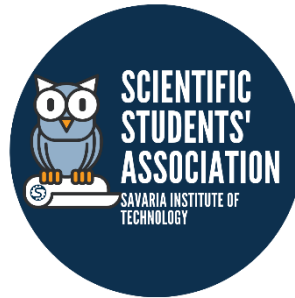




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SCIENTIFIC STUDENTS'S ASSOCIATION ESSAY

Modling the buckling of interphase spacers

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NOTATIONS

Latin letters

Notation	Name of the notation	Unit
A_{ice}	Area of the ice formed around the cable (Circular CS)	mm^2
$\mathbf{C}$	Damping matrix	1
d_c	Cable outer diameter	m
$E_{(p)}$	Complete Elliptic integral of the second kind	1
f_{antis}	Eigenfrequency of Antisymmetric in-plane motion	Hz
f_{symm}	Eigenfrequency of Symmetric in-plane motion	Hz
f_{trans}	Eigenfrequency of Transverse horizontal motion	Hz
F_h	Horizontal tensile force based on parabolic equation	N
F_H	Horizontal tensile force based on catenary equation	N
g	gravitational acceleration (9.81)	m/s^2
h	Sag of the cable	m
H	Added horizontal tensile force	N
I	Moment of Inertia	m^4
$\mathbf{K}$	Stiffness matrix	1
K	Effective length factor	1
$K_{(p)}$	Complete Elliptic integral of the first kind	1
l_0	Effective length	m
L	Span length	m
m_0	Mass per unit length	kg
$\mathbf{M}$	Mass matrix	1
P	Ice Load per unit span	N/m
P_{Euler}	Euler Buckling Load	N
w_0	Weight per unit length	N
w	Added vertical displacement (sag)	M
$X_{a(Buck.IPS)}$	Total length of IPS after deformation	m
y_a	Horizontal deformation of IPS	m

Greek letters

Notation	Name of the notation	Unit
α_{Euler}	Maximum Euler Angle	°
α_{mass}	Mass coefficient	1
β_{Stiff}	Stiffness coefficient	1
κ	Curvature of a beam	1/m
ω_i	Natural circular frequency	rad/sec
ξ_i	Damping ratio	1

Indexes, exponents

Notation	Name, explanation
FRP	Fiber-reinforce plastic
IPS	Interphase spacer

1. INTRODUCTION

1.1. Problem

Interphase Spacers (IPSs) are widely used in the power transmission system. They are made of a special composite of fiber-reinforced plastics (FRP), and the IPS is implemented between the different phases of transmission lines to prevent the danger of flashovers. Flashovers in the transmission lines may occur due to the excessively high amplitude vibrations, that are developed due to ice shedding from one of the conductors or as a result of galloping which is one form of wind-induced vibration that is characterized by high amplitudes and low frequencies. Due to this vibration, a high axial stress arises in the IPS which may cause it to buckle. The designer should calculate the axial loads during conductors' vibrations to be able to correctly design the IPS and choose the required number of spacers in a specific span length. Since the suspended conductors are loaded by their weight, their profile will be in a form of a catenary curve, and the horizontal tensile force will be calculated based on the catenary equation. This tensile force increases during ice formation, and it varies during vibration that follows the shedding of ice chunks.

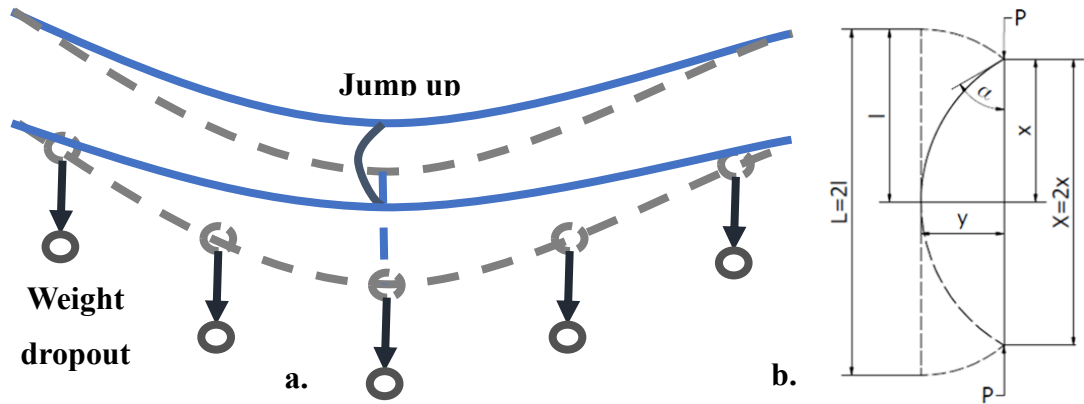


Figure 1: a. Typical set-up of snow dropout test. b. buckling model of IPS

1.2. Aims and Overview

This project aims to study the buckling of IPSs as the result of the dynamic forces acting on them during conductor vibration following ice shedding as represented in Figure 1. For reaching this goal, three different approaches will be investigated as follows:

- Experimental approach
 - Material tests on samples from the FRP used in the manufacturing of IPS
 - Buckling test

- Analytical approach
 - Defining all the equations that are needed in the literature review
 - A need for well-defined equations
 - Program an application based on these equations using MATLAB app designer
- Numerical approach
 - Study the vibration modes of the conductors
 - Simulate the vibration of two conductors that are separated by two IPSs
 - Study the buckling behavior of the IPS and compare it with the experimental
 - Simulate the ice-shedding loads on the IPS in a buckling model

At the begin of the investigation of IPS, material tests must be conducted to obtain the properties of the special FRP material used by the manufacturer, which will be used in numerical simulations. Additionally, buckling tests will be carried out in the laboratory of the Savaria Institute of Technology. These experiments will allow us to compare the results with the numerical and analytical models that we have previously developed by the author [1].

Meanwhile, an application named "Conductors' Vibration and IPS Buckling" was developed using MATLAB's App Designer. The purpose of the application is to perform all necessary analytical calculations and compare them with experimental results and also to build and validate numerical simulations.

It is also important to understand the static and dynamic behavior of the transmission line conductors and to study them, a few points are assumed of the cable's behavior. Those assumptions are represented in the following points:

- The conductors are uniform extensible cables.
- Conductors are hung between two fixed points on the same level.
- Cable elements are perfectly flexible.
- Cables can sustain only tensile forces.

As James Bernoulli said, “the action of any part of the line upon its neighbor is purely tangential” [2], which means that the cables suffer only from the tangential force in between the cable's elements. Moreover, the conductors are suspended on their weight, which will cause them to form a catenary profile curve [3].

2. LITERATURE REVIEW

2.1. Suspended cables

2.1.1. Static equilibrium of the cable

Suspended cables are now a mandatory thing in our everyday life, as they are the main method to transmit electricity over long distances, also they exist almost in every country in all around the world. We can indeed transmit the electricity via underground cables, but in this case, the cost will be 7-10 times higher [4]. The structure of the overhead transmission line conductors consists of a group of wires twisted around each other forming the cable. Those cables are not insulated because of two main reasons. The first reason is the weight of this insulating material, which accordingly will affect the tensile force needed to hold the cable meaning also that the towers will have to be designed for higher loads. While the second reason is regarding the maintenance of this material, as they will be strongly affected by the environmental effects; like sun, rain, or even the wind [4, 5].

Firstly, we must study the behavior of the static suspended cables under their weight as they are hanged from two endpoints in the transmission towers, which are well designed to support the weight of the cables with a specifically required sag. The cable length must be more than the span length [3] and be pre-tensioned. This tension force is obtained based on the catenary profile that the suspended cable forms, or if the ratio between the sag and the span length is very small (i.e. less than 1:8, and consequently the span length is very close to the cable length), the cable profile can be closely approximated by a parabolic curve profile [6, 7]:

$$F_h = w_0 \cdot L^2 / 8 \cdot h, \quad (1)$$

$$h = \frac{F_H}{w_0} \cdot \left(\cosh \left(w_0 \cdot L / 2 \cdot F_H \right) - 1 \right), \quad (2)$$

$$S = \frac{2 \cdot F_H}{w_0} \cdot \sinh \left(\frac{w_0 \cdot L}{2 \cdot F_H} \right), \quad (3)$$

$$\varepsilon = \frac{\sigma}{E} = \frac{F_H}{E \cdot A_C} \quad (4)$$

Equation 1 and Equation 2 both determine the required tensile force to suspend the cable under its weight, but each will result in a different profile, as the first equation refers to a parabolic curve profile while the last one refers to the catenary profile. The difference between their results is approximately just 0.1 %, but for precision, we will proceed with the catenary profile equation.

The weight per unit length of the cable (w_0) is considered as the load of the system and can be represented as:

$$w_0 = m_0 \cdot g \quad (5)$$

While the tension in the cable is shown as (T) and its horizontal component is represented for the parabolic and the catenary profiles as:

$$F_h \approx F_H = T \cdot \cos \vartheta \quad (6)$$

Also as represented in the Freebody diagram of Figure 2. Equation 3 can calculate the total length of the suspended cable forming the required sag, this length is obtained for the static state of the system after applying the pretension force. This tensile force will indeed cause elongation in the cable, and this elongation can be represented as a pre-strain in the conductor. This pre-strain can be calculated in Equation 4 based on Hook's law. Those equations will be needed to design the cables and the supports in the system, and also to construct the numerical model using the ANSYS workbench.

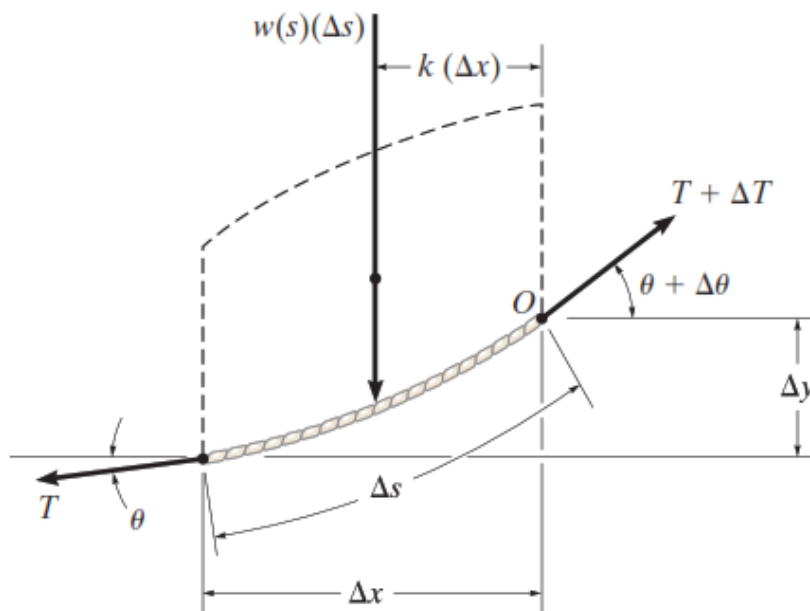


Figure 2: The Freebody diagram for a small segment Δs of a suspended cable [6]

2.1.2. Vibration modes of the conductors

Cables may vibrate due to many different reasons, and the different modes of vibrations will be discussed in this section. The natural circular frequency of the cable is based on the structure and the material of the cables, and those frequencies can be calculated after solving the equations of motion [7]. Natural circular frequency can be determined for motions in two planes, firstly in a transverse horizontal motion and secondly in the in-plane motion, which is divided into antisymmetric and symmetric modes.

i. Transverse horizontal motion

In transverse horizontal motion, for the first mode, there is no additional cable tension induced by the motion. This is consistent with experience since, for a hanging chain, the first swinging mode is the easiest to be excited. Therefore, if the disturbance has no in-plane component, it will cause only transverse horizontal motion. The natural frequency can be obtained using the following equation [7]:

$$\omega_{Trans}(n) = \frac{\pi n}{L} \sqrt{\frac{F_H}{m_o}}, n = 1, 2, 3, \dots, \quad (7)$$

It is known that the first transverse horizontal mode ($n = 1$) is associated with the lowest natural frequency of any given parabolic cable.

ii. In-plane motion

The in-plane motion has two forms, either antisymmetric vertical component and symmetric longitudinal components, and is called antisymmetric in-plane motion, or the opposite, symmetric vertical components, and antisymmetric longitudinal components which are called symmetric in-plane motion.

a. Antisymmetric in-plane motion

As it was just stated, the antisymmetric in-plane motion has a symmetric longitudinal component but an antisymmetric vertical component, and there is no additional tension in the cables. The natural frequency can be obtained by the following equation:

$$\omega_{Antis}(n) = \frac{2\pi n}{L} \sqrt{\frac{F_H}{m_o}}, n = 1, 2, 3, \dots, \quad (8)$$

We can see that the natural frequency of this antisymmetric in-plane motion is twice as much as that in the case of the transverse horizontal motion as shown in Figure 3.

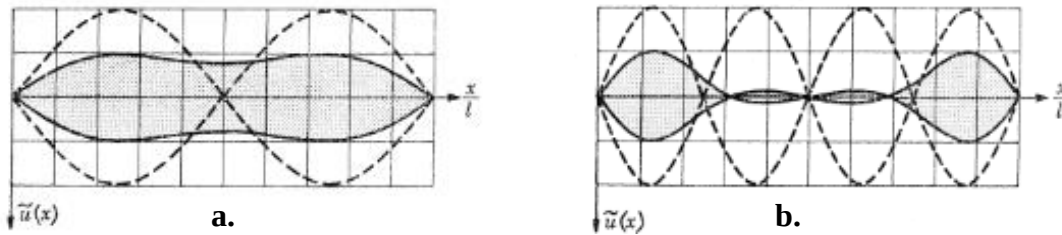


Figure 3: Longitudinal components and associated vertical components of the first two antisymmetric in-plane motion
a. First symmetric longitudinal component. **b.** Second symmetric longitudinal component. [7]

b. Symmetric in-plane motion

Here in the case of the symmetric in-plane motion, there is additional tensile force arising in the cable, and in this case, the longitudinal component is asymmetric while the vertical component is symmetric. We can also calculate the natural circular frequency based on the following equations [7]:

$$\tan\left(\frac{1}{2}\beta \cdot L\right) = \frac{1}{2}\beta L - (4/\lambda^2) \cdot \left(\frac{1}{2}\beta L\right)^3, \quad (9)$$

$$\lambda^2 = \left(\frac{8 \cdot d}{l}\right)^2 \cdot \frac{L}{(F_h \cdot L_e)/(E \cdot A_c)}, \quad (10)$$

$$L_e = L \cdot \left(1 + 8 \cdot \left(\frac{h}{L}\right)^2\right), \quad (11)$$

$$\text{If } \lambda^2 > 4\pi^2 \quad \beta_{1,2} = \frac{(2.86 \cdot \pi)}{L}, \frac{(4.92 \cdot \pi)}{L}, \beta_n \cong \frac{((2 \cdot n + 1) \cdot \pi)}{L}, n = 3, 4, 5, \dots \quad (12)$$

$$\text{Else,} \quad \beta_n = \frac{((2 \cdot n - 1) \cdot \pi)}{L}, n = 1, 2, 3, \dots \quad (13)$$

$$\omega_{\text{Symm}(n)} = \beta_n \cdot \sqrt{\frac{F_H}{m}} \quad (14)$$

It is seen that λ^2 , is an important parameter that involves the geometry and the elasticity of the cable, and it has a great influence on the roots of Equation 9.

