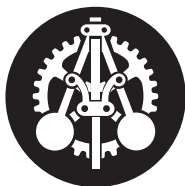


BUDAPESTI MŰSZAKI ÉS GAZDASÁGTUDOMÁNYI EGYETEM
GÉPÉSZMÉRNÖKI KAR
MŰSZAKI MECHANIKAI TANSZÉK



Appendix

Estimating the coefficient of friction based on the
memory effect of an elastic tire

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BUDAPEST, 2022 november

Deformation function parameters for three fragmentation zones

The points, where the sticking region turns into sliding, can be expressed as

$$x_R = -\frac{a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi^2 - 12 F_N a e k \mu \psi \text{sign}(\psi)} - 2 a^2 k \psi \text{sign}(\psi) \right)}{3 F_N \mu},$$

$$x_L = \frac{a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi^2 + 12 F_N a e k \mu \psi \text{sign}(\psi)} - 2 a^2 k \psi \text{sign}(\psi) \right)}{3 F_N \mu}.$$

Deformation function parameters for five fragmentation zones

The parameters of the deformation function with five zones, in case of positive angular direction can be expressed as

$$x_R^* = -\frac{a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu},$$

$$x_L^* = \frac{a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu},$$

$$m_R = -\frac{\text{num}(m_R)}{\text{denum}(m_R)},$$

$$\begin{aligned} \text{num}(m_R) = & e \psi + \\ & + \frac{3 F_N \mu \left(a^2 - \frac{a^2 \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)^2}{9 F_N^2 \mu^2} \right)}{4 a^3 k} + \\ & + \frac{a \psi \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu} \\ \text{denum}(m_R) = & e + \\ & + \frac{0.3333 a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{F_N \mu}, \end{aligned}$$

$$m_L = \frac{\text{num}(m_L)}{\text{denum}(m_L)},$$

$$\text{num}(m_L) = \frac{3 F_N \mu \left(a^2 - \frac{a^2 \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max})} \right)^2}{9 F_N^2 \mu^2} \right)}{4 a^3 k} - \frac{a \psi \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu} - e \psi + \frac{a \psi \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu},$$

$$\text{denum}(m_L) = e - \frac{0.3333 a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{F_N \mu},$$

$$x_R = \frac{a^3 k \left(\frac{\sigma_1}{\sigma_2} - \sqrt{\frac{\sigma_1^2}{\sigma_2^2} - \frac{6 F_N \mu \left(\frac{e \sigma_1}{\sigma_2} - \frac{3 F_N \mu}{2 a k} \right)}{a^3 k}} \right)}{F_N \mu},$$

$$\sigma_1 = e \psi + \frac{3 F_N \mu \left(a^2 - \frac{a^2 \sigma_3^2}{9 F_N^2 \mu^2} \right)}{4 a^3 k},$$

$$\sigma_2 = e + \frac{a \sigma_3}{3 F_N \mu},$$

$$\sigma_3 = \sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max})},$$

$$x_L = \frac{a^3 k \left(\frac{\sigma_4}{\sigma_5} + \sqrt{\frac{\sigma_4^2}{\sigma_5^2} - \frac{6 F_N \mu \left(\frac{e \sigma_4}{\sigma_5} - \frac{3 F_N \mu}{2 a k} \right)}{a^3 k}} \right)}{F_N \mu},$$

$$\sigma_4 = -e \psi + \frac{3 F_N \mu \left(a^2 - \frac{a^2 \sigma_6^2}{9 F_N^2 \mu^2} \right)}{4 a^3 k},$$

$$\sigma_5 = e - \frac{a \sigma_3}{3 F_N \mu},$$

$$\sigma_6 = \sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max})}.$$

And then, the parameters of the deformation function with five zones, in case of negative angular direction can be expressed as

$$x_R^* = - \frac{a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu},$$

$$x_L^* = \frac{a \left(\sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max})} - 2 a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3 F_N \mu},$$

$$m_R = - \frac{\text{num}(m_R)}{\text{denum}(m_R)},$$

$$\begin{aligned}
\text{num}(m_R) = & e\psi - \\
& \frac{3F_N\mu \left(a^2 - \frac{a^2 \left(\sqrt{9F_N^2\mu^2 + 4a^4k^2\psi_{\max}^2} - 12F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)^2}{9F_N^2\mu^2} \right)}{4a^3k} + \\
& + \frac{a\psi \left(\sqrt{9F_N^2\mu^2 + 4a^4k^2\psi_{\max}^2} - 12F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3F_N\mu},
\end{aligned}$$

$$\begin{aligned}
\text{denum}(m_R) = & e + \\
& + \frac{a \left(\sqrt{9F_N^2\mu^2 + 4a^4k^2\psi_{\max}^2} - 12F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3F_N\mu},
\end{aligned}$$

$$m_L = -\frac{\text{num}(m_L)}{\text{denum}(m_L)},$$

$$\begin{aligned}
\text{num}(m_L) = & e\psi + \\
& \frac{3F_N\mu \left(a^2 - \frac{a^2 \left(\sqrt{9F_N^2\mu^2 + 4a^4k^2\psi_{\max}^2} + 12F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)^2}{9F_N^2\mu^2} \right)}{4a^3k} - \\
& - \frac{a\psi \left(\sqrt{9F_N^2\mu^2 + 4a^4k^2\psi_{\max}^2} + 12F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3F_N\mu},
\end{aligned}$$

$$\begin{aligned}
\text{denum}(m_L) = & e - \\
& - \frac{a \left(\sqrt{9F_N^2\mu^2 + 4a^4k^2\psi_{\max}^2} + 12F_N a e k \mu \psi_{\max} \text{sign}(\psi_{\max}) - 2a^2 k \psi_{\max} \text{sign}(\psi_{\max}) \right)}{3F_N\mu},
\end{aligned}$$

$$\begin{aligned}
x_R &= -\frac{a^3 k \left(\frac{\sigma_7}{\sigma_8} + \sqrt{\frac{\sigma_7^2}{\sigma_8^2} + \frac{6 F_N \mu \left(\frac{e \sigma_7}{\sigma_8} + \frac{3 F_N \mu}{2 a k} \right)}{a^3 k}} \right)}{3 F_N \mu}, \\
\sigma_7 &= e \psi - \frac{3 F_N \mu \left(a^2 - \frac{a^2 \sigma_9^2}{9 F_N^2 \mu^2} \right)}{4 a^3 k} + \frac{a \psi \sigma_9}{F_N \mu}, \\
\sigma_8 &= e + \frac{a \sigma_9}{3 F_N \mu}, \\
\sigma_9 &= \sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 - 12 F_N a e k \mu \psi_{\max} \operatorname{sign}(\psi_{\max}) - 2 a^2 k \psi_{\max} \operatorname{sign}(\psi_{\max})}, \\
x_L &= -\frac{a^3 k \left(\frac{\sigma_{10}}{\sigma_{11}} + \sqrt{\frac{\sigma_{10}^2}{\sigma_{11}^2} - \frac{6 F_N \mu \left(\frac{e \sigma_{10}}{\sigma_{11}} - \frac{3 F_N \mu}{2 a k} \right)}{a^3 k}} \right)}{3 F_N \mu}, \\
\sigma_{10} &= e \psi + \frac{3 F_N \mu \left(a^2 - \frac{a^2 \sigma_{12}^2}{9 F_N^2 \mu^2} \right)}{4 a^3 k} - \frac{a \psi \sigma_{12}}{F_N \mu}, \\
\sigma_{11} &= e - \frac{a \sigma_{12}}{3 F_N \mu}, \\
\sigma_{12} &= \sqrt{9 F_N^2 \mu^2 + 4 a^4 k^2 \psi_{\max}^2 + 12 F_N a e k \mu \psi_{\max} \operatorname{sign}(\psi_{\max}) - 2 a^2 k \psi_{\max} \operatorname{sign}(\psi_{\max})},
\end{aligned}$$

where $\sigma_1 \dots \sigma_{12}$ are parameters created, in order to ease the description of the given expressions.

Analytic estimation results for simulated data

The results of the analytic estimation for simulated data, are shown here (only those cases, that were not showed in the documentation).

