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BSc Thesis



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Vortex detection in fluid flow around solid bodies

BSc Thesis

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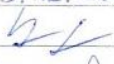
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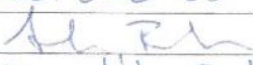
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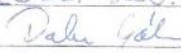
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Abstract

Flow around different objects has been studied immensely due to the number of practical applications. The main aim of the current thesis is vortex detection in fluid flow around a circular cylinder and a square cylinder, which also means the optimization of the vortex detection parameters. The thesis aims to make an existing detection method more objective by giving suggestions for the values of the vortex detection parameters. The fluid flow itself is a result of simulation, which was performed with the help of ANSYS Workbench 2022 R2. The simulations were run for different Reynolds numbers and different angles of attack. The detection was carried out in MATLAB R2020b. With the obtained simulation results, the optimization of the vortex detection parameters has been performed. Broadly speaking, these parameters determine the minimum size and required shape of a vortex. These parameters and their importance are thoroughly introduced, as well as the implementation of the parameter optimization. The effect of the parameters can be examined through numerous figures, which demonstrate the optimization procedure. As a result of my thesis, the optimal intervals, and the section of these intervals have been determined for each parameter, and the results are also presented and discussed in the current paper.

Keywords: *bluff bodies, circular cylinder, square cylinder, Reynolds number, vortex, vortex detection, parameter optimization*

Absztrakt

A különféle testek körüli áramlást a gyakorlati alkalmazások magas számából kifolyólag rendkívül alaposan tanulmányozták az áramlástan története során. Jelen értekezés fő célja örvénydetektálás egy körhenger és egy négyzet alapú henger körüli áramlásban, mely magába foglalja az örvénydetektálási paraméterek optimalizálását is. A dolgozat célja, hogy egy meglévő örvénydetektálási módszert tegyen objektívebbé azáltal, hogy javaslatokat ad az örvénydetektálási paraméterek optimális értékeire. Maga az áramlás, szimuláció eredménye, melyet az ANSYS Workbench 2022 R2 segítségével végeztem el. A szimulációk különböző Reynolds számokra és különböző megfúvási szögekre lettek lefuttatva. A detektálás MATLAB R2020b segítségével történt. A szimulációs eredményeket felhasználva elvégeztem az örvénydetektálási paraméterek optimalizálását. Általánosságban elmondható, hogy ezek a paraméterek határozzák meg az örvények minimális méretét és elvárt alakját. Ezen paraméterek és fontosságuk, valamint a paraméter-optimalizálás megvalósítása alaposan bemutatásra kerülnek. A különböző paraméterek hatása számos ábrán keresztül vizsgálható, melyek az optimalizálás folyamatát hivatottak reprezentálni. Dolgozatom eredményeként meghatározásra kerültek az egyes paraméterekre vonatkozó optimális intervallumok, illetve ezen intervallumok metszete is, az eredmények jelen értekezésben bemutatásra és tárgyalásra kerülnek.

Kulcsszavak: *tompá testek, körhenger, négyzet alapú henger, Reynolds szám, örvény, örvénydetektálás, paraméteroptimalizálás*

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Nomenclature

Latin letters	Name	Unit
a	Side length	m
A	Area	m ²
$B(t)$	Boundary of a rotationally coherent Lagrangian vortex	
c_D	Drag coefficient	1
c_L	Lift coefficient	1
C	Courant number	1
$C(t)$	Center of a rotationally coherent Lagrangian vortex	
d	Characteristic length	m
dV	Area element in 2D, volume element in 3D	m ² , m ³
D	Diameter	m
$D(t)$	Rotationally coherent Lagrangian vortex	
f	Vortex shedding frequency	Hz
f_{local}	Frequency of the up-and-down motion of the vortex street	Hz
F_D	Drag force	N
F_L	Lift force	N
h	Projected frontal height	m
$L(t)$	Lagrangian coherent structure	
N	Number of cells	1
p	Pressure	Pa
p_∞	Static pressure	Pa
p_m	Measured order of convergence	1
p_t	Total pressure	Pa
r	Refinement ratio	1
Re	Reynolds number	1
s	Variable of integration	
Sr	Strouhal number	1
t	Final time instant of the integration	s
t_0	Starting time instant of the integration	s
Δt	Time step size	s
T	Time period	s
U	Potential energy	J
$U(t)$	Time-dependent spatial domain	
$\mathbf{v}(\mathbf{x}, t)$	Unsteady velocity field	m·s ⁻¹
V	Velocity	m·s ⁻¹

$vol(U(t))$	Area in 2D, volume in 3D	m^2, m^3
x_i	Coordinate of a vortex center in the x direction	m
$\mathbf{x}(t, \mathbf{x}_0)$	Instantaneous particle positions	m
$\mathbf{x}_0$	Initial particle positions	m
Δx	Smallest cell size in the domain	m
Δx_{ij}	Difference between the centers of two neighboring vortices in the x direction	m
y_i	Coordinate of a vortex center in the y direction	m
Δy_{ij}	Difference between the centers of two neighboring vortices in the y direction	m

Greek letters	Name	Unit
ε_h	Discretization error	1
θ	Angle of incidence	°
μ	Dynamic viscosity	$kg \cdot m^{-1} \cdot s^{-1}$
ϱ	Density	$kg \cdot m^{-3}$
ϕ_h	Fine mesh result	SI
ϕ_{2h}	Intermediate mesh result	SI
ϕ_{4h}	Coarse mesh result	SI
ϕ_0	Exact value extrapolated to the infinitely fine mesh	SI
$\omega(\mathbf{x}, t)$	Vorticity	s^{-1}
$\bar{\omega}(t)$	Instantaneous spatial mean of the vorticity	s^{-1}

Abbreviations

<i>DeficiencyThresh</i>	Convexity deficiency threshold
<i>DSV</i>	Dual secondary vortices
<i>FFT</i>	Fast Fourier Transform
<i>LAVD</i>	Lagrangian-averaged vorticity deviation
<i>LCS</i>	Lagrangian coherent structure
<i>MinLength</i>	Minimal arc-length threshold
<i>MS</i>	Main separation
<i>Nct</i>	Number of contour levels
<i>RMS</i>	Root mean square
<i>S</i>	Steady
<i>SSV</i>	Single secondary vortex
<i>VM</i>	Vortex merging

1. Introduction

1.1 Aim of the work

Flow around different objects has been studied immensely due to the number of practical applications. These objects are usually divided into two groups, streamlined bodies (e.g., airfoil) and bluff bodies (e.g., cylinder, sphere). For streamlined bodies, friction drag is the dominant source of medium resistance. For a bluff body, the dominant source of drag is pressure drag. Airfoils, which are streamlined bodies, have such a shape that it increases the lift, and at the same time it decreases the drag on the wings. Whereas, cylinders mostly experience boundary layer separation, and strong flow oscillations.

In the case of bluff bodies, for a limited interval of the Reynolds number, symmetric, periodic vortex shedding can be observed, this is the so-called von Karman vortex street ([1], pp. 514-515). The periodic nature of these flows sometimes causes unwanted structural vibrations. This can be especially dangerous if the vortex shedding frequency matches one of the natural frequencies of the bodies ([1], pp. 515). Thus, it is vital that engineers take vortex shedding into account while designing skyscrapers, chimneys etc.

The main aim of the current thesis is vortex detection in fluid flow around bluff bodies (i.e., a circular cylinder and a square cylinder with different angles of incidence) which also means the optimization of the vortex detection parameters. The optimization is beneficial because the publication [2], on which the vortex detection is based, does not give suggestions for the values of these parameters, which increases the subjectivity of the method. The current thesis aims to make the vortex detection method more objective by giving suggestions for the values of the vortex detection parameters.

Firstly, a two-dimensional model will be constructed for all of the geometries, and afterwards, simulations will be run for different Reynolds numbers. The validation of the results will happen by comparing basic flow characteristics with figures, and values taken from the literature. Afterwards, with the obtained simulation results, the optimization of the vortex detection parameters is going to be performed.

The current study can be helpful in numerous ways. As a basic idea, through vortex detection one can observe the structure of the flow. Additionally, vortex detection through measurements, is a really time-consuming procedure. With the help of this numerical method, the detection time can be decreased substantially. In addition, the detection also makes it possible to follow the path of the shed vortices. For instance, if two buildings are close to each other, the effect of these buildings on one another can be examined through the vortices. As it was mentioned in the second paragraph, it is vital to take vortex shedding into account while designing different structures. Through vortex detection, it can be seen where the vortices are evolving, thus, from this data the necessary geometrical modifications can be deduced. Lastly, it is known that the von Karman vortex street causes the phenomenon of "humming/singing" telephone wires ([1], pp. 515), meaning, it causes noise. Therefore, the vortex detection method can be useful in the area of noise reduction as well.

1.2 Overview

The current thesis consists of 6 chapters, including the introduction part (*Chapter 1*). In *Chapter 1*, the main aim and the outline of the thesis are presented.

In *Chapter 2*, some basic definitions and principles are discussed with regard to flow physics. This chapter is based on [1, 3-5]. Among other things, the chapter discusses the definition of the Reynolds number, the characteristics of fluid flow around the examined geometries, and it also touches upon the topic of aerodynamic forces, and the Strouhal number.

Afterwards, in *Chapter 3* the so-called Lagrangian-averaged vorticity deviation is introduced. This chapter is based on George Haller et al., *Defining coherent vortices objectively from the vorticity*, 2016 [2]. *Chapter 3* mainly focuses on the definition of a coherent vortex, and the basic principles of vortex detection.

Chapter 4 centers around the circular cylinder geometry. The flow around the cylinder is a result of simulation which was performed with the help of ANSYS Workbench 2022 R2. The two-dimensional geometrical model, the meshing of the flow field, and the simulation settings are introduced. The simulations were run for different Reynolds numbers (12,60,120,1200). For the validation of the results, the time-averaged values of the lift and drag coefficients, along with the Strouhal number were compared with the theoretical values. A mesh independency study was also performed to ensure the accuracy of the results. Lastly, the vortex detection along with the parameter optimization is presented. The detection was carried out in MATLAB R2020b. At first, I performed the optimization process for the results obtained from the simulation with a Reynolds number of 120. This is due to the fact that for a Reynolds number of 120, the vortices are detached periodically, thus forming a von Karman vortex street ([3], pp. 22). For a von Karman vortex street, there exists a theoretical value in the literature for the distance between the neighboring vortices [6]. Thus, by calculating these values, the quality of the detection can be examined. The final step in *Chapter 4* was to check the optimized parameters for the other simulation results as well.

Chapter 5 focuses on the square cylinder arrangement. In this case as well, the simulation of the fluid flow around the square cylinder was performed with the help of ANSYS Workbench 2022 R2. The two-dimensional geometrical model, the meshing of the flow field, and the simulation settings are introduced. The simulations were run at different Reynolds numbers with various angles of incidence ($\theta = 0^\circ, Re = 50, 120, 185$; $\theta = 5^\circ, Re = 120$; $\theta = 45^\circ, Re = 120$). The discussion of the simulation results is similar to that of the circular cylinder. Afterwards, a mesh independency study ensured the accuracy of the results. The vortex detection along with the parameter optimization is presented for all of the cases by keeping the results obtained from the case of the circular cylinder in mind.

Lastly, a brief summary of the results and the conclusion sums up the current paper in *Chapter 6*. An *Appendix* closes the thesis, in which I included all of the figures that were mentioned in the paper, but due to the lack of space could not have been included.

2. Flow physics and basic definitions

The following brief theoretical summary is based on the following books: *Az áramlástan alapjai* by Lajos Tamás, 2019 [1], *Boundary-Layer Theory* by Hermann Schlichting and Klaus Gersten, 2016 [3], *Áramlások numerikus modellezése* by Dr. Kristóf Gergely, 2019 [4], and on the following publication: Dong-Hyeog Yoon et al., *Flow past a square cylinder with an angle of incidence*, 2010 [5]. The upcoming definitions and basic principles are vital to understand and interpret the simulation and its results.

2.1 Courant number

The Courant number is used to determine the appropriate time step size for transient simulations. It is a dimensionless value which basically represents the time a particle stays in one cell of a mesh. Its ideal value is below 1. For the calculation of the Courant number the following formula can be used [4]:

$$C = \frac{V \cdot \Delta t}{\Delta x} \quad (1)$$

Where:

- V : velocity of the fluid $\left[\frac{\text{m}}{\text{s}}\right]$
- Δt : time step size [s]
- Δx : the smallest cell size in the domain in the direction of V [m]

During the simulations I always assumed the maximum Courant number to be 1.

2.2 Reynolds number

The Reynolds number by definition is a dimensionless value which is the ratio of the inertial forces and the viscous forces:

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} \quad (2)$$

By formula, it can be determined in the following way ([1], pp. 373):

$$Re = \frac{V \cdot \varrho \cdot d}{\mu} \quad (3)$$

Where:

- ϱ : density $\left[\frac{\text{kg}}{\text{m}^3}\right]$
- d : characteristic length [m]
- μ : dynamic viscosity $\left[\frac{\text{kg}}{\text{m} \cdot \text{s}}\right]$

For a circular cylinder the characteristic length is the diameter of the cylinder (D). In the case of a square cylinder, the characteristic length can be calculated in the following way [5] (this is the projected frontal height):

$$h = a(\sin(\theta) + \cos(\theta)) \quad (4)$$

Where:

- a : side length [m]
- θ : angle of incidence [$^\circ$]

In Figure 1 the two geometries can be seen.

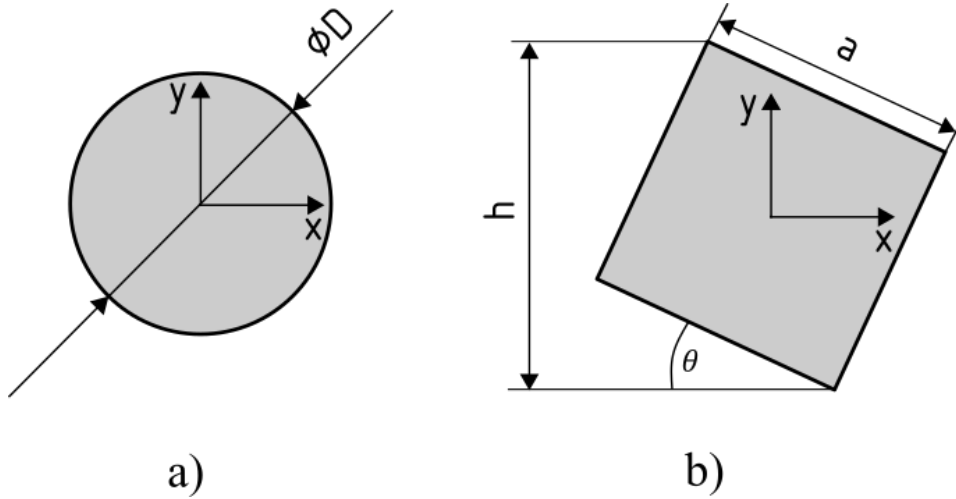


Figure 1. Schematic arrangements a) circular cylinder b) square cylinder.

Based on the Reynolds number, it is possible to predict flow patterns in different cases of fluid flow. At low Reynolds numbers, layered, laminar flow is observed, while at high Reynolds numbers, disturbed, turbulent flow is typical. In the case of laminar flow, viscous forces dominate, while in the case of turbulent flow, inertial forces dominate.

2.3 Flow patterns in the case of circular and square cylinders

According to [5] the main difference between circular and square cylinders is the fact that for circular cylinders the separation points are time-dependent, and for the latter, they are fixed at some edges. The separation point is the point on the body where the boundary layer detaches from the surface into a wake. In the case of a square cylinder, with the angle of incidence these separation points can be shifted, and this can cause a drastic change in flow topology. According to [5], the shift of separation points can alter flow characteristics such as Strouhal number of vortex shedding, drag, and lift forces on the cylinder, depending upon the angle of incidence (θ). The critical Reynolds number at which vortex shedding occurs is also greatly affected. In Figure 2 ([5], pp. 4) one can see how the critical Reynolds number changes with θ .

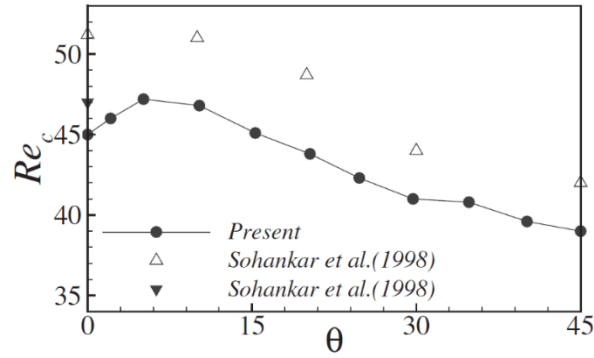


Figure 2. Change of the critical Reynolds number with the angle of incidence in the case of a square cylinder ([5], pp. 4).

In the next few paragraphs, I am going to talk about the basic flow patterns for the two geometries. In Table 1 flow regimes for a circular cylinder can be seen ([3], pp. 22).

Table 1. Flow characteristics at different Reynolds numbers in the case of a circular cylinder. Reproduced from ([3], pp. 22).

Reynolds number regime	Flow regime	Flow form	Flow characteristic	Strouhal number Sr	Drag coefficient c_D
$Re \rightarrow 0$	Creeping flow		Steady, no wake	—	see Fig. 1.12
$3 - 4 < Re < 30 - 40$	Vortex pairs in wake		Steady, symmetric separation	—	$1.59 < c_D < 4.52$ ($Re = 30$) ($Re = 40$)
$\frac{30}{40} < Re < \frac{80}{90}$	Onset of Karman vortex street		Laminar, unstable wake	—	$1.17 < c_D < 1.59$ ($Re = 100$) ($Re = 30$)
$\frac{80}{90} < Re < \frac{150}{300}$	Pure Karman vortex street		Karman vortex street	$0.14 < Sr < 0.21$	
$\frac{150}{300} < Re < \frac{10^5}{1.3 \cdot 10^5}$	Subcritical regime		Laminar, with vortex street instabilities	$Sr = 0.21$	$c_D \approx 1.2$

In the case of a circular cylinder the shape of the streamlines is symmetric around the cylinder's upper and lower side. For very small Reynolds numbers this is true around the cylinder's front and back as well. It is important to note that as the Reynolds number increases this front to back symmetry disappears. For smaller Reynolds numbers, steady, symmetric separation can be observed. As the Reynolds number increases, for a limited interval, symmetric, periodic vortex shedding can be observed, this is the so-called von Karman vortex street. With the further increase of the Reynolds number, the flow becomes more and more unstable, and the symmetric vortex shedding does not apply anymore.

In the case of a square cylinder the flow is symmetric if and only if: $\theta = 0, 45$ [°]. While staying in the laminar flow regime, in [5] they examined the flow pattern for various Reynolds numbers and angles of incidence. According to [5] the following distinct flow patterns can be identified for a square cylinder: main separation (MS), vortex merging (VM),

and steady (S). Moreover, MS pattern has two subpatterns, which are single secondary vortex (SSV), and dual secondary vortices (DSVs). In Figure 3 the corresponding domains for these flow patterns can be seen ([5], pp. 4).

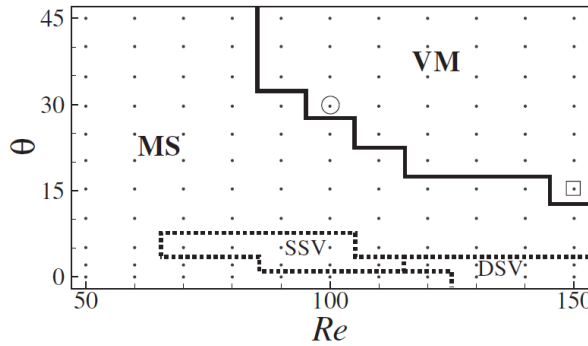


Figure 3. Flow patterns past a square cylinder with an angle of incidence ([5], pp. 4).

To summarize very briefly, in the MS pattern, main vortices of opposite sign roll up in an alternating matter, and the separation point at a downstream edge approaches an upstream edge as θ increases. In the case of SSV, vortex shedding in the near-wake region is quite similar to that of the simple MS. However, flow separates at an upstream edge, afterwards it periodically reattaches, and this causes a secondary vortex. This secondary vortex only appears on the upper side of the cylinder. The difference between SSV and DSV is that for the latter, flow separates on both upstream edges. As for vortex merging, this pattern means the generation and merging of two small vortices. Lastly, steady flow means that there is no vortex shedding, meaning, the Reynolds number (for a given angle of incidence) is below the critical value. If the flow is symmetric (i.e., $\theta = 0, 45$ [°]), the main counter-rotating vortices reattach on the cylinder surface. In the case of an asymmetric flow, these two vortices do not reattach on the surface. A detailed, and well-illustrated description of the main flow patterns can be found in ([5], pp. 4-7).

2.4 Static, dynamic, and total pressure

If a solid body is immersed in a fluid flow, then there exists a point on the body, where the velocity of the fluid drops to zero ([1], pp. 120). This is the so-called stagnation point, and the pressure at this point is the stagnation/total pressure. An example for the stagnation point can be seen in Figure 4.

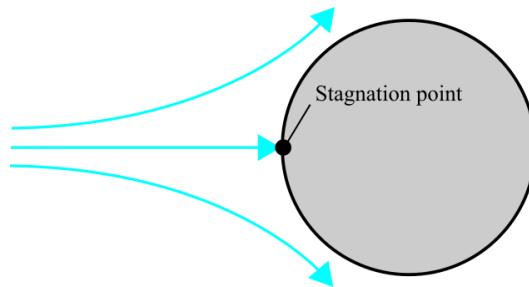


Figure 4. Stagnation point on a circular cylinder.

To determine the stagnation pressure, the simplified Bernoulli equation can be used (conditions for the simplification: the examined flow is inviscid, steady, incompressible and irrotational, also the field of force is potential) which is the following ([1], pp. 118-120):

$$\frac{p}{\varrho} + \frac{V^2}{2} + U = \text{constant} \quad (5)$$

Where:

- p : pressure [Pa]
- U : potential energy [J]

By writing up Equation (5) between the ∞ and the stagnation point, one can obtain the formula for the total pressure ([1], pp. 120-122):

$$p_{\infty} + \varrho \frac{V_{\infty}^2}{2} = p_t \quad (6)$$

Where:

- p_{∞} : static pressure [Pa]
- p_t : total pressure [Pa]
- $\varrho \frac{V_{\infty}^2}{2}$: dynamic pressure [Pa]

It can be seen from Equation (6) that the stagnation pressure, which is the pressure of the stopped flow, is higher than the static pressure, which is the pressure of the still, undisturbed flow. The difference between the total and the static pressure is called the dynamic pressure. Also, from Equation (5) one can see that if the velocity of the fluid increases, then the pressure must decrease and vice versa. Let us assume that the potential energy remains constant ($U = \text{constant}$), then Equation (5) is going to be the following:

$$\frac{p}{\varrho} + \frac{V^2}{2} = \text{constant} \quad (7)$$

Therefore, from Equation (7) it can clearly be seen that the pressure and the velocity are inversely proportional.

2.5 Aerodynamic forces

2.5.1 Lift coefficient

The lift coefficient is a dimensionless quantity which can be calculated in the following way ([1], pp. 527):

$$c_L = \frac{F_L}{\varrho \frac{V^2}{2} A} \quad (8)$$

Where:

- F_L : lift force [N]
- A : area [m²] (for a blunt body the area is the projection of the cross – section which is perpendicular to the direction of the flow)

It basically models the effects of shape, inclination, and some flow conditions on lift. The application of the lift coefficient is beneficial because if the geometry is the same, it can be applied even in the case of different sizes, velocities, and densities (on an interval of the Reynolds number, where the lift coefficient can be considered to be constant). In the case of a symmetric flow the time-averaged value of the lift coefficient is zero. Thus, in the case of the circular cylinder the expected time-averaged value of the lift coefficient is zero for every case. However, this is not true for the square cylinder, because as it was mentioned before the flow is only symmetric if $\theta = 0, 45 [^\circ]$. In Figure 5 the contour diagram of the mean lift coefficient on the $\theta - Re$ plane can be seen ([5], pp. 9).