2.2. Vibration types of the conductors

Some natural phenomena are happening on the transmission lines, which leads to different kinds of vibration. Those phenomena include the wind, ice shedding, or the impact of any object. Line compaction is primarily based on the principle of minimizing interphase spacing. As a result of this, there is a greater risk of interphase flashover with a compact design when two conductors are moved close to one another, for example, because of wind or ice shedding that might cause a short circuit; this may lead to an interruption in service. Different methods are used to prevent those flashovers, one of those methods involves the introduction of IPSs between the phases to secure the gap distance between them.

2.2.1. Ice formation and behavior of the system under the ice load

One condition of galloping is the asymmetric cross-section of the conductor, which is a consequence of the formation of ice over the conductors. Supercooled droplets or snowflakes may adhere to the conductor surface, forming a base for the large ice chunks or snow sleeves to accrete on it [8]. Ice or snow will act as extra loads on the system, as a result of this load, the conductors will suffer from extra tension force and then the towers will require extra force to overtake this load, consequently, the system will have extra vertical deformation which is considered as extra sag [3]. Referring to Figure 4 by applying that $X_2 = 0$ and $X_3 = L$, we get the following simplified equations for the additional tension and vertical displacement due to distributed load:

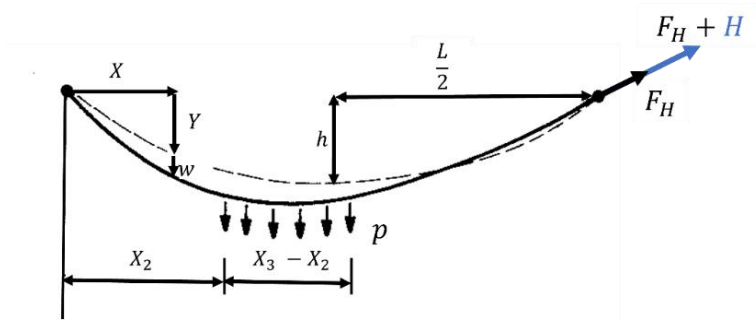


Figure 4: Uniformly distributed load on a conductor [3].

$$H^3 + \left(2 + \frac{\lambda^2}{24}\right) \cdot H^2 + \left(1 + \frac{\lambda^2}{12}\right) \cdot H - \left(1 + \frac{P}{2}\right) \cdot \frac{\lambda^2}{12} \cdot P = 0, \quad (15)$$

$$w = \frac{1}{2(1+H)} \cdot \left(1 - \frac{H}{P}\right) \cdot x \cdot (1-x), \quad (16)$$

where; $H = \frac{H}{F_H}$, $P = \frac{p}{m_o \cdot g}$, $w = \frac{w}{PL^2/F_H}$ and $x = \frac{X}{L}$.

2.2.2. Ice shedding and the sleet jump

Shedding of ice accreted on transmission-line cables is a chronic problem, especially in cold regions. Ice shedding may have similar consequences to galloping as it can lead to a tower arm failure or cascade failure of several towers in transmission lines as there is a high dynamic load induced in the lines. The ice or snow that is deposited on the conductor will lead to an increase in the conductor sag and the strain energy stored in the conductor. When this load drops from the conductors, the strain energy will be converted into kinetic energy and then as potential energy until it reaches its maximum height.

This phenomenon is called ‘sleet jumping’, the amplitude of the jump can be higher than the clearance allowance between the two phases, which will cause flashovers.

Moreover, after the ice shedding the conductor form a new static equilibrium with less sag, and if the upper conductor is still not shed yet, it might cause an air gap breakdown between the phases that will cause a closed circuit between the different phases [8]. To solve the problem of the possibility of air gap breakage, we introduce interphase spacers between the conductors, as it is shown in Figure 1.

2.3. Interphase spacers ‘IPS’

As it was mentioned in Section 2.2 the line design is required to be as compact as possible for increasing the transmission capability of the line due to lower surge impedance, which is the ratio of the amplitudes of voltage and current of a single wave propagating along the line; that is, a wave traveling in one direction in the absence of reflections in the other

direction. In addition, it reduces the electric and magnetic fields at the edges of the line corridor so that it is not expected to have any harmful effect on the environment [9].

Due to those vibrations, flashover may occur between the phases causing short circuits. This is a very dangerous case and might lead to a cut in the electric current and fires. To prevent such damages due to galloping and ice shedding, IPSs are introduced between the phases, which will ensure the clearance between those cables. During those vibrations, a very high axial load arises in the IPS, which may lead to buckling, and for that reason, the IPS needs to be designed very carefully considering the high axial loads. This design is based on the structure that also determines how many IPSs are used in a span with a specific length.

Figure 5 shows an IPS suffering from a buckling phenomenon. The old IPS were made of porcelain which is characterized by low breaking strain that leads to a catastrophic failure of the IPS as soon as the compression force exceeds the Euler load. Nowadays the IPSs are made of composite materials that can withstand loads higher than the Euler's load. They can do that as they have high breaking strain energy due to the fiber-reinforced plastic 'FRP'.



Figure 5: Highly deformed IPS [9].

2.3.1. Deflection of Beams

We need to study the buckling behavior of the IPS, which will require an explanation for the elastic curve of a beam that represents the axis of deflected beams as shown in Figure 6 (a). The aim is to find a relationship between the curve equation and the curvature of the beam after that we will be able to define the equation of the elastic curve [10].

We will consider the section *cdfe* from the beam in Figure 6 (a), which is subjected to a pure bending moment M , this bending moment will cause the fibers above the neutral axis of the beam to be tensioned and elongated while the fibers under the neutral axis will be compressed and shortened.

Let O be the center and R be the radius of curvature which is reciprocal to the curvature of the beam $\kappa = \frac{1}{R}$, also let ij be the neutral axis of the curved beam that is included in the angle θ , and let σ be the stress in the filament gh that is at distance y from the neutral axis.

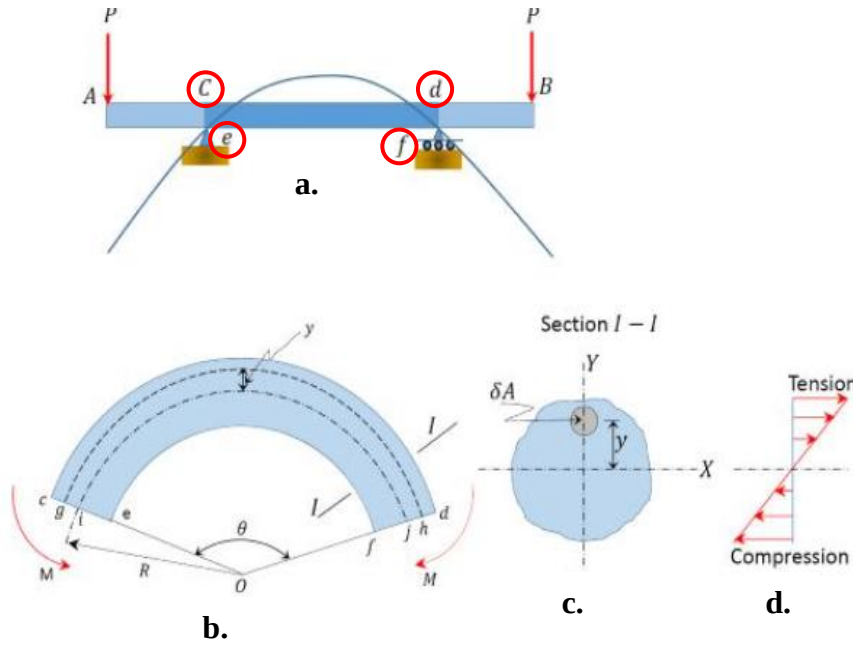


Figure 6: The elastic curve of a beam [10]

Based on the geometry, the original length of the beam L_{beam} , is the length of the neutral axis ij , and that of the elongated filament gh can be computed as follows:

$$L_{beam} = ij = R\theta \text{ and } gh = (R + y)\theta$$

$$\Delta L_{beam} = gh - ij = (R + y)\theta - R\theta \rightarrow \Delta L_{beam} = y\theta$$

This means that the strain in this filament can be described as:

$$\epsilon_{gh} = \frac{\Delta L_{beam}}{L_{beam}} = \frac{y\theta}{R\theta} = \frac{y}{R} \quad (17)$$

For a linear elastic material, we can apply Hooke's law as follows:

$$\epsilon_{gh} = \frac{\sigma}{E} = \frac{y}{R} \quad (18)$$

The external moment must be balanced by the internal forces causing a moment around the neutral axis to reach the static equilibrium state, by applying the Navier formula to find the bending stress:

$$\sigma = \frac{M}{I} \cdot y \quad (19)$$

From Equations (17-19) we get that the curvature of a beam κ is:

$$\kappa = \frac{1}{R} = \frac{M}{I \cdot E} \quad (20)$$

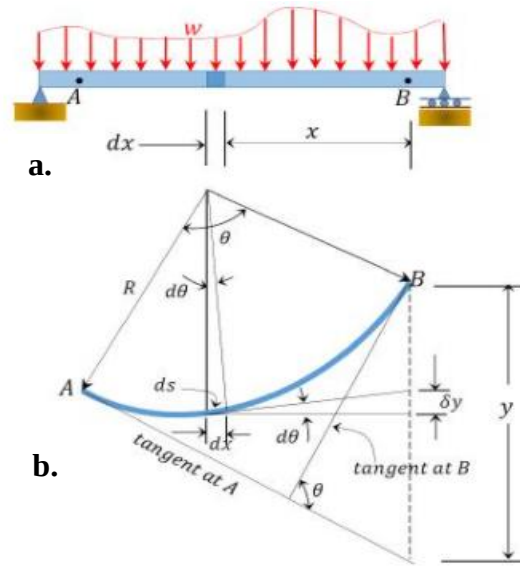


Figure 7: Deflected beam [10]

Figure 7 shows a deflected beam due to a distributed load whereas in Figure 7 (b) we consider the portion AB of the elastic curve of that beam which has a radius of curvature R that is subtending an angle $d\theta$. Based on the geometry the length of the arc ds can be defined as:

$$ds = R \cdot d\theta \rightarrow R = \frac{ds}{d\theta}, \quad (21)$$

and

$$ds \approx \sqrt{dx^2 + dy^2}, \quad (22)$$

$$ds \approx dx \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2}, \quad (23)$$

Geometrically,

$$\frac{dy}{dx} = \tan \theta, \frac{dy}{dx} = y', \quad (24)$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (\tan \theta) \rightarrow y'' = (1 + \tan^2 \theta) \cdot \frac{d\theta}{dx}, \quad (25)$$

$$d\theta = y'' \cdot \frac{dx}{(1 + \tan^2 \theta)} \quad (26)$$

From Equations (21-26) we get the radius of curvature R as:

$$R = \frac{(1 + (y')^2)^{3/2}}{y''} \quad (27)$$

And from Equation (20, 27) we get the equation of the curvature of the curve as:

$$\kappa = \frac{1}{R} = \frac{y''}{(1 + (y')^2)^{3/2}} = \frac{M}{I \cdot E} \quad (28)$$

Since $y_{(x)}$ is the deflection function of the elastic curve, thus, $y'(x)$ is the angular function of the elastic curve. Thus, at small displacements where $\theta < 1^\circ = 0.02 \text{ rad}$ Equation 26 which is the equation of the elastic curve, can be rewritten as:

$$\kappa = \frac{1}{R} = y'' = \frac{d^2y}{dx^2} = \frac{M}{I \cdot E} \quad (29)$$

2.3.2. Buckling Behavior of beams and IPS

2.3.2.1. Euler's Column Formula

The behavior of an ideal column can be used to obtain the critical Euler load of a compressed bar. The ideal column is assumed to be a perfectly straight, elastic, and centrally compressed rod, as shown in Figure 8 (a). As the load P is less than the critical load P_{Euler} , the rod remains straight and suffers only an axial compression force. This elastic equilibrium state of the straight form is stable, which means if there will be any small deflection happens due to a lateral force, this deflection must disappear by removing the lateral force and the rod comes back to its straight form. The other condition is if the P increased so that the elastic equilibrium state will be unstable, and by applying a lateral force a deflection will happen that doesn't disappear by removing this lateral force. This critical Euler load " P_{Euler} " is the load that is enough to keep the rod in such a bent form as in Figure 8 (b) [11].

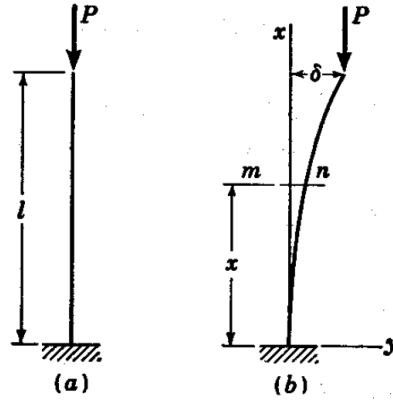


Figure 8: Fixed-end Slender Ideal Column under axial load [11]

The differential equation of the elastic curve in Equation 29 can be used to calculate the critical Euler load P_{Euler} . If we consider the coordinate system of the beam as in Figure 8 (b) and consider the beam slightly deflected, we get the moment at any section mn as follows:

$$M = -P(\delta - y) \quad (30)$$

And the differential Equation 29 can be:

$$y'' = \frac{P}{E \cdot I} \cdot (\delta - y) \quad (31)$$

Which can also be written as

$$y'' + k^2 y = k^2 \delta \quad (32)$$

In which,

$$k^2 = \frac{P}{E \cdot I} \quad (33)$$

The general solution of that differential equation is

$$y(x) = A \cdot \cos kx + B \cdot \sin kx + \delta, \quad (34)$$

Where A and B are integral constants that can be determined from the initial conditions as from the lower end: $y_{(x=0)} = 0$ & $\left. \frac{dy}{dx} \right|_{x=0} = 0$, it can be fulfilled if $A = -\delta$ and $B = 0$.

Then Equation 34 can be: $y_{(x)} = \delta \cdot (1 - \cos kx)$ (35)

While at the upper end, the condition is $y_{(x=l)} = \delta$ that can be satisfied if:

$$\delta \cdot \cos kl = 0 \quad (36)$$

Equation 36 requires one of two situations to be true. First, if $\delta = 0$ which means that there is no buckling as the bar in Figure 8 (a), while the other situation if $\cos kl = 0$ that would give us a relation for k as:

$$k \cdot l = (2n - 1) \cdot \frac{\pi}{2} \text{ (where } n = 1, 2, 3, \dots) \quad (37)$$

And based on Equations (33, 37) to get the smallest critical load we substitute $n = 1$ and get:

$$k \cdot l = l \cdot \sqrt{\frac{P}{E \cdot I}} = \frac{\pi}{2}$$

So, we get the equation for the critical Euler force as

$$P_{Euler} = \frac{\pi^2 \cdot E \cdot I}{4 \cdot l^2} \quad (38)$$

This critical load depends on the end condition of the beams, and it can be obtained from the solution of each case. In the case of the IPS where both ends are hinged, the beam under compression can be represented by the model in Figure 9 which is called the fundamental case, at which by the similarity of the member, the beam can be divided into two halves, each is the same as the entire bar represented in Figure 8 (b). So, the critical load can be described as:

$$P_{Euler} = \frac{\pi^2 \cdot E \cdot I}{l_0^2} \quad (39)$$

Where l_0 is the effective length of compression members $l_0 = K \cdot l$ depends on the end condition of the member and can be defined based on reference [12].

