



SZAKDOLGOZAT FELADAT

Balogh Csanád

villamosmérnök hallgató részére

MATLAB implementation of a new component based TPA method

As the background noise of the latest vehicles vanishes due to the lack of internal combustion engine masking noise, the growing demand for electric vehicles challenges the NVH engineers in the automotive industry to further improve the acoustic quality of every subcomponent. In order to further reduce the noise level of each independent noise source in the vehicle, new transfer path analysis methods have been developed. One of the recently developed component based TPA techniques is the in-situ blocked force method.

The student working on the thesis should accomplish the following tasks:

- Based on the available literature summarize the theoretical background of the in-situ blocked force method
- Introduce the virtual point transformation method and explain the underlying equations
- Implement the method of FRF calculations from impact measurements
- Investigate possible implementations of the virtual point transformation
- Based on available measurement results execute in-situ blocked force calculations

Tanszéki konzulens: Dr. Fiala Péter doc.

Külső konzulens: Kimpián Tibor

Budapest, 2021. március 12.

Dr. Imre Sándor
egyetemi tanár
tanszékvezető

Konzulensi vélemények:

Tanszéki konzulens: |Beadható, |Nem beadható, dátum:

Külső konzulens: |Beadható, |Nem beadható, dátum:

aláírás:

aláírás:



Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Control Engineering and Information Technology

Csanád Levente Balogh

MATLAB IMPLEMENTATION OF A NEW COMPONENT BASED TPA METHOD

SUPERVISORS

Tibor Kimpián

Dr. Péter Fiala

BUDAPEST, 2021

Contents

Abstract	6
Kivonat	7
1 Introduction.....	8
2 Theoretical background.....	10
2.1 Describing the vibration of a system	10
2.1.1 Discrete Fourier transform	10
2.1.2 Describing an oscillating system	11
2.1.3 Frequency response functions	12
2.1.4 Calculation of FRFs in practice	13
2.2 Dynamic substructuring.....	14
2.2.1 Coupling with Frequency-Based Substructuring (FBS)	15
2.2.2 Interface restrictions	16
2.3 Transfer path analysis	18
2.3.1 Traditional TPA.....	18
2.3.2 Component-based TPA.....	19
2.3.3 Transmissibility TPA	24
2.4 Virtual point approach.....	25
2.4.1 Interface displacement modes	26
2.4.2 Transformation of forces.....	29
2.4.3 Transformation of displacements	30
2.4.4 Virtual point FRFs	30
2.4.5 Transformation quality indicators	31
2.5 Singular value decomposition.....	34
2.5.1 Singular value truncation	35
2.5.2 Moore-Penrose pseudoinverse	35
3 Measurement and analysis workflow	37
3.1 Main segments	37
3.1.1 Measurement	37
3.1.2 FRF calculation	39
3.1.3 Calculation of virtual point FRFs	43
3.1.4 Transfer path Analysis	44
3.2 Virtual point transformation on model	46
3.2.1 Consistency on model.....	46

3.2.2 Virtual point FRFs of the model.....	47
4 Application on a steering gear	50
4.1 Measurement.....	50
4.2 Observation on the measured FRFs.....	51
4.3 Virtual point transformation	54
4.3.1 Condition number and overdetermination	55
4.3.2 Consistency	56
4.3.3 Virtual point FRF reciprocity	57
4.4 TPA calculation.....	58
4.4.1 Validation.....	59
5 Conclusion	63
References	64
Figures	66

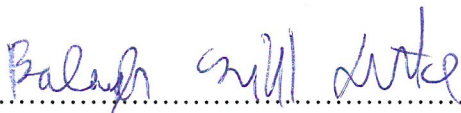
STUDENT DECLARATION

I, **Csanád Levente Balogh**, the undersigned, hereby declare that the present BSc thesis work has been prepared by myself and without any unauthorized help or assistance. Only the specified sources (references, tools, etc.) were used. All parts taken from other sources word by word, or after rephrasing but with identical meaning, were unambiguously identified with explicit reference to the sources utilized.

I authorize the Faculty of Electrical Engineering and Informatics of the Budapest University of Technology and Economics to publish the principal data of the thesis work (author's name, title, abstracts in English and in a second language, year of preparation, supervisor's name, etc.) in a searchable, public, electronic and online database and to publish the full text of the thesis work on the internal network of the university (this may include access by authenticated outside users). I declare that the submitted hardcopy of the thesis work and its electronic version are identical.

Full text of thesis works classified upon the decision of the Dean will be published after a period of three years.

Budapest, 13 May 2021


.....
Csanád Levente Balogh

Abstract

In this thesis, the theoretical background, and application of In-situ Transfer Path Analysis (TPA) will be presented. In-situ TPA is a method to characterize and predict the propagation of noise in and assembly. In this case, the TPA is applied on a steering gear, and the whole analysis is shown, augmented with measurement and calculation quality indicators every step of the way. In-situ TPA is part of the component TPA family, where the goal is to obtain interface forces that describe the active component. Doing so allows for vibration propagation predictions onto different receiver structures.

Virtual point transformation is an important part of the procedure, which makes it possible to project the FRFs describing the dynamic behavior of the gear to an arbitrary point, thus allowing for calculations in the exact connection point.

The thesis aims explain the relevant techniques and provide guidelines for the measurements and analysis that can be applied on steering gears for blocked force calculation either on a testbench or in the vehicle.

Kivonat

A dolgozatban bemutatásra kerül az In-situ TPA (Transfer Path Analysis – Átviteli Út Analízis) elméleti háttere és alkalmazása egy kormányművön. Az In-situ TPA egy rezgés és zaj terjedés elemzési módszer, ami lehetővé teszi az terjedés előrejelzését is. A későbbiekben a módszer alkalmazása és az teljes analízis egy kormánymű példáján keresztül lesz bemutatva, kiegészítve a mérés és az elvégzett számolások ellenőrzésére használt módszerekkel. Az in-situ TPA a komponens alapú TPA család tagja, ahol a cél interfész erők meghatározása, melyekkel az aktív komponens jellemezhető. Ez lehetőséget ad az aktív komponens által keltett rezgések előrejelzésére különböző fogadó struktúrákban.

A virtuális pont transzformáció a módszer fontos része, mely segítségével a kormánymű dinamikus viselkedését leíró FRF-ek egy tetszőleges pontra vetíthetők, így lehetségessé válik a kapcsolódási pont pontos leírása.

A dolgozat fő célja a releváns technikák bemutatása és egy metódus felállítása, amely minden kormánymű esetén alkalmazható interfész erők számítására akár tesztpadon, akár a járműben.

1 Introduction

The rapid development in the automotive industry created a demand for steering systems, that can help the driver, making driving easier and more comfortable. Unfortunately, steering systems (and other components in the vehicle created to help the driver) can generate noise inside the vehicle. This noise usually lowers the comfort of the passengers and may endanger their safety, by disturbing the driver. As a result, acoustic analysis became an important part in the development of auxiliary systems, such as steering gears. TPA is one of the most frequently used techniques to analyze the noise propagation in a vehicle.

Transfer path analysis (TPA) has been used in the industry for the evaluation of NVH (Noise Vibration and Harshness) problems for a long time. Every assembly can be divided into components. If a component capable of inducing vibration is present, this vibration is transferred to the rest of the assembly through the connection points. Most times this vibration induces unwanted noise and can distract the safe operation of a machine or vehicle. Although there are several studies concerning the reduction of vibration in naval [10] and aeronautic applications (and the research of vibrations transmission first occurred in these fields), analysis of vibration transfer has an increasing role in the automotive industry, as mentioned before. TPA stands for several methods designed for the evaluation of vibration transfer. The thesis mostly concentrates on the in-situ TPA. Measurements can take place with the steering gear connected to a testbench or with the gear placed in the vehicle. It is one of the component-based TPA methods, meaning the aim of the analysis is to find a set of “blocked forces” on the interface, that characterise the vibration generated by the active structure, in our case the steering gear.

In the process of obtaining blocked forces, virtual point transformation (VPT) is a crucial part and therefore has to be understood thoroughly and applied with utmost care. With VPT, the physical measurement can be transformed in the virtual point space, creating the possibility to characterise the connection points, or indeed any point chosen freely to be the virtual point. The methodology is presented in [19] and [15] and is used in blocked force calculation to obtain virtual point FRFs. With at least a factor of 1.5 or 2 times overdetermination of the forces and responses, VPT also acts as a regularisation technique, smoothing out the measurement data and compensating for example for the smaller inaccuracies in position or angle during the impulse hammer measurements. However, classical conditioning methods may also be used. In the thesis, SVD truncation will be shown in details.

Especially in an industrial environment with sharp deadlines, it is very important that errors and imprecisions in the measurement and calculations are spotted in time and can be corrected with the least effort possible. For that matter, a number of measurement and calculation quality indicators are also introduced, in several steps throughout the complete measurement and data processing workflow. Right after the impulse hammer measurement, the quality of the acquired FRFs can be checked, and also if the parts of the assembly assumed to be rigid, but contain considerable flexibility, it can be spotted. Next, there are some indicators to verify the virtual point transformation and the obtained virtual point FRFs. If all data seems sufficient, the TPA is calculated, after which on-board validation gives an opportunity to get an instant feedback of the results.

It is important to establish a procedure, that works in case of very steering gear (or other components for that matter) and is always applicable. Such procedure will be presented step by step and demonstrated on one particular steering gear at the end of the thesis.

2 Theoretical background

In the following, the important signal processing, coupling, virtual point transformation and TPA techniques are shown for in-situ blocked force calculation. Transfer path analysis or experimental dynamic substructuring alone has a very extensive literature available in themselves. Only the relevant methods are shown, based on the available instrumentation and infrastructure. In this section I mostly follow [8] to present the theory. For a more complete and detailed description see [8].

2.1 Describing the vibration of a system

The methods that are going to be examined in this thesis rely on describing the dynamic behavior of an assembly. For that purpose, a way of describing the assembly must be chosen. Luckily, there are several tools at hand, for example impulse response functions in the time domain, or Frequency Response Functions (FRFs) in the frequency domain. One could also describe the dynamic behavior in the modal or the state-space domain. In this thesis, frequency response functions are used to characterize the vibrations of a system. Here should stand a description of the theoretical derivation and practical meaning of the FRFs.

2.1.1 Discrete Fourier transform

Time domain signals can be transformed into the frequency domain using Fourier transformation. A version of Fourier transformation is Discrete Fourier Transformation (DFT), where the frequency domain samples $X(k)$ are calculated from the time domain samples $x(n)$ by the following discrete sum instead of the continuous integral:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi kn}{N}}, \quad (2.1)$$

where N is the number of time domain samples. With sampling frequency f_s , DFT calculates N samples, from 0 to f_s with equidistant bins. The frequency of a calculated value k will be $\frac{f_s}{N} k$. The frequency resolution is determined by the number of samples and the sampling frequency $\Delta f = \frac{f_s}{N}$, specifying the steps between the frequency components. In practice, for faster calculation Fast Fourier-Transform (FFT) is used. A more detailed description can be found in [1].

2.1.2 Describing an oscillating system

To get a closer look at the physical content of FRFs, let us consider a concentrated parameter oscillating system, in a similar manner as in [2]. Four masses connected serially to each other through parallel dampers and springs, where the excitations are the forces acting on the masses, and the responses are the displacements of the masses as seen in Figure 1.

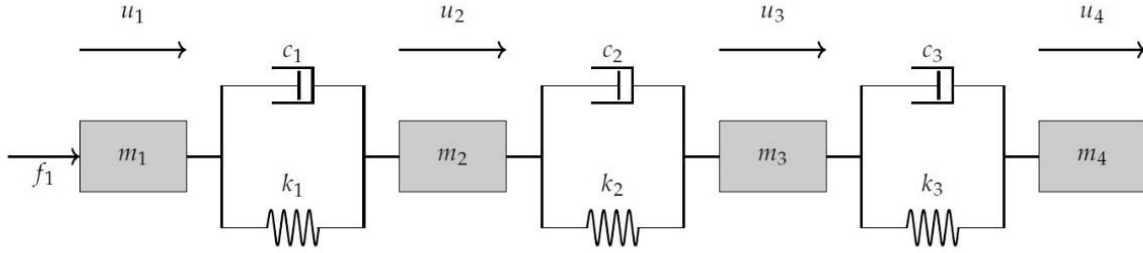


Figure 1: 4 DoF concentrated parameter oscillating system

By definition $f = ku$ for stiffness, $f = cv = c\dot{u}$ for damping and $f = ma = m\ddot{u}$ for the mass, where the dot represents derivation in time. The first mass m_1 is being accelerated by force f_1 and the forces caused by the first stiffness k_1 and the first damping c_1 . The second mass is being accelerated the forces caused by the first stiffness, the first damping, and also the forces of the second stiffness and damping. Following the same logic, 4 independent equations can be written to describe the complete system. The number of independent equations is also called Degrees of Freedom (DoF). The 4 DoFs in this case are the 4 displacements of the masses.

$$k_1(u_1(t) - u_2(t)) + c_1(\dot{u}_1(t) - \dot{u}_2(t)) + m_1\ddot{u}_1(t) = f_1(t) \quad (2.2a)$$

$$k_2(u_2(t) - u_3(t)) + c_2(\dot{u}_2(t) - \dot{u}_3(t)) + m_2\ddot{u}_2(t) + k_1(u_2(t) - u_1(t)) + c_1(\dot{u}_2(t) - \dot{u}_1(t)) = 0 \quad (2.2b)$$

$$k_3(u_3(t) - u_4(t)) + c_3(\dot{u}_3(t) - \dot{u}_4(t)) + m_3\ddot{u}_3(t) + k_2(u_3(t) - u_2(t)) + c_2(\dot{u}_3(t) - \dot{u}_2(t)) = 0 \quad (2.2c)$$

$$k_3(u_4(t) - u_3(t)) + c_4(\dot{u}_4(t) - \dot{u}_3(t)) + m_4\ddot{u}_4(t) = 0 \quad (2.2d)$$

The same equation system organised into a matrix:

$$\begin{aligned}
& \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 \\ 0 & -k_2 & k_3 + k_2 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 & 0 \\ -c_1 & c_1 + c_2 & -c_2 & 0 \\ 0 & -c_2 & c_3 + c_2 & -c_3 \\ 0 & 0 & -c_3 & c_3 \end{bmatrix} \begin{bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \\ \dot{u}_3(t) \\ \dot{u}_4(t) \end{bmatrix} \\
& + \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{bmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \\ \ddot{u}_3(t) \\ \ddot{u}_4(t) \end{bmatrix} = \begin{bmatrix} f_1(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.3)
\end{aligned}$$

The equation system of Eq. (2.3) could also be written as:

$$Ku(t) + C\dot{u}(t) + M\ddot{u}(t) = f(t) \quad (2.4)$$

This system of equations can be used for any oscillating or vibrating system. This is the base equation to characterise the vibrations of a given assembly.

2.1.3 Frequency response functions

As it has been mentioned before, in this method FRFs are used to describe the dynamic behavior of the system. First, Eq. (2.3) is transformed to the frequency domain, note that derivation in time domain is multiplying by $j\omega$ in frequency domain.

$$(K + j\omega C - \omega^2 M)u(\omega) = f(\omega)$$

$$Z(\omega)u(\omega) = f(\omega) \quad (2.5)$$

$Z(\omega)$ in Eq. (2.4) contains complex values and is called dynamic stiffness matrix. It embodies the mass, damping and stiffness attributes of the system we want to characterise. At a given frequency, one element of the dynamic stiffness matrix is given by: $Z_{ji}(\omega) = \frac{f_j(\omega)}{u_i(\omega)}$ assuming the other elements are zero. Here it can be seen, that Z represents the force one need to apply to acquire unit displacement. FRF (which is the transfer function of the system) is defined as the inverse of that Z :

$$Y_{ij}(\omega) = \frac{u_i(\omega)}{f_j(\omega)} \quad (2.6)$$

At a given frequency, the FRF matrix contains the transfer coefficients of all the responses to all the excitations. For a N DoF system:

$$Y(\omega) = \begin{bmatrix} \frac{u_1(\omega)}{f_1(\omega)} & \dots & \frac{u_1(\omega)}{f_N(\omega)} \\ \vdots & \ddots & \vdots \\ \frac{u_N(\omega)}{f_1(\omega)} & \dots & \frac{u_N(\omega)}{f_N(\omega)} \end{bmatrix} \quad (2.7)$$

Using FRFs seems to be a more suitable approach for describing the dynamics of a system. If one takes one column of the matrix of Eq. (1.6) all responses for a certain excitation can be observed. By looking at one row of the matrix, one can see all the response of one degree of freedom for all the excitations.

2.1.4 Calculation of FRFs in practice

Obtaining FRFs in practice takes some further consideration. It is important to know, that while conducting measurements, one can only get an estimation of the actual transfer function. To estimate the transfer function as accurately as possible, one can conduct several measurements and calculate the average. It is also possible to calculate H_1 or H_2 estimators, to reduce the effect of noise (2.1.4.2).

2.1.4.1 Auto and cross power spectral density

Power spectral density provides the distribution of power between the frequency components of the signal. Using power spectral density to calculate the FRFs, results in a more may be less affected by measurement noise thus providing a better estimate of the system's real FRFs. The two power spectra can be found as:

$$\begin{aligned} \text{auto: } P_{XX}(\omega) &= X(\omega)X^*(\omega) \\ \text{cross: } P_{FX}(\omega) &= F(\omega)X^*(\omega) \end{aligned} \tag{2.8}$$

In Eq. (1.8) $X(\omega)$ denotes the spectrum of the response, $F(\omega)$ denotes the spectrum of the excitation and the $*$ in $X^*(\omega)$ denotes complex conjugate. Although there are several methods for power spectra estimation, the Welch method [3] is a widely accepted and used approach, as it is a stable method with quite low computational time. During the implementation in MATLAB I used the *pwelch*, *tffestimate* and *mscoherence* functions. Each of these functions uses the Welch method.

2.1.4.2 Detecting and decreasing noise

As it has been mentioned above, in practice, only the calculation of transfer function estimators is possible, as noise is always present in the system in the case of real applications. For decreasing the effect of noise, different transfer function estimators can be of help. Two important estimators (that are also available in MATLAB) are the H_1 and H_2 estimators. H_1 estimator is useful when the noise is dominantly added to the system at the output, as it assumes noiseless input and noisy output. H_2 estimator assumes noiseless output signal and a noisy input signal. Therefore, it is handy when the noise is mainly added to the system at the input. For the calculation of the estimators, we use the power spectrum described in section 2.1.4.1.

$$H_1(\omega) = \frac{P_{XF}(\omega)}{P_{FF}(\omega)} \quad (2.9a)$$

$$H_2(\omega) = \frac{P_{XX}(\omega)}{P_{FX}(\omega)} \quad (2.9b)$$

It is also common practice to conduct more measurements for the same transfer function and calculate the average. In an ideal scenario, all the measurements would be the same, however, it is not true in a real application. Comparison of the different measurements provides an opportunity to evaluate the precision of our estimation. If the measurements yield roughly the same result at a given frequency, the estimation should be very close to the actual transfer function. If the measured transfer functions greatly differ from each other, that indicates the presence of considerable noise or nonlinearity in the system. The coherence function serves exactly this purpose. It correlates the input and the output, also using power spectra functions:

$$coh(\omega) = \frac{|P_{FX}(\omega)|^2}{P_{FF}(\omega)P_{XX}(\omega)} \quad (2.10)$$

The value of the coherence varies in the range of 0 and 1, 0 meaning no correlation between the input and the output and 1 meaning perfect correlation. The coherence function also provides an opportunity to give an estimation on the signal to noise ratio ([18]) with the following equation:

$$SNR = \frac{coh(\omega)}{1 - coh(\omega)} \quad (2.11)$$

Although there is no data available about the noise, for the above-described reasons it can ruin the value of the coherence function, which makes it possible to approximate the SNR.

2.2 Dynamic substructuring

Component-based TPA analyzes the vibration being transferred between two systems, but first, we have to describe the behaviour of the system at the connection point and investigate the way their behaviour changes as a result of connecting them. Dynamic Substructuring (DS) refers to a number of techniques developed to do exactly that. These methods divide the whole structure into substructures, analyze the models of the substructures and connect them to describe the original system. DS has extensive literature (e.g. [4], [5], [6]), in this thesis, the approach and notations of [7] and [8] are followed. Both time and frequency domain-based methods exist. Here, the use of frequency-based experimental dynamic substructuring is described in more detail. It is frequency-based, as it uses frequency response functions (shown in section 2.1.3). FRFs could be acquired in several ways. In possession of FE or other numerical models, a modal synthesis can be implemented,

providing the wanted FRFs. Although it is a plausible method, it includes the error of the modal analysis and the inaccuracy caused by omitting higher modes in the calculation of the FRFs. Instead, the data is obtained from measurements, hence the experimental motion. As the measurements reflect the actual physical system, they tend to provide more accurate results than numerical methods. On the other hand, rotational degrees of freedom (RDoF) (that are of great importance in dynamic substructuring, see [9]) are hard to measure in practice. To solve this problem virtual point transformation is introduced, as discussed in section 2.4.

