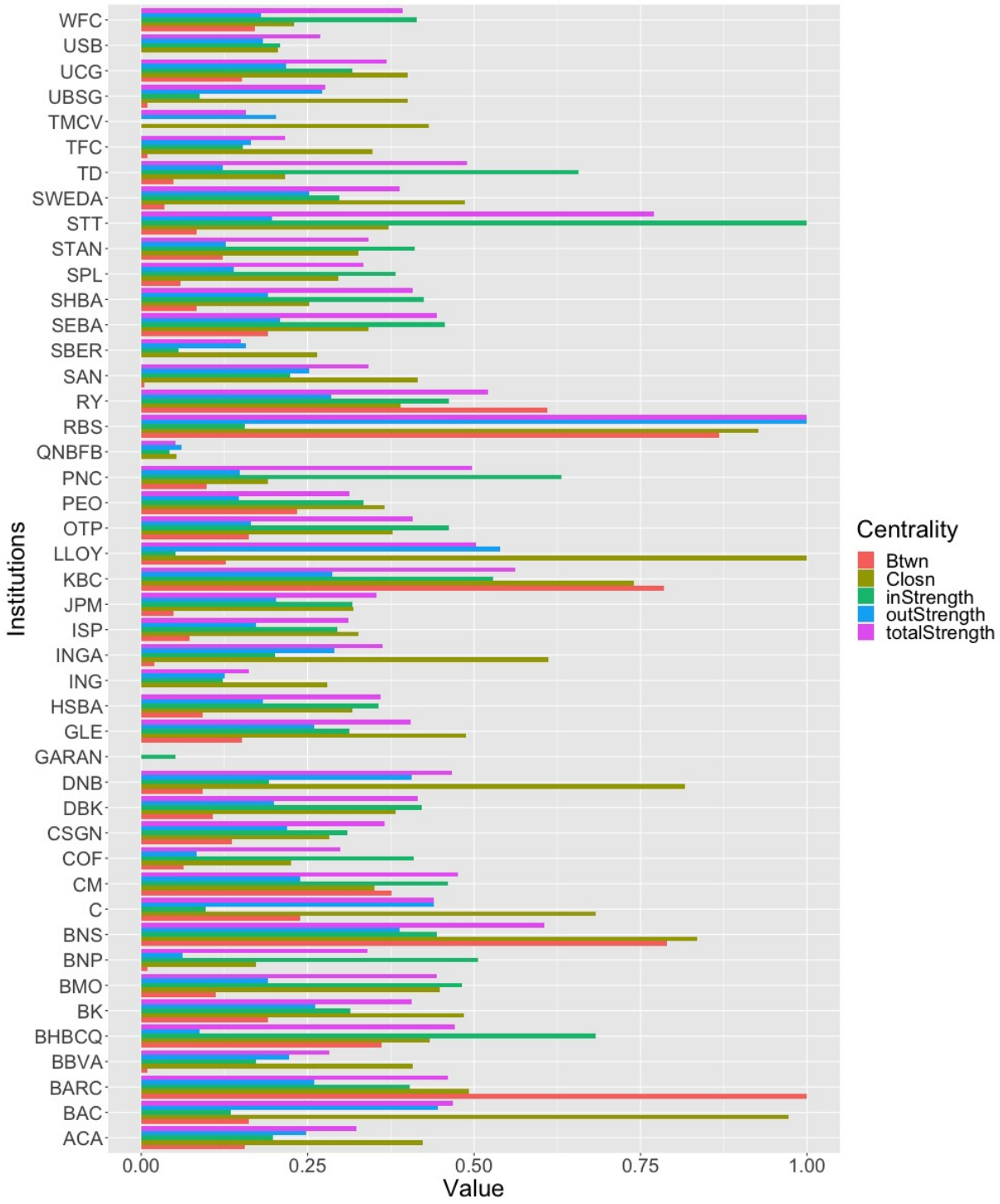

Figure 1: Centrality measures of the complete volatility network in the crisis period. The centrality measures included are betweenness, closeness, in, out and total strength of nodes. Note: all values are scaled to the [0,1] interval, where the maximum value of the variable takes 1 and the minimum takes 0.

Top 3 institutions - first period - volatility network					
	Total Str.	In Strength	Out Strength	Closeness	Betweenness
1	COF	WFC	COF	COF	UBSG
2	PNC	SHBA	RBS	PNC	RBS
3	WFC	STAN	GLE	TD	SAN
Top 3 institutions - second period					
	Total Str.	In Strength	Out Strength	Closeness	Betweenness
1	DNB	DNB	SWEDA	DNB	SWEDA
2	SWEDA	RBS	DNP	SWEDA	DNB
3	SHBA	KBC	JPM	RBS	JPM
Top 3 institutions - third period					
	Total Str.	In Strength	Out Strength	Closeness	Betweenness
1	RBS	STT	RBS	BARC	LLOY
2	STT	BHBSCQ	LLOY	RBS	BAC
3	BNS	TD	BAC	BNS	RBS

Table 4: Summary of the rankings created from the network centrality measures of the volatility network. Each period is considered, and we can see how the rankings change according to time and the measure being used.

The following figures show the three arborescence graphs for the volatility network and this section provides a brief explanation of their structure. The central node is distinguished by a red rectangle, it is the only node with no incoming edges. As stated earlier, this structure of nodes and edges can be interpreted as the most probable route that information - or in this context negative firm performance - spreads, assuming that the starting point of the chain is the root node. The arrows are the directed edges, where the tail is the source of causal relationship and the head is the target. For 45 nodes, we can see that there are 44 edges without circles, as the graph is connected.

