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Vortex detection in fluid flow around a cylinder

TDK study

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Nomenclature

Latin letters	Name	Unit
A	Area	m^2
$B(t)$	Boundary of a rotationally coherent Lagrangian vortex	
c_D	Drag coefficient	1
c_L	Lift coefficient	1
C	Courant number	1
$C(t)$	Center of a rotationally coherent Lagrangian vortex	
d	Characteristic length	m
dV	Area element in 2D, volume element in 3D	m^2, m^3
D	Diameter	m
$D(t)$	Rotationally coherent Lagrangian vortex	
f	Vortex shedding frequency	Hz
f_{local}	Frequency of the up-and-down motion of the vortex street	Hz
F_D	Drag force	N
F_L	Lift force	N
$L(t)$	Lagrangian coherent structure	
N	Number of cells	1
p	Pressure	Pa
p_∞	Static pressure	Pa
p_m	Measured order of convergence	1
p_t	Total pressure	Pa
r	Refinement ratio	1
Re	Reynolds number	1
s	Variable of integration	
Sr	Strouhal number	1
t	Final time instant of the integration	s
t_0	Starting time instant of the integration	s
Δt	Time step size	s
T	Time period	s
U	Potential energy	J
$U(t)$	Time-dependent spatial domain	
$\mathbf{v}(\mathbf{x}, t)$	Unsteady velocity field	$\text{m} \cdot \text{s}^{-1}$
V	Velocity	$\text{m} \cdot \text{s}^{-1}$
$vol(U(t))$	Area in 2D, volume in 3D	m^2, m^3
x_i	Coordinate of a vortex center in the x direction	m

$\mathbf{x}(t, \mathbf{x}_0)$	Instantaneous particle positions	m
$\mathbf{x}_0$	Initial particle positions	m
Δx	Smallest cell size in the domain	m
Δx_{ij}	Difference between the centers of two neighboring vortices in the x direction	m
y_i	Coordinate of a vortex center in the y direction	m
Δy_{ij}	Difference between the centers of two neighboring vortices in the y direction	m

Greek letters	Name	Unit
ε_h	Discretization error	1
μ	Dynamic viscosity	$\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$
ϱ	Density	$\text{kg}\cdot\text{m}^{-3}$
ϕ_h	Fine mesh result	SI
ϕ_{2h}	Intermediate mesh result	SI
ϕ_{4h}	Coarse mesh result	SI
ϕ_0	Exact value extrapolated to the infinitely fine mesh	SI
$\omega(\mathbf{x}, t)$	Vorticity	s^{-1}
$\bar{\omega}(t)$	Instantaneous spatial mean of the vorticity	s^{-1}

Abbreviations

<i>DeficiencyThresh</i>	Convexity deficiency threshold
<i>FFT</i>	Fast Fourier Transform
<i>LAVD</i>	Lagrangian-averaged vorticity deviation
<i>LCS</i>	Lagrangian coherent structure
<i>MinLength</i>	Minimal arc-length threshold
<i>Nct</i>	Number of contour levels
<i>RMS</i>	Root mean square

1. Introduction

1.1 Aim of the work

Flow around different objects has been studied immensely due to the number of practical applications. These objects are usually divided into two groups, streamlined bodies (e.g., airfoil) and bluff bodies (e.g., cylinder, sphere). Streamlined bodies have such a shape, that the surface of the body is aligned with the streamlines when the body is placed in the flow, this is not true for bluff bodies. Airfoils, which are streamlined bodies, have such a shape that it increases the lift, and at the same time it decreases the drag on the wings. Whereas, cylinders mostly experience boundary layer separation, and strong flow oscillations.

In the case of bluff bodies, for a limited interval of the Reynolds number, symmetric, periodic vortex shedding can be observed, this is the so-called Karman vortex street ([1], pp. 514-515). The periodic nature of these flows sometimes causes unwanted vibrations. This can be especially dangerous if the vortex shedding frequency matches one of the resonant frequencies of the bodies ([1], pp. 515). Thus, it is vital that engineers take vortex shedding into account while designing skyscrapers, chimneys etc.

The main aim of the current study is vortex detection in fluid flow around a cylinder, which also means the optimization of the vortex detection parameters. The optimization is beneficial because the publication [2], on which the vortex detection is based, does not give suggestions for the values of these parameters, which increases the subjectivity of the method. My study aims to make the vortex detection method more objective by giving suggestions for the values of the vortex detection parameters.

Firstly, a two-dimensional model will be constructed, and afterwards, simulations will be run for different Reynolds numbers. The validation of the results will happen by comparing basic flow characteristics with figures, and values taken from the literature. Afterwards, with the obtained simulation results, the optimization of the vortex detection parameters is going to be performed.

The current study can be helpful in numerous ways. As a basic idea, through vortex detection one can observe the microstructure of the flow. Additionally, vortex detection through

measurements, is a really time-consuming procedure, with the help of this numerical method the detection time can be decreased substantially. In addition, the detection also makes it possible to follow the path of the shed vortices. For instance, if two buildings are close to each other, the effect of these buildings on one another can be examined through the vortices. As it was mentioned in the second paragraph, it is vital to take vortex shedding into account while designing different structures. Through vortex detection, it can be seen where the vortices are evolving, thus, from this data the necessary geometrical modifications can be deduced. Lastly, it is known that the Karman vortex street causes the phenomenon of “humming/singing” telephone wires ([1], pp. 515), meaning, it causes noise. Therefore, it is possible that the vortex detection method can be useful in the area of noise reduction as well.

1.2 Overview

The current study consists of 8 chapters, including the introduction part (*Chapter 1*). In *Chapter 1*, the main aim and the outline of the study is presented.

In *Chapter 2*, some basic definitions and principles are discussed with regard to flow physics. This chapter is based on [1, 3-4]. Among other things, the chapter discusses the definition of the Reynolds number, the characteristics of a symmetric flow and it also touches upon the topic of aerodynamic forces, and the Strouhal number.

Afterwards, in *Chapter 3* the so-called Lagrangian-averaged vorticity deviation is introduced. This chapter is based on George Haller et al., *Defining coherent vortices objectively from the vorticity*, 2016 [2]. *Chapter 3* mainly focuses on the definition of a coherent vortex, and the basic principles of vortex detection.

The fluid flow around a cylinder itself, is a result of simulation which was performed with the help of ANSYS Workbench 2022 R2. The steps of the simulation are discussed in *Chapter 4*. At first, the geometrical model and the meshing of the flow field is described. Later on, the simulation settings are introduced. The simulations were run at different Reynolds numbers (12,60,120,1200), and the flow was modelled as a two-dimensional problem. The simulation which was run at a Reynolds number of 120 is special, because in this case, the vortices are detached periodically, thus, forming a Karman vortex street [3].

In *Chapter 5*, the simulation results are discussed based mostly on the obtained velocity and pressure figures. In order to see whether the obtained results are accurate or not, the time-averaged values of the lift and drag coefficients, along with the Strouhal number were compared with the theoretical values, these are presented also.

The last topic that is discussed before the vortex detection is the mesh independency. *Chapter 6* contains the mesh independency study, during which the simulations were run for three differently refined meshes. For the verification of the independency the Richardson extrapolation was used which is also discussed.

In *Chapter 7*, the vortex detection along with the parameter optimization is presented. The detection was carried out in MATLAB R2020b. I performed the optimization process for the results obtained from the simulation with a Reynolds number of 120. This is due to the fact that in the case of a Karman vortex street, there exists a theoretical value in the literature for the distance between the neighboring vortices [5]. Thus, by calculating these values, the quality of the detection can be examined. The final step in my study was to check the optimized parameters for the other simulation results as well.

Lastly, a brief summary of the results and the conclusion sums up the current paper in *Chapter 8*.

An *Appendix* closes the study which has been uploaded separately due to the large size of the document. In the *Appendix*, I included all of the figures corresponding to the parameter optimization process.

2. Flow physics and basic definitions

The following brief theoretical summary is based on the following books: *Az áramlástan alapjai* by Lajos Tamás, 2019 [1], *Boundary-Layer Theory* by Hermann Schlichting (Deceased) and Klaus Gersten, 2016 [3], *Áramlások numerikus modellezése* by Dr. Kristóf Gergely, 2019 [4]. The upcoming definitions and basic principles are vital to understand and interpret the simulation and its results.

2.1 Courant number

The Courant number is used to determine the appropriate time step size for transient simulations. It is a dimensionless value which basically represents the time a particle stays in one cell of a mesh. Its ideal value is below 1. For the calculation of the Courant number the following formula can be used [4]:

$$C = \frac{V \cdot \Delta t}{\Delta x} \quad (1)$$

Where:

- V : velocity of the fluid $\left[\frac{\text{m}}{\text{s}}\right]$
- Δt : time step size [s]
- Δx : the smallest cell size in the domain in the direction of V [m]

During the simulations I always assumed the maximum Courant number to be 1.

2.2 Reynolds number

The Reynolds number by definition is a dimensionless value which is the ratio of the inertial forces and the viscous forces:

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} \quad (2)$$

By formula, it can be determined in the following way ([1], pp. 373):

$$Re = \frac{V \cdot \rho \cdot d}{\mu} = \frac{V \cdot \rho \cdot D}{\mu} \quad (3)$$

Where:

- ρ : density $\left[\frac{\text{kg}}{\text{m}^3}\right]$
- d : characteristic length [m]
- μ : dynamic viscosity $\left[\frac{\text{kg}}{\text{m}\cdot\text{s}}\right]$
- D : diameter of the cylinder [m]

Based on the Reynolds number, it is possible to predict flow patterns in different cases of fluid flow. At low Reynolds numbers, layered, laminar flow is observed, while at high Reynolds numbers, disturbed, turbulent flow is typical. In the case of laminar flow, viscous forces dominate, while in the case of turbulent flow, inertial forces dominate. In Table 1, flow regimes at a circular cylinder can be seen ([3], pp. 22).

Table 1. Flow characteristics at different Reynolds numbers. Reproduced from ([3], pp. 22).

Reynolds number regime	Flow regime	Flow form	Flow characteristic	Strouhal number Sr	Drag coefficient c_D
$\text{Re} \rightarrow 0$	Creeping flow		Steady, no wake	–	see Fig. 1.12
$3 - 4 < \text{Re} < 30 - 40$	Vortex pairs in wake		Steady, symmetric separation	–	$1.59 < c_D < 4.52$ (Re = 30) (Re = 40)
$\frac{30}{40} < \text{Re} < \frac{80}{90}$	Onset of Karman vortex street		Laminar, unstable wake	–	$1.17 < c_D < 1.59$ (Re = 100) (Re = 30)
$\frac{80}{90} < \text{Re} < \frac{150}{300}$	Pure Karman vortex street		Karman vortex street	$0.14 < \text{Sr} < 0.21$	
$\frac{150}{300} < \text{Re} < \frac{10^5}{1.3 \cdot 10^5}$	Subcritical regime		Laminar, with vortex street instabilities	$\text{Sr} = 0.21$	$c_D \approx 1.2$

2.3 Static, dynamic, and total pressure

In the case of a circular cylinder the shape of the streamlines is symmetric around the cylinder's upper and lower side. For very small Reynolds numbers this is true around the cylinder's front and back as well. It is important to note that as the Reynolds number increases this front to back symmetry disappears. It is also important to mention, that there is one streamline which does not travel along neither the upper nor the lower regions. It moves in a horizontal direction towards the

cylinder. The point where it reaches the cylinder is the so-called stagnation point, at this point the velocity decreases to zero, and the pressure at this point is the so-called total/stagnation pressure. In Figure 1 an example for the stagnation point can be seen.

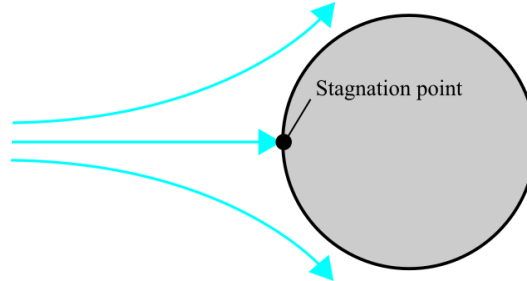


Figure 1. Stagnation point.

The formula for the static, dynamic and total pressure can be derived from the simplified Bernoulli equation (conditions for the simplification: the examined flow is inviscid, steady, incompressible and irrotational, also the field of force is potential) which is the following ([1], pp. 118-120):

$$\frac{p}{\rho} + \frac{V^2}{2} + U = \text{constant} \quad (4)$$

Where:

- p : pressure [Pa]
- U : potential energy [J]

The formula for the static, dynamic and total pressure can be derived by writing up Equation (4) between ∞ and the stagnation point, and it is the following ([1], pp. 120-122):

$$p_{\infty} + \rho \frac{V_{\infty}^2}{2} = p_t \quad (5)$$

Where:

- p_{∞} : static pressure [Pa]
- p_t : total pressure [Pa]
- $\rho \frac{V_{\infty}^2}{2}$: dynamic pressure [Pa]

It can be seen from Equation (5) that the stagnation pressure, which is the pressure of the stopped flow, is higher than the static pressure, which is the pressure of the still, undisturbed flow. The difference between the total and the static pressure is called the dynamic pressure.

Also, from Equation (4) one can see that if the velocity of the fluid increases, then the pressure must decrease and vice versa. Let us assume that the potential energy remains constant ($U = \text{constant}$), then Equation (4) is going to be the following:

$$\frac{p}{\rho} + \frac{V^2}{2} = \text{constant} \quad (6)$$

Therefore, from Equation (6) it can clearly be seen that the pressure and the velocity are inversely proportional.

2.4 Aerodynamic forces

2.4.1 Lift coefficient

The lift coefficient is a dimensionless quantity which can be calculated in the following way ([1], pp. 527):

$$c_L = \frac{F_L}{\rho \frac{V^2}{2} A} \quad (7)$$

Where:

- F_L : lift force [N]
- A : area [m^2] (for a blunt body the area is the projection of the cross – section which is perpendicular to the direction of the flow)

The lift coefficient basically models the effects of shape, inclination, and some flow conditions on lift.

The application of the lift coefficient is beneficial because if the geometry is the same, it can be applied even in the case of different sizes, velocities, and densities (on an interval of the Reynolds number, where the lift coefficient can be considered to be constant). In the case of a symmetric flow the time-averaged value of the lift coefficient is zero.

2.4.2 Drag coefficient

The drag coefficient is also a dimensionless value which can be calculated as follows ([1], pp. 527):

$$c_D = \frac{F_D}{\rho \frac{V^2}{2} A} \quad (8)$$

Where:

- F_D : drag force [N]

The drag coefficient basically models the effects of shape, inclination, and some flow conditions on drag.