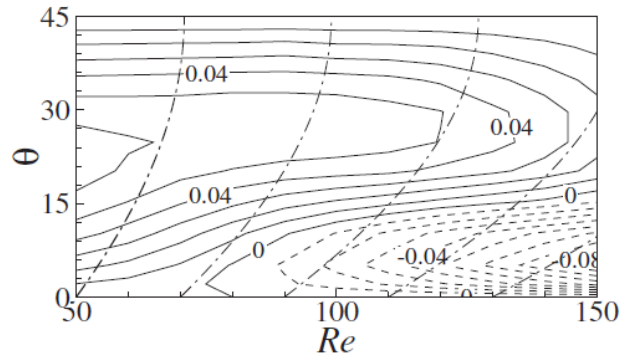


Figure 5. Contours of the mean lift coefficient in the case of a square cylinder ([5], pp. 9).

2.5.2 Drag coefficient

The drag coefficient is also a dimensionless value which can be calculated as follows ([1], pp. 527):

$$c_D = \frac{F_D}{\rho \frac{V^2}{2} A} \quad (9)$$

Where:

- F_D : drag force [N]

It basically models the effects of shape, inclination, and some flow conditions on drag. The application of the drag coefficient is beneficial because if the geometry is the same, it can be applied even in the case of different sizes, velocities, and densities (on an interval of the Reynolds number, where the drag coefficient can be considered to be constant). Let us introduce the so-called pressure drag and friction drag. According to ([3], pp. 20) one property of the boundary layer is that, under specific conditions, reverse flow can occur

directly at the wall. This can cause the separation of the boundary layer from the body and the formation of large or small eddies at the back of the body. This results in a significant change with regard to the pressure distribution at the back of the body, leading to the pressure drag. At the same time, friction drag is caused by the friction between the fluid and the surfaces over which it is flowing. In the case of bluff bodies (e.g., cylinders, spheres) the pressure drag is the main source of the drag force and the friction drag is a secondary source. In streamlined bodies (e.g., airfoil) this is exactly the other way around. The drag coefficient of a circular cylinder is shown in Figure 6 ([3], pp. 19) as a function of the Reynolds number. It can be seen that the drag coefficient is Reynolds number dependent: as the Reynolds number increases the drag coefficient decreases, and over a long interval it remains constant.

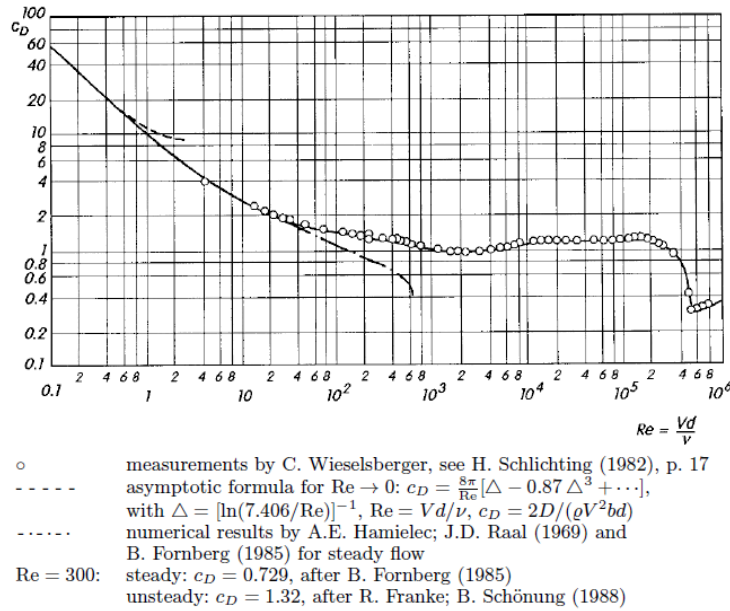


Figure 6. The effect of the Reynolds number on the drag coefficient in the case of a circular cylinder ([3], pp. 19).

As for the square cylinder, the contour diagram of the mean drag coefficient on the $\theta - Re$ plane can be seen in Figure 7(a) ([5], pp. 9). I included another figure, Figure 7(b) ([5], pp. 3), so that the mean drag coefficient values can be seen for the steady domain as well.

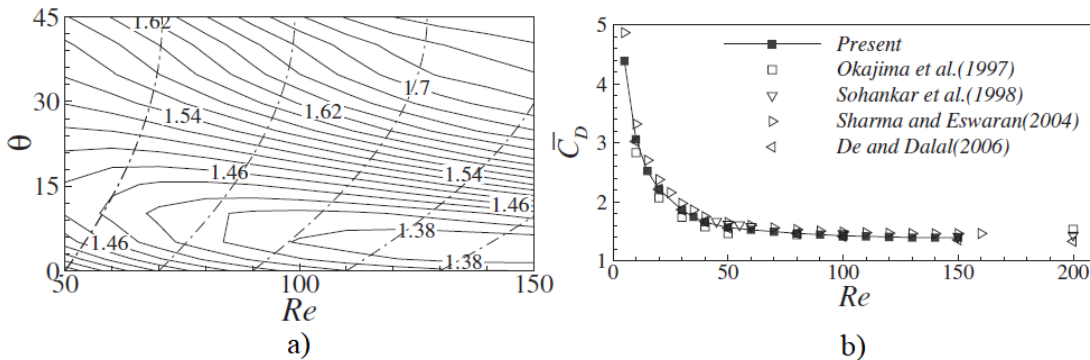


Figure 7. Mean drag coefficient values in the case of a square cylinder for a) various θ values ([5], pp. 9) b) $\theta = 0^\circ$ ([5], pp. 3).

Figure 7(b) will be important during the discussion of the simulation results, because one of the simulations is going to be a steady simulation (the only relevant geometry will be when $\theta = 0^\circ$, that is why only this case is presented (7(b)) with regard to the steady domain).

2.6 Strouhal number

The Strouhal number is a dimensionless value which describes frequency and unsteady effects. It is useful in analyzing oscillating, unsteady fluid flow problems. It can be calculated in the following way ([1], pp. 404, pp. 514):

$$Sr = \frac{f \cdot d}{V} \quad (10)$$

Where:

- f : vortex shedding frequency [Hz]

The expected values of the Strouhal number for a circular cylinder have already been presented in Table 1. In Figure 8 ([5], pp. 9) the contour diagram of the Strouhal number can be seen on the $\theta - Re$ plane in the case of a square cylinder.

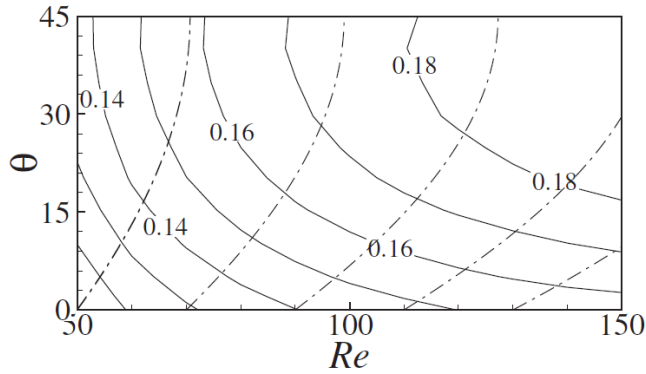


Figure 8. Contours of the Strouhal number in the case of a square cylinder ([5], pp. 9).

3. Lagrangian-averaged vorticity deviation

The following theoretical summary is based on George Haller et al., *Defining coherent vortices objectively from the vorticity*, 2016 [2].

Let us start by defining what a coherent vortex is. The definition of coherent vortices is still a disputed matter currently. However, according to [2] there are two main characteristics which are widely accepted. Firstly, vortices are concentrated regions of high vorticity. Secondly, they are usually seen as evolving domains with a high level of material invariance. As it was mentioned before, despite the fact that these characteristics are broadly accepted, different authors interpret vortices in different ways. These different kinds of interpretations are collected and can be found in [2]. The core of a coherent vortex is an impermeable material region with a high concentration of vorticity. Per contra, the interpretation of high concentration is scarcely objective. Thus, with regard to constructing an explicit vortex definition, material invariance could be a good starting point. In [2] an objective vortex criterion is proposed, this is the so-called Lagrangian-averaged vorticity deviation (LAVD).

LAVD by definition is the trajectory integral of the normed difference of the vorticity from its spatial mean:

$$LAVD_{t_0}^t(\mathbf{x}_0) := \int_{t_0}^t |\boldsymbol{\omega}(\mathbf{x}(s; \mathbf{x}_0), s) - \bar{\boldsymbol{\omega}}(s)| ds \quad (11)$$

Where:

- t_0 : starting time instant of the integration [s]
- t : final time instant of the integration [s]
- $\mathbf{x}_0$: initial particle positions [m]
- $\boldsymbol{\omega}(\mathbf{x}, s)$: vorticity $\left[\frac{1}{s}\right]$
- $\mathbf{x}(s; \mathbf{x}_0)$: instantaneous particle positions [m]
- s : variable of integration
- $\bar{\boldsymbol{\omega}}(s)$: instantaneous spatial mean of the vorticity over $U(t)$

$\boldsymbol{\omega}$ can be calculated in the following way:

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} \quad (12)$$

Where:

- $\mathbf{v}(\mathbf{x}, t)$: unsteady velocity field defined on a time – dependent spatial domain ($U(t)$) over a finite time interval $[t_0, t_1]$
- $U(t)$:
 - Physical domain with impermeable boundary or
 - Material domain formed by a set of evolving trajectories of $\mathbf{v}(\mathbf{x}, t)$

$\bar{\omega}$ is expressed as follows:

$$\bar{\omega}(t) = \frac{\int_{U(t)} \omega(\mathbf{x}, t) dV}{vol(U(t))} \quad (13)$$

Where:

- $vol(U(t))$: area (in 2D)
- dV : area element (in 2D)

According to [2] a Lagrangian vortex is a nested set of material tubes with outward decreasing tubular level sets of LAVD. The LAVD is derived from the dynamic rotation tensor, which is obtained from the so-called dynamic polar decomposition (DPD) of the deformation gradient. This provides an objective global vortex criterion which is entirely frame-invariant.

With the help of LAVD it is possible to detect objectively material tubes along which small fluid volumes have the same bulk rotation compared to the mean rotation of the fluid. The initial positions of these tubes are tubular level surfaces of the $LAVD_{t_0}^{t_1}(\mathbf{x}_0)$. This tubular set should be interpreted as a closed convex curve in 2D, and in 3D it should be understood as a convex, cylindrical, cup-shaped, or toroidal set. Convexity is required for tubular surfaces. This is due to the fact that stable vortices mostly have near-circular cross-sections.

In [2] it is stated that a rotationally coherent Lagrangian vortex has to suffice the following conditions over the examined time interval $[t_0, t_1]$:

- It is an evolving material domain $D(t)$ where $D(t_0)$ is filled with tubular level surfaces of $LAVD_{t_0}^t(\mathbf{x}_0)$ and the LAVD values are decreasing as we move outwards
- The boundary $B(t)$ of $D(t)$ is a material surface where $B(t_0)$ is the outermost tubular level surface of $LAVD_{t_0}^t(\mathbf{x}_0)$ in $D(t_0)$
- The center $C(t)$ of $D(t)$ is a material set where $C(t_0)$ is the innermost member (maximum) of $LAVD_{t_0}^t(\mathbf{x}_0)$ in $D(t_0)$

In Figure 9 ([2], pp. 143) a rotationally coherent Lagrangian vortex can be seen. The first figure is at the beginning of the examined time interval and the second figure represents the current state. $L(t)$ represents a rotational Lagrangian coherent structure (LCS), this means that along this so-called LCS fluid volumes have the same rotation in the given time interval $[t_0, t_1]$.

As it was mentioned before, convexity is a required criterion. This requirement ensures that the material vortex is unfilamented at t_0 . However, during the numerical calculations, a small convexity deficiency is allowed. Firstly, this deficiency is allowed due to the fact that LAVD level surfaces are represented by discrete polygons numerically. If a polygon has a lot of vertices, it is likely that it will not be convex. Another reason to relax strict convexity is to remove the conceptual asymmetry which can be seen in Figure 9. This way small

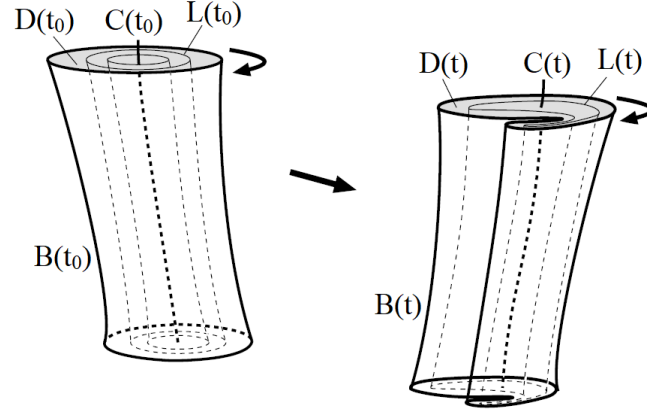


Figure 9. Initial and current positions of a Lagrangian vortex $D(t)$. Reproduced from ([2], pp. 143).

tangential filamentation can be present in the initial positions of vortex boundaries as well. Lastly, in the case of multi-scale data sets smaller-scale vortices can be near larger-scale vortices, thus a sufficiently coarse-grained notion is required for the boundaries of the larger-scale vortices.

In [2] it is also stated that small-scale local maxima of the LAVD function can arise due to numerical or observational noise in the velocity data. Due to this fact, in order to eliminate these artificial vortices, LAVD contours whose arclength falls below a minimal threshold value are ignored.

The final topic I want to mention in *Chapter 3* is the numerical implementation of the extraction procedure for rotationally coherent Lagrangian vortices in 2D. This algorithm is the following [2]:

1. For a 2D grid of initial conditions x_0 , compute the LAVD:

$$LAVD_{t_0}^t(x_0) := \int_{t_0}^t |\omega(x(s; x_0), s) - \bar{\omega}(s)| ds \quad (14)$$

2. Find initial positions $C(t_0)$ of vortex centers as local maxima of $LAVD_{t_0}^t(x_0)$
3. Find initial vortex boundaries $B(t_0)$ as outermost closed contours of $LAVD_{t_0}^t(x_0)$, these contours have to suffice the following criteria:
 - $B(t_0)$ encircles a vortex center $C(t_0)$
 - The arclength of $B(t_0)$ is higher than the given minimal threshold value
 - The convexity deficiency of $B(t_0)$ is smaller than the given maximum threshold value

The output is the initial positions of rotationally coherent Lagrangian vortex boundaries and vortex centers with respect to the time interval $[t_0, t_1]$.

4. Circular cylinder

4.1 Simulation setup

4.1.1 Geometrical model

I constructed the geometry based on [7]. The simulation is a two-dimensional (2D) simulation, thus, the cylinder was modelled as a circle. The diameter of the cylinder is $D = 0.0127 \text{ [m]}$. The physical domain has a dimension of 30 diameters in the streamwise direction ($-8D < x < 22D$) and 16 diameters in the lateral direction ($-8D < y < 8D$). I set up the geometry using SpaceClaim. The geometrical model can be seen in Figure 10.

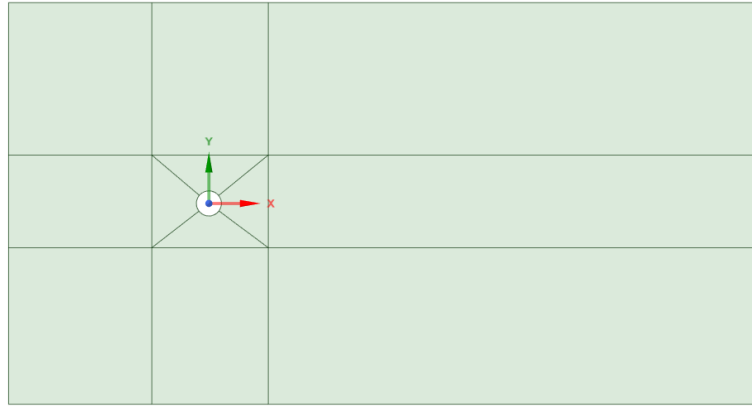


Figure 10. The geometrical model.

4.1.2 Mesh

I created the mesh using ANSYS Workbench Mesher. I refined the mesh near the cylinder with the help of edge sizing. I applied this method instead of inflation in order to be able to perform face meshing. This is due to the fact that I wanted to achieve a structured, quadrilateral mesh. I generated the mesh which can be seen in Figure 11.

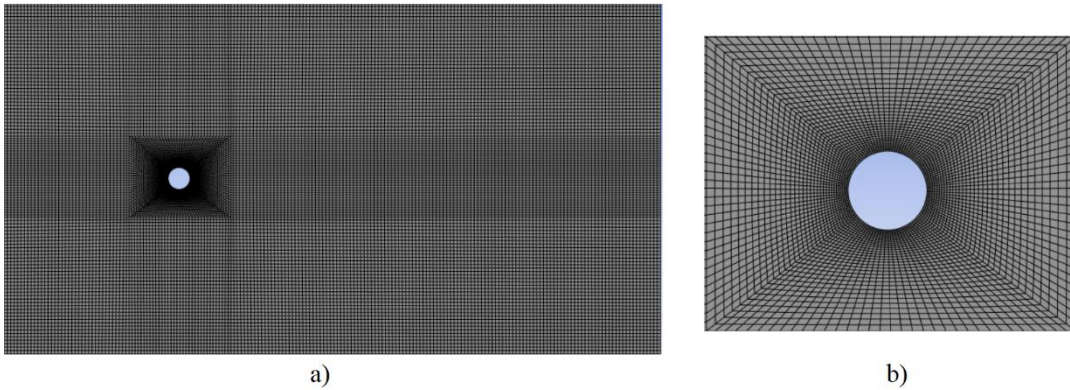


Figure 11. The generated mesh a) for the entire domain b) in the vicinity of the cylinder.

The mesh has 25252 cells and 25627 nodes. One way to check whether the quality of the mesh is appropriate or not is by examining the maximum value of the so-called skewness attribute. Skewness is based on the internal angles of an element, for regular polygon elements it is 0, if one internal angle is much smaller than the others it approaches 1 [4]. It determines the element quality based on the internal angles, with respect to the angles of ideal elements. If the skewness exceeds a given value, it can cause decreased accuracy. The maximum value of the skewness:

$$skewness_{maximum} = 0.5925 \quad (15)$$

As a general rule the maximum value should not exceed 0.85 [4]. As it can be seen, this requirement is fulfilled, thus, the quality of the generated mesh is appropriate.

One more important thing to mention is that the smallest cell size in the domain in the direction of V is approximately $\Delta x \approx 0.0004$ [m]. This was used while determining the time step size with the help of Equation (1).

4.1.3 Simulation settings

I ran the simulations using Ansys Fluent 2022 R2. For a Reynolds number of 12, 60, 120 and 1200, I selected transient simulation, and I used the Viscous Laminar model. The applied fluid was air, for which I used the following properties:

$$\rho_{air} = 1.225 \left[\frac{kg}{m^3} \right] \quad (16)$$

$$\mu_{air} = 1.7894 \cdot 10^{-5} \left[\frac{kg}{m \cdot s} \right] \quad (17)$$

I applied a constant inlet velocity, however in order to calculate the velocity, the formula for the Reynolds number had to be used. As it was mentioned before, the Reynolds number can be determined with the help of Equation (3). With this in mind, the inlet velocities for the different Reynolds numbers can be found in Table 2.

Table 2. Inlet velocities for the different Reynolds numbers.

Reynolds number	Inlet velocity $\left[\frac{m}{s} \right]$
12	0.0141
60	0.0704
120	0.1409
1200	1.4090

I set the boundary conditions and solution methods based on [7]. For the upper and lower boundaries, I set a symmetry boundary condition, this is important because in this case the solver assumes a zero shear stress at the wall, therefore the no-slip condition is not observed. For the flow outlet I applied a pressure outlet boundary condition. As for the

cylinder, wall boundary condition was applied. The wall boundary condition is used to bound fluid and solid regions, also, in viscous flows the no-slip condition is enforced at walls by default. Assigning this boundary condition was vital, because otherwise the fluid would have just flown right through the cylinder, not around it.

I set the reference values which are used to compute normalized flow-field variables, meaning they are needed for the calculation of the lift and drag coefficient.

As for the solution methods, SIMPLE was used for the pressure-velocity coupling. For the momentum equation Second Order Upwind was applied and for the pressure equation Second Order was used.

As it was mentioned before, the necessary time step can be determined with the help of Equation (1):

$$C = \frac{V \cdot \Delta t}{\Delta x} \rightarrow \Delta t \leq \frac{C \cdot \Delta x}{V} \underset{C=1}{\Rightarrow} \Delta t \leq \frac{\Delta x}{V} \quad (18)$$

The time steps for the different Reynolds numbers can be found in Table 3. Using these time steps a proper Courant number can be achieved during the simulations.

Table 3. Time step values for the different Reynolds numbers.

Reynolds number	Time step [s]
12	0.02
60	0.005
120	0.002
1200	0.0002

4.2 Results and discussion

4.2.1 Re=12

4.2.1.1 Figures of velocity and pressure

For a Reynolds number of 12 I ran the simulation for 2500 time steps. In Figure 12 the contours of the velocity magnitude can be seen.

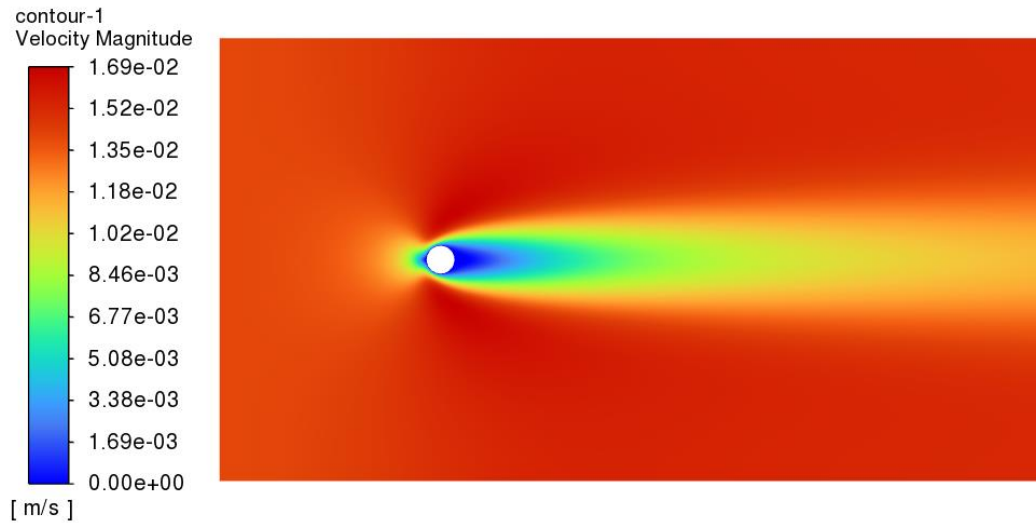


Figure 12. Contour plot of velocity at $Re=12$.

In Figure 13 the contour plot of the static pressure can be seen.

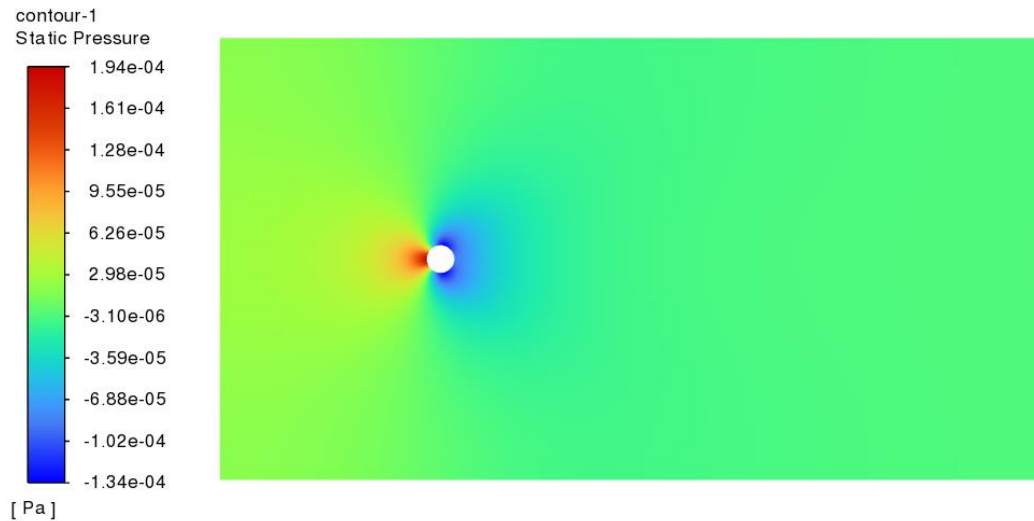


Figure 13. Contour plot of static pressure at $Re=12$.