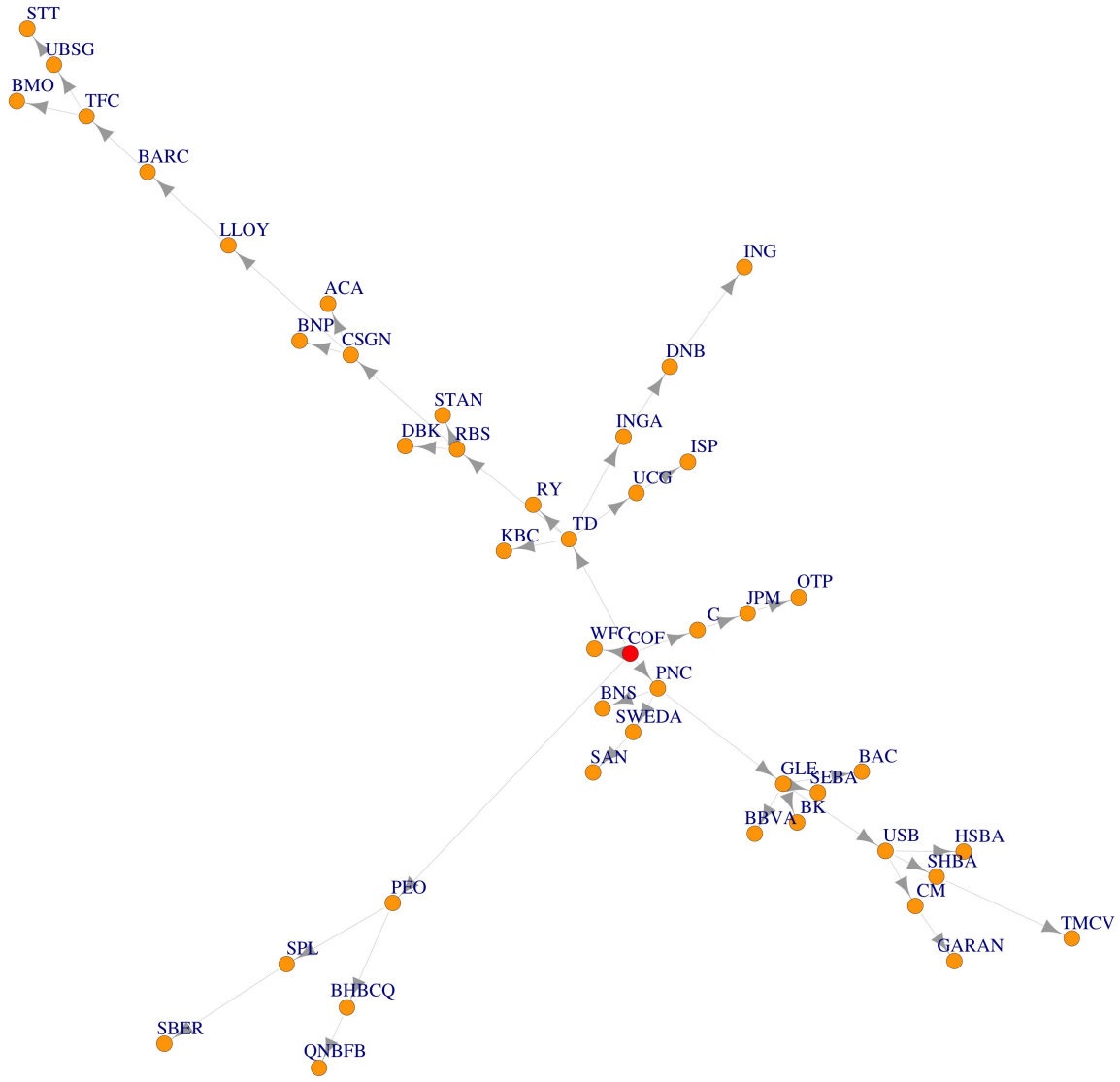

Figure 2: The arborescence graph plot from the volatility network created on the first period. The red rectangle is the root node, the arrows show outgoing connections to other vertices.

The shape of the network structure may provide information on the general connectedness properties of the firms. When an arborescence resembles a chain-like shape and only few nodes have high number of connections, the system is more stable, as there are less institutions which heavily influence many others. On the other hand, it is possible that the graph contains hubs and parts of it resemble a star structure Yao et al. (2016). In this case, there are more institutions that have large influence on others, causing the system to be less stable. The arborescence from the second period shows characteristics of the

chain structure, while the third period is more similar to a star-shape, where the root node has relatively many connections.

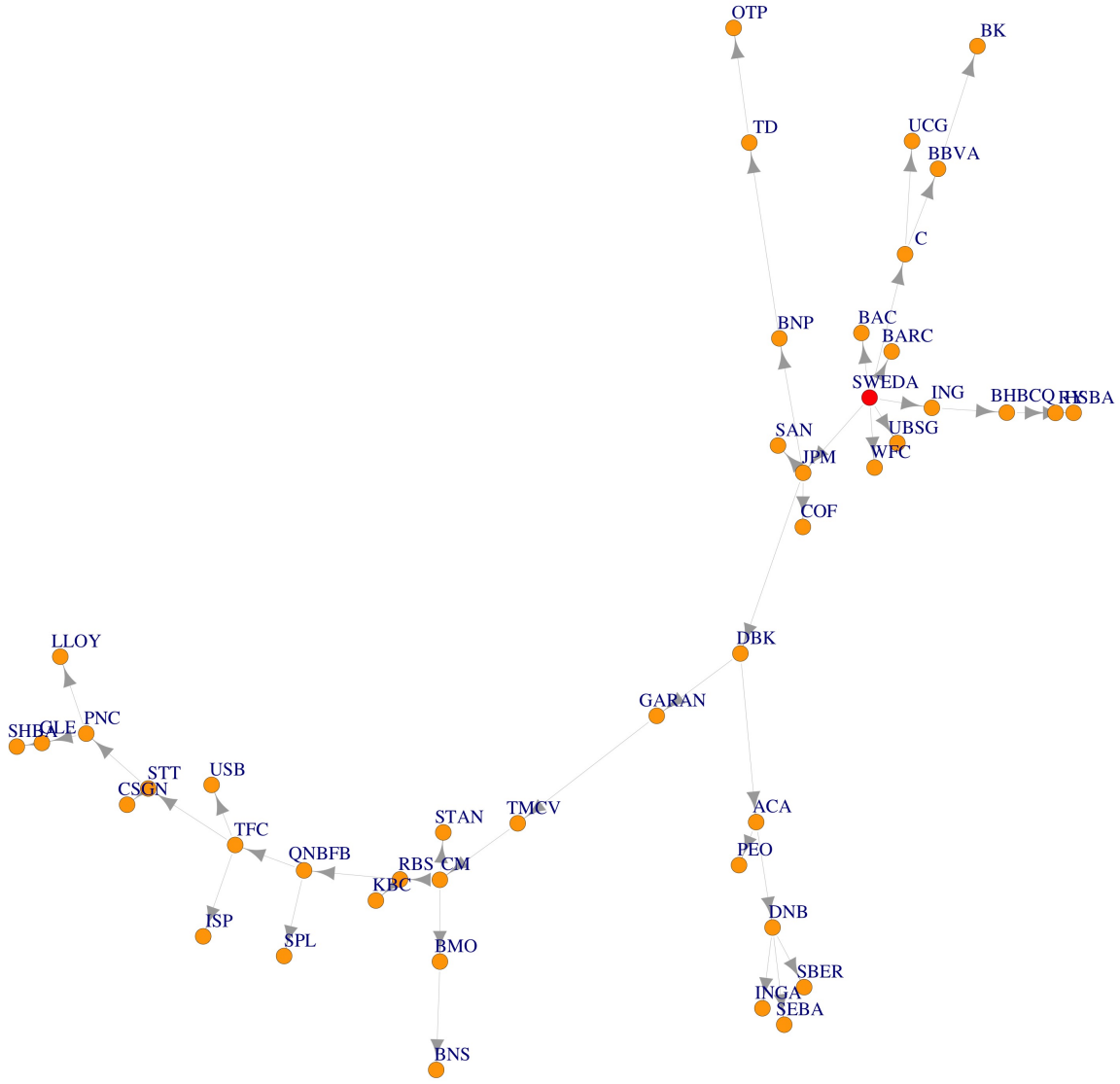

Figure 3: The arborescence graph plot from the volatility network created on the second period. The red rectangle is the root node, the arrows show outgoing connections to other vertices.

