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**GAZDASÁGI VÁLSÁGOK ELŐREJELZÉSE MAGYARORSZÁGON A
MARKOV-FÉLE REGRESSZIÓS MODELL SEGÍTSÉGÉVEL
FORECASTING RECESSIONS IN HUNGARY WITH LEADING AND
LAGGING INDICATORS USING THE MARKOV SWITCHING
DYNAMIC REGRESSION MODEL**

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1. Introduction

In their seminal work, Burns and Mitchel (1946) have framed the features of a nation's aggregate economic activity into the theoretical concept of a business cycle consisting of expansions and contractions. According to Shaw (1947), this allowed for the first time in the history of econometrics to decompose economic time series into cyclical, seasonal, trend and error components, opposing the pre-existing Keynesian unemployment equilibrium theory and its doctrine (Burns, 1946, pp: 4-11). While the idea of business cycles has been subject to rigorous analysis and criticism by Koopmans (1947), and later, by Stock's (1987) critical revision that statistically disproved the existence of a macroeconomic time scale, it nonetheless introduced a new field of research in econometrics – business cycle forecasting.

Empirical studies concerned with determining relationships between economic variables and business cycles branched into: Stock and Watson's (1991, 1993) dynamic factor modelling approach aimed at capturing comovements of coincident economic indicators (CEI), leading economic indicators (LEI) and finding composite leading indicators (CLI) for a nation's economy; Hamilton's (1989) extension on Goldfeld and Quandt's (1973) Markov switching model; and, more recently, a synthesis of the aforementioned regime switching and dynamic factor models by Diebold and Rudebusch (1996) and Chauvet (1998).

Changes in the macroeconomic environment in the form of rapid declines in output, hyperinflations and economic crises have been challenging to incorporate into a stationary linear model, due to the presence of structural breaks. When expanding the observed time horizon, it becomes evident that periods of growth and decline in the economy are recurring and cyclical. Therefore, the model in question has to incorporate a parameter that would take on different values depending on a discrete number of 'regimes' the system can theoretically be in. This enables to capture the structural breaks inherent to the system (Piger, 2009). In Hamilton's (1989) regime - switching hidden Markov model, the time series is controlled by a parameter vector that changes depending on an unobserved state variable that follows the evolution of a first-order Markov chain. The inherent property of a Markov chain is that the future value depends on its immediate value alone and not its previous values – this allows to perform accurate short-term out-of-sample forecasting and identifying 'turning points' of business cycles. There has been ample evidence on the successful implementation of univariate Markov regime-switching dynamic factor models for characterization of business cycle dynamics, particularly with datasets of developed economies. From the seminal work of

Hamilton (1989), which was based on Goldfeld and Quandt (1973) and Neftçi's (1984) analysis of unemployment and business cycle asymmetry to Filardo's (1994, 1998) time-varying transition probabilities, augmented with Layton and Smith's (2000) signalling system – the Markov regime switching model and its extensions has generated many promising results for time series of developed countries. As of writing this research, there have been few studies of business cycle modelling for Eastern European countries and developing economies. Industrial business cycles were determined through a Markov-switch method for Romania, Poland and the Czech Republic by Spulbăr (2012). Bandholz (2005) applied the univariate Markov regime-switching model to the industrial production index of Poland and Hungary as well as the BUX composite stock index, while, more recently, Siničáková (2017) analyzed the level of business cycle synchronization of the Visegrad group countries with the Euro area through a Markov-switching autoregressive model. Leon et. al (2005), however, found that the regime switching models failed to characterize the business cycle of newly industrialized South Korea and Taiwan. What is common in most of the research is that a univariate Markov switching model is applied to the economic time series. However, as it is pointed out by Blanchard and Quah (1989) and later reaffirmed in Kuan's (2002) work, a composite economic variable, such as the industrial production index, the GDP or the GNP is affected by multiple disturbances. Therefore, modelling the time series through a univariate autoregressive Markov regime switching process could yield inaccurate results. This research adopts a bivariate regression model to determine links between the Hungarian GDP growth rate and a set of key macro leading, coincident and lagging indicators, such as the unemployment rate, inflation, industrial production index, BUX composite stock market quarterly returns and 10-3-year yield spreads of Hungarian sovereign bonds. The estimation of the hidden Markov model will allow to explain the changes in the GDP using the changes in the individual macro indicators. The model is built in Python IDE and in addition to the state probability matrix and the visualization of the probabilities of regimes, a state probability distribution is plotted and analyzed in Section 5 of the research.

The objective of this paper is to assess the usefulness of common recession macroeconomic indicators in predicting changes in the GDP for the Hungarian economy through implementing a hidden Markov switching dynamic regression model, extending the initial approach of Hamilton (1989). Thereby, the contribution of this work is threefold: first, it adds to the empirical research on developing countries, since the object of the study is the Hungarian economy; secondly, an exogenous variable is included in the dynamic factor

regression model to better explain the shocks of the composite GDP indicator and thirdly, a program listing is included for future improvement and reference.

The rest of the research is structured as follows: section 2 presents a brief literature review of the application of Markov chains in econometrics, the methodology section dissects the Markov regime-switching model specifications. Section 4 provides a description and the reasoning for the choice of the macro indicators. Section 5 presents the results obtained from the model and interprets them. In section 6 the results are critically reviewed and explained in detail. Section 7 draws conclusions and possibilities of future research.

2. Literature review

Before the introduction of Markov regime-switching models in econometrics, the characterization of the relationship between economic activity indicators, such as the GNP or the GDP with the business cycle, have chiefly been through the implementation of autoregressive integrated moving average (ARIMA) models, like in the works of Nelson and Plosser (1982) and Campbell and Mankiw (1987). Simultaneously, a cointegration approach of Granger (1969, 1987) has been developed as a means of determining if the lagged values of a time series are useful in predicting the future values of the reference series. As outlined by Hamilton (1989), previous works do not take into account the fact that the time series don't follow a linear stationary process, and therefore, the approaches are unsuitable for modelling regime changes in the time series – inherent to business cycles.

In their critical work, Goldfeld and Quandt (1973) employ a Markov switching regression to determine the transition matrix of variable (z) and derive the maximized likelihood function $r(z)$ to characterize the persistence tendencies of supply-demand and the probability of regime prevalence and regime changing. Neftçi's (1984) algorithm, is adopted and formalized by the benchmark model of Hamilton (1989), where the economy is between two regimes governed by a two-state first-order Markov algorithm - the methodology which is applied is a Markov-switching autoregression model. The regime switch is dependent only on the previous state before switching occurs, this means that the model does not require any prior information from the system. The turning point, where the regime inflection occurs, can therefore be determined. Further specifications of the model are found in the succeeding part of the research.

The most prominent literature on Markov regime switching models is usually found for developed countries. As of the publication of this paper, empirical literature on the application of Markov regime-switching dynamic factor models for Central and Eastern European countries is limited to the study of asynchronous behavior of individual business cycles with the Euro area, primarily inspired by a common aim of discrediting a uniform monetary policy and further expansion of the Euro area. While Bandholz (2005), Di Giorgio (2016), Siničáková (2017), applied the Markov switching model for panel data, Artis (2004) and Darvas and Szapáry (2008) used the Hodrick-Prescott filter to measure synchronization. For both methodologies the hypotheses of CEEC and Euro Area business cycle independence have been rejected. A useful extension noted by Di Giorgio (2016) would be considering other variables within the Markov switching vector autoregression model. For stock markets in the CEEC, Moore and Wang (2007), Linne (2002) and Krolzig's (1999) works feature the application of

Markov switching for weekly stock index returns. The authors use high and low volatility states for regime switching. Recently, a Markov switches have been applied for the Turkish economy, in works of Balcilar et. al (2015) and Bilgili et. al (2020), where the relationships between globalization and environmental sustainability have been examined. Hoque et. al (2019), for instance analyzed the implementation of economic policy on stock market returns. economic However, it is determined that the level of correlation of an individual countries' endogenous variable with the greater Euro Area business cycle is not uniform across the set of panel data presented in literature. For instance, Moore and Wang (2007) determine that in case of Hungary, before the European Union integration, a low volatility regime persisted in the stock market, while in Poland, Slovenia, Slovakia and the Czech Republic, inflection points indicating a regime shift have been observed. This breaks the uniformity in business cycle characterization for other member states. Considering the various qualitative results obtained from the analysis of the aforementioned literature, this research estimates the model only for Hungary.

Further evidence of the application of Markov switching dynamic factor models to the time series of emerging economies is found in Petreski's (2011) work, where the inflation targeting policy effectiveness is analyzed through the scope of exchange rate pegging in Hungary and the Czech Republic. In the estimation equation the authors consider both endogenous and exogenous variables that enter the system. In Kuan's (2002) and Balanchard and Quah's (2002) works a bivariate regression is specified following the reasoning that supply and demand shocks on markets can't be characterized by GDP or GNP regime shifts alone. For Taiwan Chen et. al. (2000) estimates a much stronger identification capability of multivariate Markov-switching models for GDP, consumption expenditure, gross capital formation and real exports. Following the reasoning of Dueker (1997), the yield curve is an upward-sloping economic indicator – this is evident, because the longer the duration of the bond, the higher the risk, therefore, the higher the payoff – yield. From the point of view of interest rates, a higher yield ensures that the opportunity cost of investing the same amount of money in the bank is as small as possible. Slowdown in economic activity, a cooling off in financial markets, lower sales, low unemployment, low interest rates, the chase for yield – are all early signs of the end of a business cycle. Estrella and Hardouvelis (1991) explain the flattening of the yield curve with policy tightening – the result of monetary policy. During these times, the premise of investors is that long

Evidence from Hamilton and Kim's (2002) study shows strong support for the yield curve to be a predictor of GDP. As for evidence in case of European financial variables, we can note that for Germany, Plosser and Rouwenhorst (1994) found term spreads useful in predicting GDP. Switching and dynamic factor models were introduced by Diebold and Rudebusch (1996) and Chauvet (1998).

3. Methodology

Before the research moves forward with the specifications of the hidden Markov regime-switching model, it is imperative to understand the underlying basics of this statistical tool – the Markov chain and furthermore, to understand why this model is governed by a process that has no memory. Detailed introduction of Markov Switching models can be found in Hamilton (1994), Kim and Nelson (1999). A comprehensive algorithm set up guide in MATLAB is presented by Perlin (2015). The notation as well as the equations are adopted from Hamilton (1994), Perlin (2015) and Date (2022).

