

Martinovich Kristóf
MSc THESIS



BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS
FACULTY OF MECHANICAL ENGINEERING
DEPARTMENT OF APPLIED MECHANICS





BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS
FACULTY OF MECHANICAL ENGINEERING
DEPARTMENT OF APPLIED MECHANICS



Martinovich Kristóf

MSc Thesis

**Analysis of the improved car-following model based
on measured data**

Consultant:

Kiss Ádám

Research assistant

Department of Applied Mechanics

BUDAPEST, 2023

Copyright © Martinovich Kristóf, 2023.



Budapest University of Technology and Economics

Faculty of Mechanical Engineering

Department of Applied Mechanics

<http://www.mm.bme.hu/>

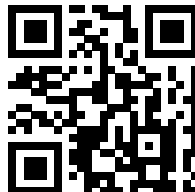
FINAL PROJECT ASSIGNMENT

Publicly Available

Identification	Name: Martinovich Kristóf		ID: 77043262253	
	Code of the Curriculum: 2N-MW0	Specialisation: 2N-MW0-SM	Document ref. number: GEMM:2023-1:2N-MW0:KHZVHP	
	Curriculum: Gépészeti modellezés mesterképzési szak			
	Final Project issued by: Department of Applied Mechanics	Final exam organised by: Department of Applied Mechanics		
	Supervisor: Kiss Ádám (77301411035), research assistant			

Project Description	Title	Analysis of the improved car-following model based on measured data. A továbbfejlesztett járműkövető modell vizsgálata mérési adatok alapján.
	Details	1. Conduct literature review on the topics of state-dependent time-delay systems and car-following models. 2. Investigate the car-following model considering headway-dependent reaction time. 3. Compare the investigated model with real measurement data. 4. Develop further the model to get better agreement with measurements in linear and nonlinear levels. 5. Summarize the results in English and in Hungarian and prepare a poster presentation.
	Advisor	Advisor's Affiliation: Advisor: ,

Final Exam	1 st subject (group)	2 nd subject (group)	3 rd subject (group)
	ZVEGEMMNWFE Finite Element Analysis	ZVEGEMMNWAM Advanced Mechanics	ZVEGEMMNWVI Nonlinear Vibrations

Authentication	Handed out: 5 September 2022		Deadline: 9 December 2022	
	Compiled by: Kiss Ádám (77301411035) Supervisor		Verified by: Dr. Tamás Insperger (signed) Head of Department	
			Approved by: Dr. Gábor Györke (signed) Vice-Dean	
	The undersigned declares that all prerequisites of the Final Project have been fully accomplished. Otherwise, the present assignment for the Final Project is to be considered invalid. Martinovich Kristóf			

DECLARATIONS

Declaration of acceptance

This thesis fully complies with all the content and form requirements prescribed by the Faculty of Mechanical Engineering of the Budapest University of Technology and Economics for the Final Project tasks, as well as those specified in the terms of reference. I consider this thesis to be suitable for public critique and public presentation.

Budapest, 2023. January 2.

Kiss Ádám

Declaration of individual work

I, *Martinovich Kristóf* (KHZVHP), the undersigned, student of the Budapest University of Technology and Economics hereby declare that the present thesis has been prepared by myself without any unauthorized help or assistance such that only the specified sources (references, tools, etc.) were used. All parts taken from other sources word by word or after rephrasing but with identical meaning were unambiguously identified with explicit reference to the sources utilized.

Budapest, 2023. January 2.



Martinovich Kristóf

Contents

Abstract	IX
Notations	X
List of figures	XI
1 Introduction	1
1.1 Objectives	2
1.2 Overview	2
2 Literature review	4
2.1 Driver reaction time	4
2.2 Next Generation Simulation (NGSIM)	6
2.3 Delay estimation	8
3 Theoretical background	14
3.1 General car-following model	14
3.2 Modeling the state-dependent time delay	17
3.2.1 The extension of the car-following model with state-dependent time delay	18
3.3 Methods of linear stability analysis	21
3.4 Theoretical background of the nonlinear analysis	23
4 Results of the road measurements	27
4.1 Measurement layout	27

4.2	Linear stability analysis	32
4.3	Nonlinear stability analysis	37
5	Summary	46
5.1	Results	46
5.2	Conclusions, outlook	46
	Összefoglaló	48
	Reference	50

Abstract

In this contribution, we aim to model the human reaction time with headway-dependent delay and validate the theoretical finding with measured data. Large quantity of highway measurement data is evaluated which validates the idea of the varying human reaction time. The extended model is based on the car-following model and the layout of the corresponding measurement contained two connected human-driven vehicles and one connected automated vehicle, all equipped with vehicle-to-vehicle communication. Stability charts and bifurcation diagrams are provided to compare the theoretical findings of the state-dependent delay model with the constant time delay model and the measurement data.

Acknowledgement

We are grateful to Stépán Gábor for his idea of introducing state-dependent time delay into the car-following model, which was the main underlying idea of the thesis. Considering the results, this proved to be a rather good intuition.

We would like to thank Orosz Gábor for drawing our attention to the correlation of measurement data and the theoretical findings of the BSc thesis and also for providing us the measurement data.

I am grateful to Kiss Ádám for the successful cooperation over several years. I thank him for all the time, help and guidance he has given me as my consultant.

Finally, I would like to thank my family and my wife Blanka, who I could always rely on. Thank you for your encouragement, without your support I could not have done it.

Budapest, 2023. January 2.

Martinovich Kristóf

Notations

Abbreviations

Notation	Meaning
ACC	adaptive cruise control
AV	automated vehicle
CACC	connected adaptive cruse control
CC	cruise control
CCC	connected cruise control
CAV	connected automated vehicle
CHV	connected human-driven vehicle
DDE	delay differential equations
HV	human-driven vehicle
MDBM	Multidimensional Bisection Method
ODE	ordinary differential equations
SD-DDE	state-dependent delay differential equations
V2V	vehicle-to-vehicle communication
V2X	vehicle-to-everything communication

Latin letters

Notation	Name, comment, value	Dimension
$i = \sqrt{-1}$	imaginary unit	1
$V(h)$	range policy/optimal velocity function	m/s

Greek letters

Notation	Name, comment, value	Dimension
α	control parameter	1/s
β	control parameter	1/s
κ	CC control parameter	1/s
λ	characteristic exponent	1
τ	constant time delay	s
$\tau(x)^1$	state-dependent time delay	s

¹here, general form is used, when the delay depends on an arbitrary x varibale

List of Figures

2.1	Evaluated road measurement dataset	7
2.2	Example of delay estimation for large headway	10
2.3	Example of delay estimation for medium headway	10
2.4	Example of delay estimation for small headway	11
2.5	Example of incorrect automatic delay estimation	12
2.6	Results of the state-dependent delay estimation	13
3.1	Model layout	14
3.2	Bifurcation diagram	24
3.3	Illustration of the central manifold	26
4.1	Measurement layout	27
4.2	The functions used in the model	32
4.3	Velocity time signals of the measurement	34
4.4	Linear stability diagrams	36
4.5	Bifurcation diagram comparing the measurements and the periodic orbits .	39
4.6	The numerical simulations at point A	41
4.7	The numerical simulations at point B	42
4.8	The numerical simulations at point C	42
4.9	The numerical simulations at point D	43
4.10	The numerical simulations at point E	43
4.11	Bifurcation diagram for tuned parameters	44

1 Introduction

We are witnessing significant changes in the road transport and the mobility sector, surrounded by huge social expectations and media interest. The development and spread of digitisation, the ever-increasing technological knowledge and the advancement in theoretical research have huge influential potential. The key areas most in need of improvement are ensuring safety by eliminating the human error factor, reducing the negative environmental impact of transport and easing traffic congestion. The transformation is not only due to the technological innovation of the industry, but also strongly influenced by the legislation of countries and intergovernmental organisation such as the European Union.

Various driving-assistance systems (DAS) are available for the consumer market, for instance, cruise control (CC) dates back decades in services, and it has already become standard equipment in most cars. Advanced driver-assistance systems (ADAS) were first introduced to luxury cars, are now becoming widespread in the mid-range and even low-end vehicle equipment, such as the adaptive cruise control (ACC) [1, 2]. The CC and ACC are mainly driver comfort functions and rely on their built in sensors, such as cameras and radars. Whereas the current state-of-the-art technologies under development are the connected cruise control (CCC) [3, 4] and the connected adaptive cruise control (CACC) [5], which gather information from other road participants too. Vehicles equipped with vehicle-to-vehicle (V2V) [6] or vehicle-to-everything (V2X) [7] communication, are able to connect with other road users in a network and even receive information from beyond their line of sight. These technologies not only help to reduce travel time and fuel consumption [8, 9], but they can also achieve safer travel by using safety-critical control [10, 11, 12, 13, 14] focusing on the critical limit situations. These technologies and ADAS systems built into human-driven vehicles (HVs) or as part of fully autonomous vehicles (AVs) can lead to significant quality of life and economic improvements at a societal scale. Especially, if the vehicles are treated as part of traffic networks and controlled at higher level, made possible by V2X technology, to achieve system-level optimisation. The trend of the formerly described transformation in traffic seems to be accelerating, and breakthrough is expected in the upcoming decade. However, these systems rely on dynamic models, which must be supported by in-depth research in mechatronics, dynamics and mathematics.

In this contribution, we aim to broaden the scientific literature by studying and expanding the so-called car-following model. Initially, it was created to study highway traffic

by modelling the behaviour of drivers [15, 16]. In order to study traffic more accurately and to obtain quantitatively accurate results, time delay was introduced into the system [17, 18], which can be described mathematically by a delay differential equation (DDE) system. Later, it became a fundamental model for researching CC and connected autonomous vehicles (CAV) [19]. CAVs are able to communicate not only with each other but also with connected human vehicles (CHV) equipped with V2V technology. The car following model is often studied in the literature in ring configuration, where N vehicles travel on a ring-road [15, 16]. If $N \rightarrow \infty$ or N is a practically large enough number, this model is suitable for studying traffic on a straight road. A comprehensive literature on the ring-road configuration can be found in [20], for which related road tests and measurements have been performed by [21, 22, 23]. Furthermore, the penetration ratio of self-driving vehicles on the stability of the system was investigated by [24].

Contrary to the previous public expectations, it may take decades to reach fully automated traffic containing only AVs [4]. The transition period requires investigating the so-called mixed traffic situations where AVs and HVs are both presented. Theoretical studies concluded [25, 26] and experiments have demonstrated [27, 28, 21, 29] that even a small number of AVs can reduce traffic congestion, thereby improving traffic flow. But to truly exploit their potential, it is necessary to model human driver behaviour in detail with sufficient accuracy.

1.1 Objectives

This thesis aims to model the reaction time as a function of the state, as firstly presented in our previous work [30], and to compare the results with measured data. A research team at the University of Michigan has performed measurements with a vehicle convoy consisting of two HVs and one AV in a ring-road configuration [23]. The provided measurement data allow us to validate the theoretical model.

1.2 Overview

The thesis is structured as follows. In Section 2 the related literature review is detailed. In Sec. 2.1 we discuss the human reaction time and its influences accepted in the literature, in Sec. 2.2 the highway traffic measurement project and the corresponding measurement data are presented and in Sec. 2.3 based on the evaluated data a conclusion is drawn on

the headway-dependent reaction time. Section 3 contains the theoretical background, the models and the methods used. The Sec. 3.1 presents the general car-following model for constant time delay and the corresponding equations. In Sec. 3.2 the state-dependent time delay is introduced and in subsection 3.2.1 the extended car-following model with the state-dependent time delay is presented. Then, the methods of linear stability analysis are shown in 3.3, followed by the theory of the nonlinear analysis in 3.4. The theoretical background is followed by a comparison of the extended model with the measurement results in Sec. 4. First, in Section 4.1 the measurement setup and conditions are described, second, in Section 4.2 the measurement data are processed and the resulting linear stability analysis is detailed in comparison with the results obtained with constant and state-dependent time delays. Lastly, in Section 4.3, the global behaviour of the system is investigated by the continuation of the periodic orbits and numerical simulations. Finally, in Section 5, we summarize the thesis, conclude the results and discuss the future challenges.

2 Literature review

In this section the literature review is discussed in detail, focusing on the relevant papers on the reaction time of human drivers and all kinds of delays associated with traffic. Then the highway traffic measurement in the framework of the NGSIM research project and the collection of the large quantity data are presented. Thereafter, we evaluate the measured data to establish the theory of modelling human reaction time with state-dependent time delay.

2.1 Driver reaction time

The reaction time associated with human drivers contains numerous factors, which are influenced by the driver's conditions and circumstances. The total reaction time shows a large variation, ranging from a minimum of 0.15 s to several seconds [31, 32]. On the one hand, the time delay includes the time of human detection of the situation, identification, decision, followed by the time of intervention (pedal depression, steering etc.) [33]. On the other hand, additional time delays result from the mechanical transmission of the vehicle, inertia, brake force build-up etc. Although it usually takes less time, it is case dependent whether this is negligible compared to human reaction time.

The time delay of self-driving vehicles is significantly shorter compared to human delays, due to the fast data processing and decision-making made possible by computers. However, the delay due to mechanics of the vehicles are also presented and additional delays have to be considered due to the sensor processing of AVs and potential message dropouts in the communication of CAVs [34, 35].

Based on the measured data according to [31, 32], it was concluded that two situations can be distinguished: in urban traffic the base reaction time is around 0.7 seconds, while outside of the urban areas in rural roads and highways (where the speed and following distances are higher) the drivers' attention is reduced, resulting in a base reaction time of at least 1 second. The reaction time increases if we have to change our focus (e.g. looking in the mirror from the road ahead) in which case it is 1.3...1.5 s. This can be followed by a further increase if we also have to make a decision, which results in a time delay of 1.6...2 s. Hence, the commonly known rule of thumb is the two-second rule, which implies that it is recommended to keep at least a two-second time headway from the predecessor vehicle. Moreover, human reaction time varies with age and it is influenced

by the driver's experience, exhaustion, concentration level and the situation to which the driver has to react.

It follows that it is not trivial to choose a suitable value for a general description of human reaction time. The simplest way to do this is to use an average constant value. It can be seen that this is a strong approximation because of the heterogeneity of drivers, and the experiences show that it can even change while driving, depending on how much they are paying attention or whether they are distracted.

Our assumption is that the reaction time of human drivers can be more realistically modelled by a time delay which depends on the circumstances. In the car-following model, drivers' intentions are mostly determined by the current following distance. This is the basis for choosing a safe speed and deciding whether to accelerate or decelerate to maintain a safe following distance. Therefore, it is reasonable to assume that the time delay depends on the following distance as a variable. This could be because drivers pay more attention when vehicles travel closer together due to the increased risk of collision, which results in a smaller time delay. Conversely, at larger following distances, their attention may be reduced, which is taken into account by a larger time delay. It may also contribute to the increased delay, that a unit change in the distance between two vehicles for a larger following distance compared to if they are closer together, is relatively smaller. Experiences show that people tend to overestimate distance and underestimate speed which makes it more difficult to accurately determine the situation and react to any given circumstance.

The variable reaction time can be mathematically expressed in the model by a state-dependent delayed differential equation (SD-DDE) system. To investigate this, advanced mathematical techniques are needed, which is an area of active research [36, 37, 38]. Although it is not yet widely used in engineering practice due to its complex modelling and computational requirements. Nevertheless, for machine tool vibration, it has been shown that more accurate modelling and consideration of multiple effects lead to state-dependent time delay, which may have a significant impact on the stability of the system [39, 40, 41]. The considerations and methods in those papers are suitable for linear and nonlinear investigations of the improved car-following model, due to their similarity in dynamics.

2.2 Next Generation Simulation (NGSIM)

The formerly described expanding trend of innovation involves the ever-increasing demand for large quantities and good quality traffic data. It is essential in the research of traffic flow models and the development of controllers to validate and refine models. However, the measurements related to highway traffic are quite challenging, expensive and time-consuming to execute. Therefore, usually tests with only a few vehicles platoon are feasible.

The Federal Highway Administration of the U.S Department of Transportation has collected a large amount of traffic data as part of the Next Generation Simulation (NGSIM) project [42, 43], to aid the academic research community, traffic scientists and microsimulation software developers. The real-world dataset contains trajectory data collected by video cameras in 4 different locations and situations at multiple times consisting of 2 highway locations, one signalized and one un-signalised intersections. Besides the freely available raw data, microsimulation algorithms focusing on the logic of car-following, lane change and their documentation are published on their website.

In this contribution, we focus on the Interstate 80 Freeway dataset, because it suits best the conditions of the car-following model we are interested in. This freeway is located in San Francisco Bay area and the studied section is approximately 500 m long consisting of 6 normal and one onramp lane, where the speed limit is 55 mph (which is 88.5 km/h or 24.6 m/s). The experiment setup was such that 7 cameras were placed on the top of a 30-story building recording synchronised video feed. This was later analysed by image-processing software to extract detailed trajectory data. However, the difficulty of the trajectory estimation, using techniques such as averaging, smoothing and filtering, and the possible errors involved were addressed before. See the comprehensive literature about the topic in [44, 45]. The most relevant information is the following: an ID was assigned to each car and its time, velocity, headway, time-headway signals and the predecessor vehicles ID along with its trajectory signals are collected. Selecting the trajectory pairs of consecutive vehicles enables us to analyse the driver behaviour as a response to the dynamics of its predecessor.

In this location a total of 45 minutes of measurement are available which was collected in three 15-minutes time periods, covering the development of traffic congestion from uncongested to peak traffic jam. For further evaluation, the 4:00 p.m. to 4:15 p.m. data set was selected during which traffic flow was not too dense and free flow was still

achievable, which best suited our needs.

There are two distinct effects regarding the forward dynamic behaviour of drivers. The first one is identified in the car-following model as the range policy, which describes the relation between the desired speed and available headway. It models how the driver tries to maintain a safe following distance from the leading vehicle. This relation can be obtained directly from the trajectory data if we plot the velocity vs. headway data of the vehicles at each time instant. The selected 15-minute long measurement consists of 1 048 575 data points. The corresponding scatter plot representation is shown in Figure 2.1 panel (a) and heatmap in panel (b), which helps to illustrate the quantity of the data. Note that, the majority of the travel happened at lower speeds and following distances indicated with yellow colour in panel (b). It is an order of magnitudes more than the rest of the data, which indicates that this period on the freeway was already somewhat crowded. Despite the large variation within the data, we can highlight some visible trends. We can observe the lack of a large number of data points in the upper left and lower right corners of the figure. It indicates, that no intentionally high-speed travel happens for small following distances, and also no low-speed driving for large headways. It means, drivers acknowledge the safe following distance and choose their speed accordingly, but if the headway is sufficiently large, they drive as fast as the speed limit allows. This trend is usually modelled in the literature either with quadratic or trigonometric piecewise functions [4]. Both functions address the nonlinear trend in the lower headway range and the limit velocity over a certain headway value, as seen in the measurement. In Fig. 2.1 panel (a) the commonly used range policies are illustrated in red and blue. These functions fit the scattered points and qualitatively indeed describe the trend for this specific dataset, although the variance of the data points is quite large.