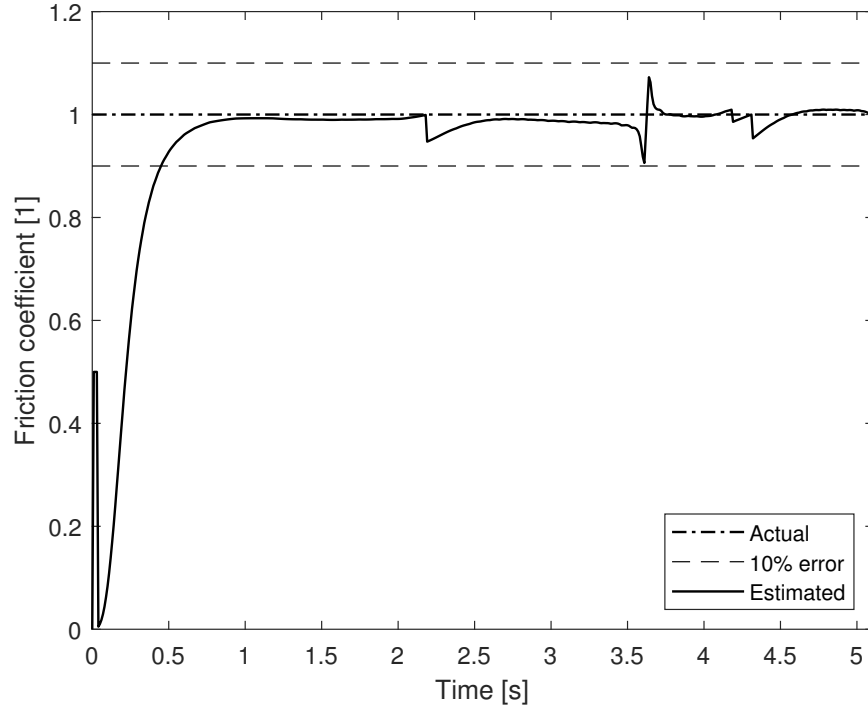


Figure 0.1: Analytic coefficient of friction estimation with simulated data for $\mu = 1$.

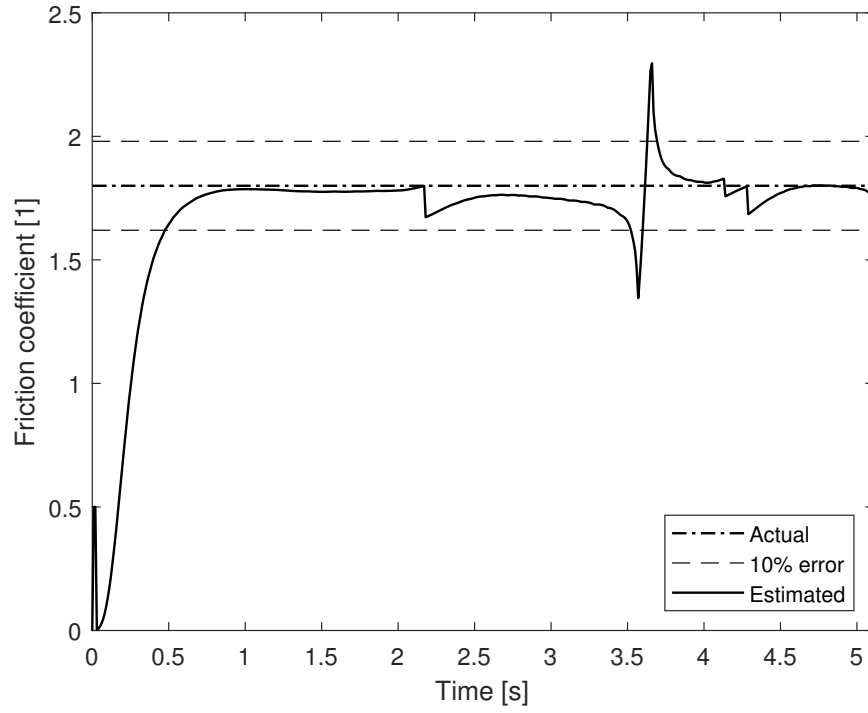


Figure 0.2: Analytic coefficient of friction estimation with simulated data for $\mu = 1.8$.

Analytic estimation results for measured data

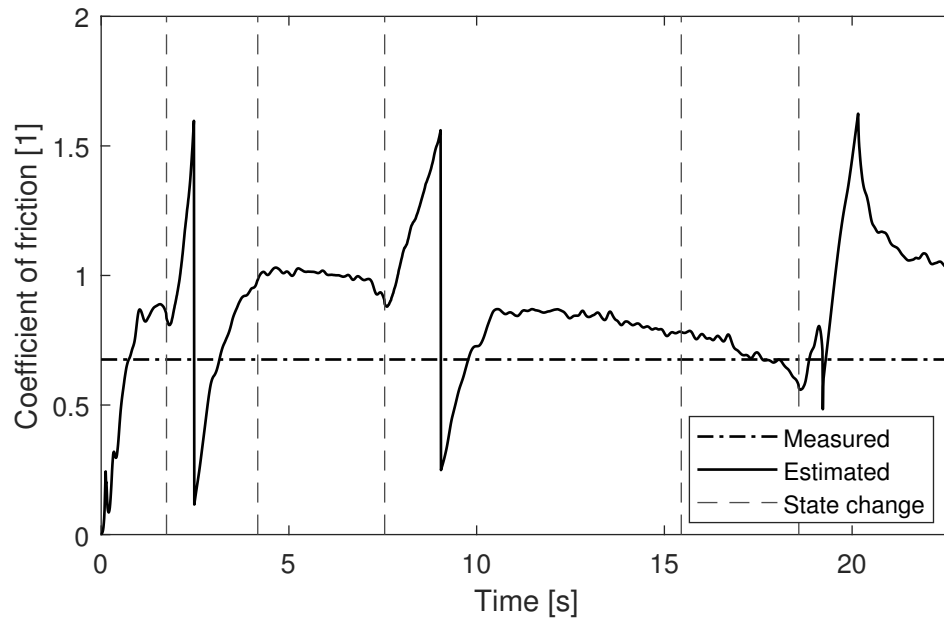


Figure 0.3: Estimation results, for the black emery paper, with contact patch half-length of $a = 40$ mm

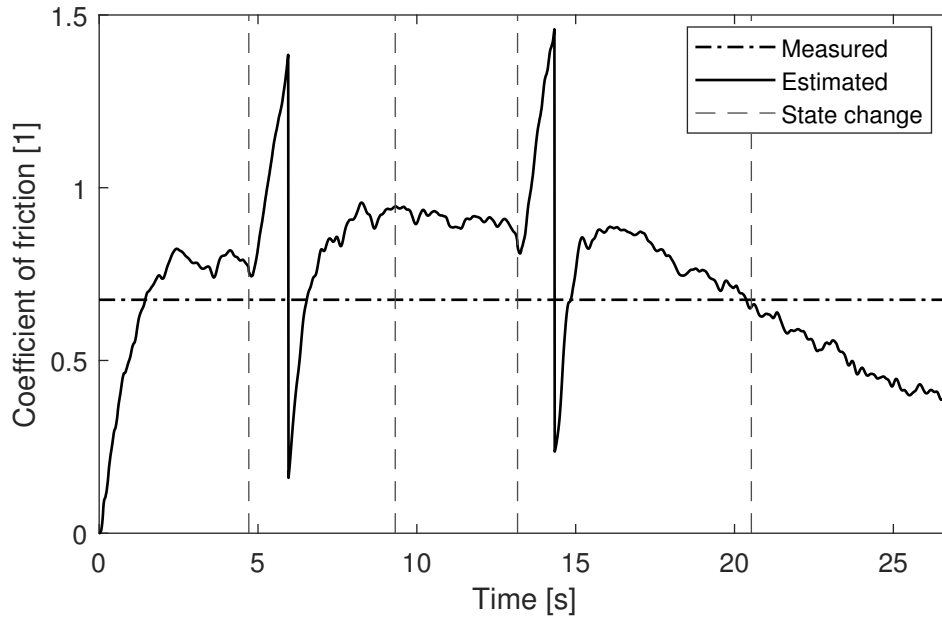


Figure 0.4: Estimation results, for the black emery paper, with contact patch half-length of $a = 40$ mm

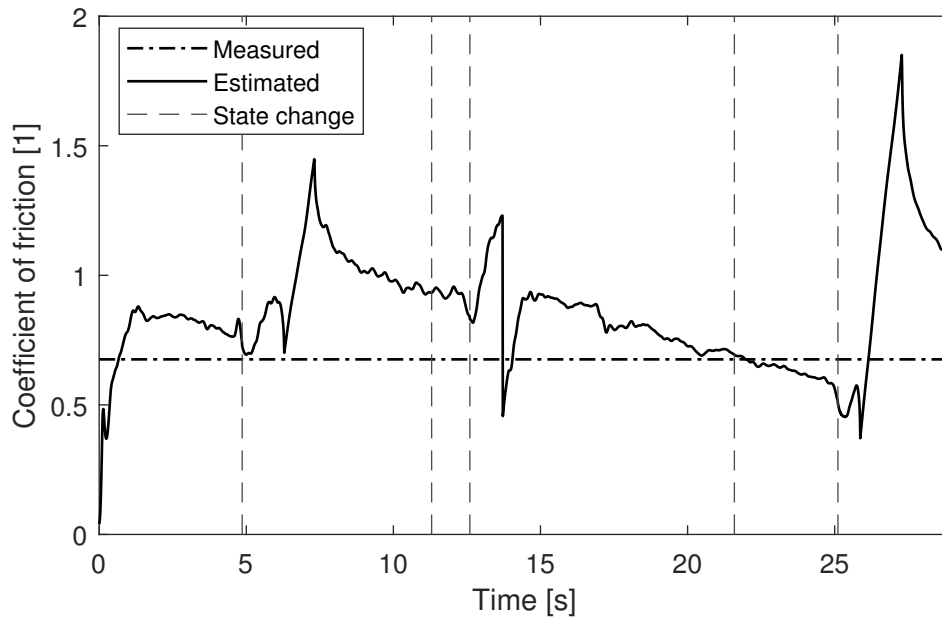


Figure 0.5: Estimation results, for the black emery paper, with contact patch half-length of $a = 40$ mm