It can be seen from Figure 12 and Figure 13 that there is a stagnation point at the front of the cylinder. As it was mentioned in 2.4, at the stagnation point, the velocity drops to zero, and this is also a high-pressure area. This is fulfilled in the case of the simulation results. In accordance with Table 1, a pair of counter-rotating vortices were created within the upper

and lower regions of the cylinder's wake and steady symmetric separation can be observed. This can be observed in Figure 14, where the pathlines colored by the velocity magnitude can be seen.

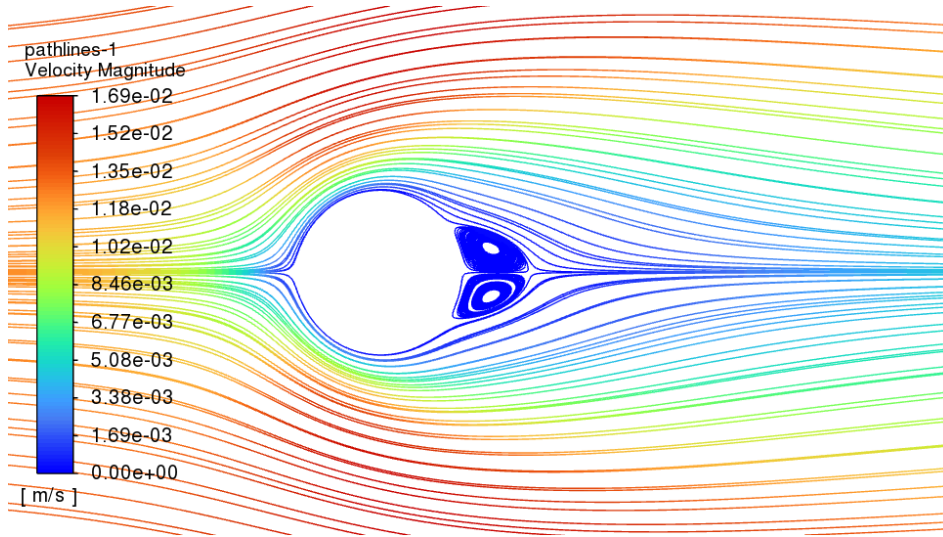


Figure 14. Pathlines at $Re=12$.

I also determined the maximum value of the cell convective Courant number:

$$C_{max} = 0.7814 \quad (19)$$

Therefore, it can be said that the results capture the aerodynamic phenomena well. In addition, the solution converged.

4.2.1.2 Lift and drag coefficient

Both the lift and the drag coefficients converged quite fast. As it was mentioned in 2.5.1, in the case of a symmetric flow the time-averaged value (in this case the converged value) of the lift coefficient is zero, this is fulfilled in this case. It is important to mention, that later on, when I am going to mention the time-averaged value, I am going to talk about the time-averaged value of the converged part.

The converged value of the drag coefficient is:

$$c_D = 2.8095 \quad (20)$$

This is in accordance with the expected result (Table 1).

4.2.2 $Re=60$

4.2.2.1 Figures of velocity and pressure

In the case of $Re = 60$ I ran the simulation for 14000 time steps and received Figure 15 for the velocity. The static pressure distribution can be seen in Figure 16.

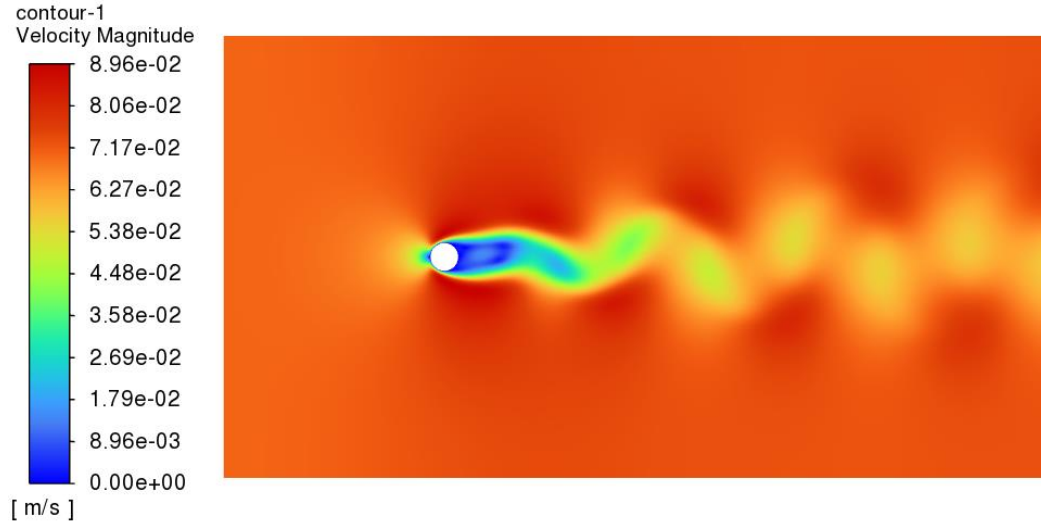


Figure 15. Contour plot of velocity at $Re=60$.

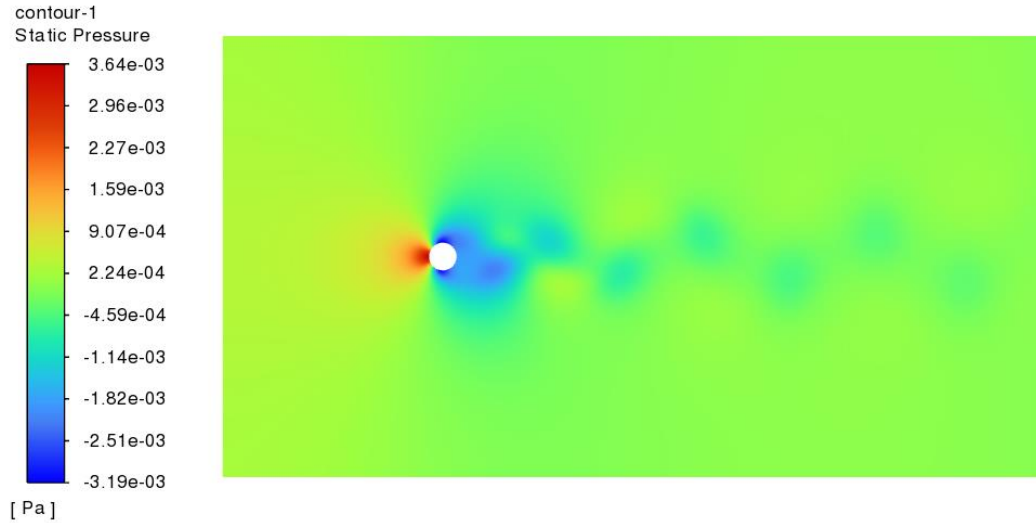


Figure 16. Contour plot of static pressure at $Re=60$.

The stagnation point which was mentioned in 4.2.1.1, can also be observed here. It can be seen from Figure 16 that the pressure difference between the front and the back of the cylinder is higher than in the case of $Re = 12$. As expected on the basis of Table 1 a laminar, unstable wake can be observed. It is also important to realize that the region surrounding the cylinder, where the velocity is less than the freestream velocity, has decreased substantially compared to the case of $Re = 12$. This can be explained by the relationship between the Reynolds number and the boundary layer thickness. The boundary layer thickness decreases as the Reynolds number increases, this explains the decrease in the aforementioned area. This decrease is observable in the case of $Re = 120$ and $Re = 1200$ as well.

The Courant number for this case is:

$$C_{max} = 1.1899 \quad (21)$$

4.2.2.2 Lift and drag coefficient

The lift coefficient oscillates, and its time-averaged value is approximately zero. The drag coefficient fluctuates a little (this is not visible in Figure 17(b)), and for the time-averaged value I received the following result:

$$c_D = 1.4942 \quad (22)$$

This is in accordance with the expected result (Table 1). It can be seen that the drag coefficient decreased as the Reynolds number increased, this is in accordance with Figure 6 and with what was stated in 2.5.2. The time evolution of the lift coefficient can be seen in Figure 17(a), and the time evolution of the drag coefficient can be seen in Figure 17(b).

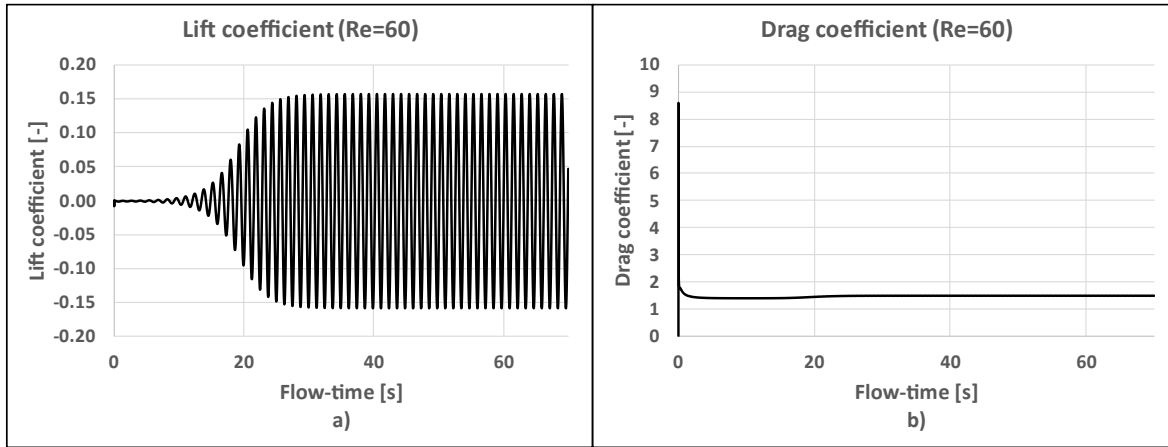


Figure 17. Time evolution of the a) lift coefficient b) drag coefficient at $Re=60$.

4.2.2.3 Strouhal number

For the validation of the results the Strouhal number can be checked also. In spite of the fact that in Table 1 a Strouhal number is not presented for $Re = 60$, I still determined the Strouhal number for this case also, because it can be seen from Figure 15 that vortex shedding is observable for this Reynolds number also. I checked another publication [8] with regard to simulation of flow past a circular cylinder, and in [8] this is confirmed. As it was mentioned in 2.6 the Strouhal number can be calculated with Equation (10). In order to calculate the Strouhal number the vortex shedding frequency had to be determined. I determined the vortex shedding frequency with the help of the obtained data for the lift coefficient. I performed the Fast Fourier Transform for the time signal of the lift coefficient. This can be seen in Figure 18. Afterwards, I looked for spikes in the low-frequency ranges, the corresponding frequency is the following:

$$f = 0.7999 \text{ [Hz]} \quad (23)$$

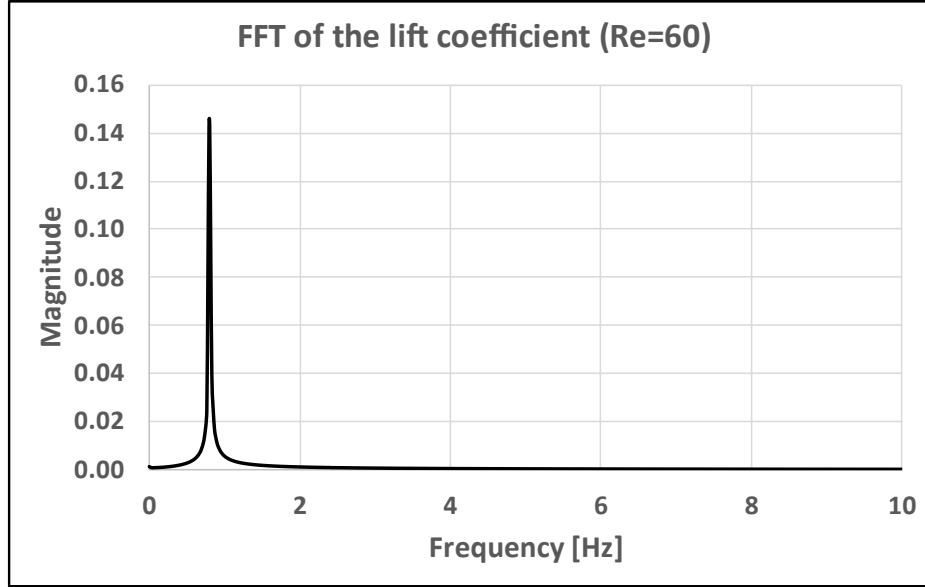


Figure 18. FFT of the lift coefficient at $Re=60$.

With this in mind the Strouhal number is the following:

$$Sr = \frac{f \cdot D}{V} = 0.1443 \quad (24)$$

This calculated value is acceptably close to the result in [8]. The slight difference can be caused by the fact that in [8] a different boundary condition was applied for the outlet, and a different domain size was used.

4.2.3 $Re=120$

4.2.3.1 Figures of velocity and pressure

For a Reynolds number of 120 I ran the simulation for 6000 time steps and received Figure 19 for the velocity. The static pressure distribution can be seen in Figure 20. Let us examine the obtained figures. It can be seen from Figure 19 that as the Reynolds number increased, the vortex region expanded and became larger in length. Also, by observing Figure 19, the aforementioned von Karman vortex street phenomenon can be recognized. As it was mentioned, the von Karman vortex street is a repeating pattern of swirling vortices, which is caused by the vortex shedding. The shedding of the vortices is responsible for the separation of flow around bluff bodies. As Table 1 shows, this is the expected result. It can be seen in Figure 20 that the pressure difference became even larger between the front and the back of the cylinder, thus, the pressure drag is increasing. The stagnation point is still present at the front of the cylinder.

The Courant number for this case:

$$C_{max} = 1.0569 \quad (25)$$

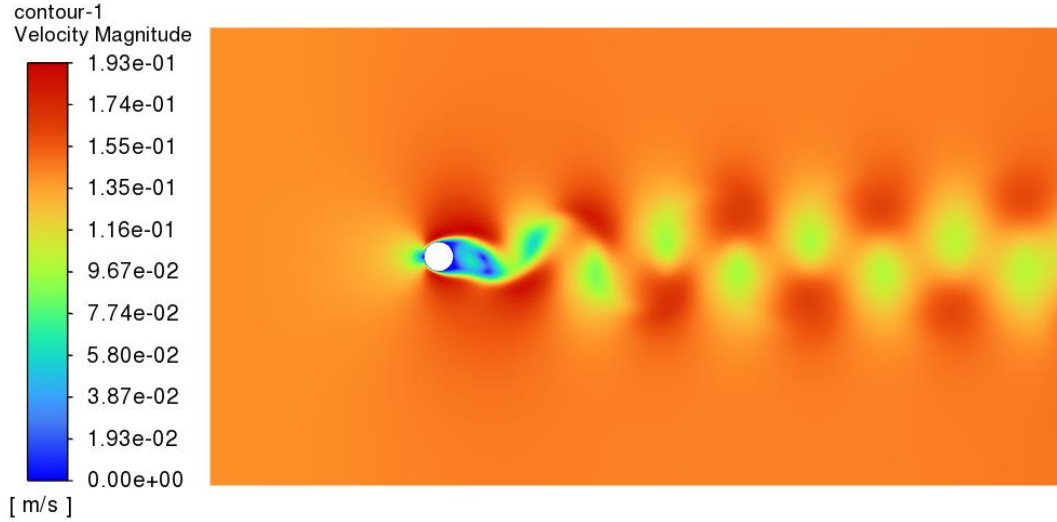


Figure 19. Contour plot of velocity at $Re=120$.

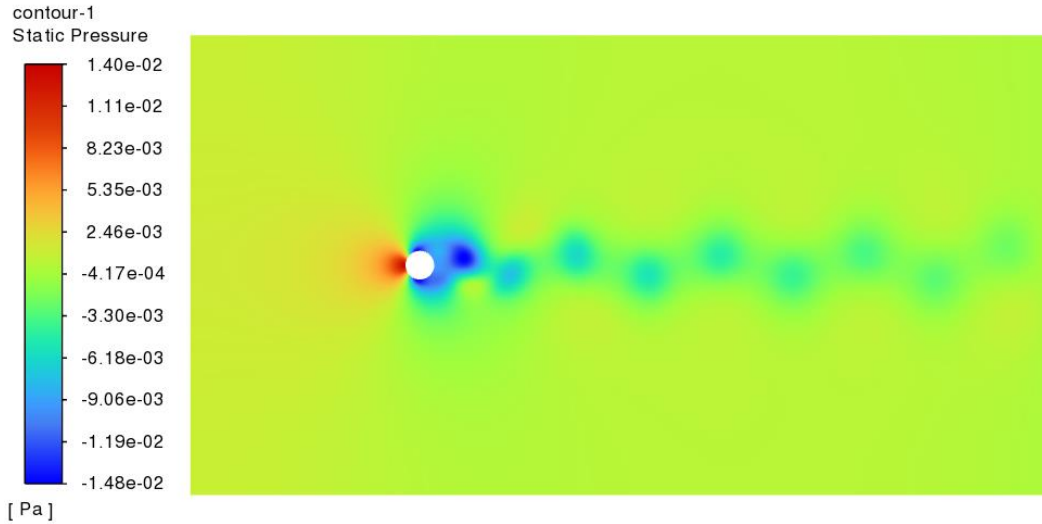


Figure 20. Contour plot of static pressure at $Re=120$.

4.2.3.2 Lift and drag coefficient

The time-averaged value of the lift coefficient is approximately zero again. The drag coefficient fluctuates, and its time-averaged value is approximately:

$$c_{D,mean} = 1.4040 \quad (26)$$

The obtained value is slightly higher than the expected result which can be seen in Table 1. This can be caused by the fact that I used a two-dimensional model instead of a three-dimensional one. Nevertheless, this result still complies reasonably well with the expected result (Table 1). In Figure 21(a) the time evolution of the lift coefficient can be seen and in Figure 21(b) the time evolution of the drag coefficient can be seen.

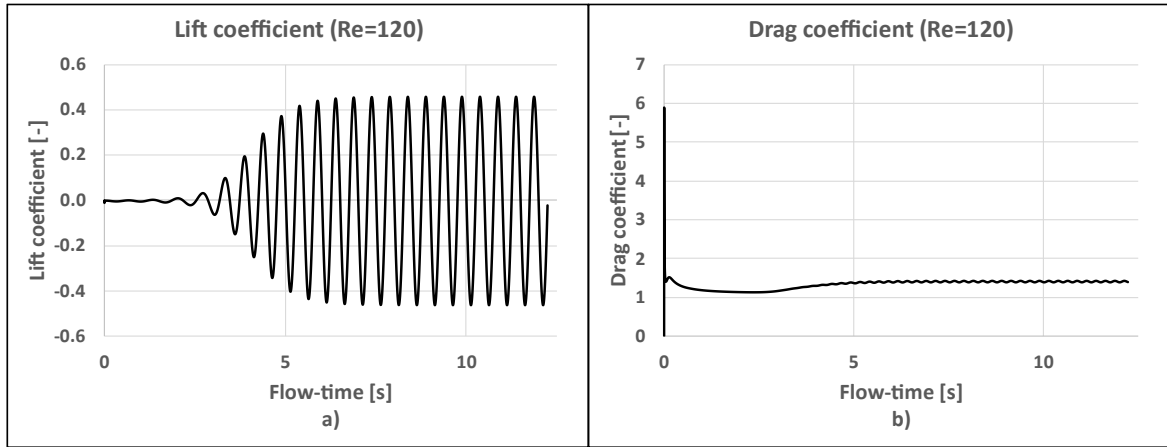


Figure 21. Time evolution of the a) lift coefficient b) drag coefficient at $Re=120$.

4.2.3.3 Strouhal number

I performed the FFT for the time signal of the lift coefficient. This can be seen in Figure 22.

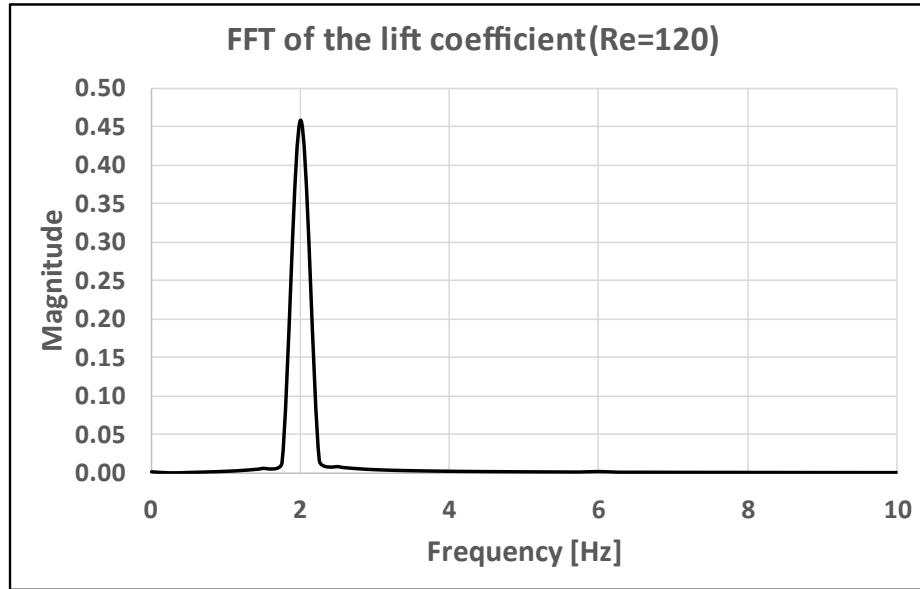


Figure 22. FFT of the lift coefficient at $Re=120$.

The vortex shedding frequency is the following:

$$f = 1.9990 \text{ [Hz]} \quad (27)$$

With this in mind the Strouhal number is the following:

$$Sr = \frac{f \cdot D}{V} = 0.1802 \quad (28)$$

This calculated value is acceptably close to the expected result which can be found in Table 1. The possible slight difference can once again be explained by the fact that a two-dimensional model was applied.

4.2.4 $Re=1200$

4.2.4.1 Figures of velocity and pressure

For a Reynolds number of 1200 the simulation was run for 10000 time steps, and I received Figure 23 for the velocity.

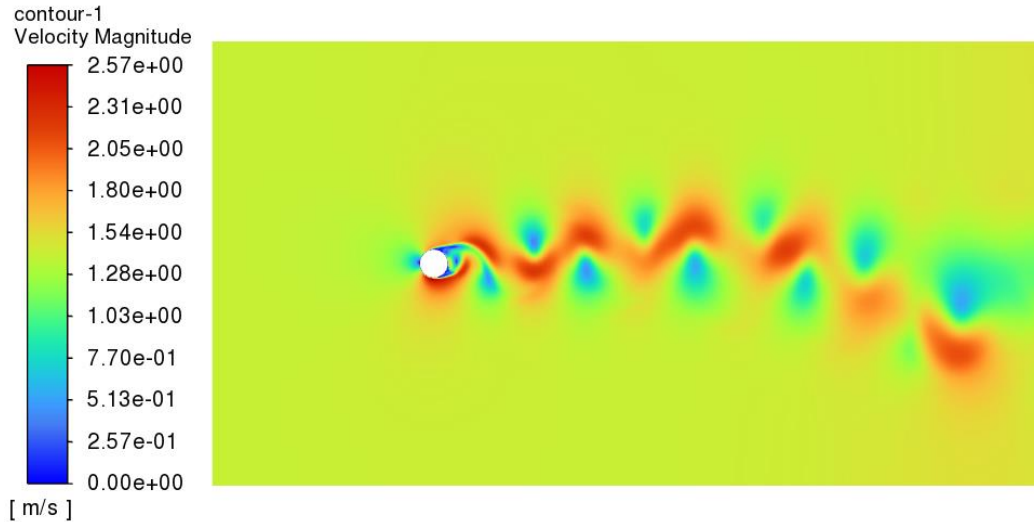


Figure 23. Contour plot of velocity at $Re=1200$.

The static pressure distribution can be seen in Figure 24.