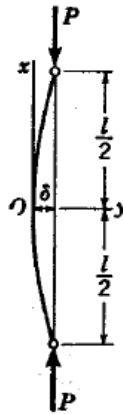
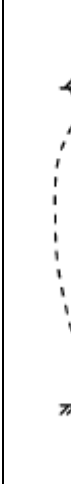
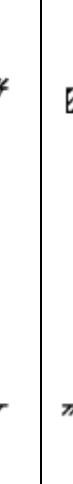
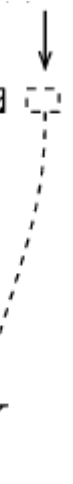


Figure 9: IPS model as a bar with hinged ends

Table 1: Theoretical and recommended effective length factor [K] for isolated column with idealized end conditions [12]

The buckled shape of a column is shown by the dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value	0.65	0.80	1.2	1.0	2.10	2.0
End condition code		Rotation fixed; translation fixed				
		Rotation free; translation fixed				
		Rotation fixed; translation free				
		Rotation free; translation free				

In all cases, the column was considered to be totally free to buckle in any planner direction, thus, $E \cdot I_{min}$ will represent the smallest flexural rigidity, but if this condition is not fulfilled and the column is free to buckle in only one plane, then we consider the $E \cdot I$ of that plane.

The critical compressive stress for buckling can be calculated if we just divide the force obtained from Equation 39 by the cross-section of the beam, also we consider the r which is the minimum radius of gyration that can be defined as:

$$r_{min} = \sqrt{\frac{I_{min}}{A}}, \quad (40)$$

And λ which is the slenderness ratio:

$$\lambda = l_0 / r_{min} \rightarrow \lambda^2 = l_0^2 \cdot A / I_{min} \quad (41)$$

So, the critical compressive stress for buckling will be:

$$\sigma_{cr} = \frac{P_{Euler}}{A} = \frac{\pi^2 \cdot E \cdot I}{A \cdot l_0^2} \rightarrow \sigma_{cr} = \frac{\pi^2 \cdot E}{\lambda^2} \quad (42)$$

2.3.2.1. Large deflection of buckled bars (The Elastica) buckling of IPS

The modern interphase spacers with composite insulators have shown a great ability to afford compressive load greater than the critical Euler Load. This endurance is due to the high breaking strain of their FRP. Thus, we need to understand the behavior of the IPS once the load on them exceeds Euler's load.

Several studies have been carried out to better understand the behavior of IPS under buckling, as a straight stiff rod of length L , modulus of elasticity E and the minimum moment of inertia I_{min} is subjected to a compressive force P which is undergone a remarkable deflection. However, the deformation can happen only in the direction of the undeformed rod since both ends are pinned as shown in Figure 10.

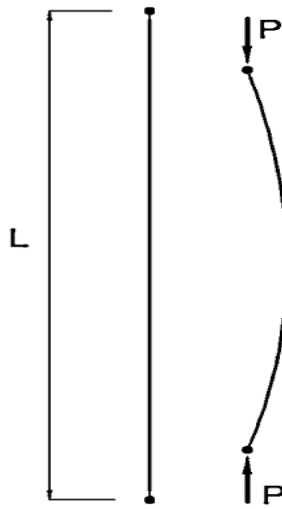


Figure 10: Static system and load of a slender IPS [9].

The effect of the Euler buckling load is to be analyzed concerning the deformed shape of the structural member, which is the rod curvature, having $y(x)$ as an expression describing the deformed shape. Considering the bending moment caused by the compressive force P in location x and the critical value of P , we get the differential equation of the curvature in the case of buckling which is similar to the conventional Euler–Bernoulli approximation that was explained in Section 2.3.1 and described in Equation 29 that can be written as:

$$y'' = \frac{d^2 y(x)}{dx^2} = -\frac{P_{crit}}{E \cdot I_{min}} y(x) \quad (43)$$

The following analysis is based on Leonard Euler's study, who, taking into consideration the symmetricity conditions, considered the examination of the deflection only on one half of the buckling rod as described before in Section 2.3.2.1 and so the l in Figure 11 represents only half the total length of the IPS rod in Figure 10 [9] so, we get $l = L_{IPS}/2$

In Figure 11 The expression of the curvature to estimate the exact value of the rod deflection during buckling $\kappa_{(x)} = \frac{d^2y}{dx^2}$ is obtained from Euler's differential equation of buckling.

$$\ddot{y} + k^2 y_{(x)} = 0 \quad (44)$$

where k^2 is the same as mentioned in Equation 33.

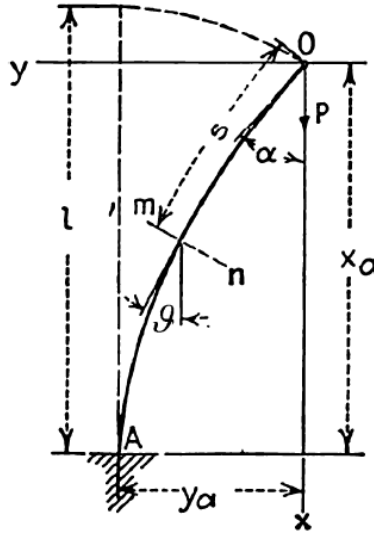


Figure 11: Parameters definition of the compressed rod [11].

By using the notations shown in Figure 11, we can re-write the exact differential equation to describe the rod deflection during buckling as [9]:

$$E \cdot I_{min} \cdot \frac{d\theta}{ds} = -P \cdot y_{(s)}, \quad (45)$$

where: $\kappa_{(x)} = y''_{(x)} = \frac{d^2y_{(x)}}{dx^2}, \kappa_{(s)} = \frac{1}{R}, R = \frac{ds}{d\theta} \rightarrow \kappa_{(s)} = \frac{d\theta}{ds}$ (46)

After the substitution of Equation 46 in Equation 45 and taking into consideration that $\frac{dy}{ds} = \sin(\theta)$ we got the exact differential equation provided by Leonard Euler and by derivation of Equation 45 concerning ds we get:

$$EI_{min} \frac{d^2\theta}{ds^2} = -P \cdot \sin(\theta) \quad (47)$$

The solution of this differential equation provides the Euler angle ' α_{Euler} ' that is the same differential equation that describes the pendulum oscillation, which is known as the Kirchhoff analogy. Elliptic integrals [13] will be required to solve the following calculations that are discussed in detail in [11]. By solving this differential equation, and according to the boundary conditions concerning the lateral displacement caused by the buckling, we define new parameters which are the Euler angle α_{Euler} and another parameter p such as:

$$p = \sin \frac{\alpha_{Euler}}{2} \quad (48)$$

In which α is the maximum Euler angle. This means that we can obtain the length of the rod as:

$$l = \frac{1}{k} \cdot \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-p^2 \cdot \sin^2 \phi}} \rightarrow l = \frac{1}{k} \cdot K(p) \quad (49)$$

where $\sin \frac{\theta}{2} = p \cdot \sin \phi = \sin \frac{\alpha}{2} \cdot \sin \phi$

Considering $k = \sqrt{\frac{P}{E \cdot I_{min}}}$ we can rewrite Equation 49 to find the load that is required to

reach any Euler angle α_{Euler} as:

$$P_{(p(\alpha))} = (K(p))^2 \cdot \frac{E \cdot I_{min}}{l^2} \quad (50)$$

Where at small deflections p and α will be very small and can be neglected, and from the equation and we can get the same Critical Euler load as in Equation 38.

From the following parameters and after several calculations, we can get that the maximum deflection in the center of the rod is given as [11]:

$$y_a = \frac{2p}{k} \quad (51)$$

And from Equation 49 we have that $k = \frac{K(p)}{l}$, So, Equation 51 can be represented as:

$$y_a / l = \frac{2p}{K(p)} \rightarrow y_a = \frac{2p}{K(p)} \cdot l \quad (52)$$

While the resultant length x_a caused by the compression in only half of the IPS rod is [11]:

$$x_a = \frac{2}{k} E(p) - l, \quad (53)$$

That can be represented based on Equation 49 as:

$$x_a / l = 2 \cdot \frac{E(p)}{K(p)} - 1 \quad (54)$$

And the total IPS length after buckling is:

$$X_{a(Buck.IPS)} = \left(2 \cdot \frac{E(p)}{K(p)} - 1 \right) \cdot 2 \cdot l \rightarrow X_{a(Buck.IPS)} = \left(2 \cdot \frac{E(p)}{K(p)} - 1 \right) \cdot L_{IPS} \quad (55)$$

$E(p)$ is the complete elliptic integral of the Second Kind.

As shown in Table 2, the value of the compressive force is dependent on both the Euler angle and the deflection on one of the pinned ends of the buckling rod.

Table 2: The relationship between the compressive load P and the deformation of the buckling rod in different directions is to be studied with the variation of the value of the Euler Angle: [11]

α_{Euler}	0°	20°	40°	60°	80°
P/P_{Euler}	1	1.015	1.063	1.152	1.293
x_a/l	1	0.97	0.881	0.741	0.56
y_a/l	0	0.22	0.422	0.593	0.719

Since there is a buckling and a horizontal deformation, there will be bending stress arises on the IPS and the maximum bending stress must be less than the maximum bending strength of the FRP core and it is deducted from the same calculations:

$$\sigma_{b,max} = \frac{P_{Euler} \cdot y_a}{I_{min}} \cdot \left(\frac{d_{IPS}}{2}\right) \quad (56)$$

Consequently, the designed IPS can endure compressive forces that are 30% greater than the Euler buckling load without any sign of permeant deformation and that is approved by the experiments whose results are shown in Table 2 at the maximum Euler angle $\alpha_{Euler} = 80^\circ$. Although the consideration of the maximum buckling force we consider just the Critical Euler Load that is calculated at $\alpha_{Euler} = 0^\circ$ or directly from Equation 39. This makes sure that there is still a load reserve according to a core diameter which assures us that the post-buckling behavior of the interphase spacer is stable.

According to the values shown in Table 2 where the Euler angle is limited to 80° for structural reasons, and with a precision of the values of E and the bending strength, a rule of thumb can be concluded, which is suitable for the selection of sensible core diameters d concerning the length of a specific IPS as:

$$d \geq 2 \cdot l / 100 = L_{IPS} / 100 \quad (57)$$

The choices for the IPS are now limited based on Equation 57, as the diameter of the IPS d_{IPS} must be greater than $\frac{L_{IPS}}{100}$, which means that gap distances between the two phases will limit the diameter of the IPS used in the system. However, this limit is not very strict, but it is approximative.

3. MATERIAL TESTING

3.1. Introduction

In Section 2.3 we explained the loads on the IPS and we introduced the fiber-reinforced plastic 'FRP' that is used in the manufacturing of the modern IPSs as it is characterized by high breaking strain, which makes it withstand high compression loads greater than the Euler load for buckling without permanent deformation. The exact polymer material is confidential information by the manufacturer, but the main goal is to understand its behavior.

Material tests are performed in the Materials Science Laboratory of the Savaria Institute of Technology on different samples of that material to obtain some of its characteristics. In particular, the modulus of elasticity and flexural strength were obtained from three-point bending tests.

These tests were followed by a series of buckling tests on other samples to see the behavior of the material under buckling and compare it with analytical and numerical models.

3.2. Three-Point Bending Test

Bending tests consist of applying a force with either a single upper load cell at the midpoint of the specimen which is a 3-point bending test, or two cell loads applied on two different points of the specimen which is the 4-point bending test.

In the 3-point bending test, the sample having either a circular or a rectangular cross-section is supported at two points, the force is applied at the midpoint, and then it is bent until fracture. The test sample is loaded in a way that creates a concave surface at the midpoint with a specified radius of curvature. During this test, plastic characteristic in brittle materials is investigated and the mechanical properties of the specimen are determined.

3.2.1. Testing operation

Zwick / Roell testing machine is used to perform the 3-point bending test. This test can be carried out after mounting three extra pieces; that are two support units and one loading unit, those three pieces should have cylindrical surfaces to avoid any stress concentration and decrease the friction during the deformation.

Tests were carried out on different samples made of FRP. Those samples have different diameters to perform different material tests on them and obtain the correct characteristics. In total 8 tests were performed on samples with two different diameters, and the setup can be seen in Figure 12.

In Figure 13, we can see the force deformation graph and it is clear that the standard force applied on the specimen is increasing during the time, and the corresponding deformation is also increasing. When time is passing, the elastic part starts to show up, which means if we stop the test at that time the material will gain back its original shape, until we see the yield point where we move from elastic to the plastic region at which the material will be permanently deformed. We are interested in this force to find the flexure strength and the modulus of elasticity of the material.

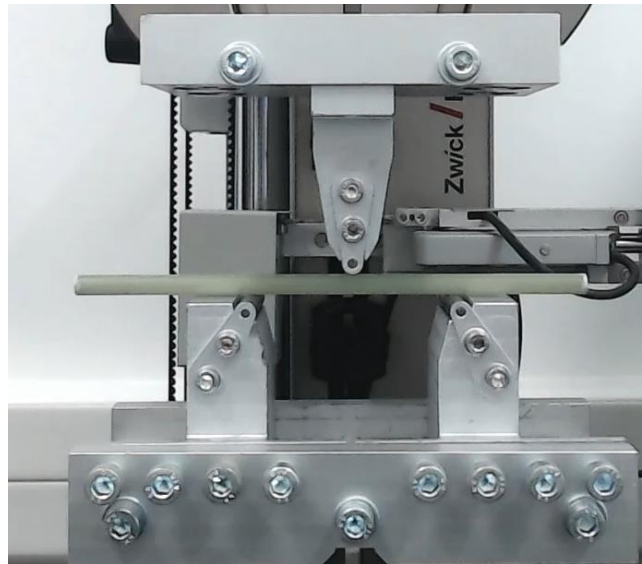


Figure 12: Tested sample of the FRP material used for IPS

During the tests as shown in Figure 13, we have the predefined load position which is 40 mm apart from each support unit, a pre-load of 10N with a speed of 3 mm/min and a test speed of 2 mm/min. The first 4 tests are applied on the samples with a diameter $d = 8$ mm while the other 4 tests were performed on samples with a diameter $d = 6$ mm.

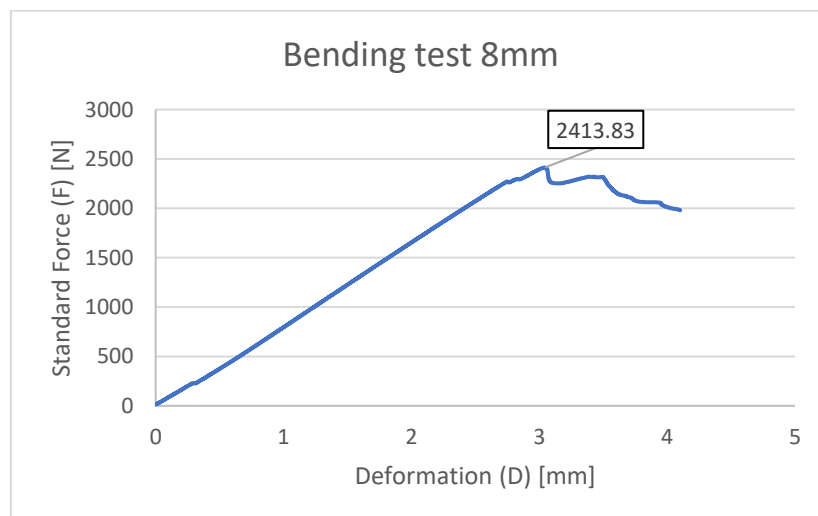


Figure 13: Two samples for the bending test of different diameters

3.2.2. Flexural Strength

The fracture stress or the flexural strength, is corresponding to the maximum bending moment, which can be calculated from the maximum standard force just before the yield point of the material.