Similarly to the lift coefficient, the application of the drag coefficient is beneficial because if the geometry is the same, it can be applied even in the case of different sizes, velocities, and densities (on an interval of the Reynolds number, where the drag coefficient can be considered to be constant).

Let us introduce the so-called pressure drag and friction drag. According to ([3], pp. 20) one property of the boundary layer is that, under specific conditions, reverse flow can occur directly at the wall. This can cause the separation of the boundary layer from the body and the formation of large or small eddies at the back of the body. This results in a significant change with regard to the pressure distribution at the back of the body, leading to the pressure drag. At the same time, friction drag is caused by the friction between the fluid and the surfaces over which it is flowing. In the case of bluff bodies (e.g., cylinders, spheres) the pressure drag is the main source of the drag force and the friction drag is a secondary source. In streamlined bodies (e.g., airfoil) this is exactly the other way around.

The drag coefficient of a circular cylinder is shown in Figure 2 ([3], pp. 19) as a function of the Reynolds number. It can be seen that the drag coefficient is Reynolds number dependent: as the Reynolds number increases the drag coefficient decreases, and over a long interval it remains constant.

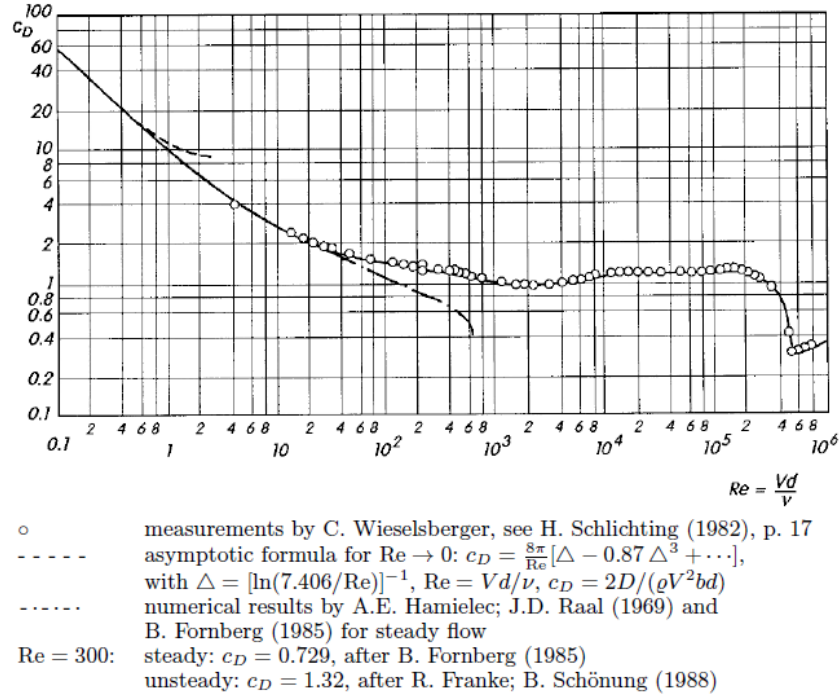


Figure 2. The effect of the Reynolds number on the drag coefficient in the case of a circular cylinder ([3], pp. 19).

2.5 Strouhal number

The Strouhal number is a dimensionless number which describes frequency and unsteady effects. It can be calculated in the following way ([1], pp. 404, pp. 514):

$$Sr = \frac{f \cdot d}{V} = \frac{f \cdot D}{V} \quad (9)$$

Where:

- f : vortex shedding frequency [Hz]

3. Lagrangian-averaged vorticity deviation

The following theoretical summary is based on George Haller et al., *Defining coherent vortices objectively from the vorticity*, 2016 [2].

Let us start by defining what a coherent vortex is. The definition of coherent vortices is still a disputed matter currently. However, according to [2] there are two main characteristics which are widely accepted. Firstly, vortices are concentrated regions of high vorticity. Secondly, they are usually seen as evolving domains with a high level of material invariance. As it was mentioned before, despite the fact that these characteristics are broadly accepted, different authors interpret vortices in different ways. These different kinds of interpretations are collected and can be found in [2]. The core of a coherent vortex is an impermeable material region with a high concentration of vorticity. Per contra, the interpretation of high concentration is scarcely objective. Thus, with regard to constructing an explicit vortex definition, material invariance could be a good starting point. In [2] an objective vortex criterion is proposed, this is the so-called Lagrangian-averaged vorticity deviation (LAVD).

LAVD by definition is the trajectory integral of the normed difference of the vorticity from its spatial mean:

$$LAVD_{t_0}^t(\mathbf{x}_0) := \int_{t_0}^t |\boldsymbol{\omega}(\mathbf{x}(s; \mathbf{x}_0), s) - \bar{\boldsymbol{\omega}}(s)| ds \quad (10)$$

Where:

- t_0 : starting time instant of the integration [s]
- t : final time instant of the integration [s]
- $\mathbf{x}_0$: initial particle positions [m]
- $\boldsymbol{\omega}(\mathbf{x}, s)$: vorticity $\left[\frac{1}{s}\right]$
- $\mathbf{x}(s; \mathbf{x}_0)$: instantaneous particle positions [m]
- s : variable of integration
- $\bar{\boldsymbol{\omega}}(s)$: instantaneous spatial mean of the vorticity over $U(t)$

$\boldsymbol{\omega}$ can be calculated in the following way:

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} \quad (11)$$

Where:

- $\mathbf{v}(\mathbf{x}, t)$: unsteady velocity field defined on a time – dependent spatial domain $(U(t))$ over a finite time interval $[t_0, t_1]$
- $U(t)$:
 - Physical domain with impermeable boundary or
 - Material domain formed by a set of evolving trajectories of $\mathbf{v}(\mathbf{x}, t)$

$\bar{\omega}$ is expressed as follows:

$$\bar{\omega}(t) = \frac{\int_{U(t)} \boldsymbol{\omega}(\mathbf{x}, t) dV}{vol(U(t))} \quad (12)$$

Where:

- $vol(U(t))$: area (in 2D)
- dV : area element (in 2D)

According to [2] a Lagrangian vortex is a nested set of material tubes with outward decreasing tubular level sets of LAVD. The LAVD is derived from the dynamic rotation tensor, which is obtained from the so-called dynamic polar decomposition (DPD) of the deformation gradient. This provides an objective global vortex criterion which is entirely frame-invariant.

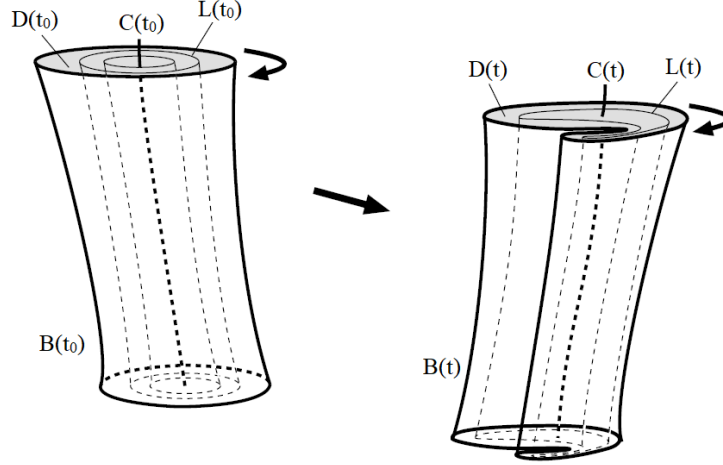
With the help of LAVD it is possible to detect objectively material tubes along which small fluid volumes have the same bulk rotation compared to the mean rotation of the fluid. The initial positions of these tubes are tubular level surfaces of the $LAVD_{t_0}^{t_1}(\mathbf{x}_0)$. This tubular set should be interpreted as a closed convex curve in 2D, and in 3D it should be understood as a convex, cylindrical, cup-shaped, or toroidal set. Convexity is required for tubular surfaces. This is due to the fact that stable vortices mostly have near-circular cross-sections.

In [2] it is stated that a rotationally coherent Lagrangian vortex has to suffice the following conditions over the examined time interval $[t_0, t_1]$:

- It is an evolving material domain $D(t)$ where $D(t_0)$ is filled with tubular level surfaces of $LAVD_{t_0}^t(\mathbf{x}_0)$ and the LAVD values are decreasing as we move outwards

- The boundary $B(t)$ of $D(t)$ is a material surface where $B(t_0)$ is the outermost tubular level surface of $LAVD_{t_0}^t(\mathbf{x}_0)$ in $D(t_0)$
- The center $C(t)$ of $D(t)$ is a material set where $C(t_0)$ is the innermost member (maximum) of $LAVD_{t_0}^t(\mathbf{x}_0)$ in $D(t_0)$

In Figure 3 ([2], pp. 143) a rotationally coherent Lagrangian vortex can be seen.



*Figure 3. Initial and current positions of a Lagrangian vortex $D(t)$.
Reproduced from ([2], pp. 143).*

The first figure is at the beginning of the examined time interval and the second figure represents the current state. $L(t)$ represents a rotational Lagrangian coherent structure (LCS), this means that along this so-called LCS fluid volumes have the same rotation in the given time interval $([t_0, t_1])$.

As it was mentioned before, convexity is a required criterion. This requirement ensures that the material vortex is unfilamented at t_0 . However, during the numerical calculations, a small convexity deficiency is allowed. Firstly, this deficiency is allowed due to the fact that LAVD level surfaces are represented by discrete polygons numerically. If a polygon has a lot of vertices, it is likely that it will not be convex. Another reason to relax strict convexity is to remove the conceptual asymmetry which can be seen in Figure 3. This way small tangential filamentation can be present in the initial positions of vortex boundaries as well. Lastly, in the case of multi-scale data sets smaller-scale vortices can be near larger-scale vortices, thus a sufficiently coarse-grained notion is required for the boundaries of the larger-scale vortices.

In [2] it is also stated that small-scale local maxima of the LAVD function can arise due to numerical or observational noise in the velocity data. Due to this fact, in order to eliminate these artificial vortices, LAVD contours whose arclength falls below a minimal threshold value are ignored.

The final topic I want to mention in *Chapter 3* is the numerical implementation of the extraction procedure for rotationally coherent Lagrangian vortices in 2D. This algorithm is the following [2]:

1. For a 2D grid of initial conditions $\mathbf{x}_0$, compute the LAVD:

$$LAVD_{t_0}^t(\mathbf{x}_0) := \int_{t_0}^t |\boldsymbol{\omega}(\mathbf{x}(s; \mathbf{x}_0), s) - \bar{\boldsymbol{\omega}}(s)| ds \quad (13)$$

2. Find initial positions $\mathcal{C}(t_0)$ of vortex centers as local maxima of $LAVD_{t_0}^t(\mathbf{x}_0)$
3. Find initial vortex boundaries $B(t_0)$ as outermost closed contours of $LAVD_{t_0}^t(\mathbf{x}_0)$, these contours have to suffice the following criteria:
 - $B(t_0)$ encircles a vortex center $\mathcal{C}(t_0)$
 - The arclength of $B(t_0)$ is higher than the given minimal threshold value
 - The convexity deficiency of $B(t_0)$ is smaller than the given maximum threshold value

The output is the initial positions of rotationally coherent Lagrangian vortex boundaries and vortex centers with respect to the time interval $[t_0, t_1]$.

4. Simulation Setup

4.1 Geometrical model

I constructed the geometry based on [6]. The simulation is a two-dimensional (2D) simulation, thus, the cylinder was modelled as a circle. The diameter of the cylinder is $D = 0.0127$ [m]. The physical domain has a dimension of 30 diameters in the streamwise direction ($-8D < x < 22D$) and 16 diameters in the lateral direction ($-8D < y < 8D$). I set up the geometry using SpaceClaim. The geometrical model can be seen in Figure 4.

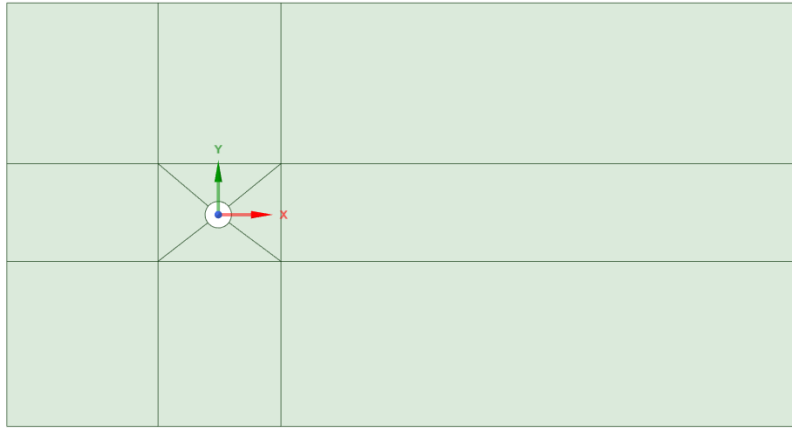


Figure 4. The geometrical model.

4.2 Mesh

In the case of the meshing the general element size is 0.002 [m]. I refined the mesh near the cylinder with the help of edge sizing. I applied this method instead of inflation in order to be able to perform face meshing. This is due to the fact that I wanted to achieve a structured, quadrilateral mesh. I generated the mesh which can be seen in Figure 5. The mesh has 25252 cells and 25627 nodes. One way to check whether the quality of the mesh is appropriate or not is by examining the maximum value of the so-called skewness attribute. Skewness is based on the internal angles of an element, for regular polygon elements it is 0, if one internal angle is much smaller than the others it approaches 1 [4]. It determines the element quality based on the internal angles, with respect to the angles of ideal elements. If skewness exceeds a given value, it can cause decreased accuracy. The maximum value of the skewness:

$$skewness_{maximum} = 0.5925 \quad (14)$$

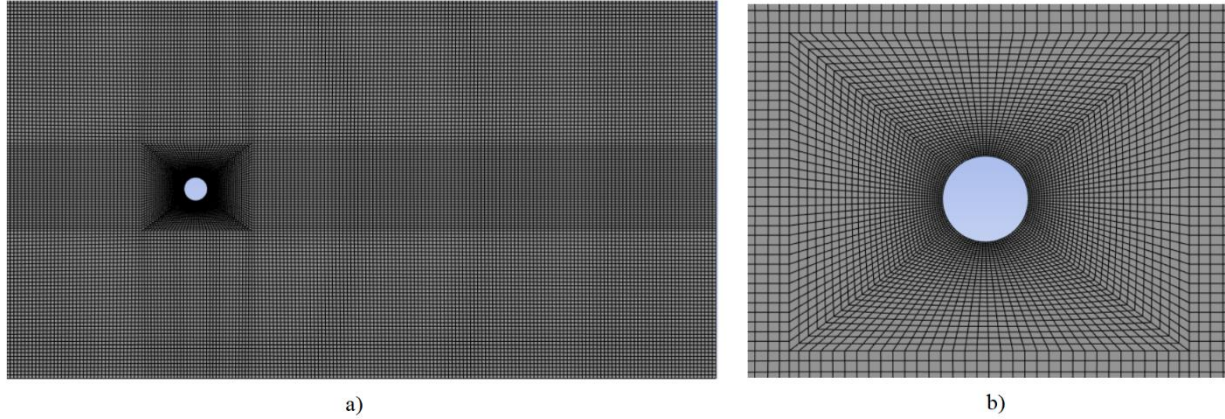


Figure 5. The quadrilateral mesh a) for the entire domain b) in the vicinity of the cylinder.