2.2.1 Coupling with Frequency-Based Substructuring (FBS)

For the connection of an active and a passive component let us consider a simple representation. Two currently uncoupled components are shown in Figure 2.

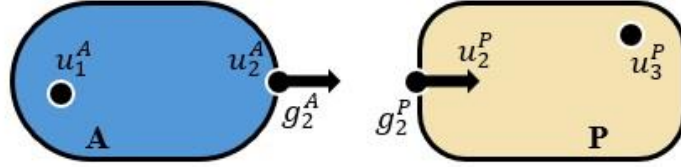


Figure 2: Uncoupled subsystems: active (A) and passive (P)

In theory, external force solely acts on the system at the first node. This is the part of the active system that generates the vibration. The interface forces by definition act on the interface (the connection point) at node 2. To represent the entire, yet uncoupled system, the vectors of (2.13a) and (2.13b) can be arranged into one vector and the matrices can be arranged into block diagonal format.

$$Y \stackrel{\text{def}}{=} \begin{bmatrix} Y_{11}^A & Y_{12}^A & 0 & 0 \\ Y_{21}^A & Y_{22}^A & 0 & 0 \\ 0 & 0 & Y_{22}^P & Y_{23}^P \\ 0 & 0 & Y_{32}^P & Y_{33}^P \end{bmatrix}; \quad u = \begin{bmatrix} u_1^A \\ u_2^A \\ u_2^P \\ u_3^P \end{bmatrix}; \quad f = \begin{bmatrix} f_1^A \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad g = \begin{bmatrix} 0 \\ g_2^A \\ g_2^P \\ 0 \end{bmatrix} \quad (2.12)$$

The attributes of the two subsystems can be written separately with admittance notation as follows:

$$Y^A \stackrel{\text{def}}{=} \begin{bmatrix} Y_{11}^A & Y_{12}^A \\ Y_{21}^A & Y_{22}^A \end{bmatrix}; \quad u^A = \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}; \quad f^A = \begin{bmatrix} f_1^A \\ f_2^A \end{bmatrix} = \begin{bmatrix} f_1^A \\ 0 \end{bmatrix}; \quad g^A = \begin{bmatrix} 0 \\ g_2^A \end{bmatrix} \quad (2.13a)$$

$$Y^P \stackrel{\text{def}}{=} \begin{bmatrix} Y_{22}^P & Y_{23}^P \\ Y_{32}^P & Y_{33}^P \end{bmatrix}; \quad u^P = \begin{bmatrix} u_2^P \\ u_3^P \end{bmatrix}; \quad f^P = \begin{bmatrix} f_2^P \\ f_3^P \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad g^P = \begin{bmatrix} g_2^P \\ 0 \end{bmatrix} \quad (2.13b)$$

In the equations and expressions above superscript A denotes the attributes of the active system, and superscript P denotes the attributes of the passive system. Y denotes the admittance matrices, u the displacements, f the external forces and g the internal (interface) forces.

The governing equation of FBS coupling is:

$$u = Y(f + g) \quad (2.14)$$

It is assumed, that the external forces and the admittances are known. The DoFs of the system are the displacements that are at this point unknown as well as the interface forces. The number of independent equations of the system is the same as the number of the displacements. Consequently, the displacements cannot be determined. One solution for this problem is to introduce two restrictions on the interface, one for the interface displacements, and one for the interface forces.

2.2.2 Interface restrictions

As mentioned before, there are two important restriction on the interface. The first, referred to as compatibility in [7] and [8] describing the connection of the interface displacements. If it can be assumed, that the connection is absolutely rigid, then u_2^A and u_2^P shall be the same. In case of more connection points, the same can be said about the corresponding displacement on the active and passive side.

$$u_2^A = u_2^P \quad (2.15)$$

A different notation with the same meaning can be introduced, using a Boolean matrix, which basically chooses the appropriate elements of the system displacement vector.

$$Bu = 0 \quad (2.16)$$

For example, in the case of the system on Figure 2, B would be defined as,

$$B \stackrel{\text{def}}{=} \begin{bmatrix} 0 & -1 & 1 & 0 \end{bmatrix} \quad (2.17)$$

Then the result of Bu would be

$$u_2^P - u_2^A = 0 \quad (2.18)$$

The second notation might be more practical when writing system equations. Boolean matrix notation is uses in section 2.2.2.1, for the dual assembly method.

The second restriction on the interface describes the connection of the forces acting on the interface. This restriction basically is the application of the third law of Newton stating $F_{12} = -F_{21}$, meaning when one object exerts a force on another object, the other object exerts a force with the same magnitude on the first object. This meant that the forces acting on the two objects act in the opposite direction.

$$g_2^A = -g_2^P \quad (2.19)$$

In a similar manner as the displacement notation, a Boolean matrix can provide a more practical approach to express the same equilibrium.

$$g = -B^T \lambda \quad (2.20)$$

For the system of Figure 2 B is already defined in Eq. (1.16), and the result of $-B^T \lambda$ would be:

$$\begin{cases} g_1^A &= 0 \\ g_2^A &= \lambda \\ g_2^P &= -\lambda \\ g_3^P &= 0 \end{cases} \quad (2.21)$$

2.2.2.1 Dual assembly

Dual assembly uses the subsystem admittances and the restrictions on the interface described above for the coupling of the subsystems. The notations of Eq. (2.16) and Eq. (2.20) are used to add the discussed restrictions to the governing equation of coupling. Eq. (2.14) augmented with the interface restrictions yields the system of equations of Eq. (2.22).

$$\begin{cases} u = Y(f - B^T \lambda) \\ Bu = 0 \end{cases} \quad (2.22)$$

Substituting the first equation of Eq. (2.22) into the second one the following equation is obtained:

$$BY(f - B^T \lambda) = 0$$

Afterwards the equation above is solved for λ .

$$\lambda = (BYB^T)^{-1}BYf$$

The result can be substituted back into Eq. (1.22) to acquire the displacements.

$$\begin{aligned} u &= Y[f - B^T(BYB^T)^{-1}BYf] \\ u &= Yf - YB^T(BYB^T)^{-1}BYf \end{aligned} \quad (2.23)$$

A FRF matrix for the coupled system can be constructed in a way that $u = Y^c f$.

$$Y^c = Y - YB^T(BYB^T)^{-1}BY \quad (2.24)$$

This admittance matrix describes the dynamic behaviour of the coupled system. Figure 2 is a simple representation of two subsystems, but the obtained equations can be used for more complex systems with several connection points. Other coupling methods exist that are adequate to describe the dynamic behaviour of the coupled system, for example primal assembly using subsystem impedances and a different logic for reducing the unknowns of the uncoupled equation system. For further discussion, see [8].

2.3 Transfer path analysis

As discussed in the Introduction, noise propagation induces comfort and safety issues in a vehicle. For instance, in a steering system, the vibration of the small electromotor, through other components, travels inside the car as noise. Apart from the discomfort the noise causes, it also may distract the driver. In this segment, the relevant TPA methods are presented and explained with a simple system of one active and one passive component starting with the traditional TPA. Later the component TPA is shown, which gives an opportunity of receiver independent characterisation and hence is an improved version of traditional TPA. For further interest [8] and [11] provides a great overview for different TPA approaches.

2.3.1 Traditional TPA

In classical transfer path problem, measurements are conducted on a coupled assembly. The coupled system can be divided into subsystems as shown in Figure 3. On the left, the coupled system, where the displacement at node 3 (u_3) is being measured caused by the excitation at node 1 (f_1). The dynamical behaviour of the system is described by the transfer function Y_{31} . The connection point (node 2) has no role in this case. Divided into an active and a passive component, the same dynamics can be described by using the coupling equations of section 2.2. Here however, the interface point at node 2 becomes important, and the attributes of the interface are included in the TPA method.

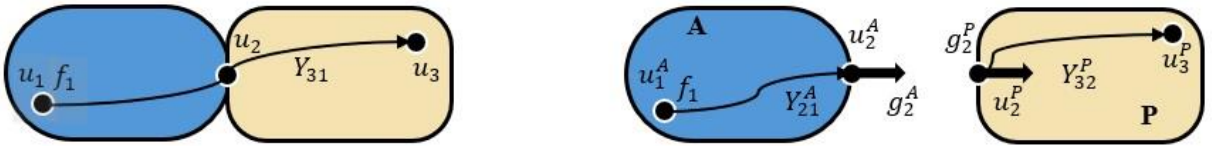


Figure 3: Traditional TPA: Coupled (left) and uncoupled (right) representation

The governing coupling equation (Eq. (2.14)) can be expressed with the subsystem admittances, forces, and displacements.

$$\begin{bmatrix} u_1^A \\ u_2^A \\ u_2^P \\ u_3^P \end{bmatrix} = \begin{bmatrix} Y_{11}^A & Y_{12}^A & 0 & 0 \\ Y_{21}^A & Y_{22}^A & 0 & 0 \\ 0 & 0 & Y_{22}^P & Y_{23}^P \\ 0 & 0 & Y_{32}^P & Y_{33}^P \end{bmatrix} \left(\begin{bmatrix} f_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2^A \\ g_2^P \\ 0 \end{bmatrix} \right) \quad (2.25)$$

Afterwards, the restrictions on the interface of Eq. (2.16) and Eq. (2.20) can be applied. Equating the second and third line, and substituting λ into the resulting equation the following is obtained:

$$Y_{21}^A f_1 + Y_{22}^A \lambda = -Y_{22}^P \lambda$$

$$-\lambda = g_2^P = (Y_{22}^A + Y_{22}^P)^{-1} Y_{21}^A f_1 \quad (2.26)$$

This formula provides the expression for the external force at the passive side knowing the admittances of the active and passive sides and the force generating the vibration. Eq. (2.26) describes the propagation of the noise from the source (node 1) to the interface. To be able to characterize the effect of the excitation at the response DoF (node 3), we need to take the dynamics of the passive system into consideration, which can be done by substituting the result of Eq. (2.26) into the last line of Eq. (2.25).

$$\begin{aligned} u_3 &= Y_{32}^P g_2^P \\ u_3 &= Y_{32}^P (Y_{22}^A + Y_{22}^P)^{-1} Y_{21}^A f_1 \end{aligned} \quad (2.27)$$

Looking at Figure 3, the picture on the left represents the whole assembly, without substructuring. The equation for the response is quite simple.

$$u_3 = Y_{31} f_1$$

Comparing that to Eq. (2.27), it is easy to see, how the system admittance can be built from the substructure admittances.

$$\begin{aligned} Y_{31} &= Y_{32}^P (Y_{22}^A + Y_{22}^P)^{-1} Y_{21}^A \\ u_3 &= Y_{32}^P (Y_{22}^A + Y_{22}^P)^{-1} Y_{21}^A f_1 \end{aligned} \quad (2.28)$$

In practice, it would be problematic to calculate the response based on Eq. (2.28) as the excitation f_1 is typically not known, and measuring the transfer function from node 1 to node 2 (Y_{21}^A) is also really difficult. All other admittances are measurable. Solutions to the traditional transfer path problem can be found for example in [8] or [11].

2.3.2 Component-based TPA

Component-based TPA introduces a different approach. While in traditional TPA the assembly is being handled as a whole, the component-based approach divides the assembly into components and attempts to describe the active component, that is generating the noise, independently from the passive structure. This way, changes in the design do not require new measurements until the active component is unchanged. This is achieved by describing the parameters of the interface. The idea is similar to finding the Thevenin and Norton equivalents of an electrical circuit.

2.3.2.1 Equivalent forces and responses

The aim is to achieve a set of forces at the interface that causes the same displacement at the response node u_3 as the original excitation f_1 . As a result, the active structure practically can be replaced with interface forces. Furthermore, the interface forces are a property solely of the active component, meaning the measurements can be conducted on any passive component, or even on a testbench. The equation for the responses calculated from the equivalent forces can be obtained from Eq. (2.28). With forces acting on the interface, the response u_3 is not influenced by the transfer function of the source and the interface.

$$u_3 = Y_{32}f_{eq} = Y_{32}^P(Y_{22}^A + Y_{22}^P)^{-1}Y_{22}^Af_{eq}$$

The equivalent force must yield the same response as the original source.

$$Y_{32}^P(Y_{22}^A + Y_{22}^P)^{-1}Y_{22}^Af_{eq} = Y_{32}^P(Y_{22}^A + Y_{22}^P)^{-1}Y_{21}^Af_1$$

$$f_{eq} = (Y_{22}^A)^{-1}(Y_{21}^A)f_1 \quad (2.29)$$

Using Eq. (2.29) the equivalent forces can be found in different cases. Later in-situ TPA is going to be used (introduced by A.T. Moorhouse, A.S. Elliot, and T.A. Evans in [12]), but for a better understanding of the underlying equations and the equivalent source concept, a few more methods is shown in the following sections.

2.3.2.2 Blocked force TPA

The blocked force method is similar to the Thevenin equivalent with discontinuity, where the voltage is analogue to the force and the current is analogue to the displacement. Here, the interface on the active component is fixed. Consequently, a restriction is given for the interface displacements. The rigid, fixed connection prevents any movement of the interface node $u_2 = 0$.

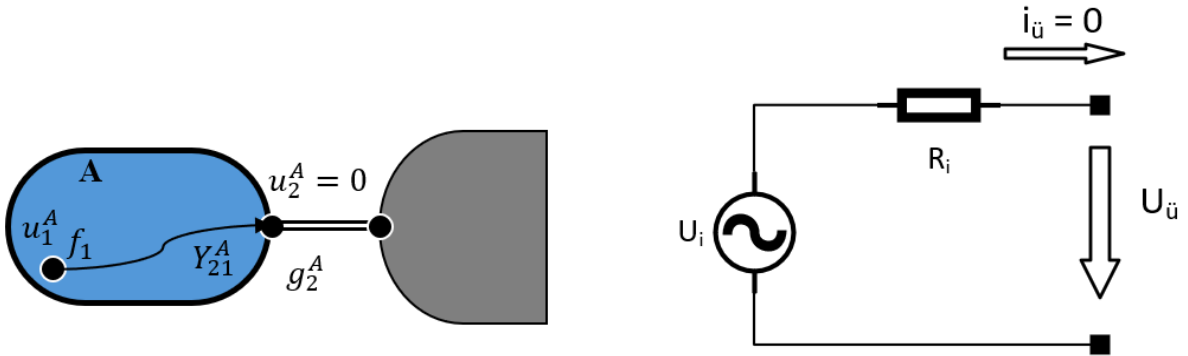


Figure 4: Blocked force TPA

The equation of the system on Figure 4:

$$\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} = \begin{bmatrix} u_1^A \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11}^A & Y_{12}^A \\ Y_{21}^A & Y_{22}^A \end{bmatrix} \begin{bmatrix} f_1 \\ g_2^A \end{bmatrix}$$

$$g_2^A = -g_2^B$$

g_2^B refers to the blocked force at the rigid connection. Solving the second line for g_2^B yields:

$$g_2^B = (Y_{22}^A)^{-1}(Y_{21}^A)f_1$$

Comparing the result to Eq. (2.29), it can be found that they are the same, meaning the expression for equivalent forces is simply:

$$f_{eq} = g_2^B \quad (2.30)$$

The resemblance between Thevenin equivalent and block forces might be more conspicuous looking at Eq. (2.30). With no displacement (no current flowing) the force (voltage) can be measured at the interface. It is a simple expression for the equivalent forces, although it is very hard in practice to create absolutely rigid, fixed connection at the interface. This is especially true for a larger assembly such as a steering gear and/or at higher frequencies.

2.3.2.3 Free displacement TPA

Another simpler concept is the free displacement TPA (analogue to the Thevenin equivalent with a sorted circuit), where the restriction is for the interface force. It can be conducted for example with the active component hung, providing free movement at the interface. It means that the interface force is zero $g_2^A = 0$.

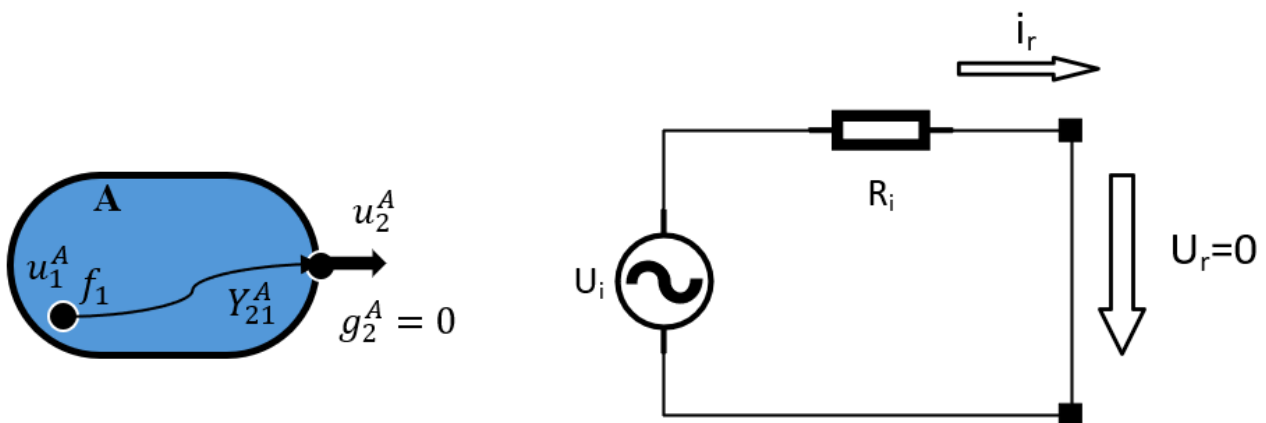


Figure 5: Free displacement TPA

Similarly to the blocked force a system of equation can be written for the active component with the restriction of the interface.

$$\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} = \begin{bmatrix} Y_{11}^A & Y_{12}^A \\ Y_{21}^A & Y_{22}^A \end{bmatrix} \begin{bmatrix} f_1 \\ 0 \end{bmatrix}$$

The second line can be solved for u_2^A .

$$u_2^A = Y_{21}^A f_1$$

Comparing the result to Eq. (2.29):

$$f_{eq} = (Y_{22}^A)^{-1}(Y_{21}^A)f_1 = (Y_{22}^A)^{-1}u_2^A \quad (2.31)$$

While the force (voltage) is zero, the displacement (current) can be measured. Again, a simple expression and a simple measurement method in theory, that can be impractical and difficult in practice. It has also been pointed out in related research, that the free displacement method is likely to work better on higher frequencies. Considering that in noise reduction the frequency band of interest is usually from about 20 Hz to a few kHz, free displacement seems less applicable than the other component-based methods. Furthermore, the free displacement condition does not allow any load to be applied on the investigated system and therefore in such a way it is not possible to investigate more operating points that are relevant in the end application of the component what is a severe disadvantage as the operating point usually influences the excitation forces significantly.

2.3.2.4 Hybrid interface TPA

A more practical approach is conducting measurements on a testbench (analogy on Figure 6). In this case, the application of strict restrictions on the interface is not possible. The measurements will not just reflect the dynamics of the active component but the dynamics of the testbench too.