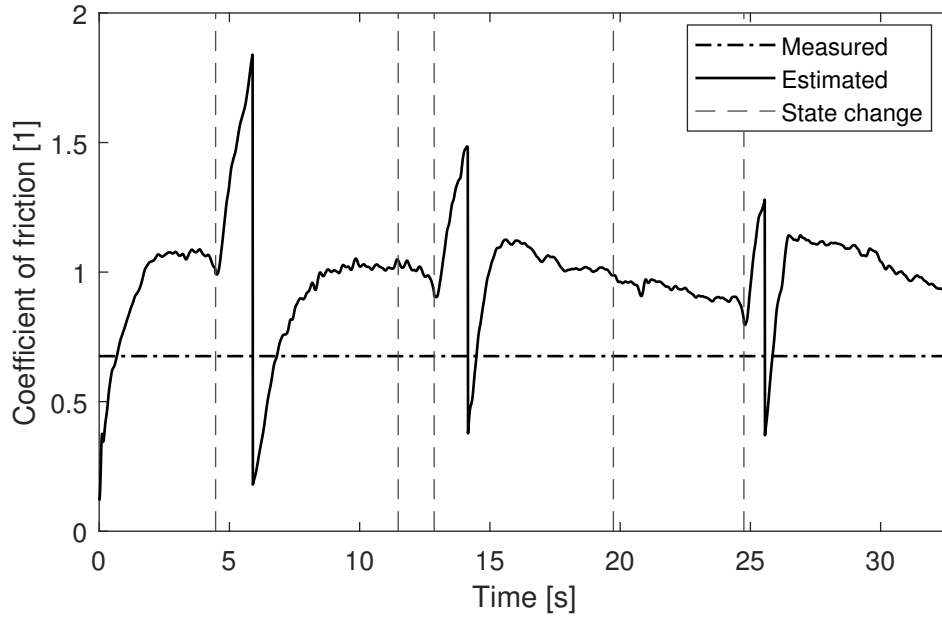


Figure 0.6: Estimation results, for the black emery paper, with contact patch half-length of $a = 45$ mm

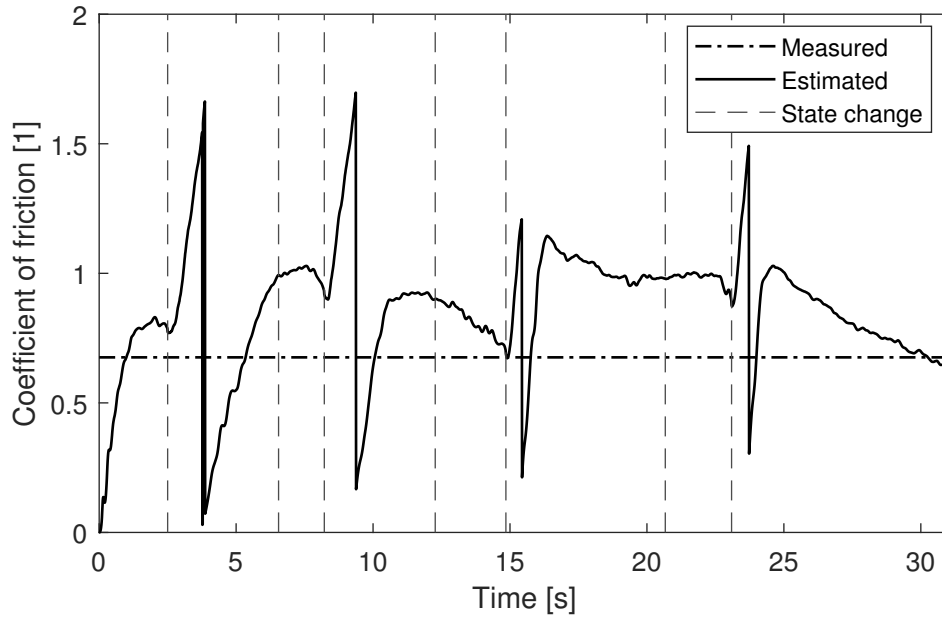


Figure 0.7: Estimation results, for the black emery paper, with contact patch half-length of $a = 45$ mm

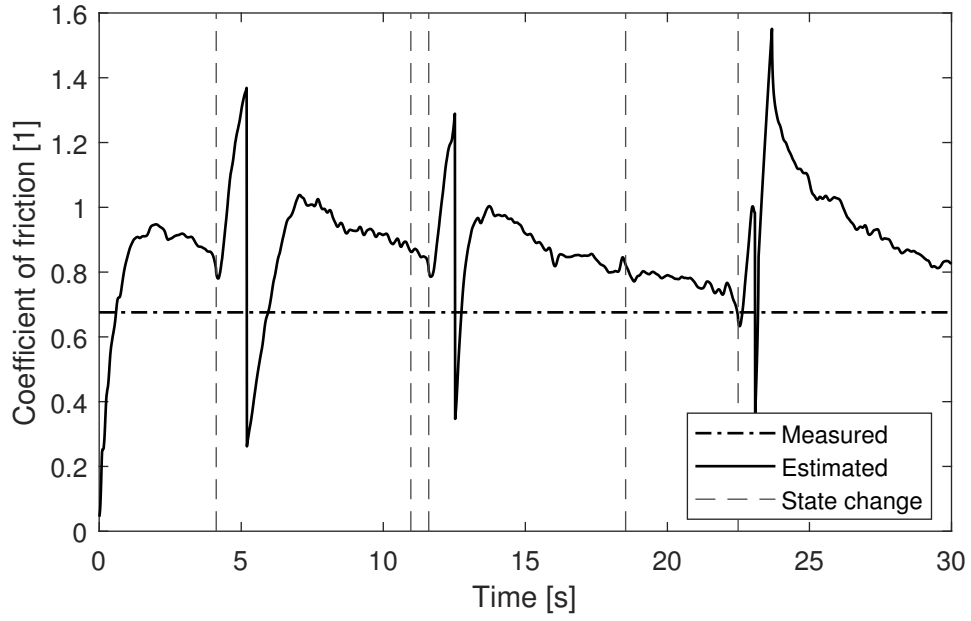


Figure 0.8: Estimation results, for the black emery paper, with contact patch half-length of $a = 45$ mm

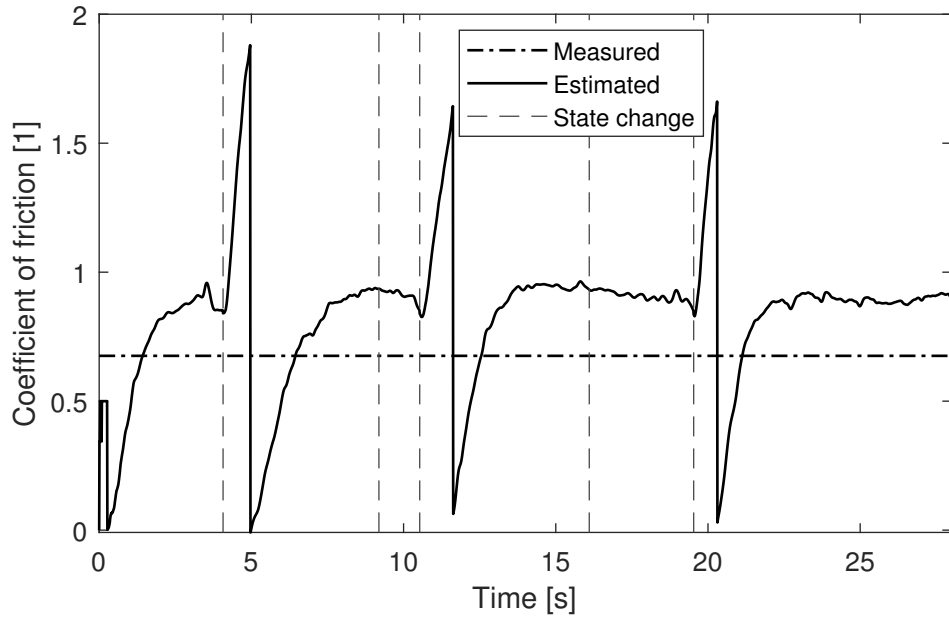


Figure 0.9: Estimation results, for the black emery paper, with contact patch half-length of $a = 50$ mm