As a general rule the maximum value should not exceed 0.85 [4]. As it can be seen from Equation (14), this requirement is fulfilled, thus the quality of the generated mesh is appropriate.

One more important thing to mention is that the smallest cell size in the domain in the direction of V is approximately $\Delta x \approx 0.0004$ [m]. This value was used while determining the time-step based on the formula for the Courant number (Equation (1)).

4.3 Simulation settings

I ran the simulations using Ansys Fluent 2022 R2. For a Reynolds number of 12, 60, 120 and 1200, I selected transient time, and I used the Viscous Laminar model. The applied fluid was air, for which I used the following properties:

$$\rho_{air} = 1.225 \left[\frac{kg}{m^3} \right] \quad (15)$$

$$\mu_{air} = 1.7894 \cdot 10^{-5} \left[\frac{kg}{m \cdot s} \right] \quad (16)$$

I applied a constant inlet velocity, however in order to calculate the velocity, the formula for the Reynolds number had to be used. As it was mentioned before, the Reynolds number can be determined with the help of Equation (3).

With this in mind, the inlet velocities for the different Reynolds numbers can be found in Table 2.

Table 2. Inlet velocities for the different Reynolds numbers.

Reynolds number	Inlet velocity $\left[\frac{m}{s}\right]$
12	0.0141
60	0.0704
120	0.1409
1200	1.4090

I set the boundary conditions and solution methods based on [6]. For the upper and lower boundaries, I set a symmetry boundary condition, this is important because in this case the solver assumes a zero shear stress at the wall, therefore the no-slip condition is not observed. For the flow outlet I applied a pressure outlet boundary condition. As for the cylinder, wall boundary condition was applied. The wall boundary condition is used to bound fluid and solid regions, also, in viscous flows the no-slip condition is enforced at walls by default. Assigning this boundary condition was vital, because otherwise the fluid would have just flown right through the cylinder, not around it.

I set the reference values which are used to compute normalized flow-field variables, meaning they are needed for the calculation of the lift and drag coefficient. As for the solution methods, SIMPLE was used for the pressure-velocity coupling. For the momentum equation Second Order Upwind was applied and for the pressure equation Second Order was used. As it was mentioned before, the necessary time step can be determined with the help of Equation (1):

$$C = \frac{V \cdot \Delta t}{\Delta x} \rightarrow \Delta t = \frac{C \cdot \Delta x}{V} \underset{C=1}{\rightarrow} \Delta t \quad (17)$$

The time steps for the different Reynolds numbers can be found in Table 3.

Table 3. Time step values for the different Reynolds numbers.

Reynolds number	Time step [s]
12	0.02
60	0.005
120	0.002
1200	0.0002

5. Results and discussion

5.1 Re=12

5.1.1 Figures of velocity and pressure

For a Reynolds number of 12 I ran the simulation for 2500 time steps. In Figure 6 the contours of the velocity magnitude can be seen.

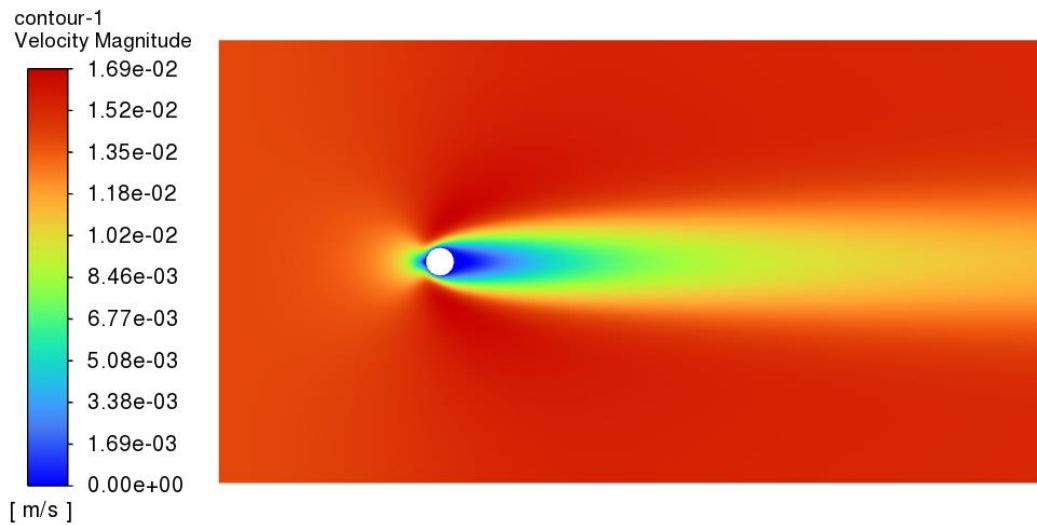


Figure 6. Contour plot of velocity at Re=12.

In Figure 7 the contour plot of the pressure can be seen.

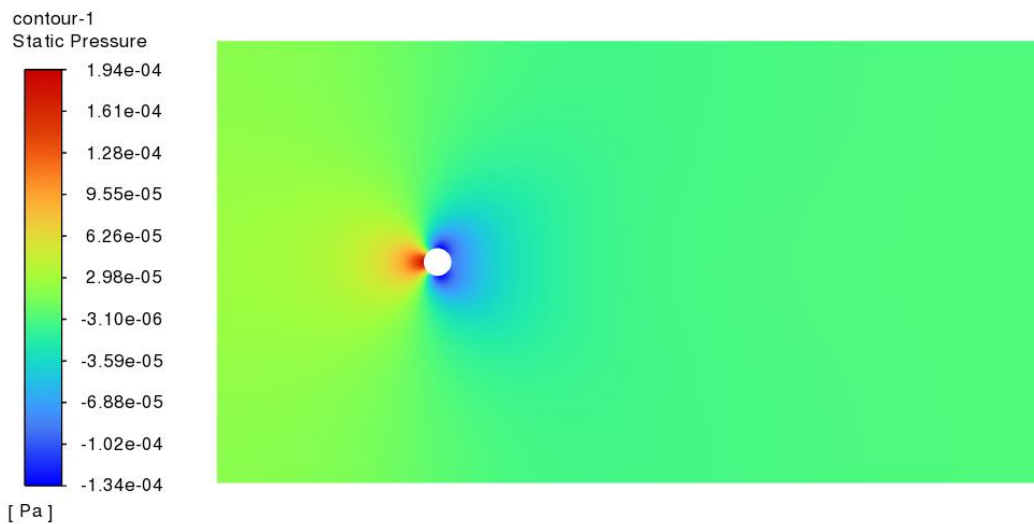


Figure 7. Contour plot of static pressure at Re=12.

It can be seen from Figure 6 and Figure 7 that there is a stagnation point at the front of the cylinder. As it was mentioned in 2.3, at the stagnation point, the velocity drops to zero, and this is also a high-pressure area. This is fulfilled in the case of the simulation results. In accordance with Table 1, a pair of counter-rotating vortices were created within the upper and lower regions of the cylinder's wake and steady symmetric separation can be observed. This can be observed in Figure 8, where the pathlines colored by the velocity magnitude can be seen.

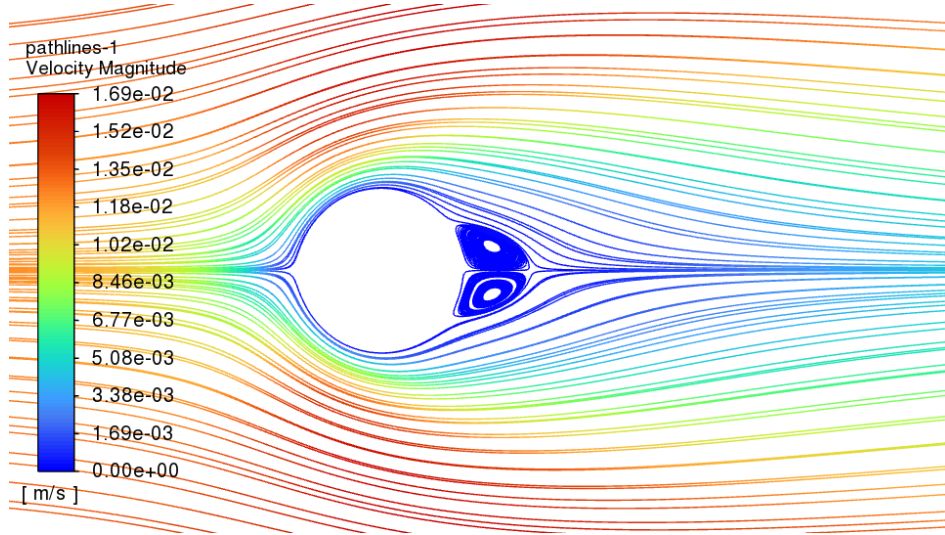


Figure 8. Pathlines at $Re=12$.

I also calculated the maximum value of the cell convective Courant number:

$$C_{max} = 0.7814 \quad (18)$$

Therefore, it can be said that the results capture the aerodynamic phenomena well, and the solutions converged.

5.1.2 Lift and drag coefficient

Both the lift and the drag coefficients converged quite fast. As it was mentioned in 2.4.1, in the case of a symmetric flow the time-averaged value (in this case the converged value) of the lift coefficient is zero, this is fulfilled in this case. The average value of the lift coefficient is approximately zero. It is important to mention, that later on, when I am going to mention the time-averaged value of the lift coefficient, I am going to talk about the time-averaged value of the converged part.

The converged value of the drag coefficient is:

$$c_D = 2.8095 \quad (19)$$

This is in accordance with the expected result (Table 1). The time evolution of the lift coefficient can be seen in Figure 9.

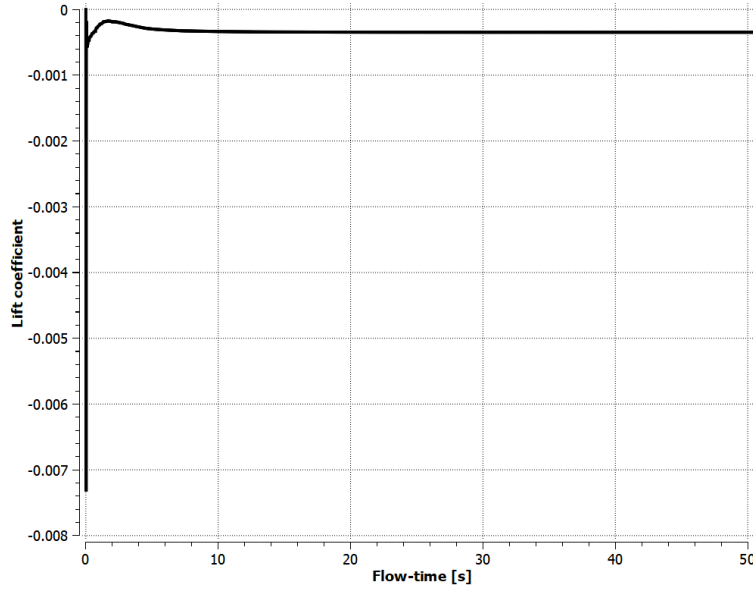


Figure 9. Time evolution of the lift coefficient at $Re=12$.

The time evolution of the drag coefficient can be seen in Figure 10.

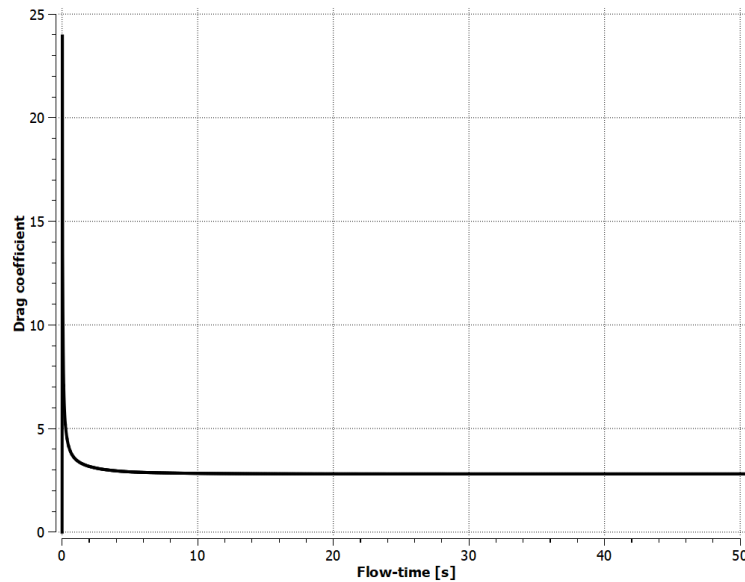


Figure 10. Time evolution of the drag coefficient at $Re=12$.

5.2 $Re=60$

5.2.1 Figures of velocity and pressure

In the case of $Re = 60$ I ran the simulation for 14000 time steps and received Figure 11 for the velocity.

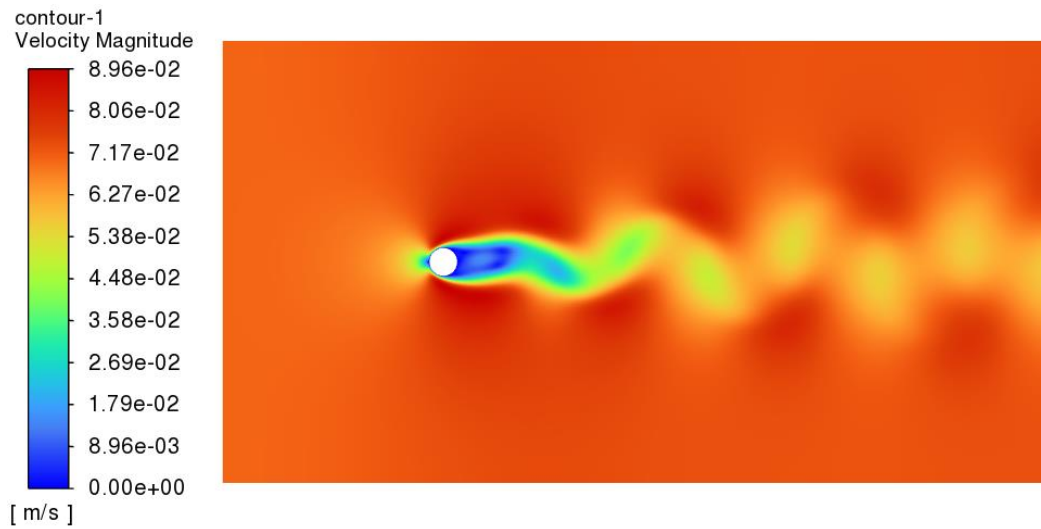


Figure 11. Contour plot of velocity at $Re=60$.