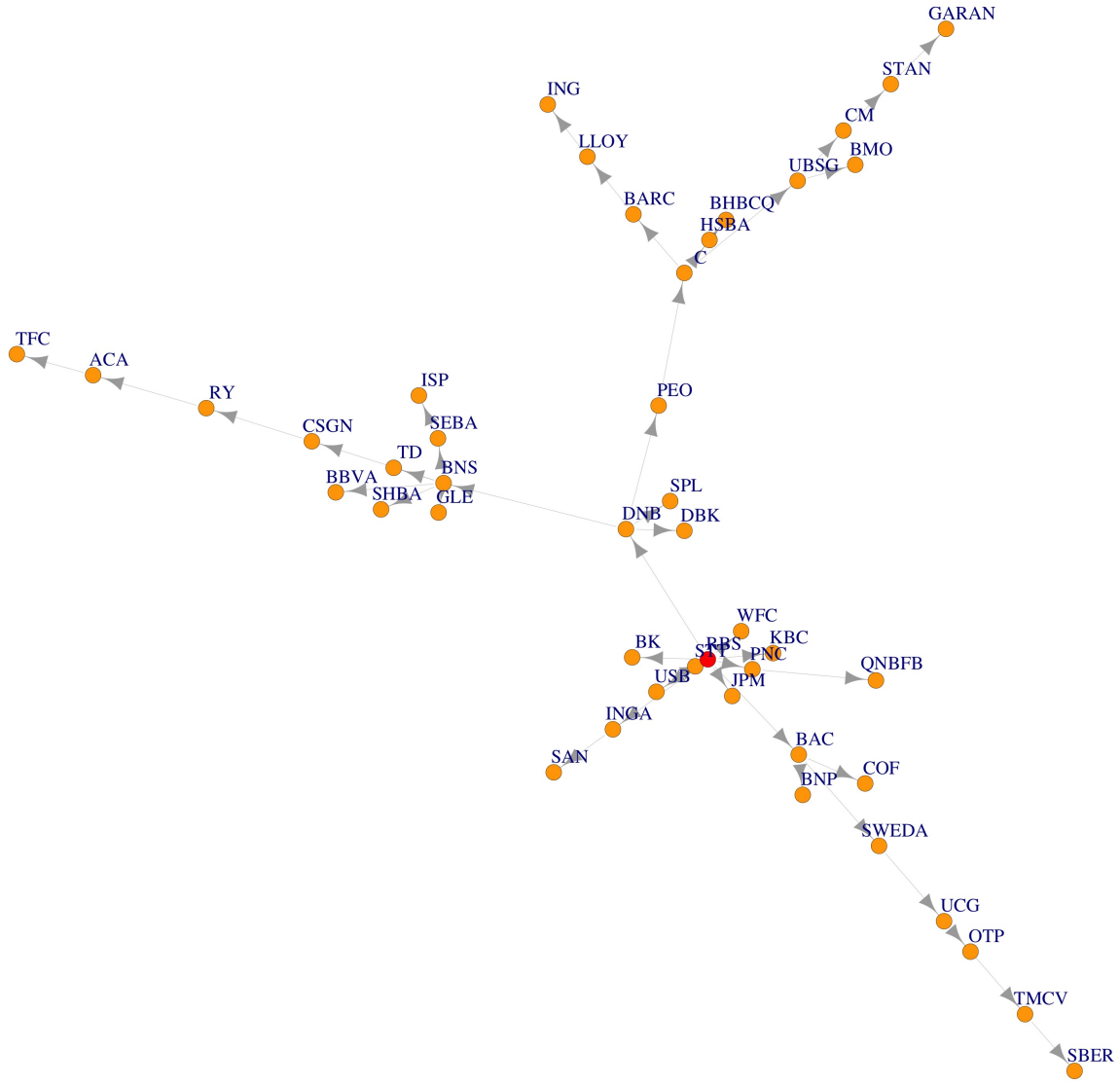

Figure 4: The arborescence graph plot from the volatility network created on the third period. The red rectangle is the root node, the arrows show outgoing connections to other vertices.

IV.6 Centrality measures of the spanning arborescence

Before turning to the regression analysis, I present the centrality measures calculated from the spanning arborescence graphs. There would be more possibilities by including simple degrees for all directions to the analysis, as the elimination of edges from the complete graph makes it possible to differentiate between nodes based on out degree. Therefore, instead of using the same variables as in the case of the complete network, I use the total

strength, total degree, closeness and betweenness measures. The reason for not including separate In and Out values is that in many cases, the Out value is 0, which means that the Total and In values are the same, and adding them to the regression separately would not add explanatory power to the estimation.

Besides the previous variables, a new one is introduced as well. The calculation process is described in chapter 3, as an aggregated measure across all possible iterations of the spanning arborescence graphs, which results in a firm-level variable. The variance of this mean edge weight variable across different versions of the arborescence is significantly lower than for other centrality measures. This may be a limitation in the regression analysis since a wider range of the explanatory variable is preferred, but it is included nevertheless. This variable has not yet been used in similar analysis, it is interesting to see whether it improves the existing centrality measures of the arborescence.

Figure (5) is similar to the one I used for the centrality measures of the complete network. It contains the 5 variables discussed in the previous paragraph, and similarly to the complete network case, the values are once again scaled to the [0,1] interval to better visualize the ranking of institutions. This plot is based on the arborescence of volatility network in the crisis period. It can be seen that the average edge weight value varies less than the other variables, and that total degrees are close to zero in many cases. The same way as with the complete network, I summarized the top three institutions according to each variable, and these two tables can be directly compared.

IV.7 Firm performance variable

In the following section, I present the results of the regression framework I used to determine whether the models augmented by the arborescence network properties provide a better prediction of firm performance during a crisis period. I described the dependent variable of the following models in the methods chapter: firm performance is measured by the average downturn returns over a certain period, following the approach of Hué et al. (2019). The explanatory variables in the models are the different properties of networks. Some specifications include only the variables of the complete network or the arborescence, while some include both. First, I present the summary statistics of the dependent variable for all three time periods. Focus is on the third period, as the idea behind using

