OTDK Thesis

2023

UNIVERSITY OF PÉCS
FACULTY OF BUSINESS AND ECONOMICS

Egy Globális Részvényportfólió Összeállítása és Menedzselése

Constructing and Managing a Global Stock Portfolio

ATTILA VAJDICS

Advisor: Ákos Tóth-Pajor

Manuscript closing date: 2022.11.09.

Contents

1 Introduction **1**

2 Asset allocation **3**

2.1 Asset allocation strategy 6

2.2 Markets 6

2.2.1 Normality analysis 7

2.2.2 Correlation analysis 9

2.3 Volatility connectedness 12

2.3.1 Measuring volatility 12

2.3.2 Penalized VAR models 13

2.3.3 Connectedness of portfolio assets 15

2.4 Constructing a risk-optimal allocation 18

2.4.1 Minimum CVaR Concentration allocation 18

2.5 Finding best performers 21

3 Asset management **24**

3.1 Biases in investment management 24

3.2 Market bubbles 25

3.2.1 History 26

3.2.2 Methods 28

3.3 Co-movement of global markets and the US economy 39

4 Conclusion **42**

Bibliography

List of Tables

2.1 Jarque-Bera test p-values 8
2.2 Penalty functions and resulting Standard Error of Out of Sample MSFE 14
2.3 Structure of connectedness tables 15
2.4 Volatility connectedness table of portfolio assets 16
2.5 Percentage CVaR contribution of global markets 20
4.1 Performance of managed and unmanaged portfolio in comparison with the S&P 500 index 43

List of Figures

2.1 Normality analysis of asset returns 7
2.2 Normality analysis of asset prices 7
2.3 46-week rolling time window Kendall correlation estimates of market returns 9
2.4 Wavelet correlation of asset returns 11
2.5 46-week rolling time window estimates of markets' net connectedness 17
2.6 Market weights according to MCC portfolio 20
2.7 Visualization of rolling estimates 21
2.8 Result of the selection method 22
2.9 Final market weights 22
2.10 Wealth trajectories of my portfolio and the benchmark 23
3.1 Outlier analytics in the PC space 29
3.2 Bayesian Change Point analytics of the markets 31
3.3 GSADF test results 33
3.4 Posterior density of estimated critical times and filtering conditions 38
3.5 Interpretation of phase differences 40
3.6 Wavelet coherence of the S&P500 and global market indices 41

Chapter 1

Introduction

This topic is a compound of my main fields of interest: macroeconomics, market risk management and portfolio theory. Investors have a penchant to look at individual securities, which often results inappropriate diversification. Furthermore, they assume the continuity of current trends in the future which makes them vulnerable to cyclical changes. The simple fact is robustness cannot be achieved by portfolio optimization techniques themselves, we must pay attention to global market events. Most strategies attempt to solve this issue by a constant and frequent rebalancing period, which seemingly performs better but transaction cost – which is important in that case – is often not considered. Instead, I attempt to develop a robust selection method to find possibly best performing stocks. Furthermore, I present several different methods to risk-monitoring as each have their own weaknesses and strengths.

The initial step is to choose the markets that we are interested in. In Chapter 2, I first present some general attributes of the markets such as the normality of their returns and prices, and the correlation between them. Additionally, I analyze their reactions to volatility shocks arriving from other markets, which is important in times of stress. I find out what a risk-optimal allocation would be and finally introduce an effective method to outperform the S&P 500 stock index.

In Chapter 3, I start with introducing common biases in investment management as in real life they have a significant impact on *consistently* beating the market. Afterwards, I present the history of the most significant market bubbles from the beginning of the 20th century. With respect to the robust allocation method, I do not make several tiny changes in our allocation rather looking for regime-switches when the whole market declines. I also probe into the co-movement of some of the global markets and the US economy. During risk-monitoring

I endeavor to present techniques outside of the Mainstream Risk Management. Finally, I draw my conclusions in Chapter 4.

Chapter 2

Asset allocation

Assets and asset classes? Assets are individual securities we can use to construct a portfolio. However, if the asset manager does not classify them into manageable classes, they might have a hard time to handle constantly changing market conditions and make reasonable decisions. Asset classes are a manageable set of assets which share similar characteristics but are unique regarding other asset classes. In professional literature the former is called “internal homogeneity” while the latter is often referred as “external heterogeneity”. In markets, investors always look for negative or low correlations, as such an asset class can help us in two ways. It either increases the portfolio’s return or reduces its risk. Additionally, they are often referred as cost-effective solutions which are liquid and well-diversified. This is what it looks like in theory. However, they are often not internally homogenous (e.g., global equities) or externally heterogenous (e.g., high-yield bonds). (Kinlaw et al, 2017)

Asset allocation is crucial, nonetheless it is not a panacea. There was an article published in the *Financial Analysts Journal* in 1986, “The Determinants of Portfolio Performance” that created this misconception. Its authors Brinson, Hood and Beebower analyzed the performance of corporate pension plans from 1974 to 1983. What their main question was is how much of their performance can be attributed to their asset allocation. They constructed regression models based on different levels of investments ranging from asset allocation policy to individual securities. They found out asset allocation policy determined 93.6 percent of total return variation, and never less than 75.5 percent. (Brinson et al, 1986) The underlying mistake is they had an untold assumption which is misleading, namely funds are either allocated as such or not invested at all. (Kinlaw et al, 2017). The foundation of asset allocation is portfolio theory, so in the next paragraph I would like to introduce the theoretical and practical aspects of modern portfolio theory.