The pressure distribution can be seen in Figure 12.

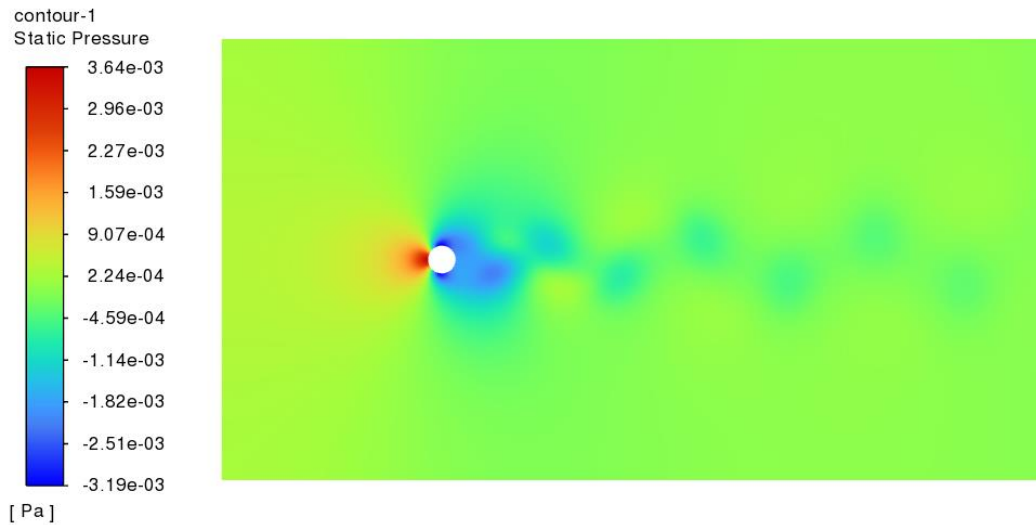


Figure 12. Contour plot of static pressure at $Re=60$.

The stagnation point which was mentioned in 5.1.1, can also be observed here. It can be seen from Figure 12 that the pressure difference between the front and the back of the cylinder is higher than in 5.1.1. As expected on the basis of Table 1 a laminar, unstable wake can be observed. It is

also important to realize that the region surrounding the cylinder, where the velocity is less than the freestream velocity, has decreased substantially compared to the case of $Re = 12$. This can be explained by the relationship between the Reynolds number and the boundary layer thickness. The boundary layer thickness decreases as the Reynolds number increases, this explains the decrease in the aforementioned area. This decrease is observable in the case of $Re = 120$ and $Re = 1200$ as well.

The Courant number for this case is:

$$C_{max} = 1.1899 \quad (20)$$

5.2.2 Lift and drag coefficient

The lift coefficient oscillates, and its time-averaged value is approximately zero. As for the time-averaged value of the drag coefficient I received the following result:

$$c_D = 1.4942 \quad (21)$$

This is in accordance with the expected result (Table 1). It can be seen that the drag coefficient decreased as the Reynolds number increased, this is in accordance with Figure 2 and with what was stated in 2.4.2. The time evolution of the lift coefficient can be seen in Figure 13.

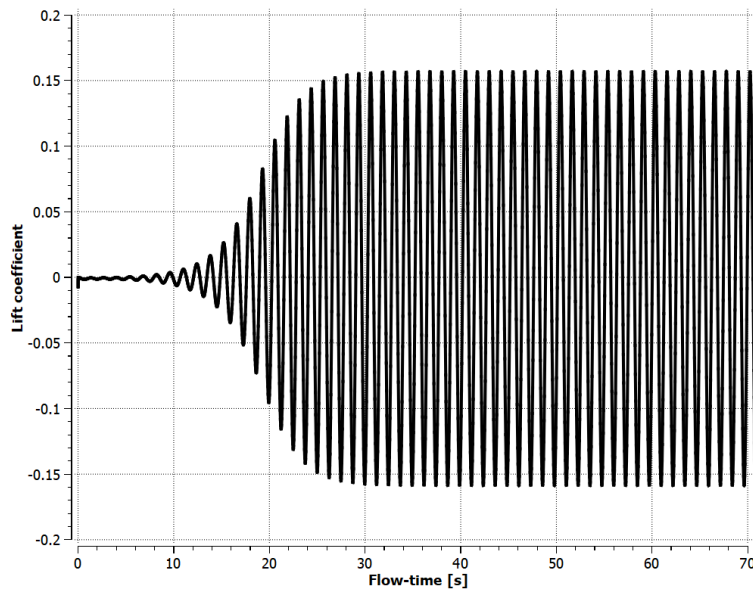


Figure 13. Time evolution of the lift coefficient at $Re=60$.

The time evolution of the drag coefficient can be seen in Figure 14.

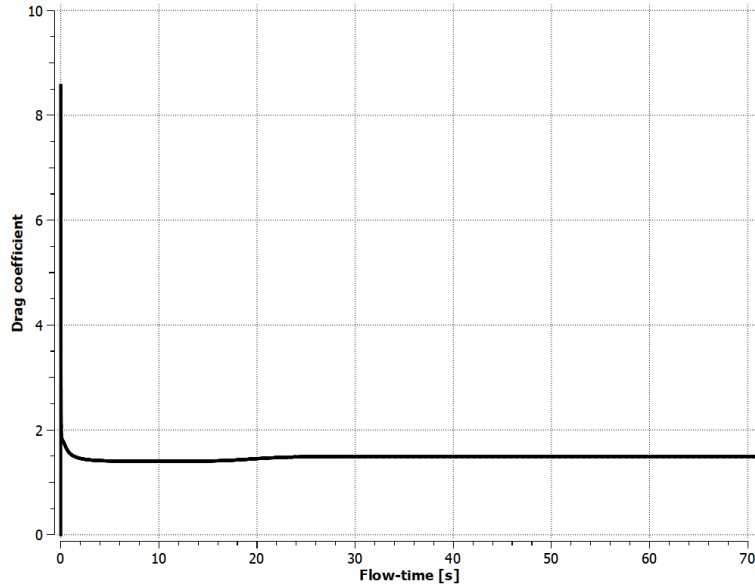


Figure 14. Time evolution of the drag coefficient at $Re=60$.

5.2.3 Strouhal number

Apart from the lift and drag coefficient, another result that can be checked is the Strouhal number. In spite of the fact that in Table 1 a Strouhal number is not presented for $Re = 60$, I still determined the Strouhal number for this case also, because it can be seen from Figure 11 that the vortices are shed for this Reynolds number as well. I checked another publication [7] with regard to simulation of flow past a circular cylinder, and in [7] this is confirmed. As it was mentioned in 2.5 the Strouhal number can be calculated with Equation (9). In order to calculate the Strouhal number the vortex shedding frequency had to be determined. I performed this with the help of the obtained data for the lift coefficient. I performed the Fast Fourier Transform of the time signal of the lift coefficient. This can be seen in Figure 15. Afterwards, I looked for spikes in the low-frequency ranges, the corresponding frequency is the following:

$$f = 0.7999 \text{ [Hz]} \quad (22)$$

With this in mind the Strouhal number is the following:

$$Sr = \frac{f \cdot D}{V} = 0.1443 \quad (23)$$

This calculated value is in accordance with the expected result [7].

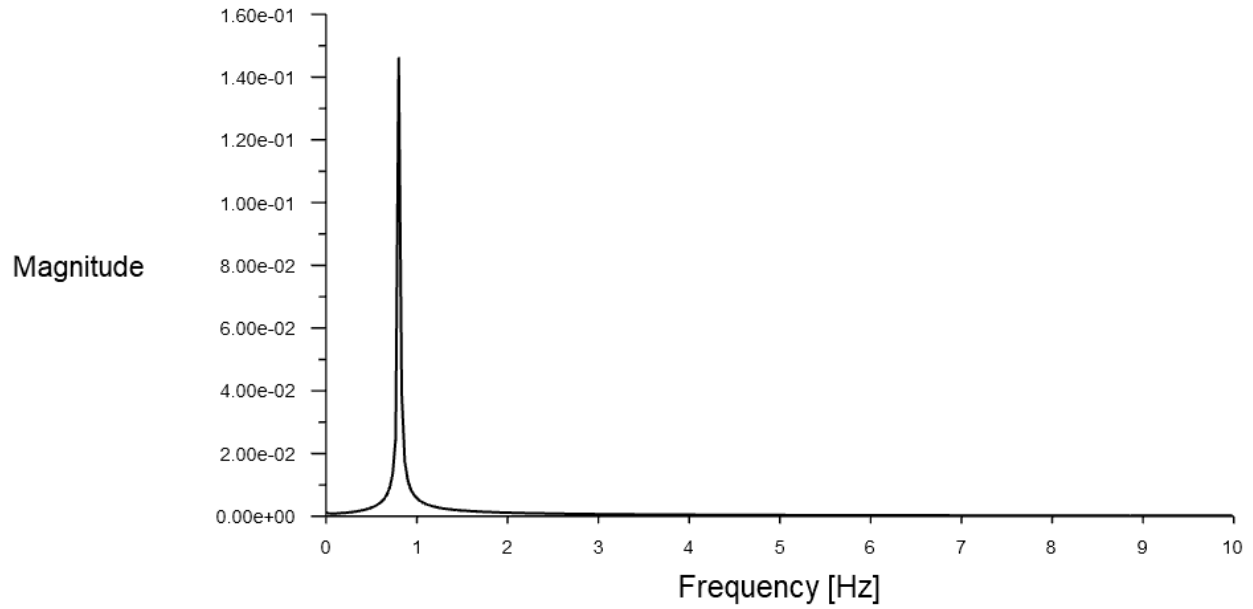


Figure 15. FFT of the lift coefficient at $Re=60$.

5.3 $Re=120$

5.3.1 Figures of velocity and pressure

For a Reynolds number of 120 I ran the simulation for 6000 time steps and received Figure 16 for the velocity.

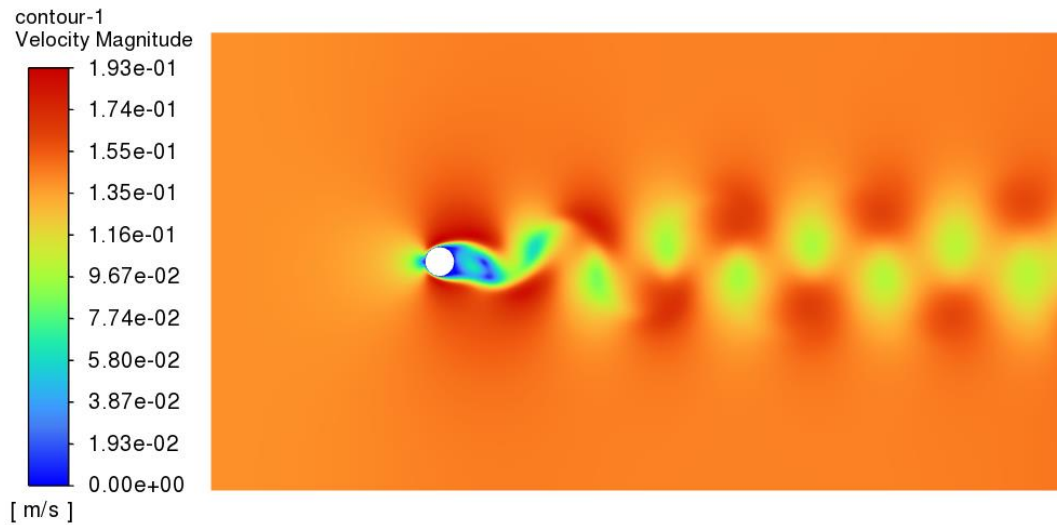


Figure 16. Contour plot of velocity at $Re=120$.

The pressure distribution can be seen in Figure 17.

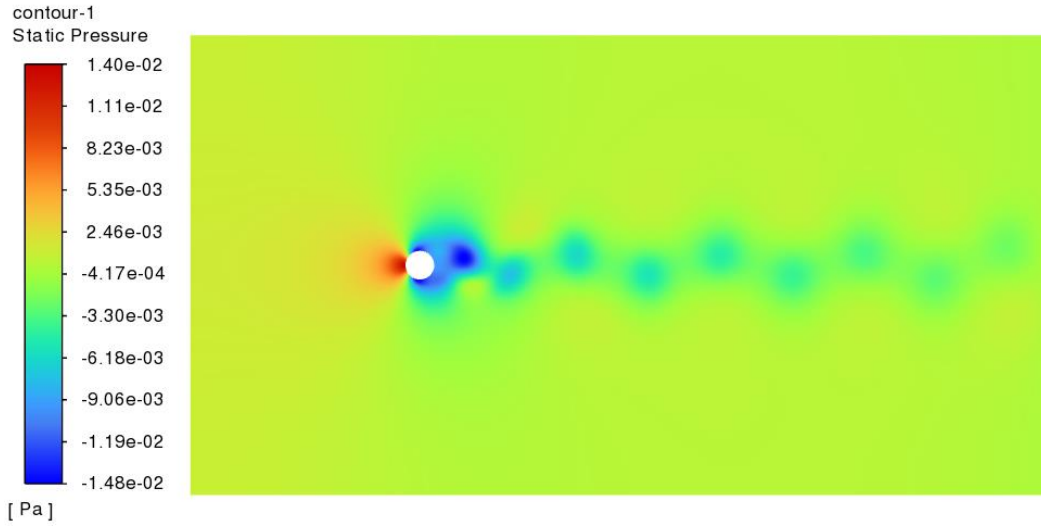


Figure 17. Contour plot of static pressure at $Re=120$.

Let us examine the obtained figures. It can be seen from Figure 16 that as the Reynolds number increased, the vortex region expanded and became larger in length. Also, by observing Figure 16, the aforementioned Karman vortex street phenomenon can be recognized. The Karman vortex street is a repeating pattern of swirling vortices, which is caused by the so-called vortex shedding. The shedding of the vortices is responsible for the separation of flow around bluff bodies. It can be seen from Table 1 that this is the expected result.

It can be seen in Figure 17 that the pressure difference became even larger between the front and the back of the cylinder, thus, the pressure drag is increasing. The stagnation point is still present at the front of the cylinder. The Courant number for this case:

$$C_{max} = 1.0569 \quad (24)$$

5.3.2 Lift and drag coefficient

The time-averaged value of the lift coefficient is approximately zero again. The drag coefficient fluctuates, and its time-averaged value is approximately:

$$c_{D,mean} = 1.4039 \quad (25)$$

The obtained value is slightly higher than the expected result which can be seen in Table 1. This can be caused by the fact that I used a two-dimensional model instead of a three-dimensional one. Nevertheless, this result still complies reasonably well with the expected results (Table 1).