Let's illustrate how a Hidden Markov Model can be used to represent a real-world data set. Let us have some real-world economic data represented in Figure 1

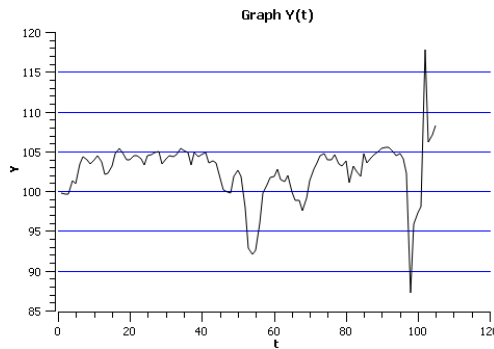


Figure 1 this graph represents data of Hungarian GDP corresponding to the same period of previous year in % - $y(t)$, instead of dates in t I used just an arbitrary serial number. Source: compiled by author

The above graph shows large scale regions of positive and negative growth together with abrupt deviations – crises. This positive and negative growth could be, for example, expansion and contraction of the economy, the ups and downs of the business cycle.

It is hypothesized that: (i) the unknown stochastic process changes the behavior of the indicator, so that the specific characteristics of the behavior are classified into regimes; (ii) there are two regimes in the Hungarian economy denoted as 1 and 2, which represent an expansion and a recession correspondingly; (iii) Every indicator can be represented in two-regime model through the maximum likelihood estimation function; (iv) the maximum likelihood function is more accurate if exogenous variables are added into the model; (v) the regime shifts of the Hungarian GDP in a multivariate model correspond to the business cycles determined by the OECD.

Finding the turning point in future effectively means we could find the starting point of the recession on the GDP time series. While modelling the panel data, we will consider a

regression model that is a mixture of the following two random variables: the observable random variable $y(t)$, which would be used to represent the observable pattern (the observable economic variable itself), and a hidden random variable $m(t)$ which is assumed to change its state or regime, and each time the regime changes, it affects the observed pattern of $y(t)$. In other words, a change in value of $m(t)$ impacts the mean and variance of $y(t)$. This is the primary idea behind Hidden Markov Models. For convenience of the reader, we present here the short summary of the relevant theory.

Let's assume that $m(t)$ switches between two regimes 1 and 2. It means that we start to use the simplest Markov chain for variable $m(t)$ – the two state Markov chain. It is obvious $m(t)$ is a 'hidden' random variable – it is 'hidden' since we do not know what state the system is in (what regime is in effect), and it is random since we do not know when it would be changed. The two states simple Markov chain is simply a minor generalization of the scheme of independent trials or the so-called Bernoulli scheme.

As it is well-known, the state transition probability matrix P in this case has the form:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \quad (1)$$

where p_{11} – is the probability of transition from state 1 to the same state 1. In other words, p_{11} probability equals to the conditional probability that event $m(t+1)=1$ will occur at $t+1$ moment if at the moment t event $m(t)=1$ has occurred, and this probability does not depend on the events that occurred at earlier moments in time: $p_{11}=P(m(t+1)=1|m(t)=1)$. Likewise for other probabilities: $p_{12}=P(m(t+1)=1|m(t)=2)$, $p_{21}=P(m(t+1)=2|m(t)=1)$, $p_{22}=P(m(t+1)=2|m(t)=2)$. Since the Markov process needs to be in some state at each time step, it follows that: $p_{11} + p_{12} = 1$, and $p_{21}+p_{22}=1$. Therefore, we can rewrite the probability matrix as follows:

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}, \quad (2)$$

So, for the definition of our scheme we need only two values: p_{11} and p_{22} . Now for our two state Markov chain random variable $m(t)$, which gains values from set $\{1,2\}$ we can write down the probability distribution function at time t , that is the unconditional probability of the system being at a certain state. It must be a two-component vector $D(t)$ with components: $d_1(t)$ – the unconditional probability that at the moment of time t the system is in the state 1, and $d_2(t)$ – the unconditional probability that at the moment of time t the system is in the state 2.

$$D(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} P(m(t) = 1) \\ P(m(t) = 2) \end{bmatrix}. \quad (3)$$

A Well-known result is that if we start with some prior (initial) probability distribution for $m(0)$, $D(0)$ then $D(t)$ can be computed by simply matrix multiplying P with itself t number of times and multiplying $D(0)$ by the matrix product P^t :

$$D(t) = D(0) \cdot P^t. \quad (4)$$

So far, we have the useful formulae of two-state Markov chain model, although one must admit, we do not know the exact time steps at which $m(t)$ makes the transition from one state to another, and we also do not know the transition probabilities P for the model.

Now we describe shortly the Markov Switching Dynamic Regression used in the current investigation. Let $y(t)$ be an observable time dependent economic indicator being explained and $x_i(t)$, $i=1,2,\dots,n$, is some observable economic indicator which explains $y(t)$. We restrict ourselves here with general linear regression model:

$$Y = X\theta + \varepsilon, \quad (5)$$

where $Y = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$ vector of sequence of $y(t)$ values at different time moments such that

$y_1=y(t_1), \dots, y_p=y(t_p)$, $X = \begin{bmatrix} 1 & x_{11} & \dots & x_{1n} \\ 1 & x_{21} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ 1 & x_{p1} & \dots & x_{pn} \end{bmatrix}$ is matrix containing the explanatory variables data

with the first column filled with ones, for x_{ij} value index i means the number of the point, while index j means the number of the explanatory variable ($x_{11}=x_1(t_1)$, $x_{21}=x_1(t_2)$, $\dots$, $x_{pn}=x_n(t_p)$), $\theta =$

$\begin{bmatrix} \theta_0 \\ \vdots \\ \theta_n \end{bmatrix}$ the vector θ contains the coefficients of the linear model. The aim of the regression model

is to determine θ . As usual the $\varepsilon = \begin{bmatrix} \varepsilon_0 \\ \vdots \\ \varepsilon_p \end{bmatrix}$ is the vector of residual error which supposed to be a

normally distributed random variable with zero mean: $\varepsilon \sim N(0, \sigma^2)$. We assume here the homoskedasticity of ε .

Now let we see how one can mix the simple regression model with two state Markov process. Depending what Markov state is in effect the linear regression coefficients set θ_i which describes data depends on what Markov state is in effect at the moment t . Namely:

$$\begin{cases} Y = X\theta^1 + \varepsilon_1, \text{ when } m = 1, \\ Y = X\theta^2 + \varepsilon_2, \text{ when } m = 2. \end{cases} \quad (6)$$

Here Y – vector of y explained data set, X – $n \times p$ matrix of explication data set (n – variables at p time moments), θ^m – vector of linear regression coefficients which corresponds to regime m (here m – is the index and not a power), and random variables $\varepsilon_m \sim N(0, \sigma_m^2)$. Now we calculate for time moment t_i

$$[1 \ x_{i1} \ \dots \ x_{in}] \cdot \begin{bmatrix} \theta_0^1 & \theta_0^2 \\ \vdots & \vdots \\ \theta_n^1 & \theta_n^2 \end{bmatrix} = [y_i^1 \ y_i^2],$$

$$\overline{y}_i = y_i^1 \cdot P(m = 1) + y_i^2 \cdot P(m = 2) \text{ or } \overline{y}_i = [y_i^1 \ y_i^2] \cdot D(t_i), \text{ where} \quad (7)$$

$$D(t_i) = \begin{bmatrix} P(m(t_i) = 1) \\ P(m(t_i) = 2) \end{bmatrix}.$$

Now we have to estimate somehow the $\hat{\theta} = \begin{bmatrix} \theta_0^1 & \theta_0^2 \\ \vdots & \vdots \\ \theta_n^1 & \theta_n^2 \end{bmatrix}$ matrix of regression coefficients and the

probability matrix for Markov chain P , and eventually the variance σ^2 of the dependent variable y . We will use the Maximum Likelihood Estimation (MLE) method (for proper estimation of the results we will compare it with the Expectation Maximization method). MLE, which finds the values of P , $\hat{\theta}$ and σ^2 , would maximize the joint probability density of observing the entire data set y . In other words, we have to maximize the following product:

$$L = \prod_{i=1}^p f(y(t_i)).$$

Since $\varepsilon_m \sim N(0, \sigma_m^2)$ it is obvious and convenient to assume that

$$f(y(t_i)) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{y_i - \overline{y}_i}{\sigma}\right)^2}.$$

So, we obtain formulae for calculations:

$$L(\sigma, \theta_t^k, P(m = 1), P(m = 2)) = \prod_{i=1}^p \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{y_i - \overline{y}_i}{\sigma}\right)^2}, \quad (8)$$

$$\overline{y}_i = y_i^1 \cdot P(m = 1) + y_i^2 \cdot P(m = 2), \quad (9)$$

$$y_i^1 = [1 \ x_{i1} \ \dots \ x_{in}] \cdot \begin{bmatrix} \theta_0^1 \\ \vdots \\ \theta_n^1 \end{bmatrix}, y_i^2 = [1 \ x_{i1} \ \dots \ x_{in}] \cdot \begin{bmatrix} \theta_0^2 \\ \vdots \\ \theta_n^2 \end{bmatrix}. \quad (10)$$

Providing the standard maximization procedure and solving the following system of equations we obtain the estimated parameters we were looking for:

$$\begin{cases} \frac{\partial L}{\partial \sigma} = 0 \\ \frac{\partial L}{\partial \theta_l^1} = 0, l = 1, \dots, n \\ \frac{\partial L}{\partial \theta_l^2} = 0, l = 1, \dots, n \\ \frac{\partial L}{\partial P(m=1)} = 0 \\ \frac{\partial L}{\partial P(m=2)} = 0 \end{cases} \quad (11)$$

To obtain numerical results from observed data we created a python program, since it has a huge functional library, although there are a lot of other options including highly developed mathematical programs like Matlab™ or Mathematica™.

3.1 Implementation in Python IDE

In Python IDE, to plot the Markov switch probabilities π_t we first import the following libraries: pandas, numpy, pyplot and statsmodels.

Then we can load the datasets, which, in case of this research, are stored on the github repository. The full program listing is provided in the Appendix. It is advisable to first plot the datapoints and visualize the time series to visually confirm possible turning points and regime switches.

The y_t dependent variable, further regarded in the research as the endogenous variable, is represented by the *GDP %Chng* variable. The regression variable – the one that we observe, is given by the individual panel data time series.

Following the statsmodels manual developed by Pektold (2022), to build perform the switching regression, we build and train the two-state Markov switching dynamic regression model by first assigning a variable to it:

```
Msdr_model = sm.tsa.MarkovRegression(endog=df['GDP %Chng'], k_regimes=2, trend='c',
exog=df['X'], switching_trend=True, switching_variance=True)
```

The “sm.tsa.MarkovRegression” takes in the endogenous variable, number of specified regimes, adds an intercept – trend ‘c’, the exogenous variable, the boolean of the switching trend and exogenous variable and outputs a table of summaries that can be viewed by running the command: “Msdr_model_results = msdr_model.fit()” and “Print(msdr_model_results.summary()).”