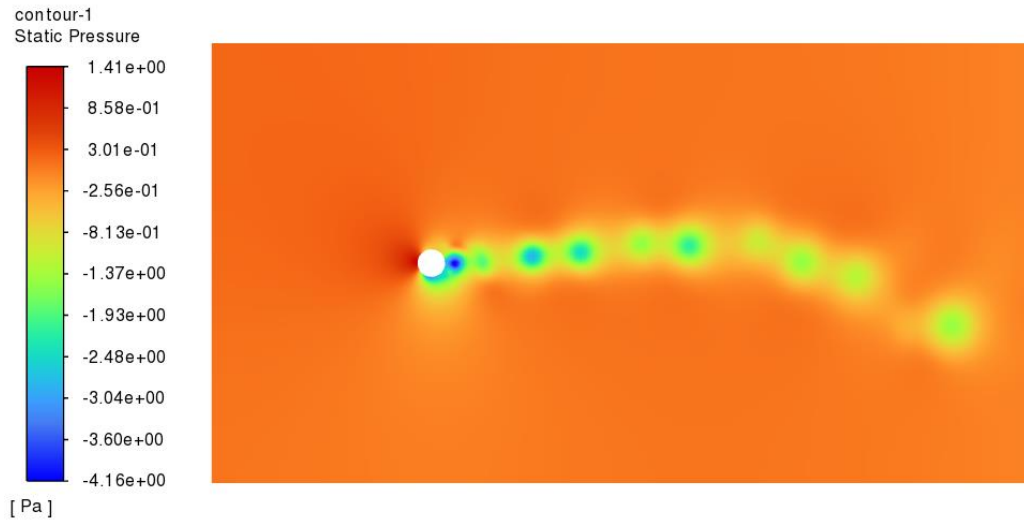


Figure 24. Contour plot of static pressure at $Re=1200$.

For a Reynolds number of 1200 the flow is still laminar, however, the vortices are mixed together, the vortex shedding is no longer symmetric. This is in accordance with Table 1. The aforementioned stagnation point can be seen in Figure 23 and Figure 24. The pressure difference between the front and the back of the cylinder increased again drastically. As for the Courant number:

$$C_{max} = 1.4957 \quad (29)$$

4.2.4.2 Lift and drag coefficient

The time-averaged value of the lift coefficient is zero. The mean value of the drag coefficient:

$$c_D = 1.7186 \quad (30)$$

This is higher than the value in Table 1. It can be seen that up to this point the drag coefficients were approximately equal to the theoretical values. According to [9] this is due to the fact that two-dimensional simulations of flow around nominally two-dimensional bluff bodies at Reynolds numbers for which the flow is intrinsically three-dimensional, lead to inaccurate lift and drag forces. In [9] it is also stated that it has been shown that the drag coefficient computed from two-dimensional numerical simulations is significantly higher than what is obtained from measurements, and also, it has been found that three-dimensional simulations lead to accurate prediction of the drag. The main reason for this is that the surface pressure distribution obtained from two-dimensional simulations is different from the pressure distribution obtained from three-dimensional simulations and measurements. However, the cause of this difference in the pressures is still unclear. In the case of a circular cylinder the critical Reynolds number above which three-dimensionality plays a vital role in determining the flow pattern is 180. Above this value a two-dimensional simulation cannot capture the effect of three-dimensional structures on the flow field. Moreover, above this critical Reynolds number two-dimensional simulations usually overpredict the drag coefficient.

I checked another publication with regard to this topic. In [10] it is also stated that for a circular cylinder, between $Re = 190$ and $Re = 260$ a primary instability mechanism occurs which causes three-dimensional disturbances. These disturbances change the structure of the wake vortices, therefore two-dimensional simulations often fail to predict the drag coefficient and the RMS (root mean square) of the lift coefficient accurately. The time evolution of the lift coefficient can be seen in Figure 25(a) and the time evolution of the drag coefficient can be seen in Figure 25(b).

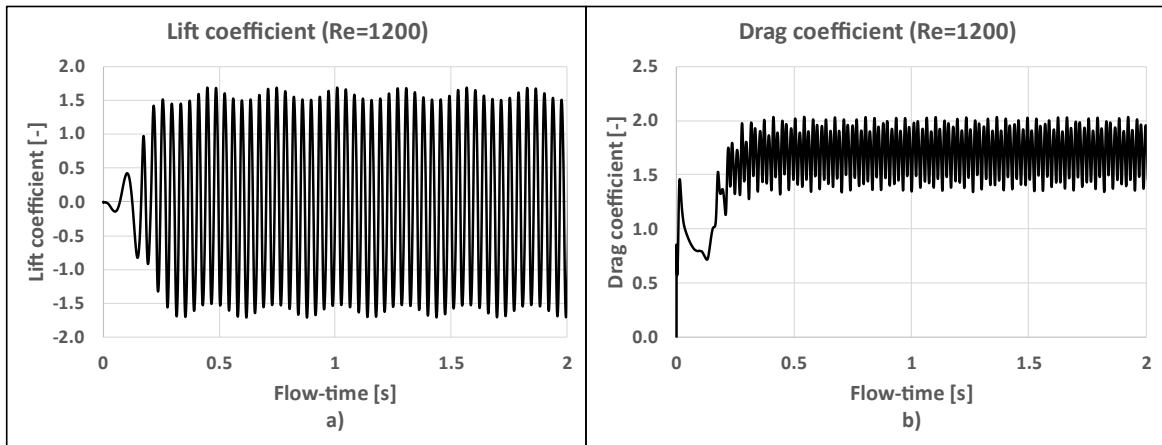


Figure 25. Time evolution of the a) lift coefficient b) drag coefficient at $Re=1200$.

4.2.4.3 Strouhal number

The FFT for the time signal of the lift coefficient can be seen in Figure 26.

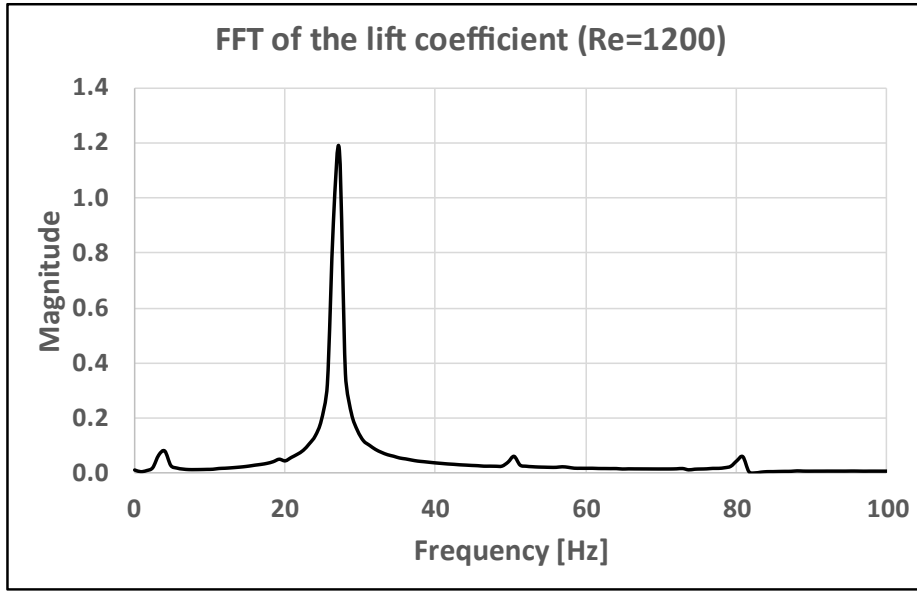


Figure 26. FFT of the lift coefficient at $Re=1200$.

The corresponding vortex shedding frequency:

$$f = 27.1956 \text{ [Hz]} \quad (31)$$

The Strouhal number is the following:

$$Sr = \frac{f \cdot D}{V} = 0.2451 \quad (32)$$

This value is higher than the value in Table 1. Due to the three-dimensional disturbances, the non-accurate value of the computed Strouhal number was expected.

In the next few paragraphs, I am going to talk about an interesting discovery with regard to the frequencies. I checked the other local maximum in the lower frequency domain, and it is the following:

$$f_{local} = 3.9994 \text{ [Hz]} \quad (33)$$

f_{local} is the frequency of the up-and-down motion of the vortex street. Thus, the T time period is the following:

$$T = \frac{1}{f_{local}} = 0.25 \text{ [s]} \quad (34)$$

From Figure 25(a) it can be seen that between $t = 0.45$ [s] and $t = 0.7$ [s] there is a whole time period. Thus, on the velocity contours with regard to these time instants one should see the following result: the vortex street is deflected in the same direction in both cases. In Figure 27 the contours of the velocity can be seen at $t = 0.45$ [s]. In Figure 28 the contours of the velocity can be seen at $t = 0.7$ [s].

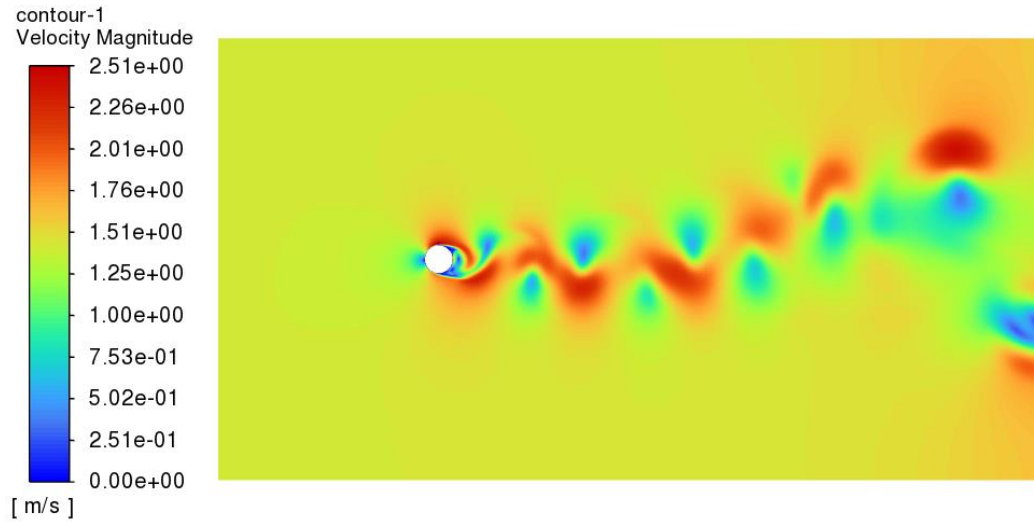


Figure 27. Contour plot of velocity at $t=0.45$ [s].

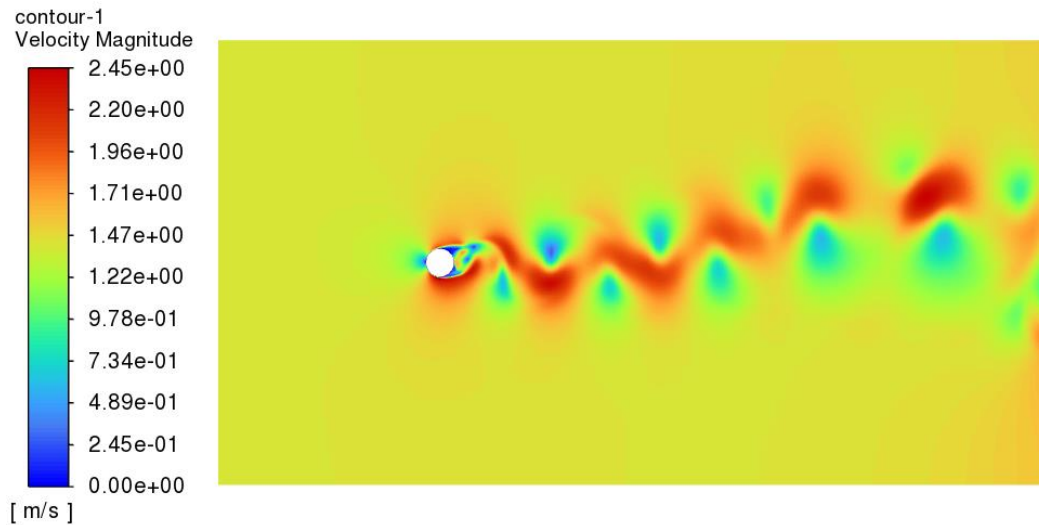


Figure 28. Contour plot of velocity at $t=0.7$ [s].

From Figures 27 and 28 it can be seen that the aforementioned theory was true, the vortex street is indeed deflected in the same direction in the two time instants. I also inserted Figure 29, which corresponds to $t = 0.58$ [s]. This time instant is approximately a half time period later than $t = 0.45$ [s], thus, in this case the vortex street should be deflected downwards. It can be seen from Figure 29 that this is fulfilled. All in all, it can be said that there are two periodic oscillations with two characteristic frequencies. One of them corresponds to the vortex shedding (Equation (31)). The other frequency (Equation (33))

corresponds to the changing of the deflection of the vortex street in the up-and-down direction.

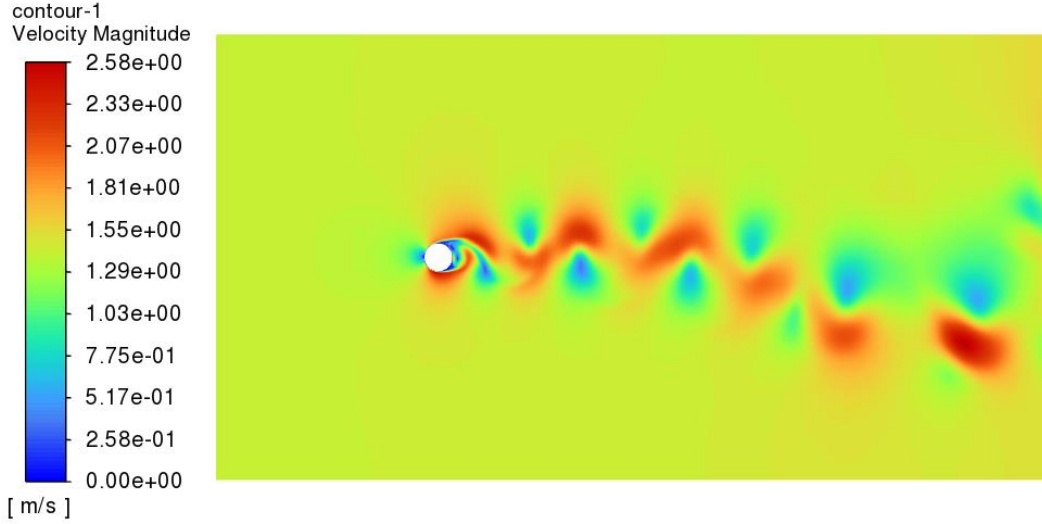


Figure 29. Contour plot of velocity at $t=0.58$ [s].

4.3 Mesh independency study

Mesh independency is a very important concept because this way it can be seen whether the results are independent of the mesh or not. One way to do this is to estimate the discretization error. The error can be estimated by the so-called Richardson extrapolation. I performed the Richardson extrapolation based on [4]. For the current mesh independency study, the refinement ratio is the following:

$$r = 2 \quad (35)$$

The first step is to double the mesh resolution in each direction, this will result in 4 times more cells (in 2D). As for the next step, this procedure is repeated, thus, in the case of the finest mesh there are 16 times more cells than in the case of the original one.

I am going to denote the results obtained from the different resolutions in the following way:

ϕ_{4h} : coarse mesh result

ϕ_{2h} : intermediate mesh result

ϕ_h : fine mesh result

With these values the discretization error can be estimated in the following way:

$$p_m = \log_r \left(\frac{\phi_{2h} - \phi_{4h}}{\phi_h - \phi_{2h}} \right) \quad (36)$$

$$\varepsilon_h \cong \frac{\phi_h - \phi_{2h}}{r^{p_m} - 1} \quad (37)$$

Where:

- p_m : measured order of convergence
- r : refinement ratio
- ε_h : discretization error

One important note is that the result for which I performed the calculations was the obtained data for the drag coefficient for a Reynolds number of 120. The results for the differently refined meshes, along with the number of cells, and the maximum Courant numbers can be found in Table 4.

Table 4. Characteristics and results in the case of the different meshes.

	Coarse mesh	Medium mesh	Fine mesh
Number of cells	25252	101008	404032
Maximum value of the Courant number	1.0569	1.0914	1.0713
Drag coefficient	1.4040	1.3963	1.3944

From the data above the necessary calculations for the Richardson extrapolation can be performed. The measured order of convergence:

$$p_m = \log_r \left(\frac{\phi_{2h} - \phi_{4h}}{\phi_h - \phi_{2h}} \right) = 1.9923 \quad (38)$$

I used second-order schemes during the simulations, thus, from Equation (38) one can see that the theoretical accuracy of the numerical schemes has been sufficiently realized.

The discretization error:

$$\varepsilon_h \cong \frac{\phi_h - \phi_{2h}}{r^{p_m} - 1} = -6.4425 \cdot 10^{-4} \quad (39)$$

From Equation (39) it can be seen that this value is sufficiently small, therefore it is safe to assume that if the coarse mesh is used for the simulations, it produces accurate results. Also, it can be seen that the Courant numbers are acceptable, and divergence did not occur during the simulations. Using these results, one can calculate the extrapolated value of the drag coefficient which is the following:

$$\phi_0 = \phi_h + \varepsilon_h = 1.3938 \quad (40)$$

The relative error of the coarse mesh result (ϕ_{4h}) with regard to the extrapolated value is less than 1 [%]. The Richardson plot can be seen in Figure 30, where N denotes the number of cells corresponding to the different meshes. For better readability the horizontal axis is

scaled with 10^6 . $1/N \rightarrow 0$ represents the (exact) value extrapolated to the infinitely fine mesh [4].

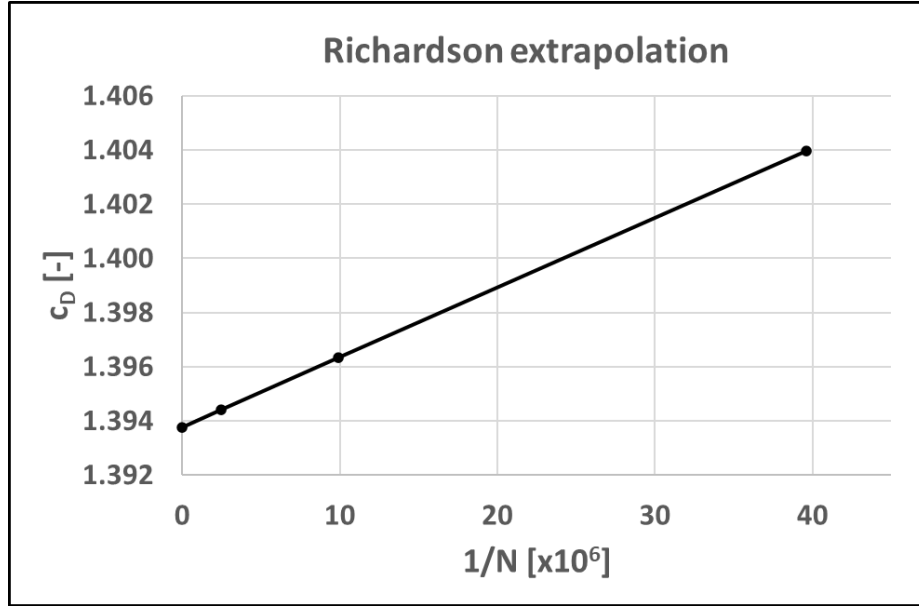


Figure 30. Richardson plot.

4.4 Vortex detection with the help of MATLAB

4.4.1 Parameter optimization and vortex detection at $Re=120$

As it was mentioned in *Chapter 1*, at first, I carried out the parameter optimization based on the results from the simulation with a Reynolds number of 120. The parameters that had to be optimized were the following: Number of contour levels, Convexity deficiency threshold (%), Minimal arc-length threshold (%). The convexity deficiency of a closed curve in the plane is defined as the ratio of the area between the curve and its convex hull to the area enclosed by the curve [2]. It is also important to note that MinLength is expressed as a percentage of the cylinder diameter. Below I inserted figures with optimal values of these parameters. In *Appendix 1* I included figures representing the detection with nonoptimal parameters, and in *Appendix 2* all of the optimization test runs can be found.

4.4.1.1 Parameter optimization

4.4.1.1.1 Number of contour levels

The first parameter I examined is denoted by N_{ct} , which is the number of contour levels intended to extract. I ran the code for the following values:

$$N_{ct} = 1; 10; 50; 100; 200; 400; 600; 800; 1000 \quad (41)$$

While doing these test runs, I kept the other parameters at a value of 10 (the examined values for the other two parameters can be seen in Equations (42) and (43)). After having finished all of the test runs, I found that the minimal value of N_{ct} should be around 100. The maximum value, however, should not be too large, because in that case the detection is not

working properly (the simulation becomes slower, and the code detects non-existent vortices). So, the proper interval for this parameter: $Nct \in [100; 200]$. In Figure 31, which is the contour plot of the computed LAVD values, a representative example can be seen with regard to Nct . It can be seen from Figure 31 that the reference system is the x-y coordinate-system (since the current problem is two-dimensional), and the origo is in the center of the cylinder. It is important to note that the red dots represent the centers of the vortices, and the red contours denote the boundaries of the detected vortices.

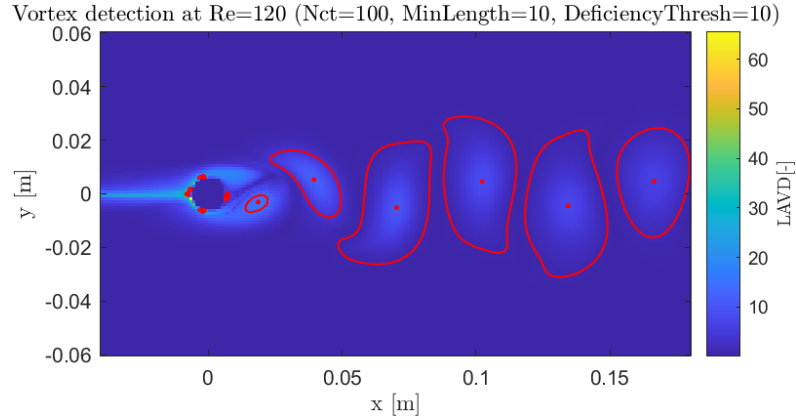


Figure 31. Vortex detection with optimal Nct value at $Re=120$.

4.4.1.1.2 Convexity deficiency threshold (%)

The next parameter that had to be optimized was the convexity deficiency threshold which is denoted by $DeficiencyThresh$. As it was mentioned in *Chapter 3* a small convexity deficiency is allowed which is characterized by the $DeficiencyThresh$. The convexity deficiency of a closed curve in the plane is defined as the ratio of the area between the curve and its convex hull to the area enclosed by the curve [2]. While trying to find the optimal value I kept Nct at an optimal value of 100, whereas the minimal arc-length threshold was kept at 10. I ran the code for the following values:

$$DeficiencyThresh = 0; 1; 2; 5; 10; 15; 20; 40; 50; 60; 80; 100 \quad (42)$$

For the $DeficiencyThresh$ I found that the minimum value should be around 1. Therefore, an optimal interval for the value of this parameter is the following: $DeficiencyThresh \in [1; 10]$. In Figure 32 the vortex detection with an optimal $DeficiencyThresh$ value from this interval can be seen.

4.4.1.1.3 Minimal arc-length threshold (%)

The last parameter is the minimal arc-length threshold, denoted by $MinLength$. As it was mentioned in *Chapter 3* this value means that the closed LAVD contours whose arclength falls below this predetermined threshold value are ignored. As it was already mentioned, $MinLength$ is expressed as a percentage of the cylinder diameter.

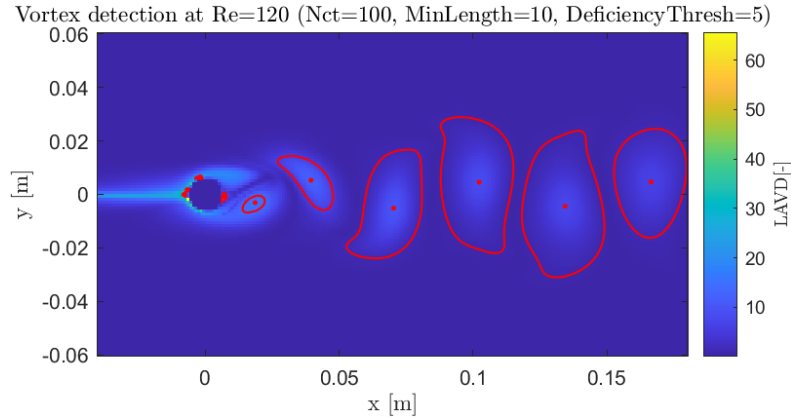


Figure 32. Vortex detection with optimal *DeficiencyThresh* value at $Re=120$.