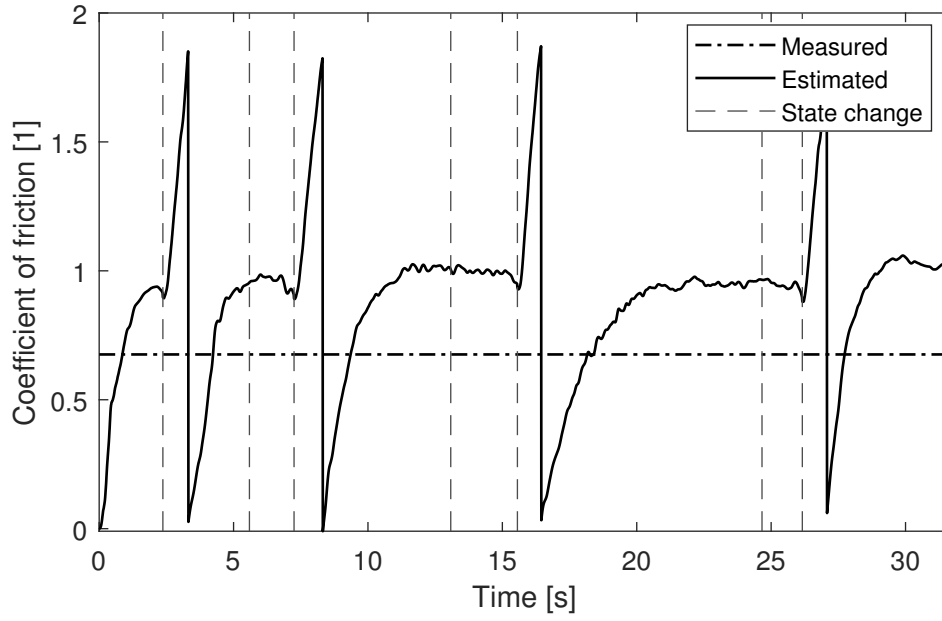


Figure 0.10: Estimation results, for the black emery paper, with contact patch half-length of $a = 50$ mm

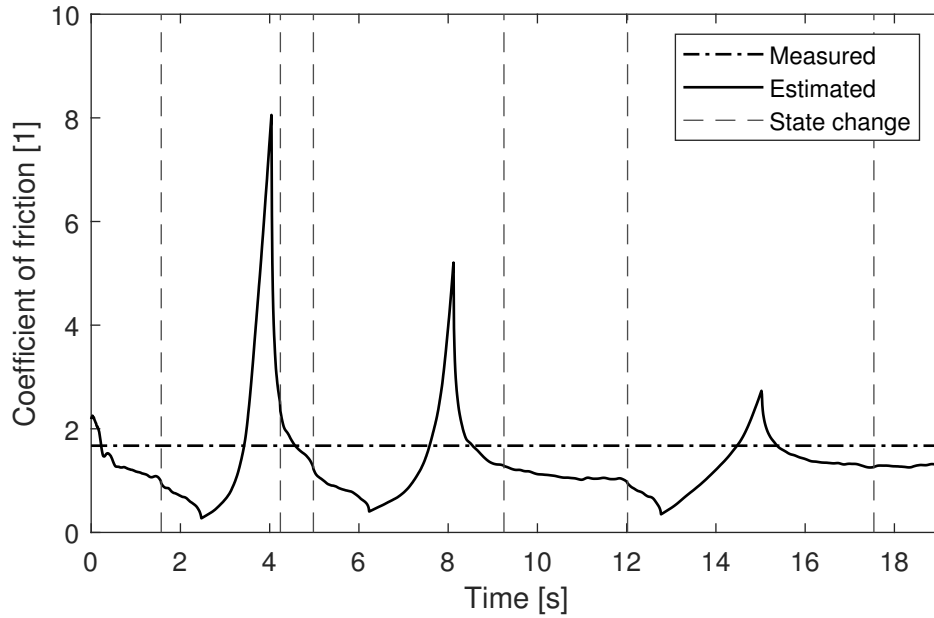


Figure 0.11: Estimation results, for the metal sheet, with contact patch half-length of $a = 40$ mm

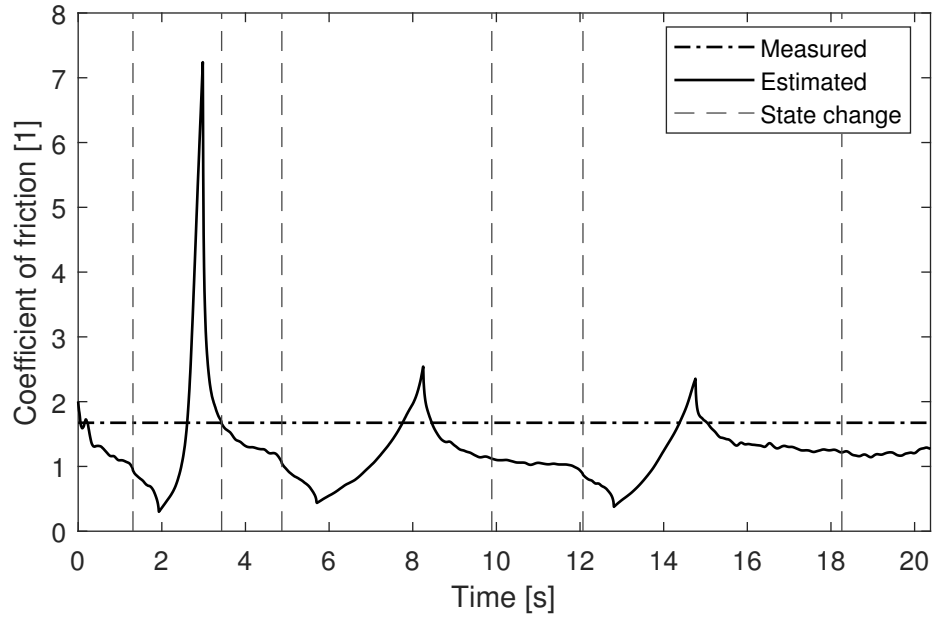


Figure 0.12: Estimation results, for the metal sheet, with contact patch half-length of $a = 40$ mm

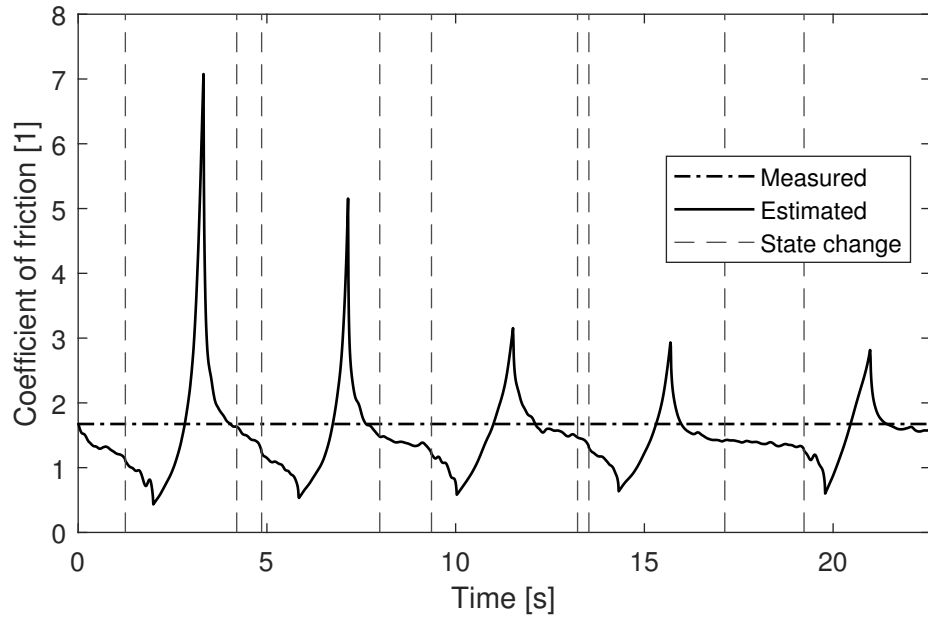


Figure 0.13: Estimation results, for the metal sheet, with contact patch half-length of $a = 45$ mm