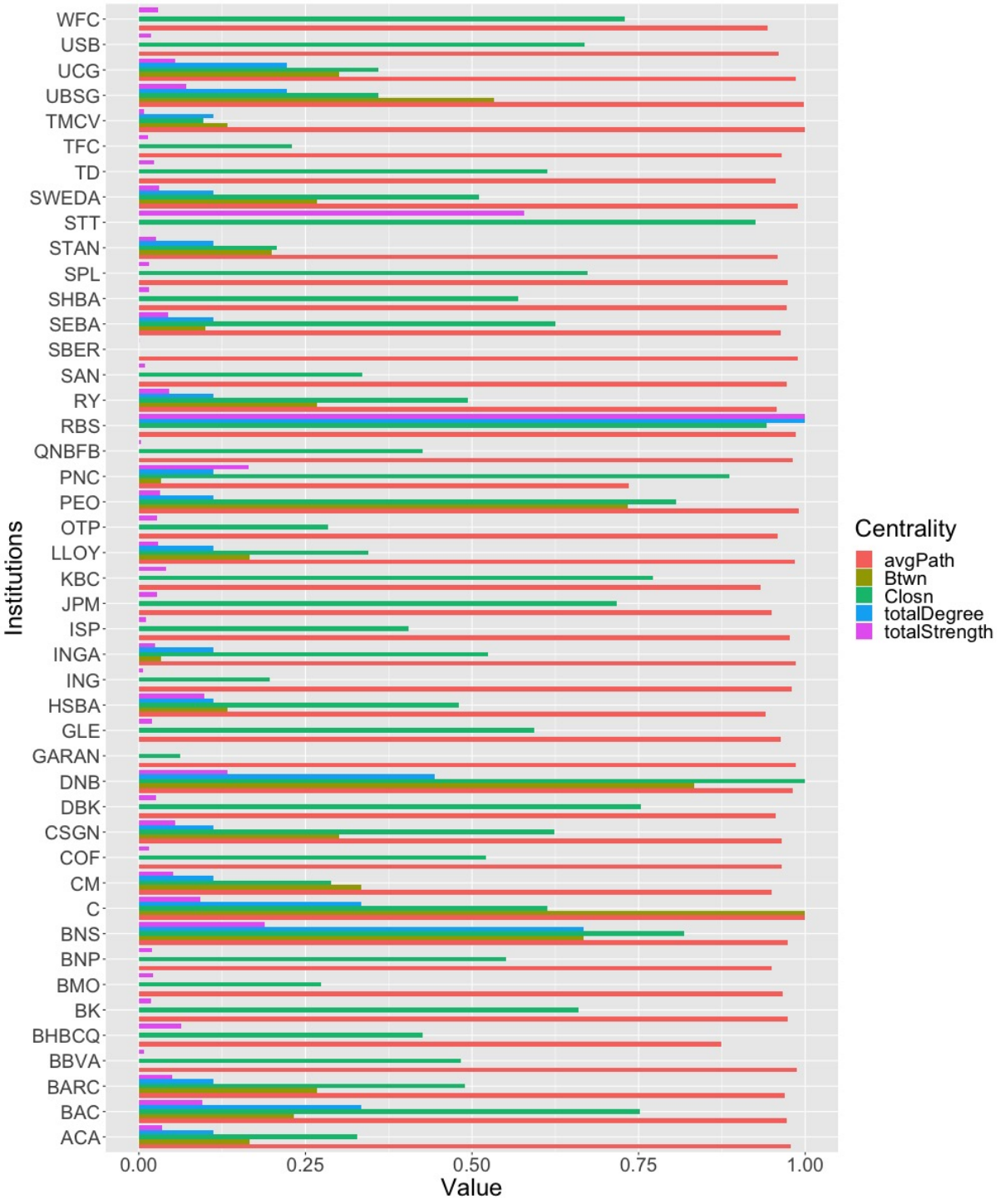

Figure 5: Centrality measures of the arborescence network in the crisis period, generated from the volatility network. The centrality measures included are average edge weight, betweenness, closeness, total degree and total strength of the nodes. Note: all values are scaled to the [0,1] interval, where the maximum value of the variable takes 1 and the minimum takes 0.

Top 3 institutions - first period - volatility arborescence					
	Total Str.	Total degree	Avg. Path	Closeness	Betweenness
1	COF	GLE	PEO	CSGN	COF
2	PNC	TD	LLOY	RBS	PNC
3	GLE	COF	SBER	LLOY	TD
Top 3 institutions - second period					
	Total Str.	In Strength	Out Strength	Closeness	Betweenness
1	SWEDA	SWEDA	TMCV	CM	DBK
2	CM	JPM	TD	RBS	JPM
3	JPM	CM	STT	QNBFB	GARAN
Top 3 institutions - third period					
	Total Str.	In Strength	Out Strength	Closeness	Betweenness
1	RBS	RBS	C	C	DNB
2	STT	BNS	TMCV	DNB	RBS
3	BNS	DNB	UBSG	PEO	STT

Table 5: Summary of the rankings created from the network centrality measures of the volatility arborescence graphs. Each period is considered, and we can see how the rankings change according to time and the measure being used.

such variables is observing firm performance under distress.

Despite there being only 45 observations of this variable for each time period and the inaccuracy of summary statistics on such low sample sizes, the important moments of the distribution have valuable insight into what to expect from firms. Another table I present here is a similar ranking of the top three and bottom three firms to the one presented before. Since the threshold for downturn and the lower tail of returns is set to -3%, that is the upper limit of the distribution. The same dynamics between different times can be seen as with the returns distribution. In times under financial distress, the distribution of average downturn losses shifts downwards.

IV.8 Results of the regression analysis

The regression design follows a pooled ordinary least squares approach for panel data. Since I presented the theoretical foundations and considerations for the regression framework in chapter 3, this section focuses more on the overview of results and their possible interpretation, along with the conclusion about the relationship between realized losses and systemic risk contribution. After merging the appropriate variables from a certain time period to a data frame, a time variables is created to indicate which part does an observation belong to. The structure of the panel is the same for the networks based

on both the returns and volatility data. It contains the firm performance explanatory variable, five centrality variables created from the complete network and five centrality variables of the arborescence graphs.

Table 4 shows an organized structure of possible calculations for the second set of models. In each cell of the table, the corresponding adjusted R squared values and the differences in root mean squared errors are presented. These values are interpreted the following way: higher R squared values imply a better in sample fit on the existing data. It is explained usually as the ratio of variance in the Y variable explained by the variance in X. The root mean squared error is the metric that compares the predicted values of the out-of-sample data and the real values. These two measures are frequently used for model selection and complement each other: the adjusted R-squared measures the goodness of fit on the in sample data, while the root mean squared error measures the goodness of fit when predicting out-of sample data. Most research papers in systemic risk analysis try to find early warning indicators for economic downturns. A systemic risk variable is a good early warning indicator when it can predict the future losses of a firm. It requires a consistency of the perceived connection between the variables, since the goodness of fit must remain high in the data that is not part of the model fitting procedure. Dividing the data into test and fit parts serves this purpose.

Returns (left) and volatility network (right) results						
	adj. R^2	RMSE	ΔRMSE	adj. R^2	RMSE	ΔRMSE
Complete network	0.095	2.49	-	0.118	2.33	-
Only arborescene	0.176	2.18	-12.4%	0.308	2.17	-6.9%
Both sets of variables	0.198	2.25	-9.6%	0.294	2.10	-9.9%
All 10 variables	0.187	2.21	-11.2%	0.323	2.10	-9.9%

Table 6: Summary of the regression results for the second set of models, that are estimated for the first two periods and the connection is tested on the out of sample third period data.

In Table 4, the measures of model performance for multiple specifications is included for both the return- and volatility-based networks. It needs to be decided which variables are included in a regression. The first three are the following: including only the variables of the complete network, including only the arborescence variables and finally, including all

ten variables. The research question of this paper is whether the models augmented by the arborescence variables provide a better fit. Therefore the first specification is the base model, to which the other two are compared. The second model explores this relationship when the number of variables are constrained to three, so that the number of variables does not increase the predictive power by itself. The third regression is the full model, using every available information from both network structures, where the increase of even the adjusted R^2 might be partly caused by the number of variables.