I performed test runs for the following values (I kept *Nct* at 100 and *DeficiencyThresh* at 10):

$$\begin{aligned} \text{MinLength} = & 1; 10; 20; 40; 60; 65; 70; 75; 80; 100; 110; \\ & 120; 130; 150; 170; 180; 185; 190; 200; 1000 \end{aligned} \quad (43)$$

Having examined the figures, I found that the minimum value should be around 75 (this is the value when only the 6 appropriate vortices are detected) and the maximum value should be in the vicinity of 180 (this is the value when the smallest vortex is still detected, it disappears around 185), thus the proper interval: $\text{MinLength} \in [75; 180]$. In Figure 33 the vortex detection with an optimal *MinLength* value can be seen.

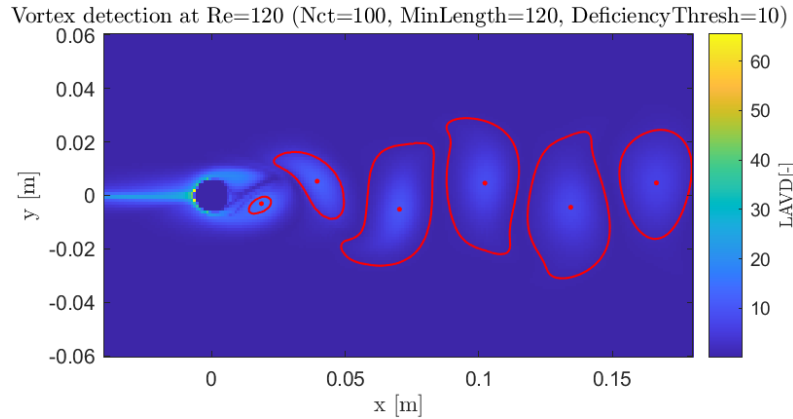


Figure 33. Vortex detection with optimal *MinLength* value at $Re=120$.

4.4.1.2 Vortex detection with optimized parameters

With the optimized parameters I ran the code and received Figure 34, one can see that the vortices are shed from alternating sides of the cylinder which is the expected result.

One way to check whether the optimized parameters are appropriate or not is to examine the centers of the vortices. The coordinates of the centers of the vortices can be seen in Table 5 (the i subscript denotes the serial number of the vortex).

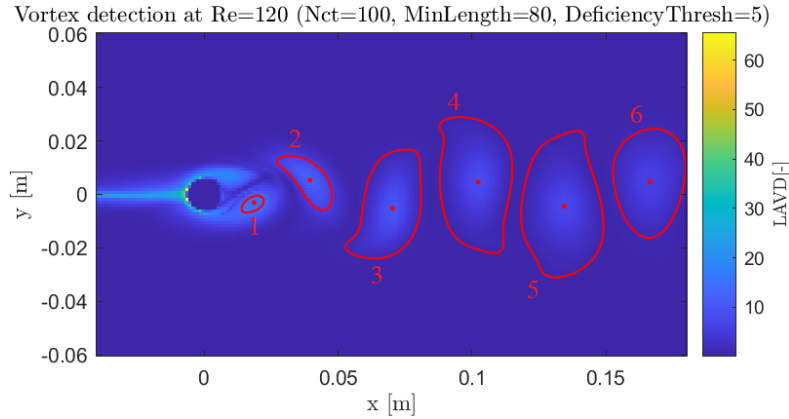


Figure 34. Vortex detection with optimized parameters at $Re=120$.

Table 5. Centers of the vortices.

Serial number of the vortex	$x_i [m], \quad i \in [1; 6]$	$y_i [m], \quad i \in [1; 6]$
1	0.0187	-0.0031
2	0.0395	0.0053
3	0.0703	-0.0052
4	0.1023	0.0046
5	0.1344	-0.0044
6	0.1664	0.0047

The next step was to check the differences between the x and y coordinates with regard to the neighboring vortices (the subscript denotes the two vortices between which I calculated the coordinate differences), this can be found in Table 6.

Table 6. Differences between the centers of the vortices.

Numbers of the neighboring vortices	$\Delta x_{ij} [m], \quad i \in [1; 5]; \quad j \in [2; 6]$	$\Delta y_{ij} [m], \quad i \in [1; 5]; \quad j \in [2; 6]$
1-2	0.0208	0.0084
2-3	0.0308	-0.0105
3-4	0.0320	0.0098
4-5	0.0321	-0.0090
5-6	0.0320	0.0091

The last thing that had to be performed was the determination of the ratio of the coordinate differences, meaning:

$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.4038, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = -0.3409, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = 0.3063 \quad (44)$$

$$\frac{\Delta y_{45}}{\Delta x_{45}} = -0.2804, \quad \frac{\Delta y_{56}}{\Delta x_{56}} = 0.2844 \quad (45)$$

The theoretical value for the $\frac{\Delta y}{\Delta x}$ ratio is 0.281 [6]. This value is basically a criterion for the von Karman vortex street. If this criterion is met, then the vortex street is stable, and this defines the von Karman vortex street. One can see from Equations (44)-(45) that the calculated results are acceptably close to the conceptual value and also, they converge to the theoretical value further downstream from the cylinder, thus, with the optimized parameters the code was able to detect the evolved vortices in a proper way.

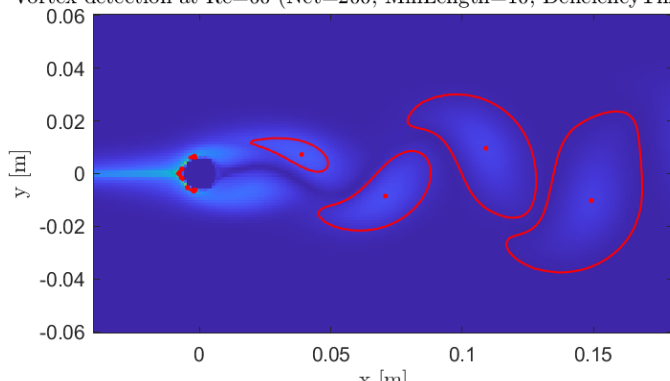
4.4.2 Parameter optimization and vortex detection at Re=12

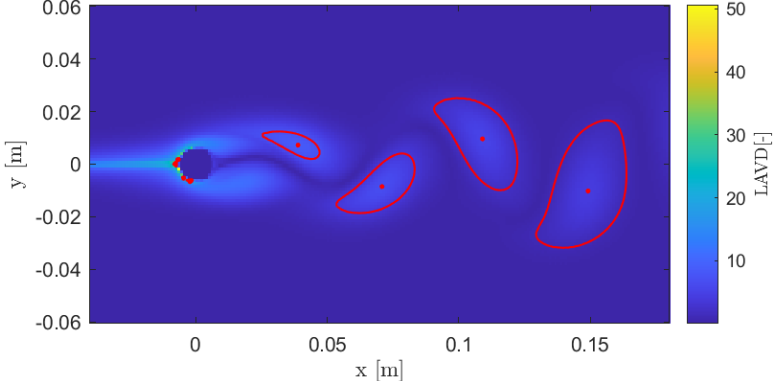
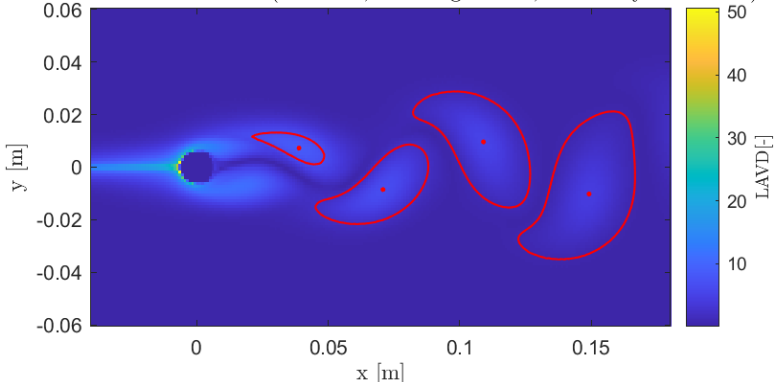
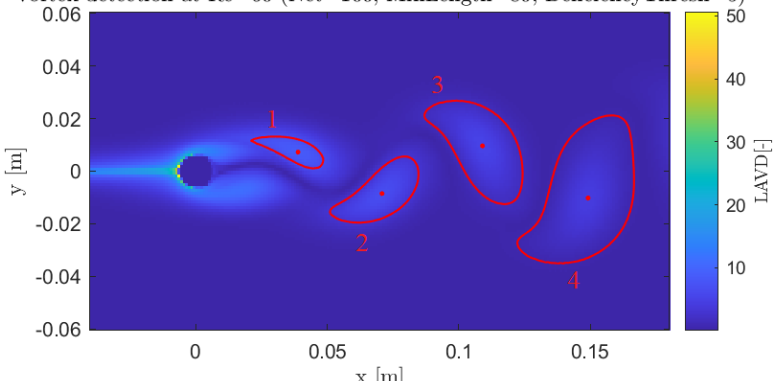
For a Reynolds number of 12, the code failed to detect the vortex pair in the cylinder's wake. This can be explained by the fact that these vortices are at a fixed place, and they are not detaching, this is a steady case of fluid flow.

4.4.3 Parameter optimization and vortex detection at Re=60

In the case of $Re = 60$ the code was able to detect the evolved vortices in a proper way. It is also important to note that the detection worked with the optimized parameters, however, I performed the optimization process again. This time I am not going to write about each parameter individually, because this has already been done in 4.4.1.1. Instead, I am going to present the ideal intervals for each parameter in a table form, along with the corresponding figures (representing the detection with optimal parameters from the aforementioned intervals). The other figures can be found in *Appendix 2*.

Table 7. Parameter optimization at Re=60.

Parameter	Nct
Test values	1;10;50;100;200;400;600;800;1000
Optimal interval	[100;200]
Vortex detection with optimal Nct parameter	Vortex detection at Re=60 (Nct=200, MinLength=10, DeficiencyThresh=10) 

Parameter	DeficiencyThresh
Test values	0;1;2;5;10;15;20;40;50;60;80;100
Optimal interval	[1;10]
Vortex detection with optimal DeficiencyThresh parameter	Vortex detection at Re=60 (Nct=100, MinLength=10, DeficiencyThresh=2) 
Parameter	MinLength
Test values	1;10;20;40;50;55;60;65;70;75;80;100;110;120;130;150;170;180;250;300;400;450;480;490;495;500;600;1000
Optimal interval	[60;495]
Vortex detection with optimal MinLength parameter	Vortex detection at Re=60 (Nct=100, MinLength=180, DeficiencyThresh=10) 
Vortex detection with optimized parameters	Vortex detection at Re=60 (Nct=100, MinLength=80, DeficiencyThresh=5) 

It can be seen that the optimal interval both for Nct and DeficiencyThresh is exactly the same as in the case of $Re = 120$. As for the MinLength parameter, I increased the upper limit of MinLength this much because the size of the evolved vortices has increased immensely also. Thus, the optimal interval can be widened, but it is still overlapping with the optimal interval for $Re = 120$. I determined the $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case as well:

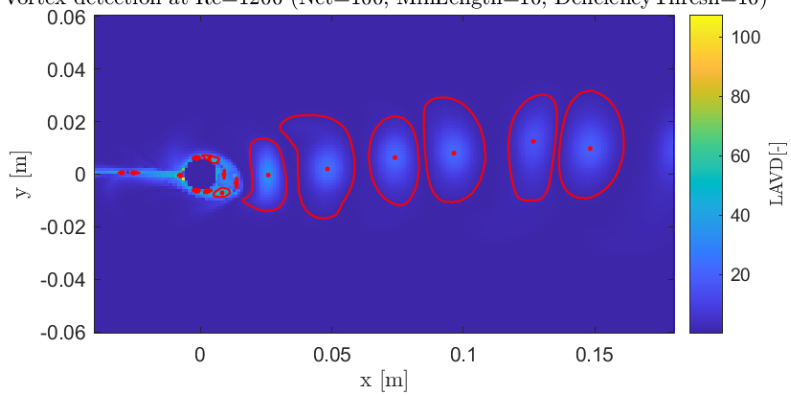
$$\frac{\Delta y_{12}}{\Delta x_{12}} = -0.4922, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = 0.4738, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = -0.4938 \quad (46)$$

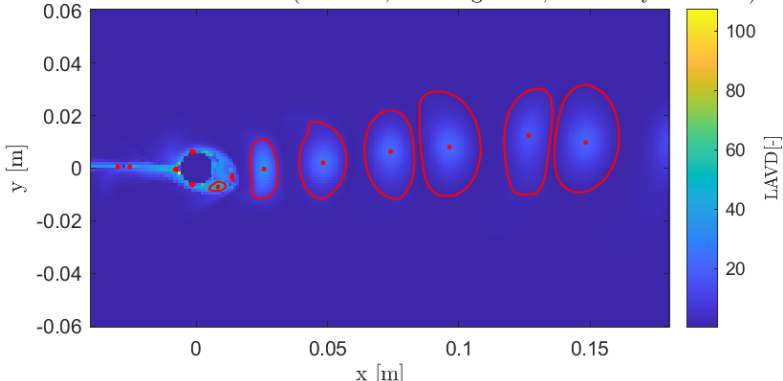
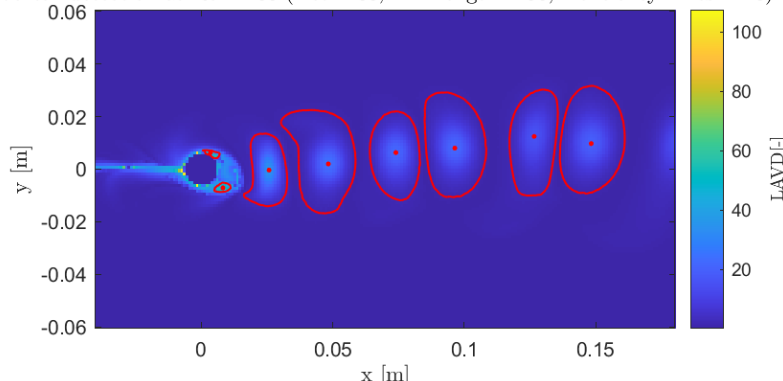
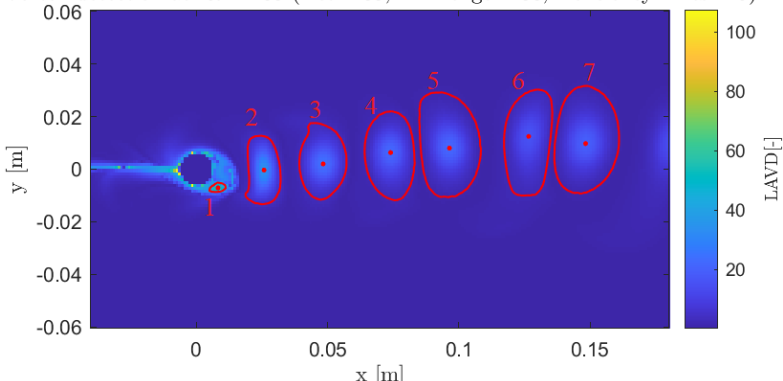
Though the value of 0.281 is only valid for a von Karman vortex, it is still good to determine these ratios, because for example one can see from Equation (46) that the vortex shedding is symmetric. The positive and negative signs are alternating, and the absolute values of the ratios are close to each other. It is important to notice that in contrast to the $Re = 120$ case, the ratios do not converge, they immediately become uniform. It seems that these ratios could be suitable for the differentiation of the different vortices and the different vortex shedding patterns.

4.4.4 Parameter optimization and vortex detection at $Re=1200$

In the case of $Re = 1200$ the code was again able to detect the evolved vortices with the optimized parameters. I performed the whole optimization process for this Reynolds numbers as well. Once again, the results are going to be presented in a table form, and the rest of the figures can be found in *Appendix 2*.

Table 8. Parameter optimization at $Re=1200$.

Parameter	Nct
Test values	1;10;50;100;200;300;400;600;800;1000
Optimal interval	[100;200]
Vortex detection with optimal Nct parameter	Vortex detection at $Re=1200$ (Nct=100, MinLength=10, DeficiencyThresh=10) 
Parameter	DeficiencyThresh
Test values	0;1;2;5;6;7;10;15;20;40;50;60;80;100
Optimal interval	[1;5]

Vortex detection with optimal DeficiencyThresh parameter	Vortex detection at Re=1200 (Nct=100, MinLength=10, DeficiencyThresh=1) 
Parameter	MinLength
Test values	1;10;20;40;60;65;70;75;80;100;110;120;125;130;150;1000
Optimal interval	[75;120]
Vortex detection with optimal MinLength parameter	Vortex detection at Re=1200 (Nct=100, MinLength=100, DeficiencyThresh=10) 
Vortex detection with optimized parameters	Vortex detection at Re=1200 (Nct=100, MinLength=80, DeficiencyThresh=5) 

The optimal interval for Nct is exactly the same as in the previous two cases. For the DeficiencyThresh parameter I had to restrict the optimal interval, because otherwise it detected vortices (while only using optimal values for each parameter) which have just been shed from the surface (e.g., at *DeficiencyThresh* = 10), and this is not favorable. Thus, I only wanted to detect vortices that had already moved from the immediate vicinity of the

cylinder (i.e., the contour of the vortex should not be touching the cylinder). Nevertheless, the optimal interval for the DeficiencyThresh at $Re = 1200$ is still overlapping with the optimal DeficiencyThresh intervals in the case of $Re = 60$ and $Re = 120$. As for the MinLength parameter, it is the most restricted for $Re = 1200$, but once again, it overlaps greatly with the optimal MinLength intervals for $Re = 60$ and $Re = 120$. The $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case:

$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.3867, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = 0.0104, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = 0.1673 \quad (47)$$

$$\frac{\Delta y_{45}}{\Delta x_{45}} = 0.0759, \quad \frac{\Delta y_{56}}{\Delta x_{56}} = 0.1490, \quad \frac{\Delta y_{67}}{\Delta x_{67}} = -0.1290 \quad (48)$$

From Equations (47)-(48) it can be seen that the vortex shedding is no longer symmetric. The positive and negative signs are not alternating, and the absolute values of the ratios are varying abruptly. One can also notice that the vortices 2-3, 4-5, and 6-7 seem as if they are shed in pairs. To see whether this can be true or not, I am going to present the distances between the centers of these vortices in the x direction:

$$\Delta x_{23} = 0.0225 [m], \quad \Delta x_{45} = 0.0224 [m], \quad \Delta x_{67} = 0.0217 [m] \quad (49)$$

From Equation (49) one can see that the assumption that these vortices are shed in pairs can be true indeed.

4.5 Summary of results

Let us start with the simulation results. For $Re = 12$ steady symmetric separation, and for $Re = 60$ a laminar unstable wake could be observed. As for $Re = 120$, symmetric periodic vortex shedding occurred, which is also known as the von Karman vortex street phenomenon. Lastly, in the case of $Re = 1200$ the flow was still laminar, however, the vortex shedding was no longer symmetric. Thus, the flow patterns were in accordance with the expected results. Moreover, a stagnation point was realized in each case, and the location of this point remained the same throughout the simulations, which was the expected result. In addition, the time-averaged values (for the converged part) of the lift and drag coefficients, as well as the Strouhal number were also determined in each case, and except for $Re = 1200$ they all complied well with the expected results. The time-averaged value of the lift coefficient was zero in all of the cases, however, the obtained drag coefficient and the Strouhal number differed from the value in the literature for $Re = 1200$. In the case of $Re = 1200$, the Reynolds number exceeds the critical Reynolds number, above which the flow transitions to three-dimensional. If this occurs, a two-dimensional simulation simply cannot capture the effect of three-dimensional structures on the flow field, and this caused the discrepancy in the results. Lastly, with the help of the Richardson extrapolation, mesh independency was ensured, thus, by using the coarse mesh the received results were sufficiently accurate.

Let us continue with the vortex detection. The code was able to detect the evolved vortices in each case, except for $Re = 12$, which is a steady case of fluid flow. In this case

the vortices are not shed, thus, it was expected that the code will not be able to detect the counter-rotating vortex pair. As for the other cases, the parameter optimization has been performed, and optimal intervals have been determined for all of the vortex detection parameters. It could be seen that in spite of the fact that the optimal intervals for the different cases are not exactly the same, each parameter has values which are optimal for all of the above cases. It is important to note that in the case of Nct all of the determined optimal intervals were the same. I also determined the intersection of the intervals for the different parameters, this can be seen in Table 9.

Table 9. Summary of the optimal intervals for the circular cylinder.

	Re=60	Re=120	Re=1200	$\cap$
Nct	[100; 200]	[100; 200]	[100; 200]	[100; 200]
DeficiencyThresh	[1; 10]	[1; 10]	[1; 5]	[1; 5]
MinLength	[60; 495]	[75; 180]	[75; 120]	[75; 120]

From Table 9 one can see that the values for Nct and DeficiencyThresh could be determined more or less independently of the Reynolds number. It is also important to mention that the value of MinLength is in great accordance with the size of the cylinder (partly, this is why it was beneficial to express the minimal arc-length threshold as the percentage of the diameter of the cylinder). What is meant by this is that the sizes of the shed vortices are not excessively smaller or larger than the cylinder itself, they are in great accordance. During the parameter optimization for the square cylinder (*Chapter 5*), I am going to take these findings into account, I am mostly going to perform test runs for values in the vicinity of the results in Table 9.

Additionally, I determined the ratios of the coordinate differences between the neighboring vortices in each case. In the case of $Re = 60$ one could see that the vortex shedding is symmetric, and the ratios do not converge, they immediately become uniform. For $Re = 120$ the ratios are close to 0.281, and they also converge to this value further downstream from the cylinder, thus, the criterion for the von Karman vortex street was met. At last, in the case of $Re = 1200$, the vortex shedding was not symmetric, although, the possibility of the vortices shedding in pairs was noticed, and by calculating the distances in the x direction this theory was supported. All in all, one can notice that these ratios could be suitable for the differentiation of the different vortices, and the different types of vortex shedding.

5. Square cylinder

5.1 Simulation setup

5.1.1 Geometrical model

I constructed the geometries based on [5], and with the help of SpaceClaim. The simulation is still a two-dimensional simulation, thus, I modelled the square cylinder with a square. The side length of the square is $a = 0.0127 \text{ [m]}$ (this value is the same as the diameter of the circular cylinder). The projected frontal height (h) can be calculated with Equation (4), as it was mentioned in 2.2. A difference in contrast to the circular cylinder is the angle of attack. I constructed three different geometries with the following angles of attack: $0^\circ, 5^\circ, 45^\circ$. The entire computational domain is defined as $-33.5h \leq x \leq 36.5h$, and $-50h \leq y \leq 50h$. As a representative example, the constructed geometry in the case of $\theta = 0^\circ$ can be seen in Figure 35 (the other two geometries can be found in *Appendix 3*).

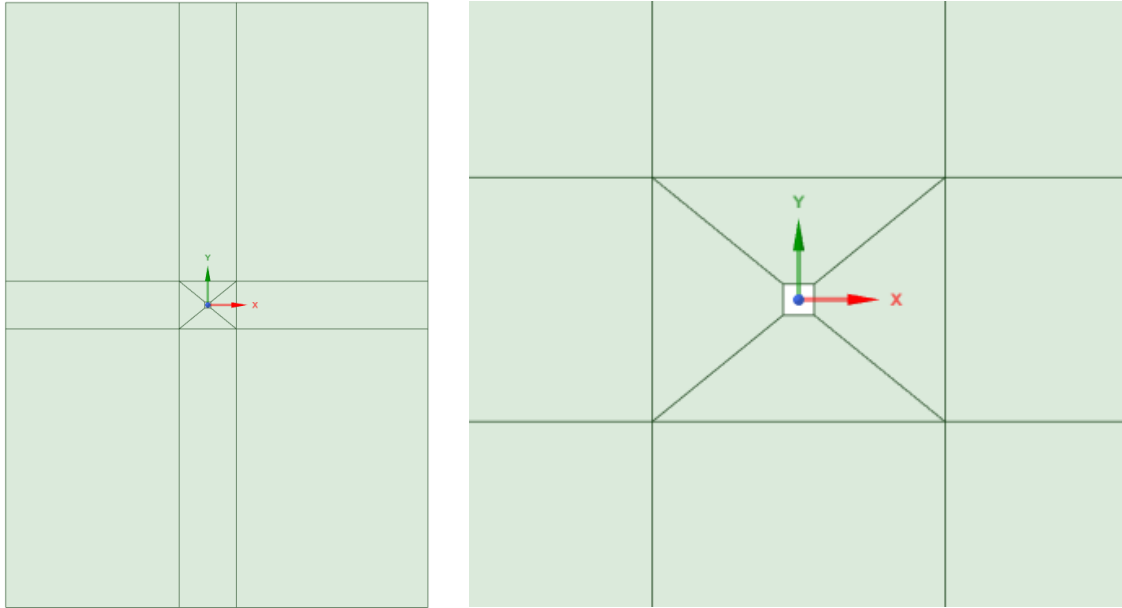


Figure 35. The geometrical model for $\theta=0^\circ$.