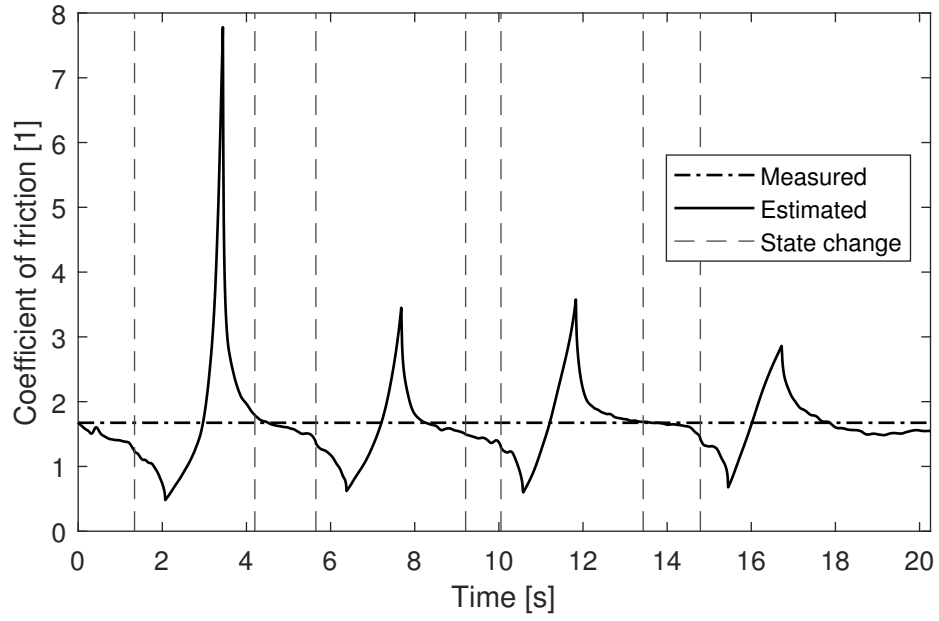


Figure 0.14: Estimation results, for the metal sheet, with contact patch half-length of $a = 45$ mm

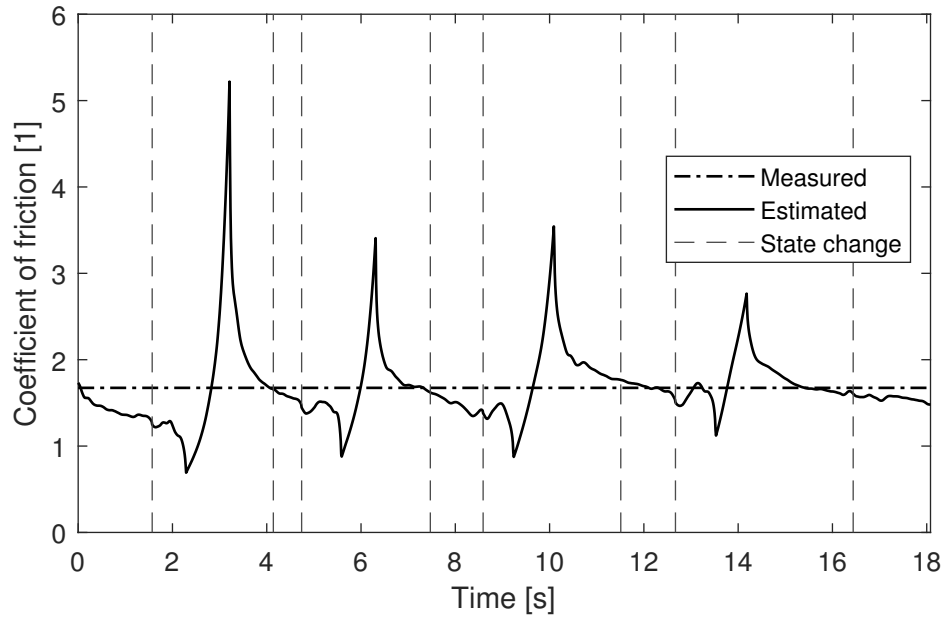


Figure 0.15: Estimation results, for the metal sheet, with contact patch half-length of $a = 50$ mm

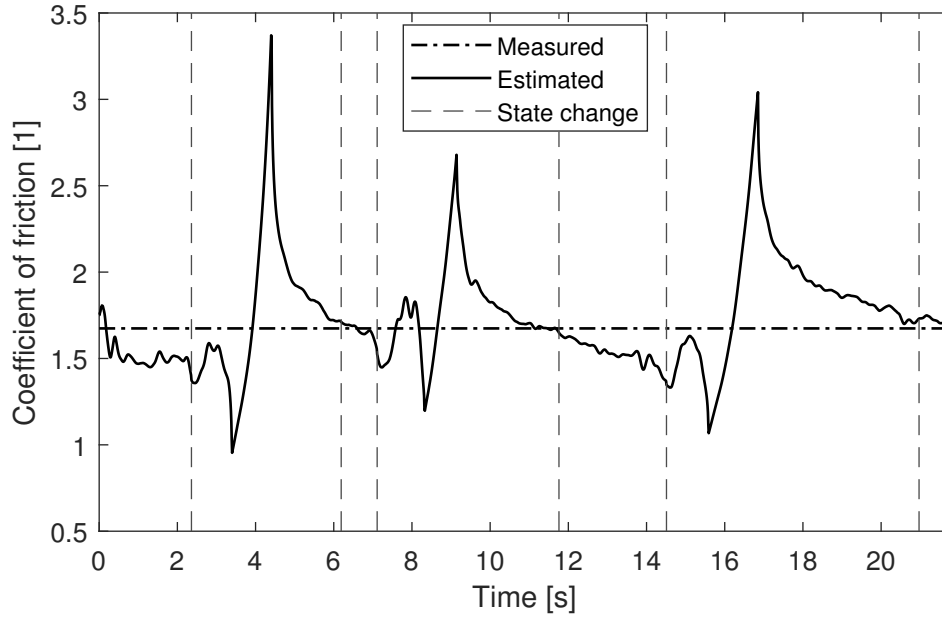


Figure 0.16: Estimation results, for the metal sheet, with contact patch half-length of $a = 50$ mm

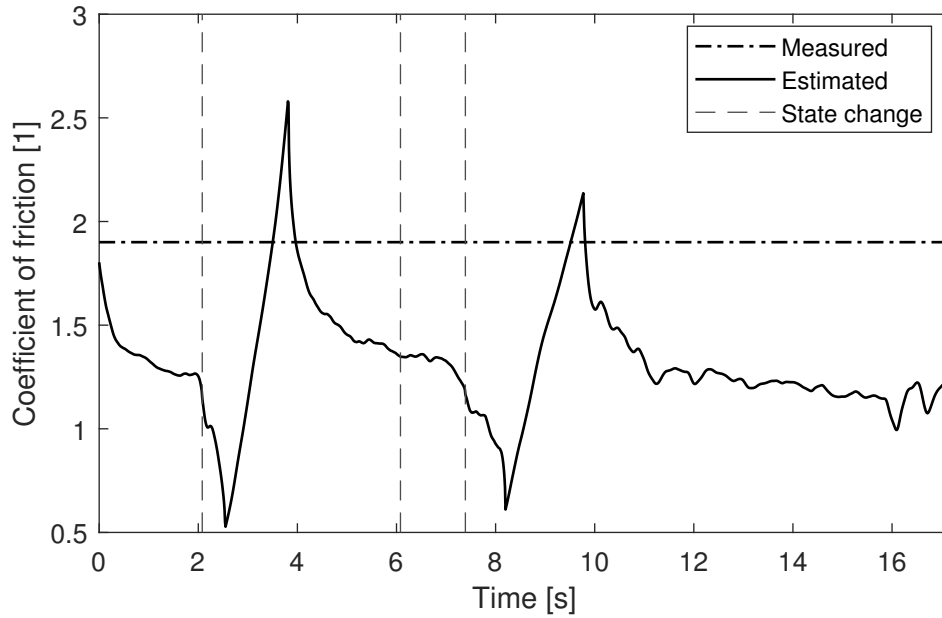


Figure 0.17: Estimation results, for the red emery paper, with contact patch half-length of $a = 40$ mm

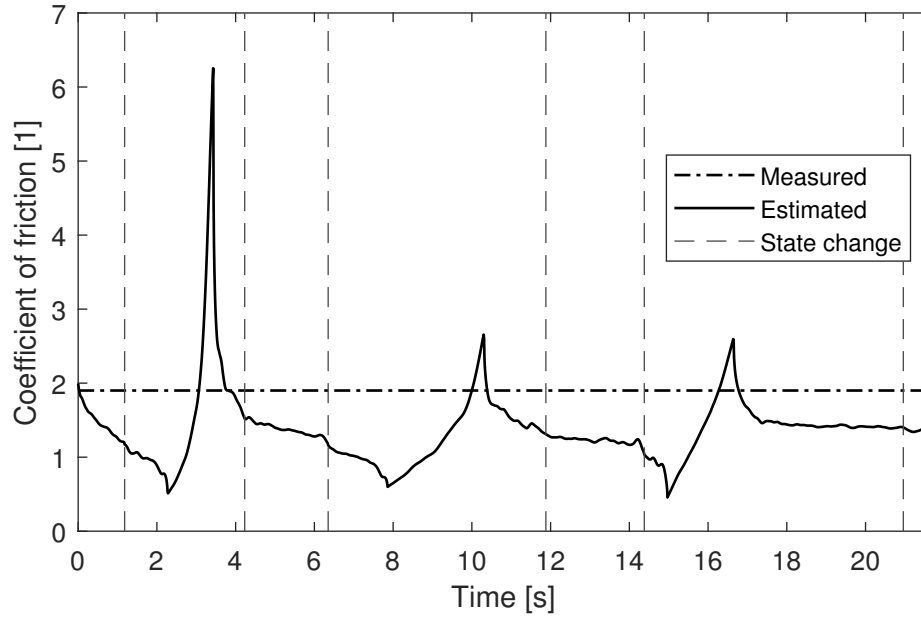


Figure 0.18: Estimation results, for the red emery paper, with contact patch half-length of $a = 40$ mm