4. Data and selection of economic variables

To satisfy the objective of the research of estimating the business cycle, the research employs. The data were collected from the OECD quarterly national accounts, the IMF International Financial statistics database, the central statistical office of Hungary (KSH) as well as the Budapest stock exchange (BÉT) data warehouse of. The binary Hungarian recession indicator is based on the OECD recession indicator retrieved from the Federal Reserve Bank of St. Louis database. 1 is assigned to recessionary periods and 0 is assigned for recovery/expansionary periods. Estrella and Mishkin (1998) propose a set of 6 indicators of economic activity, however, due to low complexity of Hungarian financial markets, this research instead focuses on classical indicators such as the unemployment rate, industrial production index, the inflation rate and the BUX composite stock market index returns. Only one of the proposed indicators is considered in this research – the 10 year and 3 year government bond yield spreads. Table 1 provides some descriptive statistics of the selected variables.

Table 1 Descriptive statistics of selected Hungarian economic time series. Source: Own calculation

	GDP %	Unemployment rate	Industrial production index	Inflation rate	BUX	Bux % change QoQ	10Y-3M yield
obs	105	105	105	105	105	105	93
mean	2.60	7.02	5.10	6.21	20575.3	0.03	0.10
std	3.58	2.33	7.71	5.50	12523.2	0.07	1.42
min	-12.8	3.3	-22.9	-1	2635.0	-0.28	-3.6
max	17.8	11.1	21.1	27.9	54197.7	0.32	2.3

The data sample covers the period between Q1-1996 to Q1-2022, with a quarterly frequency totaling 105 observations. The 10-year and 3-year yield time series samples cover a slightly smaller timespan of Q1-1999 to Q1-2022. The seasonally adjusted time series are represented in a quarterly frequency. The choice of the unemployment rate as a lagging indicator in the scope of a Markovian model is inspired by its analysis in a large number of empirical works on the US economy. In a recent study, McGrane (2022) proved that there's an asymmetry between business cycles and unemployment, whereby the unemployment rate rises faster in recessions than it falls during expansions, supporting the New Keynesian model. Nevertheless, for sake of obtaining the regime-switching probabilities of the unemployment rate, this research follows the standard notion of the business cycle and unemployment correlation and relaxes assumptions of wage stickiness. The choice for the industrial production

index as an indicator of business cycle dynamics is supported by Krolzig et. al's (2000) findings of a common cycle independent of the industrial sector, and by Madhioub's (2015) three-state regime model, as an example of a developing economy. Inflation signifies a change in the price of consumer goods – this happens as a result of an increase in money supply, due to monetary policy enacted to combat exogenous or endogenous shocks and to keep credit available in the economy. High inflation rates signal greater probabilities of recessions, hence a direct link to business cycles. The stock market is by far the most popular leading indicator. Since the current price of a stock doesn't only reflect future earnings expectations but also consumer sentiment, it is highly correlated with business cycles. The BUX composite share price index is used in Bandholz's (2005) univariate model. However, thanks to a wider timeframe and greater availability of observations in this research, the statistical inferences that can be drawn from the model become more reliable. The difference between the 10-year and 3-year Hungarian sovereign bond yields indicates the investors' outlook on future economic conditions. The wider the spread, the steeper the yield curve is, and the more positive the outlook on the economy. Conversely, the tighter the spread, the less confident investors are in the economy. Negative 10-year and 2-year yield spreads are notorious for predicting every single recession in the US. As a result, the research. The reference series the cyclical dynamics of which the research aims to predict is the quarterly GDP growth rate.



Figure 2 GDP quarterly growth rate superimposed on OECD recession periods. Source: Own calculations (OECD)

To better understand the reference series, it is appropriate to provide a brief description of the recession periods graphed in Figure 2. The grey areas represent peak-to-through recession periods in the Hungarian economy following the OECD recession indicator. Overall, seven recessionary periods are identified in the dataset. As a result of the fiscal stabilization policy in 1995 the GDP growth rate decelerated, causing a mild recession. The 1998 recession was caused by a sharp decline in demand as a result of the Russian financial crisis. Following

the burst of the dot com bubble in 2001, foreign demand decreased yet again causing a recession. The inflation rate of the Forint sharply dropped during this period. The 2008 global financial crisis caused by subprime lending, risky financial products, and the housing market collapse in the US resulted in a significant drop in the Hungarian GDP. Being on the brink of default, Hungary was bailed out by the IMF and the European Union. In 2012 the eurozone sank into a debt crisis that resulted in an unfavorable economic environment and low domestic demand. The mild decline in 2016 was partly due to a reduction in European Union financing and partly due to a weaker automobile industry. In 2020 the economy witnessed the sharpest decline in GDP growth in its history because of nationwide lockdowns caused by the COVID-19 pandemic. Altogether, the selected indicators form a strong base for testing the methodology for the Hungarian economy. The data can be retrieved in the following repository: <https://github.com/albertmolnar/Markov-chain>

5. Empirical results for Hungarian economic variables

Table 2 presents the estimates of the linear Markov-switching model per the panel data. The variance of the error term is allowed to switch.

Table 2 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the unemployment rate. Source: Own calculations based on FRED and OECD data

Unemployment	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	6.24	-0.33	0.58	0.00	0.00	0.00	0.926	0.098	441
Regime 2	5.33	-0.67	20.49	0.017	0.019	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the Unemployment rate $m(t)$:

Regime 1: $\text{GDP \%Change} = 6.24 - 0.33\text{Unemployment rate} + 0.58$;

Regime 2: $\text{GDP \%Change} = 5.33 - 0.67\text{Unemployment rate} + 20.49$

The transition probability matrix is defined as: $P = \begin{bmatrix} 0.9267 & 0.0733 \\ 0.0983 & 0.9017 \end{bmatrix}$, where the expected duration for regime 1 is 13.63 quarters, while the expected duration of regime 2 is 10.17 quarters. For additional reference on the Markov switching model result, Appendix A-E provide the model summaries and the state space probability distributions.

Given the 50% chance of regimes 1 and 50% chance of regime 2, after 100 iterations, the state probability of switching to state 2 from state one reaches 0.427156. the calculated probability of remaining in a low state while being in one is 0.57284.

The smoothed state probabilities $D(t)$ at time t , where $t = 1$ quarter, and the state is equal to 2 (a recession) are plotted below. Figure 3 illustrates a side by side comparison of the smoothed probability of recessions compared to actual recessions and The GDP %change,

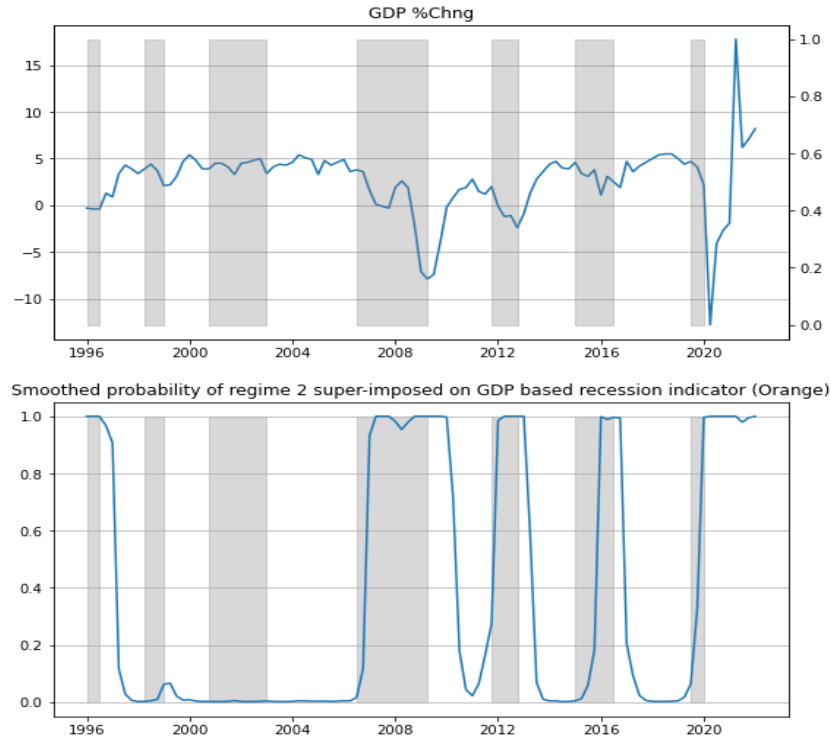


Figure 3 Smoothed probabilities of being in a recession superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

We lag the explaining variable $m(t)$ by one quarter to see if its past values could be more efficient in producing the probability estimate.

Table 3 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the 1 quarter lagged unemployment rate time series. Source: Own calculations based on FRED and OECD data

Lagged Unem-nt	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	5.44	-0.18	0.34	0.00	0.00	0.00	0.921	0.078	433
Regime 2	3.56	-0.37	19.35	0.089	0.013	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the Lagged unemployment rate $m(t)$ are as follows:

$$\text{Regime 1: GDP \%Change} = 5.44 - 0.18\text{LaggedUnemployment rate} + 0.34 ;$$

$$\text{Regime 2: GDP \%Change} = 3.56 - 0.37\text{LaggedUnemployment rate} + 19.35$$

The transition probability matrix is defined as: $P = \begin{bmatrix} 0.9206 & 0.0794 \\ 0.9214 & 0.0786 \end{bmatrix}$. Using the “msdr_model_results.expected_durations” function, we determine that the expected duration of regime 1 is 12.60 quarters, while the expected duration of regime 2 is 12.73 quarters.