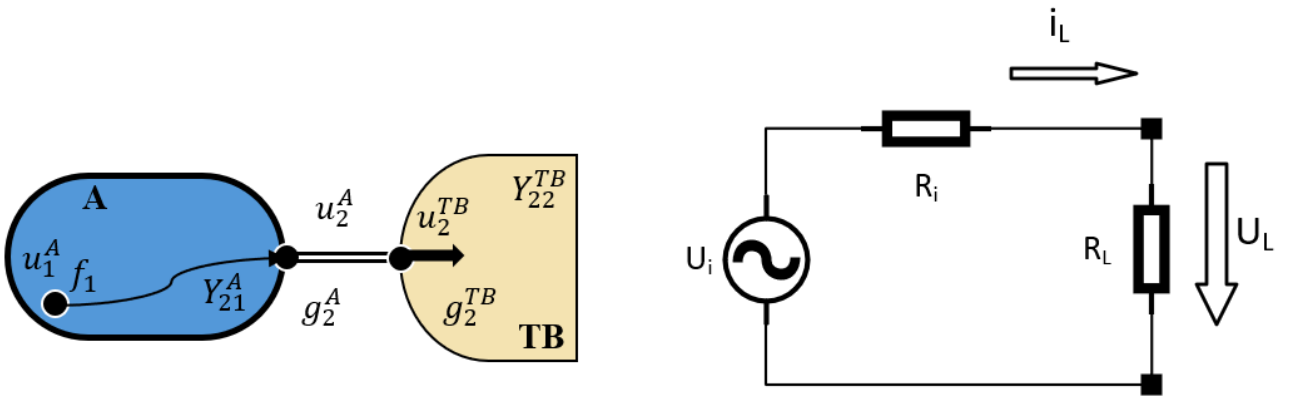


Figure 6: Hybrid interface TPA

For describing the system, the coupling equations of section 2.2 are used.

$$\begin{bmatrix} u_1^A \\ u_2^A \\ u_2^{TB} \end{bmatrix} = \begin{bmatrix} Y_{11}^A & Y_{12}^A & 0 \\ Y_{21}^A & Y_{22}^A & 0 \\ 0 & 0 & Y_{22}^{TB} \end{bmatrix} \begin{bmatrix} f_1 \\ g_2^A \\ g_2^{TB} \end{bmatrix} \quad (2.32)$$

Just as the derivation of $-\lambda$ in section 2.3.1 Eq. (2.26), u_2^{TB} is found by combining the second and third line of Eq. (2.32) with the interface restrictions of section 2.2.2.

$$g_2^{TB} = (Y_{22}^A + Y_{22}^{TB})^{-1} Y_{21}^A f_1 \quad (2.33)$$

Substituting Eq. (2.33) back to the second line of Eq. (2.32), the displacement at the interface can be found.

$$u_2^{TB} = Y_{21}^A f_1 - Y_{22}^A (Y_{22}^A + Y_{22}^{TB})^{-1} Y_{21}^A f_1 \quad (2.34)$$

The next step is eliminating Y_{22}^{TB} from Eq. (2.33) and Eq. (2.34) while combining them. The equivalent force is obtained:

$$f_{eq} = g_2^{TB} + (Y_{22}^A)^{-1} u_2 \quad (2.35)$$

The superscript of u_2 is omitted as the restriction of Eq. (2.16) stands. It is interesting to observe that the first expression on the right side of Eq. (2.35) is the blocked force, and the second expression in the same as the solution for equivalent force with the free displacement method. g_2^{TB} can be expressed with admittance and displacement, meaning that is possible to conduct TPA without measuring forces.

$$f_{eq} = (Y_{22}^{TB})^{-1} u_2 + (Y_{22}^A)^{-1} u_2 \quad (2.36)$$

Although force measurement is omissible, this way FRF measurements need to be conducted on both the testbench and the active component, which is demanding and may take a lot of time and considerable effort.

2.3.2.5 In-situ TPA

Alternatively, a passive component can be used to conduct the measurements. Interface forces can be calculated in a similar manner as the hybrid interface method. Actually, a testbench can be considered a passive side as well, if it is not too rigid.

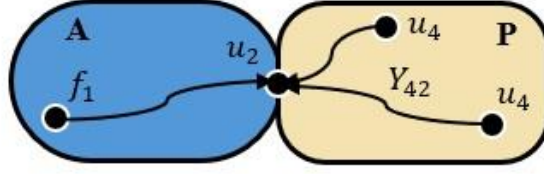


Figure 7: In-situ TPA

Switching to resistance notation, Eq. (2.36) is written as:

$$\begin{aligned}
 f_{eq} &= Z_{22}^{TB} u_2 + Z_{22}^A u_2 \\
 f_{eq} &= (Z_{22}^P + Z_{22}^A) u_2 = Z_{22}^{AP} u_2 \\
 f_{eq} &= (Y_{22}^{AP})^{-1} u_2
 \end{aligned} \tag{2.37}$$

This means that instead of the admittances of the interface on the two sides separately, the dynamic behaviour can be characterised by the admittances of the coupled system interface. This is a great improvement. Eq. (2.37) stands for in-situ TPA in an operational manner. The receiver DoFs used with the operational measurements are also the ones used for the impact hammer measurements beforehand obtaining the FRFs. This means that the receiver DoFs are placed close to the interface. Alternatively, separate receiver DoFs u_4 can be chosen for the operational measurement.

$$f_{eq} = (Y_{42}^{AP})^+ u_4 \tag{2.38}$$

Not having to replace the sensors is an advantage of the first alternative. Measurements were conducted in operational conditions and the results are presented in section 4.4. All methods discussed above impose restrictions on the interface. If the restrictions are not met, the result of the TPA will not be able to reflect the reality. For example, conducting measurements on a testbench, the rigidity of the testbench determines if the blocked force method or the in-situ method can be used.

2.3.3 Transmissibility TPA

Transmissibility TPA similarly to in-situ TPA is calculated on a coupled system. It uses indicator DoFs placed near the interface, and response DoFs at the places of interest on the passive structure.

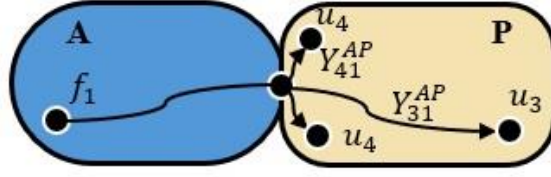


Figure 8: Transmissibility TPA

This method is usually better to identify path contributions. Calculation of forces can be skipped and FRFs are many times determined from operational measurements instead of impact hammer measurements. Results tend to be less accurate using transmissibility TPA, but with the right conditions, the accuracy can be improved to the level of component-based TPA methods. Response and indicator DoFs are calculated using the excitation f_1 and the corresponding admittance.

$$u_4 = Y_{41}^{AP} f_1 \quad (2.39)$$

$$u_3 = Y_{31}^{AP} f_1 \quad (2.40)$$

In possession of the indicator DoF data u_4 the excitation f_1 can be calculated with rearranging Eq. (2.39).

$$f_1 = (Y_{41}^{AP})^+ u_4$$

Subscript $+$ denotes Moore-Penrose pseudoinverse. Generally, Eq. (2.39) is either over or underdetermined. Substituting the result into Eq. (2.40) the responses are acquired as a function of the indicators.

$$u_3 = Y_{31}^{AP} (Y_{41}^{AP})^+ u_4 = Y_{34}^{AP} u_4 \quad (2.41)$$

Discussion of transmissibility TPA ends here. For the purpose it is used in this thesis Eq. (2.41) and the way of acquiring this result will be of importance. For more detailed discussion and deeper analysis of the method, see [8] or [13].

2.4 Virtual point approach

In sections concerning coupling and TPA, one or more interface points were used as a key part of connecting substructures and tracing the propagation of vibrations in an assembly. As mentioned in section 2.2, the use of rotational DoFs is advisable. Thus, both translational and rotational displacements should be measured precisely in the connection point. Most times placing sensors exactly at the connection point is impossible or at least extremely difficult. Instead, virtual

point transformation is performed. The so-called virtual points (VP) are the connection or interface points, where the active component is able to influence the passive component. With a spatial transformation, it is possible to transform the measured data to the VP. A set of virtual point displacements and forces are obtained, that act if the sensors were placed in the VP and the excitations were acting on it. It is possible to use rotational sensors for measuring RDoFs [14]. Alternatively, rotational DoFs can be calculated based on solely translational measurements ([8], [15]) which requires fewer sensors, thus less hardware capacity.

2.4.1 Interface displacement modes

The basis of the transformation is a spatial reduction. A virtual point is described by 6 virtual displacements and 6 virtual forces. The 6 virtual DoFs represent 6 rigid motions. Possible flexibility of the assembly is not accounted for by the virtual point transformation. Translation in the X, Y, Z direction, and rotation along the same directions. These are called interface displacement modes (IDMs). These motions are contained by the IDM matrix R . The calculation of the IDM matrix of both the displacement and the forces requires 2 types of information about the geometry of the assembly. Distances and orientations.

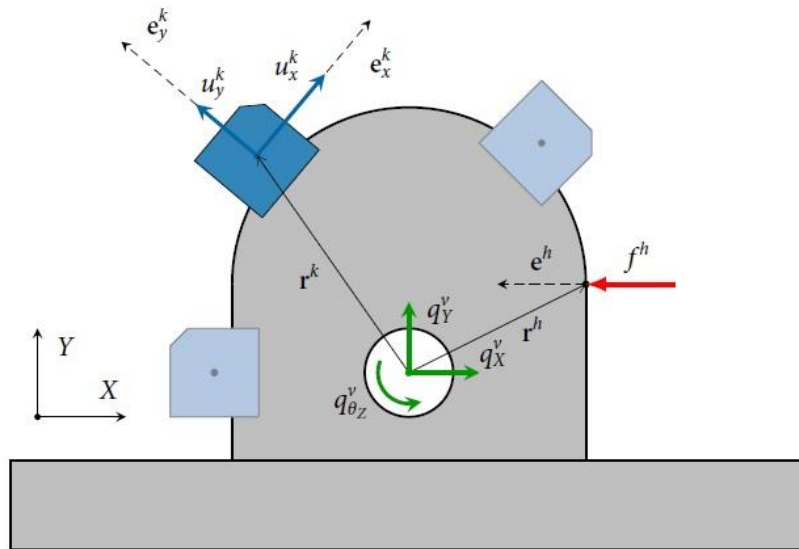


Figure 9: Illustration of virtual point transformation [8]

Mapping one force alone to the virtual point coordinate system requires 6 geometrical information. The orientation of the excitation in comparison to the virtual point coordinate system takes 3, and the other 3 is needed for the distance of the point which has been excited. One measured force is projected onto the 6 virtual forces m by a transformation matrix calculated merely from the geometrical data.

Assuming no flexible motion is present, the expression for the IDM matrix is given by an underdetermined system of equations.

$$\begin{bmatrix} m_X^v \\ m_Y^v \\ m_Z^v \\ m_{\theta_X}^v \\ m_{\theta_Y}^v \\ m_{\theta_Z}^v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -r_Z^h & r_Y^h \\ r_Z^h & 0 & -r_X^h \\ -r_Y^h & r_X^h & 0 \end{bmatrix} \begin{bmatrix} e_X^h \\ e_Y^h \\ e_Z^h \end{bmatrix} f^h$$

$$m^h = \tilde{R}^{vk^T} E f^h$$

With $R^{hv^T} = \tilde{R}^{hv^T} E^h$:

$$m^h = R^{hv^T} f^h \quad (2.42)$$

Where r^h denotes the distance of excitation h , e^h denotes the orientation of excitation h , f^h denotes the measured force of excitation h , and m^v denotes the forces of virtual point v . Lowercase letters in the subscript refer to the coordinate system of the measured data, while uppercase letters stand for the coordinate system of the virtual point. For the mapping of every measured force to the corresponding virtual point, all R^{hv} matrices can be organized into a single R matrix representing the whole transformation. As a rule of thumb, at least 3 excitations should be measured in all 3 directions. The IDM matrix of the forces is referred to as R_f . Assuming 10 excitations at every virtual point:

$$R_f = \begin{bmatrix} R_f^{1,1} \\ R_f^{2,1} \\ \vdots \\ R_f^{9,1} \\ R_f^{10,1} & R_f^{11,2} \\ & R_f^{12,2} \\ & \vdots \\ & R_f^{19,2} \\ & R_f^{20,2} & \ddots \\ & & R_f^{n_h, n_v} \end{bmatrix}$$

Using the complete R matrix, Eq. (2.43) is obtained.

$$m = R_f^T f \quad (2.43)$$

The projection of the displacements is defined in a reverse manner. For a single sensor, the distance on the sensor from the virtual point in the 3 directions and the orientation of the sensor channels (each channel compared to the coordinate system of the virtual point) is needed. Altogether, 9 orientation information and 3 distance information belong to one tri-axial sensor.

$$\begin{bmatrix} u_x^k \\ u_y^k \\ u_z^k \end{bmatrix} = \begin{bmatrix} e_{x,X}^k & e_{x,Y}^k & e_{x,Z}^k \\ e_{y,X}^k & e_{y,Y}^k & e_{y,Z}^k \\ e_{z,X}^k & e_{z,Y}^k & e_{z,Z}^k \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & r_Z^k & -r_Y^k \\ 0 & 1 & 0 & -r_Z^k & 0 & r_X^k \\ 0 & 0 & 1 & r_Y^k & -r_X^k & 0 \end{bmatrix} \begin{bmatrix} q_X^v \\ q_Y^v \\ q_Z^v \\ q_{\theta_X}^v \\ q_{\theta_Y}^v \\ q_{\theta_Z}^v \end{bmatrix} + \begin{bmatrix} \mu_x^k \\ \mu_y^k \\ \mu_z^k \end{bmatrix},$$

$$u^k = E^{kT} \tilde{R}^{kv} q^v + \mu^k$$

With $R^{kv} = E^{kT} \tilde{R}^{kv}$:

$$u^k = R^{kv} q^v + \mu^k \quad (2.44)$$

The equation is overdetermined, μ contains the error. The orientations of each channel (the rows of the E matrix) are defined as unit vectors. Using tri-axial sensors, the rows of E are orthogonal to each other. Virtual displacements can be mapped to measured displacements in one step, by organizing them all into one projection matrix R^{kv} . Assuming 3 sensors at every virtual point the IDM matrix takes the form seen below. The IDM matrix of the displacements is referred to as R_u .

$$R_u = \begin{bmatrix} R_u^{1,1} \\ R_u^{2,1} \\ R_u^{3,1} \\ & R_u^{4,1} \\ & R_u^{5,1} \\ & R_u^{6,1} \\ & & \ddots \\ & & & R_u^{n_k, n_v} \end{bmatrix}$$

$$u = R_u q + \mu \quad (2.45)$$

These displacement modes represent rigid motions of the assembly, thus any flexibility can lead to errors in the transformation. This will be elaborated on later. With the IDM matrices the transformation can be performed directly using the pseudoinverse of the IDM matrices, or with further conversions, following the guidelines of [8] and [15].

2.4.1.1 Condition number

As the first quality check of the transformation, the condition number of the $S = R^T R$ matrix can be examined for both the forces and the displacements, as proposed in [16]. Condition number is a good indicator for the precision of the transformation. It is defined as:

$$\kappa(S) = \frac{\sigma_{max}(S)}{\sigma_{min}(S)},$$

where $\sigma_{max}(S)$ denotes the largest eigenvalue of the matrix S , and $\sigma_{min}(S)$ denotes the smallest eigenvalue. Eigenvalues are closely connected to geometrical transformation. $\kappa(S) \rightarrow \infty$ means, that the transformation can not span the whole space of the virtual displacements or forces. The higher the difference between the smallest and largest eigenvalue, thus the condition number, the less the transformation is able to describe one or more DoFs of the virtual point. Small condition number does not guarantee successful transformation, but a high value indicates, that the transformation could be improved. In practice, precise sensor and excitation positioning is the key to decrease condition number.

2.4.2 Transformation of forces

Virtual point transformation is a crucial part of the whole process, and there are many sources of error that should be considered. The upcoming transformation method allows for the construction of a so-called filtering matrix which can be helpful in identifying the errors of virtual point transformation if any. The guiding equation of the force transformation is Eq. (2.43), displayed again here:

$$m = R_f^T f$$

Eq. (2.43) is normally underdetermined. A weighting matrix W can be used to choose the importance of the different forces during the transformation. Solving Eq. (2.43) for f yields:

$$\tilde{f} = WR_f(R_f^T WR_f)^+ m \quad (2.46)$$

The transformation matrix can be seen in Eq. (2.46).

$$T_f^T = WR_f(R_f^T WR_f)^+ \quad (2.47)$$

The filtering matrix represents the two transformation steps of the forces. The transformation from the virtual point space to the space of the measured data, then transforming it back to the virtual point space, composed into one.

$$F_f = T_f^T R_f^T \quad (2.48)$$

The filtering matrix will later be used as a quality indicator of the transformation.

2.4.3 Transformation of displacements

The displacement transformation follows a similar route as forces. The guiding equation (Eq. (2.45)) is solved for q .

$$q = (R_u^T W R_u)^+ R_u^T W u \quad (2.49)$$

A weighting matrix again is used. Because Eq. (2.45) is overdetermined, a minimal-quadratic solution is calculated for q . The transformation matrix is found in the following form:

$$T_u = (R_u^T W R_u)^+ R_u^T W \quad (2.50)$$

The filtering matrix is acquired by completing the two transformation steps in one step.

$$F_u = R_u T_u \quad (2.51)$$

Throughout the application of the virtual point transformation in TPA, the calculation of the actual q and m values, is not necessary. These information are embedded in the transfer functions of the assembly. During TPA calculation, only the transformation matrices T_f and T_u are used.

2.4.4 Virtual point FRFs

With the transformation matrices defined in previous sections, the measured frequency response functions can be converted into virtual point FRFs. Assuming the transformation was precise, virtual point FRFs represent the transfer function between the virtual point instead of the excitation and sensor positions where the measurement actually has been conducted.

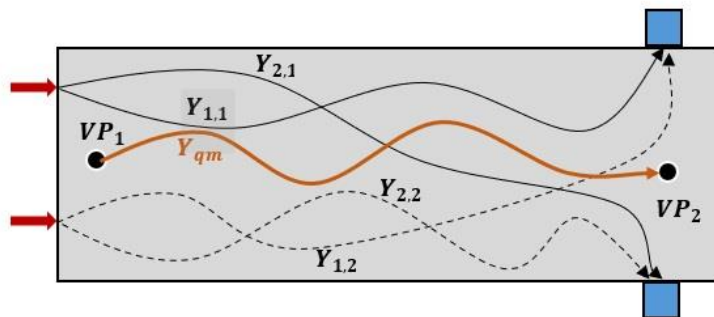


Figure 10: Virtual point FRF

The virtual point FRF matrix should satisfy the equation,

$$q = Y_{qm}m,$$

Where q is the vector of virtual point displacements, m is the vector of virtual point forces and Y_{qm} is the virtual point FRF matrix at a given frequency. Expanding the equation using the transformation matrices, Eq. (2.52) is obtained.

$$q = T_u Y T_f^T m \quad (2.52)$$

The expression for the virtual point FRF matrix can be written:

$$Y_{qm} = T_u Y T_f^T \quad (2.53)$$

It is also possible, to apply the transformation only on forces, constructing a FRF matrix where the forces act on the virtual point while the displacements measured on the interface stay the same. In any way, the description of the exact connection point became possible.

2.4.5 Transformation quality indicators

After the calculation of the virtual point FRFs everything is given to go on with the TPA. It is advisable to inspect the precision of the transformation. Generally, these quality indicators can be the courier of two things: presence of flexible body modes and inaccuracy in the geometry measurement.

2.4.5.1 Overall consistency

For both excitations and displacement, two types of consistency is available. Overall and specific consistency. Both are designed to validate the transformation based on the filtering matrix. Overall consistency examines all sensor channels or excitations which belong to the same virtual point. First, the unfiltered and filtered responses are defined:

$$u_{1,2} \stackrel{\text{def}}{=} Y_{12} f_2 \quad (2.54)$$

Subscript 1,2 means, the responses at virtual point 1 will be observed, caused by one or more forces at virtual point 2. The filtered responses are obtained by using the corresponding submatrix of F_u

$$\tilde{u}_{1,2} \stackrel{\text{def}}{=} F_{u_1} Y_{12} f_2 \quad (2.55)$$

The value of the overall consistency is given by:

$$\rho_{1,2} = \frac{\|\tilde{u}_{1,2}\|}{\|u_{1,2}\|} \quad (2.56)$$

At every frequency, the value of overall consistency varies between 0 and 1. Deviation from 1 means that the norm of the filtered response is smaller. That is caused by flexibility in the assembly. As the

transformation can only deal with rigid motions, any flexibility will be ignored, which results in the decrease of the norm. In practice, it also can mean error in the measurement of the sensor geometry. The filtered and unfiltered responses for the impact consistency are defined in a similar manner, the difference being the calculation of a weighted sum (normally just a simple sum).

$$y_{2,1} \stackrel{\text{def}}{=} wY_{21} \quad (2.57)$$

$$\tilde{y}_{2,1} \stackrel{\text{def}}{=} wY_{21}F_{f_1} \quad (2.58)$$

F_{f_1} denotes the corresponding submatrix of F_f and w is a row vector. The overall impact consistency then can be defined.

$$\rho_{1,2} = \frac{\|\tilde{y}_{2,1}\|}{\|y_{2,1}\|} \quad (2.59)$$

Besides flexible body motions, impact consistency can indicate error in the geometry measurement of the excitation positions, or in the excitation angles.