Why bother with optimization methods? As mentioned before, optimized portfolios intend to maximize return for a given risk or minimize risk for a given return. However, their out-of-sample performance is often questioned, resulting in the popularity of equal-weight portfolios. It is a simple method, which gives the same weight to every asset in the portfolio. There is an influential paper written by DeMiguel, Garlappi, and Uppal (2009). They carried out-of-sample backtests seven different datasets and used numerous methods for input estimation. Their research revealed that equal-weight portfolios gave 50 percent higher Sharpe ratios than any other optimization methods they applied. (DeMiguel et al, 2009) One year later, Kritzman, Page and Turkington did similar research in which they compared optimization methods to equal-weight portfolios. They deliberately used simple optimization methods and a wide range of asset classes. They found even these simple optimization techniques gave better Sharpe ratios than equal-weight portfolios. The difference in their research is that they used longer periods to estimate. As Kinlaw, Kritzman and Turkington write in their book (2017) “even a broken clock is right twice a day”. They used mean-variance portfolio to show equal-weight portfolio can be a possibly optimal allocation, however it is extremely unlikely, only for investors how have no expectations about the market and think all asset prices are efficient at the same time.

Harry Markowitz has been the father of modern portfolio theory since his article "Portfolio Selection," was published in the *Journal of Finance* in 1952. Several modern approaches are still based on Markowitz’s work. What his main achievement was is that risk/return concept should not be viewed on the level of individual assets but on portfolio level instead. In this respect we can differentiate two types of efficiency: maximal return for a given risk or minimal risk for a given return. The latter is much more common, so the following contains an example for this case. Consider a portfolio that consists of N assets, and their returns are normally distributed. The portfolio return is indicated with r , which is the product of weight vector ω and return vector μ. Furthermore, let Σ be the variance-covariance matrix of asset returns One can obtain the minimum variance portfolio with a given target return through the following optimization problem: (Pfaff, 2016)

$$P = \arg\min \sigma_W^2 = \omega' \Sigma \omega \tag{2.1}$$

As can be seen modern portfolio theory still have some shortcomings such as normality assumption and that portfolio weights are derived linearly from asset returns. (Pfaff, 2016)

So far, there was an underlying assumption, namely the variance-covariance matrix of returns represents diversification appropriately. In 2008 Choueifaty and Coignard presented a new metric to measure diversification for long-only optimization methods. This new measure is called simply diversification ratio (DR). (Choueifaty and Coignard, 2008)

$$DR_{\omega\in\Omega} = \frac{\omega'\sigma}{\sqrt{\omega'\Sigma\omega}} \tag{2.2}$$

Where Σ is the variance-covariance matrix of asset returns, σ is the vector of asset volatilities and ω is the weight vector. The numerator is the weighted average volatility of individual assets, and the denominator is the portfolio volatility. (Pfaff, 2016) Then, in 2011, Choueifaty et al. presented the following condition for the weights of the most diversified portfolio (P_{MDP}):

$$P_{MDP,\omega\in\Omega} = \arg\max DR \tag{2.3}$$

By maximizing DR, which technically means minimizing the denominator $\omega'\Sigma\omega$, one can obtain a more diversified portfolio. The difference is that in this case we use the correlation matrix and not the usual variance-covariance matrix. However, it still relied on a volatility-based risk measure, which assumes normal distribution. Furthermore, it reveals another difference between theory and practice, namely, it assumes that any deviation from the mean is unfavorable. Thus, it practically minimizes return, as well. Boudt et al. (2010, 2011) and Ardia et al. (2011b) introduced similar methods for downside-based risk measure (CVaR), which measures risk in pecuniary loss.

2.1 Asset allocation strategy

In my asset allocation strategy, I first choose various stock market indices to get a general overview of the markets we intend to invest in. Then, I analyze the normality of index prices and returns to find out which type of data to use for the next important step: measuring correlation. I use weekly data in case of visual normality analysis and daily data in case of parametric ones. Then I implement Kendall correlation analysis which does not assume the normality and is a non-parametric method, thus resulting a more robust solution. Additionally, to measure correlation in the time and frequency domain, as well, I implement Wavelet correlation analysis.

Afterwards I dedicate a sub-chapter to introduce a popular multivariate time series method, namely Vector Autoregression as a foundation of measuring the effect of volatility shocks on stock market indices. Then I find out which would be a robust, risk-optimal allocation of our assets across global markets. Moreover, I present its allocation's properties: portfolio level CVaR, and CVaR contribution of individual assets to the whole portfolio risk.