The probability of a recession given by the hidden regime switching process of the lagged $y(t)$ is specified in Figure 4

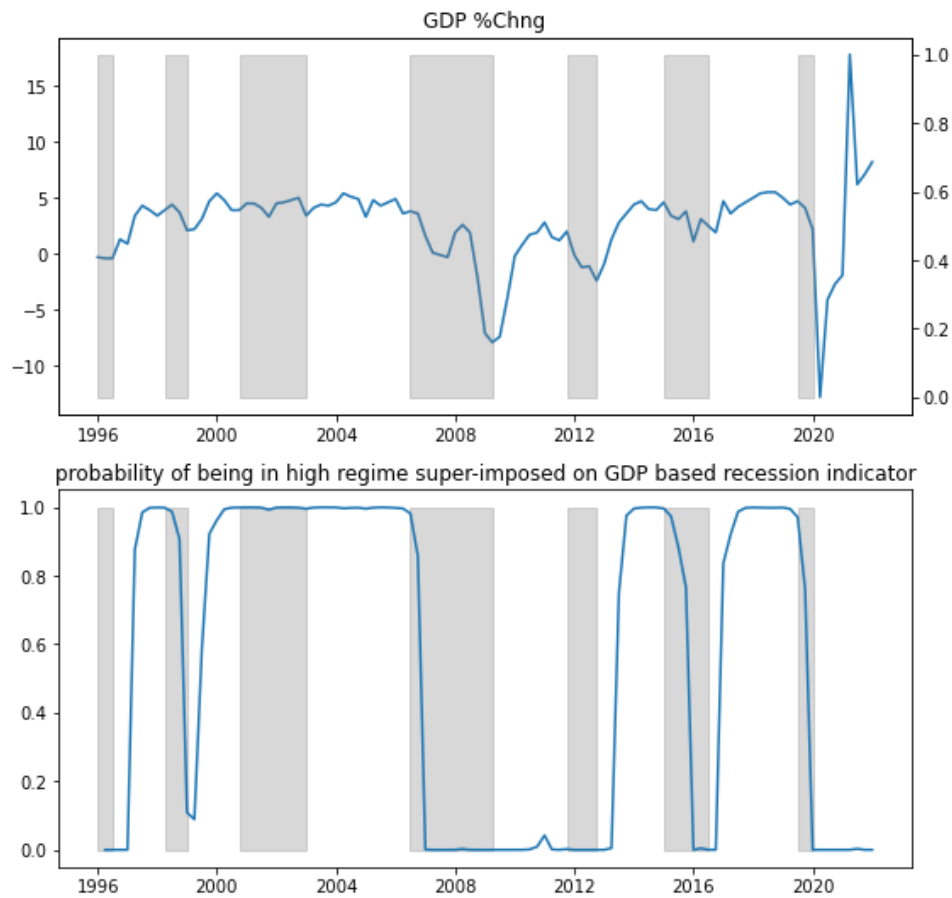


Figure 4 Smoothed probabilities of a recession, with lagged unemployment variable as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Table 4 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the inflation rate time series. Source: Own calculations based on FRED and OECD data

Inflation	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	4.41	-0.017	0.407	0.00	0.35	0.00	0.921	0.074	453.4
Regime 2	0.40	0.031	19.60	0.67	0.767	0.00			

Regression models for the regimes are as follows:

Regime 1: $GDP \%Change = 4.41 - 0.017Inflation\ rate + 0.407$;

Regime 2: $GDP \%Change = 0.40 + 0.031Inflation\ rate + 19.60$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9206 & 0.0794 \\ 0.0748 & 0.9252 \end{bmatrix}$, the expected duration for regime 1 is 12.60 quarters, the expected duration of regime 2 is 13.36 quarters.

The initial $D(t)$ is set to 0.5 and 0.5 for both regimes. Following the same 100 iterations steady state for the 1-2 regime switch is attained at 0.48508, the probability remaining in state 1 is 0.514915.

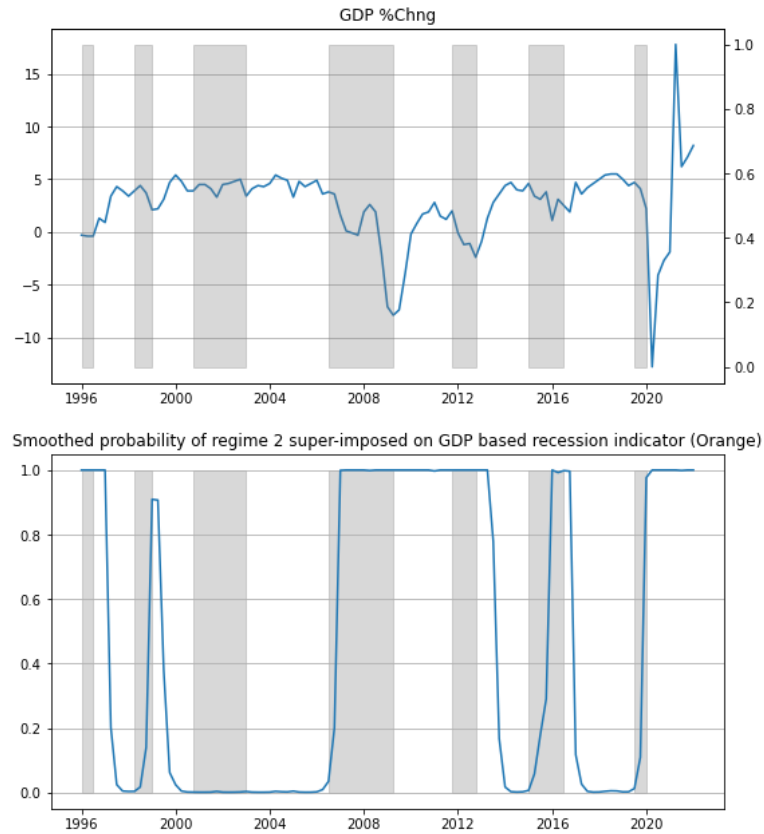


Figure 5 Smoothed probabilities of a recession, with inflation rate as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED.
Source: Own calculations

Table 5 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the lagged inflation rate time series. Source: Own calculations based on FRED and OECD data

Lagged Infl-n	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	4.42	-0.018	0.407	0.00	0.29	0.00	0.921	0.076	448.23
Regime 2	0.67	-0.004	20.03	0.494	0.967	0.00			

The regime specific equations are:

$$\text{Regime 1: GDP \%Change} = 4.42 - 0.018\text{LaggedInflation rate} + 0.407 ;$$

$$\text{Regime 2: GDP \%Change} = 0.6702 - 0.0047\text{LaggedInflation rate} + 20.03$$

Probability matrix is defined as: $P = \begin{bmatrix} 0.9207 & 0.0793 \\ 0.0761 & 0.9252 \end{bmatrix}$, the expected duration for regime 1 is 12.60 quarters, while the expected duration of regime 2 is 13.36 quarters.

The Markov state probabilities $D(t)$ at time t , where $t = 1$ quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse:

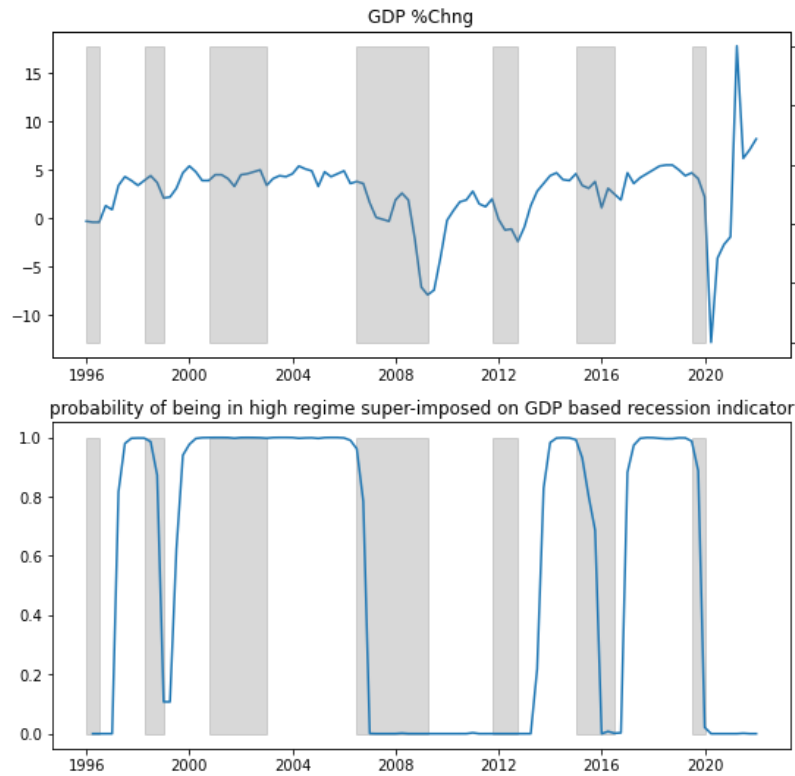


Figure 6 Smoothed probabilities of a recession, with lagged inflation rate as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED.
Source: Own calculations

Table 6 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the Industrial Production Index (IPI) time series. Source: Own calculations based on FRED and OECD data

IPI	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	-0.44	0.286	1.159	0.309	0.00	0.00	0.936	0.042	495
Regime 2	1.90	0.356	9.81	0.00	0.00	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the Industrial production index $m(t)$:

Regime 1: $\text{GDP \%Change} = -0.44 + 0.286IPI + 1.159$;

Regime 2: $\text{GDP \%Change} = 1.90 + 0.356IPI + 9.81$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9359 & 0.0641 \\ 0.0421 & 0.9579 \end{bmatrix}$, the expected duration for regime 1 is 15.61 quarters, regime 2 is 23.73 quarters.

The 1-2 transition probability steady state is, 0.6035769, while the 1-1 transition probability is 0.396423 given the 0.5 initial conditions of $D(t)$.

Figure 7 illustrates the smoothed recession probabilities with the industrial production index being the explaining variable $m(t)$ of the GDP %Change.

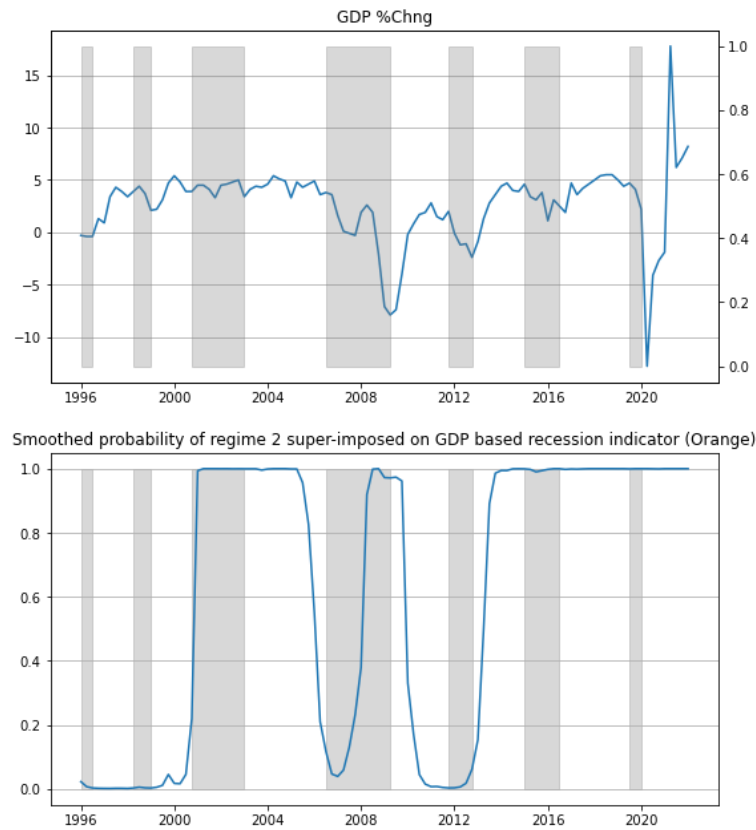


Figure 7 Smoothed probabilities of a recession, with Industrial production index (IPI) as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Table 7 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the Lagged Industrial Production Index (IPI) time series. Source: Own calculations based on FRED and OECD data

Lagged IPI	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	-0.46	0.29	1.7	0.04	0.0	0.00	0.965	0.03	473
Regime 2	2.04	0.39	7.84	0.00	0.00	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the Lagged Industrial production index $m(t)$:

$$\text{Regime 1: GDP \%Change} = -0.4657 + 0.2907\text{LaggedIPI} + 1.7076 ;$$

$$\text{Regime 2: GDP \%Change} = 2.0451 + 0.0398\text{LaggedIPI} + 7.8434$$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9659 & 0.0341 \\ 0.0302 & 0.9698 \end{bmatrix}$, the expected duration for regime 1 is 29.30 quarters, while the expected duration of regime 2 is 33.15 quarters.