In Figure 18 the time evolution of the lift coefficient can be seen.

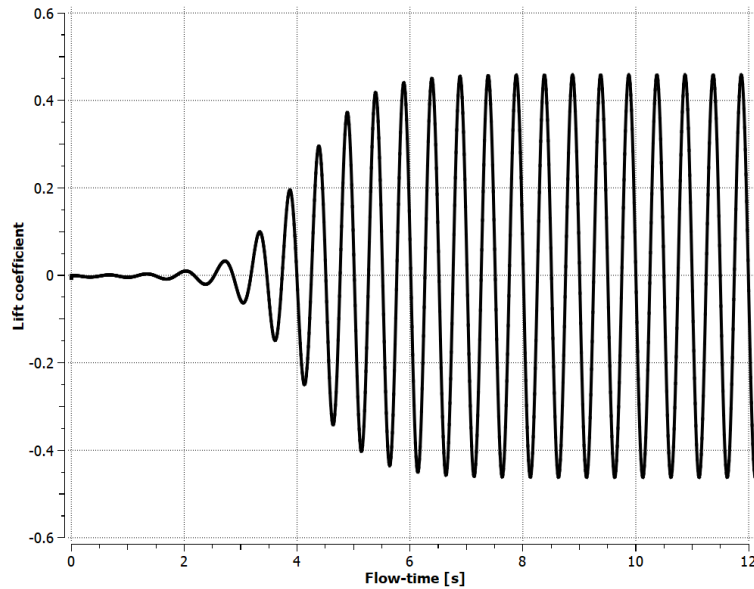


Figure 18. Time evolution of the lift coefficient at $Re=120$.

In Figure 19 the time evolution of the drag coefficient can be seen.

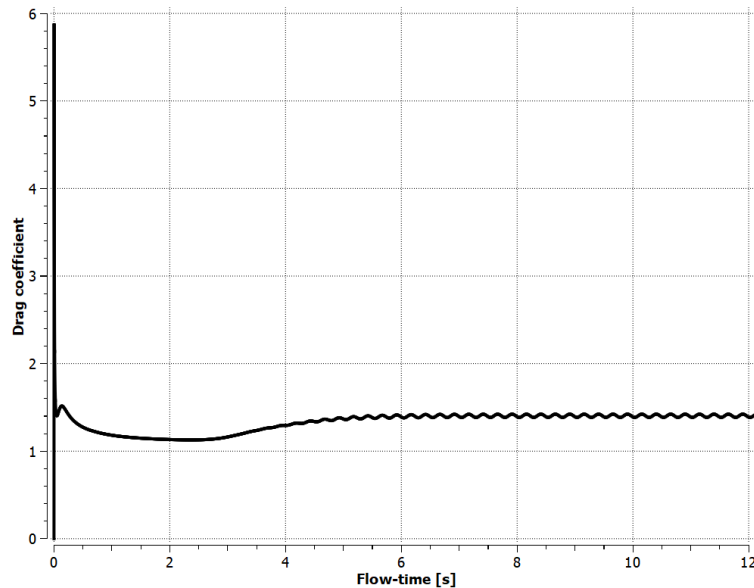


Figure 19. Time evolution of the drag coefficient at $Re=120$.

5.3.3 Strouhal number

I once again performed the Fast Fourier Transform for the time signal of the lift coefficient, this can be seen in Figure 20.

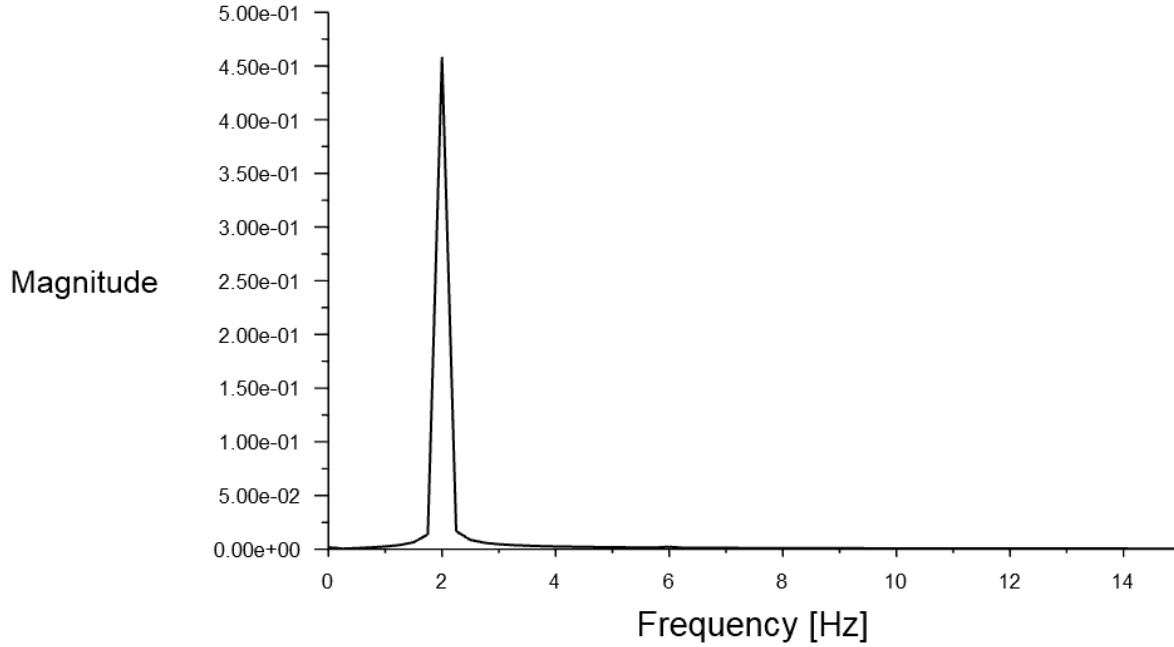


Figure 20. FFT of the lift coefficient at $Re=120$.

The vortex shedding frequency is the following:

$$f = 1.9990 \text{ [Hz]} \quad (26)$$

With this in mind the Strouhal number is the following:

$$Sr = \frac{f \cdot D}{V} = 0.1802 \quad (27)$$

This calculated value is in accordance with the expected result which can be found in Table 1.

5.4 Re=1200

5.4.1 Figures of velocity and pressure

For a Reynolds number of 1200 the simulation was run for 10000 time steps, and I received Figure 21 for the velocity.

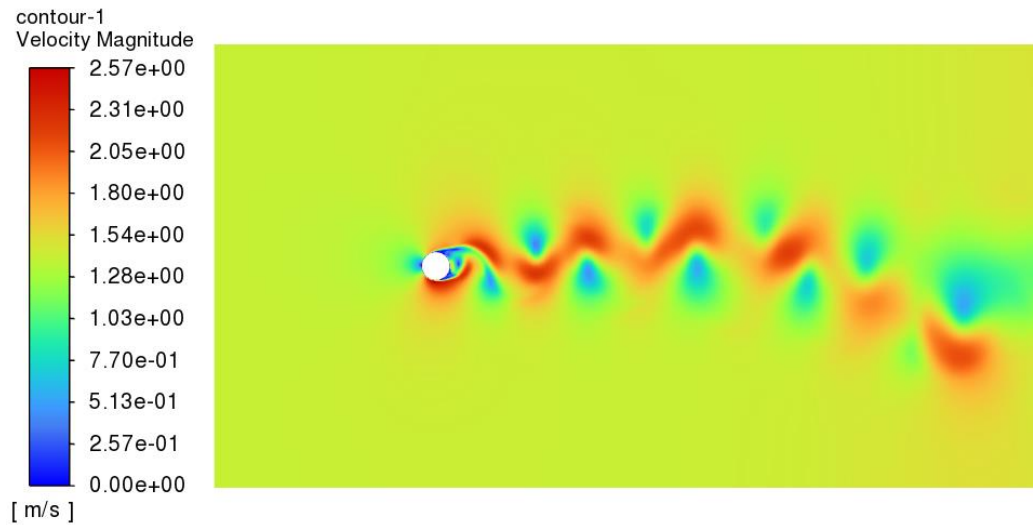


Figure 21. Contour plot of velocity at $Re=1200$.

The static pressure distribution can be seen in Figure 22.

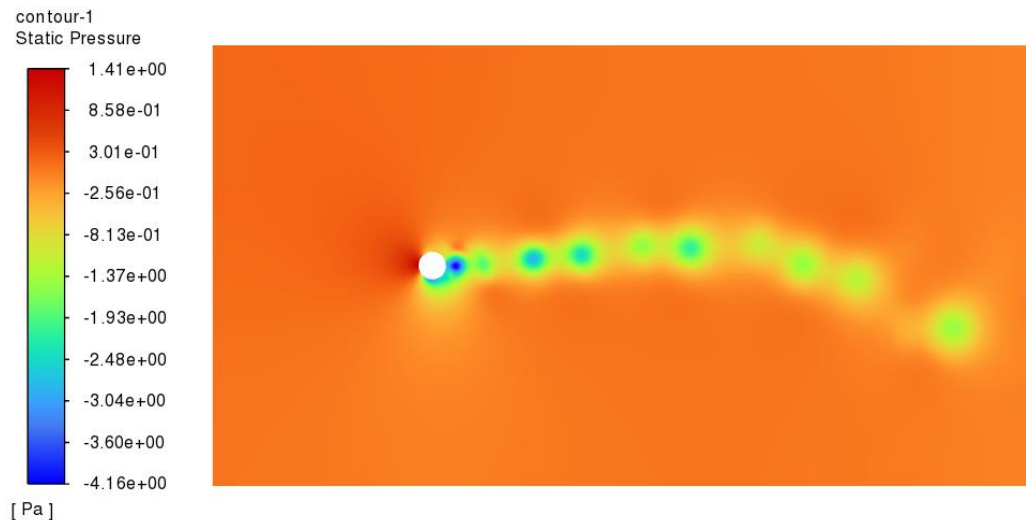


Figure 22. Contour plot of static pressure at $Re=1200$.

For a Reynolds number of 1200 the flow is still laminar, however, the vortices are mixed together, the vortex shedding is no longer symmetric. The aforementioned stagnation point can be

seen in Figures 21 and 22. The pressure difference between the front and the back of the cylinder increased again drastically. As for the Courant number:

$$C_{max} = 1.4957 \quad (28)$$

5.4.2 Lift and drag coefficient

The time-averaged value of the lift coefficient is zero. The mean value of the drag coefficient:

$$c_D = 1.7186 \quad (29)$$

This is higher than the value in Table 1. It can be seen that up to this point the drag coefficients were approximately equal to the theoretical values. According to [8] this is due to the fact that two-dimensional simulations of flow around two-dimensional bluff bodies at Reynolds numbers for which the flow is in reality three-dimensional, lead to inaccurate lift and drag forces.

In [8] it is also stated that it has been shown that the drag coefficient computed from numerical simulations is significantly higher than what is obtained from measurements, and also, it has been found that three-dimensional simulations lead to accurate prediction of the drag. The main reason for this is that the surface pressure distribution obtained from two-dimensional simulations is different from the pressure distribution obtained from three-dimensional simulations and measurements. However, the cause of this difference in the pressures is still unclear.

According to [8] the critical Reynolds number above which three-dimensionality plays a vital role in determining the flow pattern is 180. Above this value a two-dimensional simulation cannot capture the effect of three-dimensional structures on the flow field. Moreover, above this critical Reynolds number 2D simulations usually overpredict the drag coefficient.

I checked another publication with regard to this topic. In [9] it is also stated that between $Re = 190$ and $Re = 260$ a primary instability mechanism occurs which causes three-dimensional disturbances. These disturbances change the structure of the wake vortices, therefore two-dimensional simulations often fail to predict the drag coefficient and the RMS (root mean square) of the lift coefficient accurately.

The time evolution of the lift coefficient can be seen in Figure 23.

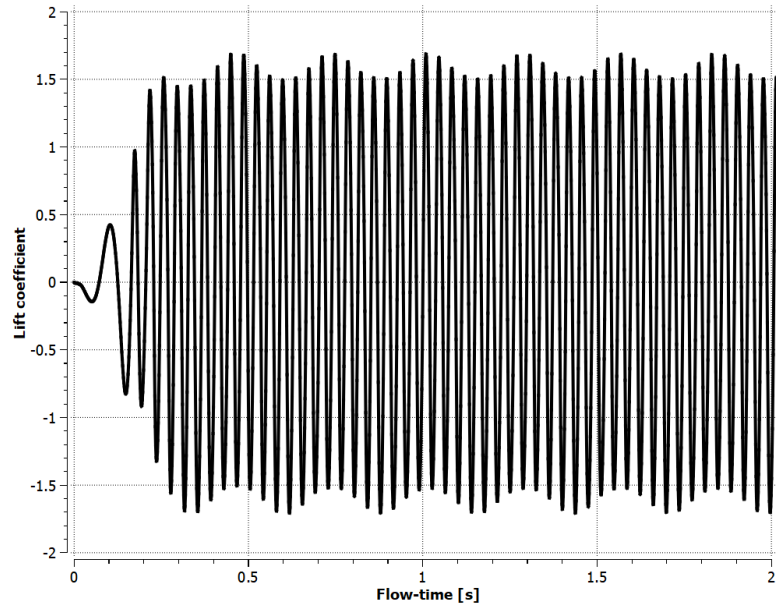


Figure 23. Time evolution of the lift coefficient at $Re=1200$.

The time evolution of the drag coefficient can be seen in Figure 24.

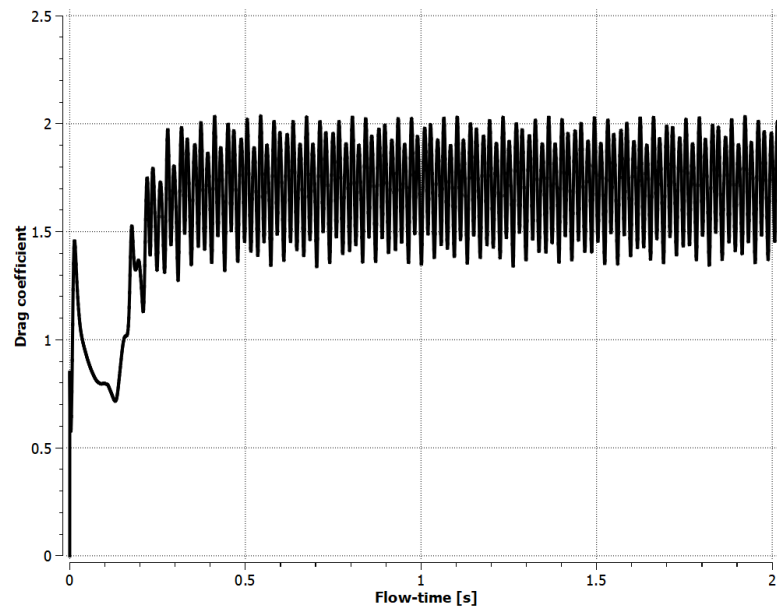


Figure 24. Time evolution of the drag coefficient at $Re=1200$.