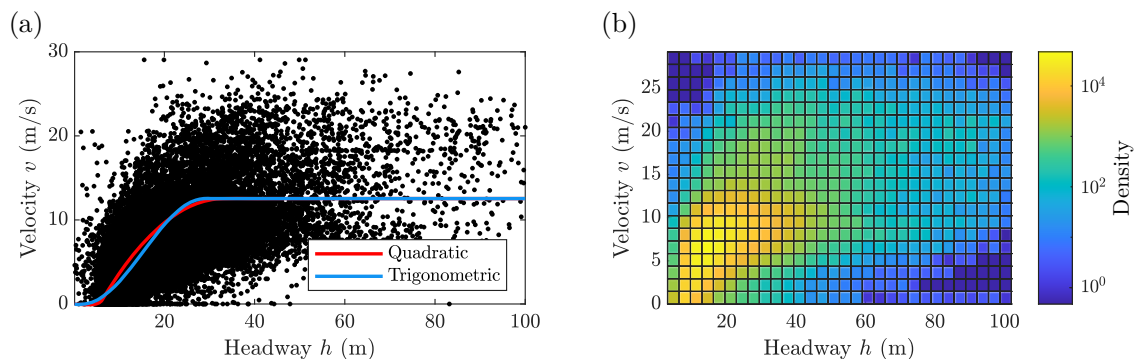


Figure 2.1: Evaluated road measurement, corresponding to NGSIM I-80 4 pm to 4:15 pm dataset. In panel (a) the scatter plot and the commonly used range policies and in panel (b) the heatmap of the data is shown.

2.3 Delay estimation

The second distinct effect is the reaction time which determines the safe following distance. However, subtracting the reaction time of a driver from the data is not straightforward. First, we have to define what to consider as the reaction time of drivers. Dedicated measurements exist where the circumstances are controlled and the stimulus and the reaction can be easily identified. However, in the traffic measurement data, neither the stimuli nor the reactions are exposed clearly, yet we have to identify the correlation between the signals of the leading and the following vehicle. In [46] two methods were proposed for delay estimation by spectrum analysis. These methods are based on Fourier analysis of the data. The coherence spectrum of the two signals is calculated, and then the maximum coherence with respect to the offset of the signal gives the estimation of the delay between the signals.

However, this method assumes, that one signal is the delayed and amplified version of the first one, thus they are similar in character when observing them in the frequency domain. Hence, it might be inaccurate for higher variance when the signals are not exact copies of each other. The theoretical car-following model also assumes, that the behaviour of the driver only relies on the leading vehicle. Whereas, in reality, other non-modelled factors might be relevant and bias the behaviour. More experienced drivers may react to vehicles in the distance or in other lanes. The drivers' behaviour also changes during an intended or realized lane change manoeuvre which would require different modelling assumptions. Furthermore, the delay is not always equal to the realizable, minimal reaction time, but drivers may purposefully postpone their action if something happens in the distance. By considering the capabilities and the brake distance of the vehicle, sometimes it is enough to start to decelerate later or at a different rate than the leading vehicle and still avoid a collision. The best indicator for drivers about the changing behaviour of the leading vehicle is the brake lamp, which objectively warns the drivers if a car decelerates. But for judging the change of other factors, such as following distance, velocity and acceleration, drivers must rely on their perception and experience.

Several algorithms were tried and tested to evaluate the measurement data. As a first approach, we focused on the changes in velocity which best visualise the correlation between the signals. To do so, we selected the extreme values of the acceleration signals and looked for the corresponding deceleration of the ego. The individual braking can be identified easily, but finding the corresponding deceleration of the other vehicle is troublesome.

This method was improved by extending the detection algorithm with multiple steps. First, we identified the negative peak values of the acceleration signal exceeding a chosen threshold for the front vehicle. Then these were used as a trigger to find the corresponding ego peak accelerations within a prescribed time. Brake events were considered to be similar if both the acceleration and the velocity of the ego and the leading vehicles were within a relative threshold. Although in specific situations it could estimate the driver's reaction time with high accuracy, on a large scale it produced false results for the majority, due to noisy data or in the case of signal pairs with less correlation. We could not eliminate this problem and improve this method by varying and tuning the threshold values either.

Finally, the delay detection was utilised using the signal delay detection algorithm in MATLAB celled *finddelay*, since it provides the most accurate and reasonable estimation for the delay. This function relies on a similar principle as in [46]. It calculates the normalized cross-correlation function between each pair of signals, using the function called *xcorr*. The delay is calculated as the offset for which the cross-correlation has the maximal absolute value. The best results could be obtained by examining the acceleration signals. However, focusing only on the acceleration signals is not adequate, but the velocity signals have to also correlate. Also, this method gives an average estimation of the delay over the whole time period and does not take into account the individual deceleration which might happen at different headways. In total 1725 vehicles' trajectory data is available, leading to 3258 different situations, when different vehicles follow each other considering the lane changes. All of these situation were evaluated using the above introduced technique, out of which some typical cases are presented in the figures below: in panel (a) the headway with respect to time is illustrated, in panel (b) the original velocity signal of the leading vehicle is marked by a dashed yellow curve, its offset signal based on the delay estimation with orange, and the velocity of the follower vehicle with a blue curve, finally in panel (c) the acceleration signals are shown similarly. Note that the follower is also called the ego, since we estimate the reaction time of the driver of this car. By good correlation we mean when the offset signals overlap each other, and the trajectory of the ego vehicle is the copied and delayed version of the leading car.

In Fig. 2.2, the time signals are related to large following distances. The average headway is 84.5 m and the estimated delay is 4.1 s. The signals only show high correlation in the last part of the data, especially in the acceleration signals. It is indicated with a green box that is the delayed curve of the leading vehicle coincidences with velocity profile of the ego vehicles. But there is higher variance in the first section. Overall, this is an acceptable estimation.

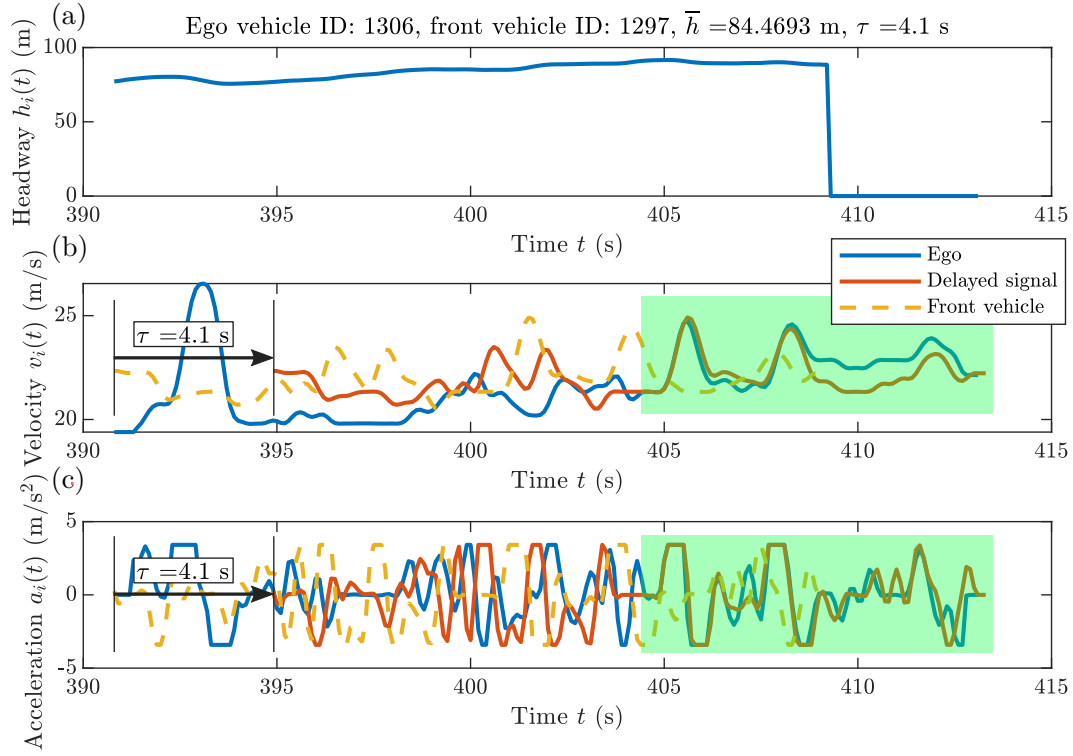


Figure 2.2: Delay estimation for large headway, in panel (a) the headway signals, in panel (b) the velocity signals and in panel (c) the acceleration signals are shown

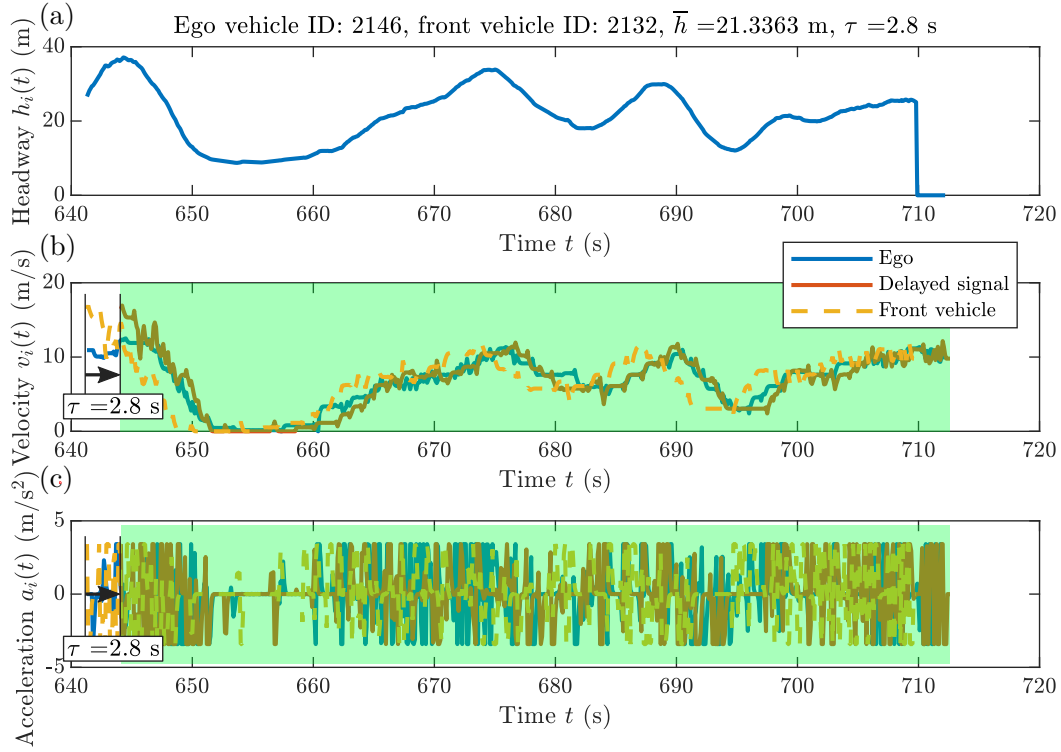


Figure 2.3: Delay estimation for medium headway. In panel (a) the headway signals, in panel (b) the velocity signals and panel (c) the acceleration signals are shown

In Fig. 2.3 the time signals are related to the medium following distance. The average headway is 21.3 m and the estimated delay is 2.8 s. The signals show a high correlation in the overall section, both for velocity and acceleration, it is an acceptable estimation.

In Fig. 2.4 a short time signal is shown for a small following distance. The average headway is 7.9 m and the estimated delay is 0.7 s. This dataset corresponds to a lane change situation, see the drop in the headway signal between $t = 319.7 - 323.2$ s. The signals show a high correlation in the overall section, hence it is an acceptable estimation.

However, In Fig. 2.5 one can see an example of an incorrect estimation. In this case, the average headway is 14.7 m and the estimated delay is 13.5 s, which is intuitively wrong. The signals show a high correlation for the acceleration as a coincidence. However, if we take a look at the velocity signals, we can see that is clearly incorrect. In this case, the delay estimation is not acceptable.

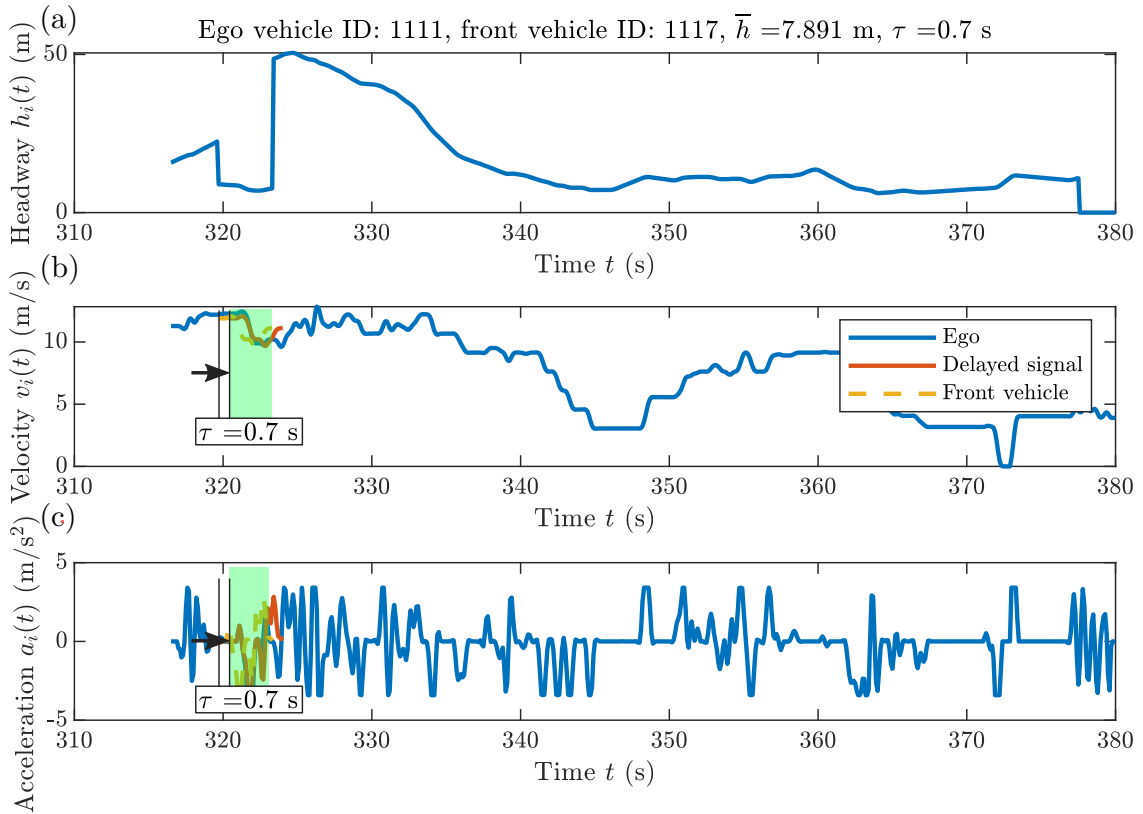


Figure 2.4: Delay estimation for small headway. In panel (a) the headway signals, in panel (b) the velocity signals and in panel (c) the acceleration signals are shown

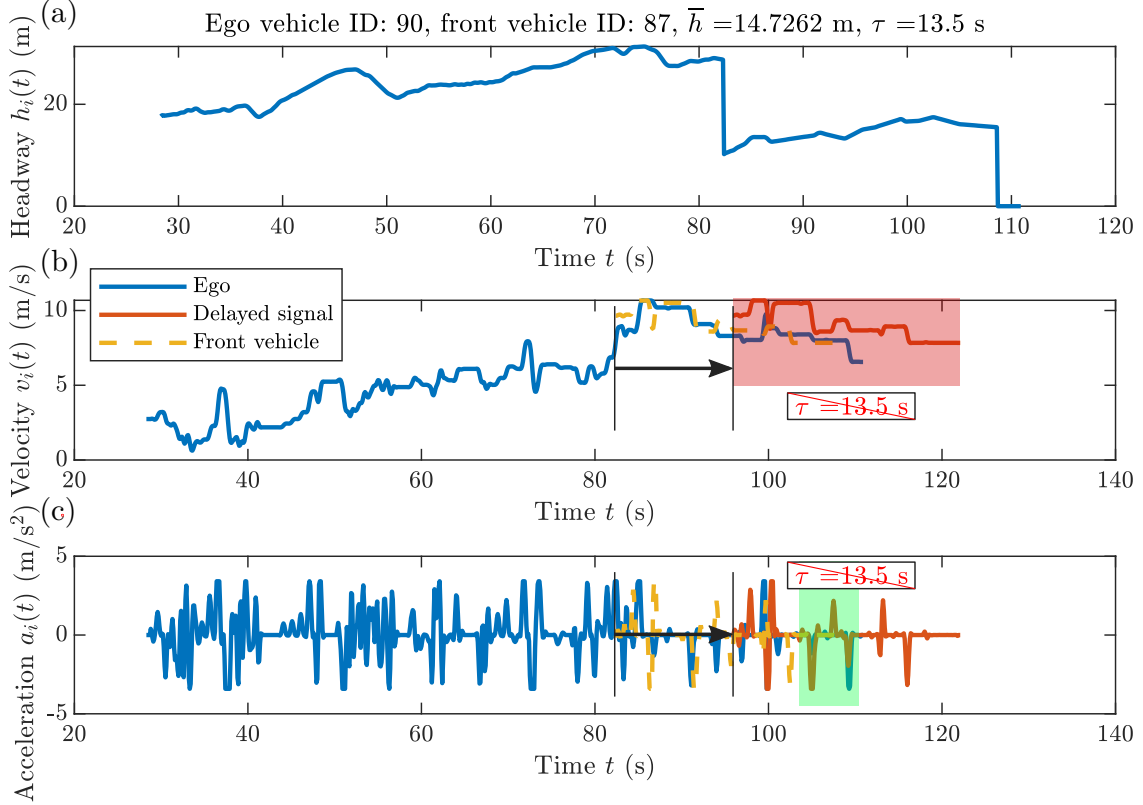


Figure 2.5: Example of incorrect automatic delay estimation, in panel (c) an accidental correlation was found automatically, but in panel (b) the velocity signal indicates that the offset is clearly wrong.

As we can see, it is not a reliable method to automatically estimate the delays. In some cases, the correlation between the signals is clearly visible, but often only some part of the time signal show correlation. In other cases, no clearly identifiable correlation exists due to the corrupted data. It can be because of frequent lane changes (indicated by a drop in the headway signals), too short measurements, or other unknown and non-modelled factors that change the drivers' behaviour. For this reason, the method requires a manual review of each resulting delay, and to visually decide if the signal offset was accurate or not. Note, that it involves subjective decisions and may lead to biased data, but it is still possible to draw a conclusion on large-scale trends. As a result, 470 evaluated delays were acceptable which is illustrated in Figure 2.6. In panels (a) and (b) the evaluated delay vs. headway data are shown, and in panels (c) and (d) the delay vs. time headway data.

The results confirm our assumption that the drivers' reaction time indeed depends on the situation. A clear increasing trend is visible in relation to the delay vs. headway. No high delay values are found for low headways and similarly, no quick reaction times for

large headways. The linear regression only resulted in a moderate coefficient of determination $R_{adj}^2 = 0.46$. Note, that statistically it is not a high value, but for human-related measurement, this variation level can be accepted. Linear relation does not follow clearly from this, but this more advanced model describes better the behaviour and shows a better correlation with the measurements than the commonly used constant delay model. Hence, it is worth extending the car-following model and it is acceptable to use linear relation for the headway-dependent delay.