The Markov state probabilities $D(t)$ at time t , where $t = 1$ quarter, are given by Figure 8.

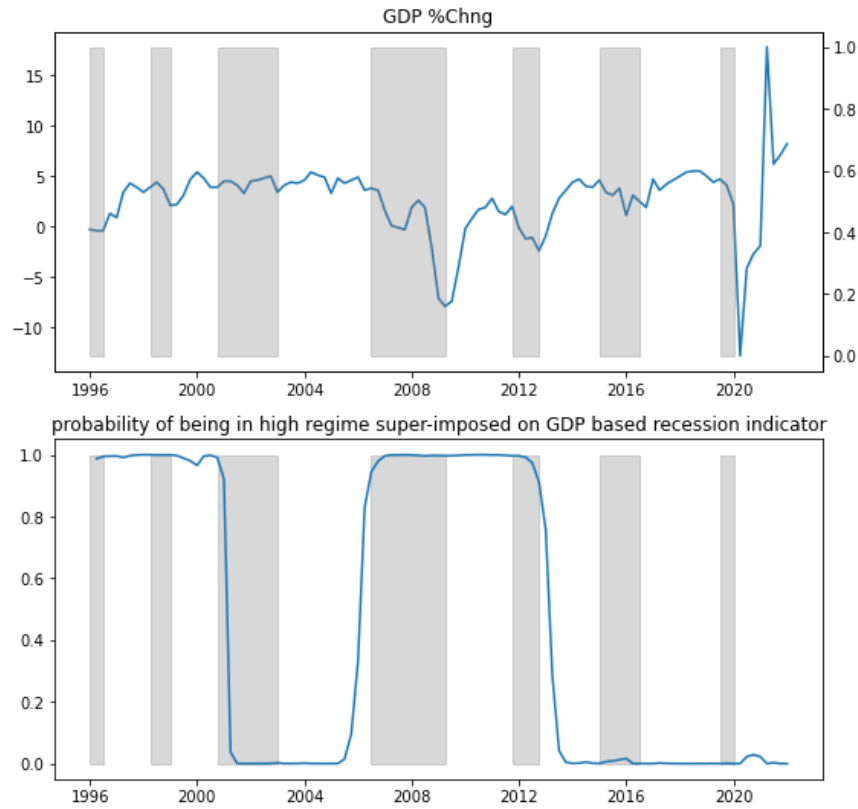


Figure 8 Smoothed probabilities of a recession, with lagged Industrial production index (IPI) as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Table 8 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the BUX year-on-year percentage change time series. Source: Own calculations based on FRED and OECD data

BUX YoY%	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	0.79	-0.42	1.99	0.097	0.55	0.07	0.894	0.035	567
Regime 2	2.732	2.415	13.744	0.00	0.10	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the BUX year-on-year change $m(t)$:

Regime 1: $\text{GDP \%Change} = 0.79 - 0.42\text{BUXYoY} + 1.99$;

Regime 2: $\text{GDP \%Change} = 2.732 + 2.415\text{BUXYoY} + 13.74$

The probability matrix is defined as: $P = \begin{bmatrix} 0.8945 & 0.1055 \\ 0.0351 & 0.9649 \end{bmatrix}$, the expected duration for regime 1 is 9.48 quarters, while the expected duration of regime 2 is 28.51 quarters.

Given the 50% chance of regimes 1 and 50% chance of regime 2, following 100 iterations, the state probability reaches the steady state: 0.2496 of probability of going to state 2 after state 1 and 0.750355 probability of remaining in state 1 after state 1.

The Markov state probabilities $D(t)$ at time t , where $t = 1$ quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse:

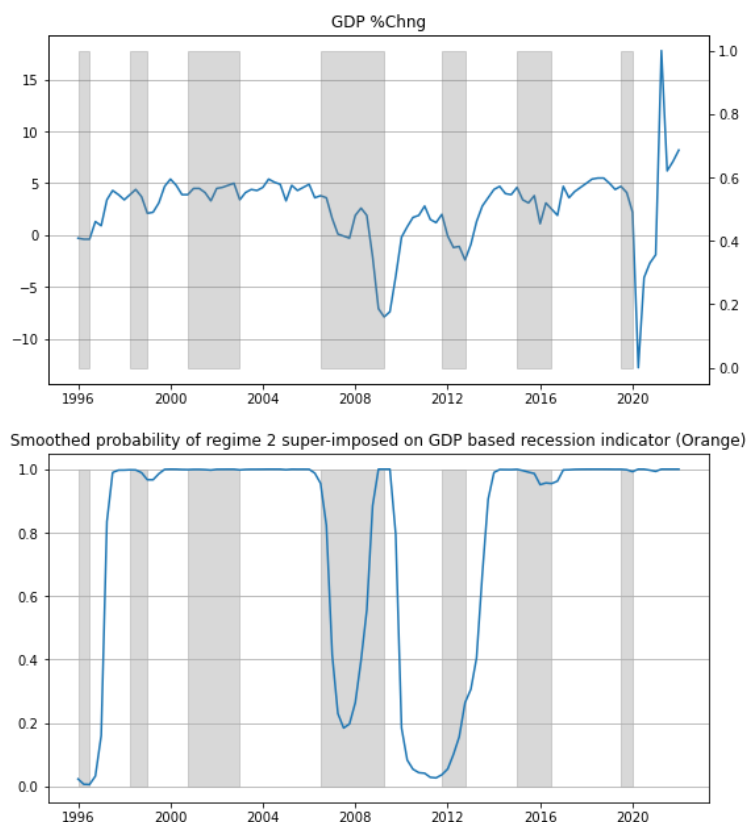


Figure 9 Smoothed probabilities of a recession, with BUX YoY% as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Table 9 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the Lagged BUX year-on-year percentage change time series. Source: Own calculations based on FRED and OECD data

Lagged BYoY	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	4.17	0.253	0.539	0.00	0.45	0.00	0.936	0.063	445
Regime 2	0.0014	2.769	19.22	0.99	0.45	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the lagged BUX year-on-year change $m(t)$:

$$\text{Regime 1: GDP \%Change} = 4.17 + 0.253\text{LaggedBYoY} + 0.539;$$

$$\text{Regime 2: GDP \%Change} = 0.0014 + 2.769\text{LaggedBYoY} + 19.22$$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9363 & 0.0637 \\ 0.0639 & 0.9361 \end{bmatrix}$, the expected duration for regime 1 is 15.70 quarters, while the expected duration of regime 2 is 15.65 quarters.

The Markov state probabilities $D(t)$ at time t , where $t = 1$ quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse:

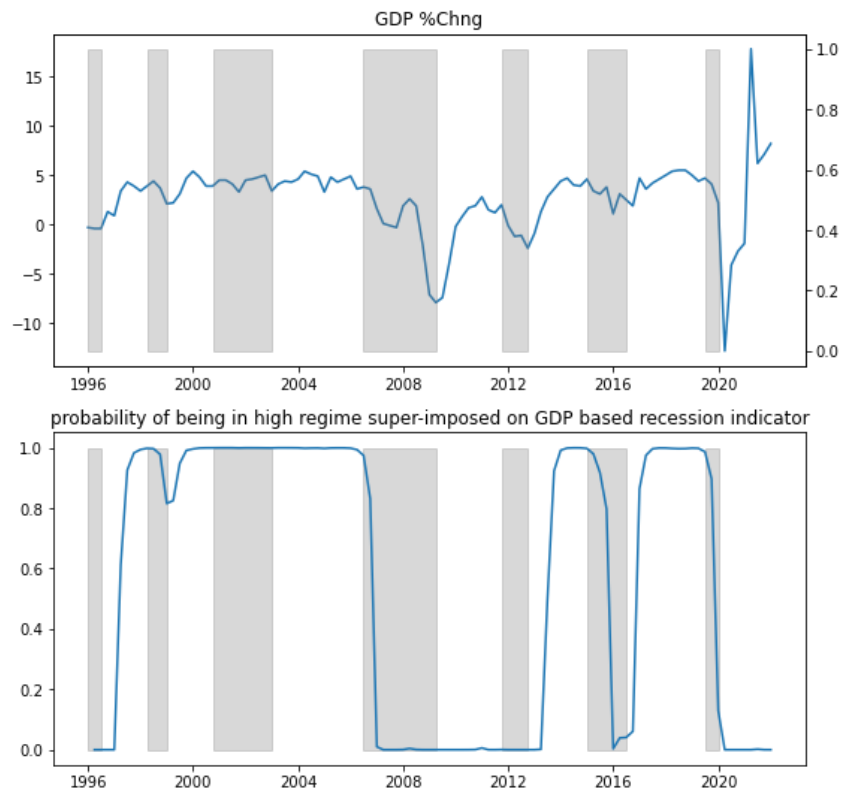


Figure 10 Smoothed probabilities of a recession, with Lagged BUX YoY% as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED.
Source: Own calculations

Table 10 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the 10Year and 3Year yield spread time series. Source: Own calculations based on FRED and OECD data

YSpread	θ_0	θ_1	σ^2	$P > z \theta_0$	$P > z \theta_1$	$P > z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	4.37	0.00	0.40	0.00	0.97	0.00	0.929	0.061	406
Regime 2	0.62	0.07	21.80	0.38	0.89	0.00			

$$\text{Regime 1: GDP \%Change} = 4.37 + 0.00241\text{YSpread} + 0.40;$$

$$\text{Regime 2: GDP \%Change} = 0.6287 + 0.07\text{YSpread} + 21.80$$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9294 & 0.0716 \\ 0.0616 & 0.9384 \end{bmatrix}$, the expected duration for regime 1 is 14.16 quarters, while the expected duration of regime 2 is 16.22 quarters.

Given the 50% chance of regimes 1 and 50% chance of regime 2, following 100 iterations, the state probability reaches the steady state: 0.4683258 of probability of going to state 2 after state 1 and 0.531674719 probability of remaining in state 1 after state 1.