5.4.3 Strouhal number

The Fast Fourier Transform for the time signal of the lift coefficient can be seen in Figure 25.

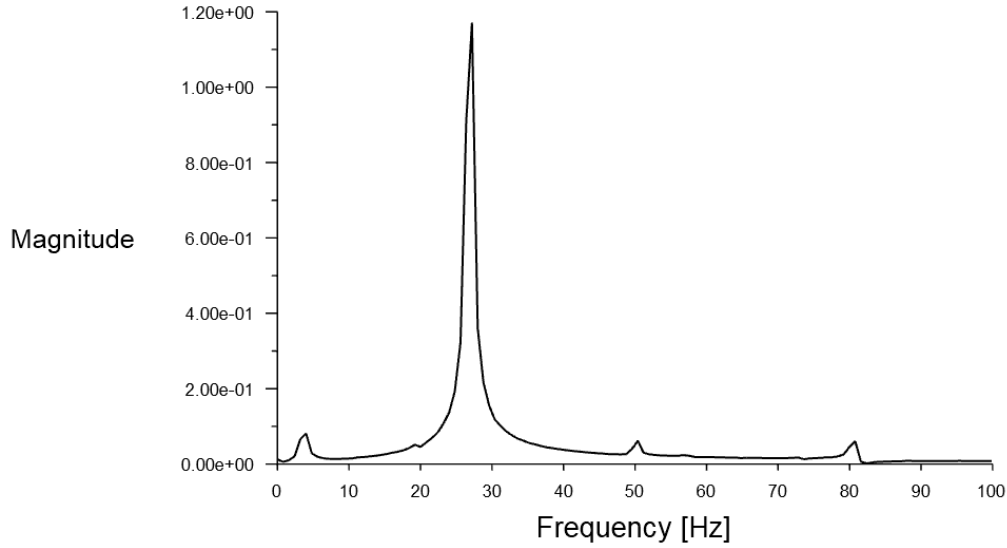


Figure 25. FFT of the lift coefficient at $Re=1200$.

The vortex shedding frequency:

$$f = 27.1956 \text{ [Hz]} \quad (30)$$

The Strouhal number is the following:

$$Sr = \frac{f \cdot D}{V} = 0.2451 \quad (31)$$

This value is higher than the value in Table 1. Due to the three-dimensional disturbances, the obtained lift and drag coefficient values are inaccurate, thus, the non-accurate value of the computed Strouhal number was expected.

In the next few paragraphs, I am going to talk about an interesting discovery with regard to the frequencies. I checked the other local maximum in the lower frequency domain, and it is the following:

$$f_{local} = 3.9994 \text{ [Hz]} \quad (32)$$

f_{local} is the frequency of the up-and-down motion of the vortex street. Thus, the T time period is the following:

$$T = \frac{1}{f_{local}} = 0.25 [s] \quad (33)$$

From Figure 23 it can be seen that between $t = 0.45 [s]$ and $t = 0.7 [s]$ there is a whole time period. Thus, on the velocity contours with regard to these time instants one should see the following result: the vortex street is deflected in the same direction in both cases. In Figure 26 the contours of the velocity can be seen at $t = 0.45 [s]$.

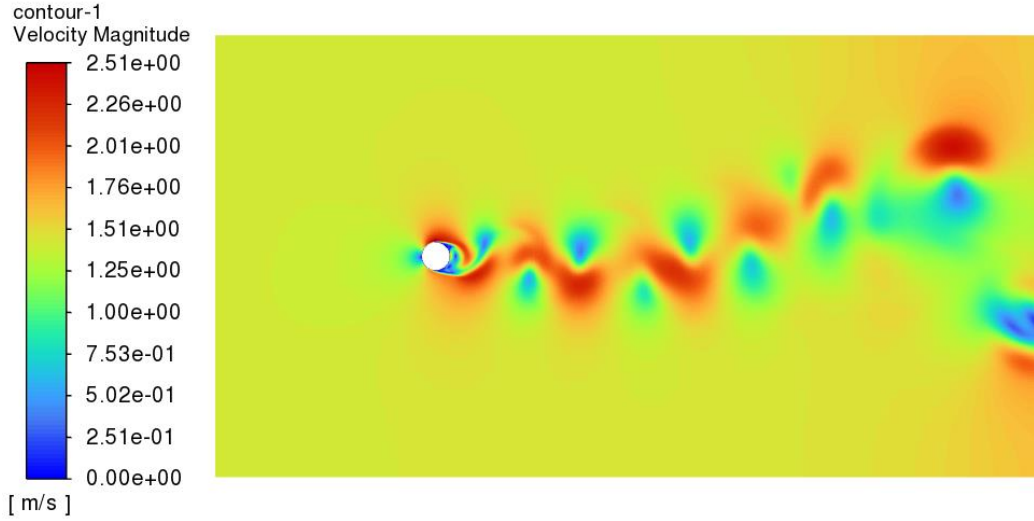


Figure 26. Contour plot of velocity at $t=0.45 [s]$.

In Figure 27 the contours of the velocity can be seen at $t = 0.7 [s]$.

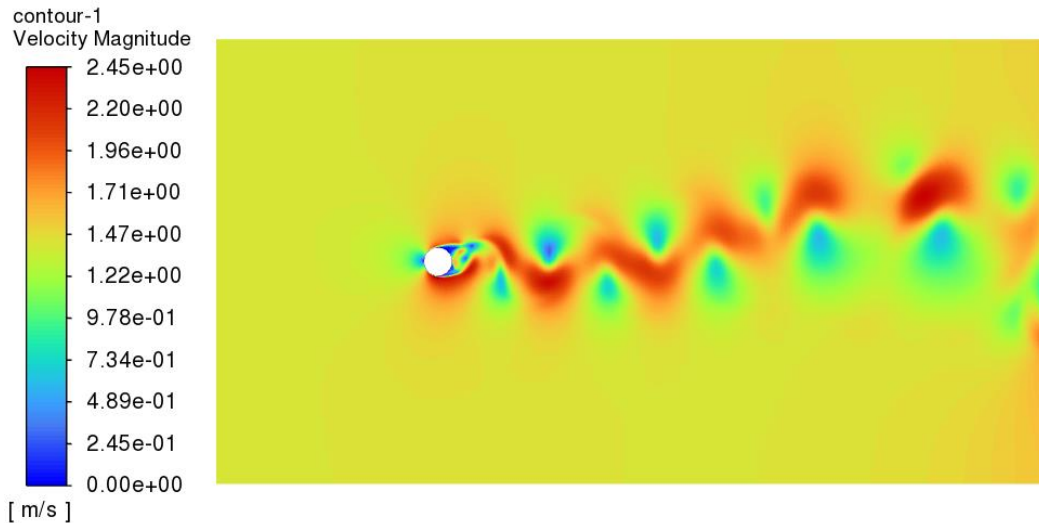


Figure 27. Contour plot of velocity at $t=0.7 [s]$.

From Figures 26 and 27 it can be seen that the aforementioned theory was true, the vortex street is indeed deflected in the same direction in the two time instants. I also inserted Figure 28,

which corresponds to $t = 0.58$ [s]. This time instant is approximately a half time period later than $t = 0.45$ [s], thus, in this case the vortex street should be deflected downwards. It can be seen from Figure 28 that this is fulfilled.

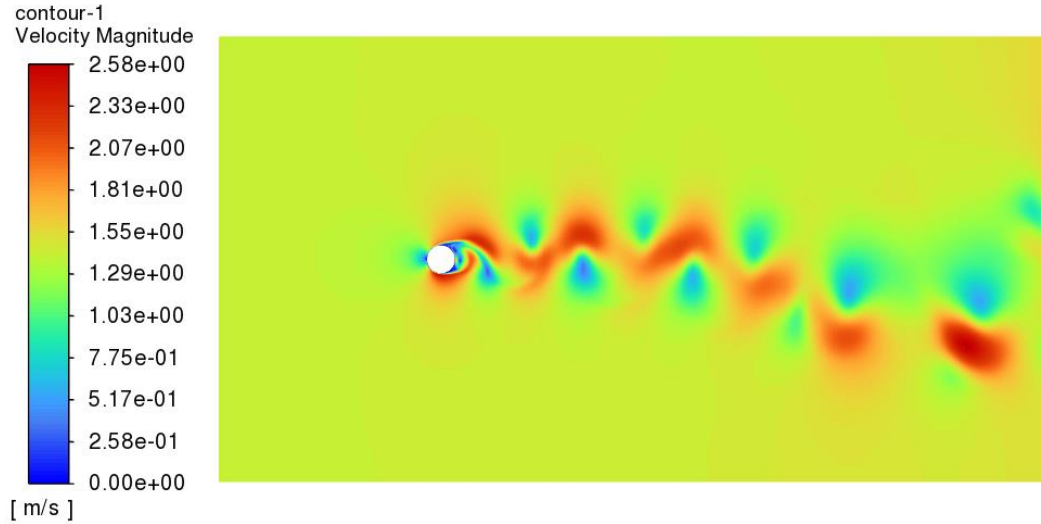


Figure 28. Contour plot of velocity at $t=0.58$ [s].

All in all, it can be said that there are two period oscillations with two characteristic frequencies. One of them corresponds to the vortex shedding (Equation (30)). The other frequency (Equation (32)) corresponds to the changing of the deflection of the vortex street in the up-and-down direction.

6. Mesh independency study

Mesh independency is a very important concept because this way it can be seen whether the results are independent of the mesh or not. One way to do this is to estimate the discretization error. The error can be estimated by the so-called Richardson extrapolation. I performed the Richardson extrapolation based on [4]. The first step is to double the mesh resolution in each direction, this will result in 4 times more cells (in 2D). As for the next step, this procedure is repeated, thus in the case of the finest mesh there are 16 times more cells than in the case of the original one. I am going to denote the results obtained from the different resolutions in the following way:

ϕ_{4h} : coarse mesh result

ϕ_{2h} : intermediate mesh result

ϕ_h : fine mesh result

With these values the discretization error can be estimated in the following way:

$$p_m = \log_r \left(\frac{\phi_{2h} - \phi_{4h}}{\phi_h - \phi_{2h}} \right) \quad (34)$$

$$\varepsilon_h \cong \frac{\phi_h - \phi_{2h}}{r^{p_m} - 1} \quad (35)$$

Where:

- p_m : measured order of convergence
- r : refinement ratio
- ε_h : discretization error

The result for which I performed the calculations was the obtained data for the drag coefficient for a Reynolds number of 120. It is important to mention that keeping the maximum value of the Courant number below 1 would have been too time-consuming with regard to the simulation time, thus, I aimed to keep the Courant number below 5 for all of the simulations. I am going to present the maximum value of the Courant-number for each case, and it will be seen that they are sufficiently low, also, all of the simulations converged, divergence did not occur.

Coarse mesh

In the case of the original mesh, the mesh consists of 25252 cells. After running the simulation, I determined the mean value of the drag coefficient and received the following result:

$$\phi_{4h} = 1.4038 \quad (36)$$

For this case the maximum value of the Courant number is the following:

$$C_{4h,max} = 1.0606 \quad (37)$$

Intermediate mesh

In Fluent I applied the so-called Manual Mesh Adaptation, with which I was able to achieve a finer mesh with a cell number of 101008. I ran the simulation again and obtained the following result:

$$\phi_{2h} = 1.3960 \quad (38)$$

The corresponding Courant number:

$$C_{2h,max} = 2.1808 \quad (39)$$

Fine mesh

As for the fine mesh, I once again applied the beforementioned Manual Mesh Adaptation and obtained the fine mesh with a cell number of 404032. For this case, the simulation result was the following:

$$\phi_h = 1.3937 \quad (40)$$

The Courant number for the fine mesh:

$$C_{h,max} = 4.2878 \quad (41)$$

Richardson extrapolation

From the data above the necessary calculations for the Richardson extrapolation can be performed. The only value that is missing at this point is the refinement ratio which is the following:

$$r = 2 \quad (42)$$

The measured order of convergence:

$$p_m = \log_r \left(\frac{\phi_{2h} - \phi_{4h}}{\phi_h - \phi_{2h}} \right) = 1.8038 \quad (43)$$

I used second-order schemes during the simulations, thus, from Equation (39) one can see that the theoretical accuracy of the numerical schemes has been sufficiently realized.

The discretization error:

$$\varepsilon_h \cong \frac{\phi_h - \phi_{2h}}{r^{p_m} - 1} = -9.0273 \cdot 10^{-4} \quad (44)$$

From Equation (40) it can be seen that this value is sufficiently small, therefore it is safe to assume that if the coarse mesh is used for the simulations, it produces accurate results.

Using these results, one can calculate the extrapolated value of the drag coefficient which is the following:

$$\phi_0 = \phi_h + \varepsilon_h = 1.3928 \quad (45)$$

The relative error of the coarse mesh result (ϕ_{4h}) with regard to the extrapolated value is less than 1 [%]. The Richardson plot can be seen in Figure 29, where 'N' denotes the number of cells corresponding to the different meshes. For better readability the horizontal axis is scaled with 10^6 . $1/N \rightarrow 0$ represents the (exact) value extrapolated to the infinitely fine mesh [4].

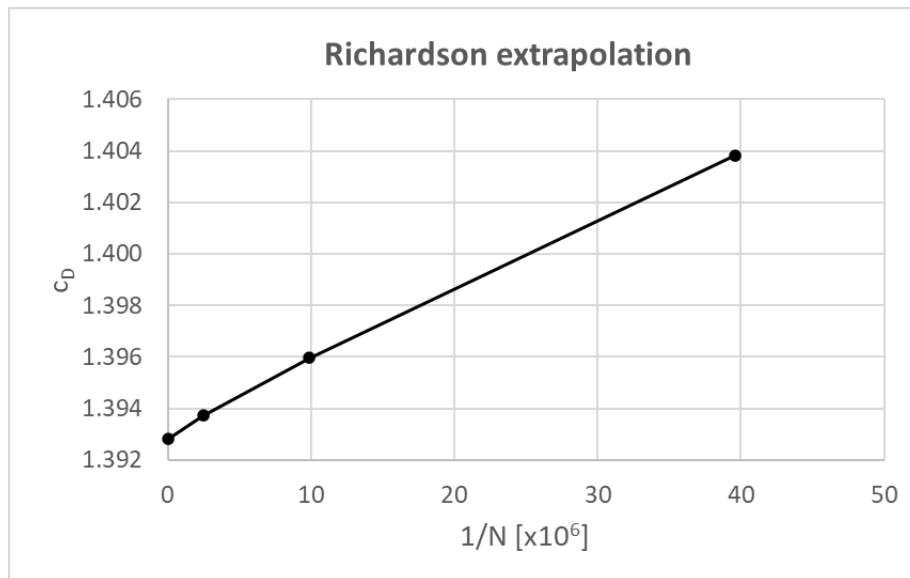


Figure 29. The Richardson plot.