Results show that the regression models including the arborescence variables outperform the base model by both measures. On the returns data, the base model adjusted R^2 is 0.09547. The RMSE of this model is 2.4923. The following results are compared to this base model. The only-arborescence equation provides a 0.1329 R^2 and a 2.1809 RMSE value. As for the hybrid model, the R^2 is 0.1982 with a RMSE of 2.2488. To summarize, these results on the returns network show that the models using arborescence methods outperform the base model by a considerable margin. The second model outperforms even the hybrid one, leading to a conclusion that arborescence variables are not only good as an extra set of information to improve prediction, but are by themselves stronger predictors of firm performance.

The results based on the volatility network follow similar trends. The theoretical proposition that volatilities tend to capture better the connections between firms and the financial risks seems to hold in this case, as the volatility network results generally provide a better in and out-of sample fit than returns networks. The three models are the same as in the previous case. Model one produces an R^2 of 0.1181 and a RMSE of 2.3321. The only-arborescence regression results in an R^2 of 0.3077 and a RMSE of 2.1746. The R^2 of the last model is 0.2936 and the RMSE is 2.0992. In conclusion, these results show that both the in sample fit and the out-of sample prediction is better for the volatility network. In contrast with the returns network, the best model regarding predictive performance is the third one, which contains both types of variables. However, the best fitting regression is the second specification, similar to the returns network.

	Dependent variable:			
	Perf			
	(1)	(2)	(3)	(4)
inStrength				0.002***
				(0.001)
outStrength				0.004***
				(0.001)
totalStrength	−0.0004		0.002***	
	(0.001)		(0.001)	
Btwn	0.0002		−0.001	−0.001
	(0.002)		(0.001)	(0.001)
Closn	−9.195		−11.446**	−20.088***
	(5.880)		(4.994)	(7.123)
totalDegree				0.197*
				(0.116)
totalStrength.1		−0.001	−0.003	−0.008**
		(0.002)	(0.002)	(0.003)
Btwn.1		−0.014*	−0.009	−0.018**
		(0.007)	(0.008)	(0.009)
Closn.1		−14.579***	−18.419***	−12.565**
		(5.558)	(5.550)	(6.149)
avgPath		−0.049**	−0.060***	−0.067***
		(0.022)	(0.023)	(0.023)
Constant	−3.959***	−2.619***	−2.103***	−1.898***
	(0.359)	(0.386)	(0.436)	(0.579)
Observations	133	133	133	133
R^2	0.066	0.332	0.386	0.416
Adjusted R^2	0.044	0.311	0.351	0.373
F Statistic	3.029**	15.927***	11.212***	9.716***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 7: Regression output on the variables of the returns network. This model is estimated on the panel data of all tree periods, describing the general explanatory power of the network variables. Columns 1 and 2 only contain the complete network and the arborescence variables respectively, while columns 3 and 4 contain both. Note: the total strength variable in column 4 is omitted due to being linearly dependent.

	Dependent variable:			
	Perf			
	(1)	(2)	(3)	(4)
totalStrength	0.00001		0.003***	
	(0.001)		(0.001)	
inStrength				0.003***
				(0.001)
outStrength				0.006***
				(0.001)
Btwn	0.001		−0.001	−0.002*
	(0.001)		(0.001)	(0.001)
Closn	−19.624***		−13.668***	−29.969***
	(5.337)		(4.855)	(7.442)
totalDegree				0.241**
				(0.105)
totalStrength.1		−0.002**	−0.004***	−0.008***
		(0.001)	(0.001)	(0.002)
Btwn.1		−0.004	−0.0001	−0.009
		(0.006)	(0.006)	(0.006)
Closn.1		−3.886	−11.802**	−9.703*
		(4.645)	(4.997)	(4.961)
avgPath		−0.073***	−0.073***	−0.074***
		(0.017)	(0.017)	(0.017)
Constant	−3.352***	−2.513***	−2.141***	−1.855***
	(0.280)	(0.316)	(0.325)	(0.378)
Observations	133	133	133	133
R^2	0.240	0.396	0.469	0.518
Adjusted R^2	0.223	0.377	0.439	0.482
F Statistic	13.613***	21.011***	15.763***	14.662***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 8: Regression output on the variables of the volatility network. This model is estimated on the panel data of all tree periods, describing the general explanatory power of the network variables. Columns 1 and 2 only contain the complete network and the arborescence variables respectively, while columns 3 and 4 contain both. Note: the total strength variable in column 4 is omitted due to being linearly dependent.

IV.9 Limitations of the regression analysis

Evidently, there are some limitations of the regression framework applied in this paper. Most issues arise with the application on the present dataset. First, the number of observations are lower than optimal for a robust regression analysis. Given the number of financial institutions, using panel data analysis techniques can fix this potential bias by multiplying the number of observations. Choosing the time dimension of the panel structure is not obvious however, as it can be divided for example yearly or into longer sections. I decided to use the same three periods of the whole time frame as in the rest of the paper.

The reason for following this strategy is that the paper is structured to show the different periods in the data. The differences in the characteristics of the networks and arborescence graphs are easier to show both visually and numerically on a smaller number of periods. The regression framework should follow this approach so that the results can be compared to the earlier parts of the analysis. The second reason is that the role of these regression models is to prove the early warning indicator function of the network variables, which can most easily be done using a distinct crisis period.

The theoretical limitations are for example the method of the estimation and choosing sets of variables for the analyzed regression specifications. The method was a simple Ordinary Least Squares linear regression approach, pooling the panel data from the first two period for the in-sample analysis. While more sophisticated regression methods exist, the number of observations and the perceived characteristics, for example the distribution, of the variables did not suggest using additional methods. Also, while model selection is an important part of regression analysis, and many methods exist to find the best fitting model to provide the best explanation for the connections between the variables. However, the focus of this paper was to determine whether the arborescence variables provide a better explanation of firm losses. This means that there is less focus for finding a set of variables that provides the best fit, instead, I checked how the results change when different variables are used, which however represent the same concepts, only in different network structures.

V Robustness analysis

In this section, I present briefly the results of a robustness analysis. This process aims to determine if the outcomes of the analysis change when some parameters of the whole estimation process change. For example, the parameters of the Granger causality estimation, the calculation of the dependent variable and the regression analysis can be changed as well. If the results do not change significantly when a set of parameters is altered, the results are considered to be robust to changing circumstances. In this paper, I only conduct a robustness analysis with changed δ parameters on the dependent variable, changing it to both -1% and -5%. As for other parameters and variables, I discuss possibilities that were not calculated here.

The detailed regression tables for the tested changes are displayed in Appendix C. Results show that both other specifications, the -5% and -1% variables hold approximately the same explanatory power (also discussed in Hué et al. (2019)). The former can be interpreted as a more strict definition of losses, not counting the smaller daily negative returns. The latter on the contrary serves as a more general approach to capture firm performance, including normal daily fluctuations as well. The differences in the explanatory power may occur mostly because of the differences in sample size. Since some firms did not have less than -5% returns through the examined period, these are not included in the regression. The number of observations for all three period was 133 in the original case, only 111 in the -5% case, while all firms were present in the -1% regressions, resulting in 135 observations. For a brief summary, the adjusted R^2 of the last specification (including all variables) on the volatility network changed from 0.482 to 0.428 in the -5% case and to 0.525 in the -1% case.