The Markov state probabilities $D(t)$ at time t , where $t = 1$ quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse:

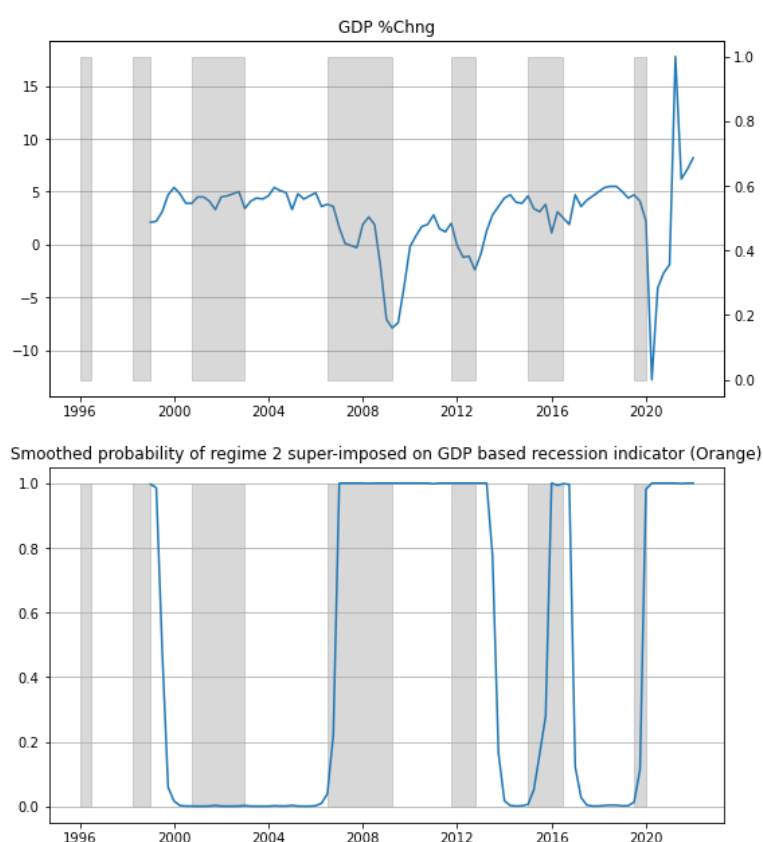


Figure 11 Smoothed probabilities of a recession, with YSpread as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED. Source: Own calculations

Table 11 Estimation of the Markov regime switching model. $y(t)$ is the GDP variable, while $m(t)$ is the Lagged 10Year and 3Year yield spread time series. Source: Own calculations based on FRED and OECD data

LaggedYSpread	θ_0	θ_1	σ^2	$P> z \theta_0$	$P> z \theta_1$	$P> z \sigma^2$	p_{11}	p_{21}	AIC
Regime 1	4.33	0.06	0.48	0.00	0.30	0.00	0.948	0.061	394
Regime 2	-0.33	1.98	19.41	0.66	0.01	0.00			

Regime 1 and 2 models of the GDP as $y(t)$ and the Lagged Yspread yields $m(t)$:

Regime 1: $\text{GDP \%Change} = 4.33 + 0.0638\text{LaggedYSpread} + 0.48$;

Regime 2: $\text{GDP \%Change} = -0.3363 + 1.9894\text{LaggedYSpread} + 19.41$

The probability matrix is defined as: $P = \begin{bmatrix} 0.9481 & 0.0529 \\ 0.0614 & 0.9386 \end{bmatrix}$, the expected duration for regime 1 is 19.27 quarters, while the expected duration of regime 2 is 16.28 quarters.

The Markov state probabilities $D(t)$ at time t , where $t = 1$ quarter, are plotted below. The GDP %change chart is plotted alongside the dates of Hungarian recessions obtained from the OECD data warehouse:

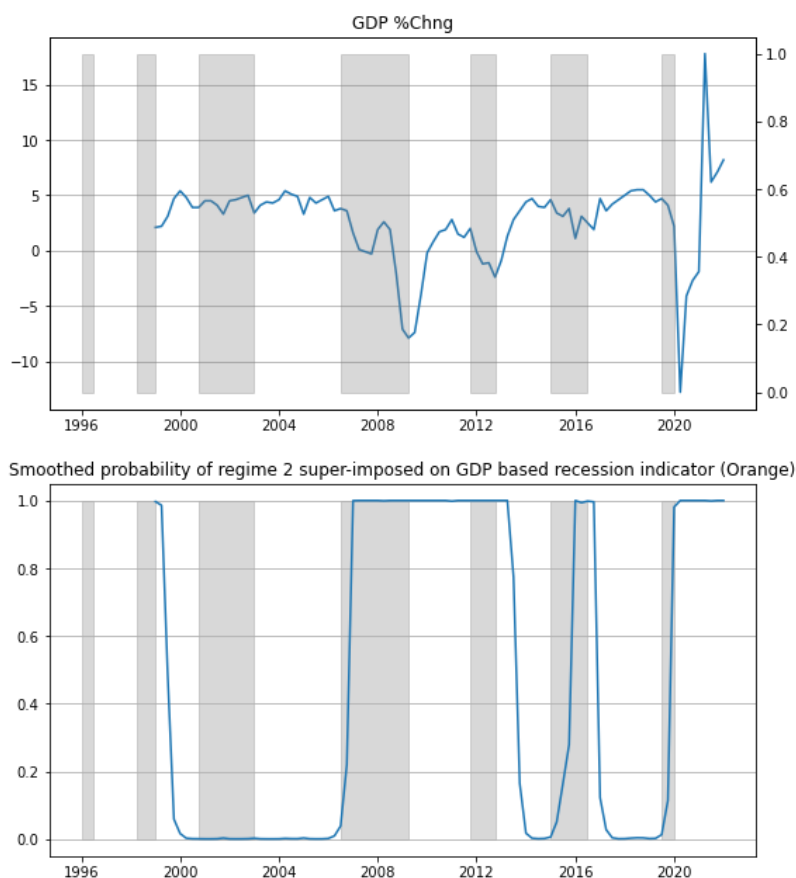


Figure 12 Smoothed probabilities of a recession, with Lagged YSpread as the exogenous variable superimposed on actual recessions determined by the OECD composite indicator obtained from FRED.
Source: Own calculations

6. Discussion

The parameter estimates of the linear MSDR model are presented in Tables 3-11. The selection of the exogenous variables is justified in section 4 of the research. The estimation period for the time series is 105 quarters or 26 years and 3 months. Figures 3-12 plot the smoothed recession probabilities compared to actual recession represented by grey bars on the plot. Let us examine the results obtained for each of the exogenous variables - $m(t)$:

Unemployment:

Firstly, the θ_1 value for regime 1 is negative, this shows that during an expansion the unemployment rate tends to drop. This follows the conventional laws of Keynesian economics, whereby, as a result of the wide availability of credit, companies can increase the labor force, decreasing the unemployment rate. However, θ_1 for regime 2 is also negative, moreover, its slope is steeper than that of regime 1. Following the previous reasoning, this means that during a recession unemployment decreases at a faster rate than during an expansion. Secondly, the variance of the error term is significantly higher during a recession – this points to stronger propagation of the shocks. The graph of the smoothed recession probabilities captures 5 out of the 7 recessionary periods of the economy – a 71% predictive power, which is by far the best result out of the panel data. The fallbacks of the smoothed probability results are that the graph doesn't capture the early 2000's recession and the 1999 recession, and for some reason, the regime remains at state 2 up till now. Could this mean that since the COVID-19 crisis the dynamics of the Unemployment rate remained in regime 1?

Inflation:

We start with the regression coefficients for regime 2: we obtain θ_1 and assert that it is positive. Indeed, this matches with theory – higher inflation leads to poor credit conditions in an economy, resulting in a decrease in the output rate. In the expansionary regime 1, θ_1 has a negative slope, indicating that inflation steadily decreases. Alternatively, when the inflation rate is lagged back 1 quarter, both of the regimes feature a negative θ_1 variable. The variance of regime 2 is high compared to regime 1. At the same time, rising inflation is not to be associated with the certainty of a recessionary state. For instance, a radical example would be that according to modern monetary theory, if the overall growth rate of the economy remains higher than that of inflation, it is possible to sustain the model indefinitely – attaining an endless period of expansion. It is important to note that the measure of expansion (regime 1) is not a low, zero, or even, negative inflation rate. Deflation is just as harmful, if not even more harmful

in an economy as inflation. The inflation variable captures 6 of the 7 recessionary periods in Hungary, which at first glance, may be an even better result than a model with the unemployment rate as the exogenous variable. However, examining the plot in detail, we can see that the recession probability remained high throughout the 2008-2012 business cycle encompassing two recessions and one expansionary period. The model, therefore, failed to predict the expansion between 2009 and 2012. We conclude that the inflation model is on par with the unemployment model in the overall signal to noise ratio.

Industrial Production Index:

The estimates provided by the MSDR training results summary indicate that notwithstanding the state of the business cycle, whether the exogenous variable is lagged or not, the slope of the regime equations remains positive. This leads to another questionable conclusion that according to our model, while the negative GDP growth slows down the growth of the IPI, it does not make it negative. The IPI showed the recession between 2001 and 2003, corresponding to the GDP decline, it also switched to the high regime in the second half of the 2008 recession, missed the 2012 European credit crisis, and, if such an interpretation can be made, predicted the 2016 GDP decline at least 6 quarters ahead. From that moment, it remained in state 2 – missing the expansionary periods between 2016 and 2020. The lagged IPI performed even worse, with only capturing 4 of the recessions along the examined time horizon.

BUX YoY% change

The regime 1 coefficient for the BUX growth rate exogenous variable is -0.42 with a statistical significance of 0.55, which is far more than the 5% threshold indicating statistical insignificance in favor of the null hypothesis. For the lagged BUX YoY variable, the p-value is also not statistically significant. According to the results in Appendix D, the duration of regime 2 is 3 times longer on average than of regime 1. The smoothed recession probability graph indicates that the model captures 5 of the 7 recessions in the examined timeframe, but at the same time misses 3 expansionary periods. It assumed in this research that the stock market is a strong leading indicator. One explanation as to why regime 2 occurs so often is related to the effect of volatility. The problem could be with the data: if the quarterly period ending values of the BUX were represented in a weekly or daily frequency, it would have been possible to smooth the data, eliminate short-term volatility and filter for a quarterly frequency. In this case, however, it is complicated to say what part of the quarterly BUX value is ‘noise’ and what part

of it is actual trend. Therefore, as a possibility of further extending this work, following the data manipulation procedure mentioned before, the test should be repeated and compared with the results obtained in this research.

10-Year and 3-Year Hungarian sovereign bond yield spreads.