Since the stress is defined as the ratio of the instantaneous bending moment over the cross-section factor, the flexural strength, which is the ratio of the maximum moment over the circular cross-section factor of the sample, can be defined as:

$$\sigma_{max} = \sigma_{fs} = \frac{M_{max}}{K}, \quad (58)$$

where:

$$M_{max} = \frac{F_{max}}{2} \cdot \frac{L}{2}, \quad (59)$$

and

$$K = \frac{\pi \cdot d^3}{32} \quad (60)$$

The value of the flexural strength was changing between the different samples, especially if we change the diameters. The maximum flexural strength was obtained from a sample with a diameter $d = 6$ mm and it was $\sigma_{max} \approx 1150$ MPa, while the minimum value was obtained from a sample with a diameter $d = 8$ mm, and it was $\sigma_{max} \approx 850$ MPa so, we take the average value of the total tests, and we get that:

$$\sigma_{max} = \sigma_{fs} \approx 1000 \text{ MPa} \quad (61)$$

3.2.3. Modulus of Elasticity

The elastic modulus is the measure of the material's resistance to being deformed elastically when stress is applied. The modulus of elasticity is a specific material constant; it does not theoretically depend on the sample's geometry, and it is calculated as the ratio of the stress in a body to the corresponding deformation.

$$E_f = \frac{m \cdot L^3}{12\pi \cdot R^4}, \quad (62)$$

that m is the slope of the graph of the force over the deformation, and defined as:

$$m = \frac{F}{D} \text{ (of the elastic region)} \quad (63)$$

The same fluctuations of the results can be observed for the modulus of Elasticity as well. The maximum value was obtained from the smaller sample, and it was $E_f \approx 53$ GPa. While the smallest value was from the larger sample and the value was around $E_f \approx 47$ GPa, so we also take the average of the 8 tests, and we get the modulus of elasticity as:

$$E_{IPS} = E_f \approx 50 \text{ GPa} \quad (64)$$

3.3. Buckling test

3.3.1. Introduction

Buckling is the mode of failure of elements when they are exposed to high axial compressive stresses, it results in a sudden sideways deflection. The buckling test can have different modes as it was mentioned in Section 2.3.2, we are interested in the first mode of buckling as it is always happening first, and with an effective length $k = 1$. To achieve this we need free rotation and fixed translation ends for the sample to perform the test, this is based on Table 1.

3.3.2. Grip designing

In the material science lab of Savaria institute of technology, we didn't have this kind of grip to hold our sample, so I had to design a sample holder using the Autodesk Inventor, during the designing stage we need to design it in a flexible way to be able to fit in samples with different diameters, but at the same time it must be tight enough in order not to let the sample to slip from the griper, so we are using two sets of grippers with different hole diameters, and there is a side hole that can ensure the sample in position, this design was manufactured in the manufacturing lab of the university, for the assembly drawing check Appendix 2 in Section 8.2.

Then we bought a rotary encoder with a resolution of 0.6° to measure the angular change during the buckling test, to mount this encoder, the best way was to 3D print a sensor holder in the project lab using and then it was mounted in the whole unit and the buckling tests was successfully performed.

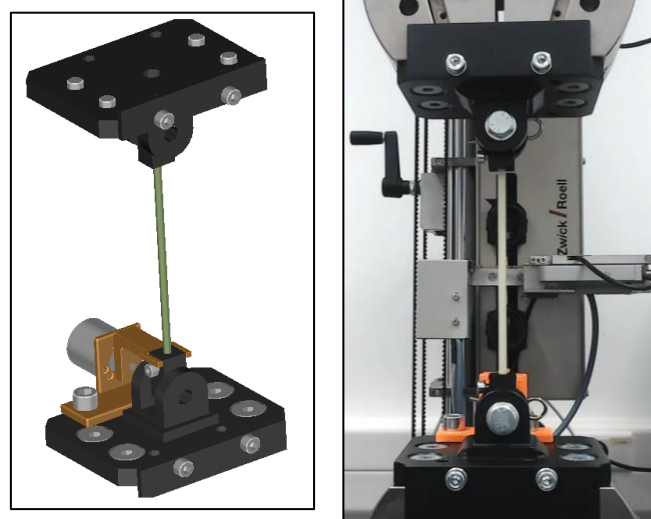


Figure 14: Designed and manufactured buckling grip

3.3.3. Buckling operation

The same Zwick / Roell testing machine is used also to perform the buckling tests, after mounting the newly manufactured sample holder and the rotary encoder as shown in Figure 14, Six buckling tests were applied to 4 samples with two different diameters to study their behavior. Those tests can be represented in Table 3.

Table 3: buckling and post-buckling tests

Sample	Buckling 1		Buckling 2		Bending test	
	Angle [°]	Force [N]	Angle [°]	Force [N]	σ_{fs} [GPa]	E_f [GPa]
S1 (8mm)	16.2	2400	30	1790	0.9 GPa	35 GPa
S2 (6mm)	25.2	780	40	740	1.1 GPa	48 GPa
S3 (6mm)	25.8	775	Not Performed			
S4 (8mm)	21.6	2365				

The first buckling test was conducted to observe any visual damage to the sample with a large buckling angle but below the sample's limit. The second buckling test was then performed, exceeding this limit, to see the deformation of the angle and force deformation graphs until the fibers of the sample began to tear apart. Finally, a 3-point bending test was conducted on the partially broken samples to observe the effect of very high buckling angles.

The buckling test performed on the sample (S2) can be seen in Figure 15, where we can see that the angle change is always following the same function in both buckling tests, and the force is also very similar in both cases after the occurrence of the buckling.

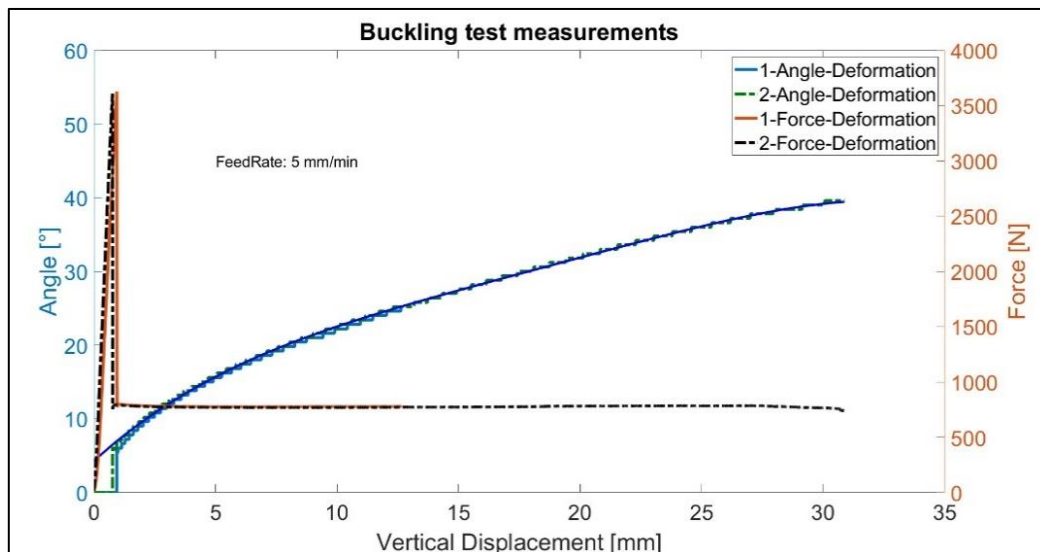


Figure 15: Buckling test measurements from MATLAB

4. NUMERICAL SIMULATIONS

4.1. Problem definition and aim of the analysis

We will study the high amplitude vibrations such as the sleet jump phenomenon which is the vibration of the cables following the ice-shedding. For this, we need to simulate ice shedding from the conductors using the ANSYS software by applying a distributed load on two conductors and removing it at a specific moment causing the system to vibrate with a high amplitude after that we can find the axial loads that cause the IPS to suffer buckling and study it in a separate model for buckling, the buckling model will be firstly used to be compared with the experimental results as well as the analytical calculations of the buckling.

4.2. Description of Geometrical Model

4.2.1. Geometry and material model of suspended conductors

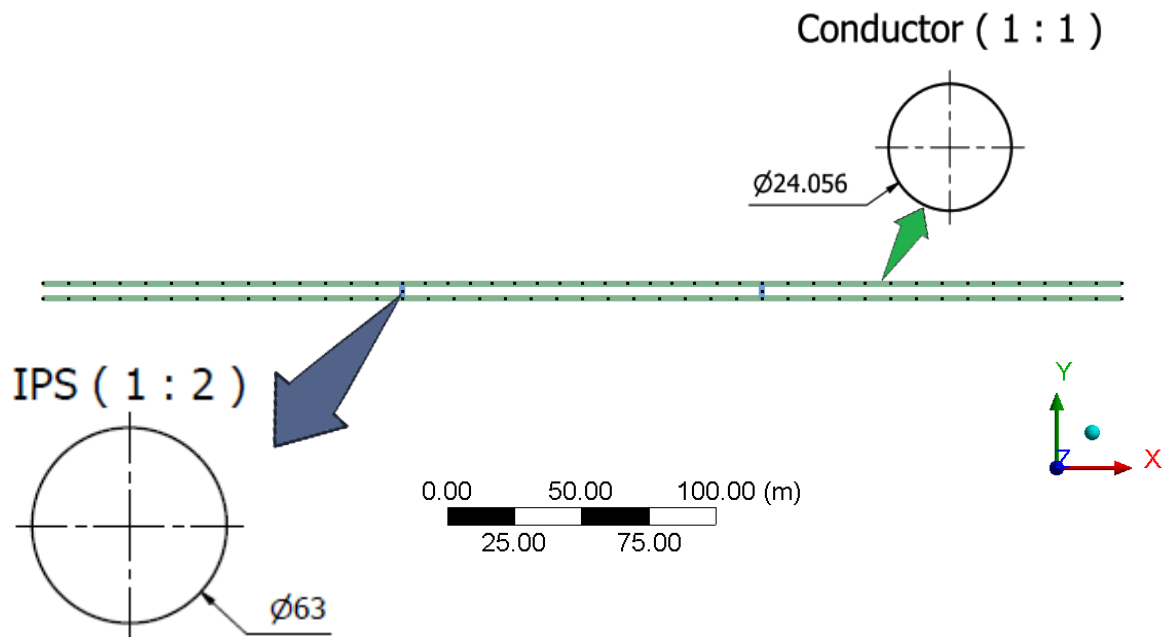


Figure 16: Full-scale model of suspended conductors.

We study the vibration of two aluminum conductors steel reinforced (ACSR) that are known as ‘Condor’ conductors. having characteristics that are represented in Table 4, those conductors are separated with two IPSs that are also characterized as mentioned in Table 5. This model is represented as a line body model as shown in Figure 16, while the real ice-loaded cross-section of the Condor conductor is represented in Figure 17.

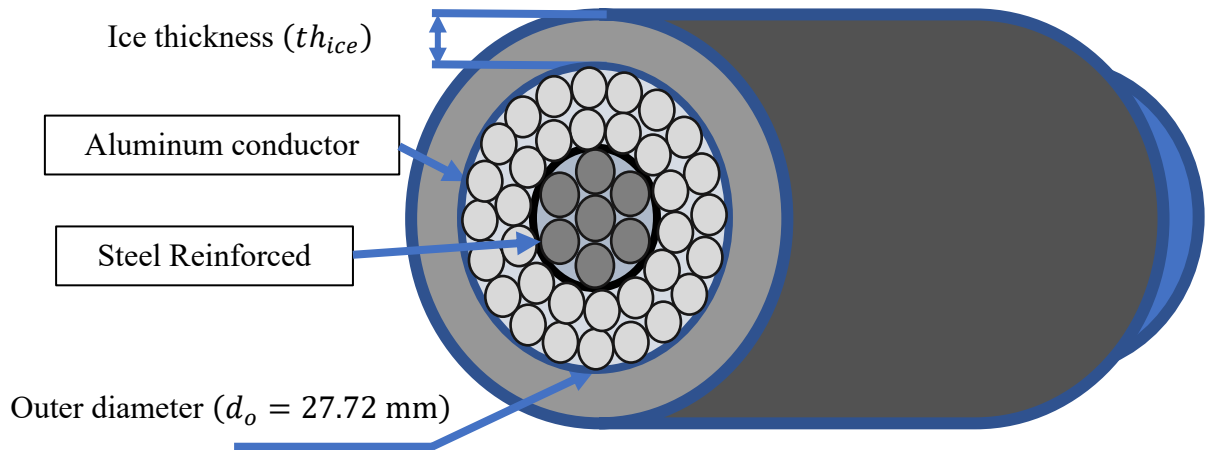
Table 4: Geometrical and material data of the Condor conductor [14]

Span length	Mass per unit length	Sag	Outer diameter	Effective diameter	Cross-sectional area	Young's modulus	Poisson's ratio
(L) [m]	(m_0) [kg/m]	(h) [m]	(d) [mm]	d_c [mm]	(A_c) [mm ²]	(E) [GPa]	(ν) [1]
400	1.457	11.09	27.72	24.055	454.5	65.727	0.33

Table 5: Geometrical and material data of IPS [14]

Length	Diameter	Density	Young's modulus	Poisson's ratio	Flexural strength
(L_{IPS}) [m]	(d_{IPS}) [mm]	(ρ_{IPS}) [kg/m ³]	(E_{IPS}) [GPa]	(ν) [1]	(σ_{fs}) [MPa]
5.5	63	2037.05	50	0.35	1000

We are using the simulation of the full structure to obtain displacement and the force generated on the IPS, and for this, a line body model would be enough to obtain those results. In this model, we have cables that are longer than the span length. Their final length is calculated based on Equation 3. It must be considered that this final length is obtained after the cable was elongated due to the tensile force that holds the cable in position, which is obtained based on Hook's law in Equation 4 and the force from Equation 1. So, based on the previous equations and the data in Table 4 and Table 5, we can calculate that the tensile force required to hang each cable is $F_H \approx 27900$ N with initial the total length $S = 400.82$ m the requires pre-strain $\varepsilon = 0.000919$.

**Figure 17:** Real cross-section of the iced Condor conductor

4.2.2. Buckling model for IPS

We first build a model that is exactly like the structure used in the buckling experiments, the model will consist of two gripping ends that are free to rotate around the z-axis, and are perfectly connected to a 20 cm sample, with a diameter of 6.2 mm.

Then, for the full-scale model, the IPSs will be connected to both conductors, there will be two connection points where the cables will go through, and in the 3D structure, the total length of the IPS will be a bit longer than the original length of the spacer to have some extra material to support those connection points, for this model we need to have asymmetric load condition because it is never perfect in the reality, that is why we shifted the position of the upper pin 4mm as in Figure 18 (b.), while for the experiment model, we need it to be as close as the test we performed so we kept the structure in a symmetric condition as represented in Figure 18 (a.)