7. Vortex detection with the help of MATLAB

7.1 Parameter optimization and vortex detection at $Re=120$

As it was mentioned in *Chapter 1*, I carried out the parameter optimization based on the results from the simulation with a Reynolds number of 120. The parameters that had to be optimized were the following: Number of contour levels (Nct), Convexity deficiency threshold (%), Minimal arc-length threshold. Below I inserted figures with optimal values of these parameters. In *Appendix 2* all of the optimization test runs can be found.

7.1.1 Parameter optimization

7.1.1.1 Number of contour levels

The first parameter I examined is denoted by Nct, which is the number of contour levels intended to extract. I ran the code for the following values:

$$Nct = 1; 10; 50; 100; 200; 400; 600; 800; 1000 \quad (46)$$

While doing these test runs, I kept the other parameters at a value of 10. After having finished all of the test runs, I found that the minimal value of Nct should be around 100. The maximum value, however, should not be too large, because in that case the detection is not working properly (the simulation becomes slower, and the code detects non-existent vortices). So, the proper interval for this parameter: $Nct \in [100; 200]$. In Figure 30, which is the contour plot of the computed LAVD values, a representative example can be seen with regard to Nct.

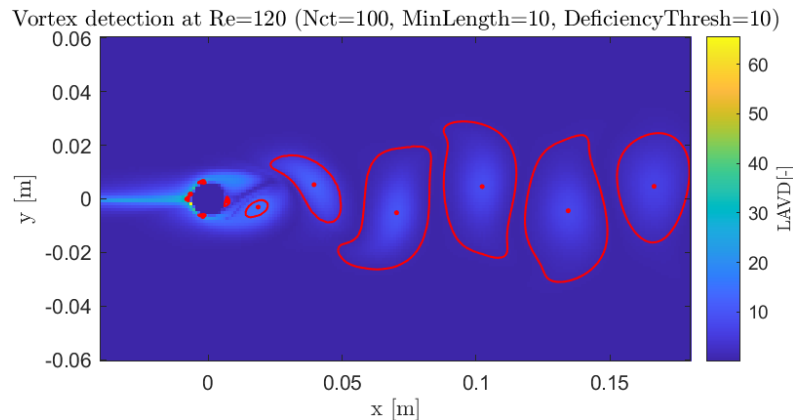


Figure 30. Vortex detection with optimal Nct value at $Re=120$.

It can be seen from Figure 30 that the reference system is the x-y coordinate-system, and the origo is in the center of the cylinder. The red dots represent the centers of the vortices, and the red contours denote the boundaries of the detected vortices.

7.1.1.2 Convexity deficiency threshold (%)

The next parameter that had to be optimized was the convexity deficiency threshold which is denoted by *DeficiencyThresh*. As it was mentioned in *Chapter 3* a small convexity deficiency is allowed which is characterized by the *DeficiencyThresh*. The convexity deficiency of a closed curve in the plane is defined as the ratio of the area between the curve and its convex hull to the area enclosed by the curve [2]. While trying to find the optimal value I kept *Nct* at an optimal value of 100, whereas the minimal arc-length threshold was kept at 10. I ran the code for the following values:

$$DeficiencyThresh = 0; 1; 2; 5; 10; 15; 20; 40; 50; 60; 80; 100 \quad (47)$$

For the *DeficiencyThresh* I found that this value should be around 1. As it was stated in *Chapter 3*, a small convexity error is allowed. Meaning, an interval for the optimal value of this parameter is the following: $DeficiencyThresh \in [1; 10]$. In Figure 31 the vortex detection with an optimal *DeficiencyThresh* value from this interval can be seen.

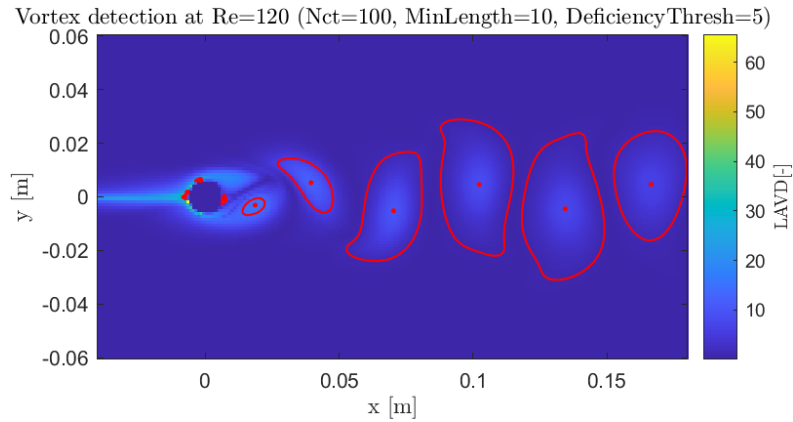


Figure 31. Vortex detection with optimal *DeficiencyThresh* value at $Re=120$.

7.1.1.3 Minimal arc-length threshold (%)

The last parameter is the minimal arc-length threshold, denoted by *MinLength*. As it was mentioned in *Chapter 3* this value means that the closed LAVD contours whose arclength falls below this predetermined threshold value are ignored. It is important to note that *MinLength* is

expressed as a percentage of the cylinder diameter. I performed test runs for the following values (I kept Nct at 100 and DeficiencyThresh at 10):

$$\begin{aligned} \text{MinLength} = & 1; 10; 20; 40; 60; 65; 70; 75; 80; 100; 110; \\ & 120; 130; 150; 170; 180; 185; 190; 200; 1000 \end{aligned} \quad (48)$$

Having examined the figures, I found that the minimum value should be around 75 (this is the value when only the 6 appropriate vortices are detected) and the maximum value should be in the vicinity of 180 (this is the value when the smallest vortex is still detected, it disappears around 185), thus the proper interval: $\text{MinLength} \in [75; 180]$. In Figure 32 the vortex detection with an optimal MinLength value can be seen.

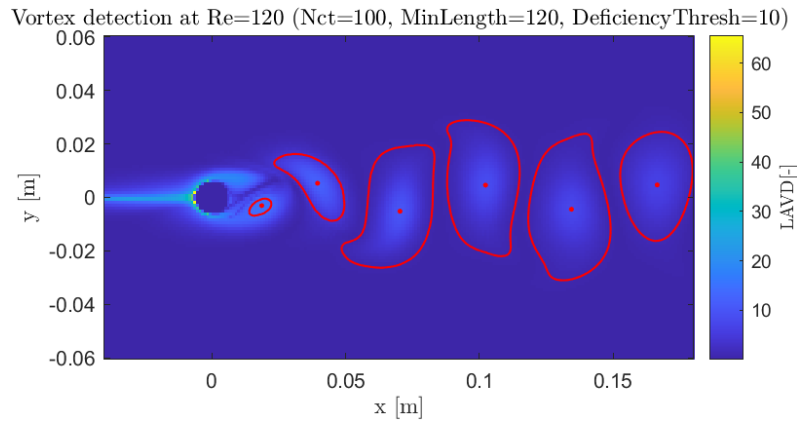


Figure 32. Vortex detection with optimal MinLength value at $Re=120$.

7.1.2 Vortex detection with optimized parameters

With the optimized parameters I ran the code and received Figure 33.

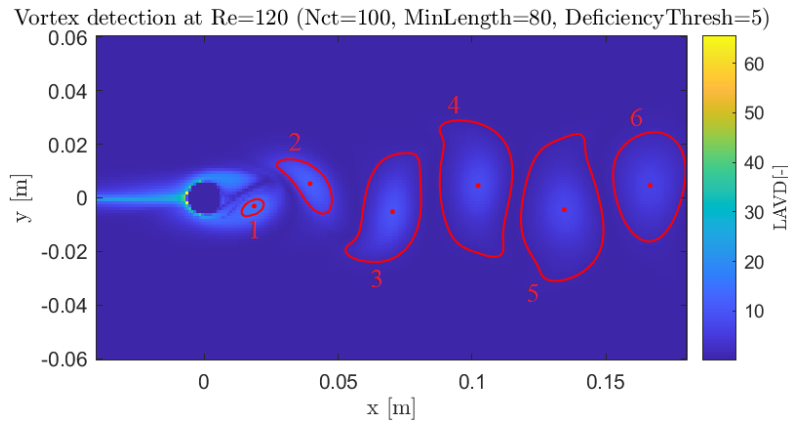


Figure 33. Vortex detection with optimized parameters at $Re=120$.

It can be seen in Figure 33 that the vortices are shed from alternating sides of the cylinder which is the expected result. One way to check whether the optimized parameters are appropriate or not is to examine the centers of the vortices. The coordinates of the centers of the vortices can be seen in Table 4.

Table 4. Centers of the vortices.

Serial number of the vortex	$x_i [m], \quad i \in [1; 6]$	$y_i [m], \quad i \in [1; 6]$
1	0.0187	-0.0031
2	0.0395	0.0053
3	0.0703	-0.0052
4	0.1023	0.0046
5	0.1344	-0.0044
6	0.1664	0.0047

The next step was to check the differences between the x and y coordinates with regard to the neighboring vortices (the subscript denotes the two vortices between which I calculated the coordinate differences), this can be found in Table 5.

Table 5. Differences between the centers of the vortices.

Numbers of the neighboring vortices	$\Delta x_{ij} [m], \quad i \in [1; 5]; j \in [2; 6]$	$\Delta y_{ij} [m], \quad i \in [1; 5]; j \in [2; 6]$
1-2	0.0208	0.0084
2-3	0.0308	-0.0105
3-4	0.0320	0.0098
4-5	0.0321	-0.0090
5-6	0.0320	0.0091

The last thing that had to be performed was the determination of the ratio of the coordinate differences, meaning:

$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.4038 \quad (49)$$

$$\frac{\Delta y_{23}}{\Delta x_{23}} = -0.3409 \quad (50)$$

$$\frac{\Delta y_{34}}{\Delta x_{34}} = 0.3063 \quad (51)$$

$$\frac{\Delta y_{45}}{\Delta x_{45}} = -0.2804 \quad (52)$$

$$\frac{\Delta y_{56}}{\Delta x_{56}} = 0.2844 \quad (53)$$

The theoretical value for the $\frac{\Delta y}{\Delta x}$ ratio is 0.281 [5], one can see from Equations (49)-(53) that the calculated results are acceptably close to the conceptual value and also, they converge to the theoretical value further downstream from the cylinder, thus, with the optimized parameters the code was able to detect the evolved vortices in a proper way.

7.2 Parameter optimization and vortex detection at Re=12

For a Reynolds number of 12, the code was not able to detect the vortex pair at the back of the cylinder neither with the optimized nor with other parameters. This can be explained by the fact that these vortices are at a fixed place, and they are not detaching, this is a steady case of flow.

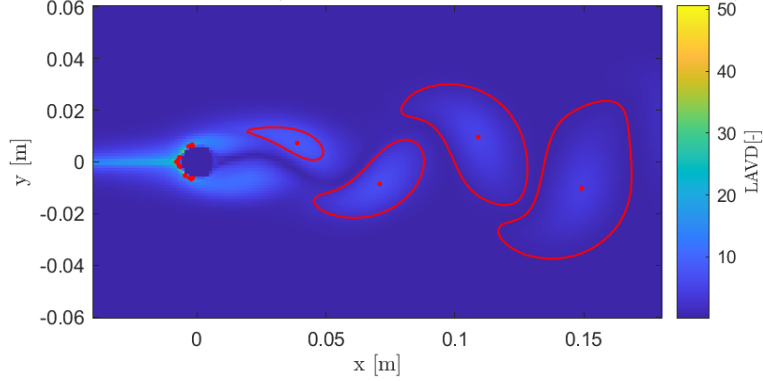
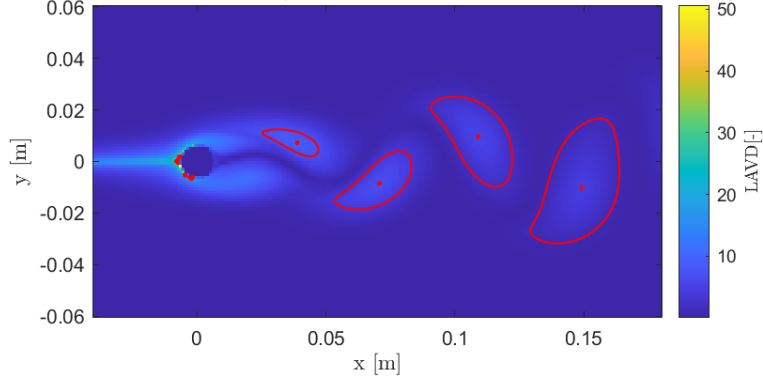
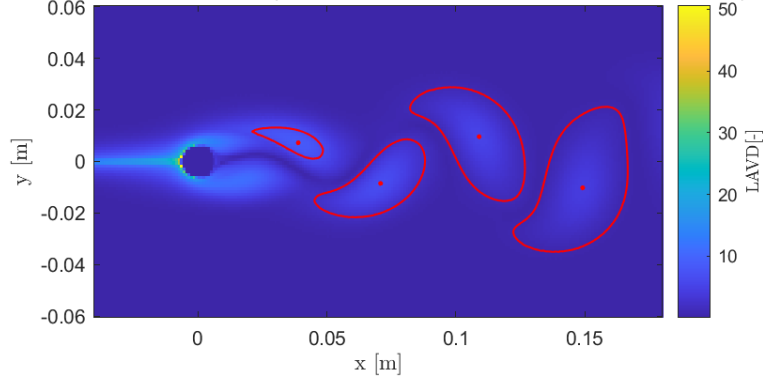
7.3 Parameter optimization and vortex detection at Re=60

7.3.1 Parameter optimization

In the case of $Re = 60$ the code was able to detect the evolved vortices in a proper way. It is also important to note that the detection worked with the optimized parameters, however, I performed the optimization process again (the results remained the same). This time I am not going to write about each parameter individually, because this has already been done in 7.1.1. Instead, I am going to present the ideal intervals for each parameter in a table form, along with the corresponding figures (representing the detection with optimal parameters from the aforementioned intervals). The other figures can be found in *Appendix 1*.

Table 6. Parameter optimization at Re=60.