Other possibilities for robustness analysis include changing the estimation parameter of the Granger causality links or the GARCH volatility models. In the pairwise Granger causality estimation, the maximum lag parameter may change the strength of the connections in some cases. Higher lag dimensions mean that further past values are expected to have an influence on the present values of the series. The GARCH specification could also be changed from the baseline model of (1,1) to a generalized (p,q) formula, where multiple lagged values are considered. Similar to the Granger causality, higher maximum lag parameters imply a connection with further past values. Another possibility is to

change the dependent variable in the analysis: different attempts exist to determine firm performance, and other variables may capture different aspects of the performance. Multiple dependent variables are compared in Gandica et al. (2020). The above possibilities are not tested in this paper, and are considered to be possible improvements in the future.

VI Conclusions

In this paper I analyzed the properties of the financial network constructed from 45 North American and European banks, using a pairwise Granger causality approach to generate a directed and weighted graph. This network represents the interconnectedness of the financial market, which is a viable tool to analyze and measure systemic risks, as well as to draw conclusions and predictions regarding the performance of the firm. I used the concept of the maximum spanning arborescence to simplify the network and remove the noise by eliminating unnecessary edges. The process results in a directed tree graph, which can be interpreted as the most probable route of information transfer between the institutions - the information in this context is the realization of risks, the propagation of the financial shock, that is, the spread of low or negative returns. The analysis of systemic risk and firm performance is rooted in the research of Billio et al. (2012) and Hué et al. (2019).

Besides applying the arborescence method to financial networks, the aim of this paper is to expand the stream of research that starts from Billio et al. (2012), by exploring further properties of the network and finding improvements in its application. That is why the main focus of this paper is whether the characteristics of the simplified network explain the performance of firms better than the characteristics of the complete network. My hypotheses were the following: (1) the spanning arborescence graph has similar properties to the complete network, meaning that simplifying the network does not alter the structure significantly, which can be tested by the rankings of the institutions according to the different centrality measures; (2) that the centrality measures derived from the spanning arborescence provide better explanatory power on the general connection with firm performance, which was tested using a multivariate regression framework, comparing specification where either one or both sets of variables are included, and finally (3) that the characteristics of the arborescence network serve as better early-warning indicators

to foresee periods of economic downturn.

Out of the above 3 hypotheses, the first two are tested in this paper, while only a discussion of the 3rd matter is presented, due to constraints on data, space and time, with the possibility of concluding further research in the topic. As for the first two hypotheses, based on the present analysis, I concluded that both of them hold. For hypothesis 1, the comparison of the arborescence and complete networks can be seen on the figures displaying the centrality measures, as well as the tables of the rankings of institutions. Hypothesis 2 is tested in the two regression frameworks: in the first, the connection between network properties and firm performance is tested on all three periods, and it concludes that both the models with only arborescence variables and the ones with both sets of variables outperform the centrality measures of the complete network and provide a better fit. The second framework tests the stability of the estimated model, by dividing the data into two parts, the pre-crisis and crisis periods. Prediction of the firm performance variable on the 3rd period based on the model fitted in the pre-crisis period shows that both the only-arborescence and the mixed models provide a more stable estimation.

VII Further research

The existing limitations of the paper raise the possibilities of further research in this topic. In this final chapter, I highlight three aspects on which the analysis can be improved or continued. The first is the set of data that is used for the analysis: in network-based systemic risk analysis papers, such as in, the importance of data selection is emphasized. The reason for this is that to properly represent the financial system, a large set of institutions is more reliable, but more importantly, the set of institutions need to be more diverse with respect to both the location, size and type of firms. Firm size is important, because research suggests, that smaller firms can have a great impact on the system as well Diebold and Yılmaz (2014). The location of the institutions is important, since that way, the connectedness of regions can be observed. Ideally, firms should be included from all continents and from several countries, as it is done by Hué et al. (2019). Finally, the sector of the firm may be included, to show how different sectors in the financial market behave and connect to each other, a great example is Billio et al. (2012) or Yang et al. (2020) In this paper, such diverse set of financial institutions is not necessary for

introducing arborescence graphs. Each possible improvement of the data can be observed in existing research, but it never occurs combined, to represent as accurately as possible the financial network, perhaps because it gets more difficult to visualize and show results on a large set of institutions.

The second aspect is the methodology of the analysis. The first thing that comes to mind is the remaining hypothesis to be tested: whether arborescence network variables serve as a better early warning indicator for economic downturn. Both more data points and using new methods may help achieve that goal. In a similar data and regression framework that was used in this paper, a possible method is a rolling window analysis. It allows for a regression design where only a marginal set of observations of the dependent variable is estimated by a moving set of observations from before that period. Instead of the mostly unreliable estimation of between the independent variables of the pre-crisis period and the dependent variable of the crisis period, this method is able to estimate how the early warning connection between the two variables changes over time. Besides improving the regression analysis, another possibility is looking for other methods to describe the financial networks, other than spanning arborescence graphs. The problem with the arborescence in this case is that multiple ones can be generated based on the choice of the root node, and the method to choose one or aggregate the information of all was so far rather simple. Further research is possible by finding more complex network structures or methods that can aggregate the information of all arborescence versions.

The third aspect is the set of economic questions and methods that build on this type of analysis. An evident possibility is economic policy, given that the field of systemic risk analysis of the financial sector is closely connected to the macroprudential policy, which is conducted by central banks. One possible use case based on the arborescence is identifying the probable routes of shock propagation in the financial network, and another one is identifying systemically important institutions. One addition to the economic analyses based on financial networks could be the inclusion of other variables, for example different sectors of the economy. The purpose of this economic analysis would be to determine how the shocks from the financial system affect the whole economy, trough the connections to stock market indices for example.

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Appendix A: List of institutions

North American (left) and European (right) institutions			
Abbr.	Full name	Abbr.	Full name
BAC	Bank of America Corporation	BARC	Barclays PLC
BK	Bank of New York	BNP	BNP Paribas
BMO	Bank of Montreal	CSGN	Credit Suisse Group
BNS	Bank of Nova Scotia	DBK	Deutsche Bank
C	Citigroup	DNB	Dun & Bradstreet Holdings
CM	Canadian Imperial Bank of Commerce	GLE	Société Générale Société anonyme
COF	Capital One Financial Corporation	HSBA	HSBC Holdings
JPM	JPMorgan Chase	INGA	ING Groep N.V.
PNC	PNC Financial Services	ISP	Intesa Sanpaolo
RY	Royal Bank of Canada	KBC	KBC Group
STT	State Street Corporation	LLOY	Lloyds Banking Group
TD	The Toronto-Dominion Bank	RBS	Realia Business
TFC	Truist Financial Corporation	SAN	Banco Santander
USB	U.S. Bancorp	SEBA	Skandinaviska Enskilda Banken
WFC	Wells Fargo & Company	SHBA	Svenska Handelsbanken
BHBCQ	Beverly Hills Bancorp	STAN	Standard Chartered PLC
TMCV	Temecula Valley Bancorp	SWEDA	Swedbank AB
ACA	Arcosa Inc.	UBSG	UBS Group AG
		UCG	UniCredit
		GARAN	Turkiye Garanti Bankasi A.S.
		ING	ING (NY)
		OTP	OTP Bank
		PEO	Bank Polska Kasa Opieki SA
		QNBFB	QNB Finansbank
		SBER	Sberbank of Russia
		SPL	SPL

Table 9: List of institutions

Appendix B: Additional centrality plots

In this section, I present additional figures about the centrality measures of the complete volatility network. These plots do not contain important results of the analysis, I included them only to have a full set of centrality plots, at least for one version of the network, where all three distinct periods are covered and may be compared.