2.2 Markets

I chose the following indices as indicators of the markets I would like to cover: S&P 500 (^GSPC), FTSE 100 (^FTSE), EURO STOXX 50 (^STOXX50E), BSE SENSEX (^BSESN), Hang Seng (^HIS) and NIKKEI 225 (^N225). As we cannot invest directly in indices just different products which track them e.g. ETF they are only used to analyze the characteristics of the chosen markets. However, since January 2018 many of these index tracking ETFs with an ISIN code starting with US are simply not tradeable for EU investors. There are existing alternatives, but they have significantly higher net expense ratio and smaller liquidity. We are interested in the highest final wealth in US Dollar, so all the other markets carry some exchange rate risk, as well. The data is from Yahoo Finance from 01/01/2010 to 01/01/2018. To give an overview about the markets I use weekly data for visualization purposes.

2.2.1 Normality analysis

I would like to emphasize the difference between using log returns or prices as input variables. To do so, I compare the empirical distribution and normal distribution of the given time series of prices and returns.

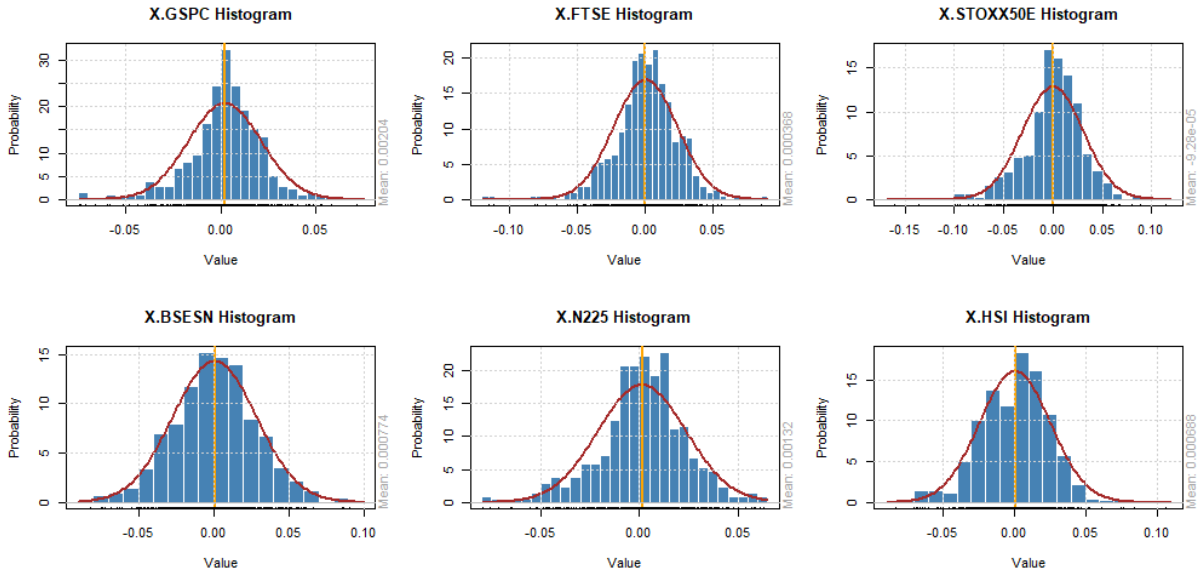


Figure 2.1: Normality analysis of asset returns

Source: own editing

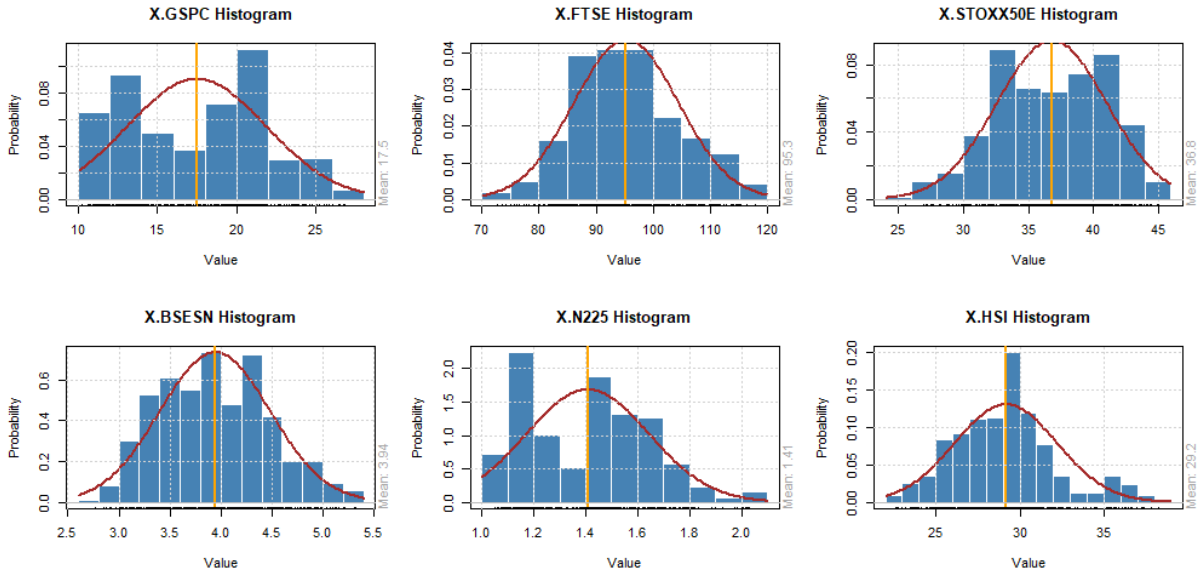


Figure 2.2: Normality analysis of asset prices

Source: own editing

As can be seen on the histograms, neither returns nor prices are normally distributed. In case of prices each time series were divided by 100 as per technical reasons. Additionally, I conduct a parametric test for normality, namely Jarque-Bera normality tests for both asset returns and prices.