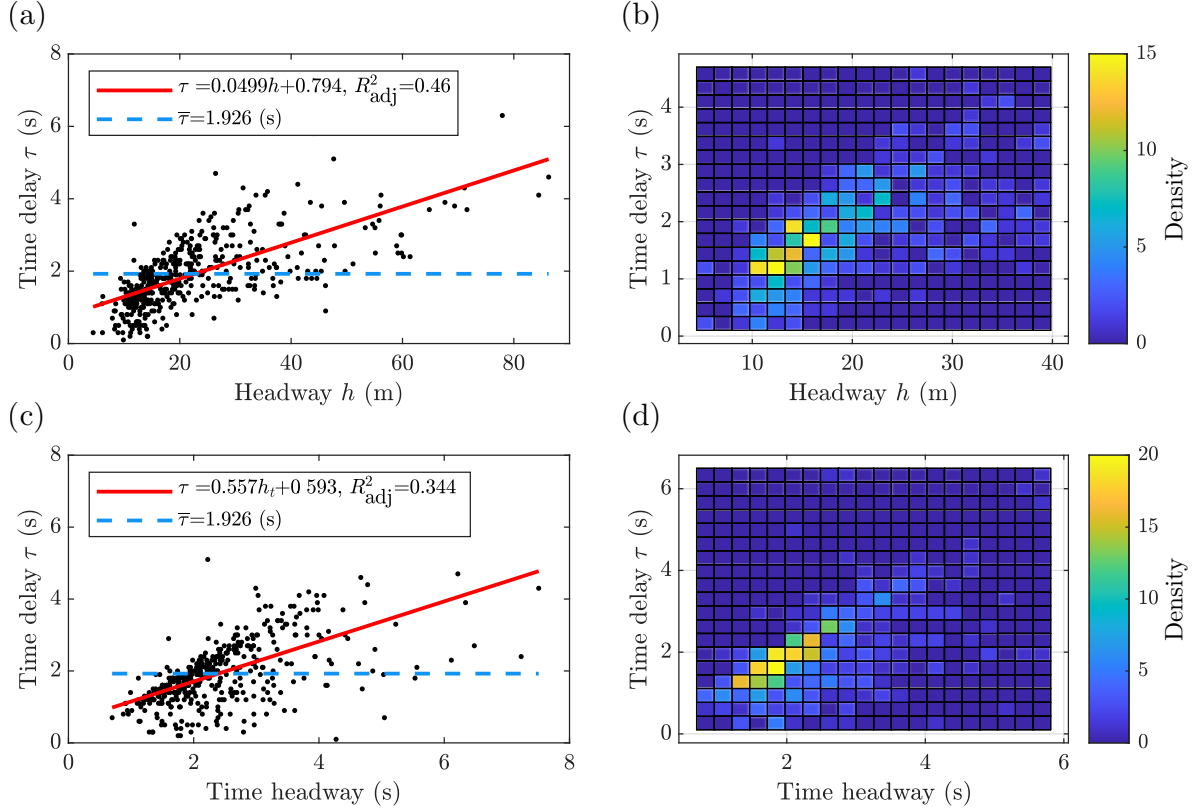


Figure 2.6: Results of the state-dependent delay estimation. In panels (a) and (b) are the scatter plot and the heatmap of the time delays vs. headways, respectively, and in panels (c) and (d) are the scatter plot and the heatmap of the time delays vs. time headways, respectively.

3 Theoretical background

In this section, the theoretical background of the commonly used car-following model with constant delay and its governing equations are presented. Then, the state-dependent model for human time delay and the extended car-following model are introduced. Finally, the most relevant theories and methods for linear and nonlinear analysis are provided.

3.1 General car-following model

In this section, we revisit the general car-following model with constant time delay. In the scientific literature, this model is often used to study the dynamics of motorways and highways and the formation of traffic congestion. The obtained results are essential for designing various CC technologies and AVs. The layout of the model is shown in Figure 3.1, where N vehicles follow each other on a ring road of circumference L . The forward dynamics of each car are described by a longitudinal control function $V(h)$. The system has $2N$ dimensions, whereas the state of the vehicles is described by two variables: the velocity $v_i(t)$ and the $h_i(t)$ headway or following distance relative to the leading car. It is called the bumper-to-bumper distance, which is the distance between the front bumper of the i th vehicle and the rear bumper of the $i + 1$ th, leading vehicle.

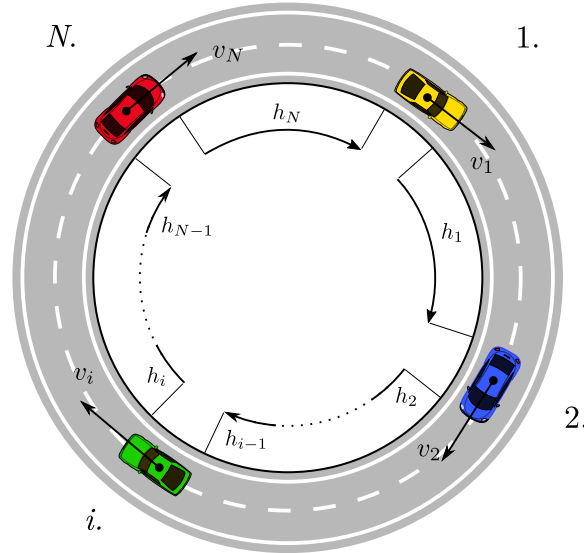


Figure 3.1: Model layout, where N vehicles follow each other on a ring-road. Each of them is described by two state variables the, the following distance h_i and the velocity

$$v_i, \text{ where } i = 1 \dots N.$$

The theoretical model can be simplified by neglecting the length of the vehicles, since

including them would only cause a spatial offset and they would drop due to the derivatives in the calculation of the stability analysis anyway. However, in the case of real vehicle controllers, for tests and road measurements they have to be considered to obtain the real position of the vehicles.

The ring configuration provides feedback in the vehicle platoon which makes it possible to study the behaviour of large systems in general, even with a small number of cars. The movement of each vehicle and its change in motion after N cars feeds back on themselves, hence due to a perturbation caused by a vehicle, braking and acceleration waves emerge along the ring propagating backwards. If $N \rightarrow \infty$ or N is a sufficiently large number, it describes accurately the behaviour of a straight road. Only the kinematics are considered, thus the forces due to continuous turning are not taken into account.

The model in general is given with a nonlinear delay differential equation system (DDE) in the following form

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{x}(t - \tau_i), \dots), \quad \mathbf{f}: \underbrace{X \times X \times \dots}_{(N+1)} \rightarrow \mathbb{R}^{2N-1}. \quad (3.1)$$

The state variables are written in a vector as

$$\mathbf{x} = \begin{bmatrix} v_1 \dots v_N & h_1 & \dots & h_{N-1} \end{bmatrix}^\top \in X \subseteq \mathbb{R}^{2N-1}, \quad (3.2)$$

where N is the number of vehicles, τ_i is the delay of the i th car, v_i is the velocity and h_i is the headway, between i th and $i + 1$ th one. The acceleration of the i th vehicle is written according to [3, 23] by two equations. The derivative of the velocity with respect to time is defined by a nonlinear control equation, which in the case of AVs written as

$$\begin{aligned} \dot{x}_i(t) = \dot{v}_i(t) = \text{sat} & \left(\alpha_i \left(V_i(h_i(t - \tau_i)) - v_i(t - \tau_i) \right) \right. \\ & \left. + \sum_{j=1}^k \beta_j W_i(v_{i+j}(t - \tau_i) - v_i(t - \tau_i)) \right), \end{aligned} \quad (3.3)$$

if $i = i_a$, where $k = 1, 2, \dots$ gives the communication range of CAV, the number of vehicles in front from which data is received. It can be used to realise an optimised control strategy by reacting earlier to the situation change at a distance. Furthermore, in the case of human drivers

$$\begin{aligned} \dot{x}_i(t) = \dot{v}_i(t) = \text{sat} & \left(\alpha_i \left(V_i(h_i(t - \tau_i)) - v_i(t - \tau_i) \right) \right. \\ & \left. + \beta_i W_i(v_{i+1}(t - \tau_i) - v_i(t - \tau_i)) \right), \end{aligned} \quad (3.4)$$

if $i = i_h$. The α_i and β_i are the i th car's control gains, $V_i(h)$ is the so-called optimal velocity function (or range policy), that gives the relation of the desired velocity in the relation of the following distance. The acceleration depends on the difference between the actual and desired velocity by α_i sensitivity parameter. Additionally, it also depends on the speed difference of the consecutive vehicles by β_i control gain. In the case of a CAV equipped with V2V technology, it can take into account the velocity difference of further vehicles by β_j control gain.

On the one hand, these are meant to model the human drivers realistically and capture the most essential factors in the longitudinal dynamics of HVs to accurately describe their behaviour on a microscopic scale. On the other hand, for AVs and DAS systems these gains and the optimal velocity function are design parameters and free to change by engineers. Tuning them enables the researchers and engineers to achieve the desired behaviour depending on the circumstances, that is optimal for the individual vehicle or even at the level of the entire traffic flow. The second half of the state variable vector $\dot{x}_{i+N}$ if $i = 1 \dots N - 1$ simply follows from the ring geometry and it is given as the derivative of the headways resulting in the velocity difference of the consecutive vehicles

$$\dot{x}_{i+N}(t) = \dot{h}_i(t) = v_{i+1}(t) - v_i(t). \quad (3.5)$$

Also note that the following distance of the N th vehicle can be detached, since it can be calculated by the length of the ring L (in theory it is a chosen arbitrary value, whereas in practice is a given geometric property) and the following distance of all the other vehicles.

$$h_N(t - \tau_N) = L - \sum_{i=1}^{N-1} h_i(t - \tau_N), \quad (3.6)$$

thus, the acceleration of the N th vehicle is given via the equation

$$\begin{aligned} \dot{v}_N(t) = \text{sat} & \left(\alpha_N \left(V_N \left(L - \sum_{i=1}^{N-1} h_i(t - \tau_N) \right) - v_N(t - \tau_N) \right) \right. \\ & \left. + \beta_N W_N \left(v_1(t - \tau_N) - v_N(t - \tau_N) \right) \right). \end{aligned} \quad (3.7)$$

This results in the reduction of the dimension of the system to $2N - 1$. If we do not account for it, a trivial constant result would be a solution. This corresponds to a rigid body-like motion, when all vehicles travel at the same speed, accelerating and decelerating in equal amounts. In general, the so-called optimal velocity function or range policy is a nonlinear continuous piecewise function, expressed as

$$V_i(h) = \begin{cases} 0 & \text{ha } h \leq h_{\text{stop},i}, \\ F_i(h) & \text{ha } h_{\text{stop},i} < h < h_{\text{go},i}, \\ v_{\text{max},i} & \text{ha } h \geq h_{\text{go},i}. \end{cases} \quad (3.8)$$

where the lower limit $h_{\text{stop},i}$ corresponds to the minimum following distance under which the driver attempt to stop the vehicle, and over the upper limit $h_{\text{go},i}$ they tend to drive with $v_{\text{max},i}$ the maximal allowed speed. As traffic congestion builds up, the vehicles collectively slow down to maintain a safe following distance. In the case of uncongested condition, fewer vehicles and larger headway allow higher overall travel speed. The behaviour between the two limits is given by the function $F(h)$, which can be different for the individual drivers. Linear, polynomial or trigonometric functions are also commonly used in the literature. Furthermore, the speed policy function is often considered in the models

$$W_i(v) = \begin{cases} v & \text{ha } v \leq v_{\text{max},i}, \\ v_{\text{max},i} & \text{ha } v > v_{\text{max},i} \end{cases} \quad (3.9)$$

to prevent vehicles from speeding, even if the leading car drives above the permitted v_{max} speed limit. The acceleration and braking capabilities of the vehicles can be modelled more realistically by utilizing acceleration saturation. It is taken into account by a continuous piecewise function written as

$$\text{sat}(a) = \begin{cases} a_{\text{min}} & \text{ha } a \leq a_{\text{min}}, \\ a & \text{ha } a_{\text{min}} \leq a \leq a_{\text{max}}, \\ a_{\text{max}} & \text{ha } a \geq a_{\text{max}}, \end{cases} \quad (3.10)$$

where a_{min} and a_{max} are the minimal and maximal possible acceleration limits, respectively. We presented in detail the effect of the acceleration saturation on the behaviour of the car-following model in [47], which may significantly alter the global stability of the system.

In the rest of this work for the derivation of the SD-DDE system and the evaluation of the measurement data, we assume the following simplifications: the speed policy $W(v)$ and the acceleration saturation $\text{sat}(a)$ are neglected, to be able to observe the distinctive effect of the state-dependent time delay.

3.2 Modeling the state-dependent time delay

Based on the literature review in Section 2 and the evaluated measurement data in Section 2.2 regarding the factors affecting the reaction time, we introduced the time delay of the

i th HV as headway dependent in [30], given in general as

$$\tau = g(h(t - \tau)) := \tau(h_t), \quad (3.11)$$

if $i = i_h$, where $g(h)$ is an arbitrary chosen function, h is the headway and $h_t(\tau) = h(t + \theta)$, $\theta \in [-\tau, 0]$ represents the past of the state in the $[-\tau, 0]$ time period. The prescribed time delay is implicit since τ appears in both the left and the right side of the equation in the argument of the following distance. The reason for modelling the delay in such a way is that the driver observed the following distance at an earlier $t - \tau$ time instant, due to his reaction time. For AVs, the τ_i time delay is assumed to be constant regardless of the circumstances, since its variance is negligible compared to the variability of the human drivers' reaction time.

3.2.1 The extension of the car-following model with state-dependent time delay

Generally, the SD-DDE system is given, similarly to Equation (3.1), by the following

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t), \mathbf{x}(t - \tau_1(\mathbf{x}_t)), \mathbf{x}(t - \tau_2(\mathbf{x}_t)), \dots, \mathbf{x}(t - \tau_N(\mathbf{x}_t))\right), \quad (3.12)$$

where $\mathbf{x}_t(\theta) = \mathbf{x}(t + \theta)$, $\theta \in [-\tau, 0]$ represents the continuous function describing the extended state variable. Every vehicle has a different τ_i delay, even if the function given in (3.11) is the same because their individual headway is different at a given time. The vector of the state variables can be expressed as

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} v_1(t) & \dots & v_N(t) & h_1(t) & \dots & h_{N-1}(t) \end{bmatrix}^\top \\ \mathbf{x}(t - \tau_i(\mathbf{x}_t)) &= \begin{bmatrix} v_1(t - \tau_i(h_{i,t})) & \dots & v_N(t - \tau_i(h_{i,t})) & \dots & \\ h_1(t - \tau_i(h_{i,t})) & \dots & h_{N-1}(t - \tau_i(h_{i,t})) \end{bmatrix}^\top, \end{aligned} \quad (3.13)$$

where $\mathbf{x}(t)$ is the non-delayed and $\mathbf{x}(t - \tau_i(\mathbf{x}_t))$ is the delayed state variables, given with a separate vector for each $i = 1 \dots N$. The variable h_N can be detached from the system similarly to Equation (3.6) in the case of the constant delay system and it can be calculated as

$$h_N(t - \tau_N(h_N)) = L - \sum_{k=1}^{N-1} h_k(t - \tau_N(h_N)) := h_{Nt}. \quad (3.14)$$

Note, that each h_k , where $k = 1 \dots N - 1$, is delayed by the reaction time of the N th vehicle τ_N and not with τ_k . The derivative of the variables of the i -th vehicle is given by

the following equations, as before

$$\begin{aligned}\dot{v}_i(t) = \dot{x}_i(t) = & \alpha_i \left(V_i \left(h_i(t - \tau_i) \right) - v_i(t - \tau_i) \right) \\ & + \sum_{j=1}^k \beta_j \left(v_{i+j}(t - \tau_i) - v_i(t - \tau_i) \right),\end{aligned}\tag{3.15}$$

for AVs if $i = i_a$ for and by

$$\begin{aligned}\dot{v}_i(t) = \dot{x}_i(t) = & \alpha_i \left(V_i \left(h_i(t - \tau_i(h_{i,t})) \right) - v_i(t - \tau_i(h_{i,t})) \right) \\ & + \beta_i \left(v_{i+1}(t - \tau_i(h_{i,t})) - v_i(t - \tau_i(h_{i,t})) \right),\end{aligned}\tag{3.16}$$

if $i = i_h$ for HVs. However, the derivative of the headway does not depend on the delayed state

$$\dot{h}_i(t) = \dot{x}_{i+N}(t) = v_{i+1}(t) - v_i(t)\tag{3.17}$$

since it is a consequence of the geometry of the ring road. Note, that SD-DDE systems are always nonlinear since the state variable is included implicitly in its own argument through the delay. Moreover, the solution of the system cannot be differentiated with respect to the state-dependent delay. Consequently, the "true" linearization, as in the case of DDE systems, is not possible either. Instead, an associated linear DDE system is chosen, which has the same local stability properties as the original SD-DDE system. [36, 39].