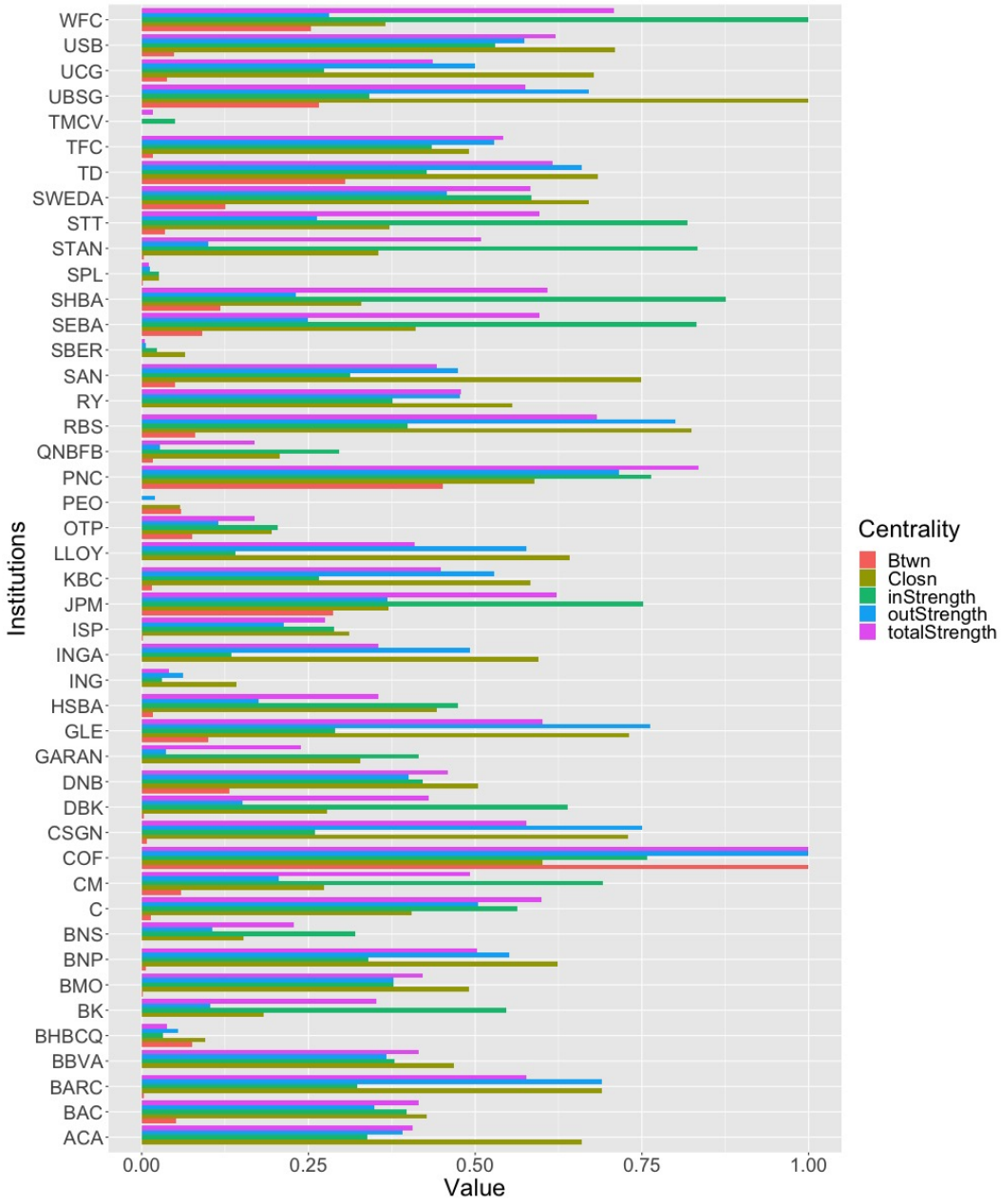

Figure 6: Centrality measures of the complete volatility network in the first period. The centrality measures included are betweenness, closeness, in, out and total strength of nodes. Note: all values are scaled to the [0,1] interval, where the maximum value of the variable takes 1 and the minimum takes 0.