2.4.5.2 Specific consistency

Overall consistency is a good tool to spot errors, however it points to a group of sensors or excitations to search through for finding said errors. Specific consistency deals with the single channels and excitations using a modified version of the spectral coherence function, to narrow the possibilities where the error could occur. Here I use the notation *coh* (as with coherence) to be consistent with the literature. (The reason for this notation, is that this function is a modified version of the spectral coherence function)

$$coh(x, y) = \frac{(x + y)(x^* + y^*)}{2(xx^* + yy^*)} \quad (2.60)$$

In the expression above x and y are complex numbers. It could be seen as comparing two vectors. The nominator holds the length of the vectors sum, while the denominator is the sum of the length of the two vectors, both squared. The denominator contains an additional constant multiplier. When the two vectors are identical, than the value is 1. For small changes in the length or the angle, the value begins to drop below 1, but it always stays positive. If the vectors are the same length but are pointing in the opposite direction, the value is 0. To summarize, the value of *coh* is an indicator for the similarity between two complex numbers. Using the filtered and unfiltered response of Eq. (2.55) and Eq. (2.54) the formula to calculate the specific sensor consistency is acquired.

$$\rho_{u_{i,2}}(\omega) = coh(\tilde{u}_{i,2}(\omega), u_{i,2}(\omega)) \quad (2.61)$$

All u_i is one of the sensor channels around the virtual point under examination. The same formula is applicable in case of the specific impact consistency.

$$\rho_{f_i,2}(\omega) = coh(\tilde{y}_{2,i}(\omega), y_{2,i}(\omega)) \quad (2.61)$$

f_i denotes one of the excitations around a given virtual point. With the specific consistency function, it is easier to identify faulty channels and excitations specifically.

2.4.5.3 FRF reciprocity

As a result of virtual point transformation, all six virtual forces act on the same point. Also, all displacement describe the motion of the same point. Consequently, the virtual point FRF matrix should be reciprocal at every frequency.

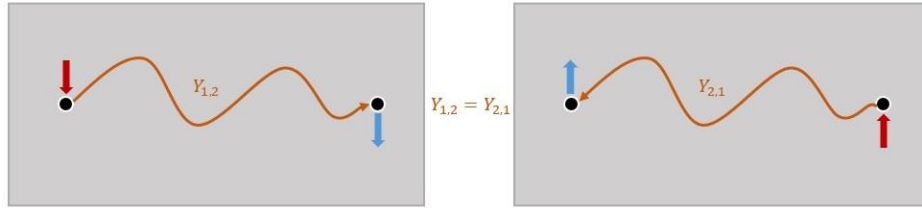


Figure 11: FRF reciprocity. Excitation and response in the same direction at different virtual points.

For example, the transfer function for excitation at VP1 in X direction, and response at VP2 at Z direction should be the same as the transfer function for excitation at VP2 in Z direction, and response at VP1 in X direction. The reciprocity is calculated with the formula of Eq. (2.60).

$$\chi(\omega)_{ij} = coh(Y_{ij}(\omega), Y_{ji}(\omega)), \quad (2.62)$$

where Y_{ij} and Y_{ji} are elements of the virtual point matrix Y_{qm} . For the diagonal elements of Y_{qm} the value of the reciprocity is always 1, by definition. For the evaluation of diagonal values, passivity can be used.

$$angle(Y_{ii}) = \begin{cases} \in [-180,0] & \text{if displacement is measured} \\ \in [-90,90] & \text{if velocity is measured} \\ \in [0,180] & \text{if acceleration is measured} \end{cases} \quad (2.63)$$

The measurements shown in section 4 are conducted with accelerometers, thus the last line is considered. The application of quality indicators is mostly practical to spot any possible error before continuing the analysis, so certain measurements can be repeated and corrected, before using the result for further calculation. As physical measurements always bear some mistakes, the quality indicators will never show perfect results.

2.5 Singular value decomposition

Singular value decomposition (SVD) has a widespread field of applications. It is used as a data reduction tool in many cases. It is possible to find the dominant attributes of a dataset with the use of SVD and sort out all the data that has no or very little contribution to the information stored by the dataset. Underdetermined and overdetermined systems of equations can be solved with the Moore-Penrose pseudoinverse, which is a method to invert non-square matrices with the use of SVD. Furthermore, it is the basis of the principal component analysis (PCA). For more detailed discussion of SVD its applications, see [17]. A given matrix X can be decomposed into three matrices U , Σ and V .

$$X = U\Sigma V^T \quad (2.64)$$

where U and V are unitary matrices and Σ is a diagonal matrix containing the singular values in decreasing order. All singular values are non-negative, and they refer to the importance, or dominance of the corresponding column in the U and V matrices. If X is size $n \times m$, U is $n \times n$, and V is $m \times m$.

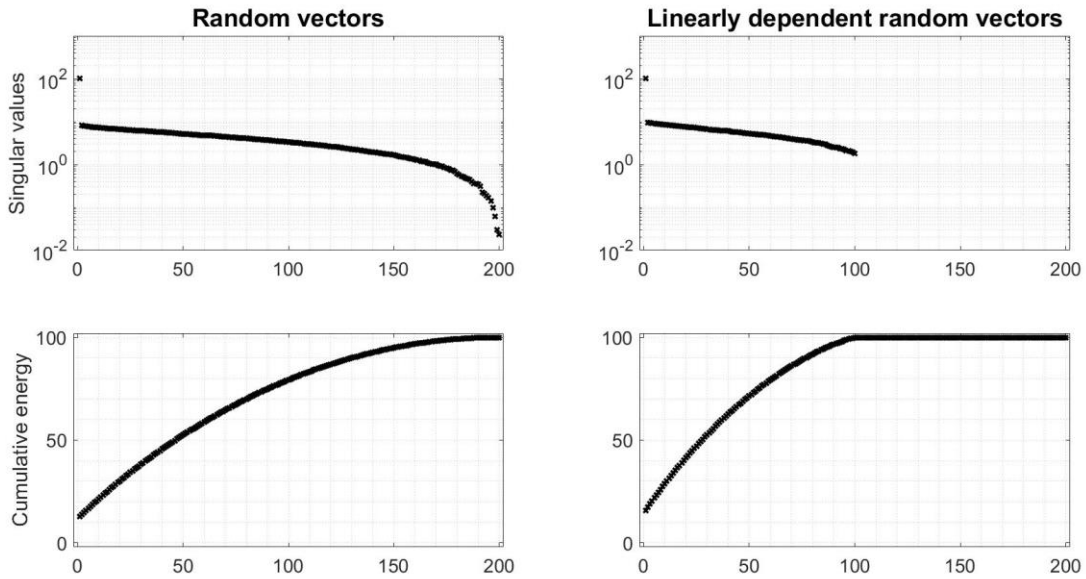


Figure 12: Examples for singular values

Two examples are shown in Figure 12. On the left, the singular values of a random matrix of size 200×200 can be seen, and the cumulative energy up to the corresponding singular values. The same goes for the right side for a matrix. This matrix is constructed by generating a 100×200 random matrix, then concatenating it with the same matrix multiplied by a scalar. This way, the number of linearly independent vectors is 100. As it can be observed in the top right picture, the singular value corresponding to the linearly dependent vectors is zero, because they contain no valuable information.

It also means that all energy is going to be contained by the by the first 100 singular values, as seen in the bottom right picture.

2.5.1 Singular value truncation

During many problems, calculations (such as matrix inversion) are done with badly conditioned matrices, where some singular values get lost in the noise, causing numerical inaccuracies in the calculation. In these cases, it is possible to set the diagonal values of the Σ matrix to 0 under a certain threshold. If the threshold is chosen well, the original matrix reconstructed with the truncated Σ matrix will contain all the important information, while becoming better conditioned. Consequently, operations (that otherwise would be heavily influenced by noise) applied to the truncated matrix yield smoother results.

2.5.2 Moore-Penrose pseudoinverse

Many physical systems are described by a linear system of equations, which can be written in the following form:

$$Ax = b, \quad (2.65)$$

where x and b are column vectors. If A is a square matrix, the equation can be solved for x by a simple inversion.

$$x = A^{-1}b$$

The situation is a bit more difficult when the rows and columns of A are not the same size. Applying SVD on A in Eq. (2.65) yield

$$U\Sigma V^T x = b$$

U and V are unitary matrices, meaning $U^{-1} = U^T$ and $V^{-1} = V^T$. This equation can be solved for x by executing the inversions in the correct order.

$$\tilde{x} = V\Sigma^{-1}U^T b \quad (2.66)$$

The expression on the left side of b is called the Moor-Penrose pseudoinverse and is denoted by a $+$ in the superscript.

$$A^+ = V\Sigma^{-1}U^T \quad (2.67)$$

The solution in Eq. (2.66) is denoted by $\tilde{x}$ because it is not the exact solution, just the best estimation for x . Since A is not square and the number of elements in x and b are not the same, the equation can not be solved exactly. There are two main types: underdetermined and overdetermined system of

equations. In the underdetermined case, the number of elements in x is larger than in b , thus there are infinite solutions for x in theory. For the overdetermined system of equations, the opposite is true, the number of elements in b , is larger than in x . As a result, there is no solution for x . $\tilde{x}$ provides the least mean square solution, minimizing the value of $\|A\tilde{x} - b\|$ for the best estimation.

3 Measurement and analysis workflow

In the previous section, all the important tools of this method have been presented. In this section, the steps of the measurement and analysis are shown with a few practical considerations and guidelines to achieve valuable results. Use of other TPA method for example, can lead to deviation from the workflow seen in Figure 13, but the structure is always similar. For the steering system measurements, this workflow is followed.

3.1 Main segments

The process can be divided into four major parts. The first part is conducting the physical measurements and the rest falls under the category of analysis.

3.1.1 Measurement

The first step is the determination of the virtual point locations. Although the calculations can be conducted for arbitrary locations, the obvious place which always yields useful results is the exact connection point of two assemblies. For instance, if the steering system is screwed to the car, the middle of the screw thread is an excellent choice.

After that, the sensor and excitation positions should be chosen. In both cases, the physical DoFs shall be adequate to describe the virtual point DoFs. To that matter, the overdetermination of both transformations is advisable. As a rule of thumb, at least 3 tri-axial sensors should be used at every virtual point, providing 9DoFs to be transferred to 6 virtual DoFs. In the case of the excitations, minimum of 3 excitations in all directions (X, Y, Z) provides 9 DoFs. For even better placement, the condition number mentioned in section 2.4.1.1 can be taken into consideration. Doing so requires the geometry data to be set before the measurements and sensor placements. Alternatively, the geometry data can be measured anytime, before the virtual point transformation and after the sensor placement, although one can choose sensor positions a bit less carefully this way.

Next, the responses of the sensor channels are measured for force impacts generated by an impact hammer. Exact excitations are important, location and angle errors may lower the measurement and transformation quality. Errors in geometry measurements can be spotted with the virtual point transformation quality indicators. The operational measurement for the TPA can be conducted at any point before the TPA calculation, although might be practical to use the same sensor positions and conduct it right away.

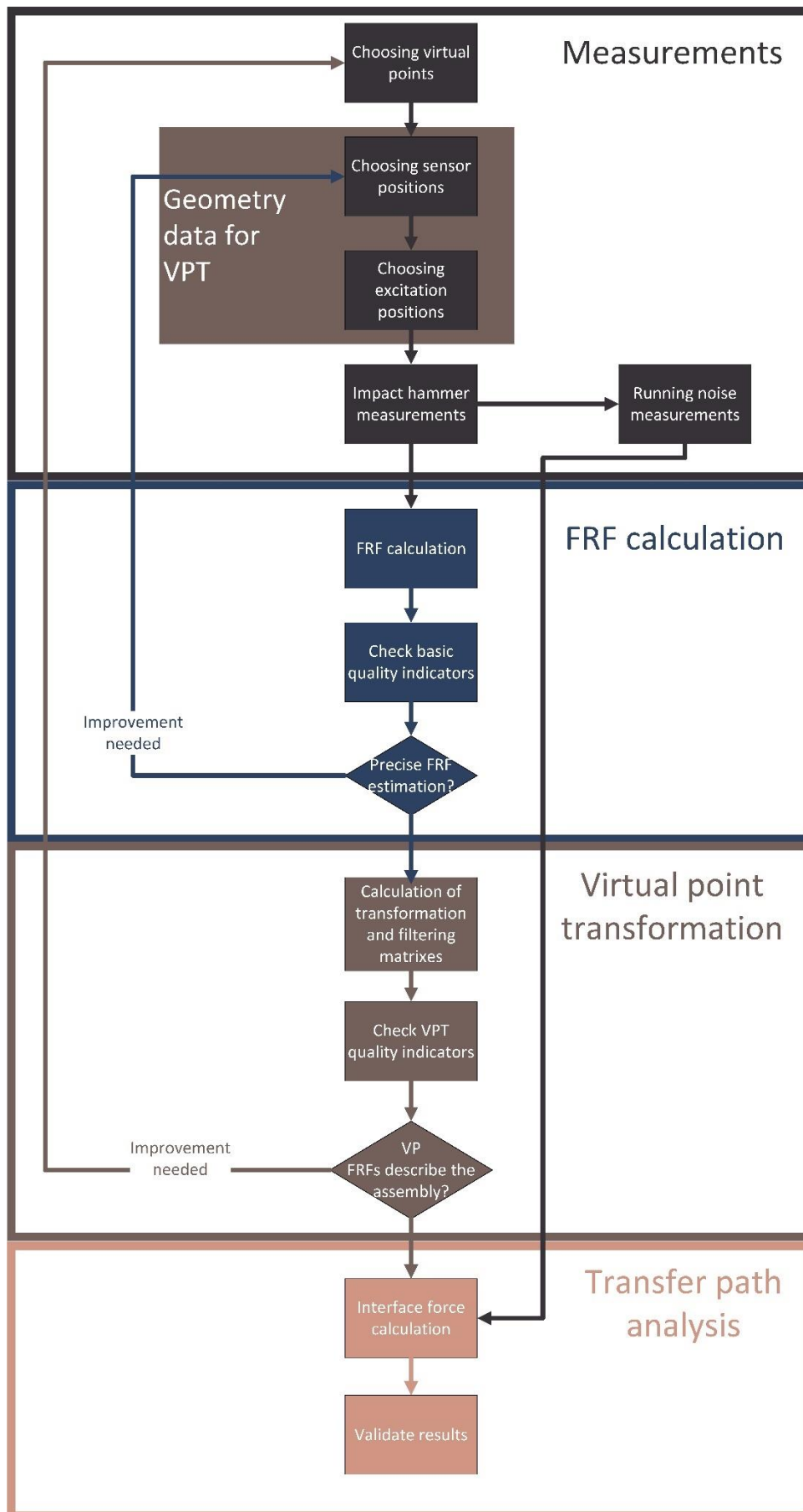


Figure 13: Workflow

3.1.2 FRF calculation

The role of FRFs has already been established as representations of global dynamic attributes. With closer examination, the frequency range can be established where the data reflects the nature of the physical system adequately. The first and perhaps most important indicator is the coherence function (and SNR derived from it) described in section 2.1.4.2, expressing the precision of the FRF estimations. On higher frequencies, the coherence is likely to plummet, but at least for 2-3 kHz the coherence value is expected to be around 100%. When the coherence is poor measurements should be repeated, as the imprecision of the transfer function estimation will bring errors to further calculations. On Figure 14, the FRFs and coherences of all sensor channels for one excitation are shown with the average coherence. Around 1.5 kHz a significant decrease of the coherence function can be observed at every channel.

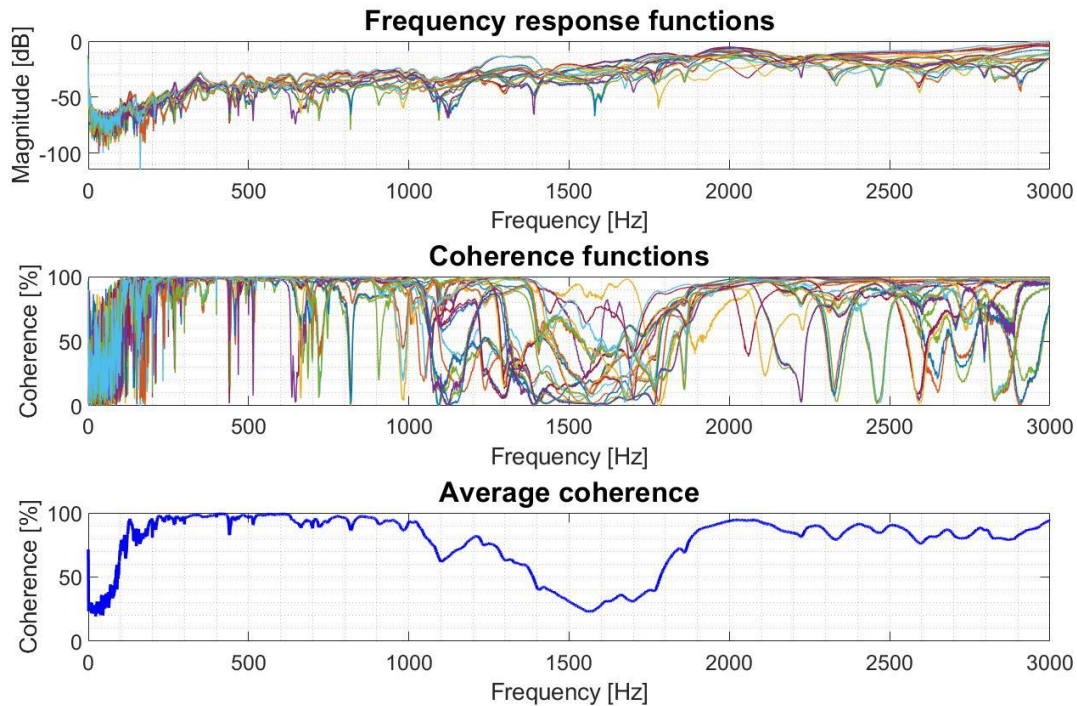


Figure 14: Example for poor coherence on lower frequencies

As discussed in [8], SVD can also be used to decide, if the assembly cannot be considered rigid enough at a certain virtual point by looking at the singular values of the corresponding FRF matrix at every frequency. At a given frequency, if only 6 dominant singular values are present, then rigidity can be presumed, although it is not certain. However, if there are more than 6 dominant singular values, the assembly cannot be considered rigid. On Figure 15, the singular values of one foot of a

steering gear are shown. It can be seen, that there are no more than 6 significant singular values, the rest gets lost in the noise.

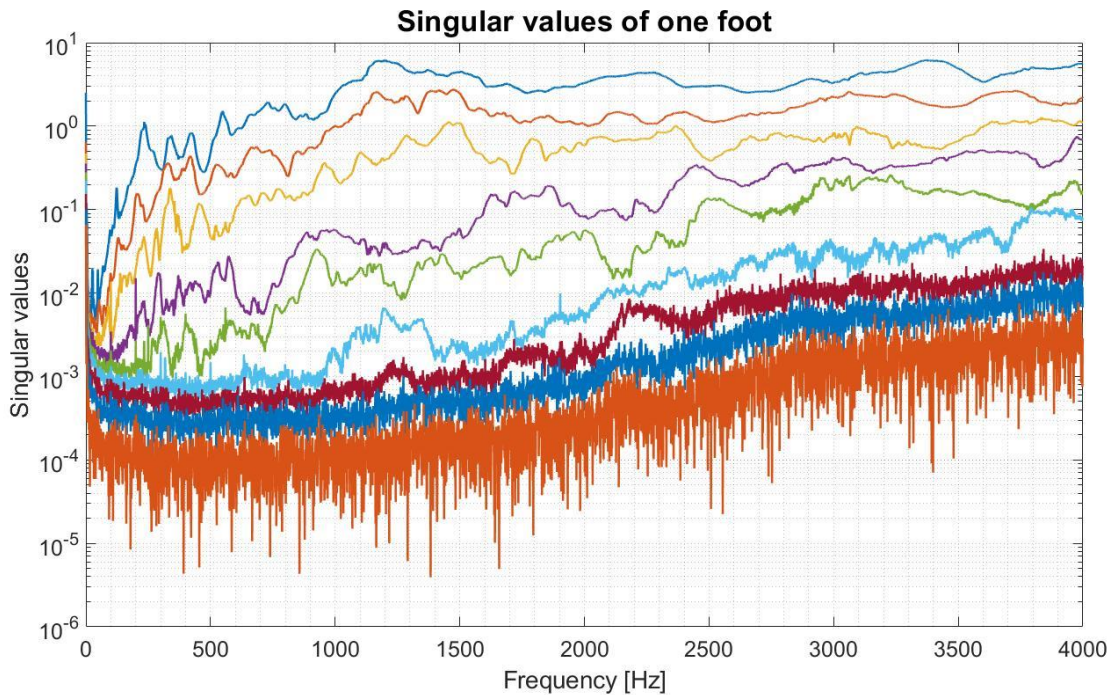


Figure 15: Singular values of a subset of the whole FRF matrix that belongs to one foot of the steering gear

Looking at the SVD of the whole assembly (Figure 16, left side), one can see more than 6 dominant singular values, as the whole steering gear does not behave rigidly. Also, on higher frequencies, the singular values of one foot (Figure 16, right side) get close to each other to a point where there are no 6 dominant singular values, thus rigidity cannot be presumed.