If the system is at its equilibrium state, the equilibrium headway is $h_i(t - \tau_i(\mathbf{x}_t)) \equiv h^*$, hence the equilibrium delay $\tau_i(h_{i,t}) \equiv \tau_i^*$ is also constant in time. Note, that if $i = i_a$ the delay of the AV is already constant by definition $\tau_i^* \equiv \tau_i$. Consequently, the linear DDE system associated with the state-dependent system shown in Equation (3.12), when $\mathbf{x}(t) = \mathbf{x}^*$ is given as

$$\dot{\tilde{\mathbf{x}}}(t) = \underbrace{D_0 \mathbf{f}^*}_{\mathbf{L}} \tilde{\mathbf{x}}(t) + \sum_{i=1}^N \underbrace{D_i \mathbf{f}^*}_{\mathbf{R}_i} \tilde{\mathbf{x}}(t - \tau_i^*) + \sum_{i=1}^N D_{\tau_i} \mathbf{f}^* D_i \tau_i \tilde{\mathbf{x}}_t,\tag{3.18}$$

where we introduce the $\mathbf{f}^* = \mathbf{f}(\mathbf{x}^*, \mathbf{x}^*, \dots)$ notation for the nonlinear differential equation system at the $\mathbf{x}^*$ equilibrium state. $D_0 \mathbf{f}^*$ denotes the derivative of $\mathbf{f}$ with respect to the non-delayed terms, $D_1 \mathbf{f}^*, \dots, D_N \mathbf{f}^*$ are the derivative of $\mathbf{f}$ with respect to the delayed state variable, D_{τ_i} is the derivative of the system with respect to the delays, and $D_i \tau_i$ denotes the so-called Frechét derivative (the infinite-dimensional gradient) of each τ_i time delay with respect to $\mathbf{x}_t$. However, since the system in Eq. 3.12 does not depend on the delay explicitly, therefore the $\sum_{i=1}^N D_{\tau_i} \mathbf{f}^* D_i \tau_i \tilde{\mathbf{x}}_t = \mathbf{0}$ term containing the Frechét derivatives

drops out. The vector of the state variables of the associated linear system is

$$\begin{aligned}\tilde{\mathbf{x}}(t) &= \begin{bmatrix} \tilde{v}_1(t) & \dots & \tilde{v}_N(t) & \tilde{h}_1(t) & \dots & \tilde{h}_N(t) \end{bmatrix}^\top, \\ \tilde{\mathbf{x}}(t - \tau_i(h_{i,t})) &= \begin{bmatrix} \tilde{v}_i(t - \tau_i(h_{i,t})) & \dots & \tilde{v}_N(t - \tau_t(h_{Nt})) & \dots & \tilde{h}_i(t - \tau_i(h_{i,t})) & \dots & \tilde{h}_{N-1}(t - \tau_{N-1}(h_{N-1t})) \end{bmatrix}^\top,\end{aligned}\quad (3.19)$$

where $i = 1 \dots N$. The vector of the variables at the equilibrium state is given as

$$\mathbf{x}^* = [v^* \dots v^* h^* \dots h^*]^\top, \quad (3.20)$$

which elements are determined as follows

$$\begin{aligned}v_i(t) &\equiv v_i(t - \tau_i^*) \equiv v^* = V(h_i^*), \\ h_i(t - \tau(h_{it})) &\equiv h_i(t) \equiv h_i^*, \\ \tau(h_{it}) &\equiv \tau_i(h_i^*) = \tau_i^*,\end{aligned}\quad (3.21)$$

where $i = 1 \dots N$ and the length of the ring road is $L = \sum_{i=1}^N h_i^*$. Note that, in the case of real-world measurements the circumference of the ring is given and the equilibrium following distance h_i^* is calculated thereafter. Whereas usually in the theoretical calculation the dynamics with respect to h^* parameter is investigated, then, if necessary, the original ring road can be calculated. The linear coefficient matrix in Equation (3.18) corresponding to the non-delayed terms are denoted by $\mathbf{L} \in \mathbb{R}^{2N-1 \times 2N-1}$:

$$\mathbf{D}_0 \mathbf{f}^* = \mathbf{L} = \left[\begin{array}{ccc} \frac{\partial f_1}{\partial v_1(t)} & \dots & \frac{\partial f_1}{\partial h_{N-1}(t)} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{2N-1}}{\partial v_1(t)} & \dots & \frac{\partial f_{2N-1}}{\partial h_{N-1}(t)} \end{array} \right] \bigg|_{\mathbf{x}(t)=\mathbf{x}^*}, \quad (3.22)$$

and the coefficient matrices for the delayed term are given in the following form

$$\mathbf{D}_i \mathbf{f}^* = \mathbf{R}_i = \left[\begin{array}{ccc} \frac{\partial f_1}{\partial v_1(t-\tau_i(\mathbf{x}_t))} & \dots & \frac{\partial f_1}{\partial h_{N-1}(t-\tau_i(\mathbf{x}_t))} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{2N-1}}{\partial v_1(t-\tau_i(\mathbf{x}_t))} & \dots & \frac{\partial f_{2N-1}}{\partial h_{N-1}(t-\tau_i(\mathbf{x}_t))} \end{array} \right] \bigg|_{\mathbf{x}(t)=\mathbf{x}^*}, \quad (3.23)$$

respectively, where $\mathbf{R}_i \in \mathbb{R}^{2N-1 \times 2N-1}$ and $i = 1, \dots, N$. We seek the solution of the linear system in the form of exponential terms, such as

$$\begin{aligned}\tilde{\mathbf{x}}(t) &= \mathbf{c} e^{\lambda t}, \\ \tilde{\mathbf{x}}(t - \tau(h^*)) &= \mathbf{c} e^{\lambda(t - \tau_i^*)} = \mathbf{c} e^{\lambda t} e^{-\tau_i^* \lambda}, \\ \dot{\tilde{\mathbf{x}}}(t) &= \lambda \mathbf{c} e^{\lambda t}\end{aligned}\quad (3.24)$$

where $\lambda \in \mathbb{C}$ is the characteristic exponent and $\mathbf{c} \in \mathbb{C}^{2N-1}$ is the characteristic vector. Substituting into Equation (3.18) gives

$$\lambda \mathbf{c} e^{\lambda t} = \mathbf{L} \mathbf{c} e^{\lambda t} + \sum_{i=1}^N \mathbf{R}_i \mathbf{c} e^{\lambda t} e^{-\tau_i^* \lambda}, \quad (3.25)$$

then by rearranging the equation we get the following condition for the stability

$$\left(\lambda \mathbf{I} - \mathbf{L} - \sum_{i=1}^N \mathbf{R}_i e^{-\tau_i^* \lambda} \right) \mathbf{c} e^{\lambda t} = \mathbf{0}. \quad (3.26)$$

It is fulfilled if one of the multipliers is $\mathbf{0}$. The term $e^{\lambda t} > 0$ is always satisfied, and we are interested in the solution other than the trivial one $\mathbf{c} = \mathbf{0}$, hence the characteristic equation is obtained as

$$\det \left(\lambda \mathbf{I} - \mathbf{L} - \sum_{i=1}^N \mathbf{R}_i e^{-\tau_i^* \lambda} \right) = 0. \quad (3.27)$$

By solving the characteristic equation one can determine the linear stability of the system. It is possible using one of the methods detailed in the following section.

3.3 Methods of linear stability analysis

The characteristic equation of DDE systems such as in Eq. (3.27) is a transcendental equation having infinitely many solutions. Hence determining the characteristic roots is not straightforward and it leads to computational challenges. The DDE systems are infinite dimensional since λ is in the index of the exponent function. Note that, if we would have not detached the $2N$ th variable in Equation (3.14) then $\lambda = 0$ characteristic exponent would be a solution leading to $\dot{\mathbf{x}} = \mathbf{0}$.

In general, the characteristic exponent is a complex number which can be decomposed into real and imaginary parts such as $\lambda = \text{Re}\lambda + i\text{Im}\lambda = \gamma + i\omega$, where $i^2 = -1$ is the imaginary unit. The linear stability condition of the system is that all characteristic exponents must be on the left side of the imaginary axis, or reformulated, and the rightmost characteristic exponent has to have a negative real part. This implies that the real part for all infinitely many roots must be negative $\gamma < 0$. The so-called critical root corresponds to the stability limit, where the stability of the system changes and its real part fulfils $\max(\gamma_i) \equiv \gamma_{\text{cr}} = 0$ condition. In the case of a DDE system, the critical roots of the characteristic equation (3.27) often form complex conjugate root pairs. Hence, at the stability boundary, a root pair loses its stability simultaneously: $\lambda_{\text{cr}} = \pm i\omega_{\text{cr}}$ while crossing the imaginary axis. It is a dynamical stability loss called Hopf bifurcation, from

which a periodic orbit arises [48]. The critical roots can be determined, by substituting them into (3.27) resulting in

$$D(i\omega) = \det\left(i\omega_{\text{cr}}\mathbf{I} - \mathbf{L} - \mathbf{R}e^{-\tau i\omega_{\text{cr}}}\right) = 0. \quad (3.28)$$

One possible method to draw the exact stability curves of the system is the so-called D-subdivision method [49]. The equation corresponding to the stability boundary (3.27) is separated into real $\text{Re}D(\lambda) = 0$ and imaginary $\text{Im}D(\lambda) = 0$ parts. The solution of the two equations leads to the stability boundaries with respect to the system parameters. To compute these curves efficiently, the semi-discretisation method [50] or the multi-dimensional bisection method (MDBM) can be applied [51].

Another but less efficient way to compute the solution is by brute force calculation when the parameter space is mapped in discrete points. The 3.12 system is simulated in these points, e.g. by built SD-DDE solver in MATLAB, time signals are analysed and the stability boundary can be drawn in discrete points. Despite its simplicity, deciding the stability behaviour is not straightforward, especially in the vicinity of the stability boundaries where the signals may decay too slowly. Additionally, mapping the parameter space in high enough resolution and running the simulation long enough is computationally demanding and time-consuming.

Finally, one can use numerical continuation methods such as the MATLAB DDE-BIFTOOL package [52, 53] to calculate the roots and the stability boundaries. First, this algorithm computes the steady state and its stability along a chosen parameter to obtain the Hopf points by an iterative process. Then, the Hopf point is continued in the space of two arbitrary parameters by pseudo-arclength continuation (using secant approximations for tangents). Meanwhile, the roots of the characteristic equation are approximated and corrected by the Floquet multipliers method [52] resulting in the stability charts in the space of the chosen parameters.

In the case of a state-dependent delay system, the continuation of the stability boundaries is more complicated than for constant time delay models. The DDE-BIFTOOL algorithm can be applied only for explicitly defined state-dependent delays. However, in the car-following model, the delay is defined with implicit equation (3.11). One solution is to define the delay $\tau(\mathbf{x}_t)$ as an additional state variable. However, the realisation of the additional algebraic equation and the modification of the system is rather difficult.

One can see that the stability of the system in the case of explicit and implicit time

delay are identical, since in both cases the equilibrium headway is the same $h(t - \tau(h_t)) \equiv h(t) \equiv h^*$, hence the equilibrium delay is also equal. Utilizing suitable modification of the continuation algorithm in DDE-BIFTOOL makes it possible to force it to calculate the stability boundaries for implicit delay without the need for the algebraic equation.

The linear stability theory of dynamics stability loss implies that an unstable system results in an ever-growing oscillation which amplitude tends to infinity. However, for large amplitude vibrations, nonlinear effects may modify the behaviour. Since the car following model contains multiple nonlinearities, therefore, it is necessary to consider them to be able to analyse the global behaviour of the system.

3.4 Theoretical background of the nonlinear analysis

As it was aforementioned, usually the stability loss of the critical roots happens via Hopf bifurcation in the car-following model. In this case, periodic orbits emerge from the equilibrium starting with infinitely small amplitude and initially with ω_{cr} frequency [54, 55].

If a stable periodic orbit grows over the unstable equilibrium, then it is called supercritical Hopf bifurcation as illustrated in Fig. 3.2 in panel (a). Usually, it is undesirable to operate in this state due to the unstable equilibrium. Its counterpart is the so-called subcritical Hopf bifurcation, when an unstable periodic orbit rises over the stable equilibrium, as shown in Fig. 3.2 panel (b). This state may seem safe to operate from a linear point of view if we are not aware of the periodic orbits in the system. In practice and simulation, an oscillation caused by a small perturbation (e.g. a slight braking) will decay. However, if the perturbation "pushes" the oscillation outside of the region of attraction of the unstable periodic orbit, then the system cannot return to its equilibrium state and the amplitude of the oscillation will keep increasing. Note that theoretically the amplitude tends to infinity, but another stable periodic orbit might exist over the unstable one, thus limiting the vibrations. If so, the stability of the orbit changes and the unstable periodic orbit will become stable through the so-called fold bifurcation point. If in the subcritical case the periodic orbit folds back, then the system will be bistable over this region, because a stable equilibrium and a stable periodic solution co-exist. Taking this into consideration subcritical bifurcation can be generally considered more dangerous than a supercritical one, in engineering applications.

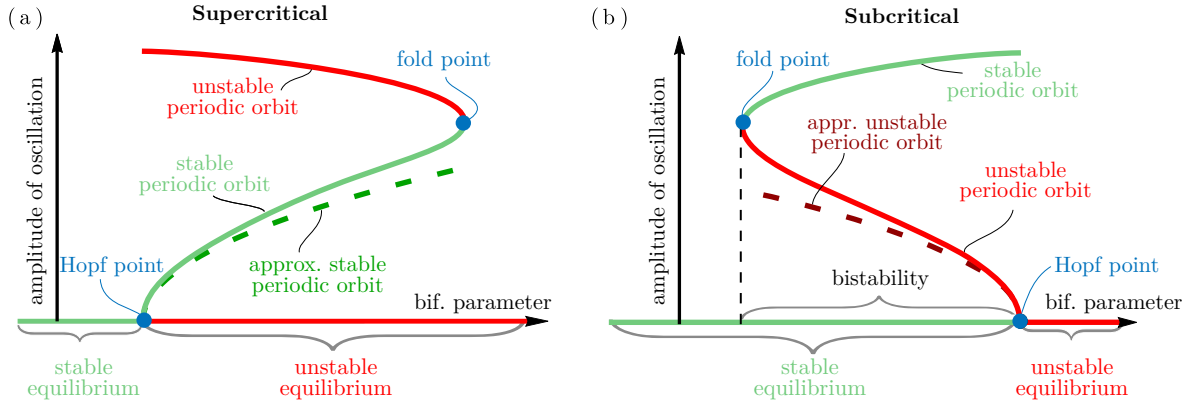


Figure 3.2: Bifurcation diagram illustrates the periodic orbit (the steady state oscillation) amplitude with respect to the bifurcation parameter. The periodic orbit rises from the Hopf point, and in some cases, it may change stability in the fold bifurcation point. Panel (a) shows the supercritical bifurcation, where a stable periodic orbit grows "over" the unstable equilibrium state, whereas its counterpart the subcritical bifurcation is in panel b, where the unstable periodic orbit rise over the stable equilibrium.

By applying the theory of Hopf bifurcation analysis for DDEs, one can obtain the second-order analytical approximation of the amplitude of the periodic orbit [54], which is illustrated by a dashed parabolic curve in Fig. 3.2. To do so, (3.18) has to be expanded into Taylor series up to the third order. Thereafter, via the infinite-dimensional centre manifold reduction, [56, 57, 58, 59, 60, 61] the infinite-dimensional system is reduced into two dimensions, called the direction of the long term (or slow) dynamics which correspond to the complex conjugate root pairs at the Hopf bifurcation point. The other dimensions can be detached, since the vibrations corresponding to those settle down faster [62], hence, it does not influence the steady-state solution.

To have a better understanding of the dynamics in the infinite-dimensional state space, the three-dimensional qualitative representation of the centre manifold reduction is shown at the vicinity of a Hopf bifurcation point in Fig. 3.3. The two slow directions are ξ_1 and ξ_2 corresponding to the long-term dynamics while ξ_i illustrates the other infinitely many dimensions. By linearisation, the slowest spectral subspace is approximated by a tangent plane at the equilibrium spanned by the directions of ξ_1 and ξ_2 . Furthermore, the above referred nonlinear analytical calculation provides the second-order approximation of the so-called centre manifold as a surface, on which the long-term dynamics can be characterised by periodic orbits.

In Fig. 3.3, typical behaviours are illustrated starting with different initial conditions

for super- and subcritical cases. Panel (a) illustrates the supercritical case, where an unstable equilibrium (red) is in the origin and a stable periodic orbit (green) exists on the centre manifold. In the case of small/large perturbation as the transient vibrations decay, the trajectories settle on the surface of the centre manifold and converge towards the stable periodic orbit.

By comparison, the subcritical case is illustrated in panel (b) of Fig. 3.3. Here, a stable equilibrium is in the origin and an unstable periodic orbit lies on the centre manifold. In this case, a manifold exists, which divides the infinite-dimensional state space and determines the region of attraction, illustrated by the red shaded surface. In the case of the blue trajectory, the initial condition is close to the equilibrium starting within the region of attraction and converging to the equilibrium. As it was aforementioned, the subcritical case may be dangerous since it looks like a stable, well-operating system. However, starting the initial condition outside of the region of attraction (yellow trajectory), the solution converges to the centre manifold, but it is repelled by the unstable periodic orbit and continues to oscillate with ever-growing amplitude. Note that the approximation of the region of attraction for time delay systems is a challenging problem due to the infinite-dimensional state space and out of the scope of this work, although there exist some promising techniques in the literature [63, 64, 65, 66].

Although the scientific literature is quite extensive and comprehensive on the bifurcation theory of delayed systems, to the best of our knowledge, no nonlinear analytic bifurcation theory exists for state-dependent time delay systems that can be used to approximate periodic orbits.

Therefore, in this thesis for further calculations, the DDE-BIFTOOL software package is used [52, 53], which is able to simulate SD-DDE systems, hence to investigate the global behaviour. Similarly to linear stability, it is possible to continue the steady-state periodic trajectories along chosen parameters and determine their stability with the very same pseudo-arclength continuation technique. If the stability of the orbits is calculated, then by iterative correction steps the fold point can be determined. Additionally, these fold points can be continued along two parameters to determine and draw the bistable areas. However, for implicit SD-DDE, the continuation of nonlinear periodic orbits is only possible with an extra condition as described in Section 3.3. For this reason, we simplify the calculation of periodic trajectories and give the time delay in explicit form for the nonlinear calculations.

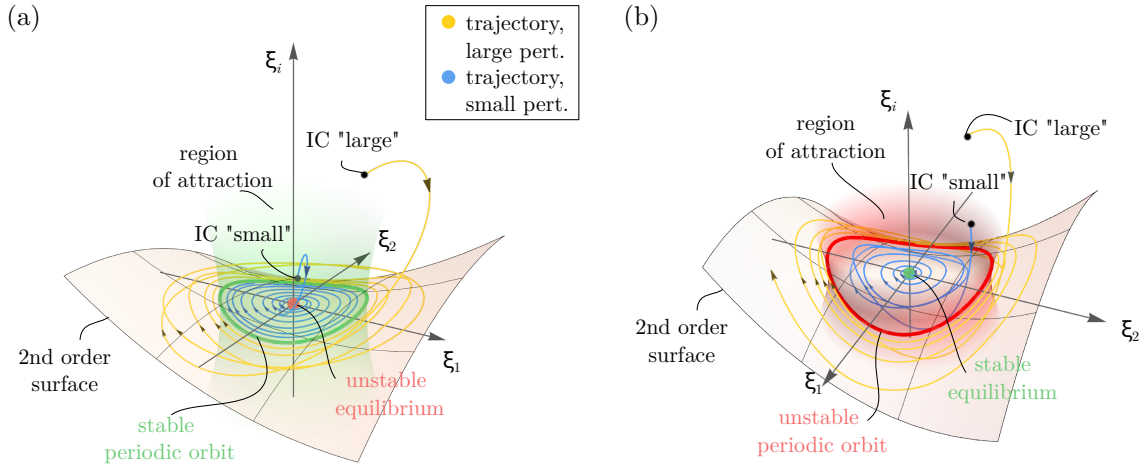


Figure 3.3: Illustration of the central manifold approximation in the vicinity of the Hopf bifurcation point. The trajectories correspond to simulations with small and large perturbations, with yellow and blue respectively. In panel (a) a supercritical and panel (b) a subcritical bifurcation is shown.

4 Results of the road measurements

In this section, we revisit the measurement setup and its implementation of the [23], then analyse the model extended with the state-dependent time delay introduced in Section 3.2.1. The results are compared with the constant time delay model and the measurement data, using a real case study to perform the linear and nonlinear analyses detailed in the previous sections.