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{2.4}$$

Where n is the number of observations, S is the skewness and K is the kurtosis of the sample. This kind of normality test is often preferred in finance because it is reliable with large sample size, as we often deal with one in finance. Additionally, it has general understanding, we reject null hypothesis – normality – if the p-value is smaller than the given level of significance.

Jarque-Bera test p-values

Indices	Returns	Prices
S&P 500	< 2.2e-16	4.026e-6
FTSE 100	< 2.2e-16	0.031
EURO STOXX 50	< 2.2e-16	0.001
BSE SENSEX	0.461	0.008
Hang Seng	2.973e-5	3.35e-6
NIKKEI 225	9.427e-7	0.0002

Table 2.1: Jarque-Bera test p-values

Source: own editing

As can be seen, we can reject null hypothesis in most cases, at all reasonable significance level. Some of them like the BSESN index showed normality, at least at weekly data. This means neither prices nor returns are normally distributed. For further analysis I use weekly returns of the stock market indices.

2.2.2 Correlation analysis

Diversification is one of the most important things in investment management. When choosing our portfolio assets, it can be traced by various correlation measures. In this section I would like to introduce two aspects of it: input variable selection and different correlation measures. It could be an interesting question what to use as input variables in correlation measurement: prices or returns? The problem is that asset prices at different point of time can be written as: $P_0, P_1, P_2, \dots, P_T$. From this the log return at a certain point of time can be written as: $R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$. Since prices are dependent on all the previous returns, it amplifies correlation in both directions positive and negative, as well. Additionally, financial markets have an "upward bias" in general, which means prices move upward in the long term, by nature. (QuantDare: Correlation with prices or returns: that is the question, 2018)

Kendall correlation

To analyze the correlation of my portfolio assets I use *Kendall rank correlation coefficient*. It is often referred as Kendall's τ, and for two variables it is calculated as the difference of probabilities of concordant (c) and discordant (d) pairs: $\tau = \frac{c-d}{\binom{n}{2}} = \frac{c-d}{\frac{1}{2}n(n-1)}$

Figure 2.3: 46-week rolling time window Kendall correlation estimates of market returns

Source: own editing

It can be seen most portfolio assets have modest correlation based on full sample values (green), however the rolling window estimation reveals significant changes over time. It is important because it shows mutual increased correlation in all pairs at the end of our training period, as so full sample values would have been misleading in our case.

Wavelet correlation

Time series analysis in time and frequency space is becoming more and more popular as this way we can extract unique features unlike in any other way of measuring correlation. When we would like to measure correlation with wavelets, we use discrete wavelet transform (DWT) .First, we need a father wavelet $\phi_{j,k}(t) = 2^{-\frac{j}{2}}\phi\left(\frac{t-2^j k}{2^j}\right)$ and a mother wavelet $\psi_{j,k}(t) = 2^{-\frac{j}{2}}\psi\left(\frac{t-2^j k}{2^j}\right)$ which are transformation functions. Where 2^j is the scale factor that is necessary to partition frequency and $2^j k$ is the translation (shift) parameter which locates wavelet in the time domain. In this sense each time series (X_t) is regarded as a signal. (In and Kim, 2013)

$$x_t \approx \sum_k S_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{J-1,k}(t) \tag{2.5}$$

where

$$S_{J,k} = \int \phi_{J,k}(t)x_t dt \quad \text{and} \quad d_{J,k} = \psi_{J,k}(t)x_t dt \quad \text{where } j = 1, 2, \dots, J$$

In this context J is the number of scales. Practically, $S_{J,k}$ are the ones that can capture the trend component, they are often referred as "smooth coefficients", and $d_{J,k}$ are the ones that can capture scale deviations (oscillations) from the smoothed trend. To estimate correlation, we need to define two univariate time series X_t and Y_t. Next, we need to calculate the covariance of wavelet coefficients of X_t and Y_t at scale λ_j, and wavelet variances of X_t and Y_t at scale λ_j. Once the above-mentioned transformations are done for each time series, one can obtain the covariance and variances of their wavelet transforms. (In and Kim, 2013)

$$\tilde{\rho}_{XY}(\lambda_j) \equiv \frac{Cov_{XY}(\lambda_j)}{\tilde{v}_X(\lambda_j)\tilde{v}_Y(\lambda_j)} \tag{2.6}$$

As can be seen the way it handles variables resulting from wavelet analysis is *identical to* the way how the traditional one handles its variables. Afterwards, we can calculate the wavelet correlation for different scales λ_j, which is also known as the "depth of the decomposition". In the next plot we can see wavelet correlations of portfolio asset returns, with a depth of decomposition of 8.