5.1.2 Mesh

I generated a structured, quadrilateral mesh in each case using the ANSYS Workbench Mesher. I refined the mesh near the cylinder with the help of edge sizing, and I applied face meshing. Some of the main characteristics of the different meshes can be found in Table 10, and the mesh for the $\theta = 0^\circ$ case can be seen in Figure 36 (the mesh in the case of the other two geometries can be found in *Appendix 4*). The entire mesh is not shown due to the large size of the domain. I inserted figures closer to the cylinder, so that the mesh can be observed in the more critical parts of the fluid domain.

Table 10. Characteristics of the different meshes.

	$\theta = 0^\circ$	$\theta = 5^\circ$	$\theta = 45^\circ$
Number of cells	42225	42225	41590
Maximum skewness	0.5652	0.6279	0.5004
Smallest cell size [m]	$8.6685 \cdot 10^{-5}$	$8.1263 \cdot 10^{-5}$	$8.0438 \cdot 10^{-5}$

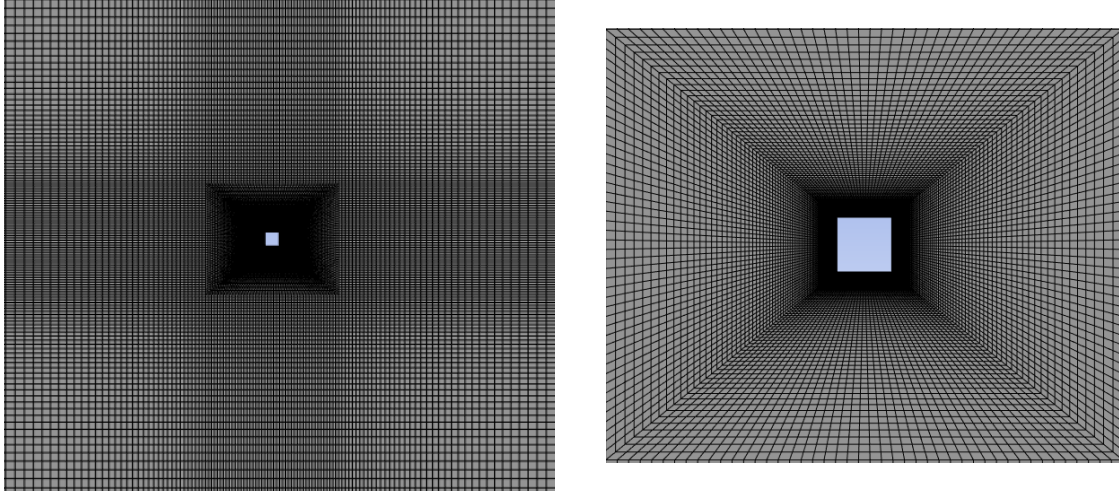


Figure 36. The quadrilateral mesh for $\theta=0^\circ$.

5.1.3 Simulation settings

The simulation settings are completely identical to the ones which were described in 4.1.3. The simulations were run using Fluent 2022 R2. I ran simulations for various θ and Re pairings. I mostly chose θ and Re pairings in order to examine the different flow patterns (Figure 3), but also, to test the findings in [5] by choosing values which are close to the boundaries of the different regions. These θ and Re combinations, along with the inlet velocities, the time step sizes, and the number of time steps can be found in Table 11.

Table 11. Simulation settings for the different cases.

		Reynolds number	Inlet velocity $\left[\frac{m}{s}\right]$	Time step [s]	Number of time steps
Angle of incidence $[\circ]$	0°	50	0.0564	0.001	40000
		120	0.1409	0.0006	35000
		185	0.2113	0.0004	37500
	5°	120	0.1297	0.0006	35000
	45°	120	0.0996	0.0008	37500

5.2 Results and discussion

5.2.1 Velocity profiles and flow patterns

For the discussion of the simulation results I inserted pathline plots colored by the velocity magnitude in order to be able to examine the different flow patterns (S, MS, SSV, DSV, VM) properly. At first, it is important to talk about the location of the stagnation point. For the circular cylinder, one could observe that the stagnation point is always located at a fixed position, this comes from the fact that for a circular cylinder an angle of incidence cannot be defined (the geometry is always the same). However, this is not true for the square cylinder. With the introduction of θ one will be able to observe how the stagnation point is shifted while changing the angle of incidence. Naturally, for a given θ the stagnation point remains at the same position.

In Figure 2 one could observe the critical Reynolds number (as a function of θ) above which vortex shedding occurs. According to the authors of [5] in the case of $\theta = 0^\circ$ and $Re = 50$ the Reynolds number is above the critical Reynolds number. However, one can see from Figure 37 that the flow is steady and symmetric ($\theta = 0^\circ$), a counter-rotating vortex pair can be observed. Nevertheless, in Figure 2 one can also observe the findings of other authors, and one of these findings is in accordance with the current result. As for the stagnation point, it is located at the center of the front edge (this is true for all of the $\theta = 0^\circ$ cases).

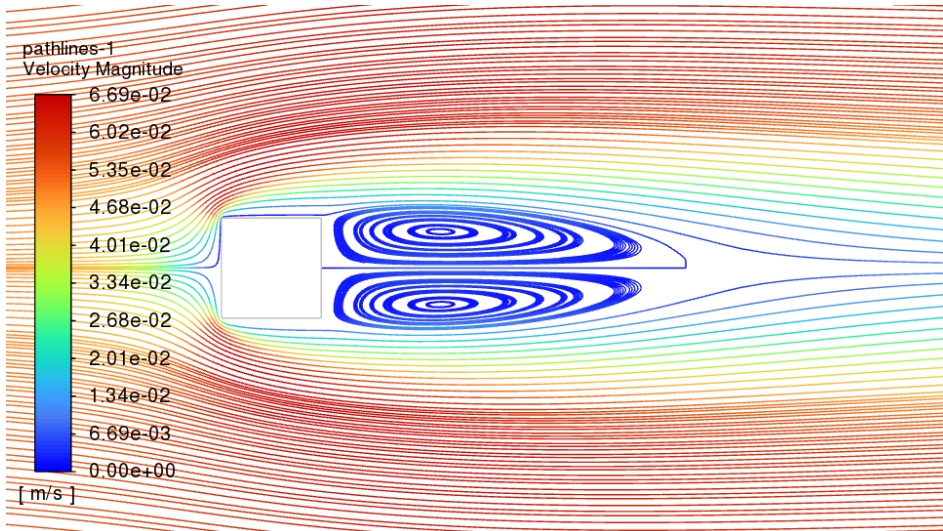


Figure 37. Pathlines for the steady pattern.

The second case to be examined is the $\theta = 0^\circ$ and $Re = 120$ case. According to Figure 3, this is the MS pattern. By observing Figure 38, one can see that the vortices of opposite sign are shed in an alternating manner, which was the expected result.

I ran one additional simulation whilst θ remained zero. In this case $Re = 185$. I chose this value in order to have a simulation which has not been examined in [5]. In addition, for this case a three-dimensional instability is expected ([5], pp. 11). By taking a look at Figure 39, it can be seen that the flow is separated on both upstream edges, and the secondary

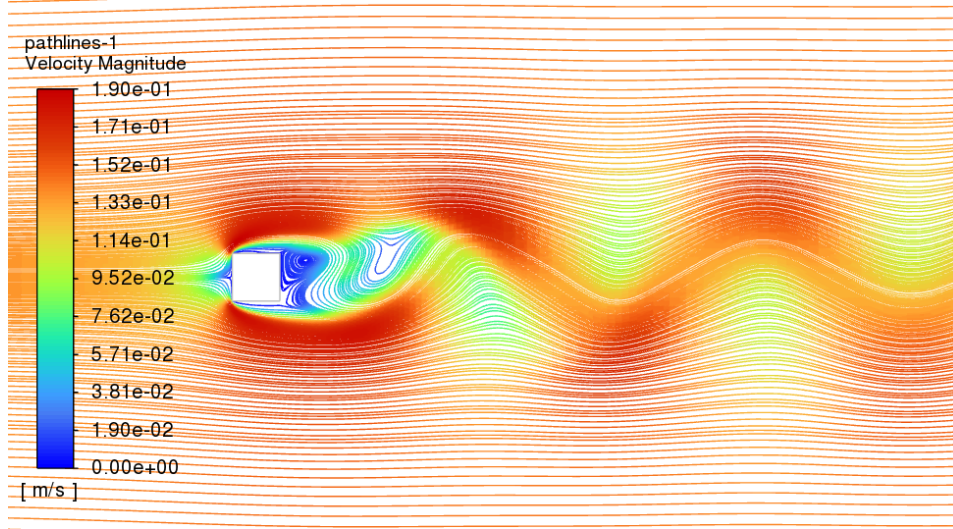


Figure 38. Pathlines for the MS pattern.

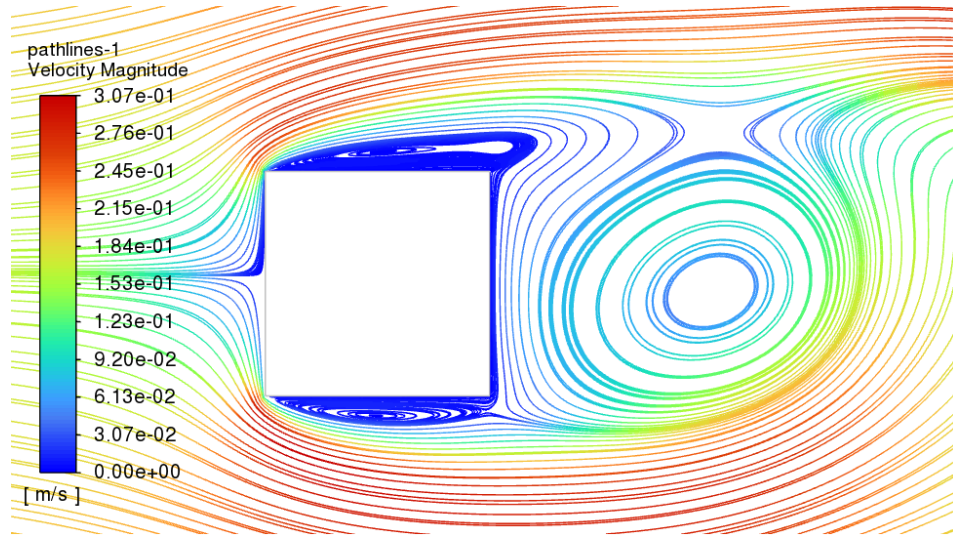


Figure 39. Pathlines for the DSV pattern.

vortices appear both on the lower and upper sides of the cylinder. Thus, by observing Figure 39, one can identify the DSV pattern. The DSV pattern is found when θ is very small, and Re is high ([5], pp. 5).

In the next case $\theta = 5^\circ$ and $Re = 120$. One can see from Figure 40 that the flow is only separated at one of the upstream edges, thus, the secondary vortex only appears on the upper side of the cylinder. Therefore, though according to Figure 3 this case corresponds to the MS pattern, one can see that according to the current results it corresponds to the SSV pattern. In ([5], pp. 5) it is stated that the SSV pattern appears approximately when $\theta \leq 5.1^\circ$ and $70 \leq Re \leq 110$, but one can deduce from Figure 40 that according to the current findings it is still valid for $Re \approx 120$. It can also be observed in Figure 40 that the stagnation point moved towards the lower edge of the cylinder with the increase of θ , which was the expected result ([5], pp. 8). This will be observable for the last case also.

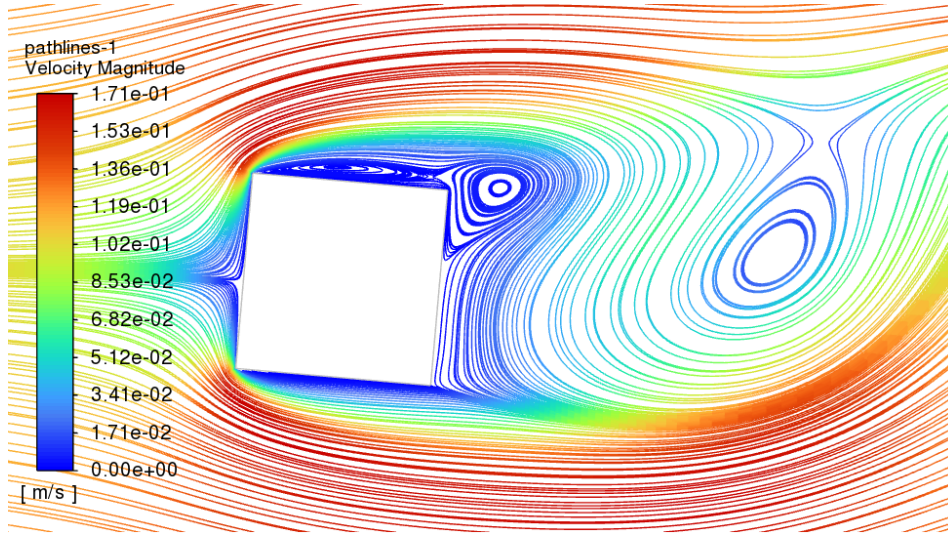


Figure 40. Pathlines for the SSV pattern.

According to Figure 3, for $\theta = 45^\circ$ and $Re = 120$, the so-called VM pattern should occur. This means the generation and merging of two small vortices. The pathlines for the VM pattern can be seen in Figure 41. Once again, the stagnation point is shifted towards the lower edge of the cylinder.

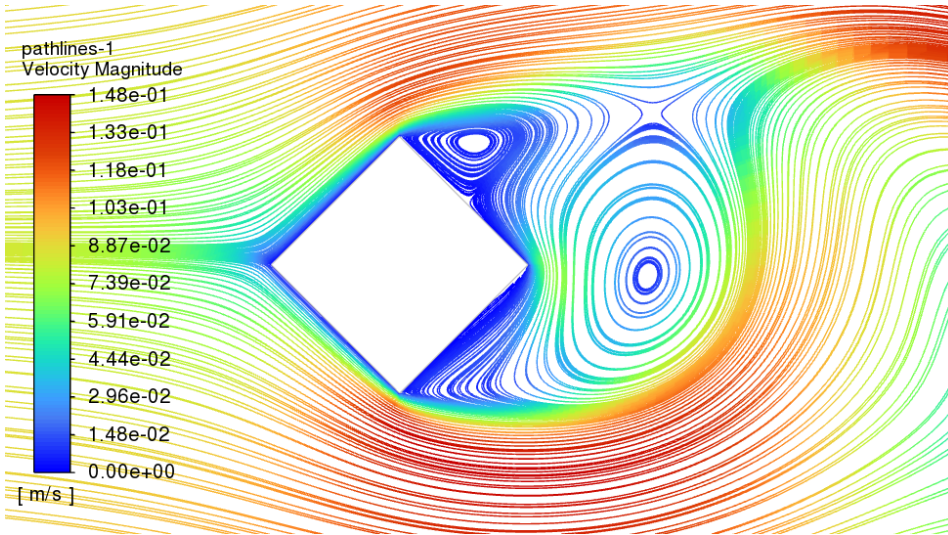


Figure 41. Pathlines for the VM pattern.

From the flow patterns one can make a few deductions. At first, the boundaries of the regions should not be treated too strictly. Secondly, the difference between the current results and the results in [5] can be caused by the fact that in [5] they used a different solver with a different boundary condition on the outlet.

At last, I determined the Courant numbers for the different cases. These values can be found in Table 12, and one can see that they are sufficiently low.

Table 12. Courant numbers for the different cases.

		Reynolds number	Maximum value of the Courant number
Angle of incidence [°]	$\theta = 0^\circ$	50	0.9198
		120	1.9906
		185	2.1920
	$\theta = 5^\circ$	120	2.0117
	$\theta = 45^\circ$	120	1.1705

5.2.2 Lift and drag coefficient

As it was mentioned in 2.5.1, the time-averaged value of the lift coefficient is zero if the flow is symmetric, for the square cylinder the time-averaged lift coefficient is zero if $\theta = 0^\circ, 45^\circ$. This was fulfilled throughout the simulations, as can be seen from Table 13.

Table 13. The time-averaged lift coefficient values.

		Reynolds number	Time-averaged value of the lift coefficient
Angle of incidence [°]	$\theta = 0^\circ$	50	0
		120	0
		185	0
	$\theta = 5^\circ$	120	-0.0784
	$\theta = 45^\circ$	120	0

The time-averaged lift coefficient value for $\theta = 5^\circ$ is slightly smaller than the expected value in Figure 5. The difference can be caused by the already mentioned different solver, and the different boundary condition on the outlet. Nevertheless, this value still complies well with the expected result. The obtained drag coefficient values can be found in Table 14.

Table 14. The time-averaged drag coefficient values.

		Reynolds number	Time-averaged value of the drag coefficient
Angle of incidence [°]	$\theta = 0^\circ$	50	1.5561
		120	1.4716
		185	1.5231
	$\theta = 5^\circ$	120	1.4102
	$\theta = 45^\circ$	120	1.8414

Let us look at the drag coefficient results. For the steady case ($\theta = 0^\circ$, $Re = 50$), the time-averaged drag coefficient value is in accordance with the expected result (Figure 7(b)). For $\theta = 0^\circ$ the drag coefficient decreases as Re increases ([5], pp. 8). For $Re = 120$ this is true, and even though the time-averaged drag coefficient is slightly higher than the expected result (Figure 7(a)), it still complies well with the expected value. Once again, this small difference between the results can be caused by the different solver, and the different boundary condition on the outlet. For $Re = 185$ the decline of the drag coefficient is not observable. As it was mentioned in 4.2.4.2, if Re exceeds a given critical value, the two-dimensional flow transitions to a three-dimensional one, and in this case, a two-dimensional simulation cannot capture the three-dimensional effects, which leads to inaccurate lift and drag forces. Above this critical value the two-dimensional simulations usually overpredict the drag coefficient. According to ([5], pp. 11) for $\theta = 0^\circ$ this three-dimensional transition should occur around $Re \approx 170$. I checked another publication with regard to this topic. According to ([11], pp. 44), in the case of a square cylinder several critical Re values have been determined for the transition from a two-dimensional flow to a three-dimensional flow, one of these values is the following: 162 ± 12 (for $\theta = 0^\circ$). In addition, in [11] the authors also determined the critical Reynolds numbers for different θ at which three-dimensional instability happens, and their findings were also similar ([11], pp. 49). In the present case $Re = 185$, thus, the discrepancy in the drag coefficient can once again be explained by the three-dimensional transition. For the last two cases the same thing can be said as for the $\theta = 0^\circ$ and $Re = 120$ case, the obtained values are slightly higher, but this might be due to the different boundary condition on the outlet, and the different solver. Nonetheless, the results are acceptably close to the expected results in Figure 7(a). It is also important to note that according to ([5], pp. 9), in the unsteady range the maximum drag for a given Re is achieved at $\theta = 45^\circ$. It can be seen from Table 14 that this is fulfilled in this case.

5.2.3 Strouhal number

I performed the FFT for the time signal of the lift coefficient in each unsteady case. Afterwards, I determined the vortex shedding frequencies, and computed the Strouhal numbers with Equation (10). The results can be seen in Table 15.

Table 15. Strouhal number values for the different cases.

		Reynolds number	Vortex shedding frequency [Hz]	Strouhal number
Angle of incidence [°]	$\theta = 0^\circ$	120	1.6363	0.1475
		185	2.3749	0.1427
	$\theta = 5^\circ$	120	1.4443	0.1537
		120	0.9999	0.1803

It can be seen from Figure 8 that the Strouhal number monotonically increases with increasing θ and Re . From Table 15 one can see that this is not fulfilled only in one case,

when the aforementioned three-dimensional transition happens ($\theta = 0^\circ, Re = 185$). Apart from the $\theta = 0^\circ, Re = 185$ case, the values in Table 15 are almost in perfect accordance with Figure 8, and the possible causes of the accidentally observable small differences have already been explained before.

5.3 Mesh independency study

I performed the mesh independency study for the $\theta = 0^\circ$ arrangement, and also, for a Reynolds number of 120. As for $\theta = 5^\circ$ and $\theta = 45^\circ$, in those cases the mesh is almost completely identical to the $\theta = 0^\circ$ arrangement. Thus, it can be said that if mesh independency is fulfilled for $\theta = 0^\circ$, then it is fulfilled for $\theta = 5^\circ$ and $\theta = 45^\circ$ as well.

Contrary to 4.3, this time I created a coarse, and a fine mesh, and the original mesh is considered to be the medium one. Another difference compared to 4.3 is that this time the refinement ratio is the following:

$$r = 1.5 \quad (50)$$

This means that the coarse mesh is going to consist of 2.25-times less cells than the medium one, and the fine mesh is going to consist of 2.25-times more cells than the medium one. I did not use Manual Mesh Adaptation this time, instead I manually created the coarse and the fine mesh, while aiming the refinement ratio to be 1.5.

In this case, I performed the Richardson extrapolation for the RMS value of the lift coefficient. This is due to the fact that for the drag coefficient I received a divergent result for the Richardson extrapolation. However, in that case too, the relative error of the coarse mesh result with regard to the fine mesh result was less than 1 [%].

In Table 16 the number of the cells, the maximum values of the Courant numbers and the RMS values of the lift coefficients can be seen for the different meshes.

Table 16. Characteristics and results in the case of the different meshes.

	Coarse mesh	Medium mesh	Fine mesh
Number of cells	18740	42225	95840
Maximum value of the Courant number	1.8047	1.9906	1.8037
RMS value of the lift coefficient	0.2563	0.2458	0.2398

The measured order of convergence is the following:

$$p_m = \log_r \left(\frac{\phi_{2h} - \phi_{4h}}{\phi_h - \phi_{2h}} \right) = 1.3755 \quad (51)$$

I used second-order schemes during the simulations, thus, from Equation (51) one can see that the theoretical accuracy of the numerical schemes has not been that sufficiently realized, however the value of the discretization error is satisfactory. The discretization error:

$$\varepsilon_h \cong \frac{\phi_h - \phi_{2h}}{r^{p_m} - 1} = -4.7665 \cdot 10^{-3} \quad (52)$$

The discretization error is sufficiently small. Moreover, the Courant numbers are also acceptable, and divergence did not occur during the simulations. Using these results the extrapolated value of the RMS value of the lift coefficient can be calculated:

$$\phi_0 = \phi_h + \varepsilon_h = 0.2351 \quad (53)$$

The Richardson plot can be seen in Figure 42.

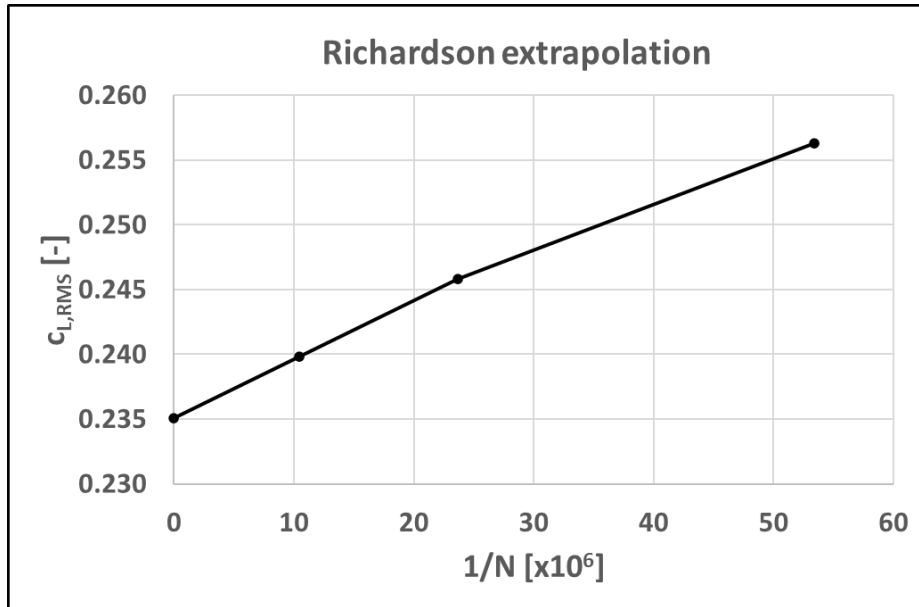


Figure 42. Richardson plot.