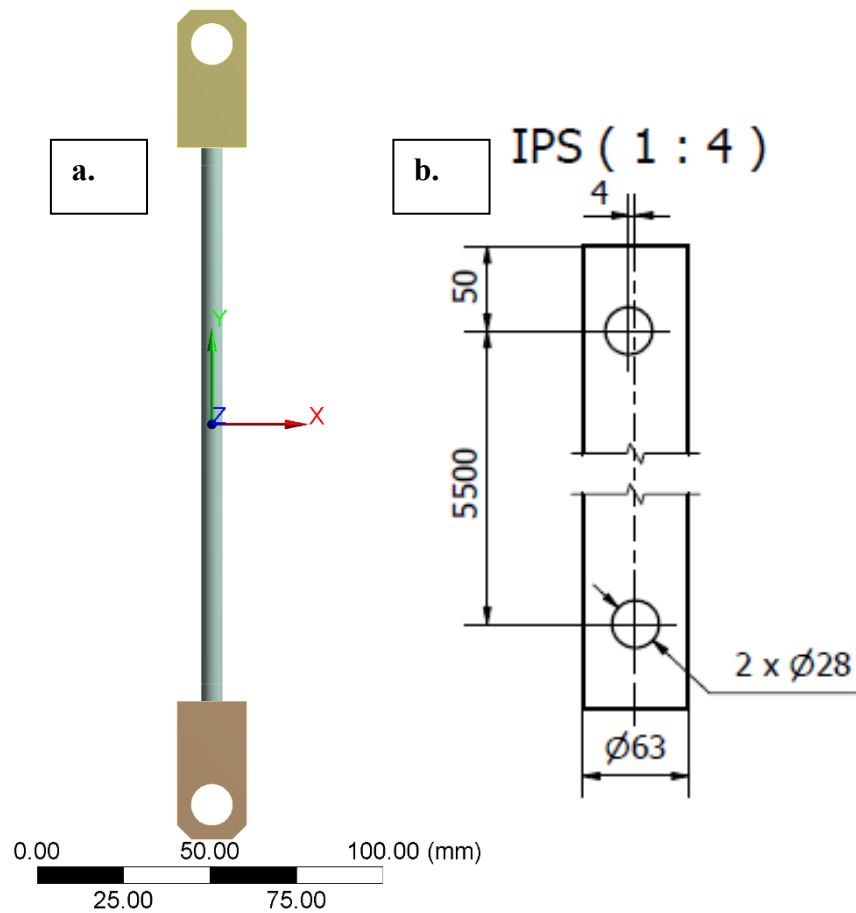


Figure 18: a. Experimental model sample. **b.** Simplified full-scale model of IPS

4.3. Description of the load model

4.3.1. Free suspended conductors

4.3.1.1. The self-weight of the cables

At the beginning of the analysis, we have bare conductors without any extra load on them. However, in the case of transmission lines, their weight affects their profile significantly since they are suspended over a long distance, and their weight act as a load that will elongate them due to strain energy in them as well as affects the supports.

4.3.1.2. Applying pre-strain in the cables

We need to have elongated cables that are strained due to pretension. To do so, we should pre-strain the cables by writing a command in the numerical model for the desired geometries (i.e., the two cables):

```
INISTATE, SET, CSYS, -2
```

```
INISTATE, SET, DTYP, EPEL
```

```
INISTATE, DEFINE ,,,,0.000919
```

Table 6: Command description for initial strain [\[15\]](#)

Code	Meaning
INISTATE	Is used to apply initial state in the model
SET	To designate initial state coordinate system, data type, and material type
CSYS	Coordinate system
-2	Element Coordinate System
DTYP	data that will be set on the subsequent INISTATE, DEFINE command
EPEL	Is Strain Data
DEFINE	To specify the actual state values

4.3.2. Ice loaded conductors

The ice is gradually formed on both cables adding external loads on them that will also cause higher tension force on the supports. The most dangerous scenario is simulated when ice sheds from the lower conductor, and the upper one is still loaded. Then, ice also sheds from the upper conductor due to the vibration of the lower conductor, causing a sleet jump of the system, which keeps it under the danger of flashovers and short circuits. The IPS helps prevent

flashovers, so it will suffer from a high compression force. Ice thickness and density are given in Table 6, and further parameters are calculated by Equations (65)-(67).

Table 7: Properties of Ice [14]

Ice thickness (<i>th</i>) [mm]	Ice density (ρ_{Ice}) [$\frac{\text{kg}}{\text{m}^3}$]
18	735

$$A_{ice} = ((d + 2 \cdot th)^2 - d^2) \cdot \frac{\pi}{4} \cdot 10^{-6} \text{ m}^2 \quad (65)$$

$$m_{o_{ice}} = A_{ice} \cdot \rho_{ice} = 1.9 \text{ kg/m} \quad (66)$$

$$F = m_{o_{ice}} \cdot L \cdot g = 1.9 \cdot 400 \cdot 9.81 \approx 7450 \text{ N} \quad (67)$$

4.4. Description of Mesh of the FEM model

4.4.1. Line body

Based on the iterative method by changing the mesh sizing for a line body model, we can conclude that the change in the mesh above 45 elements does not have any significant effect on the final static equilibrium position of the system, so we use the default mesh size with a default element order. We have a total of 135 nodes and 45 elements.

4.4.2. Buckling modeling for the IPS

Higher precision results are required for the IPS as we aim to study the stress region on them. For this reason, a finer mesh is applied to the 3D geometry of the IPS in the separate buckling model. This mesh would be in a quadratic order with a total node number of 28670 and elements 14988, and for the experimental model, we also used a fine mesh that result in 43344 nodes and 16862 elements.

4.5. Boundary conditions

4.5.1. The vibration of suspended conductors

4.5.1.1. Ice-shedding Phenomenon

We need to define loads and constraints that can correctly describe the problem of ice shedding and perform the sleet jump phenomenon. For that, we need to apply both kinematic and dynamic boundary conditions, where the lower cable is loaded, and then the ice will shed from it causing the sleet jump phenomenon to take place, the details of the boundary conditions are mentioned in the last year and this conference paper [1].

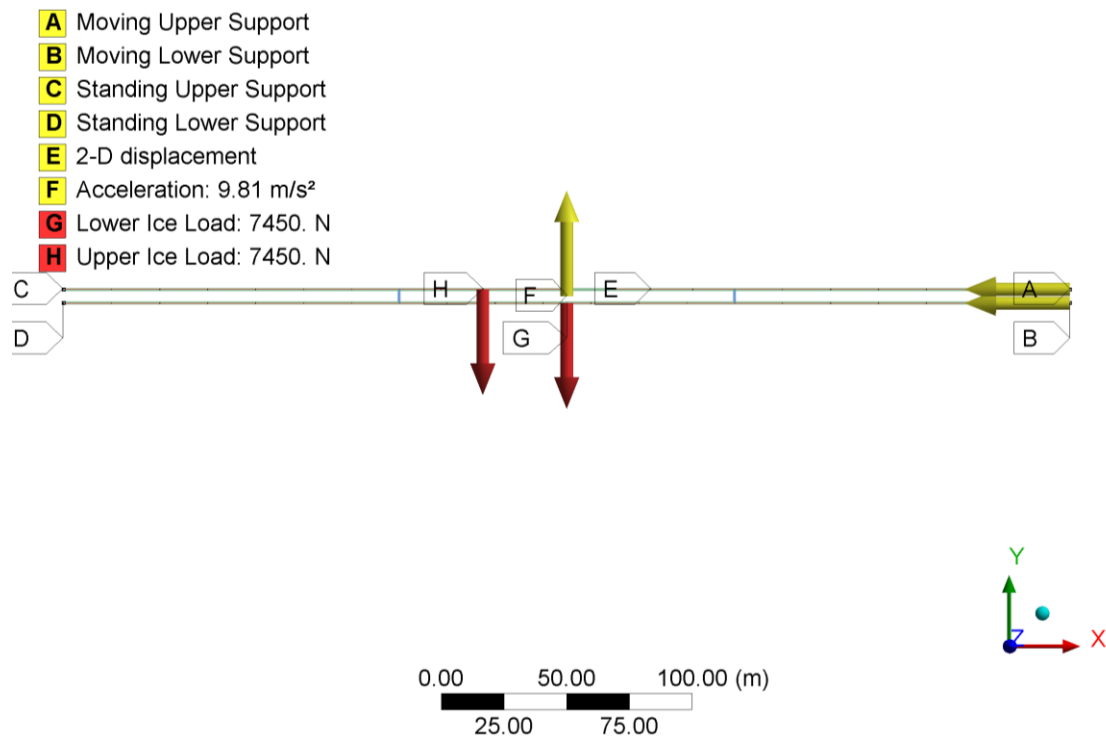


Figure 19: Boundary conditions of the loaded suspended cables

We can define that we have got four main steps to apply the loads, the first step is where we apply ramped acceleration to simulate the standard earth gravity, then the second step we wait to reach the static equilibrium of the conductors, then during the 3rd step, we apply the ramped loads on both conductors then we also wait to reach the static equilibrium of the system. After that, we remove the load from the lower cable, and this will cause the sleet jump to happen in the lower cable respectively the upper one which also leads to shedding the ice from it. Then we wait until we reach the final static equilibrium state of the structure after a minute.

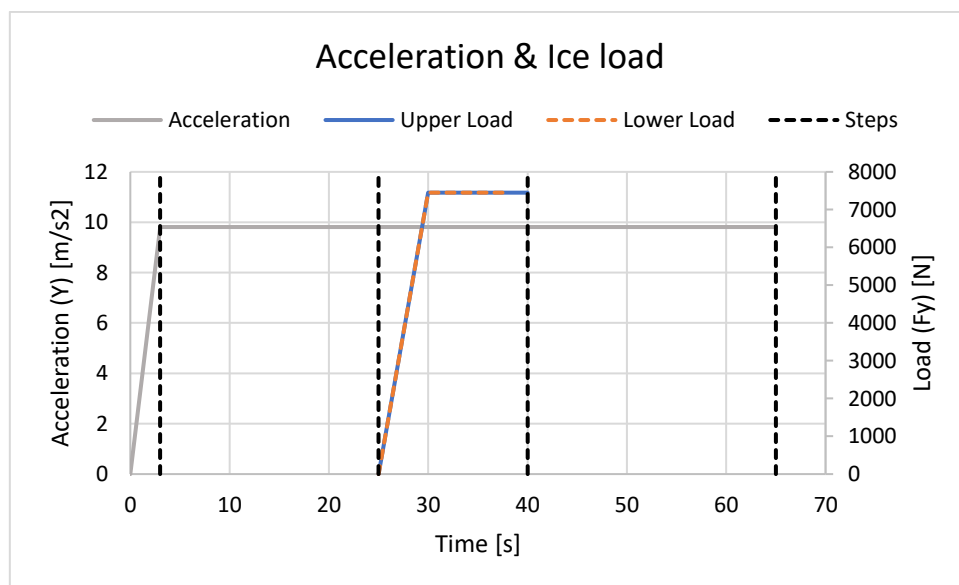


Figure 20: Dynamic boundary conditions for conductors

In Figure 20 and Figure 21 we can see how the boundary conditions of the analysis are applied over time, firstly the self-weight will be considered and because of this weight the moving supports on the right side will be moving toward the center of the cables, after that the system was left to reach the equilibrium state then the ice load is being added to the cables then stays constant for a while until the ice-shedding happens the ice weight is removed causing a vibration of the system.

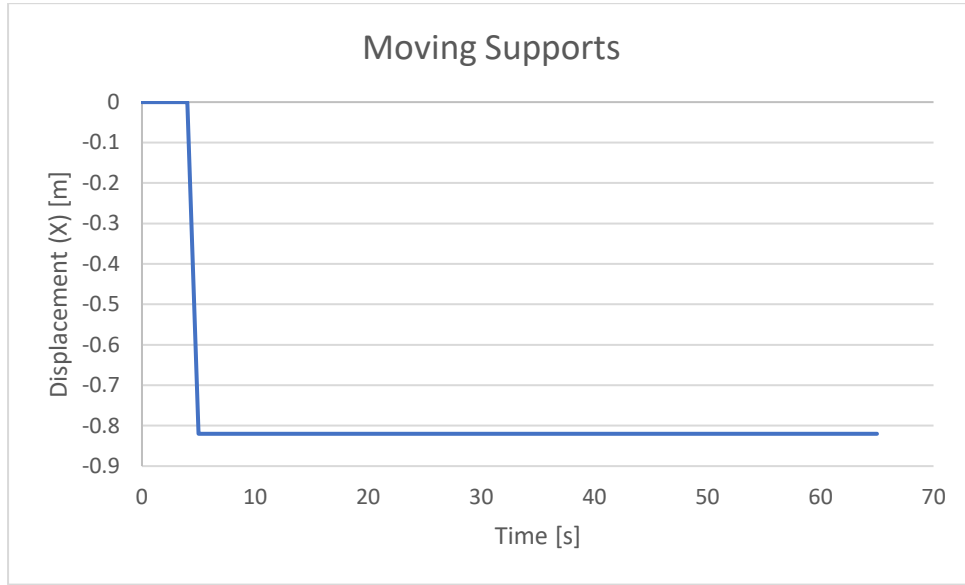


Figure 21: Kinematic boundary conditions of the moving supports

4.5.1.2. Rayleigh damping for the conductor's vibration

The structural damping of the conductor contributes to vibration attenuation. This damping is considered by Rayleigh damping, where two constants must be defined. For that we will need to apply two coefficients; the mass coefficient ' α_{mass} ' and the stiffness coefficient ' β_{Stiff} ' based on their matrices ' $\mathbf{M}$ ' & ' $\mathbf{K}$ ' respectively that can be obtained based on Equations 68-71 [16, 17]:

$$\mathbf{C} = \alpha_{mass}\mathbf{M} + \beta_{Stiff}\mathbf{K}, \quad (68)$$

$$\alpha_{mass} + \beta_{Stiff} \cdot \omega_i^2 = 2\omega_i \cdot \xi_i, i = 1, 2, \dots, \quad (69)$$

$$\alpha_{mass} = 2\omega_1 \cdot \xi_1 - \beta_{Stiff} \cdot \omega_1^2, \quad (70)$$

$$\beta_{Stiff} = 2 \cdot \frac{(\omega_2 \cdot \xi_2 - \omega_1 \cdot \xi_1)}{\omega_2^2 - \omega_1^2}, \quad (71)$$

So, to calculate the two coefficients we will need two natural circular frequencies that can be obtained from two vibration modes: the first mode of the symmetric and the same of the antisymmetric in-plane motions. Those analytical results were derived based on a free single conductor, for this reason, the modal analysis will be applied to a single conductor to compare the numerical results with the analytical calculations, then the modal analysis of the full structure can be developed.

The created MATLAB application will be developed to calculate different kinds of natural frequencies. The basics of the natural frequency of a suspended cable were discussed in detail in Section 2.1.2, which is used to obtain Equations 72-74. Also, by considering Equations 68-71, and after calculating the frequencies of the first mode of symmetric and antisymmetric in-plane motions, the mass coefficient ' α_{mass} ' and the stiffness coefficient ' β_{stiff} ' can be obtained.

$$f_{trans.} = \frac{n}{2 \cdot L} \cdot \sqrt{\frac{F_H}{m}}, \quad (72)$$

$$f_{antis.} = \frac{n}{L} \cdot \sqrt{\frac{F_H}{m}}, \quad (73)$$

$$f_{symm.} = \frac{\beta}{2 \cdot \pi} \cdot \sqrt{\frac{F_H}{m}}, \quad (74)$$

The relative damping ratio is assumed to be 2 % for bare conductors and 10 % for iced conductors [18], we use 5 %, which means that $\xi_1 = \xi_2 = 0.05$, also we use the $f_{symm.1}$ and $f_{antis.1}$ to calculate the Rayleigh damping so the mass coefficient $\alpha = 0.1254$ and the stiffness coefficient $\beta = 0.01914$. The validation process was done previous in my thesis and we can see that these equations are perfectly good to use in the model [1].