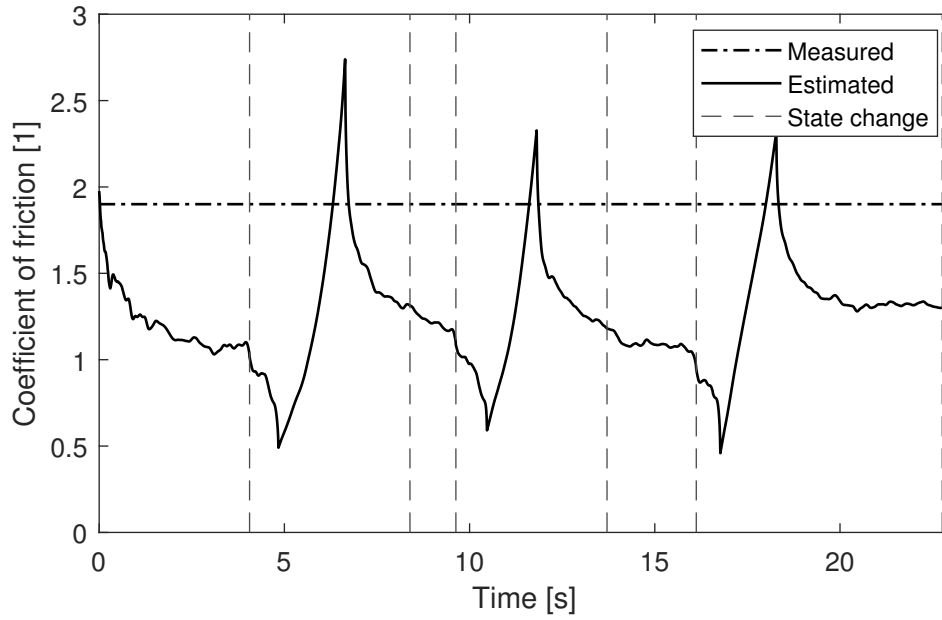


Figure 0.19: Estimation results, for the red emery paper, with contact patch half-length of $a = 40$ mm

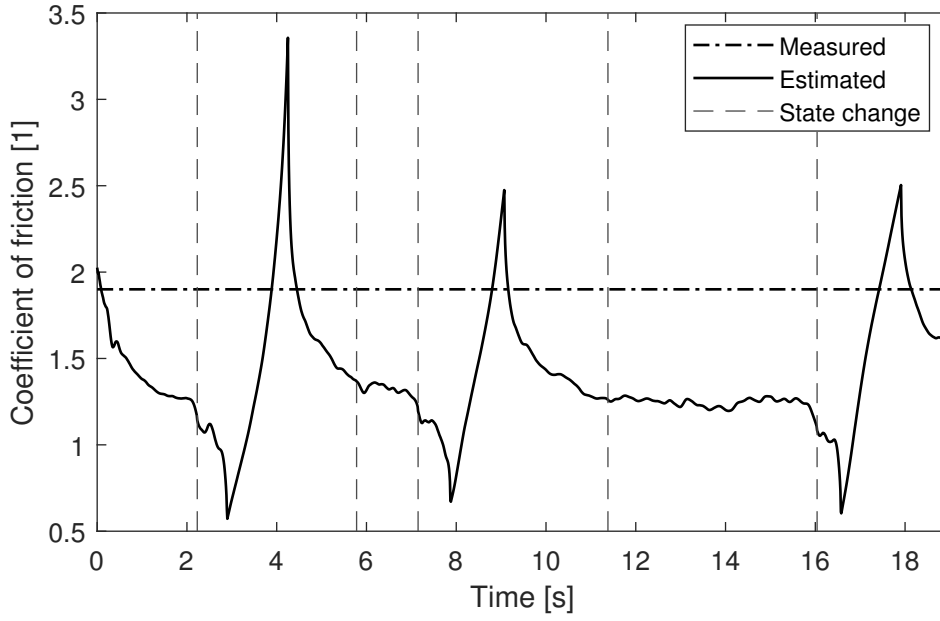


Figure 0.20: Estimation results, for the red emery paper, with contact patch half-length of $a = 45$ mm

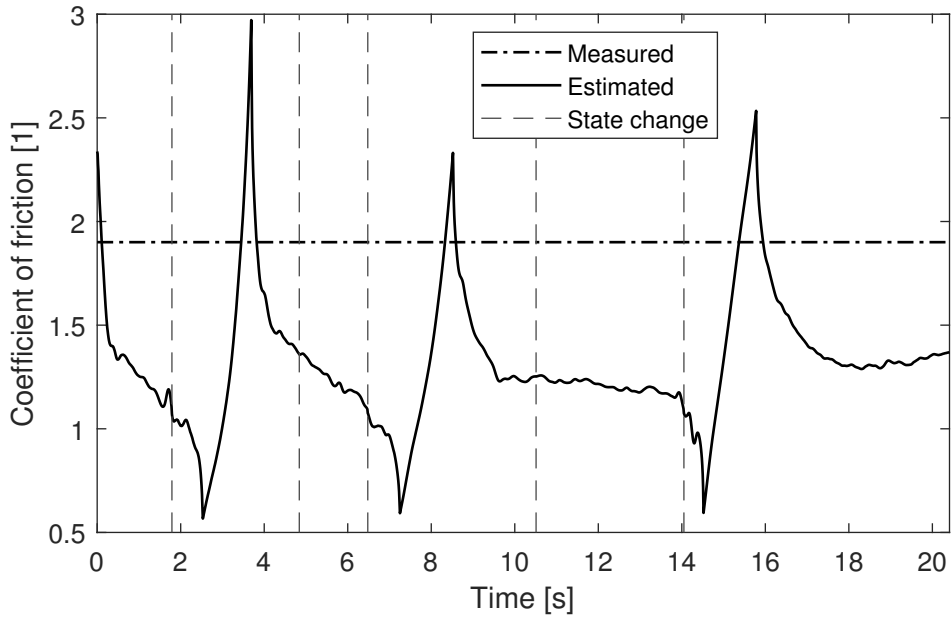


Figure 0.21: Estimation results, for the red emery paper, with contact patch half-length of $a = 45$ mm

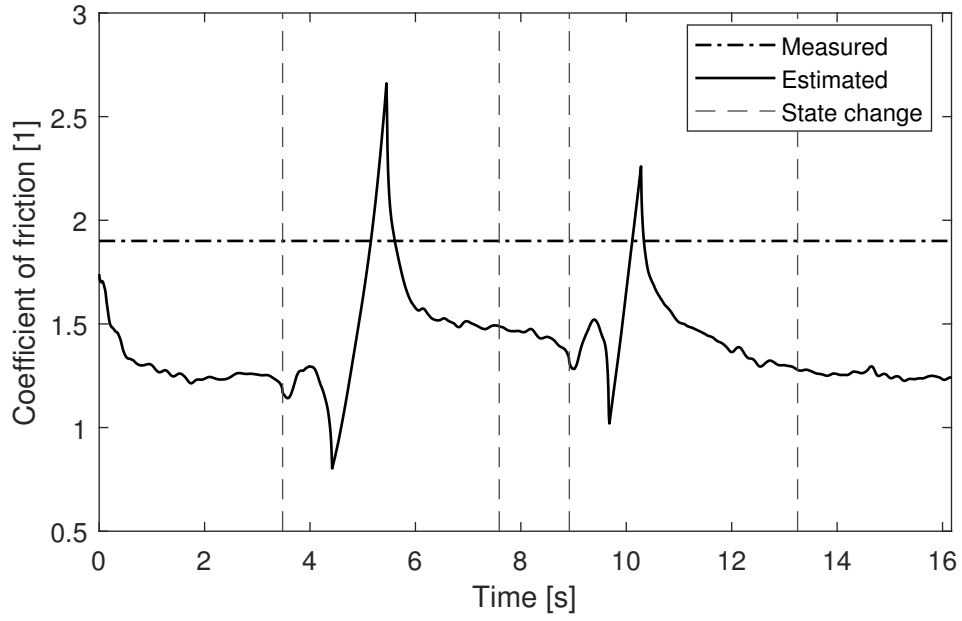


Figure 0.22: Estimation results, for the red emery paper, with contact patch half-length of $a = 50$ mm