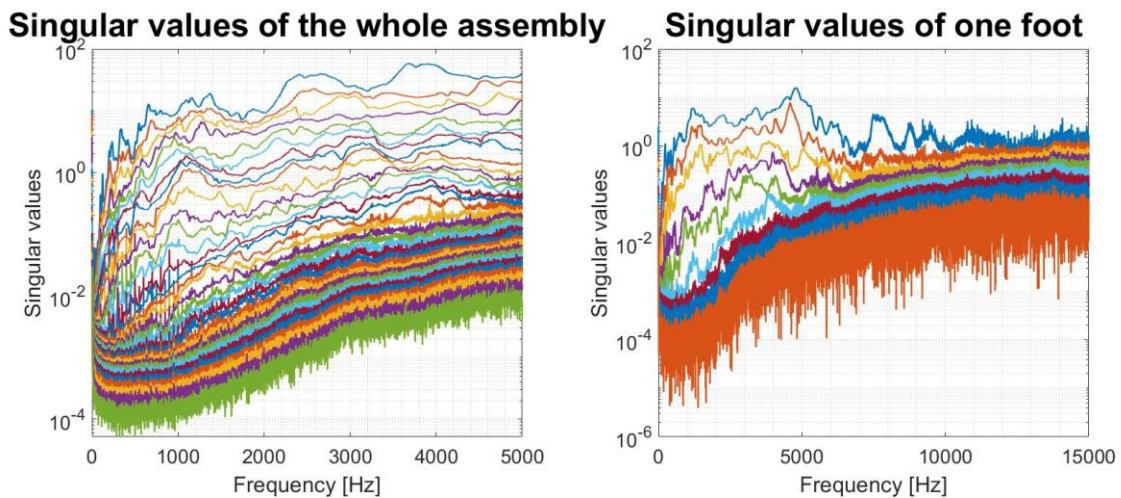


Figure 16: Examples for more than 6 dominant singular values

Besides these, other attributes can be examined and compared to the expected result, for example if the modal analysis of the steering gear is at hand, the position of peaks in the transfer functions should be very close to the frequencies of the modal components.

3.1.2.1 SNR

To see how precisely Eq. (2.11) approximates the signal-to-noise ratio from coherence, I created ideal transfer functions with second order oscillators, normalized the impulse response of the system dividing by the RMS value, and then added noise with known amplitude and deviation. With the normalization, the noise can be set easily using the *randn* function of Matlab. By multiplying the random noise with 0.1, the value of SNR will be 20, by multiplying with 0.01, the value will be 40, and so on. The same transfer function is repeated a chosen number of times, then the noise is added to simulate different measurements for the same transfer function. After that, the transfer function is estimated from the noisy simulations with built in MATLAB functions *tfestimate* and the coherence with *mscoherence* (both use auto-and cross power spectra for the calculation as described in section 2.1.4.1).

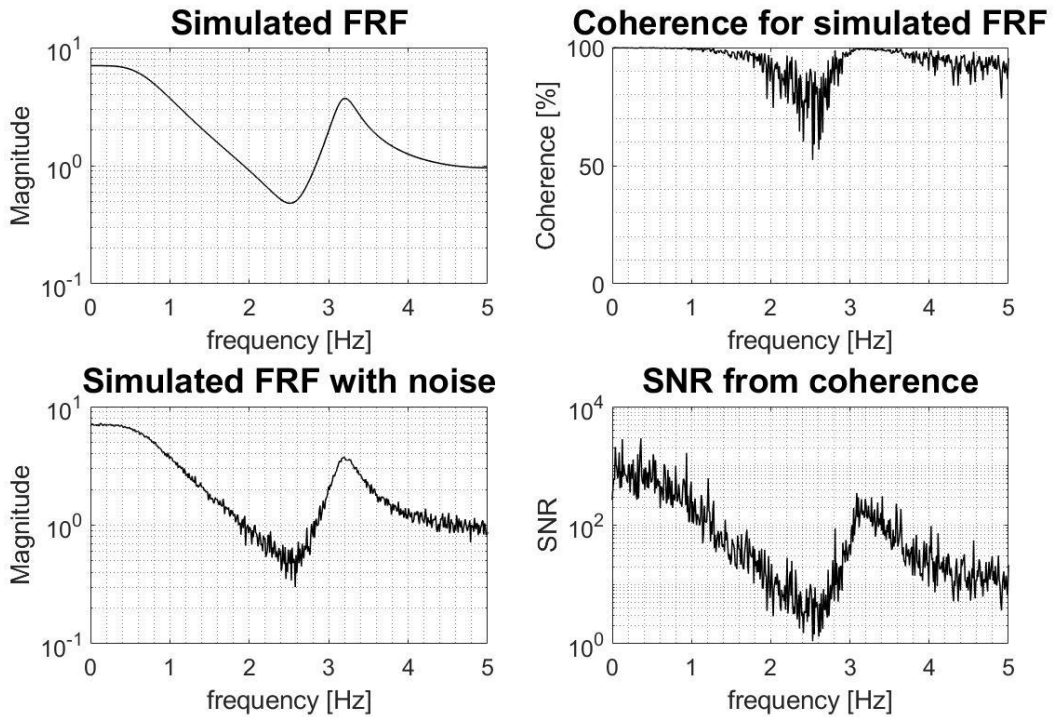


Figure 17: SNR calculated from simulated transfer function with SNR = 20dB

The influence of noise is more considerable where the transfer function has lower values, due to the smaller difference between the amplitude of the noise and the amplitude of the signal. After experimenting with the parameters, I observed that the estimation for SNR gets closer to the actual

value when increasing the number of “measurements” the transfer functions are calculated from. Since the transfer functions estimation improves with more measurements, consequently the coherence too, it is no surprise the calculated SNR approximates the real SNR better. To see how the precision of SNR depends on the number of samples, 8000 transfer function with randomized parameters have been evaluated for every number of measurements from 2 to 10, and for 50. This is no mathematical proof, but it can be helpful to know how close one can get to the real SNR calculated from actual measurement data.

Table 1: SNR estimation precision in function of the number of samples

Number of samples	Maximal difference [dB]	Average difference [dB]	Standard deviation [dB]
2	50.919	11.173	4.0775
3	31.656	4.7009	1.4985
4	15.608	3.0178	0.9253
5	11.427	2.2344	0.7096
6	11.232	1.7955	0.5974
7	9.0092	1.4964	0.5359
8	7.8397	1.283	0.4780
9	7.4232	1.1269	0.4292
10	6.2261	1.0088	0.4121
50	3.6769	0.2261	0.2034

A rapid improvement can be seen at the beginning of the table. On average there are four measurements in the data analyzed in section 4, so the SNR calculated from the data is to be handled with caution, but it can give a rough estimate. Normally one should calculate the FRFs from at least 6-8 impacts. It also might improve the quality of the FRF estimation, to use impact hammers with different tips, with which one can conduct precise measurements in different frequency ranges.

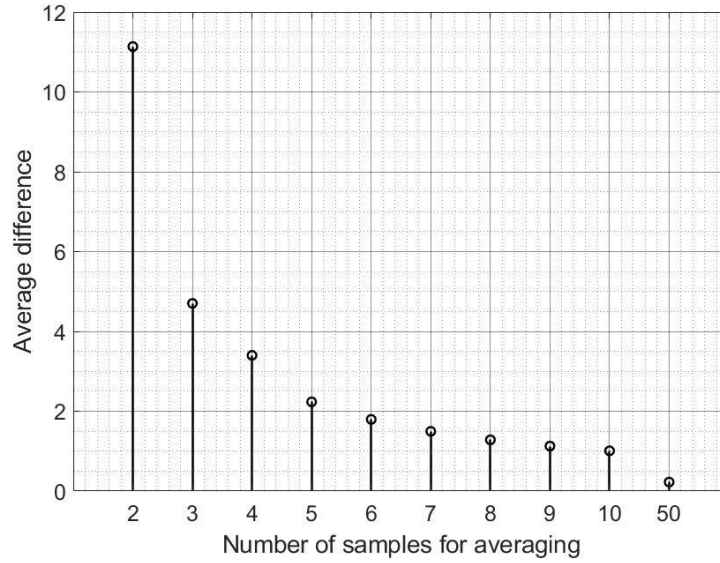


Figure 18: The influence of number of samples to SNR precision

3.1.3 Calculation of virtual point FRFs

In this part, the equations and formulas of section 2.4 are used to determine whether the geometry measurements were precise enough for the virtual point transformation to lead to useful results. Firstly, the overall sensor consistency is evaluated, which gives a general description of the sensor geometry measurement and can also give feedback on the rigidity of the assembly. If the overall sensor consistency has a high value in the frequency range of interest at a given virtual point, the process can be continued. If the consistency is poor, the precision of geometry measurement should be checked. There are two main causes for low consistency values that can be corrected by the experimentalist: errors in geometry measurements and flexibility of the assembly. In the first case, a new geometry measurement can help. In the second case, the consistency should have a fairly high value at the beginning, and it should plummet on higher frequencies due to the modal behavior. New sensor positions might satisfy the rigidity assumption, thus improve the consistency values. The overall impact consistency is very similar, poor values can imply faulty geometry measurement or flexibility. In both cases, specific consistency is a tool to find the source of error easier, by getting some information of the sensor channels or excitations separately.

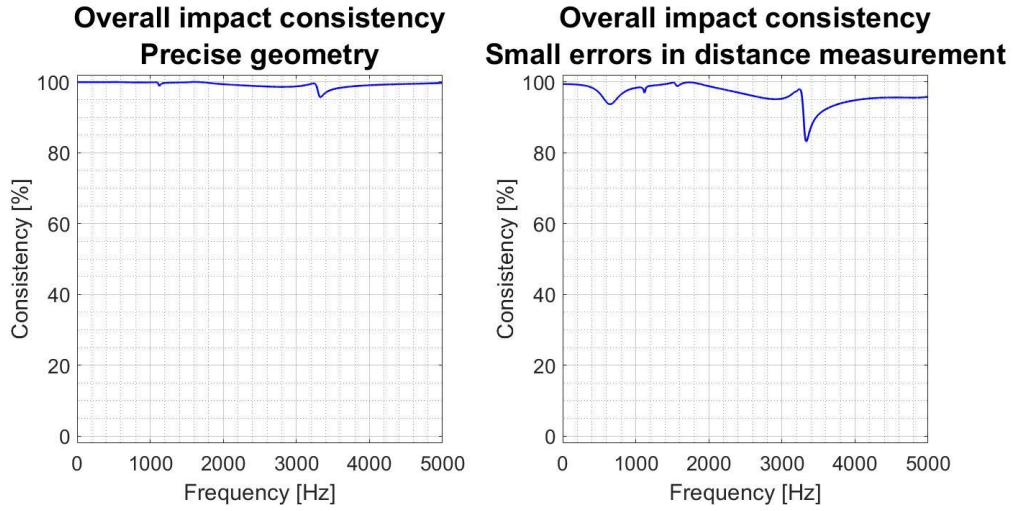


Figure 19: Overall impact consistency with correct geometry data (left) and faulty data (right)

When all consistency is acceptable the FRF reciprocity and passivity are evaluated. Similarly to consistency lower values of FRF reciprocity can indicate distance or orientation measurement errors (which also can be because of error in the impact hammer measurement) or the lack of overdetermination, in which cases the problem is easier to solve. Another possibility is the presence of nonlinearities and flexibility in the assembly. Again, changing the sensor placements can be of help, however if the sensors cannot be placed any closer and the assembly still behaves rigidly, it is the attribute of solely the assembly and can not be changed.

3.1.4 Transfer path Analysis

After acquiring the virtual FRF matrix and the operational measurements, transfer path analysis can be conducted following the equations of section 2.3.2.5. The final goal is to calculate the equivalent forces at the interface with Eq. (2.38):

$$f_{eq} = (Y_{42}^{AP})^+ u_4$$

In practice, instead of displacements, the calculations are performed with accelerations:

$$f_{eq} = (Y_{42}^{AP})^+ a_4 \quad (3.1)$$

In Eq. (3.1), a_4 represent the results of the operational measurements from triaxial accelerometers. $(Y_{42}^{AP})^+$ stands for the pseudo-inverse of the virtual point FRF matrix defined by Eq. (2.53). As mentioned in section 2.4.4, if the position of the acceleration sensors is not changed, a one-sided transformation can be calculated, leaving out the transformation of accelerations:

$$Y_{am} = Y T_f^T \quad (3.2)$$

In this case Eq. (2.37) is used with accelerations:

$$f_{eq} = (Y_{22}^{AP})^+ a_2 \quad (3.3)$$

After the set of equivalent forces f_{eq} are calculated, the results can be validated with a procedure introduced in [8]. The main idea is to define a set of response DoFs, which means placing one or more accelerometers on the assembly. The TPA of Eq. (3.3) is performed with the indicator DoFs, and then, the response for the calculated equivalent forces f_{eq} are acquired with the assembled vehicle FRFs:

$$a_3 = Y_{32}^{AP} f_{eq} \quad (3.3)$$

The calculated responses a_3 can be compared to the validation data, which are the direct results of the operational measurements. If the results fit the validation data, it means that the TPA process yields correct equivalent forces for given measurement results. At this point, transmissibility TPA (section 2.3.3) might be useful, as there is no virtual point transformation is performed on the FRF matrices, so any deviation from validation data caused by VPT is left out. Since the acceleration sensor positions are unchanged, it can be used for the same validation loop as in-situ TPA.

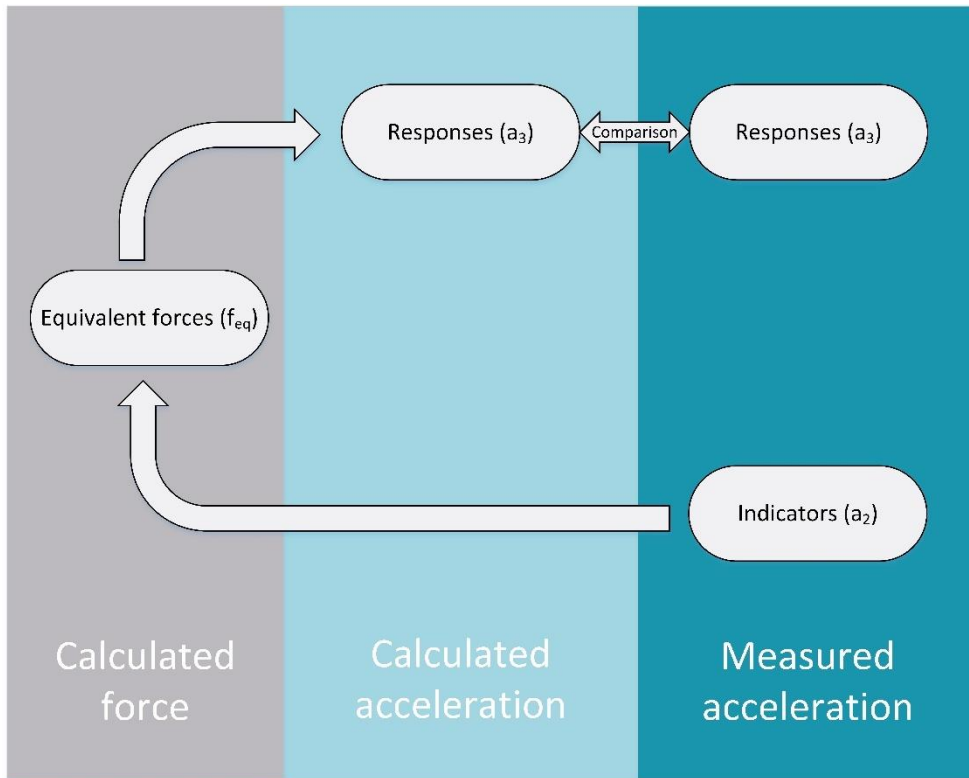


Figure 20: TPA validation process

Validation (or response) sensors should not be placed on the interface. Positions right after the interface are advisable. Validation sensors can be placed anywhere on the passive side, but one should be cautious in respect to the signal to noise ratio, which can be quite low at some parts of the testbench.

3.2 Virtual point transformation on model

For a deeper examination of the virtual point transformation, a simple model was created in ANSYS (by an engineer at the company) shown in Figure 21. 9 excitation positions and 3 sensor positions were defined at both leg of the assembly. The excitations on the active side, responses at the passive side. Care were taken, so the positions are able to describe all 6 virtual point DoFs.

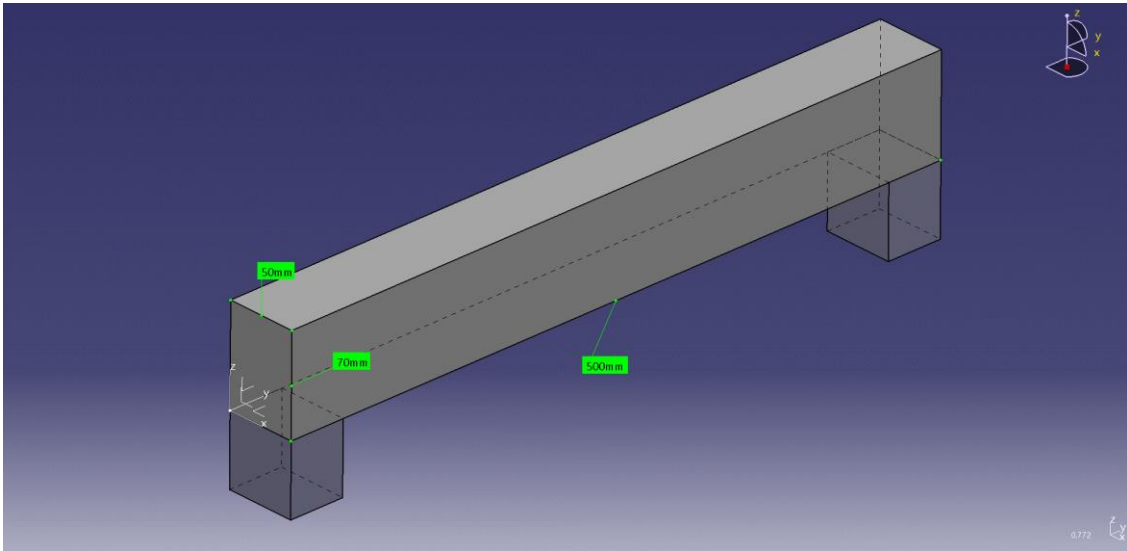


Figure 21: Simple model for experimentation

Ideal FRFs were acquired from the modal behavior for the VPT quality indicator calculations. The geometry data is absolutely precise, as every parameter of the model is known exactly, but it can be intentionally “ruined” to see the changes it causes in the quality indicators.

3.2.1 Consistency on model

It was observed, that orientation errors cause a more significant decrease in the consistency value. This can be explained by the flexibility it indicates. Faulty measurements can be spotted with consistency because the transfer function of the given (mismeasured) point would look somewhat different than the transfer function of the original point, and this difference occurs during the transformation as flexibility is neglected by the transformation itself, but noticed by the consistency. Therefore, a slight change in sensor position for example probably introduces less flexibility to the system, then rotation along some axis of the sensor. It also explains the difference between coordinate directions.