5.4 Vortex detection with the help of MATLAB

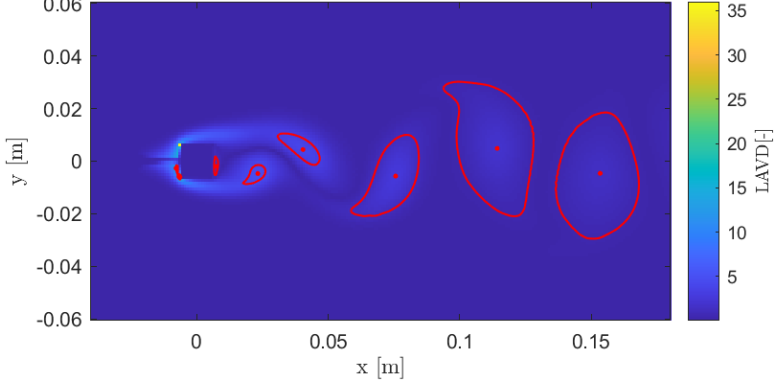
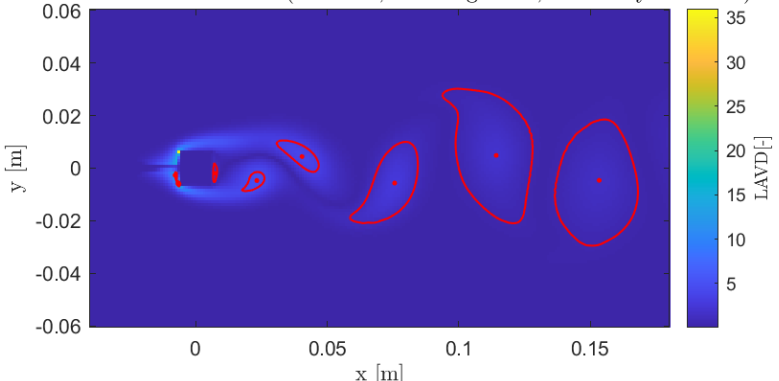
Similarly to 4.4.3 and 4.4.4 I will present the detection procedure in a table form. An essential thing to mention is the fact that for the square cylinder the MinLength parameter is expressed as the percentage of the projected frontal height (Figure 1(b)). The detection worked for all of the cases, the only exception (similarly to the circular cylinder) is the steady case of fluid flow. I performed the parameter optimization taking into consideration the results obtained from the circular cylinder case. Meaning, I mostly ran the detection for values which are close to or part of the already determined optimal intervals. The ideal intervals for each parameter, along with the corresponding figures (representing the detection with optimal parameters from the optimal intervals) can be seen in the tables below, and the rest of the figures can be found in *Appendix 2*.

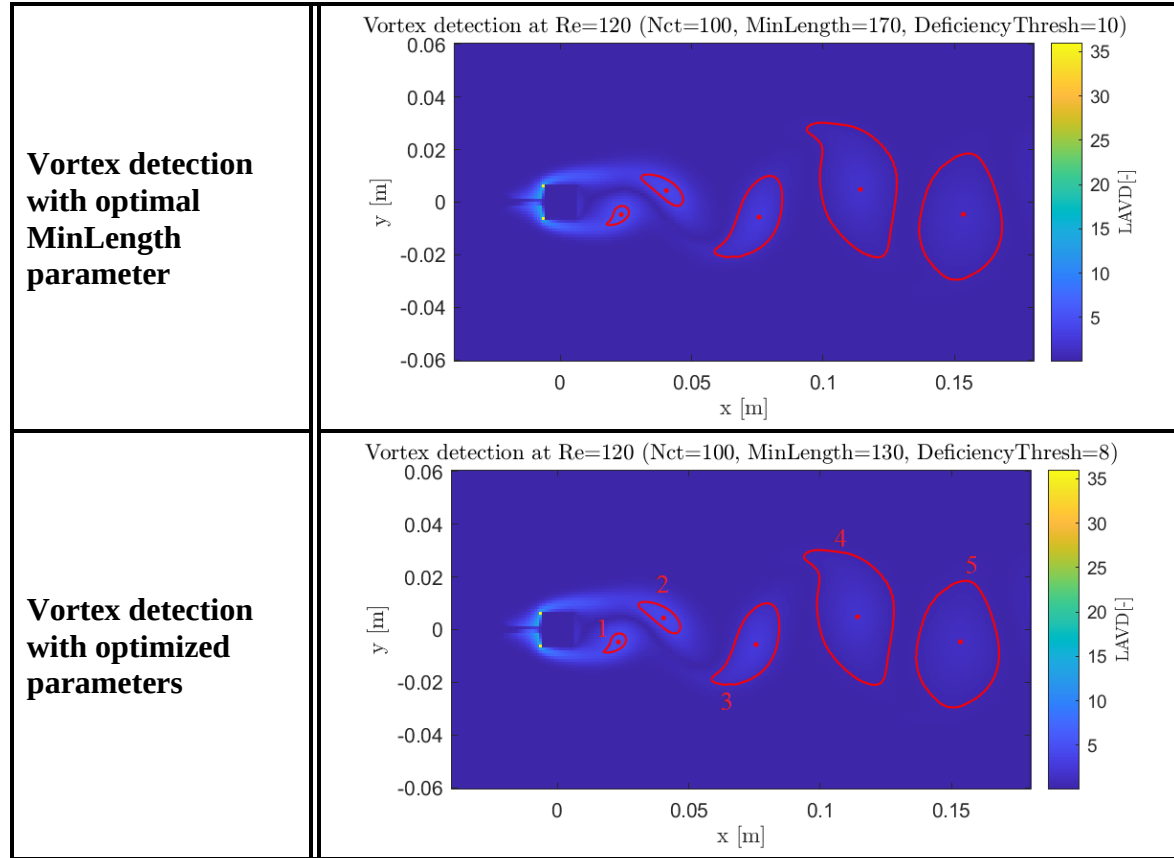
5.4.1 $\theta=0^\circ$ and $Re=50$

Similarly to 4.4.2 the code was not able to detect the counter-rotating vortex pair at the back of the square cylinder. In this case too, a steady fluid flow is present, thus, these vortices are at a fixed place, and they are not shed.

5.4.2 $\theta=0^\circ$ and $Re=120$

Table 17. Parameter optimization at $\theta=0^\circ$ and $Re=120$.

Parameter	Nct
Test values	100;200;400;600;800
Optimal interval	[100;200]
Vortex detection with optimal Nct parameter	Vortex detection at $Re=120$ (Nct=100, MinLength=10, DeficiencyThresh=10) 
Parameter	DeficiencyThresh
Test values	0;5;6;7;8;10;15;20
Optimal interval	[7;10]
Vortex detection with optimal DeficiencyThresh parameter	Vortex detection at $Re=120$ (Nct=100, MinLength=10, DeficiencyThresh=7) 
Parameter	MinLength
Test values	20;40;60;70;80;90;120;125;130;150;170;190;195;200
Optimal interval	[125;190]



The $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case:

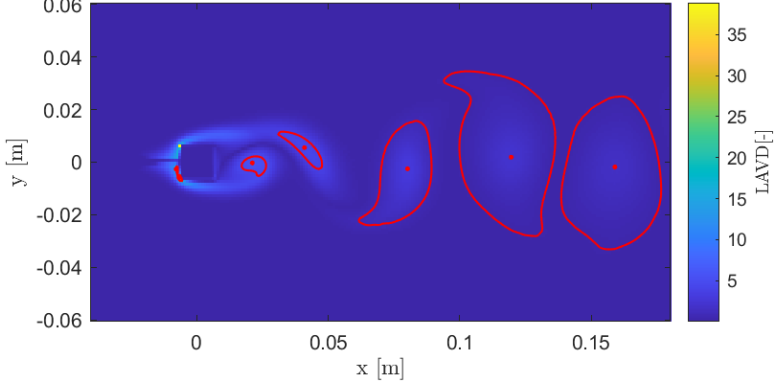
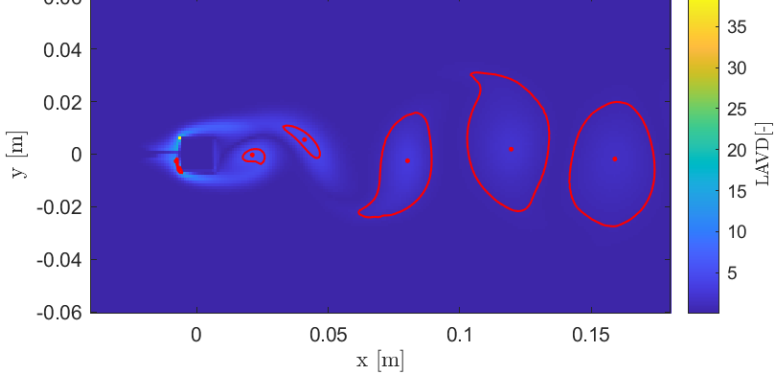
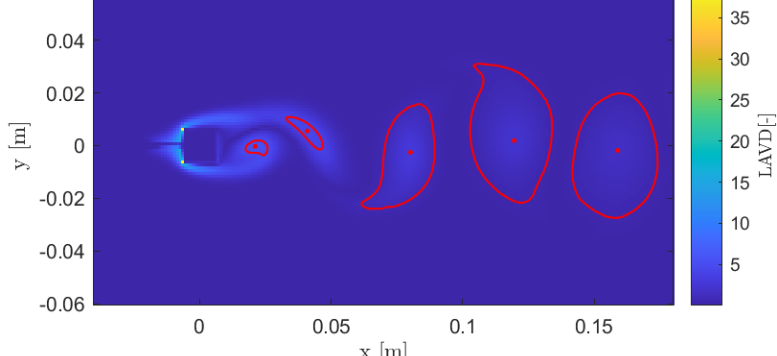
$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.5322, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = -0.2869, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = 0.2720, \quad \frac{\Delta y_{45}}{\Delta x_{45}} = -0.2344 \quad (54)$$

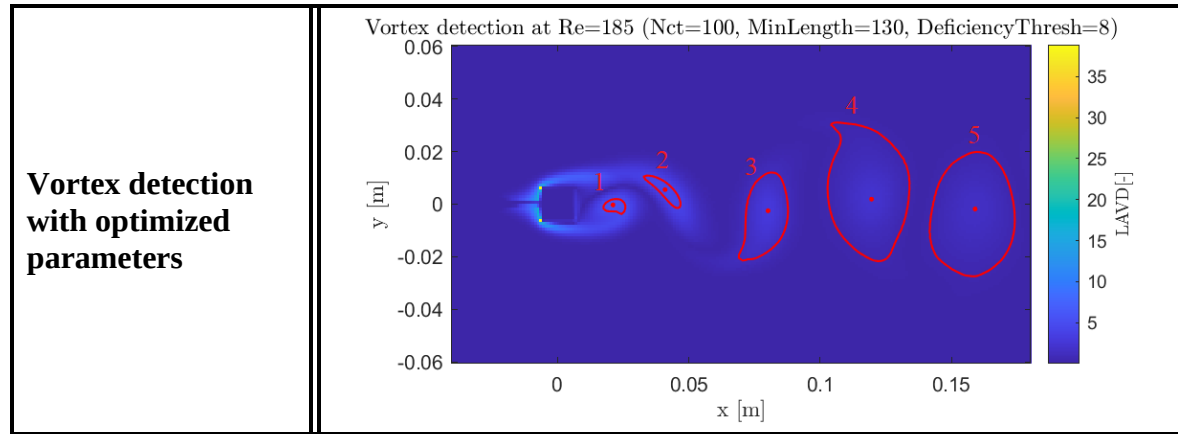
From Equation (54) it can be seen that the ratios are close to the ideal value of 0.281, and similarly to 4.4.1.2 they converge to the theoretical value further downstream from the cylinder.

5.4.3 $\theta=0^\circ$ and Re=185

Table 18. Parameter optimization at $\theta=0^\circ$ and Re=185.

Parameter	Nct
Test values	100;200;400;600;800
Optimal interval	[100;200]

Vortex detection with optimal Nct parameter	Vortex detection at Re=185 (Nct=200, MinLength=10, DeficiencyThresh=10) 
Parameter	DeficiencyThresh
Test values	0;5;6;7;8;10;15;20
Optimal interval	[6;10]
Vortex detection with optimal DeficiencyThresh parameter	Vortex detection at Re=185 (Nct=100, MinLength=10, DeficiencyThresh=10) 
Parameter	MinLength
Test values	20;40;50;55;60;80;100;140;160;170;175;180
Optimal interval	[60;175]
Vortex detection with optimal MinLength parameter	Vortex detection at Re=185 (Nct=100, MinLength=140, DeficiencyThresh=10) 



The $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case:

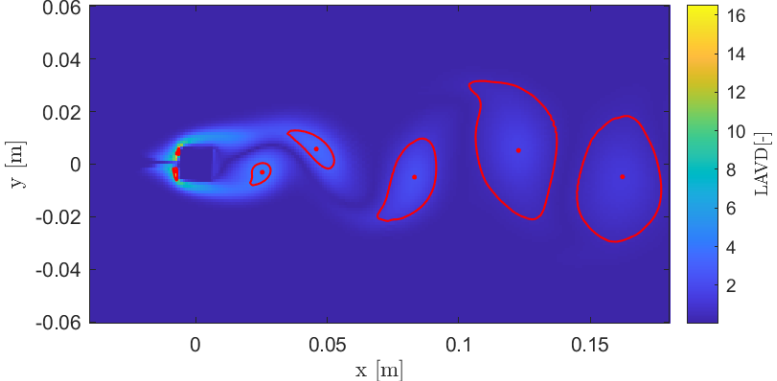
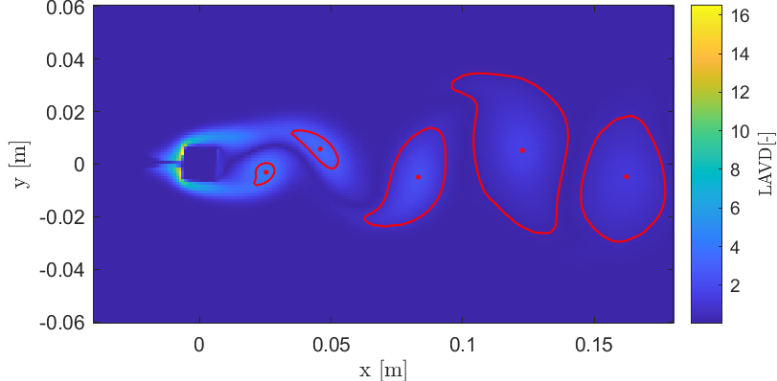
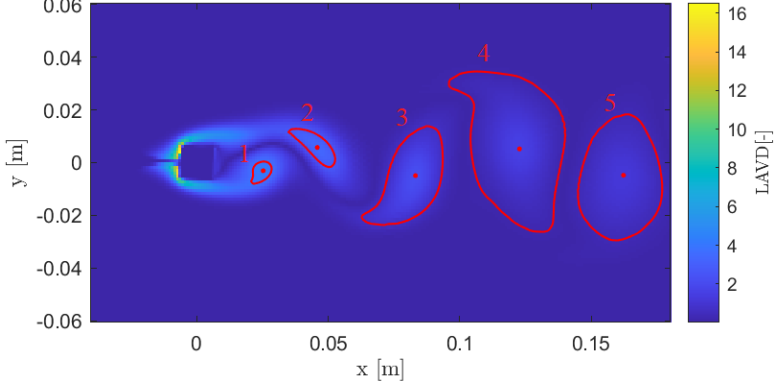
$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.2961, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = -0.2066, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = 0.1117, \quad \frac{\Delta y_{45}}{\Delta x_{45}} = -0.0939 \quad (55)$$

From Equation (55) one can see that the ratios differ from the conceptual value of 0.281. Thus, the results do not meet the criterion for the von Karman vortex street. This can be caused by the fact that in this case Re exceeds the critical Reynolds number above which the flow transitions to three-dimensional. As it was mentioned, if this occurs, a two-dimensional simulation cannot capture the effect of three-dimensional structures on the flow field, and this explains why the 0.281 criterion was not met.

5.4.4 $\theta=5^\circ$ and $Re=120$

Table 19. Parameter optimization at $\theta=5^\circ$ and $Re=120$.

Parameter	Nct
Test values	100;200;400;600;800
Optimal interval	[100;200]
Vortex detection with optimal Nct parameter	Vortex detection at Re=120 (Nct=100, MinLength=10, DeficiencyThresh=10)

Parameter	DeficiencyThresh
Test values	0;1;5;10;15;20
Optimal interval	[1;10]
Vortex detection with optimal DeficiencyThresh parameter	Vortex detection at Re=120 (Nct=100, MinLength=10, DeficiencyThresh=5) 
Parameter	MinLength
Test values	20;40;60;80;120;125;130;140;160;180;185;190;200
Optimal interval	[130;185]
Vortex detection with optimal MinLength parameter	Vortex detection at Re=120 (Nct=100, MinLength=160, DeficiencyThresh=10) 
Vortex detection with optimized parameters	Vortex detection at Re=120 (Nct=100, MinLength=130, DeficiencyThresh=8) 

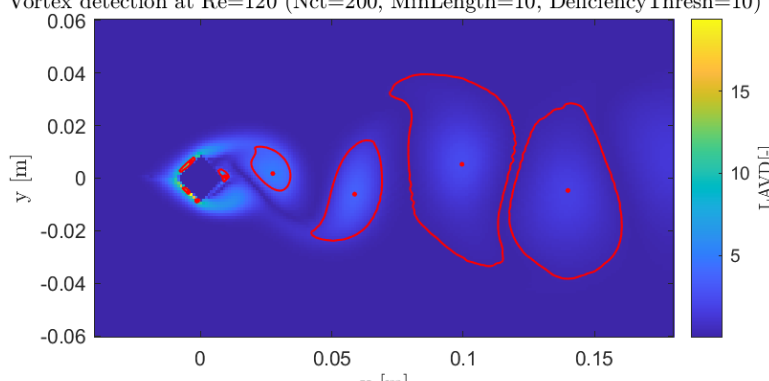
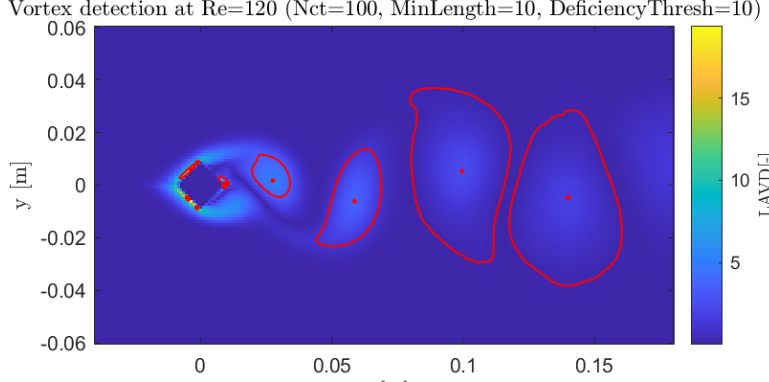
The $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case:

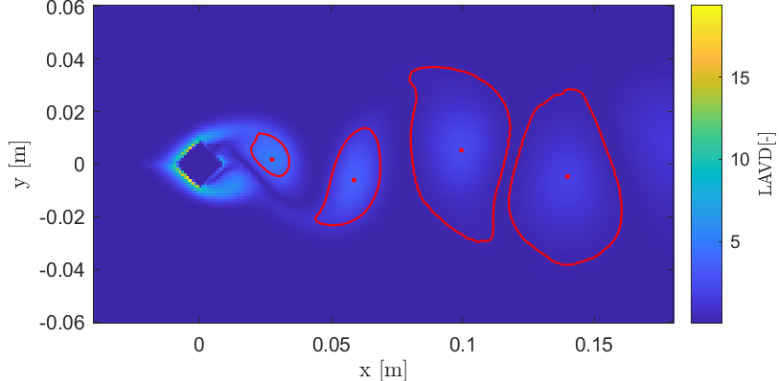
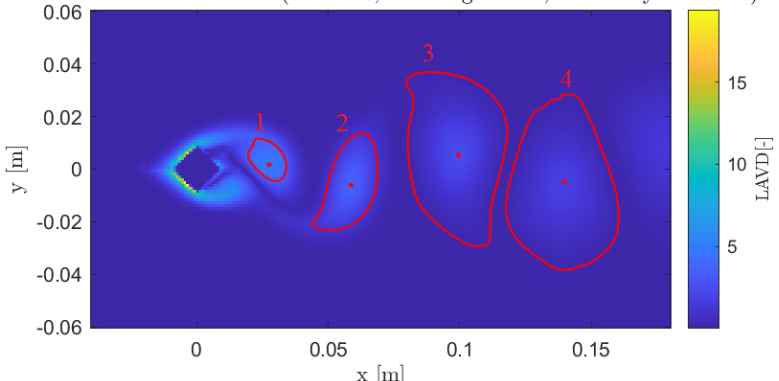
$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.4341, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = -0.2869, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = 0.2551, \quad \frac{\Delta y_{45}}{\Delta x_{45}} = -0.2532 \quad (56)$$

From Equation (56) it can be seen that the ratios are close to 0.281, and they converge to the theoretical value.

5.4.5 $\theta=45^\circ$ and $Re=120$

Table 20. Parameter optimization at $\theta=45^\circ$ and $Re=120$.

Parameter	Nct
Test values	100;200;400;600;800
Optimal interval	[100;200]
Vortex detection with optimal Nct parameter	Vortex detection at Re=120 (Nct=200, MinLength=10, DeficiencyThresh=10) 
Parameter	DeficiencyThresh
Test values	0;1;5;10;15;20
Optimal interval	[1;10]
Vortex detection with optimal DeficiencyThresh parameter	Vortex detection at Re=120 (Nct=100, MinLength=10, DeficiencyThresh=10) 

Parameter	MinLength
Test values	20;40;60;80;90;95;100;160;200;220;260;265;270;280;300
Optimal interval	[95;260]
Vortex detection with optimal MinLength parameter	Vortex detection at Re=120 (Nct=100, MinLength=220, DeficiencyThresh=10) 
Vortex detection with optimized parameters	Vortex detection at Re=120 (Nct=100, MinLength=130, DeficiencyThresh=8) 

The $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case:

$$\frac{\Delta y_{12}}{\Delta x_{12}} = -0.2540, \quad \frac{\Delta y_{23}}{\Delta x_{23}} = 0.2794, \quad \frac{\Delta y_{34}}{\Delta x_{34}} = -0.2475 \quad (57)$$

From Equation (57) it can be seen that in this case as well, the calculated ratios are close to 0.281, and also, they converge to this value.

5.5 Summary of results

Let us start with the obtained flow patterns. The aim was to choose θ and Re values such a way that the main flow patterns (MS, SSV, DSV, VM, S) which were mentioned in 2.3 and can be seen in Figure 3, could be visualized. In addition, I wanted to test the boundaries of the different regions, and I aimed to simulate a case in which three-dimensional instability occurs. From the simulation results one could see that the obtained flow patterns were mostly in accordance with the expected results, however, it was also deduced that the boundaries of the different regions should not be treated too strictly, moreover, the difference between the

results can also be caused by the fact that in [5] they used a different solver, and a different boundary condition on the outlet. In addition, a stagnation point was realized in each case, and one could see that the stagnation point moved towards the lower edge of the cylinder with increasing θ , which was the expected result. Moreover, the time-averaged values (for the converged part) of the drag and lift coefficients, as well as the Strouhal numbers were determined also. These values were in accordance with the expected results except for the $\theta = 0^\circ$, $Re = 185$ case, when the already mentioned three-dimensionality caused the discrepancy in the results. Before the vortex detection, a mesh independency study ensured grid independency with the help of the Richardson extrapolation. Thus, by using the medium mesh the received results were sufficiently accurate.

As for the parameter optimization, similarly to the case of the circular cylinder, despite the fact that the optimal intervals for the different cases are not exactly the same, each parameter has values which are optimal for all of the examined cases. Once again, in the case of Nct all of the determined optimal intervals are the same, and the values for Nct and DeficiencyThresh could be determined almost independently of the Reynolds number. It can be observed for the square cylinder also, that the value of MinLength is in accordance with the size of the cylinder, more accurately, the projected frontal height. It is observable that for the $\theta = 45^\circ$ arrangement, the shed vortices are bigger than in the case of $\theta = 0^\circ$ and $\theta = 5^\circ$. In Table 21 I summarized the optimal intervals for the vortex detection parameters for the different cases. In addition, I determined the intersection of the intervals.