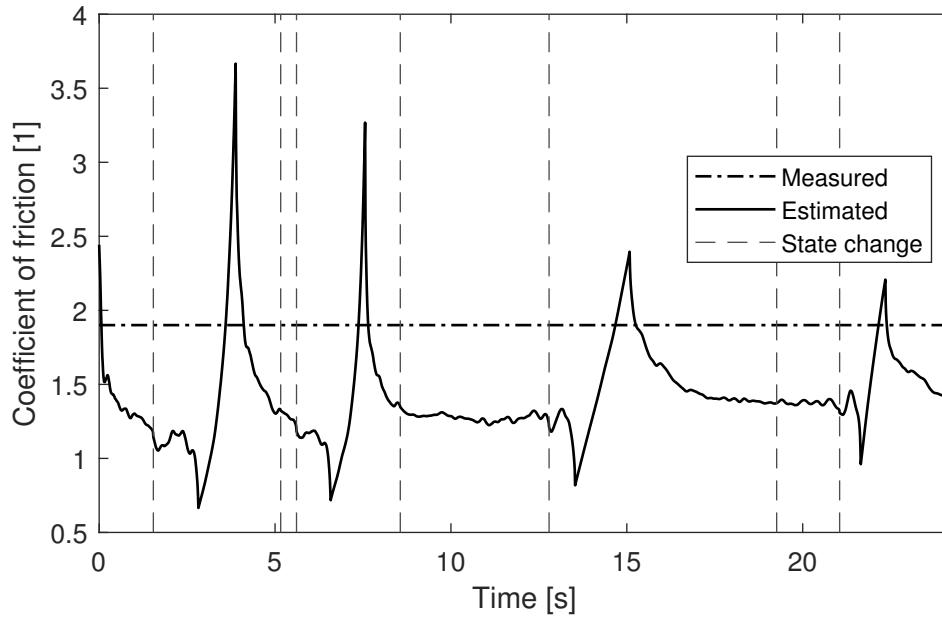


Figure 0.23: Estimation results, for the red emery paper, with contact patch half-length of $a = 50$ mm

Comparing charts of the estimation results for the validation measurements

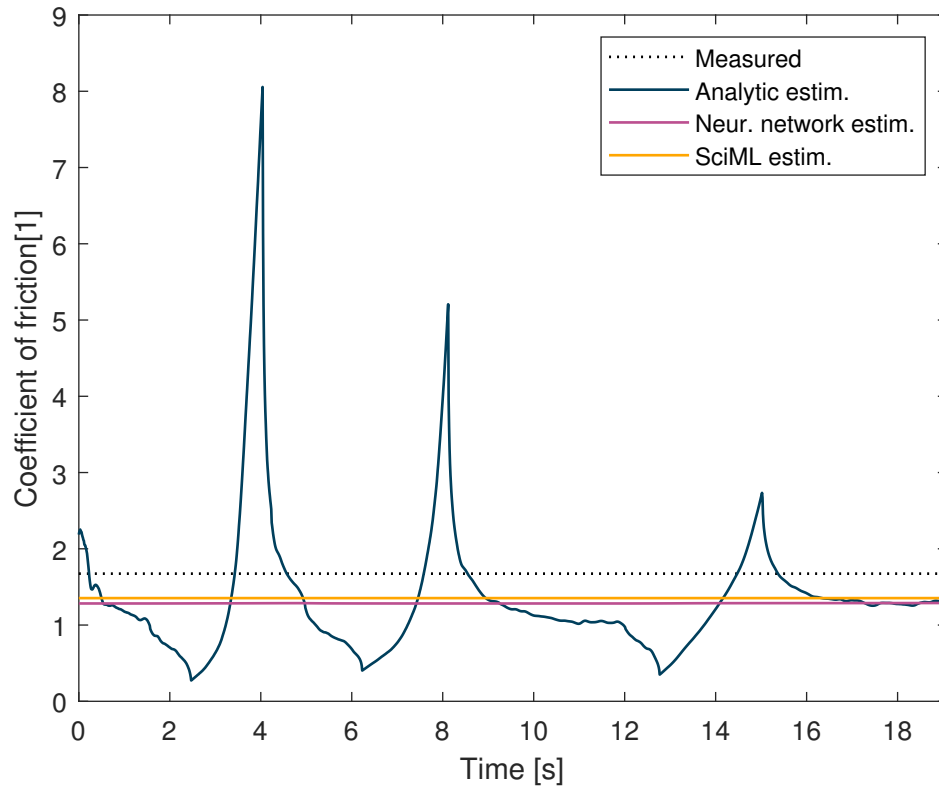


Figure 0.24: Comparing the results of each estimation method for the first validation measurement.

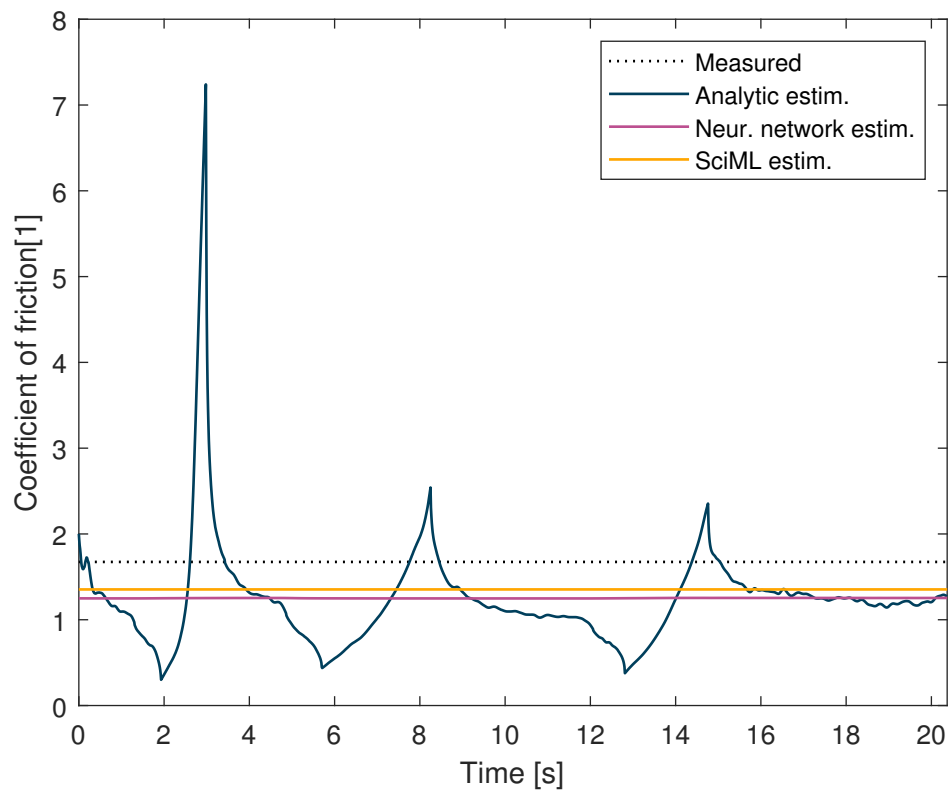


Figure 0.25: Comparing the results of each estimation method for the second validation measurement.

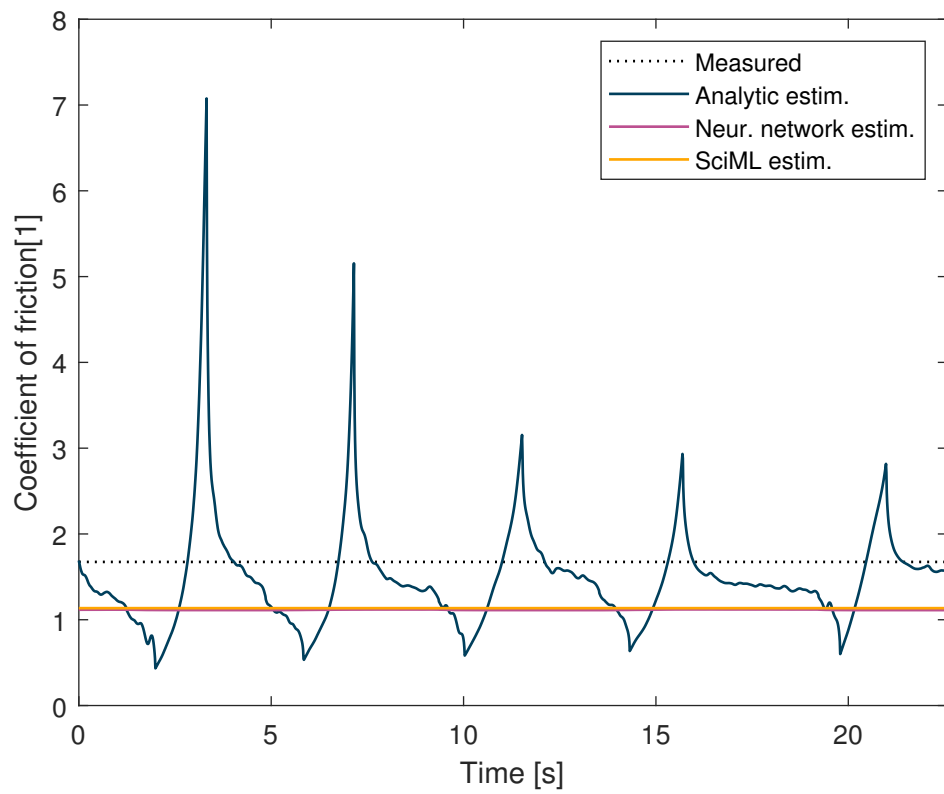


Figure 0.26: Comparing the results of each estimation method for the third validation measurement.