The MSDR training model produced one of the most interesting outputs for the yield spread exogenous variable. To reiterate, yield curve inversions detected from spreads of long and short maturity bonds have a capability of predicting recessions. Therefore, the theoretical specifications of regimes 1 and 2 couldn't have been clearer: regime 1 for positive spreads and regime 2 for negative spreads. The θ_1 value for the yield spread time series in both regimes 1 and 2 was extremely close to 0. This can be interpreted as: yield spreads have a negligible influence on GDP growth. When the economy is in an expansion, the yield spread is very small, when the economy is in a recession, the spread grows, but according to our model, only slightly. The probability of the persistence of a low regime is nearly 95%, while the persistence of a high regime is 94%. The smoothed recession probabilities capture 5 of the 6 recessions but miss 2 expansionary periods. As a possible extension to the dataset and the approach, it might be worth to do the same procedure with the 15 year and 3 month spreads, the 15 year and 3 year spreads, and the 10 year and 3 month spreads. By expanding the maturities of the sovereign bonds, it may be possible to better ascertain how investor sentiment correlates with Hungarian business cycles.

From the methodological standpoint, there are a few remarks to be made:

We set the initial $D(t)$ probability to 0.5 on both regimes, indicating that our model has an equal chance of being in a recession and an expansion at time t_0 . We follow this approach only for purposes of obtaining the state probability distribution. It can be argued, however, that the initial probability on the regimes is not 50%. For example, we can look back at recessionary periods for a given country's economy and count the average duration of a recession against the average duration of an expansion and adjust the initial probabilities by that value. In this aspect this research needs further extensions.

Homoscedasticity has a normal distribution – this is a questionable assumption within the model. What is the concrete factor in the economy $m(t)$ that is referred to as the 'hidden variable' – what is the economic significance of the regime switch particularly for the Hungarian economy?

A key characteristic of the applied Markov regime switching dynamic factor model is the assumption that within the transition matrix the transition probabilities are assumed to be constant. Filardo (1994) addresses this issue by allowing the probabilities to vary over time. For example, p_{22} in time t does not equal p_{22} in $t + n$ steps, where n is the time-step. Exploring regime switching within the time varying transition probability (TVTP) framework can yield some interesting results for analyzing the regime switches of yield curves and GDP or GNP time series.

7. Conclusions

This paper reevaluates the idea that the state of the business cycle can be determined based on the co-movements of macroeconomic indicators. By applying the Markov regime-switching dynamic model, it is possible to isolate the hidden variable and the one which is directly observed by the econometrician. The hidden variable is modeled through a two-state Markov chain, while the observed variable is modeled through a regression. We build a linear regression model with a mean, intercept and error term, where the mean is replaced by the product of some explanatory variable and a coefficient. The transition matrix for the time series is estimated through the MLE and the smoothed recession probabilities are graphed against actual recession periods given by the OECD for visual inspection and conclusions. The Empirical estimation shows that the panel data have different statistical parameters depending on the growth or recessionary state of the system, which in our case is the Hungarian economy. The hidden part of the Markov switching dynamic regression model explained most of the variance in the GDP % change, that a simple regression with panel data variables wouldn't be able to. We have determined the expected durations of the recessions by plugging in the panel data variables as the exogenous term of our model. The GDP corresponds to the business cycles the most. when the exogenous variable is the unemployment rate. To determine possible long-term correlations of the GDP with the business cycle, the exogenous variables were lagged by 1 quarter. Generally, this extension did not result in a higher predictive capability, than by not lagging the variable.

The further development of the Markov chain method is important because this allows us to analyze the unobservable variable in greater detail. The lagged maximum likelihood function of the 10 and 3 year spreads indicates that the Hungarian economy is in a recession, which at the time of writing this paper it is in a recession. However, the unemployment rate has been determined to be the most accurate indicator, which is accurate for 8 out of the 13 cycles. From a legislative standpoint, it is worth to integrate a constantly updating maximum likelihood function that would switch if a recessionary or expansionary period is being detected. Also, it is worth considering more frequent time series. Within the scope of this research, we have worked with quarterly time series, if for instance, we were to take the jobless claims instead of the quarterly unemployment rate, which is a monthly indicator and eliminate the noise and smooth it, we believe that it would be a much better 'live' version of whether the economy is bound for a recession. Further methodological developments in the case of

Hungarian business cycle estimation could be in applying Filardo's TVTP framework for the panel data.

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Appendix A: Additional results, calculations, and figures for the MSDR model with Unemployment as exogenous variable.

Table 12 Markov switching dynamic regression model for Unemployment model results

Markov Switching Model Results						
Dep. Variable:	GDP %Change		No. Observations:		105	
Model:	MarkovRegression		Log Likelihood		-212.516	
Date:	Mon, 08 Aug 2022		AIC		441.033	
Time:	21:41:59		BIC		462.264	
Sample:	01-01-1996		HQIC		449.636	
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	6.2440	0.406	15.369	0.000	5.448	7.040
x1	-0.3373	0.063	-5.338	0.000	-0.461	-0.213
sigma2	0.5870	0.126	4.657	0.000	0.340	0.834
	Regime 1 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	5.3394	2.230	2.394	0.017	0.969	9.710
x1	-0.6711	0.285	-2.353	0.019	-1.230	-0.112
sigma2	20.4992	4.722	4.341	0.000	11.244	29.754
	Regime transition parameters					
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.9267	0.033	28.268	0.000	0.862	0.991
p[1->0]	0.0983	0.050	1.963	0.050	0.000	0.196

Table 13 Markov switching dynamic regression model for lagged Unemployment model results

Markov Switching Model Results						
=====						
Dep. Variable:	GDP %Change		No. Observations:		104	
Model:	MarkovRegression	Log Likelihood		-208.705		
Date:	Mon, 08 Aug 2022	AIC		433.409		
Time:	22:16:15	BIC		454.564		
Sample:	04-01-1996	HQIC		441.980		
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	5.4436	0.305	17.875	0.000	4.847	6.041
x1	-0.1820	0.048	-3.797	0.000	-0.276	-0.088
sigma2	0.3339	0.082	4.063	0.000	0.173	0.495
	Regime 1 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	3.5640	2.093	1.702	0.089	-0.539	7.667
x1	-0.3746	0.251	-1.492	0.136	-0.867	0.118

sigma2	19.3554	4.044	4.786	0.000	11.429	27.282
Regime transition parameters						
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.9206	0.036	25.254	0.000	0.849	0.992
p[1->0]	0.0786	0.040	1.957	0.050	-0.000	0.157

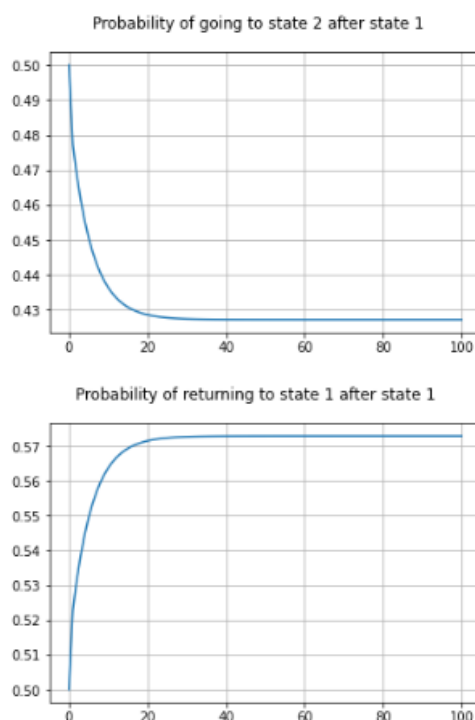


Figure 13 State probability distribution for GDP %Change & Unemployment time series.

Appendix B: Additional results, calculations, and figures for the MSDR model with Inflation rate as exogenous variable.

Table 14 Markov switching dynamic regression model for Inflation rate model results

Markov Switching Model Results						
=====						
Dep. Variable:	GDP %Change		No. Observations:		105	
Model:	MarkovRegression		Log Likelihood		-218.690	
Date:	Mon, 08 Aug 2022		AIC		453.380	
Time:	23:39:30		BIC		474.611	
Sample:	01-01-1996		HQIC		461.983	
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	4.4122	0.146	30.292	0.000	4.127	4.698
x1	-0.0177	0.019	-0.929	0.353	-0.055	0.020
sigma2	0.4076	0.104	3.907	0.000	0.203	0.612
	Regime 1 parameters					

	coef	std err	z	P> z	[0.025	0.975]
const	0.4096	0.964	0.425	0.671	-1.480	2.299
x1	0.0313	0.106	0.296	0.767	-0.176	0.239
sigma2	19.6046	4.017	4.881	0.000	11.732	27.477
Regime transition parameters						
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.9206	0.037	25.206	0.000	0.849	0.992
p[1->0]	0.0748	0.038	1.962	0.050	6.88e-05	0.150

Table 15 Markov switching dynamic regression model for Lagged inflation rate model results

Markov Switching Model Results						
=====						
Dep. Variable:	GDP %Change		No. Observations:		104	
Model:	MarkovRegression		Log Likelihood		-216.118	
Date:	Tue, 09 Aug 2022		AIC		448.236	
Time:	00:14:49		BIC		469.391	
Sample:	04-01-1996		HQIC		456.807	
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	4.4227	0.145	30.476	0.000	4.138	4.707
x1	-0.0189	0.018	-1.038	0.299	-0.055	0.017
sigma2	0.4072	0.110	3.693	0.000	0.191	0.623
	Regime 1 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	0.6702	0.979	0.684	0.494	-1.250	2.590
x1	-0.0047	0.112	-0.042	0.967	-0.224	0.215
sigma2	20.0300	4.159	4.816	0.000	11.878	28.182
	Regime transition parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

p[0->0]	0.9207	0.037	24.986	0.000	0.849	0.993
p[1->0]	0.0761	0.039	1.951	0.051	-0.000	0.153

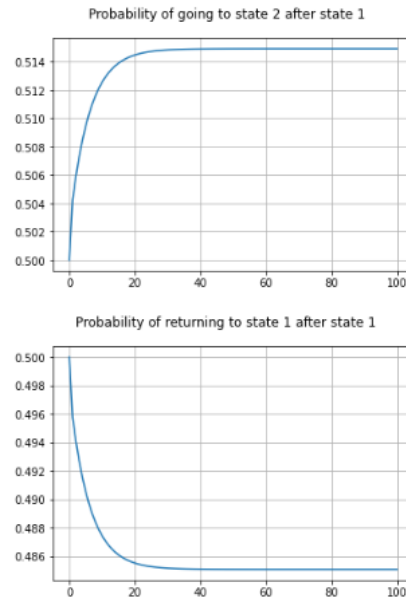


Figure 14 State probability distribution for GDP %Change & lagged Unemployment time series.

Appendix C: Additional results, calculations, and figures for the MSDR model with IPI as exogenous variable.