Figure 2.4: Wavelet correlation of asset returns

Source: own editing

The codes are the following: S&P 500 (C1), FTSE 100 (C2), EURO STOXX 50 (C3), BSE SENSEX (C4), NIKKEI (C5), Hang Seng (C6). The scale is in weeks. As can be seen S&P 500, FTSE 100 and EURO STOXX 50 are strongly correlated with each other. The correlation between S&P 500 and Hang Seng amplifies as we consider longer and longer time scales. We can also observe increasing correlation between the European and Indian stock market, as well as the European and Chinese stock market. We can observe the same in case of S&P 500 and FTSE 100, so the US and European stock market are also strongly correlated with developed (HSI) and developing (BSESN) Asian markets at longer time scales. However, the correlation between Indian stock market and Chinese stock market is also present but weaker than the correlation with US and European markets and is weakening at longer time scales. NaN cells mean there is no correlation at the given frequency with 95% confidence level.

2.3 Volatility connectedness

Firstly, we need to calculate the daily volatilities of our portfolio assets. To do this I am going to use Yang-Zhang volatility estimation method. Secondly, we need to construct a certain type of vector autoregressive (VAR) models as it is going to be the foundation to forecast error variance decomposition (FEVD). Thirdly, we can calculate the directional connectedness measures for portfolio assets and the NET connectedness of the portfolio.

2.3.1 Measuring volatility

The most widely used volatility estimation method is the Close-to-close (historical) volatility, which uses only the closing prices of the given instrument. However, it has some drawbacks such as it has no regard for opening jumps.

$$\sigma_{cl} = \sqrt{\frac{N}{n-2}\sum_{i=1}^{n-1}(r_i - \bar{r})^2} \tag{2.7}$$

Therefore, I use the Yang-Zhang volatility estimation method which is currently one of the most accurate ones. It can handle opening jumps and grid, which means mean return is not zero. It is the compound of three volatility estimation methods: Rogers-Satchell volatility $\sigma_{rs}^2 = \frac{N}{n}\sum_{n=1}^{N}\left(\ln\frac{h_i}{c_i}\ln\frac{h_i}{o_i} + \ln\frac{l_i}{c_i}\ln\frac{h_i}{o_i}\right)$, close-to-open (overnight) volatility $\sigma_o^2 = \frac{N}{n-1}\sum\left(ln\frac{o_i}{c_{i-1}} - \mu_o\right)^2$ and open-to-close volatility, $\sigma_c^2 = \frac{N}{n-1}\sum\left(ln\frac{c_i}{o_i} - \mu_c\right)^2$ with a certain weight $k = \frac{\alpha-1}{\alpha+\frac{n+1}{n-1}}$.Rogers-Satchell allows for drift but still assumes no opening jumps.

$$\sigma_{YZ} = \sqrt{\sigma_o^2 + k\sigma_c^2 + (1-k)\sigma_{rs}^2} \tag{2.8}$$

2.3.2 Penalized VAR models

Vector autoregressive (VAR) models are very popular and effective methods in multivariate time series analysis. But they are often criticized for the fact that they are heavily overparameterized. However, research showed that incorporating more variables improves their efficiency. Nicholson, Matteson, and Bien (2017b) introduced several structured regularization techniques for large vector autoregressions. Their main interest is in macroeconomic time series incorporating dozens of variables, but I use their improved large, penalized estimation methods to VAR models, which serves as the foundation of connectedness measuring. (Diebold and Yilmaz, 2015, Barunik and Krehlik, 2018)

The basic VAR model is:

$$y_t = v + \sum_{\ell=1}^{p} \Phi^{(\ell)} y_{t-\ell} + u_t \tag{2.9}$$

where v is a k-dimensional constant intercept vector, $\Phi^{(\ell)}$ is a $k \times k$ matrix of endogenous coefficients, and u_t is the Gaussian white noise, and p indicates the lag order of the process. In this model we do not have exogenous variables as they are only the Yang-Zhang volatilities of portfolio assets. But the penalties presented by Nicholson, Matteson, and Bien (2017b), are separable. So, there is a penalty function for endogenous variables and there is another one for exogenous variables. We can use the ones for endogenous variables since in a vector autoregressive model we assume that coefficient matrices contain all the information about the connection between variables. Next, we can write the summation of the VAR process with lag order p with a certain penalty as (VAR-L):

$$\underset{v,\Phi}{\operatorname{argmin}} \sum_{t=1}^{T} \left\| y_t - v - \sum_{\ell=1}^{p} \Phi^{(\ell)} y_{t-\ell} \right\|_2^F + \lambda \mathcal{P}_y(\Phi)$$

where $\|A\|_F$ is the Frobenius norm of the A matrix, λ is the penalty parameter and $\mathcal{P}_y(\Phi)$ is the penalty function for the endogenous coefficients. As penalty functions I implement the ones presented by Nicholson, Matteson, and Bien, and backtest them to find out which performs the best with my data. Afterwards we can check the sparsity matrix of the model to find out if there

is within-group sparsity. Which would suggest the use of Sparse-type penalty functions. This is the graphical way to do it. The numeric way to make the final decision is to calculate the h-step ahead mean-square forecast error (MSFE) of each model with different penalties. (Banbura et al., 2010) This is called rolling cross-validation, and to do it we need to divide the data into three sections: initialization, training, and forecast evaluation. Assuming the length of the original dataset is T, the above-mentioned sections can be separated at the following points: $T_1 = \left\lfloor \frac{T}{3} \right\rfloor, T_2 = \left\lfloor \frac{2T}{3} \right\rfloor$. We measure it as a function of $\hat{\lambda}$, the estimated penalty parameter.