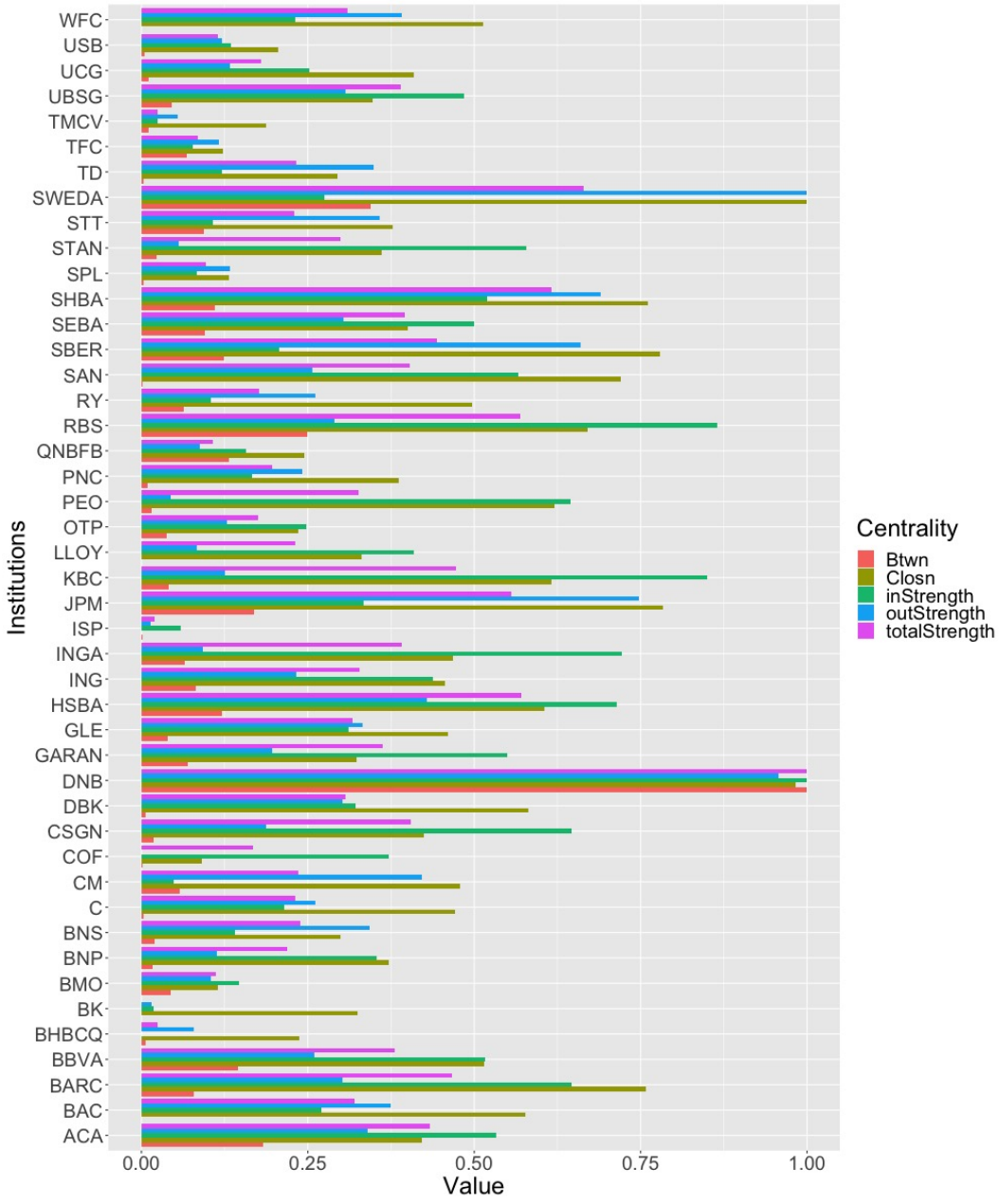

Figure 7: Centrality measures of the complete volatility network in the second period. The centrality measures included are betweenness, closeness, in, out and total strength of nodes. Note: all values are scaled to the [0,1] interval, where the maximum value of the variable takes 1 and the minimum takes 0.

Appendix C: Robustness analysis

In this section, I show how the results of the regression analysis change when the dependent variable is modified as the average losses below -5% and -1% returns.

	Dependent variable:			
	Perf			
	(1)	(2)	(3)	(4)
inStrength				0.002**
				(0.001)
outStrength				0.002*
				(0.001)
totalStrength	0.0001		0.002**	
	(0.001)		(0.001)	
Btwn	−0.00001		−0.0004	−0.001
	(0.002)		(0.002)	(0.002)
Closn	−20.636***		−16.373**	−19.486*
	(7.565)		(6.727)	(10.015)
totalDegree				0.177
				(0.200)
totalStrength.1		−0.002	−0.003	−0.007
		(0.002)	(0.003)	(0.005)
Btwn.1		−0.025**	−0.021**	−0.027**
		(0.010)	(0.010)	(0.012)
Closn.1		−11.163	−13.363*	−11.002
		(7.747)	(7.756)	(8.782)
avgPath		−0.070**	−0.072**	−0.077**
		(0.032)	(0.032)	(0.033)
Constant	−5.696***	−4.503***	−3.839***	−3.806***
	(0.455)	(0.591)	(0.651)	(0.853)
Observations	111	111	111	111
R^2	0.131	0.310	0.353	0.361
Adjusted R^2	0.107	0.284	0.309	0.304
F Statistic	5.393***	11.931***	8.041***	6.344***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 10: Full sample regression models on the returns network with modified dependent variable: the threshold for losses is changed to -5% from the original -3%

	Dependent variable:			
	Perf			
	(1)	(2)	(3)	(4)
totalStrength	−0.001		0.002*	
	(0.001)		(0.001)	
inStrength				0.002**
				(0.001)
outStrength				0.006***
				(0.002)
Btwn	0.001		−0.001	−0.003*
	(0.001)		(0.001)	(0.001)
Closn	−14.848**		−10.443	−33.522***
	(6.808)		(6.654)	(9.907)
totalDegree				0.425***
				(0.143)
totalStrength.1		−0.004***	−0.005***	−0.010***
		(0.001)	(0.002)	(0.002)
Btwn.1		−0.006	−0.002	−0.019*
		(0.009)	(0.009)	(0.010)
Closn.1		−5.621	−9.498	−6.445
		(5.956)	(6.720)	(6.516)
avgPath		−0.082***	−0.078***	−0.081***
		(0.023)	(0.025)	(0.024)
Constant	−5.610***	−4.369***	−4.125***	−3.736***
	(0.382)	(0.496)	(0.526)	(0.568)
Observations	111	111	111	111
R^2	0.226	0.367	0.388	0.475
Adjusted R^2	0.204	0.343	0.347	0.428
F Statistic	10.401***	15.340***	9.348***	10.140***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 11: Full sample regression models on the volatility network with modified dependent variable: the threshold for losses is changed to -5% from the original -3%

	Dependent variable:			
	Perf			
	(1)	(2)	(3)	(4)
inStrength				0.002***
				(0.001)
outStrength				0.003***
				(0.001)
totalStrength	−0.0004		0.002***	
	(0.001)		(0.001)	
Btwn	0.001		−0.0001	0.0001
	(0.001)		(0.001)	(0.001)
Closn	−7.796		−10.188**	−18.830***
	(5.266)		(4.320)	(6.140)
totalDegree				0.139
				(0.100)
totalStrength.1		0.001	−0.001	−0.006**
		(0.001)	(0.002)	(0.003)
Btwn.1		−0.012*	−0.008	−0.016**
		(0.006)	(0.007)	(0.007)
Closn.1		−15.172***	−18.418***	−12.858**
		(4.809)	(4.793)	(5.321)
avgPath		−0.046**	−0.055***	−0.063***
		(0.019)	(0.020)	(0.020)
Constant	−1.906***	−0.562*	−0.119	0.172
	(0.318)	(0.329)	(0.372)	(0.493)
Observations	135	135	135	135
R^2	0.062	0.372	0.424	0.452
Adjusted R^2	0.041	0.353	0.392	0.412
F Statistic	2.911**	19.284***	13.363***	11.435***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 12: Full sample regression models on the returns network with modified dependent variable: the threshold for losses is changed to -1% from the original -3%

	Dependent variable:			
	Perf			
	(1)	(2)	(3)	(4)
totalStrength	0.00003		0.003***	
	(0.001)		(0.001)	
inStrength				0.003***
				(0.001)
outStrength				0.006***
				(0.001)
Btwn	0.001		−0.001	−0.002**
	(0.001)		(0.001)	(0.001)
Closn	−18.086***		−11.626***	−28.760***
	(4.736)		(4.216)	(6.360)
totalDegree				0.215**
				(0.090)
totalStrength.1		−0.001	−0.003***	−0.007***
		(0.001)	(0.001)	(0.001)
Btwn.1		−0.002	0.001	−0.007
		(0.005)	(0.005)	(0.005)
Closn.1		−3.714	−10.969**	−8.576**
		(4.043)	(4.346)	(4.250)
avgPath		−0.070***	−0.071***	−0.072***
		(0.015)	(0.015)	(0.014)
Constant	−1.319***	−0.498*	−0.155	0.184
	(0.246)	(0.269)	(0.277)	(0.319)
Observations	135	135	135	135
R^2	0.251	0.427	0.498	0.557
Adjusted R^2	0.234	0.410	0.470	0.525
F Statistic	14.665***	24.233***	17.984***	17.472***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 13: Full sample regression models on the volatility network with modified dependent variable: the threshold for losses is changed to -1% from the original -3%