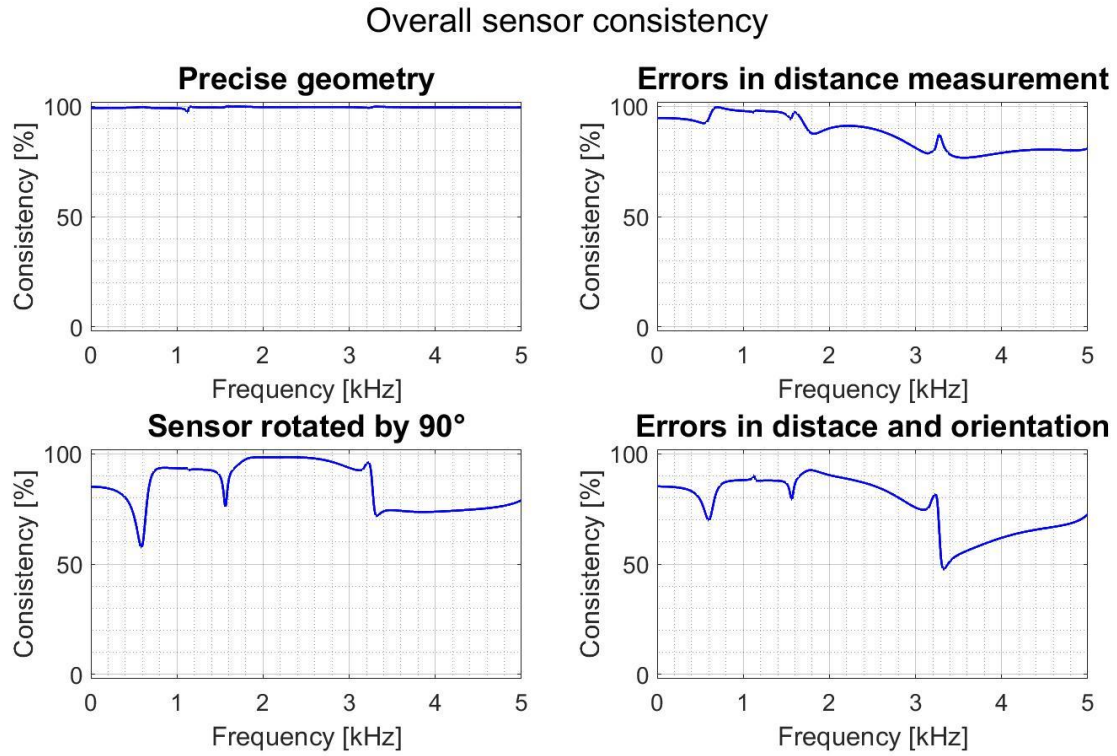


Figure 22: Measurent errors reflectd by consistency

There are directions (where the assembly is very rigid) where position inaccuracy brings very little error to the consistency, but in other directions (with more flexibility) the inaccuracy causes a more noticeable decline of consistency.

3.2.2 Virtual point FRFs of the model

This model allowed checking the transformation matrices in a different way. Normally, consistency involves the filtering matrices and FRF reciprocity the involves transformation matrices, but with this model, it is possible to place two response points into both virtual points and compare the calculated VP FRFs to the simulations. A consistency function can be calculated in every direction of every sensor for every excitation to both virtual points. Consistency is given by the formula of Eq. (2.60).

Table 2: Consistency of the transformation

Virtual point\Average consistency	Average of all calculated consistency after transformation
VP1	91.9723%
VP2	96.0493%

The calculated average values can be seen in Table 2. The consistency shows quite high values for both virtual points. In Figure 23, an example for the comparison is shown. In the first row, the amplitudes of the simulated responses are compared to those acquired by virtual point transformation. The second row shows the phases, and the last one in the consistency comparing amplitude and phase.

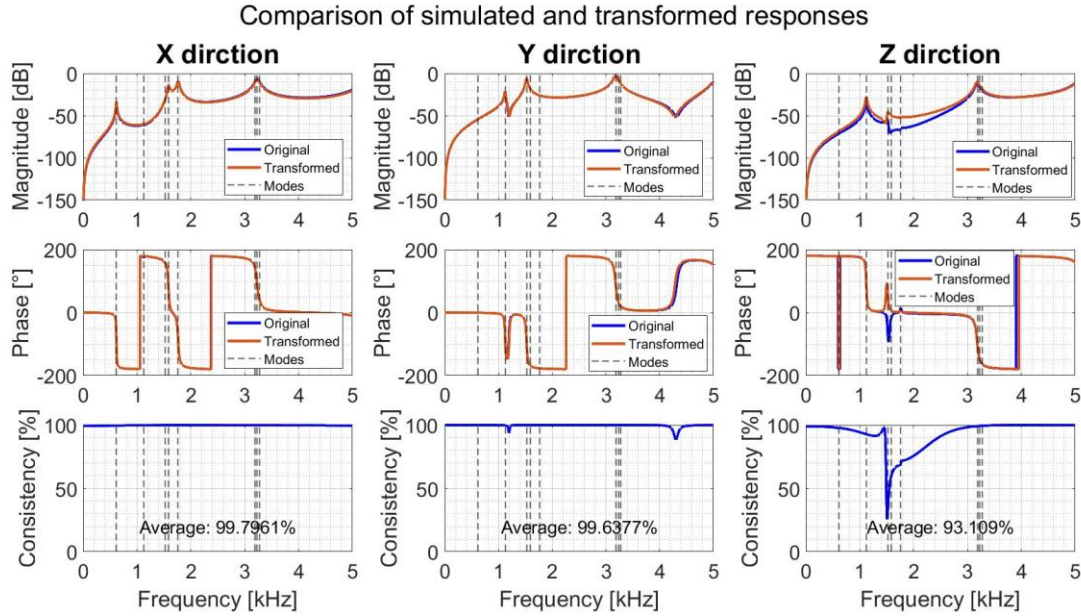


Figure 23: Comparison of transformed and simulated responses for a given excitation

All the required parameters are known exactly and all measurement errors are eliminated. Still, there is no perfect consistency between the simulated and transformed transfer functions. After deeper analysis, two kinds of errors were determined. First, the flexibility of the assembly, where the problem is the lack of flexible interface displacement modes. In the second case, some excitations were not able to have an effect on some responses as a result of improper sensor placement. When the excitation is aligned with the sensor in a way, that it cannot generate acceleration in some DoFs it is supposed to, the consistency value not just drops due to modal behavior, but starts with a lower value. On Figure 23 it is observable, that the amplitudes and phases, also the consistency tends to change rapidly around the modes of the model. Another example with poorer consistency values is shown in Figure 24, where the same effect appears.

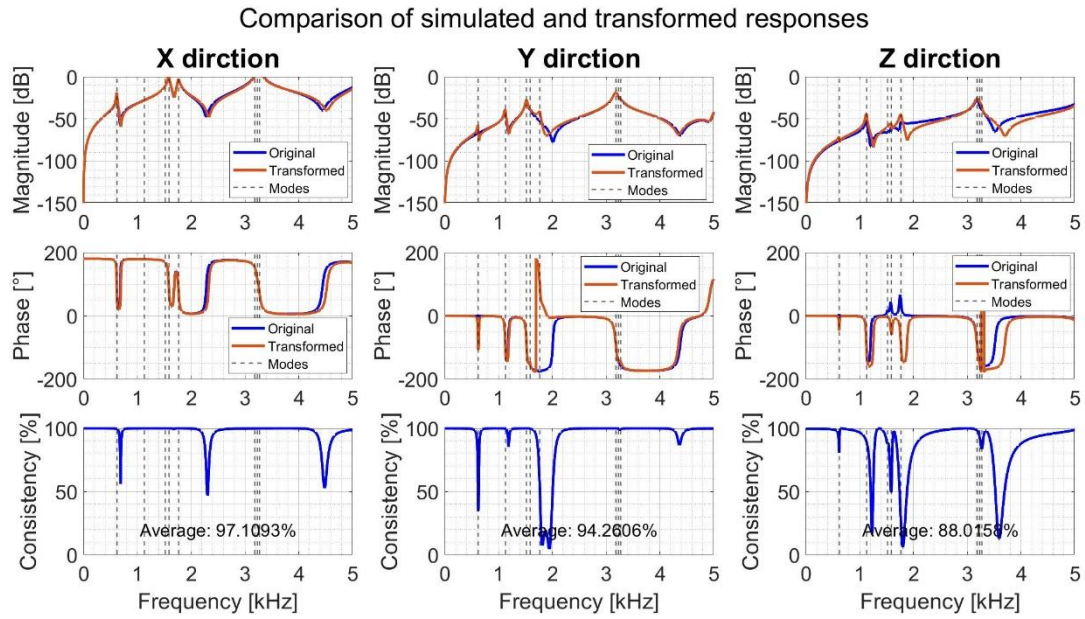


Figure 24: Example for the influence of modal behavior on VP transformation

All things considered, the transformed FRFs describe the model dynamics quite well. This experimentation pointed out the importance of careful sensor placement.

4 Application on a steering gear

The methods presented in section 2 were applied on a steering gear in Thyssenkrupp Components Technology Hungary following the guidelines of section 3. The measurements were conducted in Liechtenstein and the measurement data was sent to Hungary for analysis. In this section, I am going to present the result of the analysis at every step of the process.

4.1 Measurement

The steering gear on Figure 25 is connected to the rest of the car with four legs. A virtual point is defined at every leg in the connection point. The measurements were conducted on a testbench. Three tri-axial accelerometers are glued to the test bench around each virtual point making a whole of 12 accelerometers meaning 36 measurement channels. Nine hammer excitations took place at the first feet of the assembly, eight at the third, and seven at the last one, altogether 33 excitations meaning 1188 transfer functions and a 36×33 FRF matrix at very frequency. The geometry data was measured afterward in CAD. All sensors are placed in a way that the sensor channels are aligned with the global (virtual point) coordinate system, but the excitations could not be placed like this.

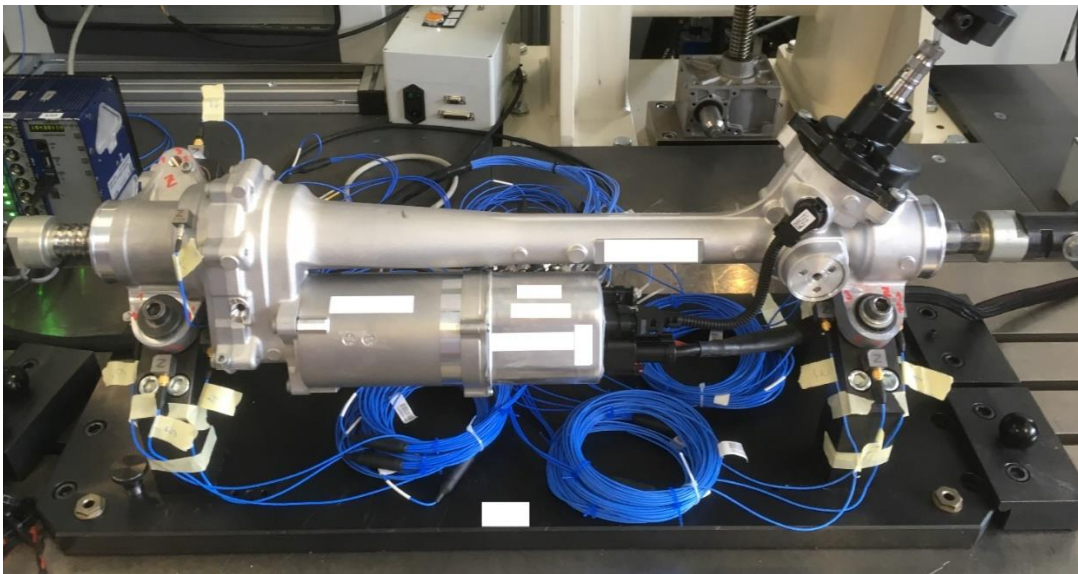


Figure 25: Steering gear

After the impulse hammer and geometry measurements, the running noise (operational) measurements were conducted with the sensor positions unchanged. The interface forces and the end of the process are calculated for these measurements.

4.2 Observation on the measured FRFs

The 1188 FRFs were calculated with the corresponding coherence functions. To determine some overall characteristics, the average is shown in Figure 26. Some global trends can be seen on the average of FRFs, but the average of coherences show more useful information about the measurement data.

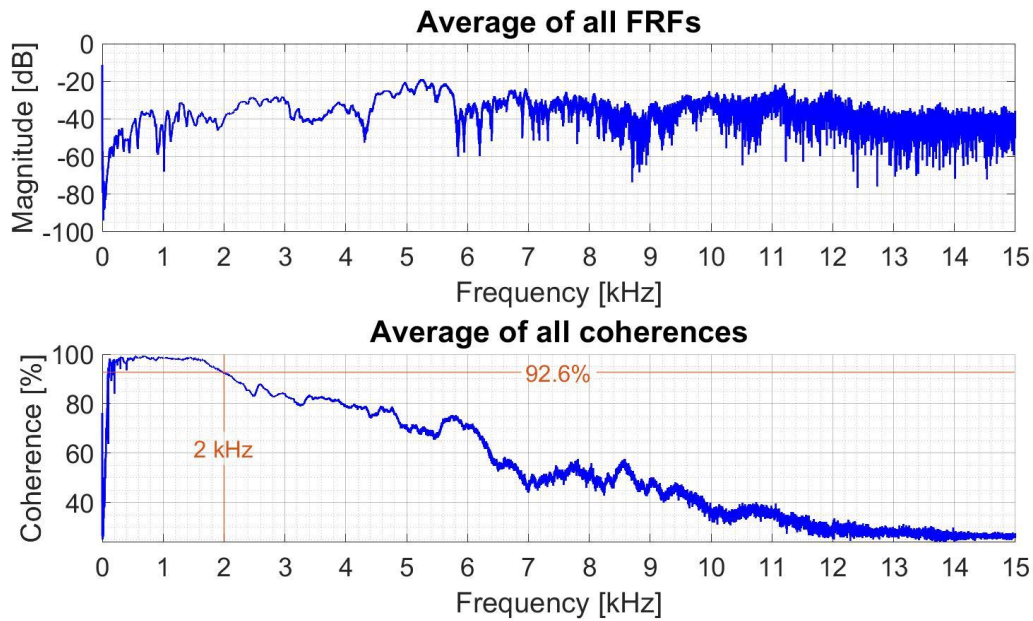


Figure 26: Average FRF and coherence

The frequency range of interest is up to about 2 kHz, where the average coherence is 92.6%, which is a fine value. It means that in this range, the transfer functions acquired from the measurement data are precise enough for further calculations. At 4 kHz, the average coherence reaches 80% that still can give roughly good results, but as the coherence value continues decreasing, the FRFs become more and more noisy and imprecise. Closer examination of the single FRFs showed the same results. Amongst modal behavior, the decline of the excitation is also a cause of the decline of the coherence. The spectrum of an ideal Dirac impulse is constant 1, but as the excitation is not ideal, the spectrum does not hold a constant value in the whole frequency range.

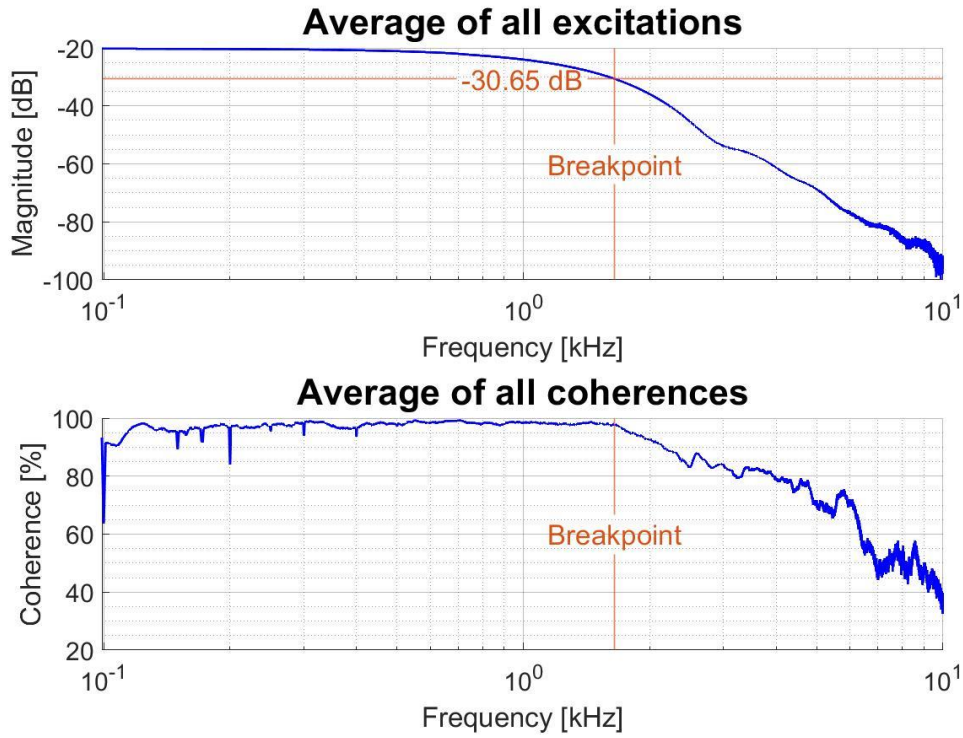


Figure 27: Average of excitations and coherences

As shown in Figure 27, soon after the excitation starts to plummet, the coherence value drops under its previous level too. It happens around the -10 dB decline of the excitations. As the H_1 estimator contains the auto spectra of the excitation in the denominator, very low values of the excitation can produce noisy values due to division with a very small value. As modal behavior is not significant in the lower frequency range, it is possible to check if the FRFs satisfy some simpler expectations. For example, since accelerations are being measured, the beginning of the transfer functions of Z-direction responses for Z-direction excitation at a given VP should increase with 40 dB/decade as the assembly acts as a mass on a spring at lower frequencies. Between 20 Hz and 200 Hz, the 40 dB/decade increase is roughly satisfied. The peak at 210 Hz probably indicates the presence of a modal component at that frequency. Unfortunately, there was no modal analysis at hand to confirm this assumption.

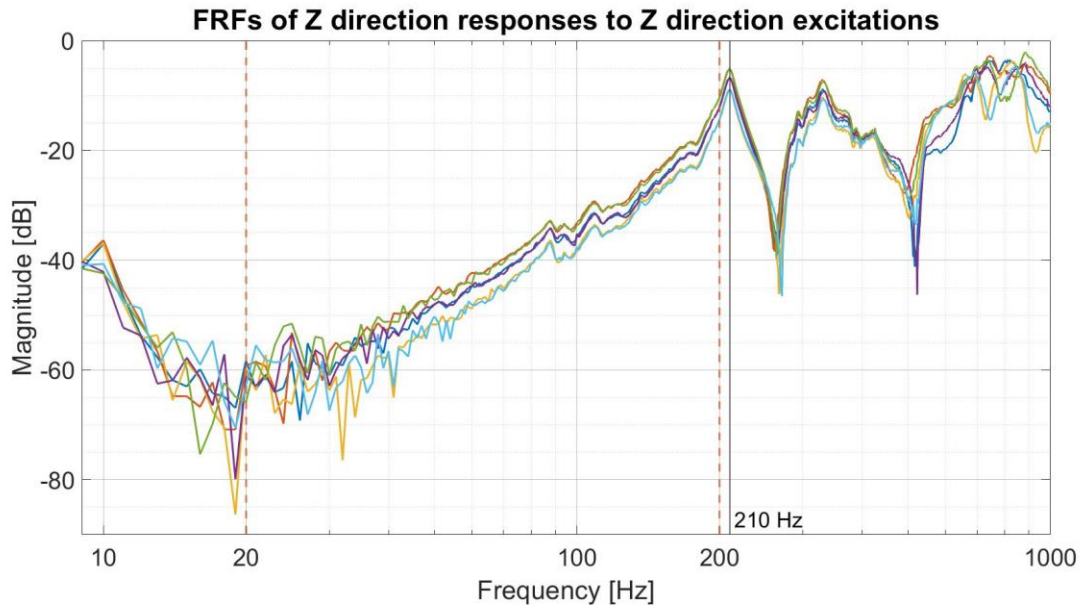


Figure 28: 40 dB/decade increase

An estimation of the signal to noise ratio was calculated for all FRFs with Eq. (2.11).

Table 3: SNR values of the steering gear

SNR calculated from coherence	From 0 Hz to 2 kHz	From 0 Hz to 4 kHz	From 0 Hz to 15 kHz
Average SNR [dB]	26.3299	23.9819	18.5268

SNR data also reflect that the measurements cannot be used for high frequencies but are adequate in the lower frequency range. As discussed in the previous chapter, it is possible to evaluate the rigidity of the feet of the assembly with SVD. If the FRFs of one foot show more than six singular values, that indicates flexibility. If they do not, there is still no guarantee, that rigidity is satisfied. The virtual point transformation quality indicators give more reliable feedback about rigidity, however, they all require the geometry data and the transformation and filtering matrices to be calculated. On the other hand, the only inputs for SVD are the adequate submatrices of the FRF matrix. On Figure 29 it can be seen, that all four legs contain no more than six dominant singular values. Now all the accessible basic quality indicators have been checked and it can be stated that the measurement data satisfies every requirement up to 2 kHz to be used for further calculations. Also, the 2 kHz – 4 kHz range probably yields acceptable results.