4.5.2. Buckling Model

The buckling study requires two kinds of simulations, the first is the eigenvalue buckling and the second is a static structural model to simulate the buckling test. The eigenvalue buckling model is required to have a better understanding of the buckling behavior and buckling modes of the system, that model was presented previously [1], and the results obtained were also similar to the experimental and analytical results, it can be seen in Table 8.

Table 8: Euler load of buckling from Experimental, Analytical, and Numerical approaches

Model	Euler's load for buckling
Experiment model	$F_{Buckling} = 780 \text{ N}$
Analytical model	$F_{Buckling} = 770 \text{ N}$
Numerical model	$F_{Buckling} = 790 \text{ N}$

The axial force that was obtained from the line body model of the ice-shedding that is represented in Figure 23 will be used as the dynamic boundary condition to study the behavior of the IPS under the ice-shedding, then applied in the buckling model of the full-scale structure represented in Figure 22.

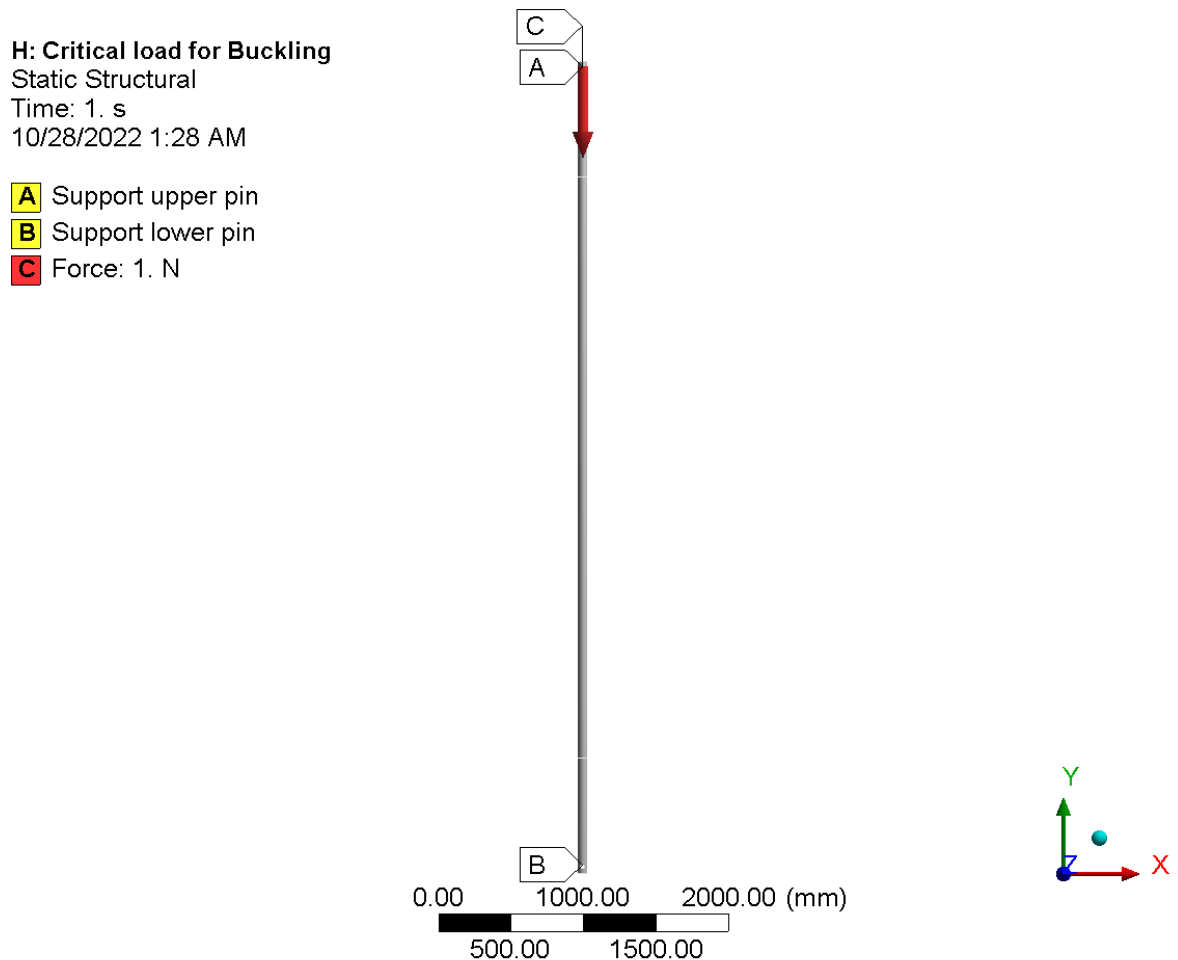


Figure 22: Boundary conditions of the Eigenvalue Buckling model

- A. Remote displacement in the upper pinhole. (Kinematic boundary condition)
 - Allow vertical displacement [Y-axis] and rotation about the Z-axis.
 - Block any other possible motion and rotation.
- B. Remote displacement in the lower pinhole. (Kinematic boundary condition)
 - Allow only rotation about the Z-axis.
 - Block any other possible motion and rotation.
- C. Axial force in the upper pinhole. (Dynamic boundary condition)
 - An axial force is introduced in the direction of only the Y-axis.
 - Other components are free

4.6. Results and discussion

Firstly, to check the results of the numerical analysis we need to validate our model by comparing the numerical results with the analytical ones for both deformation and the force reactions on the supports. Then, we can work with the numerical model for further investigations [1].

4.6.1. Numerical ice-shedding model validation

The force reaction on the supports, as well as the vertical deformation, which is the sag of the system during the unloaded and the loaded cables, are calculated at both time instants $t = 37$ & 65 s, using Equations 1-4 in Section 2.1.1 and Equations 15, 16 in Section 2.2.1.

Table 9: Analytical results for model validation at specific times

Time [s]	Force Reaction [N]		Vertical deformation (Y-axis) [m]	
	Analytical	Numerical	Analytical	Numerical
$t = 37$ s	51800 N	52600 N	13.07 m	13.064 m
$t = 65$ s	27400 N	27600 N	11.09 m	11.109 m

Table 9 shows that in the case of the loaded cables we have total tension force in the supports $F_H + H = 52600$ N and in case of the unloaded cables the tensile force reaches $F_H = 27600$ N which means that we have less than a 2% error which is accepted for further analysis.

The vertical displacements seem to be very close to the analytical calculations, that the loaded cables have $Y = 13.064$ m and after unloading them, the deformation reaches the $Y = 11.109$ m which means the error is less than 1%.

4.6.2. Loads generated on the IPS of a full-scale model

The main aim of the analysis is to check the axial load and the deformation of the IPS. Before analyzing the IPS it is needed to do some analytical calculations and set a limitation for the maximum allowed stress. The analytical calculations are done based on the equations in Section 2.3.2, the material properties obtained from the material test in Section 3.2, and the geometry of the IPS that is defined as $d_{IPS} = 63$ mm. The MATLAB application was developed to solve those equations. The equation showed the results for the critical stress of the IPS as $\sigma_{critical} \approx 4$ MPa. Based on Equation 42 the allowed stress for a safety factor of $n = 2$ is defined as $\sigma_{allowed} = \frac{\sigma_{Flexure}}{n} = 500$ MPa, which is corresponding to vertical load $P = 13110$ N and Euler angle $\alpha_{Euler} = 30^\circ$.

4.6.2.1. Line body results of the IPS

We can see in Figure 23 the change in the axial load on the IPS over time, and the absolute maximum value is after $t = 38.75$ s with a value $F = -1470$ N. We consider the absolute maximum value, as the sign shows whether it is compression or tension force, The compression force, which causes buckling, is negative. After we check the line body results, we will create a sub-modeling for the IPS to study the stress region more precisely.

Firstly, we analyze the axial loads arising in the IPS during the sleet jump in the line body structure for both scenarios that we discussed at the beginning of Section 4.5. After that, we will use the finer mesh sub-model of the IPS with the higher axial force

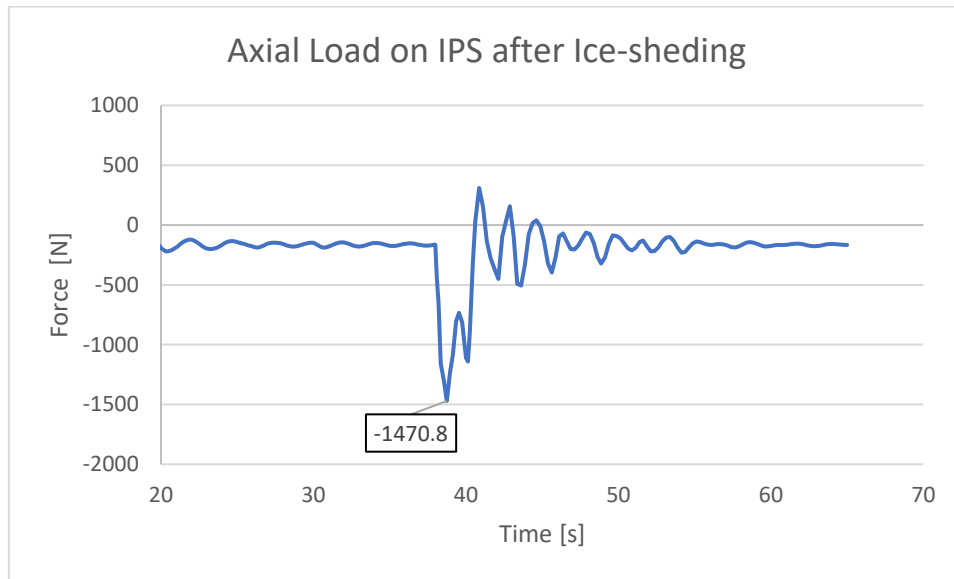


Figure 23: Axial load [N] on IPS over time [s] in the first scenario

Figure 23 shows the expected results, where the maximum compression force arises a few moments after the ice-shedding then it decays.

4.6.2.2. Stress generation in the IPS

Before we do the stress analysis on the IPS we need to do some analytical calculations and set a limitation for the maximum allowed stress. The analytical calculations are done based on three things; the equations in Section 2.3.2, the material properties obtained from the material test in Section 3.2, and the geometry of the IPS that is defined as $d_{IPS} = 63$ mm and $L_{IPS} = 5.5$ m . The MATLAB application was developed to solve those equations. The equation showed the results for the critical stress of the IPS as $\sigma_{critical} \approx 4$ MPa. The allowed stress for a safety factor of $n = 2$ is defined as $\sigma_{allowed} = \frac{\sigma_{Flexure}}{2} = 500$ MPa, which is corresponding to vertical load $P = 13110$ N and Euler angle $\alpha_{Euler} \approx 30^\circ$.

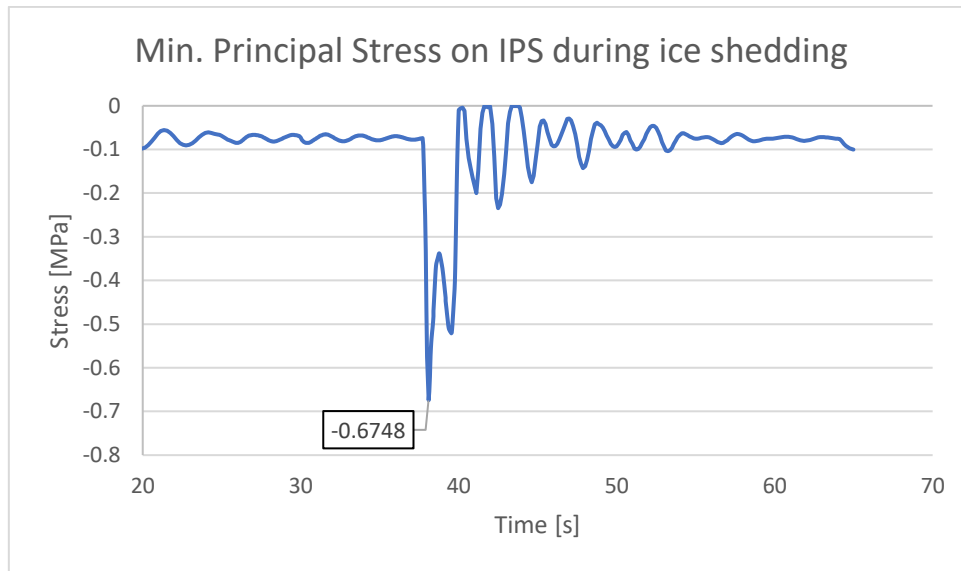


Figure 24: Stress during the Ice-shedding

Figure 24 shows that the stress on the IPS is not increasing a lot during the ice-shedding analysis. If we compare the maximum stress with the critical stress for buckling, then we will have a safety factor of more than $n = 3$.

Table 10: Analytical and numerical results of the ice-shedding analysis of IPS with $d_{IPS} = 63 \text{ mm}$

Result	Axial load [N]	Direct stress [MPa]	Euler load (P_{Euler}) [N]	Bending stress [MPa]	Horizontal deformation [mm]	Vertical deformation [mm]
Analytical	1470	0.47	12615	No buckling	0	0
Numerical	1470	0.67	12609	No buckling	0.3	0.06

It is seen in Table 10 that the results of the analytical and the numerical calculations do not fit exactly. This is due to several reasons. First, the loading surface, as theoretically we consider it to be evenly distributed on the whole circular surface, and in the numerical simulation, we consider it to be applied through a cylindrical hole. Also, the maximum value of the stress occurs for a very short time, this period is not enough to cause deformation in the spacer.

4.6.2.3. Comparison of IPSs with different diameters

The selected IPS with a diameter of $d_{IPS} = 63 \text{ mm}$ is perfectly fine on that transmission system where we use two IPSs between the second and the third phases in an equidistant from each other and the towers as well. Based on the thickness of ice adhered on the cables followed

by ice-shedding, the stress on the IPS is very small so that it does not cause any buckling or damage for the flexural strength of 1000 MPa provided by Equation 61. Hence, we can check the smaller diameter IPSs and see how they will behave under the same ice load.

Table 11: Results of the ice-shedding analysis of smaller IPSs

Results	$d_{IPS} = 52 \text{ mm}$		$d_{IPS} = 36 \text{ mm}$	
	Numerical	Analytical	Numerical	Analytical
Euler Load (P_{Euler}) [N]	5854.7	5855	1313.1	1345
Critical stress ($\sigma_{Critical}$) [MPa]	2.76	2.76	1.29	1.321
Axial load [N]	1403	1403	1317	1345.13
Direct stress [MPa]	0.87	0.66	-	1.322
Horizontal deformation [mm]	0.72	0	48.15	45.18
Vertical deformation [mm]	0.08	0	1.7	1.04
Euler angle (α_{Euler}) [$^{\circ}$]	$\alpha_{Euler} \approx 0^{\circ}$		$\alpha_{Euler} < 2^{\circ}$	
Buckling Stress (σ_b) [MPa]	No Buckling	No Buckling	Buckling 18.56	Buckling 14.15

In Table 11 the numerical results in the case of $d_{IPS} = 36 \text{ mm}$, it is seen that the maximum axial load obtained from the line body model is higher than the critical Euler load that was obtained from the Eigenvalue model. Thus, the maximum load following ice shedding causes buckling, but with a very small Euler angle that does not exceed two degrees, and for this we will study the analytical results with a force that is just higher than the Euler load. Even though the spacer is suffering a buckling, the stress is much smaller than the flexural strength of the material which means that the spacer will retain its original shape after the load is removed, but still, we want to avoid any buckling in the spacers.