Table 21. Summary of the optimal intervals for the square cylinder.

	$\theta=0^\circ$ and Re=120	$\theta=0^\circ$ and Re=185	$\theta=5^\circ$ and Re=120	$\theta=45^\circ$ and Re=120	$\cap$
Nct	[100; 200]	[100; 200]	[100; 200]	[100; 200]	[100; 200]
DeficiencyThresh	[7; 10]	[6; 10]	[1; 10]	[1; 10]	[7; 10]
MinLength	[125; 190]	[60; 175]	[130; 185]	[95; 260]	[130; 175]

As for the obtained ratios, one could see that in the case of $\theta = 0^\circ$, $Re = 120$; $\theta = 5^\circ$, $Re = 120$, and $\theta = 45^\circ$, $Re = 120$, the ratios were close to 0.281, thus, the criterion for the von Karman vortex street was met in all of these cases. However, for $\theta = 0^\circ$, $Re = 185$ the ratios were varying abruptly, they did not converge to 0.281, the criterion for the von Karman vortex street was not met in this case. This occurrence was explained by the fact that in this case Re exceeds the critical Reynolds number above which the flow transitions to three-dimensional. If this occurs, a two-dimensional simulation cannot capture the effect of three-dimensional structures on the flow field, and this explains why the 0.281 criterion was not met.

6. Summary of results

6.1 Summary in English

The main aim of the thesis was to optimize the parameters of an existing vortex detection method and detect the evolved vortices in fluid flow around a circular cylinder and a square cylinder. The data, for which I performed the detection, was a result of two-dimensional simulation for different Reynolds numbers (and angles of attack in the case of the square cylinder). The validation of the simulation results happened by comparing the obtained velocity and pressure distributions with figures from the literature, in addition, the lift coefficient, the drag coefficient, and the Strouhal number were presented for all of the cases, and they were compared with values taken from the literature.

In the case of the circular cylinder, the received flow patterns for the different Reynolds numbers were in accordance with expected results. For a Reynolds number of 12, 60, and 120 the obtained lift and drag coefficient values, as well as the Strouhal numbers for $Re = 60$ and $Re = 120$ were acceptably close to the values taken from the literature. The time-averaged value of the lift coefficient was zero in all of the three cases, and the drag coefficient decreased as the Reynolds number increased. For $Re = 1200$, the time-averaged lift coefficient was zero, however, the drag coefficient, and the Strouhal number were higher than the values taken from the literature. In the case of $Re = 1200$, the Reynolds number exceeds the critical Reynolds number, above which the flow transitions to three-dimensional. If this occurs, a two-dimensional simulation cannot capture the effect of three-dimensional structures on the flow field, and this caused the discrepancy in the results. Later on, a mesh independency study ensured grid independency. The parameter optimization was at first performed for the $Re = 120$ case due to the fact that in the case of a von Karman vortex street, there exists a theoretical value in the literature for the ratio of the coordinate differences between the neighboring vortices, which is 0.281 [6]. With the optimized parameters, after performing the detection, the calculated values were close to this theoretical value. Thus, the detection worked properly. For a Reynolds number of 12, the code failed to detect the evolved counter-rotating vortex pair, however, this was expected due to the fact that this is a steady case of flow, meaning, the vortices are not shed, they are at a fixed place. The detection worked for a Reynolds number of 60 and 1200, and after performing the optimization process repeatedly, I found the optimal intervals for these two cases as well. All in all, though the intervals are not exactly the same for the different Reynolds numbers, they are overlapping, meaning, each parameter has values which are optimal for all of the examined cases. The intersection of these intervals has been determined also.

As for the square cylinder, the obtained flow patterns were mostly in accordance with the expected results, however, it was also deduced that the boundaries of the different regions should not be treated too strictly, and the difference between the results can be caused by the different solver, and the different boundary condition on the outlet. The time-averaged values of the lift and drag coefficients, as well as the Strouhal numbers were sufficiently close to the values taken from the literature, except for the $\theta = 0^\circ$, $Re = 185$ case. Once again, the

already mentioned three-dimensionality caused the discrepancy in the results. Before the vortex detection, a mesh independency study ensured grid independency with the help of the Richardson extrapolation. Afterwards, the parameter optimization was performed by keeping the results obtained from the case of the circular cylinder in mind. For $\theta = 0^\circ$, $Re = 50$ the code failed to detect the evolved vortices, however, this was expected, because this is a steady case of fluid flow. The detection worked for all of the other cases. After having performed the optimization for each case, I also determined the intersection of the obtained intervals. Similarly to the case of the circular cylinder, despite the fact that the optimal intervals for the different cases are not exactly the same, each parameter has values which are optimal for all of them.

As a result of my thesis, the fluid flow around a circular cylinder and a square cylinder was simulated and examined at different Reynolds numbers by using a proper two-dimensional model. In addition, the optimization of the vortex detection parameters was performed, and with these parameters the vortex detection was carried out properly. The current study aimed to make the applied vortex detection method more objective by giving suggestions for the optimal values of the different vortex detection parameters. Some future plans for the current study include three-dimensional simulations and vortex detection. In addition, testing the determined optimal intervals for other geometries is also a possible field for improvement.

6.2 Summary in Hungarian (Az eredmények összefoglalása)

Jelen értekezés fő célja egy létező örvénydetektálási módszer paramétereinek az optimalizálása volt, valamint a kialakult örvények detektálása egy körhenger és egy négyzet alapú henger körüli áramlásban. Az adatok, melyekre a detektálást végeztem, kétdimenziós szimuláció eredményeként jöttek létre különböző Reynolds számok esetén (és a négyzet alapú henger esetén különböző megfűvási szögek esetén). A szimulációs eredmények validálása a kialakult sebesség- és nyomáseloszlások szakirodalomból vett ábrákkal való összehasonlításával történt, illetve, valamennyi esetben bemutatásra került a felhajtóerőtényező, az ellenállástényező és a Strouhal-szám is, majd ezek összehasonlításra kerültek a szakirodalomból vett értékekkel.

A körhenger esetében a kialakult áramképek az egyes Reynolds számokra összhangban voltak a várt eredményekkel. 12-es, 60-as és 120-as Reynolds-szám esetén a kapott felhajtóerőtényező és ellenállástényező értékek, valamint 60-as és 120-as Reynolds szám esetén a Strouhal számok kellően közel álltak a szakirodalomi értékekhez. A felhajtóerőtényező időbeli átlaga mindhárom esetben nulla volt, az ellenállástényező pedig a Reynolds szám növekedésével csökkent. 1200-as Reynolds szám esetén a felhajtóerőtényező időbeli átlaga nulla volt, azonban az ellenállástényező és a Strouhal szám magasabb volt, mint a szakirodalomból vett értékek. Ez a jelenség azzal magyarázható, hogy az 1200-as Reynolds szám meghaladja azt a kritikus Reynolds számot, mely fölött az áramlás háromdimenzióssá alakul át, és ebben az esetben a két-dimenziós szimulációk nem képesek követni a háromdimenziós hatásokat, ez okozta az eltérést az eredményekben. Ezt követően Richardson extrapoláció segítségével becslést adtam a diszkretizációs hiba mértékére, mely

által biztosítottam a hálófüggetlenséget. A paraméter optimalizálást elsőként a 120-as Reynolds számú esetre végeztem el, mivel Kármán-féle örvénysor esetén a szakirodalomban létezik egy elméleti érték a szomszédos örvények közötti koordinátakülönbségek arányára, ami 0,281 [6]. Az optimalizált paraméterekkel a detektálás elvégzését követően a számított értékek megközelítették ezt az elméleti értéket. Tehát az örvénydetektálás megfelelően működött. 12-es Reynolds szám esetén a program nem tudta detektálni a kialakult ellenforgó örvénypárt, azonban ez várható volt, mivel egy stacionárius áramlási esetről van szó, tehát az örvények nem válnak le, hanem egy fix, rögzített helyen vannak. A detektálás 60-as és 1200-as Reynolds szám esetén megfelelően működött, és az optimalizálási folyamat újbóli elvégzését követően erre a két esetre is meghatároztam az optimális intervallumokat. Összességében elmondható, hogy noha az intervallumok nem teljesen azonosak a különböző Reynolds számok esetén, vannak átfedések, vagyis minden paraméternek vannak olyan értékei, melyek valamennyi vizsgált esetre optimálisak. Ezen optimális intervallumok metszete is meghatározásra került.

Ami a négyzet alapú hengert illeti, a kialakult áramképek többnyire összhangban voltak a várt eredményekkel, viszont két következtetésre jutottam. Először is, az egyes áramképekre vonatkozó tartományok határait nem szabad túl szigorúan kezelni, másrészt, az eredmények közötti kismértékű eltérést okozhatta az eltérő peremfeltétel a kilépő keresztmetszeten, illetve a különböző megoldó használata. A felhajtóerőtényezők és az ellenállástényezők időbeli átlaga, valamint a Strouhal számok kellően közel álltak a szakirodalomból vett értékekhez, kivéve a $\theta = 0^\circ$, $Re = 185$ esetet. Ebben az esetben újfent a háromdimenziós áramlás okozta az eltérést az eredményekben. Az örvénydetektálás előtt ismét elvégeztem a hálófüggetlenségi vizsgálatot a Richardson-extrapoláció segítségével. Ezt követően a paraméter-optimalizálást a körhenger esetében kinyert eredmények szem előtt tartásával végeztem el. $\theta = 0^\circ$, $Re = 50$ esetén a kód nem tudta detektálni a kialakult örvényeket, de ez várható volt, mivel ismét egy időben állandó áramlásról van szó. A detektálás az összes többi esetben megfelelően működött. Miután minden esetre elvégeztem a paraméter-optimalizálást, meghatároztam a kapott optimális intervallumok metszetét is. A körhengerhez hasonlóan, annak ellenére, hogy az optimális intervallumok a különböző esetekre nem teljesen azonosak, minden paraméternek vannak olyan értékei, melyek minden esetre megfelelőek.

Dolgozatom eredményeként, megfelelő kétdimenziós modell alkalmazásával szimuláltam, majd vizsgáltam egy körhenger és egy négyzet alapú henger körül kialakuló áramlást különböző Reynolds számok esetén. Ezen kívül elvégeztem az örvénydetektálási paraméterek optimalizálását, és ezekkel a paraméterekkel megfelelően végrehajtottam az örvénydetektálást. Jelen tanulmány fő célja az volt, hogy az ismertetett és alkalmazott örvénydetektálási módszert objektívebbé tegye, méghozzá azáltal, hogy javaslatokat ad a különböző örvénydetektálási paraméterek optimális értékeire. Jelen tanulmány lehetséges továbbfejlesztéseként olyan témák kerülhetnek előtérbe, mint például háromdimenziós áramlások szimulációja és az így kinyert adatok felhasználásával örvénydetektálás. Ezen túlmenően, egy lehetséges fejlesztési terület továbbá a meghatározott optimális intervallumok tesztelése más geometriákra, elrendezésekre.

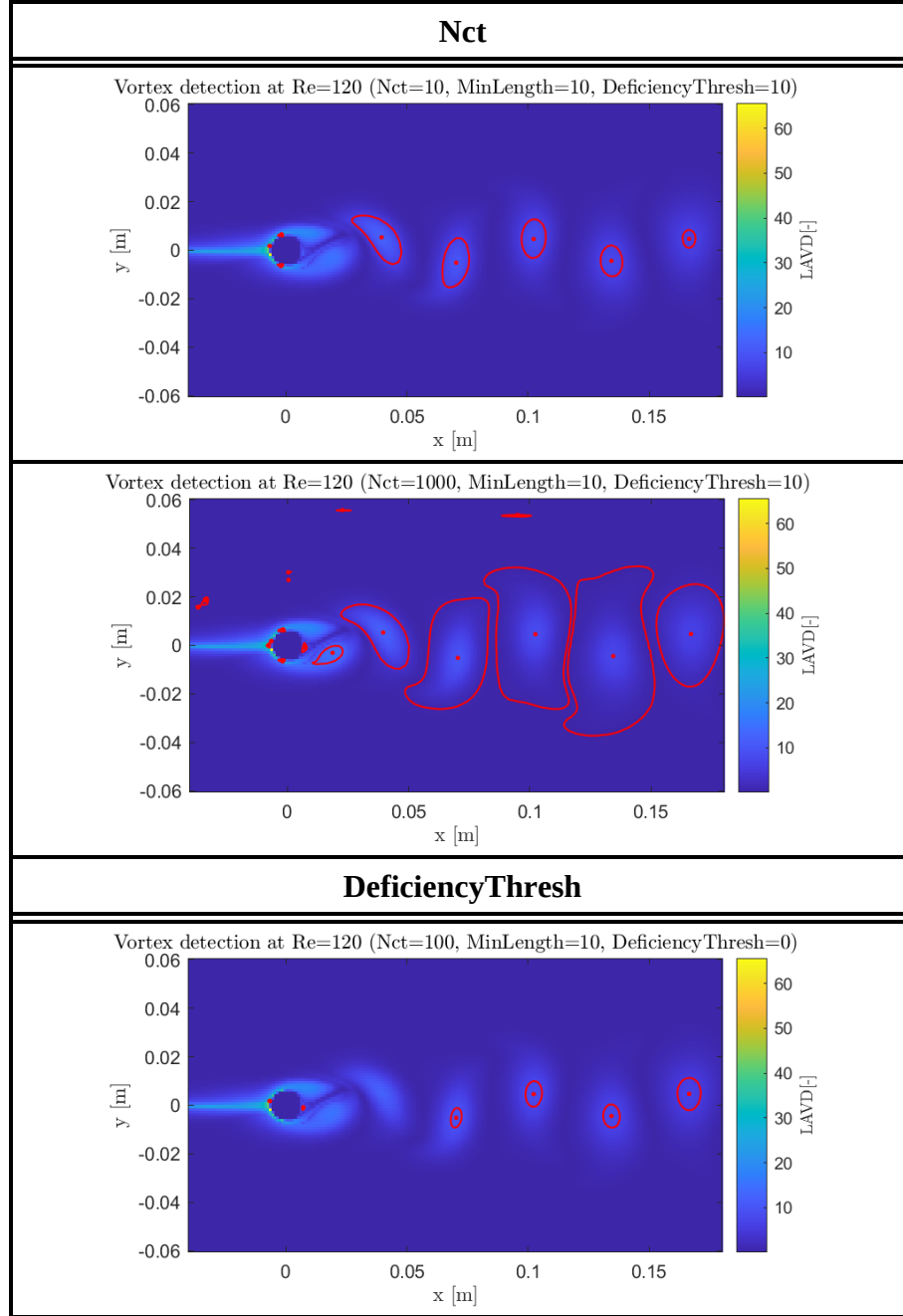
References

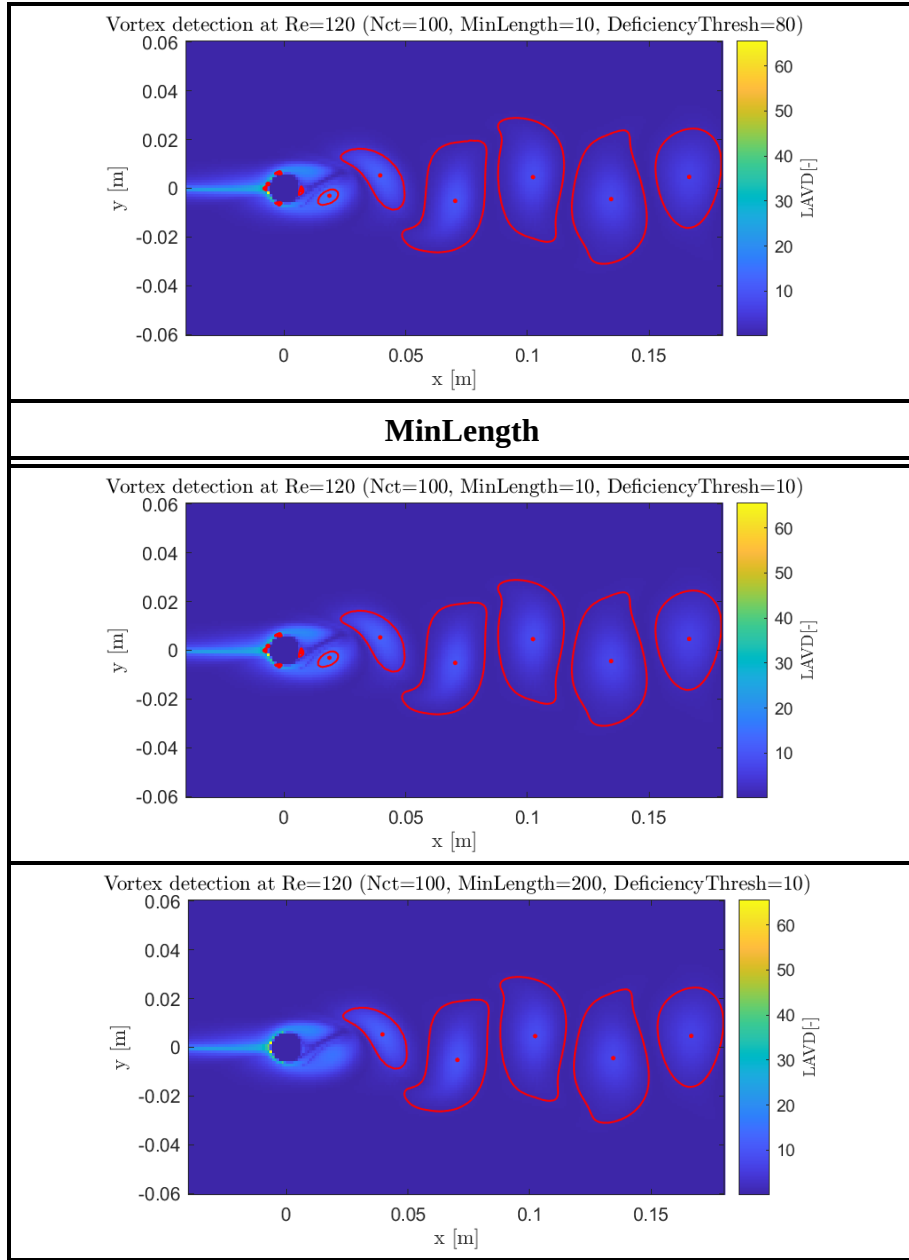
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Appendix

Appendix 1

In *Appendix 1* the figures corresponding to the vortex detection with nonoptimal parameters can be seen at $Re = 120$ in the case of the circular cylinder.





Appendix 2

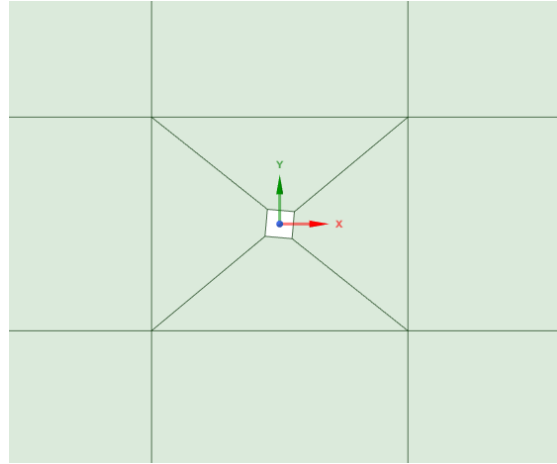
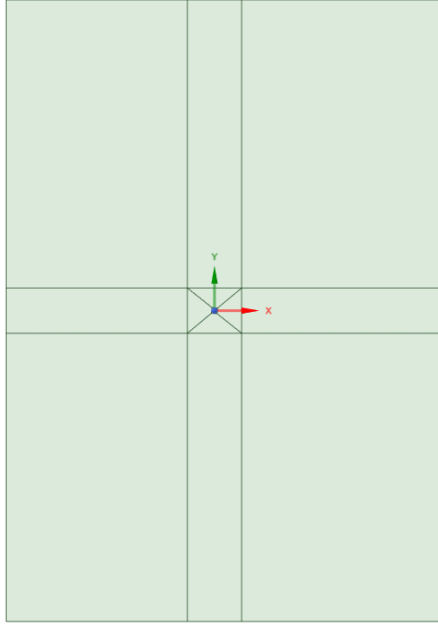
Due to the large number of figures, I uploaded all of the figures corresponding to the different cases of parameter optimization in a separate file to Google Drive, for this file the link is the following:

https://drive.google.com/drive/folders/1fPHvUsOBzAL_u-Kr0O9EFvNeESMBUQ3g?usp=share_link

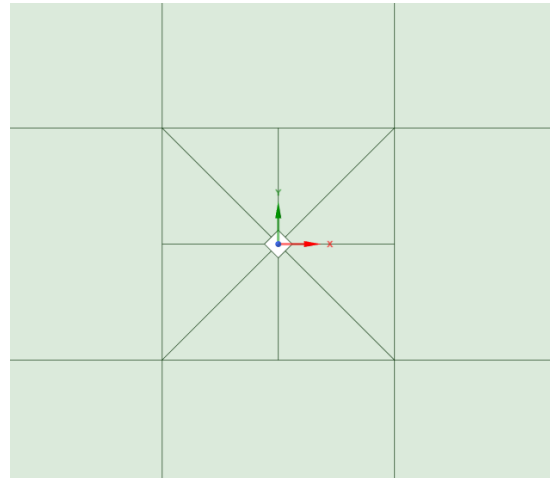
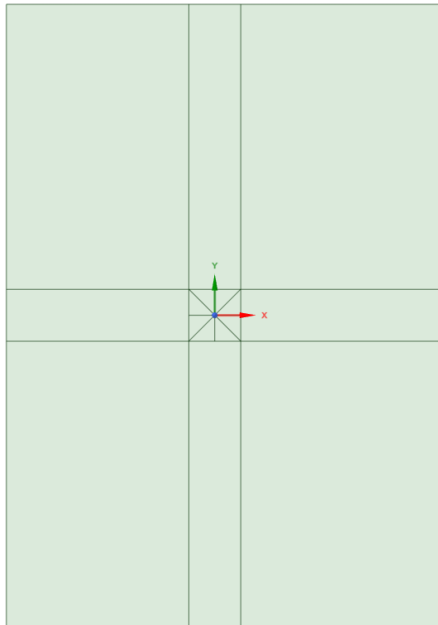
Appendix 3

In *Appendix 3* the geometrical models for $\theta = 5^\circ$ and $\theta = 45^\circ$ can be found in the case of the square cylinder.

$\theta = 5^\circ$



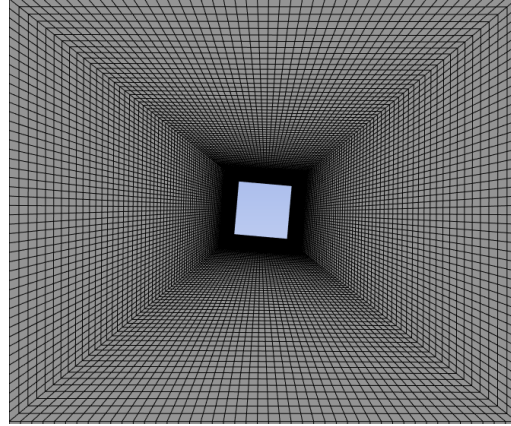
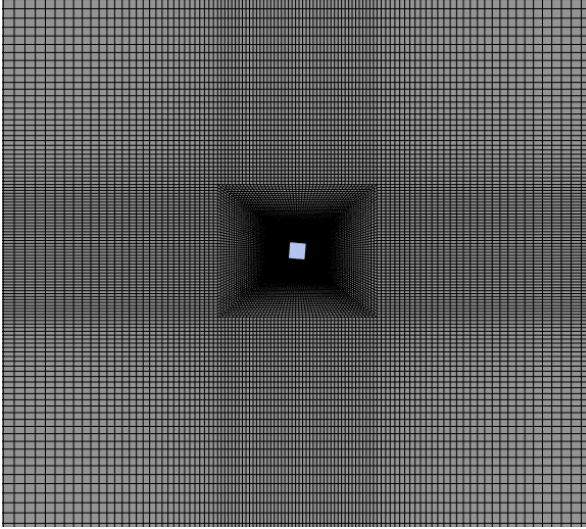
$\theta = 45^\circ$



Appendix 4

In *Appendix 4* the generated mesh for $\theta = 5^\circ$ and $\theta = 45^\circ$ can be found in the case of the square cylinder.

$$\theta = 5^\circ$$



$$\theta = 45^\circ$$