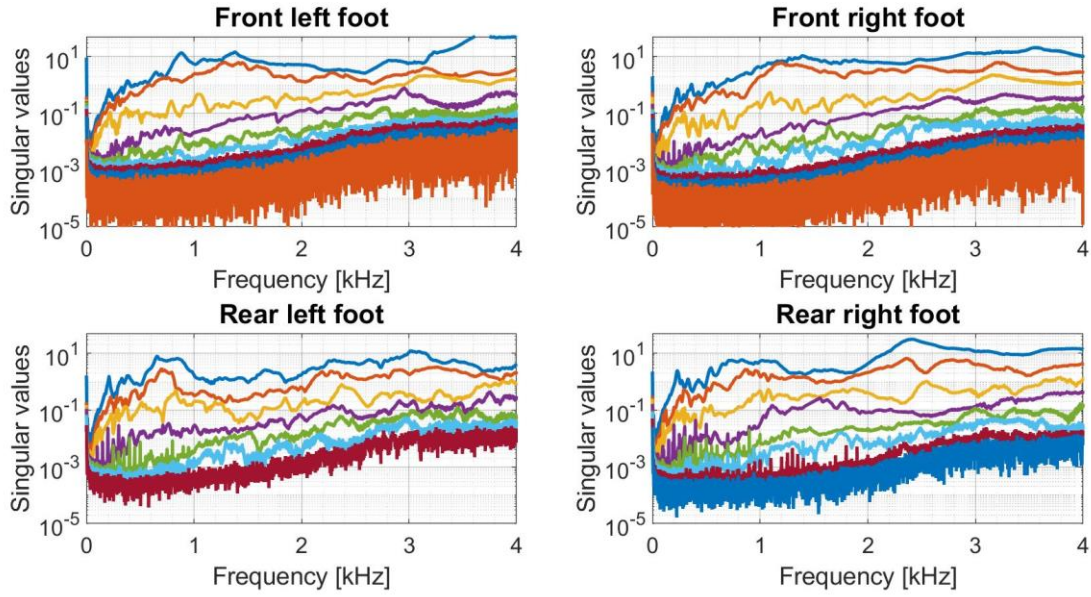


Figure 29: Singular values of every feet of the assembly

The next step is checking the geometry settings and measurements.

4.3 Virtual point transformation

Virtual point transformation was performed as discussed in section 2.4, altogether four filtering and transformation matrix was calculated, one for each virtual point. The geometry measurement was conducted on a FE CAD model of the steering gear.

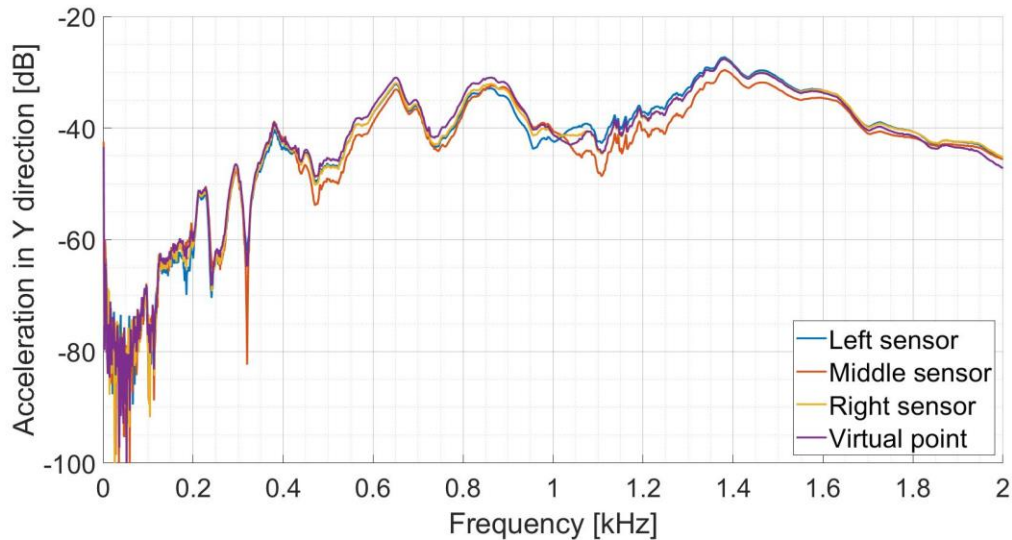


Figure 30: Example for virtual point translation result

Figure 30 shows one example for the result of virtual point translation. The virtual point acceleration follows the acceleration of the physical sensors, as it should be. One can notice, that even the

acceleration of the physical sensors deviates from each other to an extent. In those places either the rotational DoFs account for the difference, or flexibility is present.

4.3.1 Condition number and overdetermination

Typically, what one wants to see on the reduction matrix, is that all virtual DoFs are constructed from more than one physical DoFs. As a rule of thumb, it could be said that if at least 3 physical DoFs are contributing to the virtual DoFs, all directions have been adequately excited and measured with the accelerometers.

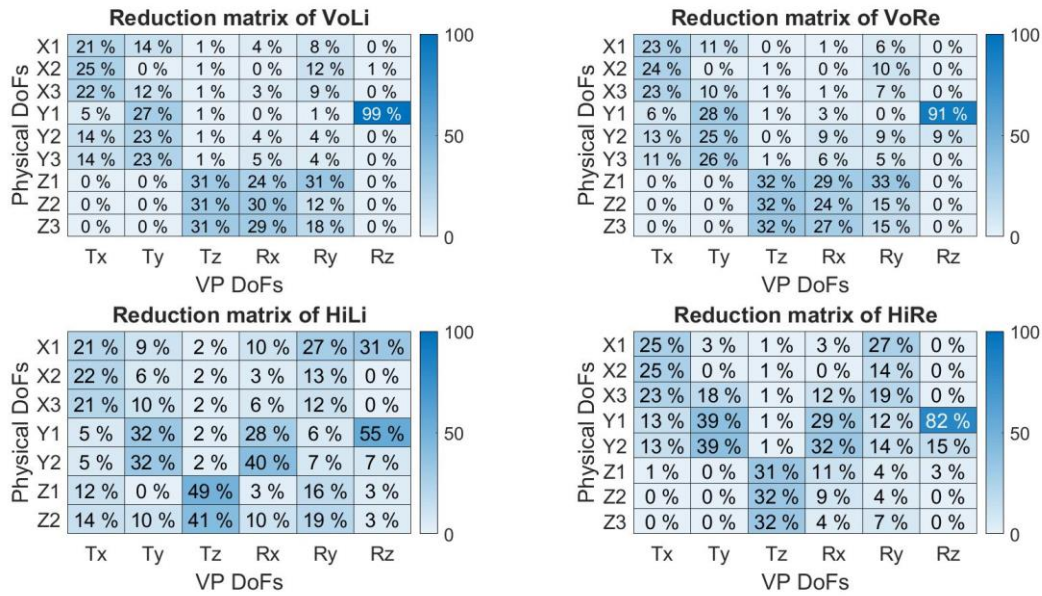


Figure 31: Contribution of physical excitations to the VP excitations

In Figure 31, Each column represents a VP DoF. Translational (T) in all directions, and rotational (R). Not every column satisfies the motion to have at least 3 contributing excitations. It is unfortunate, but many times, not all DoFs are necessary to describe the dynamic behavior, so might not be a big problem, however, translational DoFs usually are more important, so instances, where translations are not overdetermined enough, will probably cause some inaccuracy.

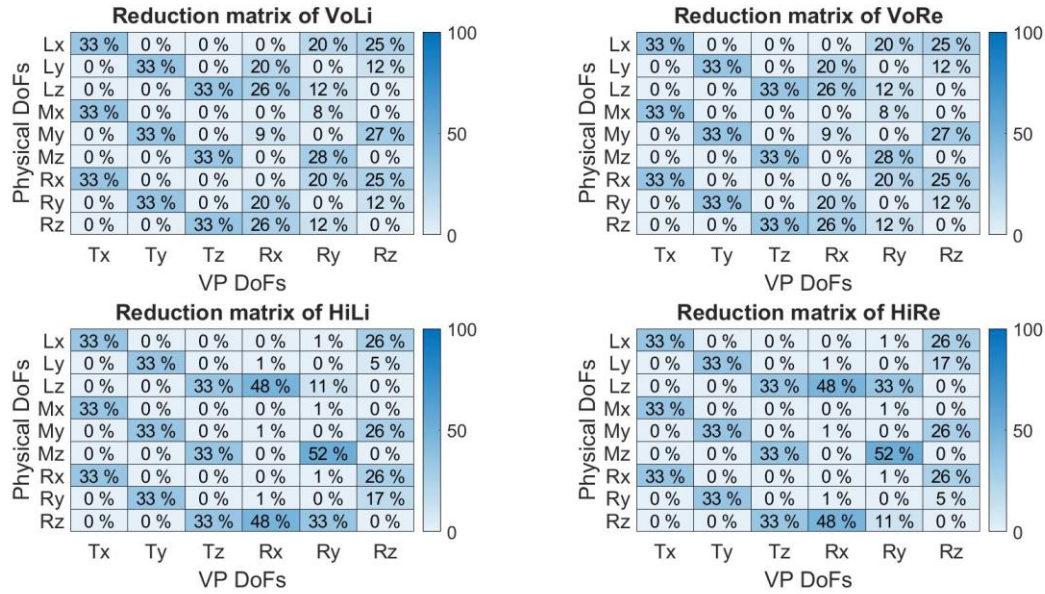


Figure 32: Contribution of physical acceleration to VP accelerations

Sensor positions seem to be chosen better (Figure 32), as there are just a few instances with less than 3 contributing physical DoFs, and all the translations are defined by 3 DoFs. Also, there is no VP DoF that is defined solely by one physical DoF.

4.3.2 Consistency

First of all, the impact consistency was evaluated as shown in Figure 33.

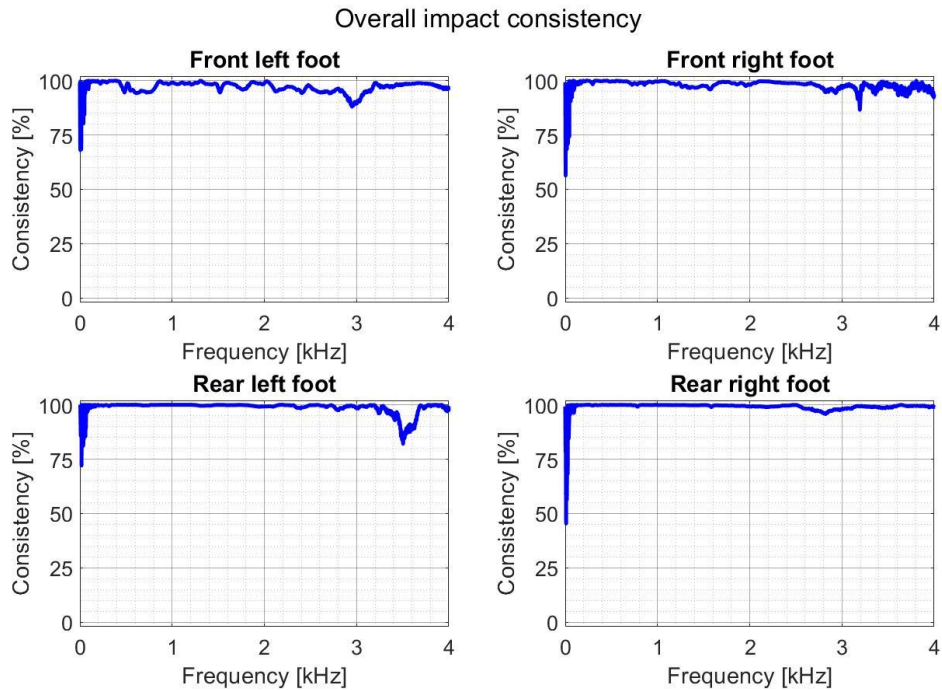


Figure 33: Overall impact consistency at every feet of the assembly

Although there are small decreases in the consistency value, this result shows that we are good to go on with the process. Especially in the 2 kHz range the data is nearly a 100% consistent at all virtual points. This means, that the area where the excitations are chosen around each virtual point can be considered rigid, and the geometry measurement was precise. As the overall consistency shows great results, no further analysis is needed. Following, the overall sensor consistency was evaluated. The results are shown in Figure 34. At two feet of the assembly, the consistency value is absolutely fine, but the other two feet show poorer results. The geometry of the sensor placement is very similar at all feet, and the previous indicators did not show significant flexibility, it was concluded that the reason for the poor consistency is faulty geometry measurement. To find where exactly the error is, specific sensor consistency could be helpful, as described below, there is no need for that here.

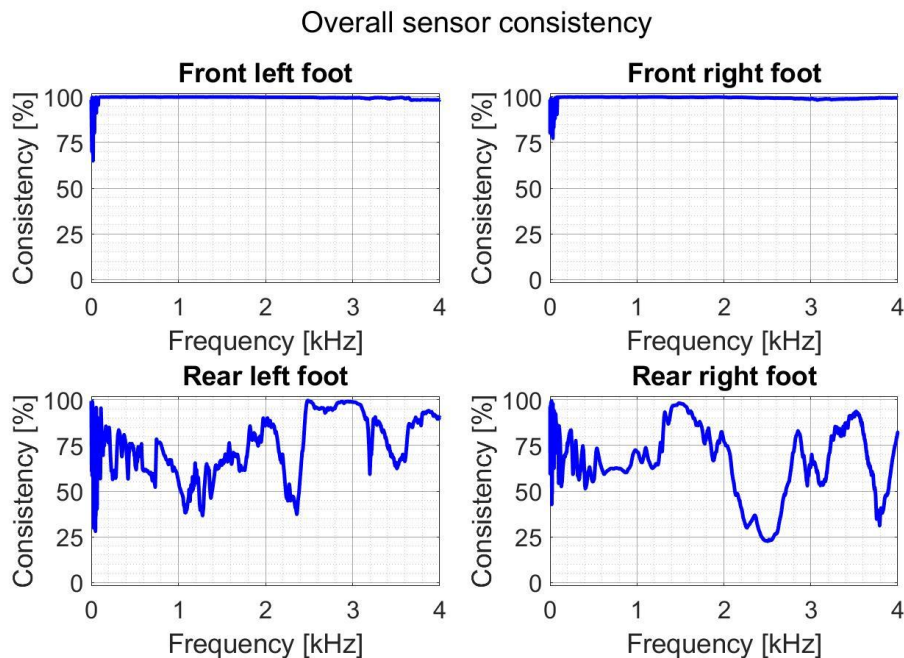


Figure 34: Overall sensor consistency at every feet of the assembly

Normally, the measurements should be repeated, but in this case, the sensor positions were unchanged consequently a one-sided transformation can be performed, omitting the transformation of the accelerations.

4.3.3 Virtual point FRF reciprocity

Although simply not using the acceleration transformation matrix requires no further time and resource to measure the geometry again, evaluation of FRF reciprocity and passivity is expected to yield lower values. With a complex assembly such as a steering gear it is not realistic to anticipate close to 100% reciprocity or passivity even with precise geometry measurement, but in this case, it is

known that the geometry contains errors. Therefore, these quality indicators are not very useful now. As the transformation of the forces seems to be adequate and the basic quality indicators showed that the FRFs are precise too, the interface force calculation can be done with transfer path analysis.

4.4 TPA calculation

It has been established before, that the one-sided transformation should be performed. It means that 36 accelerations stay unchanged while 33 excitations are transformed to 24 virtual point forces with Eq. (3.2). The resulting FRF matrix is used for the TPA, where Eq. (3.3) is applied. As discussed in section 2.5.1, the results can be improved by using singular value truncation. With cutting out some singular values, information is lost about the FRFs, but if the number of omitted singular values is correct, noise reductions is achieved, and no useful information is lost. After experimentation, 5 singular values seemed to be enough to get rid of the noise while not losing important information.

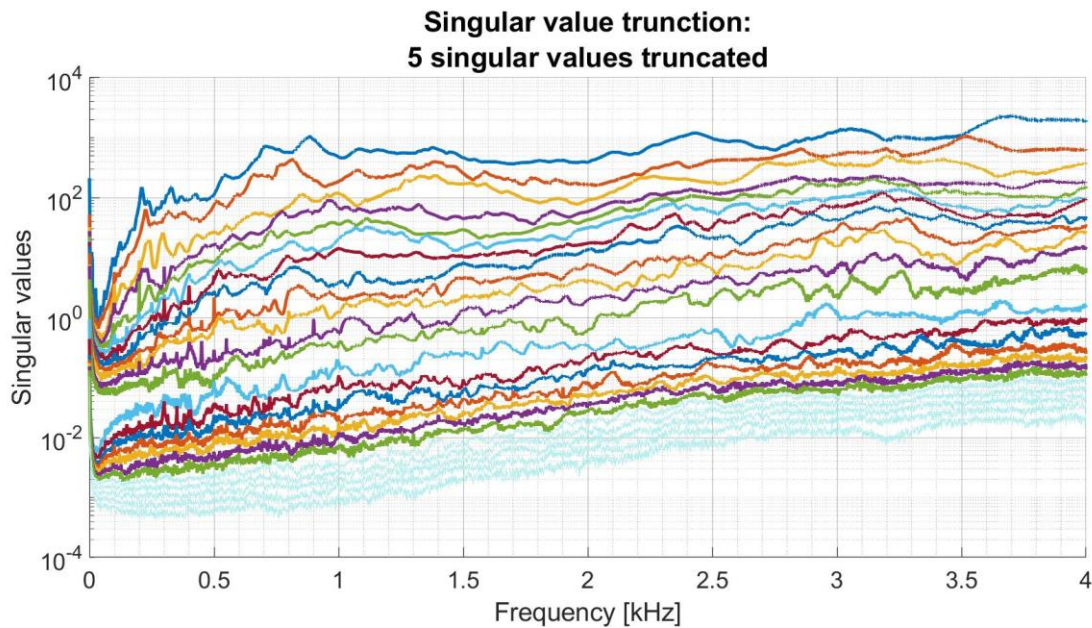


Figure 35: Singular value truncation

The picture above shows the truncated singular values with pale blue. The transfer functions reconstructed from the truncated singular value matrix fits the original, the remaining 19 singular values carry all the useful information. Although in the following 5 truncated values are used, omitting or keeping 1 or 2 more values do not make a great difference. It can also be observed, that the translational and rotational DoFs are nicely separated from each other, as the translational DoFs tend to bear the most information. It is important to note here, that SVD truncation or other matrix conditioning methods can be useful, but it is more advisable to use a 1.5-2 times overdetermination

for the forces and accelerations as conditioning, and pay attention for the precision of the measurement data. This way conditioning methods can become unnecessary to achieve great results.

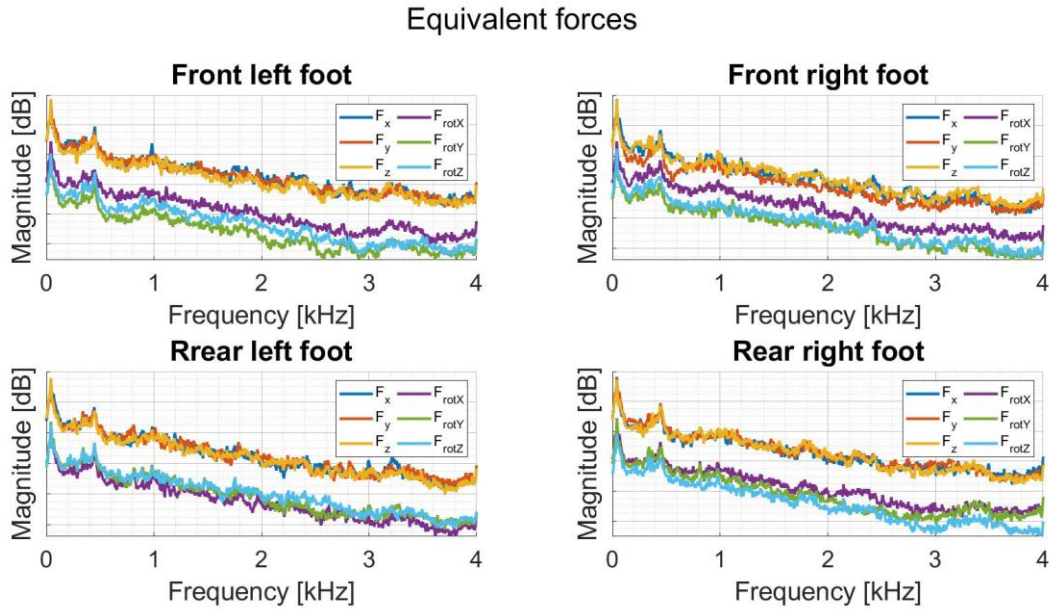


Figure 36: Equivalent forces to 600°s RL measurement

To confirm the results, of the equivalent force calculation the validation process discussed in section 3.1.4 is performed.