Hence, based on Table 11 and the rule for the selection of the IPS, which was discussed at the end of Section 2.3.2.1, and represented by Equation 57 that resulted that the $d_{IPS} \geq 55 \text{ mm}$ approximately. It is suggested not to use the IPS with $d_{IPS} = 36 \text{ mm}$, while if we use the IPS with $d_{IPS} = 52 \text{ mm}$ it will give a very close value to the limit of the rule, and it can safely be used according to the calculations as well. Thus, it is suggested to use spacers with $d_{IPS} = 52 \text{ mm}$ in this configuration.

4.6.3. Buckling model of the experimental setup

In this model, the boundary conditions are similar to the one that was represented in Figure 22, but instead of a force, a displacement will be applied, just to be simulating the same as the material test explained in Section 3.3. The results obtained from the experiment and the numerical simulations will be compared with the expected analytical theoretical behavior of buckling of beams that was explained in Section 2.3.2.

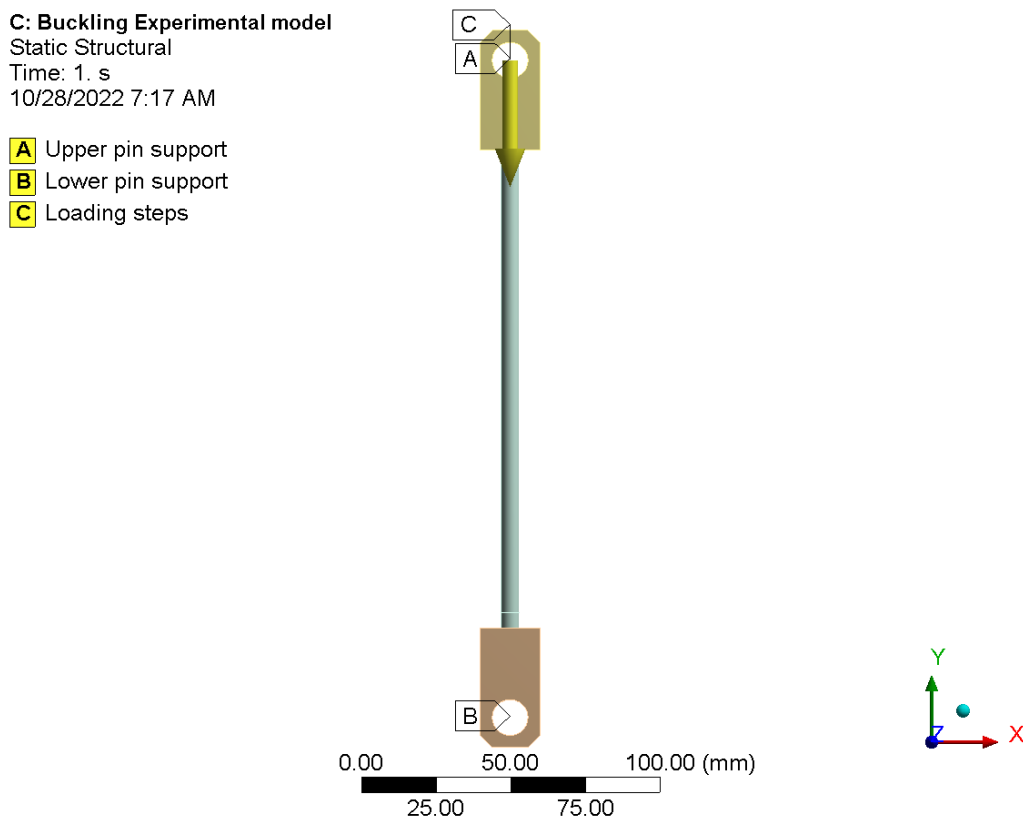


Figure 25: Boundary conditions of experimental buckling test

4.6.3.1. Force-Deformation comparison

In Figure 26, we can see the three graphs of the Force-Deformation curves obtained from the Experimental buckling test, calculated buckling force using MATLAB, and from the Numerical simulations. The experimental and the numerical results both have high peak force reactions, those are happening due to the symmetric load condition that we have, so in the beginning, the load is considered just axial compression force, then it is followed by a fast drop in the force reaching the Euler's load for buckling, and this is the moment when the buckling happens. While the analytical calculations don't consider that peak, and it considers the buckling happening from the beginning of the calculation, so we can consider the buckling results after the dashed black line seen in the graph.

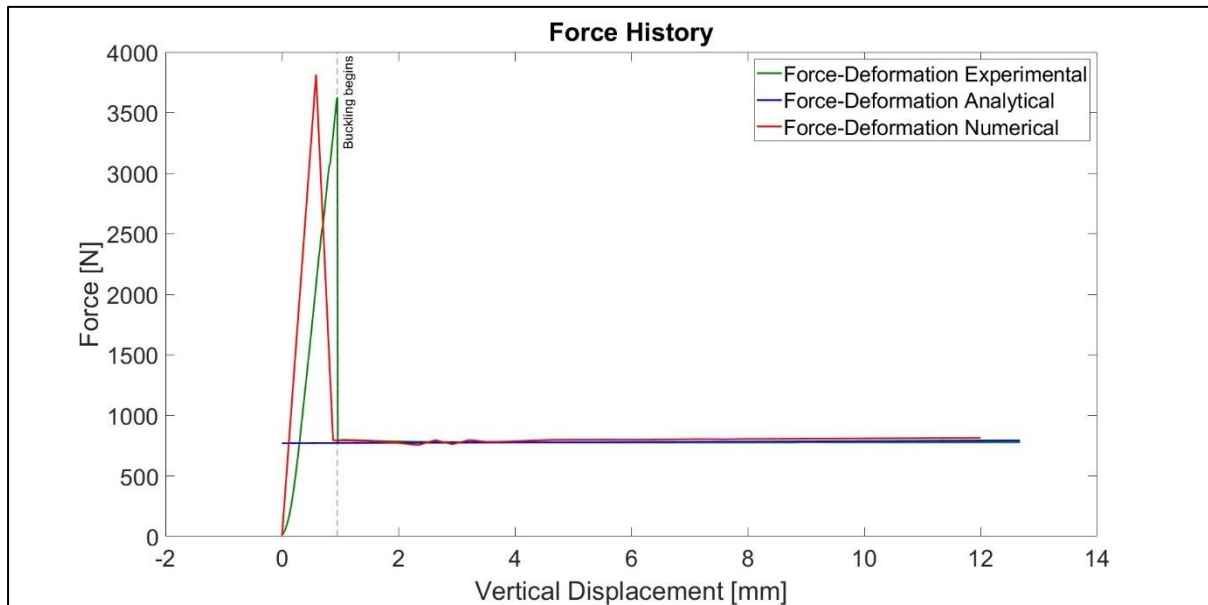


Figure 26: Force-Deformation comparison; Experimental, Analytical and Numerical

4.6.3.2. Angle-Deformation comparisons

The same phenomenon appears again in the Angle-Deformation comparisons represented in Figure 27, as it is clear, the experimental and the numerical simulation results, are almost 0° and then the angle increases suddenly to a high value corresponding to the force reaction behavior, while the analytically calculated curve is increasing linearly then it is following the function of the angle deformation behavior for buckling.

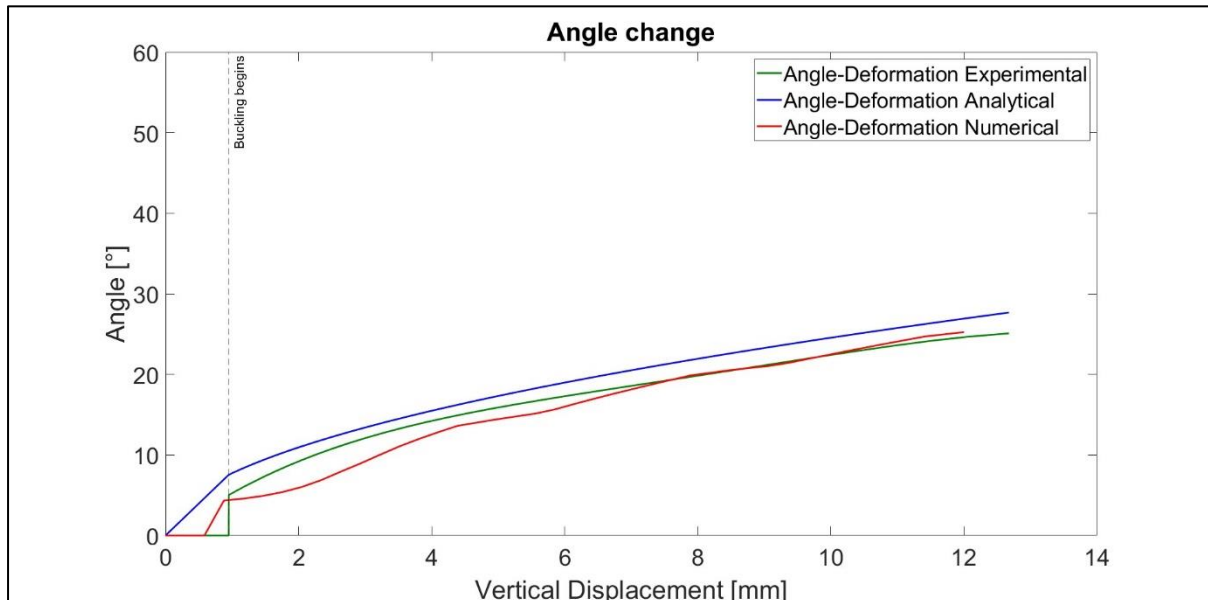


Figure 27: Angle-Deformation comparison; Experimental, Analytical and Numerical

5. SUMMARY

This project was initially started to optimize the usage of the IPS in the transmission line systems, since the power lines now are transmitting very high voltages that make a higher risk of arc initiation between the phases, so we need to perfectly separate the phases, especially during the high amplitude vibrations, such as the “Gallop” and the “Ice-shedding”. The modern spacers are made from fiber glasses that are known as FRP that have good behavior during the buckling, as it can suffer from buckling and when the load is removed it retains its original shape with no permanent damage in it.

During this project, the effect of Ice-shedding on the IPS was studied. The system consists of 2 conductors with a span length of 400 m that are separated by two spacers. The system has a sag of 11 m and phases clearance of up to 5.5 m. To reach the goal, three approaches are preferred to go through, as follows: Experimental, Analytical, and Numerical, in which the last two approaches are going simultaneously.

The experimental approach is needed to reveal the material properties of the FRP and to understand its behavior during buckling. These properties mainly are the flexural strength and the Modulus of Elasticity. To obtain those properties, we applied a 3-point bending test on 8 samples, 4 of them have an average diameter of 6 mm and the others have 8 mm.

More experiments were performed after designing and manufacturing the required grip for the buckling tests, at which 4 samples were tested in 6 buckling tests, as some samples were undergone two buckling tests then followed by a 3-point bending test to reveal if there any damage happened to the sample. Those samples showed almost the same behavior in the bending test similar to the other samples which didn’t undergo the buckling tests. From this, it is proved that this material can undergo high buckling angles, and retain its original shape without any damage.

The analytical approach involves building a solver that can solve all the analytical calculations that are to build and validate the numerical simulations. To build that solver, I used the MATLAB app designer to design and code the application “Conductor with IPS”. This application has five main sections; the first section studies the static behavior of the suspended conductor and the pretension force required to hold it, then visualized graphs can be used to determine the profile of the suspended conductors. After that, the frequencies of the vibration modes can be shown in a separate section as well as the loaded behavior of the conductors can be shown. The last section involves the buckling studies of the IPSs.

The last approach is the numerical one that involves the study of the vibration modes of the conductors that are required to validate and build the dynamic model, from which we obtain

the axial loads on the IPSs. Then we study the buckling behavior of the IPS that will define the critical load of buckling. Finally, we simulate the loaded IPS in a buckling model.

I have created three different geometrical models that are used in five main numerical simulations on the ANSYS software to study the behavior of the suspended conductors and the buckling of the IPS.

The first simulation mainly involves the vibration followed by ice-shedding which is known as the sleet-jump phenomenon, for which the line body model of the full system is used that is loaded over a specific time with the same force that describes the formation of ice based on the ice thickness and density on the conductor. Then, this force is removed after a specific time that is simulating the shedding of the ice from the conductors.

The second geometrical model is describing the 3D geometry of the IPS, and it is used for two more simulations. The first one is developed to study the behavior of the spacers under the axial loads obtained from the line body model.

The results of the suggested spacer has $d_{IPS} = 63$ mm showed that it was very safe to use, and it is far from suffering any buckling in it. Then, the same procedure was done on two smaller spacers with diameters $d_{IPS} = 52$ mm and $d_{IPS} = 36$ mm. The IPS $d_{IPS} = 52$ mm was also safe to use with no buckling, while the smallest spacer suffered buckling under a load that was a little bit smaller than the calculated Euler load. This error was a reason to develop a new model to study the Eigenvalue buckling.

The Eigenvalue buckling model will use the same geometry of the 3D IPS, but with a static load of just 1 N. The results obtained from this simulation show the force multiplier that is required for each buckling mode. We are most interested in the lowest buckling mode and the load multiplier required for it as it will happen first causing the buckling of IPS.

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7. SHORT SUMMARY

Interphase Spacers (IPSs) are widely used in the power transmission system. They are made of a special composite of fiber-reinforced plastics (FRP), and the IPS is implemented between the different phases of transmission lines to prevent the danger of flashovers. Flashovers in the transmission lines may occur due to the excessively high amplitude vibrations, that are developed because of ice shedding from one of the conductors or as a result of galloping which is a form of wind-induced vibration characterized by high amplitudes and low frequencies.

This project aims to study the buckling of IPSs as the result of the dynamic forces acting on them during conductor vibration following ice shedding. For reaching this goal, three different approaches will be applied.

Firstly, we need to start with the experimental approach, which involves material tests on FRP material, mainly performing 3-point bending tests; to reveal the modulus of elasticity and the flexural strength of the material. Furthermore, a set-up is developed for buckling tests that are carried out on samples made of the same material as the IPS to study the behavior of the material during the buckling and after the load is removed. It is also important to note that the FRP material is characterized with high breaking strain, that makes it withstand high compression loads greater than the Euler load for buckling and it retains its original geometry without permanent damage.

The second approach is building and coding an analytical solver using the MATLAB app designer to solve analytical equations to find the natural frequencies in different vibration modes, the tensile force, the sag of the loaded and the unloaded conductors, as well as find the Euler load for buckling. This application can also calculate the maximum Euler angle during buckling of the IPS also it is used to process the data obtained from the experiments. The analytical calculations can also be used to validate the results obtained from the numerical tests.