4.1 Measurement layout

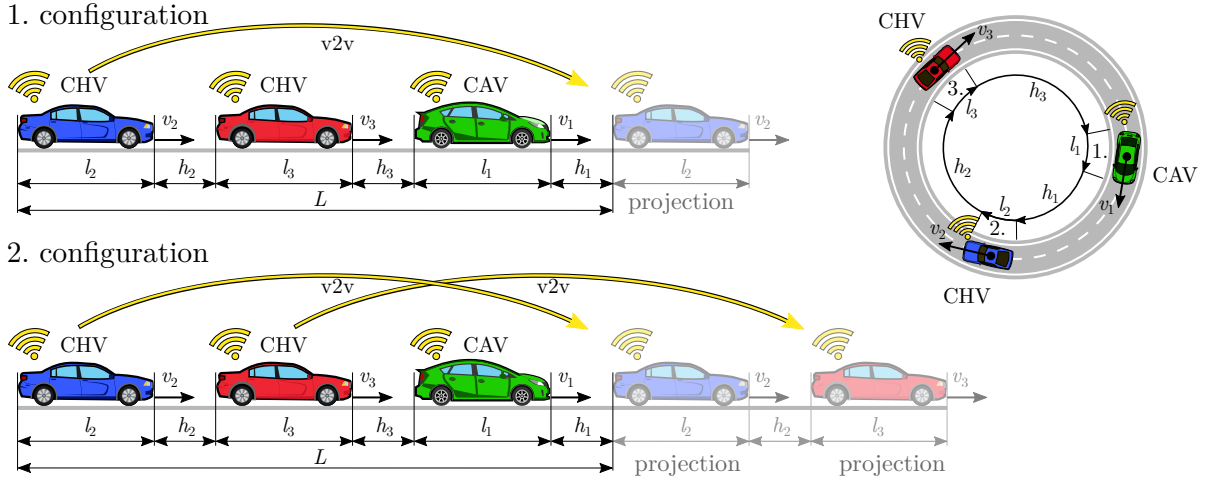


Figure 4.1: Measurement layout corresponding to [23], where a three vehicles platoon equipped with V2V technology, of which two is CHV (blue and red) and the third is CAV (green). Panel (a) illustrates the first measurement configuration realizing a virtual ring by the CAV projecting the last vehicle front of itself, in panel (b) is the second configuration shown, the CAV projects both CHVs front of itself, in panel (c) is the associated theoretical model of the virtual ring road.

Road tests were conducted in ring-road configuration of the car-following model to validate the theoretical findings in [22]. During this measurement, the so-called stop-and-go wave was detected, which propagates opposite to the direction of travel, along the vehicle platoon causing braking and acceleration waves. This phenomenon is one of the main causes of traffic congestion on highways appearing over a certain level of traffic density. However, this measurement could only be realised at low speeds, while the car-following model is primarily intended to model high-speed highway traffic. But on a relatively small circuit, it is extremely difficult for human drivers to lane keep the

vehicle at high speed due to the centrifugal force, also while maintaining a safe following distance and behaving as in normal highway circumstances. One solution is to test on a sufficiently long test track, although it would also require a large number of test vehicles to create the required traffic density, to be able to investigate the stop-and-go waves and phantom congestion phenomenon. Although, as it was aforementioned, the theoretical model only considers the longitudinal dynamics and the ring-road layout is only needed for the feedback in the system.

To overcome these practical measurement issues, a team of researchers at the University of Michigan conducted straight road measurements with a convoy of one CAV and two CHVs [23]. Taking advantage of the V2V technology, the CAV in front was able to project the CHV at the end of the platoon in front of itself and reacted to the behaviour of the CHV. This enabled them to carry out tests in an associated virtual ring of arbitrary length L following the theoretical model while driving with a realistic speed for highway conditions. Figures 4.1 panel (a)-(b) shows the measurement layout and panel (c) the corresponding virtual ring setup. The measurements were performed in two configurations for a total of four measurement series, details of which are given in Table 4.3. The 1st configuration is shown in Figure 4.1 panel (a), where the CAV was driving in the front of the vehicle convoy and projected the last car in front of itself. The 2nd configuration is illustrated in Figure 4.1 panel (b), where the longitudinal control of the CAV responded to both vehicles, utilized by projecting both vehicles. In the control law, it was realized by an additional $\beta_{a,2}$ gain in Equation (3.15), taking advantage of the CCC technology.

The average headway distance h^* is extracted from the data by the length of the virtual road and the length of each vehicle

$$h^* = \frac{1}{N} \left(L - \sum_{i=1}^3 l_i \right). \quad (4.1)$$

Thereafter, the equilibrium is calculated according to (3.21), when all vehicles travel with the same speed v^* , but with different headways due to the different human range policies $V_i(h)$. The equilibrium state can be determined by solving the following nonlinear system of equation

$$\begin{cases} V_a(h_1^*) = v^*, \\ V_2(h_2^*) = v^*, \\ V_3(h_3^*) = v^*, \\ h_1^* + h_2^* + h_3^* = 3h^*, \end{cases} \quad (4.2)$$

which is realized by the default MATLAB Newton–Raphson solver. For the human range

Vehicle	Parameter fitting		
	$h_{\text{stop},i}$ (m)	$h_{\text{go},i}$ (m)	$v_{\text{max},i}$ (m/s)
1, AV	5	$v_{\text{max},i}/\kappa + h_{\text{stop},1}$	24
2, HV	1.56 ± 0.12	29.1 ± 0.92	24.6 ± 1.2
3, HV	-0.2 ± 0.11	33.9 ± 0.64	24.0 ± 0.6

Table 4.1: The parameters of the optimal velocity function. In the case of CAVs these are technological parameters, thus tunable, and for HVs these were fitted to the measurement data according to [23]

policy, we use a continuous, quadratic piecewise function as follows

$$V_i(h) = \begin{cases} 0 & \text{ha } h \leq h_{\text{stop},i}, \\ v_{\text{max},i} \left(1 - \left(\frac{h_{\text{go},i} - h}{h_{\text{go},i} - h_{\text{stop},i}} \right)^2 \right) & \text{ha } h_{\text{stop},i} \leq h \leq h_{\text{go},i}, \\ v_{\text{max},i} & \text{ha } h \geq h_{\text{go},i}. \end{cases} \quad (4.3)$$

We use the parameters according to [23, 67], which were determined by parameter fitting and minimizing the scribed error function. In this process, they simulated the model with the fitted parameter and compared them with the measurement data. However, this method involves averaging of the behaviour of the human during the time of the measurement. Hence, it might not describe the behaviour of the system on a microscopic scale and the simulation cannot reproduce the exact measurement in all circumstances accurately. The fitted parameter and the associated uncertainty are collected in Table 4.1 and the following simulations, the nominal, and mean values of the parameters were used. For sake of simplicity, the control gains of the drivers were considered to be identical $\alpha_h = \alpha_2 = \alpha_3$, $\beta_h = \beta_2 = \beta_3$, and also the human delays $\tau_h = \tau_2 = \tau_3$ were identical. These parameters were also fitted by the measurement, however, they show high uncertainty, hence it is possible to tune them within a reasonable range for more accurate modelling. In the work of [67] a more sophisticated fitting method was used and as a result, slightly modified parameters were recommended, which are collected in Table 4.2. Their fitted model and parameter reproduced the averaged behaviour well, but it could not describe the linear and nonlinear stability of the system. Therefore, in this contribution instead of the constant delay system, we use the above-introduced state-dependent time delay. Although mathematically it is more complex to handle and simulate, it is expected to describe the behaviour of the individual drivers more accurately. These results are discussed in detail in section 4.2 and 4.3.

In the simplest case, the variable nature of the human reaction time with respect to

headway can be modelled via a linear function introduced in [30]. The headway-dependent delay is described by two parameters

$$\tau_i = ah_i(t - \tau_i) + b \implies \tau_i = ah_i(t) + b := \tau_i(h_i), \quad (4.4)$$

where $a > 0$ (s/m) is the slope of the time delay and b (s) is the constant offset. Oppose to the considerations in (3.11), for sake of simplicity explicit delay is used instead of the implicitly defined one. Then, on the right-hand side of the equation, instead of the delayed headway, the non-delayed variable $h_i(t)$ appears. This is done to facilitate the calculations, so that the linear and nonlinear analysis, and the continuation of the stability boundaries and periodic orbits in DDE-BIFTOOL can be realised without the need for the additional algebraic condition to account for the implicit equations. Hence, the introduced headway-dependent delay is shown in Figure 4.2 panel (c). Note that if we choose the parameter to be $a = 0$, then $\tau_i \equiv b$ we obtain the constant delay model as before. As a further simplification, the state-dependent reaction times of the two human drivers were chosen to be identical, similar to the control gain parameters.

The technological parameters of the CAV are tuneable, design parameters. In the measurement, the optimal velocity function in the longitudinal control was set to be linear between $h_{\text{stop},1}$ and $h_{\text{go},a}$. To facilitate numerical continuation, this function was smoothed as in [68], using the following formula

$$V_a(h) = \begin{cases} 0 & \text{ha } h \leq h_{\text{stop},1} - c, \\ \kappa(h - h_{\text{stop},1}) + \frac{(c - \kappa(h - h_{\text{stop},1}))^2}{4c} & \text{ha } h_{\text{stop},1} - c < h < h_{\text{stop},1} + c, \\ \kappa(h - h_{\text{stop},1}) & \text{ha } h_{\text{stop},1} + c \leq h \leq h_{\text{go},a} - c, \\ \kappa(h - h_{\text{stop},1}) - \frac{(v_{\text{max},a} - \kappa(h - h_{\text{stop},1}) - c)^2}{4c} & \text{ha } h_{\text{go},a} - c < h < h_{\text{go},a} + c, \\ v_{\text{go},a} & \text{ha } h \geq h_{\text{max},a} + c, \end{cases} \quad (4.5)$$

where $\kappa = \frac{1}{t_{\text{set}}}$ is the sensitivity. For ACC controllers, the inputs set by the driver are $v_{\text{max},a}$ desired speed and t_{set} time-headway. Parameter c is the smoothness range, which

Human control parameters			
	α_h (1/s)	β_h (1/s)	τ_h (s)
Original	0.14	0.54	1
Fine tuned	0.12	0.54	1

Table 4.2: The human control gains and constant delays corresponding to the original measurement fitted and the fined tuned parameters.

describes the section around the abrupt change at $h_{\text{stop},1}$ and $h_{\text{go},a}$ values, where the function is smoothed by a second-order polynomial. The parameters corresponding to the two measurement configurations and the used parameters are collected in Table 4.3.

	Name	Notation	Value	Dimension
	CAV delay	τ_a	0.5	s
	CAV control parameter	α_a	0.4	1/s
	Smoothness range	c	0.45	m
1. configuration	CAV control parameter	$\beta_{a,1}$	0.5	1/s
	CAV control parameter	$\beta_{a,2}$	0	1/s
	1. measurement data			
	CC sensitivity	κ	0.6	1/s
	Equilibrium headway	h^*	5:5:30	m
	2. measurement data			
	CC sensitivity	κ	0.8	1/s
	Equilibrium headway	h^*	5:5:40	m
	3. measurement data			
	CC sensitivity	κ	0.6	1/s
	Equilibrium headway	h^*	5:5:30	m
2. configuration	CAV control parameter	$\beta_{a,1}$	0.2	1/s
	CAV control parameter	$\beta_{a,2}$	0.3	1/s
	4. measurement data			
	CC sensitivity	κ	1	1/s
	Equilibrium headway	h^*	5:5:30	m

Table 4.3: Parameter list of the measurement, detailing the two measurement configurations and the four CAV settings in total and other technological parameters therein.

The functions of the human and autonomous vehicles are illustrated in Figure 4.2. In panel (a) is the optimal velocity function of CAV in case of the three different $\kappa = 0.6, 0.8, 1$ (s) values, in panel (b) are the fitted range policies corresponding to the HVs and in panel (c) the introduced headway dependent time delay is illustrated.

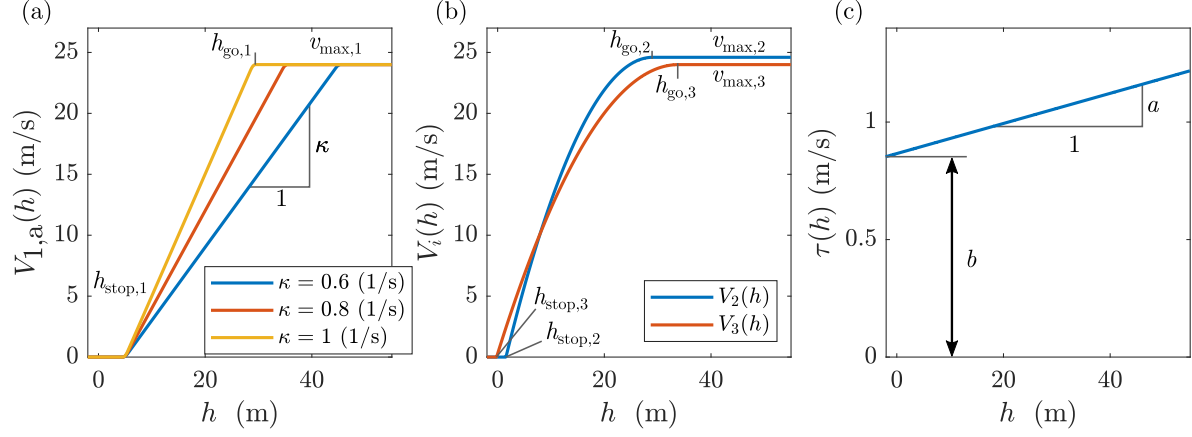


Figure 4.2: The functions used in the model. Whereas, in panel (a) is the optimal velocity function of the CAV, $i = 1$, illustrated for three $\kappa = 0.6, 0.8, 1$ (1/s) parameter values, in panel (b) are the fitted range policies corresponding to the HVs if $i = 2, 3$ and in panel (c) the introduced headway dependent time delay of the human driver is in the form of $\tau(h) = ah + b$.

4.2 Linear stability analysis

In this section, we present the measurement result, and the stability boundaries obtained by the previous model assuming constant time delay and we compare it to the results of the extended, state-dependent time model and investigate the linear stability diagrams as part of a case study. The system of differential equations based on (3.12) can be given for this particular case corresponding to the measurement layout such as

$$\begin{bmatrix} \dot{v}_1(t), \dot{v}_2(t), \dot{v}_3(t), \dot{h}_1(t), \dot{h}_2(t) \end{bmatrix}^T = \begin{bmatrix} \alpha_a \left(V_a(h_1(t - \tau_a)) - v_1(t - \tau_a) \right) + \sum_{j=1}^k \beta_j \left(v_{1+j}(t - \tau_a) - v_1(t - \tau_a) \right) \\ \alpha_h \left(V_2(h_2(t - \tau_2(h_2(t)))) - v_2(t - \tau_2(h_2(t)))) \right) + \beta_h \left(v_3(t - \tau_2(h_2(t))) - v_2(t - \tau_i(h_{2t})) \right) \\ \alpha_h \left(V_3(h_3(t) - v_3(t - \tau_3(h_3(t)))) \right) + \beta_h \left(v_1(t - \tau_3(h_3(t))) - v_3(t - \tau_3(h_3(t))) \right) \\ v_2(t) - v_1(t) \\ v_3(t) - v_2(t) \end{bmatrix}, \quad (4.6)$$

where $k = 1, 2$ and $h_3 = 3h^* - h_1\left(t - \tau_3(h_3(t))\right) - h_2\left(t - \tau_3(h_3(t))\right)$. The associated system in (3.18), linearised around the equilibrium state, has the same linear stability properties as the original SD-DDE system, given as

$$\dot{\tilde{\mathbf{x}}}(t) = \mathbf{L}\tilde{\mathbf{x}}(t) + \mathbf{R}_1\tilde{\mathbf{x}}(t - \tau_a) + \mathbf{R}_2\tilde{\mathbf{x}}(t - \tau_2(h_2^*)) + \mathbf{R}_3\tilde{\mathbf{x}}(t - \tau_3(h_3^*)), \quad (4.7)$$

Where, the matrices are $\mathbf{L}$, $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3 \in \mathbb{R}^{5 \times 5}$, and they are calculated as it was detailed in (3.22) and (3.23) resulting in

$$\mathbf{D}_0 \mathbf{f}^* = \mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}, \quad (4.8)$$

$$\mathbf{D}_1 \mathbf{f}^* = \mathbf{R}_1 = \begin{bmatrix} -\alpha_a - \beta_{a,1} & \beta_{a,1} & 0 & \alpha_a V'_a(h_2^*) & 0 \\ 0 & -\beta_{a,2} & \beta_{a,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.9)$$

$$\mathbf{D}_2 \mathbf{f}^* = \mathbf{R}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_h - \beta_h & \beta_h & 0 & \alpha_h V'_2(h_2^*) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.10)$$

$$\mathbf{D}_3 \mathbf{f}^* = \mathbf{R}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta_h & 0 & -\alpha_h - \beta_h & -\alpha_h V'_3(h_3^*) & -\alpha V'_3(h_3^*) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (4.11)$$

Then, these matrices are substituted into the characteristic equation (3.27), and the linear stability of the system can be calculated in the desired parameter space, by solving this equation. The measurement data is recorded in two configurations when the CAV utilizes information beyond its line of sight and when it does not. It is realized in the model by different $\beta_{a,1}$ and $\beta_{a,2}$ gains for 4 different κ parameter settings in discrete h^* measurement points. In total, the stability of 26 measurement points was classified manually, analysing the velocity profiles. Due to the measurement errors and the non-modelled phenomenons, it is not straightforward and it involves subjective judgement. In Figure 4.3 three typical

cases are characterised. In panel (a) vehicles travel at almost identical speeds throughout the whole period, following each other behaviour. If one car brakes then the other two decelerate too and their velocity is quickly set on the new equilibrium speed without any excessive and undesired oscillation, which indicates linearly stable equilibrium. In panel (b) their speed continuously varies and oscillates, which does not settle down. Since the vehicles cannot follow each other properly indicating unstable equilibrium. Finally, in panel (c) one can see that after the transient acceleration phase, the vehicles follow each other properly maintaining the same speed for about 50 s long. However, excessive braking leads to emerging large amplitude periodic oscillation. Presumably, it indicates bistability, assuming that both stable equilibrium and a stable periodic orbit are present.

Proposed human parameters			
α_a (1/s)	β_a (1/s)	a (s/m)	b (s)
0.1	0.32	0.023637	1.0152

Table 4.4: Proposed parameters for human drivers, containing the control gains and the parameters of the state-dependent time delay

In this contribution, the impact of the human driver parameters on the stability properties of the model was thoroughly investigated. As a result, the newly proposed values are collected in Table 4.4 for the control parameters of human drivers and the state-dependent time delay modelling their reaction times. Our method is detailed in the following. Following the measurement results, in the h^* - κ discrete stability diagram two pairs of points

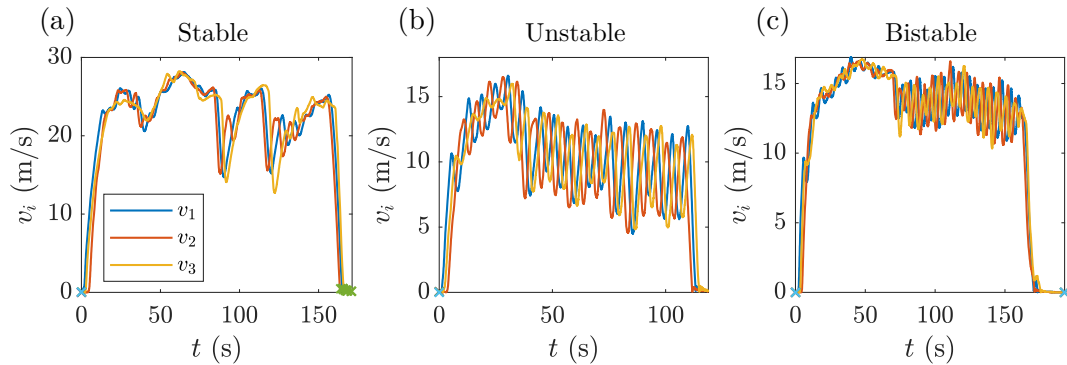


Figure 4.3: Velocity time signals of the measurement, in panel (a), is a stable equilibrium for $\kappa = 0.8$ 1/s and $h^* = 35$ m, in panel (b) an unstable equilibrium for $\kappa = 1$ (1/s) and $h^* = 15$ m and in panel (c) bistable behaviour with stable equilibrium and stable steady-state oscillation for $\kappa = 0.8$ 1/s and $h^* = 15$ m

on either side of the stability boundary were chosen for arbitrary α_h and β_h parameters. Then we search for two constant delay values so that the corresponding stability boundaries lay exactly between the chosen point pairs. Thereafter, by the two known h^* - τ pairs a headway-dependent linear function is fitted resulting in the specific a and b parameters. Thereby, ensuring that the stability of the SD-DDE system agrees with the measurement results, at least in these two points. Thereafter this method was repeated and mapped in high detail for the α_h β_h parameter space. Finally, the parameters shown in Table 4.4 were chosen, so that the obtained stability boundaries best fitted the result considering all measurement points. The resulting state-dependent delay function is shown in Figure 4.4(e).