$$MSFE(\lambda_i) = \frac{1}{(T_2 - T_1 - h + 1)} \sum_{t=T_1}^{T_2-h} \left\| \hat{y}_{t+h}^{\lambda_i} - y_{t-h} \right\|_2^F \tag{2.10}$$

Name of the penalty function	$\mathcal{P}_y(\Phi)$	SEOOSMSFE
LASSO	$\|\Phi\|_1$	0.03415
Elastic Net	$\alpha\|\Phi\|_1 + (1-\alpha)\|\Phi\|_2^2$	0.03784
Lag	$\sqrt{k^2}\sum_{\ell=1}^{p} \left\|\Phi^{(\ell)}\right\|_F$	0.03560
Own/Other	$\sqrt{k}\sum_{\ell=1}^{p} \left\|\Phi_{ON}^{(\ell)}\right\|_F + \sqrt{k(k-1)}\sum_{\ell=1}^{p} \left\|\Phi_{OFF}^{(\ell)}\right\|_F$	0.03499
Sparse Lag	$(1-\alpha)\sqrt{k^2}\sum_{\ell=1}^{p} \left\|\Phi^{(\ell)}\right\|_F + \alpha\|\Phi\|_1$	0.03453
Sparse Own/Other	$(1-\alpha)\left(\sqrt{k}\sum_{\ell=1}^{p} \left\|\Phi_{ON}^{(\ell)}\right\|_F + \sqrt{k(k-1)}\sum_{\ell=1}^{p} \left\|\Phi_{OFF}^{(\ell)}\right\|_F\right) + \alpha\|\Phi\|_1$	0.03043

Table 2.2: Penalty functions and resulting Standard Error of Out of Sample MSFE

Source: own editing, based on Nicholson, Matteson, and Bien, 2017

As can be seen the sparse own/other penalized VAR model is the most accurate method with my dataset. So, in the next section I am going to use it as a foundation of volatility connectedness measures.

2.3.3 Connectedness of portfolio assets

In their work "Financial and Macroeconomic Connectedness (2015), Diebold and Yilmaz analyzed the volatility connectedness of four asset classes in four countries, from January 1998 to June 2013. They analysis incorporated the United States, European Union, United Kingdom, and Japan. They analyzed the following asset classes: stocks, long-term sovereign bonds, foreign exchange rate with respect to US dollar, and the commodity market was depicted by the Dow Jones – UBS Commodity index. Their full-sample total volatility (Garman-Klass) connected was 42.2%, with a simple VAR model. They had a previous research paper in this topic (2012) before the book was published. First, we need a generalized variance decomposition:

$$\Theta_{L_{i,j}} = \frac{\sigma_{jj}^{-1} \sum_{\ell=0}^{L} ((\Psi_\ell \Sigma)_{i,j})^2}{\sum_{\ell=0}^{L} (\Psi_\ell \Sigma \Psi_\ell')_{i,i}} \tag{2.11}$$

Where Ψ_ℓ is the approximated version of the lag polynomials matrix from the moving average (MA) representation of the VAR model. (Barunik and Krehlik, 2017) We can define total directional connectedness ***from*** other variables to i as: (Xiao and Huang, 2018)

$$C_{i\leftarrow\star}^{L} = \sum_{\substack{j=1 \\ j\neq i}}^{N} d_{ij}^{L} \tag{2.12}$$

and the total directional connectedness ***to*** other variables from j as:

$$C_{\star\leftarrow j}^{L} = \sum_{\substack{i=1 \\ i\neq j}}^{N} d_{ij}^{L} \tag{2.13}$$

	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	⋯	$\boldsymbol{X_N}$	**FROM**
x_1	d_{11}^L	d_{12}^L	⋯	d_{1N}^L	$\sum_{j=1}^{N} d_{1j}^L, j \neq 1$
x_2	d_{21}^L	d_{22}^L	⋯	d_{2N}^L	$\sum_{j=1}^{N} d_{2j}^L, j \neq 2$
⋮	⋮	⋮	⋱	⋮	
X_N	d_{N1}^L	d_{N2}^L	⋯	d_{NN}^L	$\sum_{j=1}^{N} d_{Nj}^L, j \neq N$
TO	$\sum_{i=1}^{N} d_{i1}^L, i \neq 1$	$\sum_{i=1}^{N} d_{i2}^L, i \neq 2$	⋯	$\sum_{i=1}^{N} d_{iN}^L, i \neq N$	$\frac{1}{N}\sum_{i=1}^{N} d_{i2}^L, i \neq j$