4.4.1 Validation

Equivalent forces were calculated to a number of different operational measurements. The results of 600°s RL are shown here, which means constant wheeling to the right. The calculated equivalent forces are used with Eq. (3.3) for validation. There is a validation sensor positioned on the Rear left foot of the assembly as shown in Figure 37. The simple TPA is compared to the TPA calculated with truncation to see the effect of the truncation.

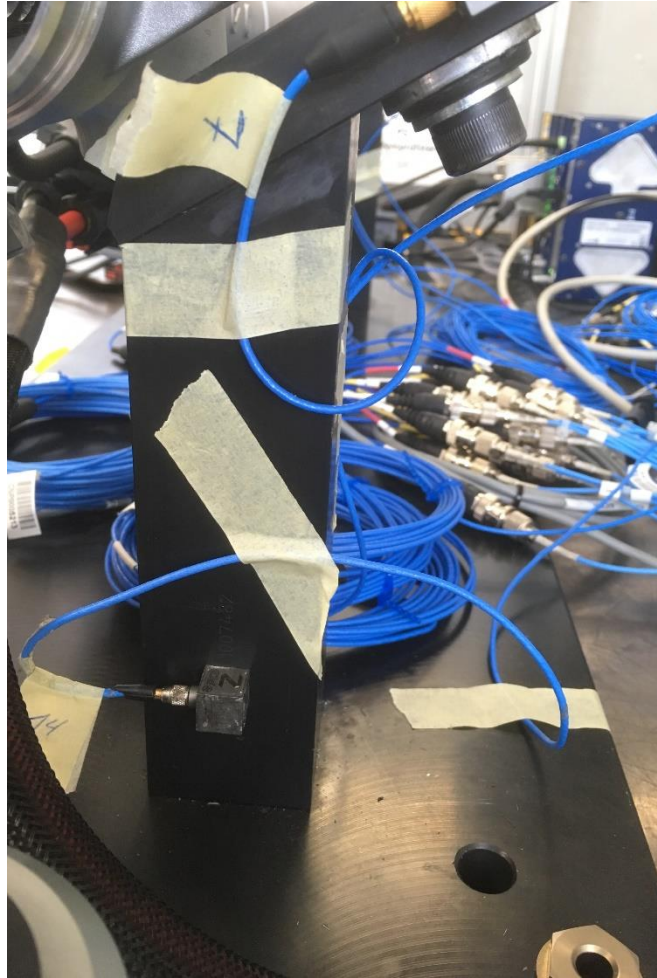


Figure 37: Validation sensor position

The results of the validation are shown in Figure 38 on the X and Z direction of the validation sensor. TPA calculation results do not fit the validation data perfectly. There are several reasons for that. One, is that the validation sensor is close to the bottom of the testbench, which is very rigid, so the signal to noise ratio is not ideal (To handle that, the implementation of a more flexible testbench fixture is in progress). Higher overdetermination of forces would lead to better conditioned data, and better on-board validation as well. The frequency range of precise FRF estimation, thus precise calculations depend greatly on the tip of the hammer used for measurements. In the future it is planned to conduct the measurements with more than one type of hammer tip in order to provide better signal to noise ratio over the entire bandwidth. All in all, the validation shows that the calculated blocked forces describe the steering gear well on the frequency range of interest. It can also be seen, that on higher frequencies, the difference between the validation data and the calculations are getting higher. That is because or FRF calculation in more reliable on lower frequencies, and also as we go in the direction of higher frequencies, the gear behaves more and more flexible.

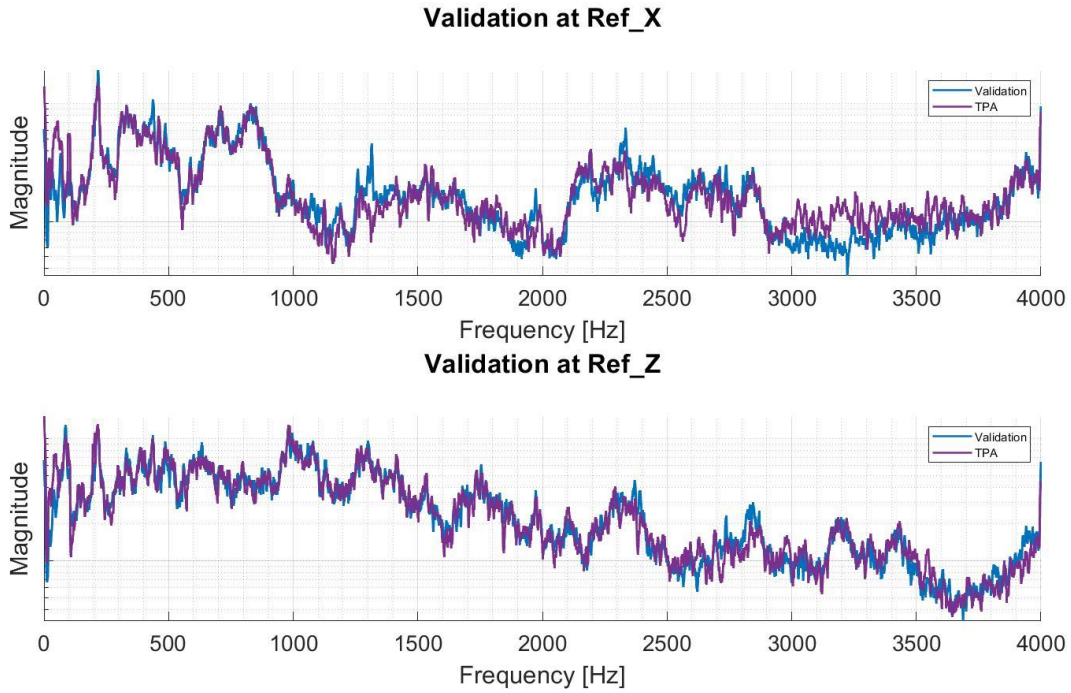


Figure 38: Validation of TPA in X and Z direction

The reason, why the sensor should not be paced on the interface, is because the interface is considered to be rigid, so validation calculated there should have a perfect fit. Although in reality, because of flexibility and noise, it will not be a perfect fit. Interface sensors are not good for validation purposes, they might just be good to show the difference between truncated and original virtual point FRFs.

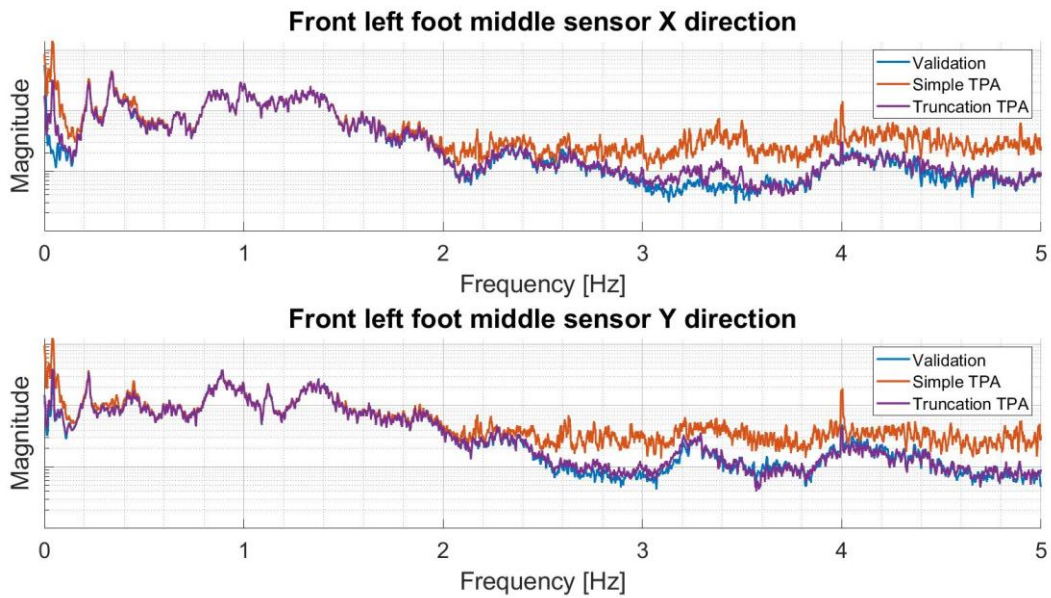


Figure 39: Effect of truncation

In Figure 39 blocked forces are calculated with the middle sensors excluded from the calculation and used as validation sensors. In the low frequency range, the TPA matches the validation nearly perfectly, but on higher frequencies the results become poorer. It is very clear however, that application of truncation allows for less noisy results.

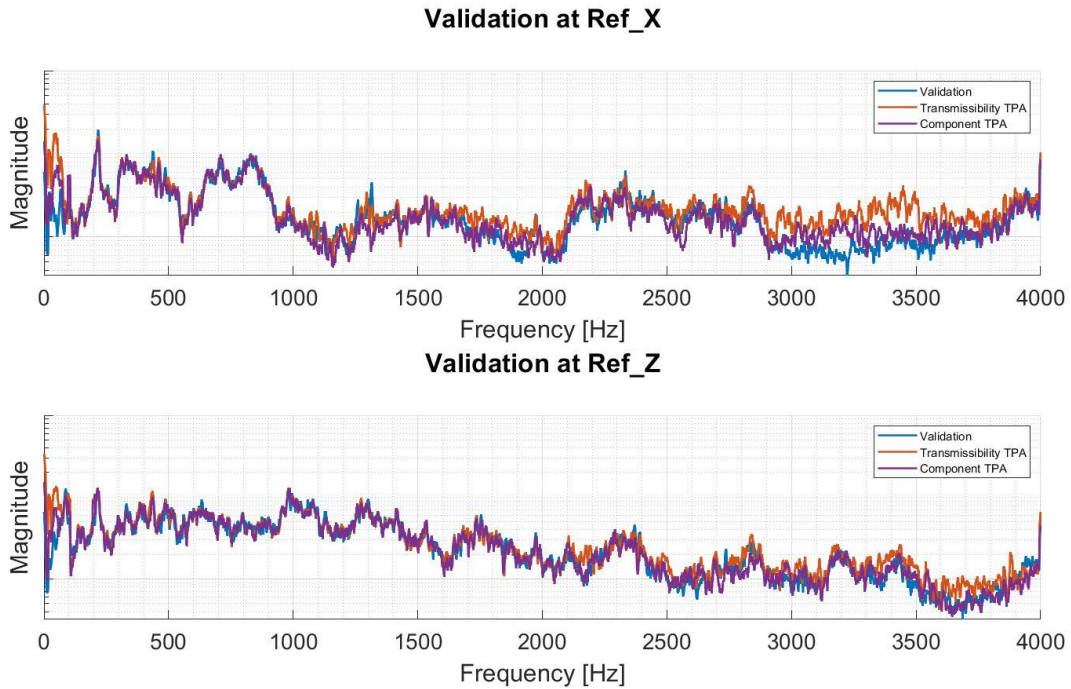


Figure 40: Comparison of component and transmissibility TPA

As shown in Figure 40, the the transmissibility TPA yields similar results as the component TPA. Based on the comparisons, the truncation is successful and the one-sided VPT is quite accurate too.

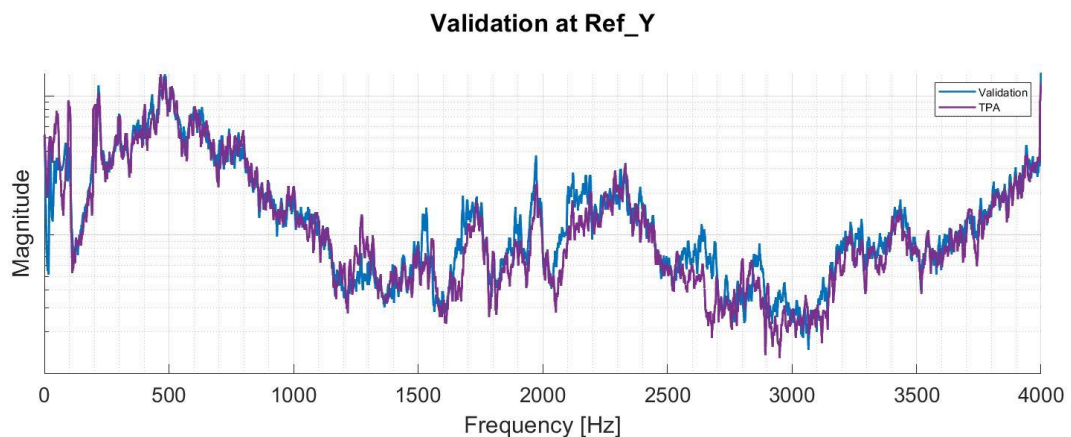


Figure 41: Validation in the Y direction

5 Conclusion

The designed workflow has been successfully applied to the steering gear. Blocked forces were acquired and validated, showing a quite good prediction on the on-board validation sensor. Experimentation with a simple model allowed for a more detailed view on virtual point transformation, which was later applied on the steering gear resulting in adequate one-side transformed virtual point FRFs. A variety of quality indicators are shown and used during the whole process.

To improve the results, the design of a new testbench fixture is in progress, optimized for blocked force calculation. For easier and quicker measurement and analysis, the implementation of a graphical user interface is planned to contain a TPA calculation module with optional regularization and conditioning methods, as well as validation, a virtual point transformation module with all the quality indicators shown in the thesis, and an on-line measurement module that provides the opportunity of quick feedback on the measurements. It is also planned to link the Matlab GUI to CAD, so the geometry is more easily measured and set. For future measurements, using higher overdetermination is also advisable.

References

- [1] G. Orosz, L. Sujbert, Elosztott rendszerek és szenzorhálózatok 1., BME-MIT, 2008
- [2] Péter Fiala, Móduselezés, Mérési leírás
- [3] P. Welch, The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms, in IEEE Transactions on Audio and Electroacoustics, vol. 15, no. 2, pp. 70-73, June 1967, doi: 10.1109/TAU.1967.1161901.
- [4] J.H. Gordis, R.L. Bielawa, W.G. Flannelly, A general theory for frequency domain structural synthesis, Journal of Sound and Vibration, Volume 150, Issue 1, 1991, Pages 139-158, ISSN 0022-460X, [https://doi.org/10.1016/0022-460X\(91\)90407-B](https://doi.org/10.1016/0022-460X(91)90407-B)
- [5] Y. Ren, C.F. Beards, On substructure synthesis with FRF data, Journal of Sound and Vibration, Volume 185, Issue 5, 1995, Pages 845-866, ISSN 0022-460X, <https://doi.org/10.1006/jsvi.1995.0421>
- [6] Armin Drozg, Gregor Čepon, Miha Boltežar, Full-degrees-of-freedom frequency based substructuring, Mechanical Systems and Signal Processing, Volume 98, 2018, Pages 570-579, ISSN 0888-3270, <https://doi.org/10.1016/j.ymssp.2017.04.051>
- [7] Klerk, Dennis & Rixen, Daniel & Jong, J. (2006). The Frequency Based Substructuring Method reformulated according to the Dual Domain Decomposition Method
- [8] Van der Seijs, Maarten. (2016). Experimental Dynamic Substructuring: Analysis and design strategies for vehicle development. 10.4233/uuid:28b31294-8d53-49eb-b108-284b63edf670.
- [9] Gialamas, T. & Tsahalidis, Demosthenes & Bregant, Luigi & Otte, Dirk & Van der Auweraer, Herman. (1996). "Substructuring by Means of FRFs: Some Investigations on the Significance of Rotational DOFs", https://www.researchgate.net/publication/266346944_Substructuring_by_Means_of_FRFs_Some_Investigations_on_the_Significance_of_Rotational_DOFs
- [10] Moon, Seok-Jun & Kwak, Jeong-Seok & Chung, Jung Hoon & Ji, Yong-Jin & Yoon, Jeong-Sik & Choi, S. & Lee, Hyun-Yup & Jung, Woo-Jin & Ki, Dock-Jong. (2010). A Study on the Hybrid Mount against Vibration and Shock for Naval Ships. Shock and Vibration. 17. 269-283. 10.1155/2010/390453.
- [11] Maarten V. van der Seijs, Dennis de Klerk, Daniel J. Rixen, General framework for transfer path analysis: History, theory and classification of techniques, Mechanical Systems and Signal Processing, Volumes 68–69, 2016, Pages 217-244, ISSN 0888-3270, <https://doi.org/10.1016/j.ymssp.2015.08.004>.
- [12] A.T. Moorhouse, A.S. Elliott, T.A. Evans, In situ measurement of the blocked force of structure-borne sound sources, Journal of Sound and Vibration, Volume 325, Issues 4–5, 2009, Pages 679-685, ISSN 0022-460X, <https://doi.org/10.1016/j.jsv.2009.04.035>.
- [13] Oriol Guasch, Carlos García, Jordi Jové, Pere Artís, Experimental validation of the direct transmissibility approach to classical transfer path analysis on a mechanical setup, Mechanical

Systems and Signal Processing, Volume 37, Issues 1–2, 2013, Pages 353-369, ISSN 0888-3270, <https://doi.org/10.1016/j.ymssp.2013.01.006>.

- [14] Bregar, Tomaž & Holeček, Nikola & Čepon, Gregor & Rixen, Daniel & Boltežar, Miha. (2019). Including directly measured rotations in the virtual point transformation. *Mechanical Systems and Signal Processing*. 141. 106440. 10.1016/j.ymssp.2019.106440.
- [15] Van der Seijs, Maarten & van den Bosch, Daniël & Rixen, Daniel & Klerk, Dennis. (2013). An improved methodology for the virtual point transformation of measured frequency response functions in dynamic substructuring. 4334-4347. 10.7712/120113.4816.C1539.
- [16] PASMA, E. A. Towards a Robust Component Transfer Path Analysis Method: Application and Validation in Automotive Research. 2014. <http://resolver.tudelft.nl/uuid:50cd45f4-e7ec-40c4-bc0e-d544e95a0e67>
- [17] Brunton, Steven & Kutz, J. (2019). Chapter 1: Singular Value Decomposition. <http://databookuw.com/>
- [18] HEYLEN, Ward, et al. *Modal analysis theory and testing*. Leuven, Belgium: Katholieke Universiteit Leuven, 1997.
- [19] Klerk, Dennis & Rixen, Daniel & Voormeeren, Sven & Pasteuning, F. (2008). Solving the RDoF Problem in Experimental Dynamic Substructuring. *Conference Proceedings of the Society for Experimental Mechanics Series*.

Figures

Figure 1: 4 DoF concentrated parameter oscillating system.....	11
Figure 2: Uncoupled subsystems: active (A) and passive (P).....	15
Figure 3: Traditional TPA: Coupled (left) and uncoupled (right) representation	18
Figure 4: Blocked force TPA	20
Figure 5: Free displacement TPA.....	21
Figure 6: Hybrid interface TPA	22
Figure 7: In-situ TPA.....	24
Figure 8: Transmissibility TPA.....	25
Figure 9: Illustration of virtual point transformation [8]	26
Figure 10: Virtual point FRF	30
Figure 11: FRF reciprocity. Excitation and response in the same direction at different virtual points.	33
Figure 12: Examples for singular values	34
Figure 13: Workflow	38
Figure 14: Example for poor coherence on lower frequencies	39
Figure 15: Singular values of a subset of the whole FRF matrix that belongs to one foot of the steering gear.....	40
Figure 16: Examples for more than 6 dominant singular values	40
Figure 17: SNR calculated from simulated transfer function with SNR = 20dB.....	41
Figure 18: The influence of number of samples to SNR precision	43
Figure 19: Overall impact consistency with correct geometry data (left) and faulty data (right)	44
Figure 20: TPA validation process.....	45
Figure 21: Simple model for experimentation	46
Figure 22: Measurement errors reflected by consistency	47
Figure 23: Comparison of transformed and simulated responses for a given excitation	48
Figure 24: Example for the influence of modal behavior on VP transformation.....	49
Figure 25: Steering gear.....	50
Figure 26: Average FRF and coherence	51
Figure 27: Average of excitations and coherences.....	52
Figure 28: 40 dB/decade increase	53

Figure 29: Singular values of every feet of the assembly	54
Figure 30: Example for virtual point translation result	54
Figure 31: Contribution of physical excitations to the VP ecitations.....	55
Figure 32: Contribution of physical acceleration to VP accelerations	56
Figure 33: Overall impact consistency at every feet of the assembly	56
Figure 34: Overall sensor sonsistency at every feet of the assembly	57
Figure 35: Singular value truncation	58
Figure 36: Equivalent forces to 600°s RL measurement	59
Figure 37: Validation sensor position.....	60
Figure 38: Validation of TPA in X and Z direction	61
Figure 39: Effect of truncation.....	61
Figure 40: Comparison of component and transmissibility TPA.....	62
Figure 41: Validation in the Y direction.....	62