The results classified by the method described above are shown in Figure 4.4. The green, red and cyan colour mark the stable, unstable and bistable evaluated measurement points, respectively. The stability boundaries, following the measurements, must be between an unstable and stable point or a bistable one (linearly it has stable equilibrium too). However, its exact position is uncertain between the points due to the given $\Delta h^* = 5$ m and $\Delta \kappa = 0.2$ 1/s resolution. The stability boundaries for constant delay are in 4.4 panel (a)-(b). Orange colour denotes the original stability boundaries corresponding to $\alpha_h = 0.12$ 1/s. In this case, in panel (a) only for $\kappa = 1$ 1/s lays the boundary at a correct position. The "c" shaped curve completely disagrees with the trend of the measurement data, because for lower κ values it bends and goes towards the higher h^* values. But in this case, for the second configuration, illustrated in panel (b) the stability boundary fits the evaluated data. In the case of the fine-tuned for the constant delay and $\alpha_h = 0.14$ 1/s, the stability boundaries are represented with yellow colour. In panel (a) the stability in $\kappa = 1$ 1/s section still satisfies the requirements. Moreover, in this case, the shape of the stable region looks more like the expected shape based on the measurements. However, for lower h^* values it still provides qualitatively bad results over a wide section. Despite the better-looking curve, in panel (b) for the second configuration, the calculated stability boundary becomes completely false. Note that these are the results of [67], where the effect of changing every possible parameter in this model was investigated, and still the results were accurate in a few points and completely wrong in the most part. This indicates, that the constant delay model cannot reproduce the real behaviour of the system since the results disagree with the measurements.

The results of the state-dependent delay model with the proposed parameters are shown in panels (c)-(d). In contrast to the above detailed constant delay model, in this case, the stability boundaries fit the measurement data very accurately, satisfying all the

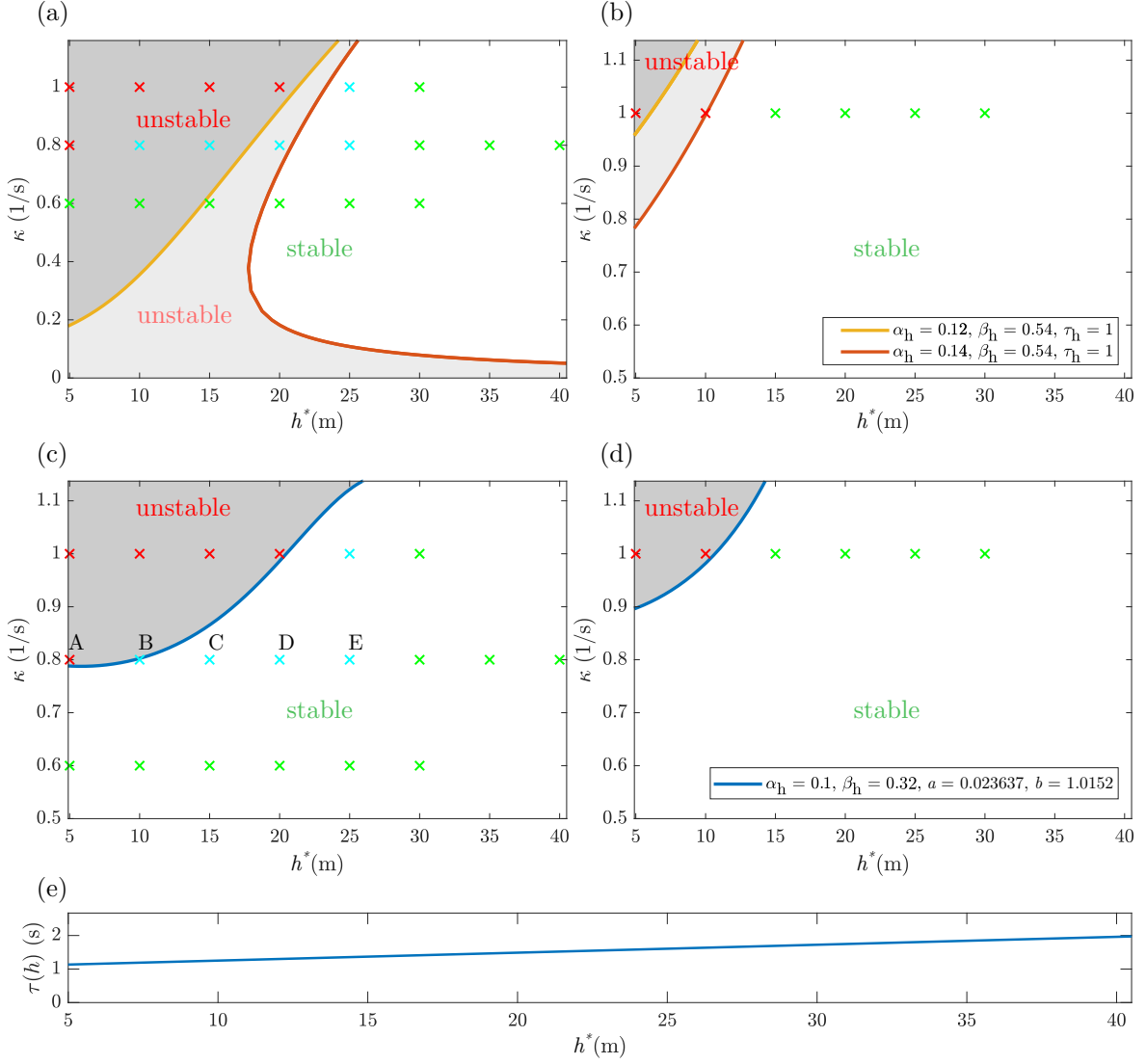


Figure 4.4: The evaluated measurement data and the results of the theoretical model are shown in the linear stability diagrams. In panels (a) and (b) the constant delay system for the 1st and 2nd configuration are placed, respectively, and in panels (c) and (d) the results for the state-dependent delay model for the 1st and 2nd configuration are shown and in panel (e) the corresponding headway dependent human time delay is illustrated.

evaluated data points in both configurations. The unstable region is illustrated with grey and all the unstable measurement points, by red crosses, are within whereas the stable and unstable points are outside of this region. Moreover, the average value of the delay in the examined $h^* = 5 - 40$ m range is $\bar{\tau} = 1.547$ s. This matches very well with the advised mean reaction time recommended in [31, 32] and also with the revealed delay range therein. Moreover, comparing it to the evaluated results of the NGSIM data, it fits within the time delay and headway measurement points shown in 2.6. Even though

the proposed a and b parameters differ from the calculated linear regression model, we can assume it is a qualitatively good fit, considering the moderate R_{adj}^2 value, especially compared to the constant delay model. Furthermore, the obtained human gain parameter $\alpha_h = 0.1$ 1/s is within the uncertainty range of the fitter parameters shown in [23, 67] and $\beta_h = 0.32$ 1/s is also consistent with the values recommended in the literature [21, 68].

In the measured signals, bistable behaviour was detected shown in Figure. 4.3. As the stability diagram was evaluated from the collected data large domain of bistable area was located in the parameter space as it was illustrated in Figure 4.4 marked by cyan colour. It implies nonlinear behaviour in the system, which cannot be examined by the linear, small perturbation theory. Therefore, it is worth performing further nonlinear analysis and comparing the theoretical findings with the results of the measured data.

4.3 Nonlinear stability analysis

As was discussed in Section 4.2, the study of the constant time system concluded that the model cannot describe the linear stability of the real system. Consequently, the periodic orbits rising from the stability boundaries are also located at incorrect places on the parameter space. Furthermore, [67] demonstrated that the shape of the orbits is wrong, neither quantitative nor qualitative similarity was observed to the measured results. Based on the constant delay model, the oscillations should have large amplitudes even at small h^* values and it was unable to produce the detected large bistable regions either, which is contradictory to the results of the measurements.

In this section, the state-dependent delay model is examined by nonlinear analyses to examine whether it is any more realistic. In the nonlinear analysis, the measurement data is processed similarly to the previous linear analysis, including manual evaluation of the collected data. The time signal corresponding to the measurement points is analysed. Perturbations cause transient oscillation after which it either settles down or steady-state oscillation remains, as it is illustrated in Figure 4.3. Then, via sufficient care and engineering considerations, the so-called "peak-to-peak" amplitudes of the oscillations are identified as the differences between the local maxima and minima. Note that, this method might be subjected to large errors due to the small number of measurements, the finite length of the measured signals and the subjectivity that the evaluation process involves.

The analysis of the theoretical model and the continuation of the periodic orbits were

carried out using DDE-BIFTOOL as it was detailed in Section 3.4. The vibration amplitudes determined at each measurement point as seen in [67] and the result of the periodic orbit continuation is shown in Figure 4.5. in the bifurcation diagrams. The stable conditions are indicated with green and the unstable with red. The different driving behaviour of the CAV and human drivers resulted in different vibration amplitudes for each vehicle. Since all the points corresponding to $\kappa = 0.6$ 1/s are globally stable, so further nonlinear tests are not relevant. The vertical columns in the figure show the bifurcation diagrams for a given parameter set for the three vehicles, whereas in each row the results corresponding to one vehicle, in the case of different situations, are placed. Overall, it can be stated that the calculated orbits fit the measured points both trend-wise and qualitatively. We observed the best match for configuration 2 in panels (c) and (f), where the amplitudes were even quantitatively accurate. In the case of configuration 1 for $\kappa = 1$ 1/s the results of all three vehicles show good agreement with the measured data between the stability boundaries in the range of $h^* = 5$ to $h^* = 20$ m, seen in panel (b), (e) and (h). However, the assumed bistable behaviour does not appear for any of the vehicles at $h^* = 25$ m. For configuration 1 and $\kappa = 0.8$ 1/s in panels (a), (d) and (g), the value and the shape of the first two points in $h^* = 5$ and $h^* = 10$ m show qualitatively good trend compared to the measurements. However, the expected large bistable region between $h^* = 10 - 25$ m does not appear.

The stable behaviour can be determined with sufficient certainty from measurements even considering the subjectivity factor. Hence, we can assume that linearly stable points were chosen correctly and the corresponding results in Fig. 4.4 and Fig. 4.5 are indeed valid. However, there is considerable uncertainty in the determination of bistable behaviour and actual amplitudes. Since the SD-DDE model resulted in an accurate linear stability diagram, it is worthwhile to further investigate points A to E marked in Fig. 4.4 panel (a) and 4.5 panel (a). Note, that the continuation of the periodic orbit only provides information about the steady-state oscillation. To investigate the transient oscillations, numerical simulations of the original SD-DDE system (4.6) are performed along the given parameters. The initial conditions are calculated for the $t < 0$ time interval by (4.2) and set to be v^* , h_1^* , h_2^* and h_3^* equilibrium state. To be able to analyse the nonlinear effect, in each point two simulations with extreme initial conditions are performed. In the first case simulating gentle braking, the velocity of the first vehicle is suddenly changed at the first time instant to $v_1(0) = 0.99v^*$, hence only a small perturbation is introduced to the system operating in the equilibrium state. In the second case, harsh braking is simulated by greatly perturbing the equilibrium state, again the velocity of the first vehicle suddenly

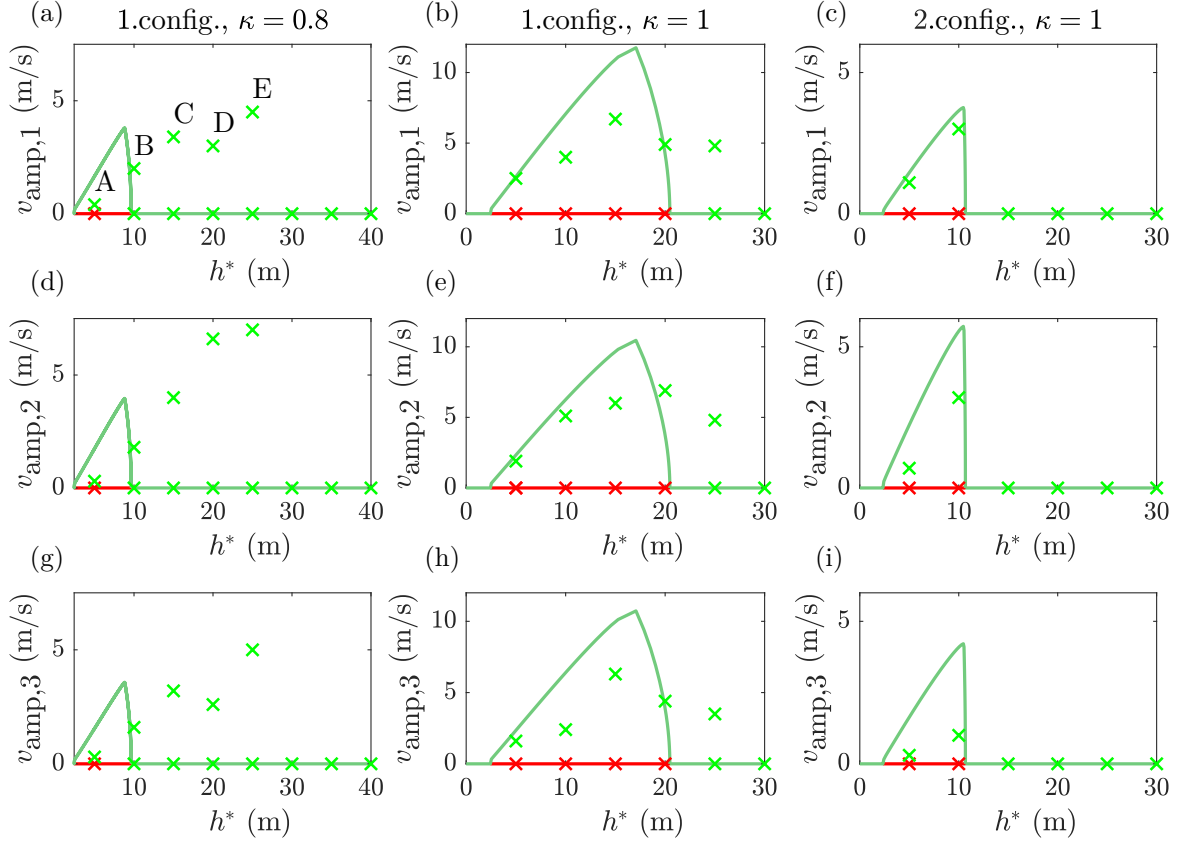


Figure 4.5: Bifurcation diagram comparing the measurements and the periodic orbits, for different configurations, parameters and vehicles, in panel (a)-(c) the first CAV, in panel (d)-(f) the second CHV and in panel (g)-(i) the third CHV is shown, respectively.

drops to $v_1(0) = 0$, and the vehicle comes to a complete stop. The simulations run for 100 s, which is similar in length to the observed large amplitude oscillation.

The measurement signals and the results of the simulations are compared in Fig. 4.6-4.10, at the points identified as bistable. In the case of a nonlinear system, it is difficult to come to a conclusion about the actual behaviour of the system based on the numerical simulation. It is especially hard close to the stability boundaries where the transients may decay slowly due to the low damping as it was discussed in Section 3.4. The small and large perturbations are shown in panels (a) and (b), respectively, and the measured time signals in panel (c). The oscillation amplitudes in the last segment of each simulation from 40 – 100 s were examined to only focus on the steady state solution and detach the transient oscillation from the beginning of each simulation. Thereafter, linear regression was fitted onto the evaluated amplitude of the oscillation to help to identify the long-term behaviour of the system. These are located in panels (d) and (e), under the simulations corresponding to the simulation with small and large perturbations. We

make the following considerations regarding the trend of the regression. If the fitted line shows an increasing trend for low perturbation, the equilibrium is unstable in all cases. Furthermore, if the amplitudes are increasing for large perturbations but not to infinity, then most certainly; whereas, if they are decreasing but tend to a non-zero value, then the existence of a periodic orbit (or limit cycle) is likely. Note that the latter can be identified if the linear regression fitting shows a low correlation and the rate of the decline is slowing. Finally, if for small perturbations the amplitudes decrease, and for large perturbations, the oscillation tends to (large) amplitude limit cycle, the examined point is bistable. This method is not completely robust, however, it provides more insight into the behaviour of the system by analysing the trend of the oscillation, then if only the amplitudes would be compared to a threshold value or other less advanced methods would be used.

In Figure 4.5 panel (a) the numerical simulations are presented at point A, for $\kappa = 0.8$ (1/s) and $h^* = 5$ (m) parameters corresponding to the unstable equilibrium shown in panel (a) of Figure 4.6. Based on panel (a) for small perturbation, the systems seem to be stable. However, if we consider the trend of the amplitude in panel (d), the linear regression reveals that the amplitudes are slowly increasing, which implies that the equilibrium is actually unstable. For large perturbations, the amplitudes are decreasing but based on their trend, most likely the oscillation tends to a nonzero value, to an existing limit cycle. This case corresponds to supercritical bifurcation when a stable periodic orbit is "over" the unstable equilibrium. These are consistent both with the result of the continuation of the periodic orbits and the measured signals in terms of the unstable equilibrium, but a difference is revealed in terms of the realised oscillation amplitudes.

However, one can see that the magnitude of the oscillation amplitude only changes ever slow slightly at a very slow rate during the examined 100 s period. If the simulation for the small perturbation runs for sufficiently long the amplitude will tend to the limit cycle and it will oscillate like the system for large perturbation since their long-term dynamics are identical. Moreover, the amplitude realised in the measurement is between the amplitude of the extreme cases, for simulations with small and large perturbations. Hence, presumably by an adequately perturbed simulation, we could reproduce the same amplitude and similar behaviour just as the measured signals.