Table 16 Markov switching dynamic regression model for Industrial production index (IPI) results

Markov Switching Model Results						
=====						
Dep. Variable:	GDP %Change	No. Observations:	105			
Model:	MarkovRegression	Log Likelihood	-239.833			
Date:	Tue, 09 Aug 2022	AIC	495.665			
Time:	00:32:43	BIC	516.897			
Sample:	01-01-1996	HQIC	504.269			
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	-0.4494	0.442	-1.017	0.309	-1.315	0.417
x1	0.2860	0.041	6.915	0.000	0.205	0.367
sigma2	1.1597	0.341	3.399	0.001	0.491	1.828
	Regime 1 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	1.9046	0.446	4.273	0.000	1.031	2.778
x1	0.3561	0.055	6.481	0.000	0.248	0.464
sigma2	9.8122	1.774	5.531	0.000	6.335	13.289
	Regime transition parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

p[0->0]	0.9359	0.039	23.816	0.000	0.859	1.013
p[1->0]	0.0421	0.025	1.672	0.095	-0.007	0.092
=====						

Table 17 Markov switching dynamic regression model for lagged Industrial production index (IPI) results

Markov Switching Model Results						
=====						
Dep. Variable:	GDP %Change		No. Observations:		104	
Model:	MarkovRegression		Log Likelihood		-228.897	
Date:	Tue, 09 Aug 2022		AIC		473.795	
Time:	01:09:43		BIC		494.950	
Sample:	04-01-1996		HQIC		482.365	
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	-0.4657	0.232	-2.011	0.044	-0.920	-0.012
x1	0.2907	0.020	14.623	0.000	0.252	0.330
sigma2	1.7076	0.367	4.654	0.000	0.988	2.427
	Regime 1 parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

const	2.0451	0.492	4.156	0.000	1.081	3.009
x1	0.3981	0.073	5.467	0.000	0.255	0.541
sigma2	7.8434	1.501	5.225	0.000	4.901	10.786
	Regime transition parameters					
=====						
	coef	std err	z	P> z	[0.025	0.975]

p[0->0]	0.9659	0.026	36.659	0.000	0.914	1.018
p[1->0]	0.0302	0.023	1.317	0.188	-0.015	0.075

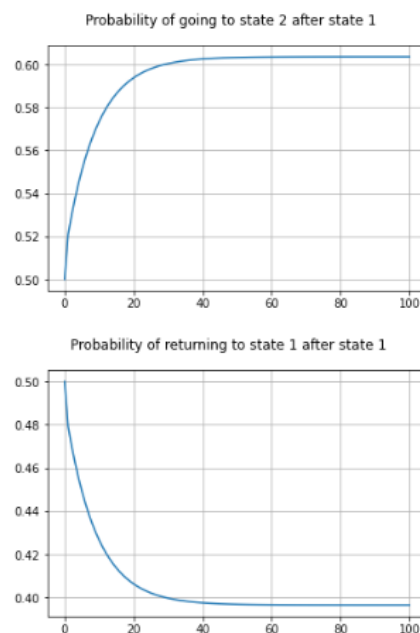


Figure 15 State probability distribution for GDP %Change & IPI time series.

Appendix D: Additional results, calculations, and figures for the MSDR model with BUX as exogenous variable.

Table 18 Markov switching dynamic regression model for BUX year-on-year percentage change results

Markov Switching Model Results						
Dep. Variable:	GDP %Change	No. Observations:				105
Model:	MarkovRegression	Log Likelihood				-275.466
Date:	Tue, 09 Aug 2022	AIC				566.932
Time:	01:54:52	BIC				588.164
Sample:	01-01-1996	HQIC				575.536
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	0.7908	0.476	1.660	0.097	-0.143	1.725
x1	-0.4256	0.720	-0.591	0.555	-1.837	0.986
sigma2	1.9972	1.111	1.797	0.072	-0.181	4.175
	Regime 1 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	2.7321	0.486	5.627	0.000	1.780	3.684
x1	2.4154	1.491	1.620	0.105	-0.506	5.337
sigma2	13.7441	2.268	6.060	0.000	9.299	18.189
	Regime transition parameters					
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.8945	0.071	12.591	0.000	0.755	1.034
p[1->0]	0.0351	0.022	1.576	0.115	-0.009	0.079

Table 19 Markov switching dynamic regression model for Lagged BUX year-on-year percentage change results

Markov Switching Model Results						
Dep. Variable:	GDP %Change	No. Observations:				104
Model:	MarkovRegression	Log Likelihood				-214.958
Date:	Tue, 09 Aug 2022	AIC				445.915
Time:	02:05:13	BIC				467.070
Sample:	04-01-1996	HQIC				454.486
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	4.1780	0.147	28.340	0.000	3.889	4.467
x1	0.2537	0.342	0.741	0.459	-0.417	0.925
sigma2	0.5390	0.141	3.810	0.000	0.262	0.816
	Regime 1 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	0.0014	0.728	0.002	0.999	-1.425	1.428
x1	2.7699	1.388	1.995	0.046	0.049	5.491

sigma2	19.2273	4.111	4.677	0.000	11.170	27.285
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Regime transition parameters						
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.9363	0.034	27.661	0.000	0.870	1.003
p[1->0]	0.0639	0.038	1.667	0.095	-0.011	0.139

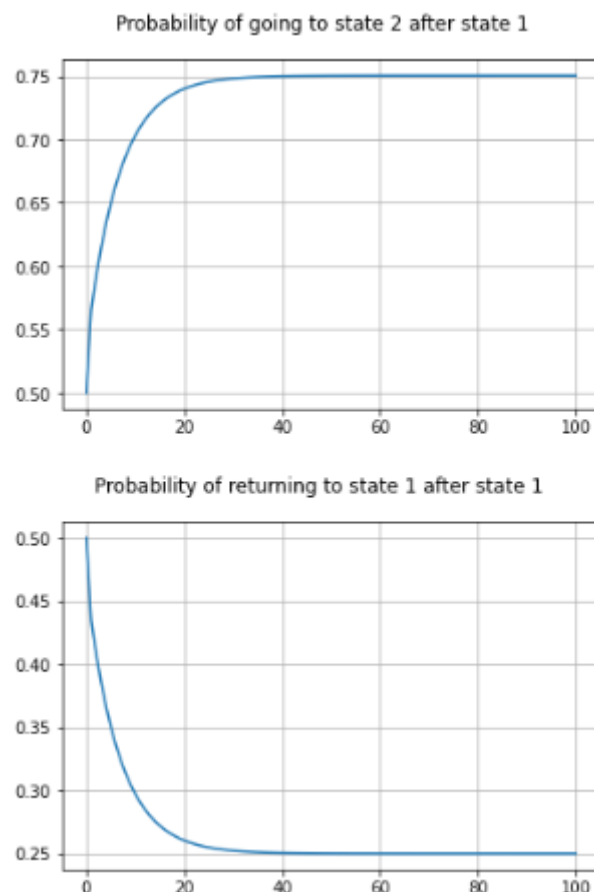


Figure 16 State probability distribution for GDP %Change & BUX YoY change time series.

Appendix E Additional results, calculations, and figures for the MSDR model with 10Year and 3Year sovereign bond yield spread as exogenous variable.

Table 20 Markov switching dynamic regression model for 10 and 3 year yield spreads results

Markov Switching Model Results			
Dep. Variable:	GDP %Change	No. Observations:	93
Model:	MarkovRegression	Log Likelihood	-195.000
Date:	Tue, 09 Aug 2022	AIC	406.000

Time:	02:13:45		BIC	426.261		
Sample:	01-01-1999	HQIC	414.181			
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	4.3779	0.097	45.168	0.000	4.188	4.568
x1	0.0024	0.066	0.036	0.971	-0.127	0.132
sigma2	0.4058	0.090	4.521	0.000	0.230	0.582
	Regime 1 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	0.6287	0.729	0.862	0.389	-0.801	2.058
x1	0.0772	0.572	0.135	0.893	-1.044	1.199
sigma2	21.8013	4.702	4.637	0.000	12.586	31.016
	Regime transition parameters					
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.9294	0.036	26.031	0.000	0.859	0.999
p[1->0]	0.0616	0.036	1.707	0.088	-0.009	0.132

Table 21 Markov switching dynamic regression model for lagged 10 and 3 year yield spreads results

Markov Switching Model Results						
Dep. Variable:	GDP %Change	No. Observations:	92			
Model:	MarkovRegression	Log Likelihood	-189.412			
Date:	Tue, 09 Aug 2022	AIC	394.824			
Time:	02:47:01	BIC	414.999			
Sample:	04-01-1999	HQIC	402.967			
	- 01-01-2022					
Covariance Type:	approx					
	Regime 0 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	4.3366	0.105	41.282	0.000	4.131	4.543
x1	0.0638	0.063	1.020	0.308	-0.059	0.187
sigma2	0.4872	0.104	4.681	0.000	0.283	0.691
	Regime 1 parameters					
	coef	std err	z	P> z	[0.025	0.975]
const	-0.3363	0.780	-0.431	0.666	-1.865	1.192
x1	1.9894	0.770	2.583	0.010	0.480	3.499
sigma2	19.4171	4.357	4.457	0.000	10.878	27.956
	Regime transition parameters					
	coef	std err	z	P> z	[0.025	0.975]
p[0->0]	0.9481	0.031	31.027	0.000	0.888	1.008
p[1->0]	0.0614	0.037	1.681	0.093	-0.010	0.133

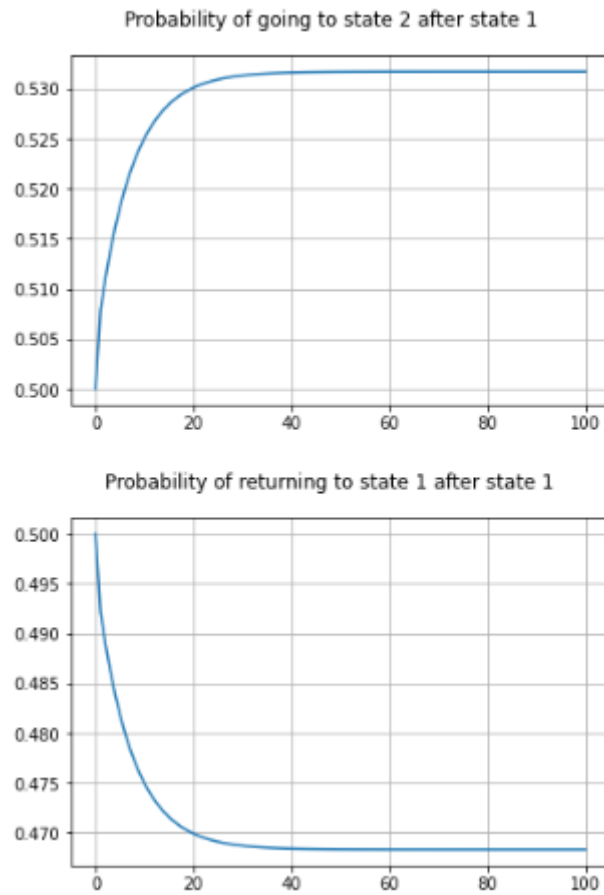


Figure 17 State probability distribution for GDP %Change & BUX YoY change time series.