Table 2.3: Structure of connectedness tables

Source: own editing, based on Xiao and Huang, 2018

Whenever we calculate such measures, we are interested in volatility spillovers, which means the transmission of instability from one market to another market. It would have been good if I could have measured this kind of financial contagion not only on asset but country level via interbank market matrix of assets and liabilities – maybe buffers as well – but interbank bilateral exposure data is not public at all. The FROM type means what it causes to other markets and TO type is what it receives from other markets. As the title of their original paper says, "Better to give than to receive" (2012), so the desired case is negative net connectedness. It shows that one influences the market, rather than being influenced by the market. The following table contains the connectedness measures calculated for my own portfolio. I calculated them with the presented VAR-L model, however, to make a fair comparison to Diebold's and Yilmaz's results in connection with the connectedness of global asset classes, I also calculated the total directional connectedness of my portfolio with their basic VAR model.

	GSPC	FTSE	STOXX50	BSESN	HSI	NIKKEI	**FROM**
GSPC	35.47	15.66	17.56	8.53	14.57	8.22	10.75
FTSE	17.7	39.28	18.52	4.46	13.37	6.68	10.12
STOXX50	18.62	18.56	36.53	8.42	12.89	4.99	10.58
BSESN	12.58	6.35	12.18	51.16	14.03	3.71	8.14
HSI	16.8	14.8	13.97	9.91	36.96	7.58	10.51
NIKKEI	14.64	13.54	9.64	7.25	11.51	43.42	9.43
TO	13.39	11.48	11.98	6.43	11.06	5.2	59.53
NET	**2.64**	**1.36**	**1.40**	**-1.71**	**0.55**	**-4.23**	**9.92 (9.35)**

Table 2.4: Volatility connectedness table of portfolio assets

Source: Own editing

We can obtain the net total directional connectedness indicated with blue color as from general formulas as $C^L = \frac{1}{N}\sum_{\substack{ij=1 \\ j\neq i}}^{N} d_{ij}^L$ which is analogous to the *from* columns or *to* rows divided by square of the number of assets: $\frac{1}{N^2}\sum_{i=1}^{N} d_{i2}^L, i \neq j$. The total directional connectedness measures are the blue ones, where the number in the brackets is the one calculated with simple VAR model. All values are in percentage, and we can see low FROM and TO connectedness in each case. We can see that US and European markets are gross receivers of volatility shocks from other markets. They also contribute to these shocks more than Asian markets, but they still receive more than they give, considering the full sample. On the other hand, Asian markets are net receivers of volatility shocks, except in case of China, which is neutral. As the net connectedness (0.55) is not statistically significant from zero. Additionally, we can see a full-

sample total net directional connectedness of 9.92%. However, these are the full-sample measures, which are valid at the end of the period, but in many cases, one is rather interested in the dynamics of the connectedness over time. As the penalized and simple VAR models were slightly different with respect to our measure of interest (net connectedness), to do the rolling estimation of connectedness measures, I use simple VAR model for computational efficiency.