In Figure 4.5 panel (a) the numerical simulations are presented at point B for $\kappa = 0.8$ (1/s) és $h^* = 10$ m parameters as seen in Fig. 4.7. Based on the considerations detailed above, taking into account panels (d) and (e), it can be concluded, that the mechanical model indeed does not have a limit cycle and the system is globally stably at the linearly

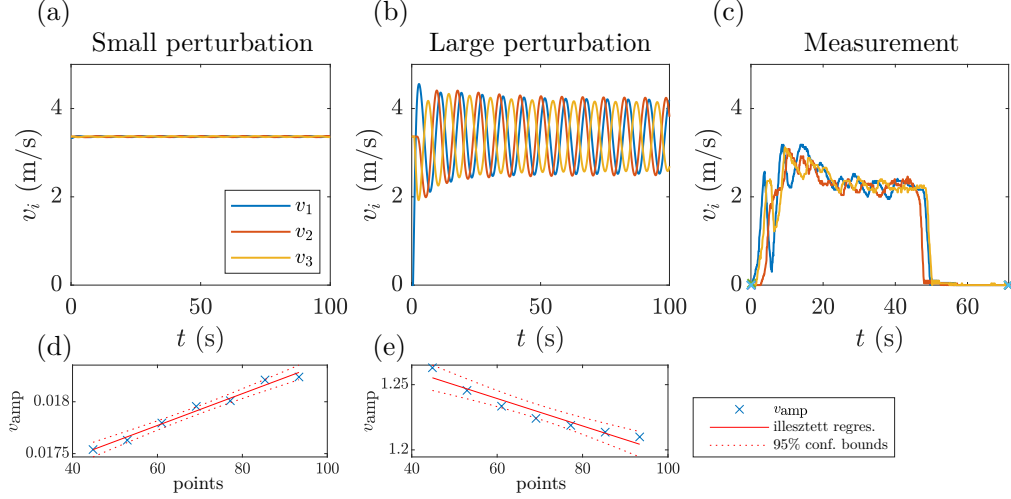


Figure 4.6: The numerical simulations at point A, corresponding to the unstable equilibrium state in Fig. 4.5 panel (a) for $\kappa = 0.8$ 1/s and $h^* = 5$ m parameters. In panels (a) and (d) is the time signals and the trend of the amplitude corresponding to the simulation for small perturbation, in panel (b) and (e) are the time signals and the trend of the amplitude corresponding to the simulation for large perturbation and in panel (c) are the measurement signals.

stable equilibrium. Similarly, globally stable behaviour is examined in points C, D and E, illustrated in Figure 4.8, 4.9 and 4.10, respectively. In all of these cases, for large perturbation, the oscillation decay, and its amplitude at a very slow rate, but steadily tends to zero. However, one can observe that the decreasing amplitude of the simulation for large perturbation and the measured signals are similar in magnitude and they even look alike.

At these points, the simulated and measured signals have qualitatively and even quantitatively similar behaviour. This suggests that the system is in fact globally stable and that the apparent bistable observed in the measurement, is probably due to the low damping of the transient oscillation. After a perturbation introduced by braking, the oscillation decays very slowly and the relatively short measurement did not allow the measured signals to decay and to obtain an accurate picture of the long-term behaviour.

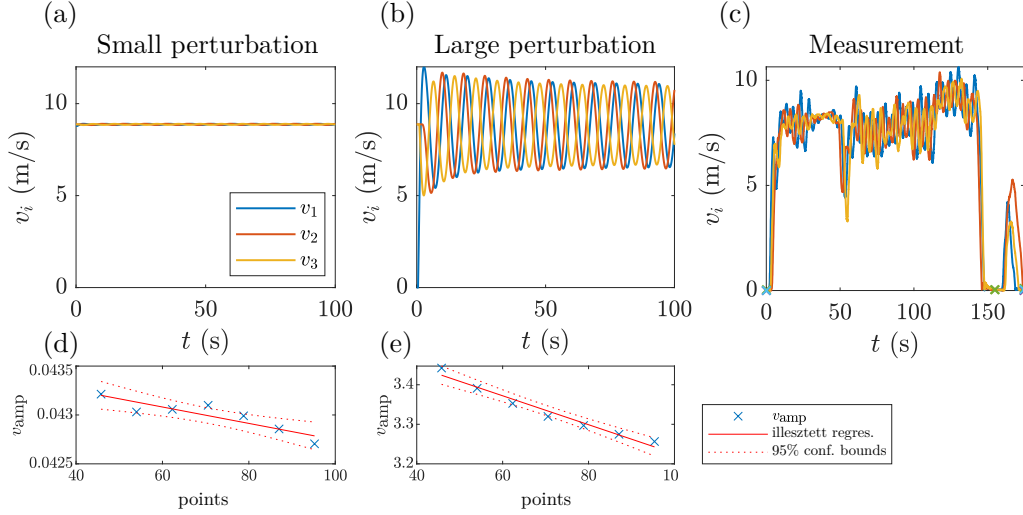


Figure 4.7: The numerical simulations at point B, corresponding to the stable equilibrium state in Fig. 4.5 panel (a) for $\kappa = 0.8$ 1/s and $h^* = 10$ m parameters. In panels (a) and (d) are the time signals and the trend of the amplitude corresponding to the simulation for small perturbation, in panels (b) and (e) are the time signals and the trend of the amplitude corresponding to the simulation for large perturbation and in panel (c) are the measurement signals.

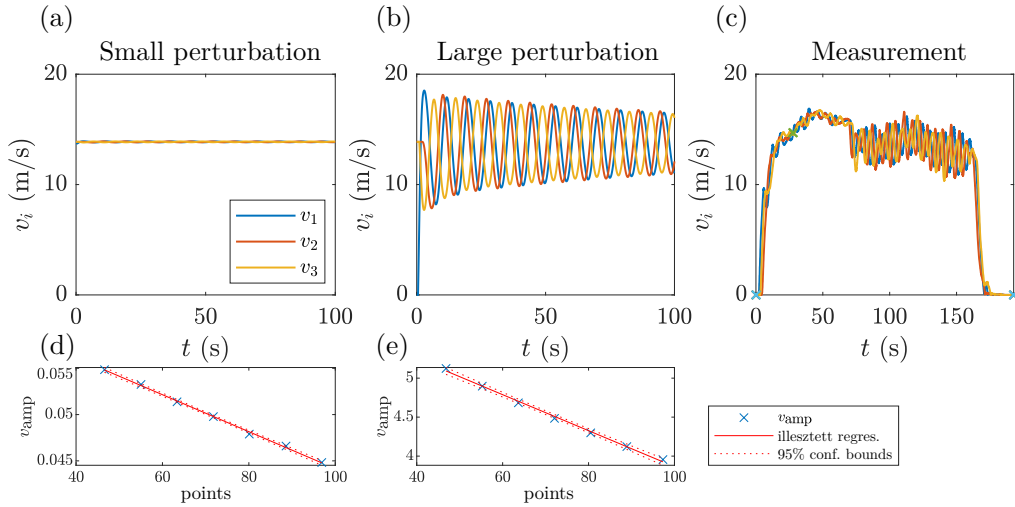


Figure 4.8: The numerical simulations at point C, corresponding to the stable equilibrium state in Fig. 4.5 panel (a) for $\kappa = 0.8$ 1/s and $h^* = 15$ m parameters. In panels (a) and (d) are the time signals and the trend of the amplitude corresponding to the simulation for small perturbation, in panels (b) and (e) are the time signals and the trend of the amplitude corresponding to the simulation for large perturbation and in panel (c) are the measurement signals.

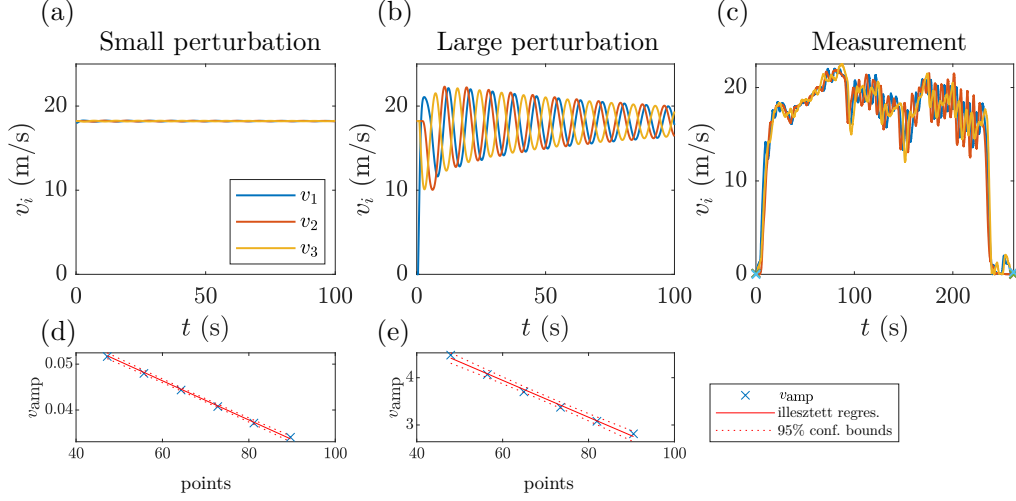


Figure 4.9: The numerical simulations at point D, corresponding to the stable equilibrium state in Fig. 4.5 panel (a) for $\kappa = 0.8$ 1/s and $h^* = 20$ m parameters. In panels (a) and (d) are the time signals and the trend of the amplitude corresponding to the simulation for small perturbation, in panels (b) and (e) are the time signals and the trend of the amplitude corresponding to the simulation for large perturbation and in panel (c) is the measurement signals.

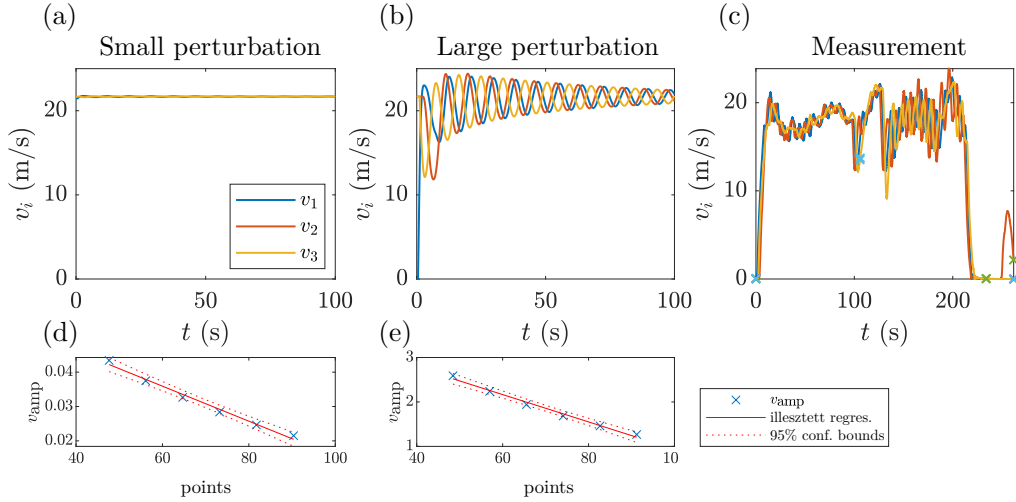


Figure 4.10: The numerical simulations at point E, corresponding to the stable equilibrium state in Fig. 4.5 panel (a) for $\kappa = 0.8$ 1/s and $h^* = 25$ m parameters. In panels (a) and (d) are the time signals and the trend of the amplitude corresponding to the simulation for small perturbation, in panels (b) and (e) are the time signals and the trend of the amplitude corresponding to the simulation for large perturbation and in panel (c) are the measurement signals.

Another possible explanation for the discrepancy between the measurement and the theoretical model could be the effect of so-called "stochastic resonance". In this case near the stability limit, disturbances during the measurement may produce a certain amount of oscillations even in the case of stable equilibrium. This effect often makes it difficult to identify the stability limits in machine tool vibration by measurement only. This is still an area of active research [69, 70], but it might be relevant for the measurement of the car-following model as well.

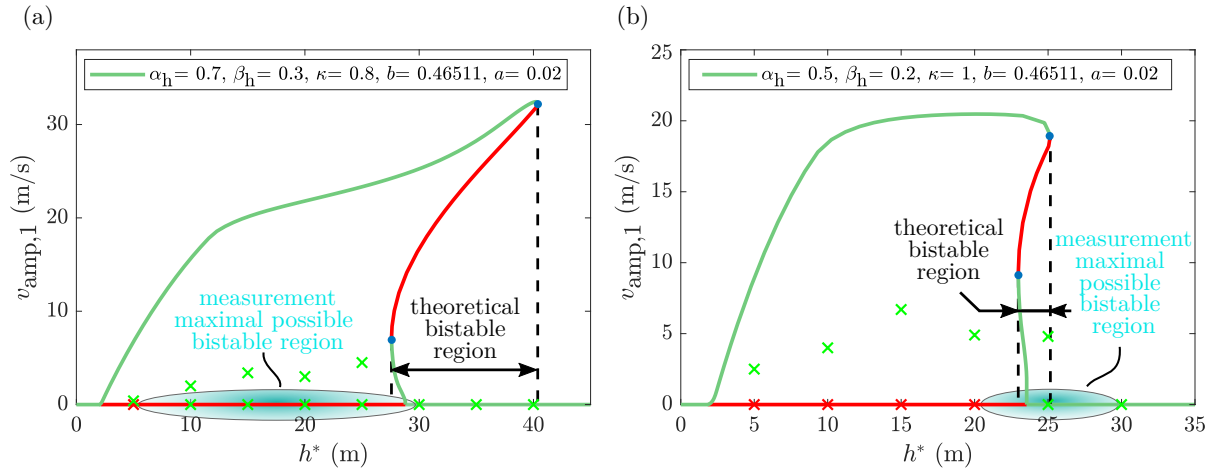


Figure 4.11: Bifurcation diagram for tuned parameters, in panel (a) for $\kappa = 0.8$ 1/s a large size bistable region appears, in panel (b) for $\kappa = 1$ (1/s) a small size bistable region appears following the measurement.

Nevertheless, if we accept that these points were indeed bistable, it is worthwhile to further examine the SD-DDE system to understand whether this extended car following model can produce large bistable areas. Among the others in the model, κ is a technological parameter, which can be freely tuned while setting up the self-driving vehicle. The equilibrium following distance h^* is related to the geometry, calculated by the length of the ring-road and the range policies of human drivers. The behaviour of humans is described using 4 parameters (see. 4.4), which are further tuneable to obtain bifurcation diagrams such as in Figure 4.11. The results show that the SD-DDE system is qualitatively able to produce a significantly large bistable area, unlike the constant delay model. However, it is important to point out, that by the parameter tuning, we do not obtain accurate stability behaviour using either linear or nonlinear analysis. In the case of panel (b), a bistable region is around $h^* = 25$ m which is consistent in nature and size of the bistable with the collected data. However, the amplitude of the periodic orbit is much larger than the measured values. Examining panel (a) a very large bistable area is obtained. However, in this case, the linear stability does not fit the measurement data and the amplitude of

the periodic orbit is almost larger by order than the measured one. Despite these deviations, the state-dependent delay model can produce a comparable bistable region to the measurement. While it is worth pointing out that the constant time-delay model was not able to do this even for any parameter tuning.

However, based on the currently available small number of collected data and the relatively short length of the measurement it is not possible to decide which of the three described effects explains the nonlinear behaviour of the system exclusively and with the highest accuracy.

5 Summary

5.1 Results

In the course of this thesis, we have presented a state-dependent time delay system that more realistically models the varying nature of human reaction time. We have discussed the corresponding literature regarding human reaction time. The NGSIM project, including traffic data collection in multiple locations, was presented. We have discussed the suitable delay recognition methods and presented our process for human reaction time estimation. The disadvantages of the process were discussed, and we have proposed a solution involving manual decision and selection of the results. By evaluating the relation of the accessed time delay with respect to headway supported by large quantity data, we could establish our assumption regarding the headway-dependent human reaction time. Based on these, we have introduced a suitable mathematical model as a system of state-dependent delayed differential equations. Its results were compared with the measurement data and the results of the constant time delay model, utilizing linear and nonlinear stability analysis and numerical simulations. As a result, the linear stability charts and bifurcation diagrams were provided. We have successfully determined the fine-tuned human parameters for the best fit for the measurement. These parameters complied with the values agreed in the literature for modelling human drivers for control gains and the time delay.

5.2 Conclusions, outlook

The received results present a qualitative leap compared to the constant delay model, which was only accurate for a few measurement points and could not fit the data qualitatively. In contrast, the SD-DDE system presented in this thesis perfectly reproduced the linear stability even quantitatively at all points. Also, it could model the nonlinear properties trend-wise sufficiently. All these results justify the state-dependent time delay for more realistically modelling the human reaction time. Therefore, it is worthwhile to investigate the future models of mixed traffic situations considering the behaviour of humans with state-dependent delay.

Several simplifications were made during the thesis, so in the future, it may be worthwhile to analyse the implicit time delay. In addition, we have also neglected the acceler-

ation saturation and the "speed policy" coupling functions. By including these nonlinear functions it may lead to more realistic models.

The optimal velocity functions and their parameters determined from the measured data in [23] revealed high uncertainty. To overcome this, it would be necessary to carry out further road measurements at several measurement points with higher resolution. This would allow us to accomplish a better fit not only for the human control gains but also for the optimal velocity function. Furthermore, the linear stability map and the nonlinear bifurcation diagram calculated with the theoretical model could be fitted more accurately with the help of higher resolution measurement data.

In Section 4.3 three ideas were presented to explain the discrepancy in the model and the collected data regarding the steady-state solutions. First, it was concluded that the bistable behaviour assumed by the measurement data is indeed globally stable and the observed large amplitude oscillations result from the low damping of the system and it would decay after a sufficiently long measurement. With specific measurements, we could explore the possibility of "stochastic resonance". And as a third explanation, we could investigate all aspects of the system focusing on the effect of other non-modelled nonlinearities to accurately obtain the bistable regions and other measurement results.

Feeding the measurement data in a simulation-in-the-loop algorithm would allow us to perform a thorough investigation of the state-dependent model. This would allow the measured position and velocity of the CAV to be prescribed in the SD-DDE model to investigate the correlation between the measured and simulated results of the HVs.

Our long-term plans involves further evaluation of the unused data in NGSIM to better understand the processes involved in human reaction time and to investigate other possible relevant parameters.

Összefoglaló

Kulcsszavak: állapotfüggő időkésés, járműkövető modell, mérés, nemlinearitás

A dolgozat során bemutattuk az emberi váltakozó reakcióidőt realisztikusabban modellező állapotfüggű időkésést. Szót ejtettünk a szakirodalomban fellelhető emberi reakcióidőre vonatkozó ismeretekről. Bemutattuk az NGSIM projekt keretében végzett nagymintás forgalmi méréseket, melyek kapcsán szót ejtettünk a fellelhető időkésés meghatározására alkalmas módszerekről is. A mérési adatok feldolgozása által megkaptuk az időkésés és a követési távolság kapcsolatát, amely megalapozta az állapotfüggő időkésés bevezetését. Ezek alapján felállítottunk egy megfelelő, állapotfüggő időkésést tartalmazó késleltetett differenciálegyenlet-rendszerrel felírt modellt. Összehasonlítottuk a közúton végzett mérési adatokkal és a konstans időkésésű modellel, bemutatva a lineáris stabilitási térképeket és nemlineáris vizsgálatok során kapott bifurkációs diagramokat, végül numerikus szimulációt is végeztünk a modellel.

Sikerült olyan, az embereket leíró paraméter értékeket meghatároznunk, amelyek esetén pontos egyezést mutat az elméleti modell a mérési eredményekkel. Ráadásul így a kontroll paraméterekre és az időkésésre is realisztikus értékeket kaptunk, amelyek megfelelnek az emberi sofőrök viselkedésére vonatkozó előzetes ismereteinknek. Az így kapott eredmények minőségi ugrást jelentenek a korábban konstans időkésést feltételező modellhez képest, amely kvalitatíve sem volt képes illeszkedni az adatokra, mindössze egy-egy mérési pont eredményét tudta visszaadni. Ezzel szemben a dolgozatban bemutatott SD-DDE rendszer a lineáris stabilitást tökéletesen, kvantitatíve is minden pontban visszaadta és a nemlineáris tulajdonságokat is trend szerűen jól modellezte. Mindezek megalapozzák az állapotfüggő időkésés létjogosultságát az emberi időkésés realisztikusabb modellezésének érdekében. Érdemes tehát a jövőben a vegyes forgalmi szituációk modellezését az általunk javasolt, kiterjesztett állapotfüggő időkésésű járműkövető modellel vizsgálni.