Parameter	Nct
Test values	1;10;50;100;200;400;600;800;1000
Optimal interval	[100;200]

Figure with optimal parameter	Vortex detection at Re=60 (Nct=200, MinLength=10, DeficiencyThresh=10) 
Parameter	DeficiencyThresh
Test values	0;1;2;5;10;15;20;40;50;60;80;100
Optimal interval	[1;10]
Figure with optimal parameter	Vortex detection at Re=60 (Nct=100, MinLength=10, DeficiencyThresh=2) 
Parameter	MinLength
Test values	1;10;20;40;50;55;60;65;70;75;80;100;110;120;130;150;170;180;250;300;400;450;480;490;495;500;600;1000
Optimal interval	[60;495]
Figure with optimal parameter	Vortex detection at Re=60 (Nct=100, MinLength=180, DeficiencyThresh=10) 

It can be seen that the optimal interval both for N_{ct} and $DeficiencyThresh$ is exactly the same as for $Re = 120$. With regard to the $MinLength$ parameter, I increased the upper limit this much, because the size of the evolved vortices has increased immensely also. Thus, the optimal interval can be widened, but it is still overlapping with the optimal interval for $Re = 120$.

7.3.2 Vortex detection with optimized parameters

With the optimized parameters I ran the code and received Figure 34.

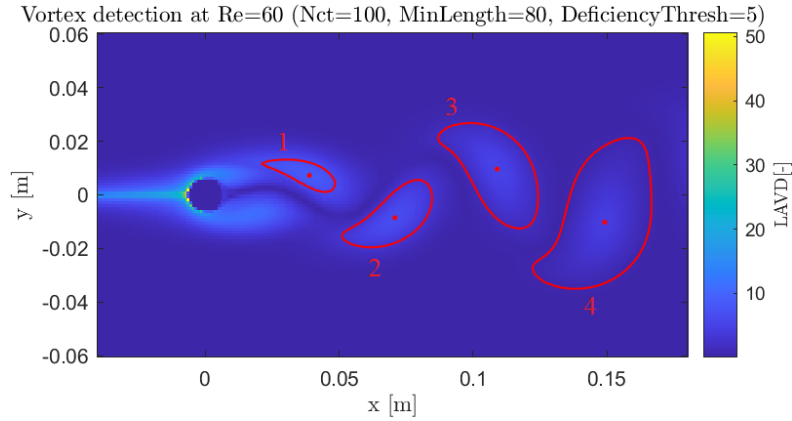


Figure 34. Vortex detection with optimized parameters at $Re=60$.

I determined the $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case as well:

$$\frac{\Delta y_{12}}{\Delta x_{12}} = -0.4922 \quad (54)$$

$$\frac{\Delta y_{23}}{\Delta x_{23}} = 0.4738 \quad (55)$$

$$\frac{\Delta y_{34}}{\Delta x_{34}} = -0.4938 \quad (56)$$

In this case there is not a theoretical value for the $\frac{\Delta y}{\Delta x}$ ratio. However, it is good to determine these values, because for example one can see from Equations (54)-(56) that the vortex shedding is symmetric. The positive and negative signs are alternating, and the absolute values of the ratios are close to each other. It is important to notice that in contrast to the $Re = 120$ case, the ratios do not converge, they immediately become uniform. It seems that these ratios are suitable for the differentiation of the different vortices.

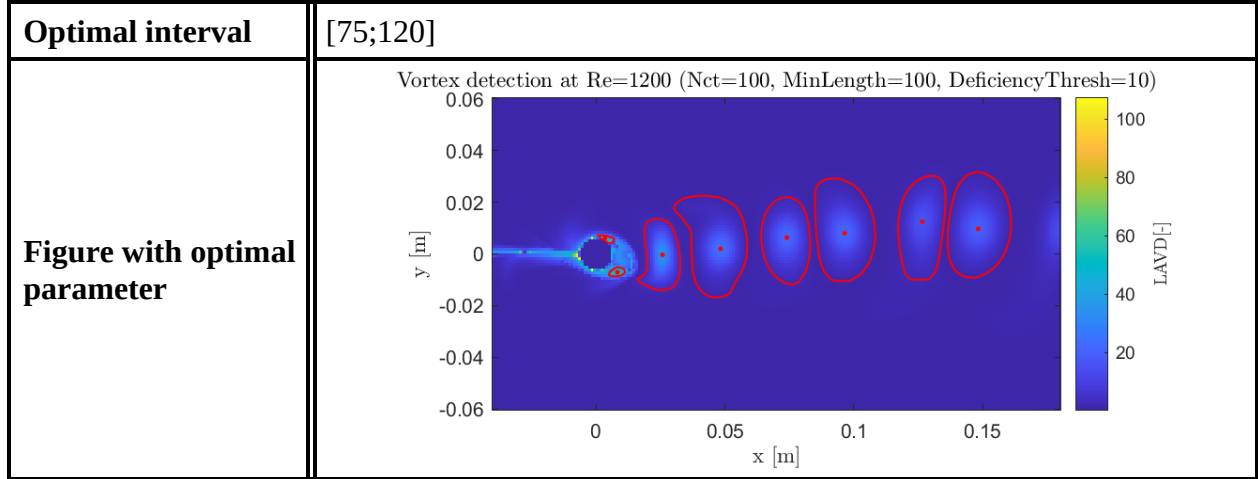
7.4 Parameter optimization and vortex detection at $Re=1200$

7.4.1 Parameter optimization

In the case of $Re = 1200$ the code was again able to detect the evolved vortices with the optimized parameters. I performed the whole optimization process for this case as well. Once again, the results are presented in a table form and the rest of the figures can be found in *Appendix 3*.

Table 7. Parameter optimization at $Re=1200$.

Parameter	Nct
Test values	1;10;50;100;200;300;400;600;800;1000
Optimal interval	[100;200]
Figure with optimal parameter	Vortex detection at $Re=1200$ (Nct=100, MinLength=10, DeficiencyThresh=10)
Parameter	DeficiencyThresh
Test values	0;1;2;5;6;7;10;15;20;40;50;60;80;100
Optimal interval	[1;5]
Figure with optimal parameter	Vortex detection at $Re=1200$ (Nct=100, MinLength=10, DeficiencyThresh=1)
Parameter	MinLength
Test values	1;10;20;40;60;65;70;75;80;100;110;120;125;130;150;1000



The optimal interval for Nct is exactly the same as in the previous two cases. For the DeficiencyThresh parameter I had to restrict the optimal interval, because otherwise it detected vortices (while only using optimal values for each parameter) which have just been shed from the surface (e.g., at *DeficiencyThresh* = 10), and this is not favorable. Thus, I only wanted to detect vortices that had already moved from the immediate vicinity of the cylinder (i.e., the contour of the vortex should not be touching the cylinder). Nevertheless, the optimal interval for the DeficiencyThresh at $Re = 1200$ is still overlapping with the optimal DeficiencyThresh intervals in the case of $Re = 60$ and $Re = 120$. As for the MinLength parameter, it is the most restricted for $Re = 1200$, but once again, it overlaps greatly with the optimal MinLength intervals for $Re = 60$ and $Re = 120$.

7.4.2 Vortex detection with optimized parameters

With the optimized parameters I ran the code and received Figure 35.

The $\frac{\Delta y_{ij}}{\Delta x_{ij}}$ ratios for this case:

$$\frac{\Delta y_{12}}{\Delta x_{12}} = 0.3867 \quad (57)$$

$$\frac{\Delta y_{23}}{\Delta x_{23}} = 0.0104 \quad (58)$$

$$\frac{\Delta y_{34}}{\Delta x_{34}} = 0.1673 \quad (59)$$

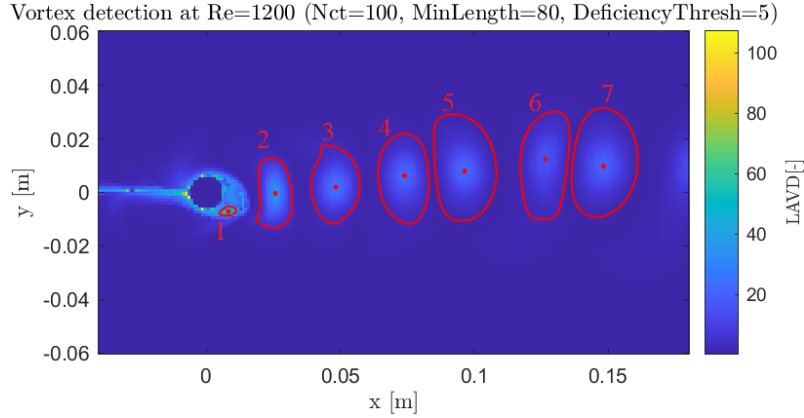


Figure 35. Vortex detection with optimized parameters at $Re=1200$.

$$\frac{\Delta y_{45}}{\Delta x_{45}} = 0.0759 \quad (60)$$

$$\frac{\Delta y_{56}}{\Delta x_{56}} = 0.1490 \quad (61)$$

$$\frac{\Delta y_{67}}{\Delta x_{67}} = -0.1290 \quad (62)$$

From Equations (57)-(62) it can be seen that the vortex shedding is no longer symmetric. The positive and negative signs are not alternating, and the absolute values of the ratios are varying abruptly.

One can also notice from Figure 35, that the vortices 2-3, 4-5, and 6-7 seem like they are shed in pairs. To see whether this can be true or not, I am going to present the distances between the centers of these vortices in the x direction:

$$\Delta x_{23} = 0.0225 [m] \quad (63)$$

$$\Delta x_{45} = 0.0224 [m] \quad (64)$$

$$\Delta x_{67} = 0.0217 [m] \quad (65)$$

From Equations (63)-(65) one can see that the assumption that these vortices are shed in pairs can be true indeed.

7.4 Summary of the vortex detection results

The code was able to detect the evolved vortices in each case, except for $Re = 12$, which is a steady case of fluid flow. In this case the vortices are not shed, thus, it was expected that the code will not be able to detect the counter-rotating vortex pair. As for the other cases, the parameter optimization has been performed, and optimal intervals have been determined for all of the vortex detection parameters. It could be seen that in spite of the fact that the optimal intervals for the different cases are not exactly the same, each parameter has values which are optimal for all of the above cases. It is important to note that in the case of Nct all of the determined optimal intervals were the same. I also determined the intersection of the intervals for the different parameters, this can be seen in Table 8.

Table 8. Summary of the optimal intervals.

	Re=60	Re=120	Re=1200	Re=60$\cap$Re=120$\cap$Re=1200
Nct	[100; 200]	[100; 200]	[100; 200]	[100; 200]
DeficiencyThresh	[1; 10]	[1; 10]	[1; 5]	[1; 5]
MinLength	[60; 495]	[75; 180]	[75; 120]	[75; 120]

From Table 8 one can see that the values for Nct and DeficiencyThresh could be determined more or less independently of the Reynolds number. It is also important to mention that the value of MinLength is in great accordance with the size of the cylinder (partly, this is why it was beneficial to express the minimal arc-length threshold as the percentage of the diameter of the cylinder). What is meant by this is that the sizes of the shed vortices are not excessively smaller or larger than the cylinder itself, they are in great accordance.

Additionally, I determined the ratios of the coordinate differences between the neighboring vortices in each case. In the case of $Re = 60$ one could see that the vortex shedding is symmetric, and the ratios do not converge, they immediately become uniform. For $Re = 120$ the ratios are close to 0.281, and they also converge to this value further downstream from the cylinder, thus, the criterion for the von Karman vortex street was met. At last, in the case of $Re = 1200$, the vortex shedding was not symmetric, although, the possibility of the vortices shedding in pairs was

noticed, and by calculating the distances in the x direction this theory was supported. All in all, one can notice that these ratios could be suitable for the differentiation of the different vortices, and the different types of vortex shedding.

8. Summary of results

Flow around bluff bodies, especially cylinders, has been studied immensely due to the number of practical applications. Through the flow around circular cylinders several phenomena can be observed with regard to fluid mechanics. For instance, boundary layer separation, and vortex shedding. For a given interval of the Reynolds number, a repeating pattern of swirling vortices can be observed, this is the Karman vortex street. This phenomenon is responsible for the “singing” of telephone wires. Vortex induced vibrations can seriously damage the structures of tall buildings, therefore, during the design process, it is essential to take these effects into account. Vortex detection can be useful for a myriad of things, from merely observing the microstructure of the flow, through geometrical optimization, to noise reduction.

The main aim of this study was to optimize the parameters of an existing vortex detection method and detect the evolved vortices in fluid flow around a cylinder. The data, for which I performed the detection, was a result of two-dimensional simulation for different Reynolds numbers. The validation of the simulation results happened by comparing the obtained velocity and pressure figures with figures from the literature. Also, the drag coefficient, the lift coefficient, and the Strouhal number was presented for all of the cases, and they were compared with values taken from the literature. For a Reynolds number of 12, 60 and 120 the obtained values were acceptably close to the values taken from the literature. The time-averaged value of the lift coefficient was zero in all of the three cases, and the drag coefficient decreased as the Reynolds number increased. For $Re = 1200$, the time-averaged lift coefficient was zero, however, the drag coefficient, and the Strouhal number were higher than the values taken from the literature. In the case of $Re = 1200$, the Reynolds number exceeds the critical Reynolds number, above which the flow transitions to three-dimensional. If this occurs, a two-dimensional simulation cannot capture the effect of three-dimensional structures on the flow field, and this caused the discrepancy in the results. Later on, a mesh independency study ensured grid independency.

The parameter optimization was at first performed for the $Re = 120$ case due to the fact that in the case of a Karman vortex street, there exists a theoretical value in the literature for the ratio of the coordinate differences between the neighboring vortices, which is 0.281 [5]. With the optimized parameters, after performing the detection, the calculated values were close to this

theoretical value. Thus, the detection worked properly. For a Reynolds number of 12, the code failed to detect the evolved counter-rotating vortex pair, however, this was expected due to the fact that this is a steady case of flow, meaning, these vortices are not shed, they are at a fixed place. The detection worked for a Reynolds number of 60 and 1200, and after performing the optimization process repeatedly, I found the optimal intervals for these two cases as well. All in all, though the intervals are not exactly the same for the different Reynolds numbers, they are overlapping, meaning, each parameter has values which are optimal for all of the examined cases. The intersection of these intervals has been determined also.

As a result of my study, the fluid flow around a circular cylinder was simulated and examined at different Reynolds numbers by using a proper two-dimensional model. In addition, the optimization of the vortex detection parameters was performed, and with these parameters the vortex detection was carried out properly. The current study aimed to make the vortex detection method more objective by giving suggestions for the optimal values of the different vortex detection parameters. Some possible future plans for the current study include three-dimensional simulations and vortex detection. In addition, testing the determined optimal intervals for other geometries is also a possible field for improvement.

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