The third approach is the numerical tests that are carried out using the ANSYS software. The line body model consists of two conductors and two IPSs, and this model is used to obtain the displacements and the forces acting on the IPS during vibration following ice shedding from one of the conductors. Then, a fine-meshed 3D sub-model of the IPS is constructed to study whether buckling occurs or not. Also, the 3D model of the IPS is used to simulate the buckling test performed in the lab. The practical benefit of the study is that the numerical model can help us to decide the most adequate choice for the IPS in each specific configuration of the transmission line.

Keywords: *ice shedding, interphase spacer [IPS], Fiber Reinforced Plastic [FRP], buckling, material test, ANSYS, vibration, Euler's equations, Transmission line*

8. APPENDIX

8.1. Introduction to the MATLAB application (Conductors with IPS)

The app designer in MATLAB software was used to create an analytical and graphical calculator for building the simulations on the ANSYS, as well as to check the numerical results obtained from those simulations, and to help in choosing the best IPS.

The application has two main pages that are divided into 5 sub-sections. The first page considers all the calculations of the conductors, starting with the study of the static state, at which we can calculate the required tensile force by both catenary and parabolic profiles and find the difference between them, as well as find the pre-strain value to achieve the total length of the conductor. The equations in the section were obtained based on Section 2.1.1, check Figure 28.

Visualized graphs for the conductors might be useful to see the difference between the parabolic and the catenary profiles, also it is used to export the coordinates of the cables based on the catenary profile, as it can be used as input geometry in the ANSYS software. Check Figure 29.

The second part deals with the dynamic of the system, starting with the natural frequency of the free suspended cable, that is used in defining the Rayleigh damping of the system, the full explanation for the equations was discussed in Section 4.5.1.2 and it is shown in Figure 30.

The third section involves the calculations of the loaded cables. This is used to obtain the extra sag and tensile force for the validation of the line body analysis, the calculations are based on the theoretical background introduced in Section 2.2.1 and the interface of the application is shown in Figure 31.

The other page of the application is specified in the studying the IPS. This section calculates the Euler load for buckling, the critical stress, and the Euler angle as well in the case of buckling. The theoretical background depends on Section 2.3.2. The results can be found in detail in Figure 32.

MATLAB App

Conductors ☒ IPS

Sag of the Conductor (h) [m]

Span Length (L) [m]

Mass per unit length (mu) [kg/m]

Effective area of the cable (Ac) [mm²]

Young modulus (E) [GPa]

Standard Earth gravity (g) [m/s²]

Density of the cable (ρ) [kg/mm³] **Catenary force (FH) [N]**

Parabolic approx. (FP) [N] **Total length of the conductor (S) [m]**

Error difference [%] **Effective diameter (dc) [mm]**

Pre-strain (ε) [-]

Figure 28: “Conductor with IPS” app interface

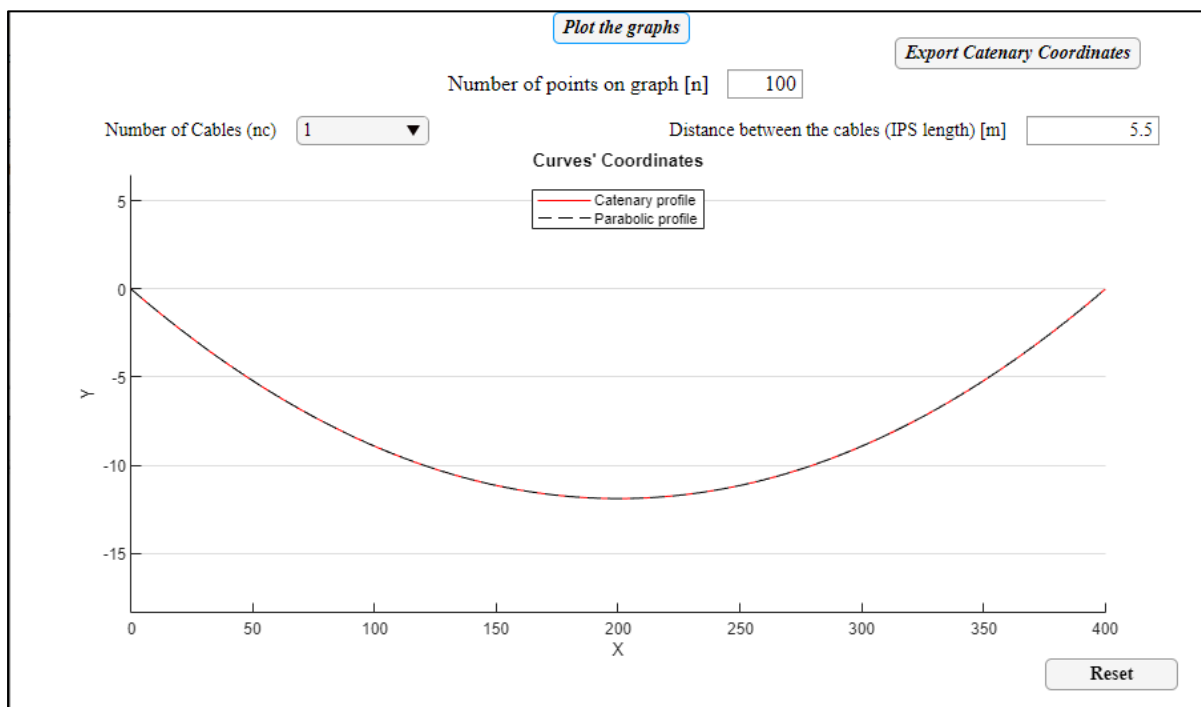


Figure 29: Suspended cable based on the catenary and parabolic profiles.

Damping Ratio (ξ_1) [-]	0.05	Damping Ratio (ξ_2) [-]	0.05
Transverse horizontal motion			
Antisymmetric in-plane motion	f trnas.(1) [Hz]	0.16635	Symmetric in-plane motion
f antis.(1) [Hz]	0.33269	n (th) mode	3
f antis.(2) [Hz]	0.66539	Find	f symm.(1) [Hz]
f antis.(n) [Hz]	0.99808	λ^2 / π^2	5.566
f1 [Hz]	0.3327	Calculate Rayleigh damping	f2 [Hz]
Mass Coeff (α) [-]	0.1254	Stiffness Coeff (β) [-]	0.01914
Reset			

Figure 30: Calculations for the natural frequencies

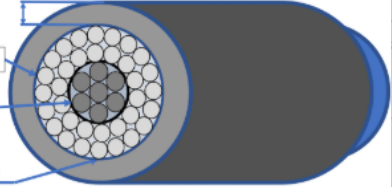
Cable's outer diameter (do) [mm]	0.000	Ice thickness (th)	0.000
Ice density (ρ_{ice}) [kg/m ³]	0.000		
Ice theckness (th_ice) [mm]	0.000		
Locatoin of added deformation (X_w) [m]	0.000		
Lambda^2 (λ^2)	0.000		
Find extra loads and deformation			
		Area of the ice (A_ice) [mm ²]	0.000
		Distributed load due to ice (Pu) [N/m]	0.000
		Total load due to ice (P) [N]	0.000
		Vertical position in catenary at xw (Y_wc) [m]	0.000
		Added tension force (h) [N]	0.000
		Additional deformation (w) [m]	0.000
Total tension Force. (Ft) [N]	0.000	Final positoin at xw (Y_w)	0.000
Reset			

Figure 31: Ice Loaded Conductor Section

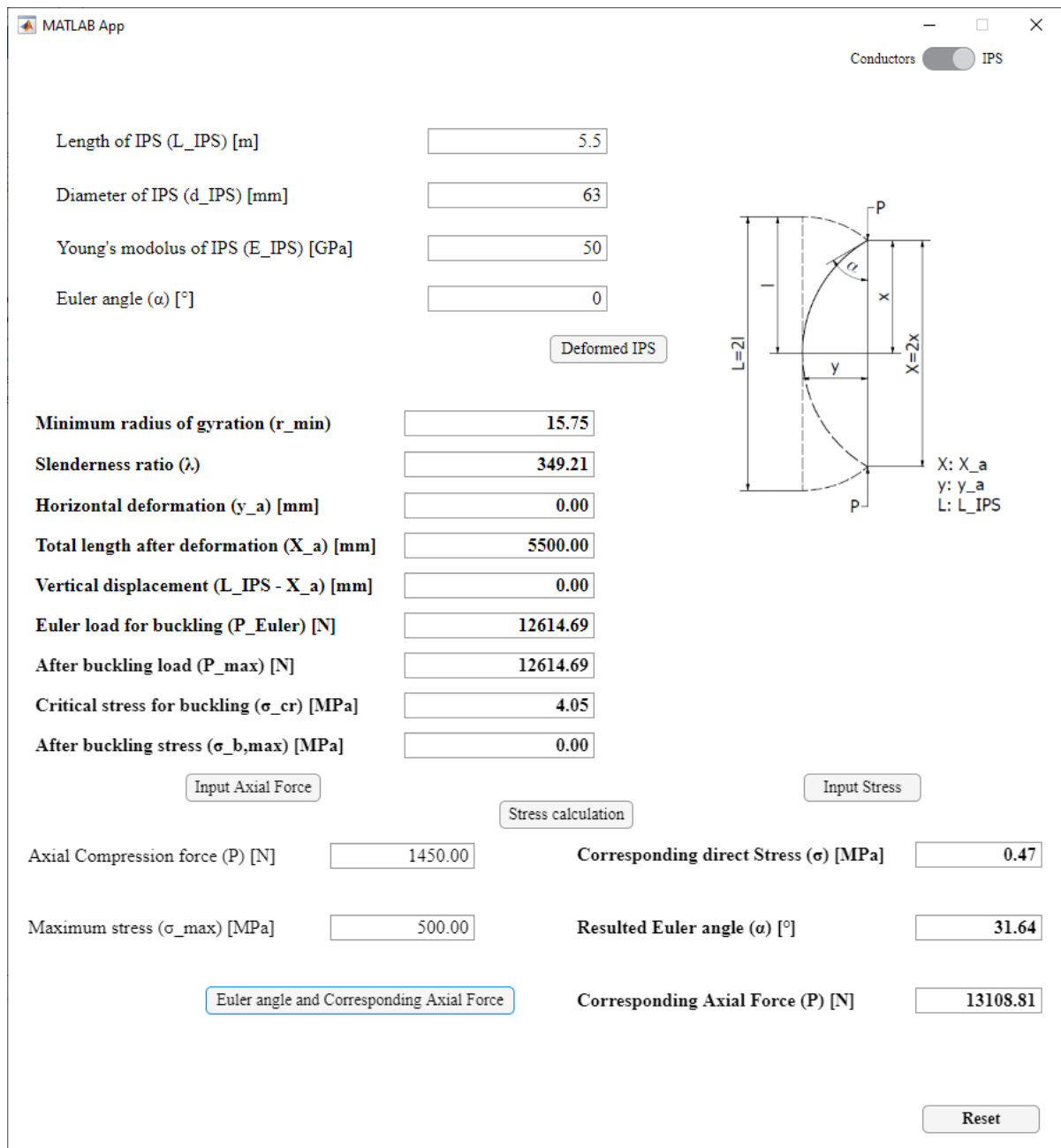


Figure 32: Buckling of IPS

8.2. Design of the buckling grip

We need to perform the buckling of the IPS in our university lab, which is why we need to consider our testing machine capabilities to design and perform a buckling test on the small samples. The grip must be designed to let the tested samples be free in rotation in one direction but fixed in transitions, as we need an effective length factor $k = 1$. Check Table 1.

We used the Autodesk Inventor software to design the parts of the grip and its technical drawing can be represented in Figure 33.

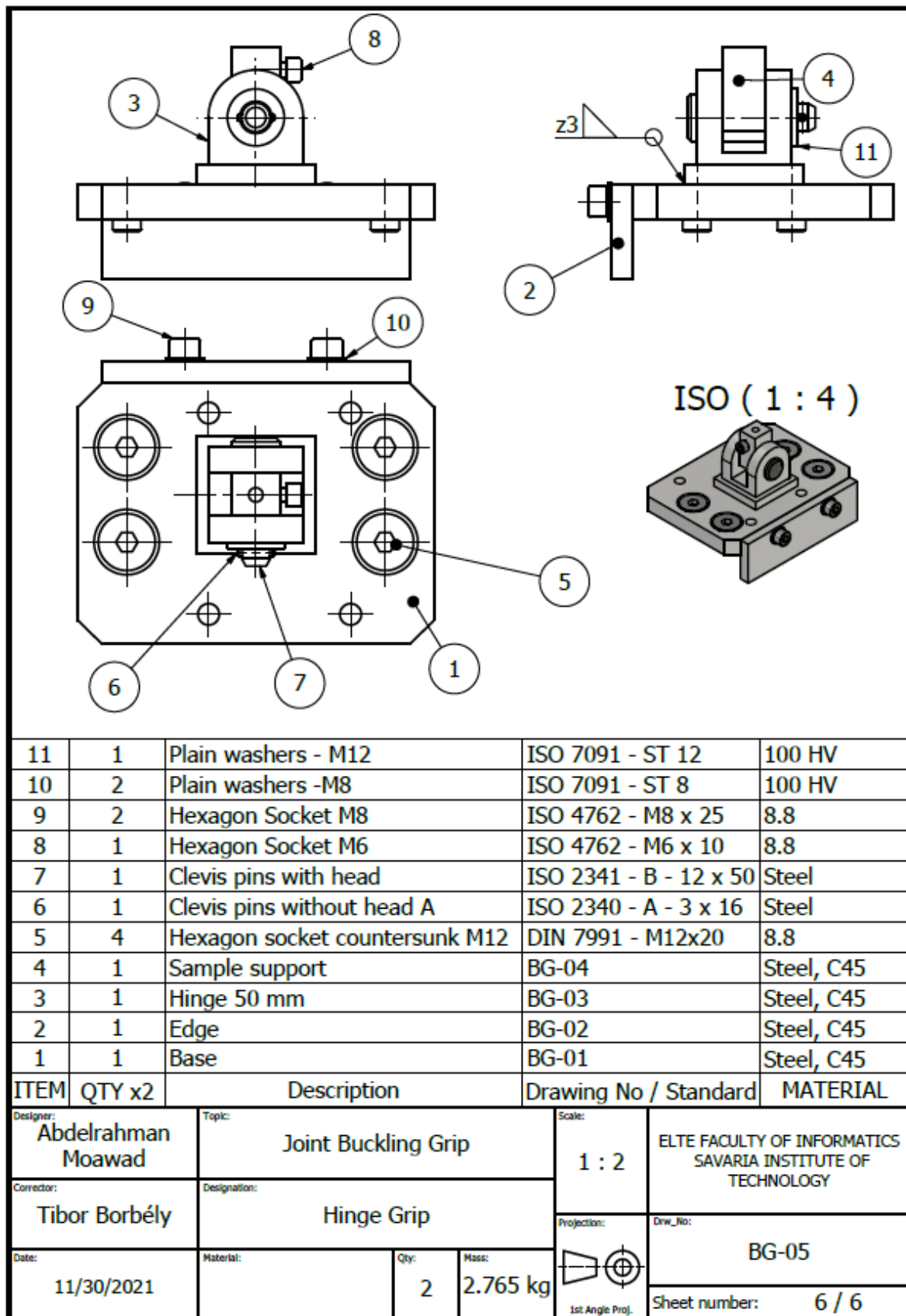


Figure 33: Technical drawings of the assembly of joint buckling grip