A dolgozatban vizsgált SD-DDE rendszer a nemlineáris vizsgálatok során nem minden pontban felelt meg a mérési adatoknak. A dolgozat során több elhanyagolással is éltünk, így a jövőben a pontosabb modellezés érdekében érdemes lehet a rendszert implicit időkéséssel vizsgálni. Ezen kívül szintén elhanyagoltuk a gyorsulás szaturáció és a "speed policy" kapcsolófüggvényeket. Ezen nemlineáris függvények figyelembevételével, vélhetően még realisztikusabb eredményeket lehetne kapni.

A mérési adatokból meghatározott optimális sebesség függvények és paramétereik nagy

bizonytalanságot mutattak. Ennek orvoslása érdekében érdemes lenne a közúti méréseket több mérési pontban, nagyobb felbontás mellett is elvégezni. Így lehetőség lenne nemcsak az emberi kontroll paraméterek, hanem az optimális sebesség függvény jobb illesztésére is. Továbbá az elméleti modellel számított lineáris stabilitási térképet és a nemlineáris bifurkációs diagramot is pontosabban lehetne illeszteni a nagyobb felbontású mérési adatok segítségével.

A 4.3 fejezetben új meglátással éltünk, miszerint a mérési adatok alapján feltételezett bistabil tartomány valójában globálisan stabil. A periodikus pályákhoz hasonlatos nagy amplitúdójú rezgések pedig a rendszer kis csillapításából következnek és kellően hosszú mérés után lecsengenek. A mérési adatokat úgynevezett "simulation-in-the-loop" számításban felhasználva lehetőség nyílna ennek a feltételezésének az alaposabb vizsgálatára. Ennek során a CAV mért pozícióját és sebességét az SD-DDE modellben előírva meg lehetne vizsgálni, hogy mennyire korrelálnak a HV-k mért és az ilyen módon szimulált eredményei.

Hosszútávú terveink közé tartozik az állapotfüggő időkésésű modell nagyobb mintán való vizsgálata, a modell validációja, a lejátszódó folyamatok alaposabb megértése, és az egyéb időkésést befolyásoló tényezők vizsgálatának érdekében.

Reference

- [1] M. Wang, W. Daamen, S. P. Hoogendoorn, B. van Arem, Rolling horizon control framework for driver assistance systems. part i: Mathematical formulation and non-cooperative systems, *Transportation Research Part C: Emerging Technologies* 40 (2014) 271–289. doi:10.1016/j.trc.2013.11.023.
- [2] M. Wang, W. Daamen, S. P. Hoogendoorn, B. van Arem, Rolling horizon control framework for driver assistance systems. part ii: Cooperative sensing and cooperative control, *Transportation Research Part C: Emerging Technologies* 40 (2014) 290–311. doi:10.1016/j.trc.2013.11.024.
- [3] G. Orosz, Connected cruise control: modelling, delay effects, and nonlinear behaviour, *Vehicle System Dynamics* 54 (8) (2016) 1147–1176. doi:10.1080/00423114.2016.1193209.
- [4] T. Ersal, I. Kolmanovsky, N. Masoud, N. Ozay, J. Scruggs, R. Vasudevan, G. Orosz, Connected and automated road vehicles: state of the art and future challenges, *Vehicle System Dynamics* 58 (5) (2020) 672–704. doi:10.1080/00423114.2020.1741652.
- [5] E. Van Nunen, J. Reinders, E. Semsar-Kazerooni, N. Van De Wouw, String stable model predictive cooperative adaptive cruise control for heterogeneous platoons, *IEEE Transactions on Intelligent Vehicles* 4 (2) (2019) 186–196. doi:10.1109/TIV.2019.2904418.
- [6] SAE J3216 (2021), Taxonomy and Definitions for Terms Related to Cooperative Driving Automation for On-Road Motor Vehicles. doi:10.4271/j3216_202107.
- [7] SAE J2735 (2020), V2X Communications Message Set Dictionary. doi:10.4271/J2735_202007.
- [8] H. Jiang, J. Hu, S. An, M. Wang, B. B. Park, Eco approaching at an isolated signalized intersection under partially connected and automated vehicles environment, *Transportation Research Part C: Emerging Technologies* 79 (2017) 290–307. doi:10.1016/j.trc.2017.04.001.
- [9] C. R. He, H. Maurer, G. Orosz, Fuel Consumption Optimization of Heavy-Duty Vehicles With Grade, Wind, and Traffic Information, *Journal of Computational and Nonlinear Dynamics* 11 (6), 061011 (07 2016). doi:10.1115/1.4033895.

- [10] D. Liu, B. Besselink, S. Baldi, W. Yu, H. L. Trentelman, On structural and safety properties of head-to-tail string stability in mixed platoons, *IEEE Transactions on Intelligent Transportation Systems* (2022) 1–13 [doi:10.1109/TITS.2022.3151929](#).
- [11] A. D. Ames, J. W. Grizzle, P. Tabuada, Control barrier function based quadratic programs with application to adaptive cruise control, in: *53rd IEEE Conference on Decision and Control*, 2014, pp. 6271–6278. [doi:10.1109/CDC.2014.7040372](#).
- [12] C. R. He, G. Orosz, Safety guaranteed connected cruise control, in: *21st IEEE International Conference on Intelligent Transportation Systems*, 2018, pp. 549–554. [doi:10.1109/ITSC.2018.8569979](#).
- [13] T. G. Molnar, A. K. Kiss, A. D. Ames, G. Orosz, Safety-critical control with input delay in dynamic environment, *IEEE Transactions on Control Systems Technology* (2021). [doi:10.48550/arXiv.2112.08445](#).
- [14] T. G. Molnar, A. Alan, A. K. Kiss, G. Orosz, Input-to-state safety with input delay in longitudinal vehicle control, *17th IFAC Workshop on Time Delay Systems* (2022). [doi:10.48550/arXiv.2205.14567](#).
- [15] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, Y. Sugiyama, Dynamical model of traffic congestion and numerical simulation, *Physical Review E* 51 (1995) 1035–1042. [doi:10.1103/PhysRevE.51.1035](#).
- [16] H. von Allwörden, I. Gasser, On a general class of solutions for an optimal velocity model on an infinite lane, *Transportmetrica A: Transport Science* 17 (3) (2021) 258–277. [doi:10.1080/23249935.2020.1778813](#).
- [17] M. Bando, K. Hasebe, K. Nakanishi, A. Nakayama, Analysis of optimal velocity model with explicit delay, *Physical Review E* 58 (5) (1998) 5429.
- [18] G. Orosz, R. E. Wilson, G. Stépán, Traffic jams: dynamics and control, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 368 (1928) (2010) 4455–4479. [doi:10.1098/rsta.2010.0205](#).
- [19] J. Wang, Y. Zheng, Q. Xu, J. Wang, K. Li, Controllability analysis and optimal control of mixed traffic flow with human-driven and autonomous vehicles, *IEEE Transactions on Intelligent Transportation Systems* PP (2020) 1–15. [doi:10.1109/TITS.2020.3002965](#).

- [20] T. G. Molnar, M. Hopka, D. Upadhyay, M. Van Nieuwstadt, G. Orosz, Virtual rings on highways: Traffic control by connected automated vehicles (2022). doi:10.48550/ARXIV.2204.11177.
- [21] J. I. Ge, S. S. Avedisov, C. R. He, W. B. Qin, M. Sadeghpour, G. Orosz, Experimental validation of connected automated vehicle design among human-driven vehicles, *Transportation Research Part C: Emerging Technologies* 91 (2018) 335–352. doi:10.1016/j.trc.2018.04.005.
- [22] R. E. Stern, S. Cui, M. L. Delle Monache, R. Bhadani, M. Bunting, M. Churchill, N. Hamilton, H. Pohlmann, F. Wu, B. Piccoli, et al., Dissipation of stop-and-go waves via control of autonomous vehicles: Field experiments, *Transportation Research Part C: Emerging Technologies* 89 (2018) 205–221. doi:10.1016/j.trc.2018.02.005.
- [23] S. S. Avedisov, G. Bansal, A. K. Kiss, G. Orosz, Experimental verification platform for connected vehicle networks, in: *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, IEEE, 2018, pp. 818–823. doi:10.1109/ITSC.2018.8569520.
- [24] V. Giammarino, S. Baldi, P. Frasca, M. L. Delle Monache, Traffic flow on a ring with a single autonomous vehicle: An interconnected stability perspective, *IEEE Transactions on Intelligent Transportation Systems PP* (2020) 1–11. doi:10.1109/TITS.2020.2985680.
- [25] S. Cui, B. Seibold, R. Stern, D. B. Work, Stabilizing traffic flow via a single autonomous vehicle: Possibilities and limitations, in: *2017 IEEE Intelligent Vehicles Symposium (IV)*, 2017, pp. 1336–1341. doi:10.1109/IVS.2017.7995897.
- [26] S. E. Li, Y. Zheng, K. Li, Y. Wu, J. K. Hedrick, F. Gao, H. Zhang, Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities, *IEEE Intelligent Transportation Systems Magazine* 9 (3) (2017) 46–58. doi:10.1109/MITS.2017.2709781.
- [27] W. B. Qin, G. Orosz, Experimental validation of string stability for connected vehicles subject to information delay, *IEEE Transactions on Control Systems Technology* 28 (4) (2020) 1203–1217. doi:10.1109/TCST.2019.2900609.
- [28] J. Ploeg, B. T. Scheepers, E. Van Nunen, N. Van de Wouw, H. Nijmeijer, Design and experimental evaluation of cooperative adaptive cruise control, in: *2011 14th*

- International IEEE Conference on Intelligent Transportation Systems (ITSC), 2011, pp. 260–265. doi:10.1109/ITSC.2011.6082981.
- [29] Y. Zhou, S. Ahn, M. Wang, S. Hoogendoorn, Stabilizing mixed vehicular platoons with connected automated vehicles: An h-infinity approach, *Transportation Research Part B: Methodological* 132 (2020) 152–170, 23rd International Symposium on Transportation and Traffic Theory (ISTTT 23). doi:10.1016/j.trb.2019.06.005.
- [30] K. Martinovich, Követési távolságtól függő reakcióidő hatása a járműkövető modellre, Szakdolgozat, Budapesti Műszaki és Gazdaságtudományi Egyetem (2021).
- [31] M. Green, "how long does it take to stop?" methodological analysis of driver perception-brake times, *Transportation Human Factors* 2 (2000) 195–216. doi:10.1207/STHF0203_1.
- [32] G. Melegh, Gépjárműszakértés, 2004.
- [33] R. Sipahi, F. M. Atay, S.-I. Niculescu, Stability of traffic flow behavior with distributed delays modeling the memory effects of the drivers, *SIAM Journal on Applied Mathematics* 68 (3) (2008) 738–759. doi:10.1137/060673813.
- [34] J. I. Ge, G. Orosz, Optimal control of connected vehicle systems with communication delay and driver reaction time, *IEEE Transactions on Intelligent Transportation Systems* 18 (8) (2017) 2056–2070. doi:10.1109/TITS.2016.2633164.
- [35] S. Beregi, S. S. Avedisov, C. R. He, D. Takacs, G. Orosz, Connectivity-based delay-tolerant control of automated vehicles: theory and experiments, *IEEE Transactions on Intelligent Vehicles* (2021) 1–1doi:10.1109/TIV.2021.3131957.
- [36] F. Hartung, J. Turi, On differentiability of solutions with respect to parameters in state-dependent delay equations, *journal of differential equations* 135 (2) (1997) 192–237.
- [37] F. Hartung, T. Krisztin, H.-O. Walther, J. Wu, Functional differential equations with state-dependent delays: theory and applications, *Handbook of differential equations: ordinary differential equations* 3 (2006) 435–545.
- [38] F. Hartung, On differentiability of solutions with respect to parameters in neutral differential equations with state-dependent delays, *Annali di Matematica Pura ed Applicata* 192 (1) (2011) 17–47. doi:10.1007/s10231-011-0210-5.

- [39] T. Insperger, G. Stépán, J. Turi, State-dependent delay in regenerative turning processes, *Nonlinear Dynamics* 47 (1-3) (2006) 275–283. doi:10.1007/s11071-006-9068-2.
- [40] T. Insperger, D. A. Barton, G. Stépán, Criticality of hopf bifurcation in state-dependent delay model of turning processes, *International Journal of Non-Linear Mechanics* 43 (2) (2008) 140–149.
- [41] D. Bachrathy, G. Stépán, J. Turi, State Dependent Regenerative Effect in Milling Processes, *Journal of Computational and Nonlinear Dynamics* 6 (4), 041002 (03 2011). doi:10.1115/1.4003624.
URL <https://doi.org/10.1115/1.4003624>
- [42] FHWA, Ngsim - next-generation simulation (Nov. 2020).
URL <https://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm>
- [43] V. G. Kovvali, V. Alexiadis, L. Zhang PE, Video-based vehicle trajectory data collection, Tech. rep. (2007).
- [44] V. Punzo, M. T. Borzacchiello, B. Ciuffo, On the assessment of vehicle trajectory data accuracy and application to the next generation simulation (ngsim) program data, *Transportation Research Part C: Emerging Technologies* 19 (6) (2011) 1243–1262. doi:<https://doi.org/10.1016/j.trc.2010.12.007>.
URL <https://www.sciencedirect.com/science/article/pii/S0968090X10001701>
- [45] B. Coifman, L. Li, A critical evaluation of the next generation simulation (ngsim) vehicle trajectory dataset, *Transportation Research Part B: Methodological* 105 (2017) 362–377. doi:<https://doi.org/10.1016/j.trb.2017.09.018>.
URL <https://www.sciencedirect.com/science/article/pii/S0191261517300838>
- [46] X. Ma, I. Andréasson, Estimation of driver reaction time from car-following data: Application in evaluation of general motor-type model, *Transportation research record* 1965 (1) (2006) 130–141.
- [47] K. Martinovich, A. K. Kiss, Nonlinear effects of saturation in the car-following model, *Nonlinear Dynamics* (Oct. 2022). doi:10.1007/s11071-022-07951-0.
- [48] G. Stépán, *Retarded dynamical systems: stability and characteristic functions*, Longman Scientific & Technical, 1989.

- [49] G. Stépán, Retarded Dynamical Systems: Stability and Characteristic Functions, Longman, UK, 1989.
- [50] T. Insperger, G. Stépán, Semi-discretization method for delayed systems, *International Journal for numerical methods in engineering* 55 (5) (2002) 503–518.
- [51] D. Bachrathy, G. Stépán, Bisection method in higher dimensions and the efficiency number, *Periodica Polytechnica Mechanical Engineering* 56 (2) (2012) 81–86. doi:10.3311/pp.me.2012-2.01.
- [52] K. Engelborghs, T. Luzyanina, D. Roose, Numerical bifurcation analysis of delay differential equations using DDE-biftool, *ACM Transactions on Mathematical Software* 28 (1) (2002) 1–21. doi:10.1145/513001.513002.
- [53] J. Sieber, K. Engelborghs, T. Luzyanina, G. Samaey, D. Roose, DDE-Biftool manual - bifurcation analysis of delay differential equations, Tech. rep. (2014). doi:10.48550/arXiv.1406.7144.
- [54] J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer, New York, 1983. doi:10.1115/1.3167759.
- [55] Y. A. Kuznetsov, Elements of applied bifurcation theory, Vol. 112, Springer Science & Business Media, 2013.
- [56] S. A. Campbell, Calculating centre manifolds for delay differential equations using maple, in: *Delay differential equations*, Springer, 2009, pp. 1–24. doi:10.1007/978-0-387-85595-0_8.
- [57] T. Kalmár-Nagy, G. Stépán, F. C. Moon, Subcritical hopf bifurcation in the delay equation model for machine tool vibrations, *Nonlinear Dynamics* 26 (2) (2001) 121–142. doi:10.1023/A:1012990608060.
- [58] G. Orosz, G. Stépán, Subcritical hopf bifurcations in a car-following model with reaction-time delay, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 462 (2073) (2006) 2643–2670. doi:10.1098/rspa.2006.1660.
- [59] Z. Dombovari, R. E. Wilson, G. Stepan, Estimates of the bistable region in metal cutting, *P Roy Soc A Math Phy* 464 (2008) 3255–3271. doi:10.1098/rspa.2008.0156.
- [60] T. G. Molnar, Z. Dombovari, T. Insperger, G. Stepan, On the analysis of the double hopf bifurcation in machining processes via centre manifold reduction, *Proceedings*

- of the Royal Society A: Mathematical, Physical and Engineering Sciences 473 (2207) (2017) 20170502. doi:10.1098/rspa.2017.0502.
- [61] B. Szakasz, G. Stépán, Delay-induced bifurcations in collocated position control of an elastic arm, *Nonlinear Dynamics* 107 (2022). doi:10.1007/s11071-021-06812-6.
 - [62] G. Haller, S. Ponsioen, Nonlinear normal modes and spectral submanifolds: existence, uniqueness and use in model reduction, *Nonlinear dynamics* 86 (3) (2016) 1493–1534. doi:10.1007/s11071-016-2974-z.
 - [63] D. Melchor-Aguilar, S.-I. Niculescu, Estimates of the attraction region for a class of nonlinear time-delay systems, *IMA Journal of Mathematical Control and Information* 24 (4) (2007) 523–550.
 - [64] R. Villafuerte, S. Mondié, On improving estimate of the region of attraction of a class of nonlinear time delay system, *IFAC Proceedings Volumes* 40 (23) (2007) 227–232.
 - [65] T. H. Scholl, H. Veit, G. Lutz, On norm-based estimations for domains of attraction in nonlinear time-delay systems, *Nonlinear Dynamics* 100 (3) (2020) 2027–2045.
 - [66] A. K. Kiss, T. G. Molnar, D. Bachrathy, A. D. Ames, G. Orosz, Certifying safety for nonlinear time delay systems via safety functionals: A discretization based approach, in: *American Control Conference*, 2021, pp. 1055–1060. doi:10.23919/ACC50511.2021.9482688.
 - [67] M. Sirák, *Forgalomdinamikai modell globális vizsgálata és validációja mérés alapján*, TDK dolgozat, Budapesti Műszaki és Gazdaságtudományi Egyetem (2019).
 - [68] A. K. Kiss, S. S. Avedisov, D. Bachrathy, G. Orosz, On the global dynamics of connected vehicle systems, *Nonlinear Dynamics* 96 (3) (2019) 1865–1877. doi:10.1007/s11071-019-04889-8.
 - [69] E. Buckwar, R. Kuske, B. L’esperance, T. Soo, Noise-sensitivity in machine tool vibrations, *International Journal of Bifurcation and Chaos* 16 (08) (2006) 2407–2416.
 - [70] D. Bachrathy, H. T. Sykora, D. Hajdu, B. Beri, G. Stepan, Why is it hard to identify the onset of chatter? a stochastic resonance perspective, *CIRP Annals* 70 (1) (2021) 329–332.