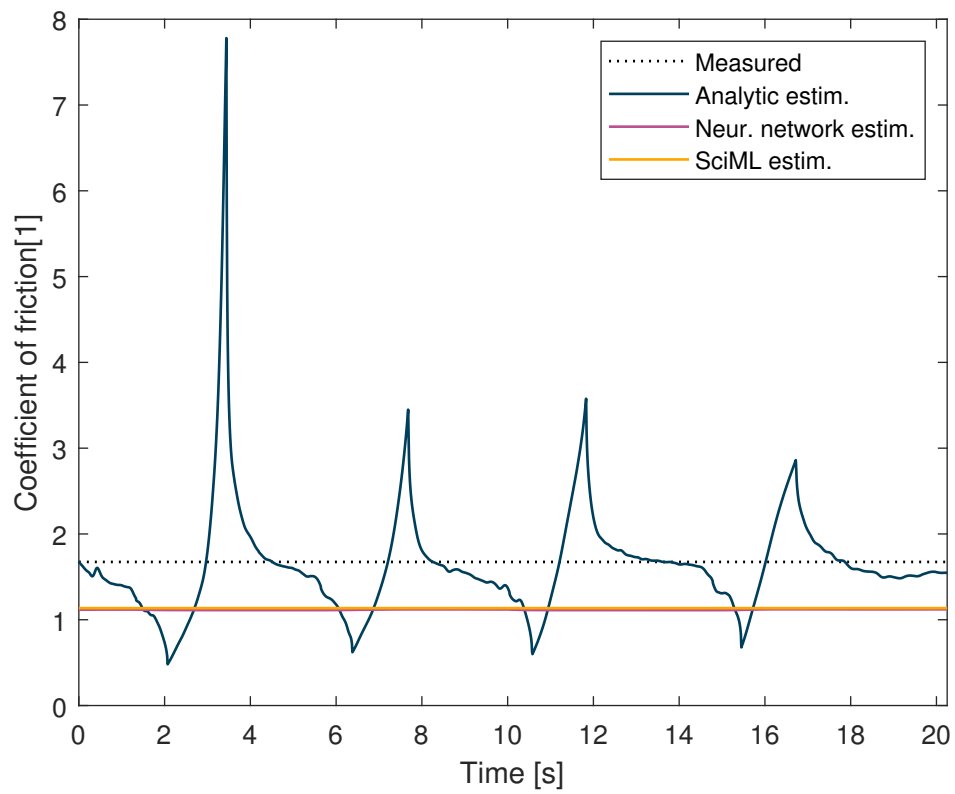


Figure 0.27: Comparing the results of each estimation method for the fourth validation measurement.

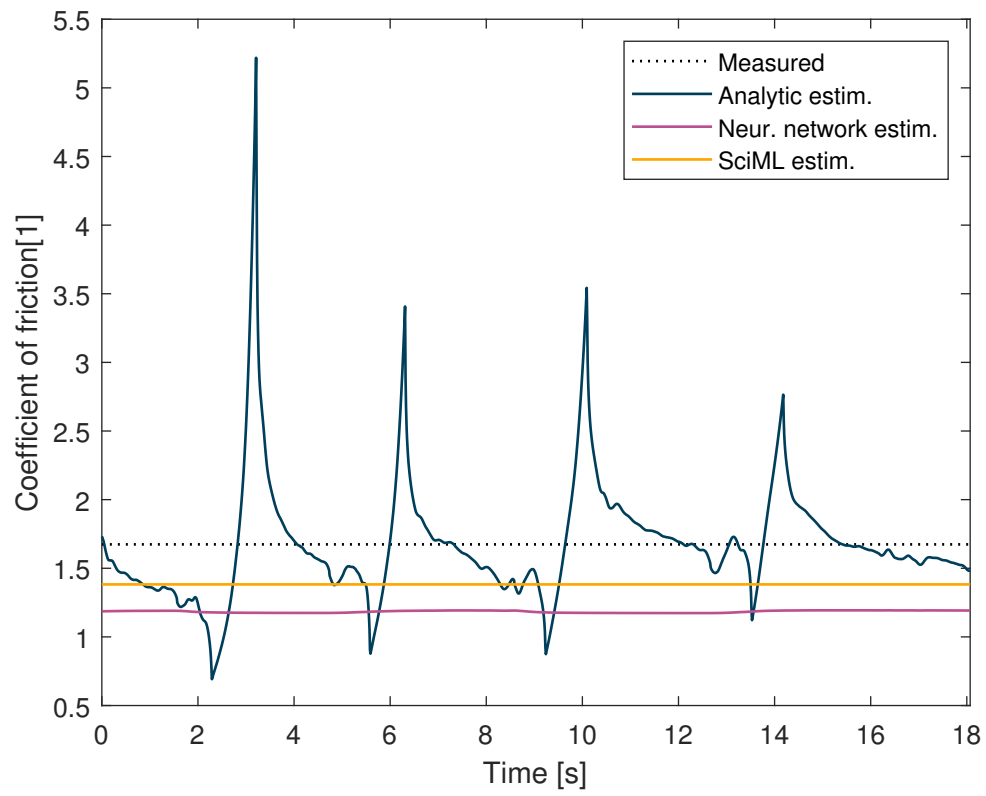


Figure 0.28: Comparing the results of each estimation method for the fifth validation measurement.

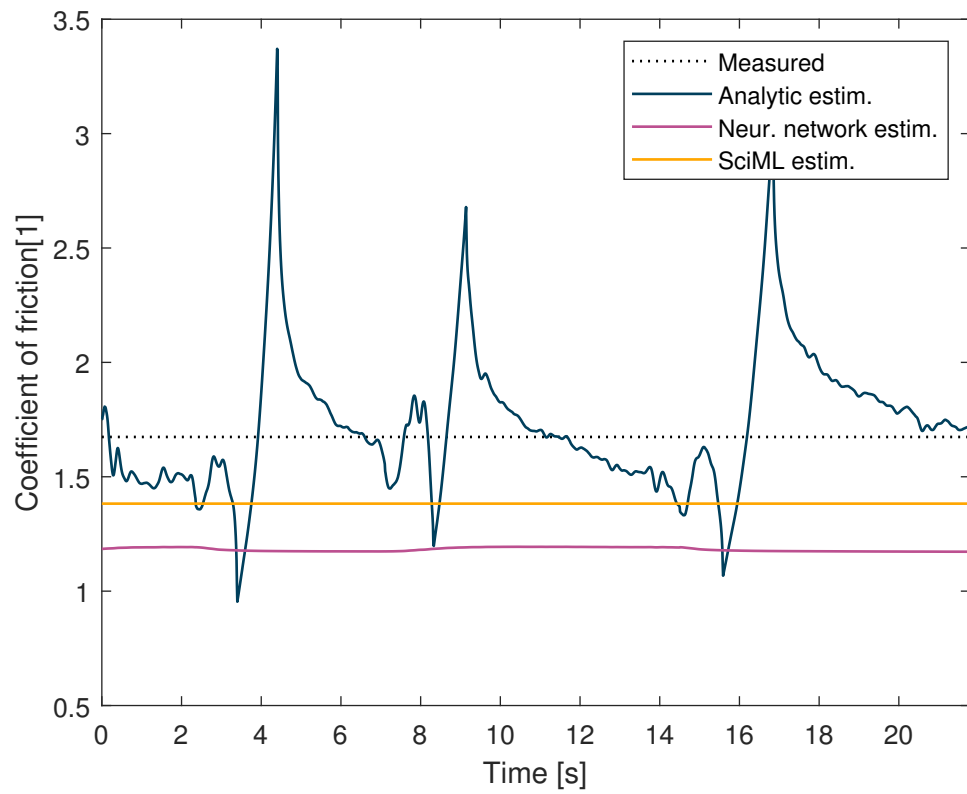


Figure 0.29: Comparing the results of each estimation method for the sixth validation measurement.