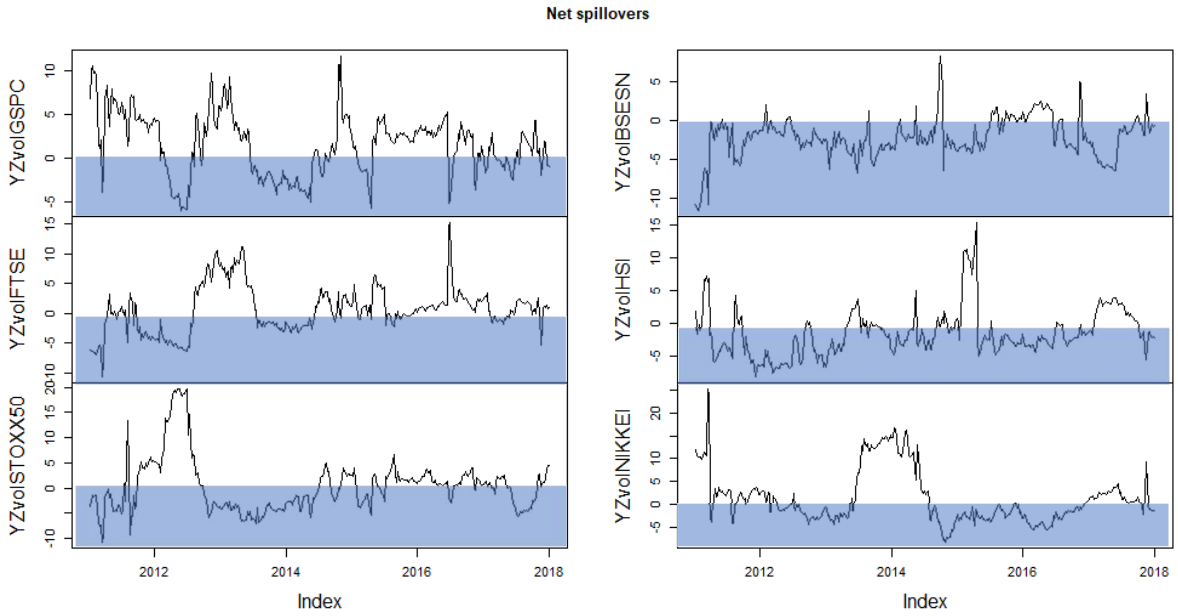


Figure 2.5: 46-week rolling time window estimates of markets' net connectedness

Source: own editing

In Figure 2.5 we can see the rolling net connectedness of our markets of interest from 2010 to 2018. As can be seen, US stock market is generally a gross receiver of volatility shocks, except some shorter periods. We can observe similar characteristics in case of British stock market which is also a gross receiver of volatility shocks after 2012, and in a similar, but generally bigger extent. In 2015 the US market experienced a big unfavorable shock; however, it affected the UK market stronger in the beginning of 2016. The European stock market shows to be more stable, however the volatility effect of the previous shock was two times stronger than in the US market. We can see the Indian stock market is nearly always a net receiver of volatility shock, even in times of shock it gives very modest reactions like in 2015. The Japanese stock market shows dissimilarities to the European market with respect to the timing of volatility shocks. China is more interesting as it was in a very beneficial position – either net receiver or neutral – until the effect of the US shock reached it in 2015.

2.4 Constructing a risk-optimal allocation

Risk management has always been – or should have been – an important aspect of portfolio management. It has been gaining ground for a long time, since investors are generally interested in the risk-adjusted return of their portfolio. In this sense we use risk as a measure to diversify our portfolio. As mentioned, we can use volatility-based or downside-based risk measures to do so. I use a downside-based one, namely Conditional Value at Risk (CVaR) at 97.5% confidence level as suggested by the Basel Committee on Banking and Supervision.

Based on the easy-to-manage property of simple equal-weight portfolio, one could consider doing so for downside risk contributions of each asset, it is called the equal risk contribution (ERC) portfolio. As each asset can be managed based on the deviations from the desired risk contribution to portfolio risk. However, for many practical applications it is too restrictive to apply such a constraint. We are likely not to find feasible set of the optimization problem, when dealing with multiple portfolio assets. A good alternative to find a workaround is to use the Minimum CVaR Concentration portfolio. (Boudt et al. 2013)

2.4.1 Minimum CVaR Concentration allocation

The instruction of the method is heavily based on Boudt et al (2013) and Pfaff (2016). First, we need a risk budget to decide how portfolio risk and thus risk contribution is measured. In this case we can turn towards game theory, Denault (2001) has shown that we should measure the risk contribution as the weight of the individual asset in the portfolio times the partial derivative of the portfolio risk as a derivative of that certain weight:

$$C_{(i)}R_w = w_{(i)}\frac{\partial R_w}{\partial w_{(i)}} \tag{2.14}$$

Where it is true that the total portfolio risk is still equal to the sum of the risk contribution of individual assets. Afterwards the CVaR risk with a given significance level α, which is 2.5% in our case, and with a given return at a given point of time r_w, with weight vector w is measured as:

$$CVaR_w(\alpha) = -\mathbb{E}[r_w | \, r_w \leq -VaR_w(\alpha)] \tag{2.15}$$

Then we can define CVaR contribution of a portfolio asset to the whole portfolio as the weight of the individual asset $w_{(i)}$ in the portfolio times the partial derivative of the portfolio CVaR as a derivative of that certain weight

$$C_{(i)}CVaR_w(\alpha) = w_{(i)} \frac{\partial CVaR_w(\alpha)}{\partial w_{(i)}} \tag{2.16}$$

Then we standardize the CVaR contributions by the portfolio (total) CVaR, this way we can obtain the percentage CVaR contribution, which is analogous to the ratio of the expected return on the position when there is a beyond-VaR loss to the expected value of the beyond-VaR loss of the portfolio:

$$\%C_{(i)}CVaR_w(\alpha) = \frac{C_{(i)}CVaR_w(\alpha)}{CVaR_w(\alpha)} = \frac{\mathbb{E}\left[w_{(i)}r_{(i)} \middle|\, r_w \leq -VaR_w(\alpha)\right]}{\mathbb{E}[r_w |\, r_w \leq -VaR_w(\alpha)]} \tag{2.17}$$

The word "concentration" is a summary statistic of CVaR allocation, which is the largest CVaR of all positions:

$$C_w(\alpha) = \max_i C_{(i)}CVaR_w(\alpha) \tag{2.18}$$

Then the Minimum CVaR Concentration (MCC) portfolio allocation, with a downside risk objective can be defined as:

$$w^{MCC} = arg\min_{w \in W} C_w(\alpha) \tag{2.19}$$

Where W means feasible portfolio weights, with a full-investment constraint. To sum up, Minimum CVaR Concentration (MCC) portfolio gives a higher portfolio CVaR than the more traditional minimum CVaR portfolio, but it is a more robust version of a similar kind. The higher the percentage CVaR values indicate that portfolio's downside risk is concentrated on that certain asset, so the portfolio's and the given asset's losses are similar in extent.

The following figure shows the market weights according to the MCC portfolio. The solution was found with the Generalized Simulated Annealing (GSA a.k.a. GenSA) method instead of the more traditional Differential Evolution (DE), with a maximum of 500 iterations and 200 000 search size.

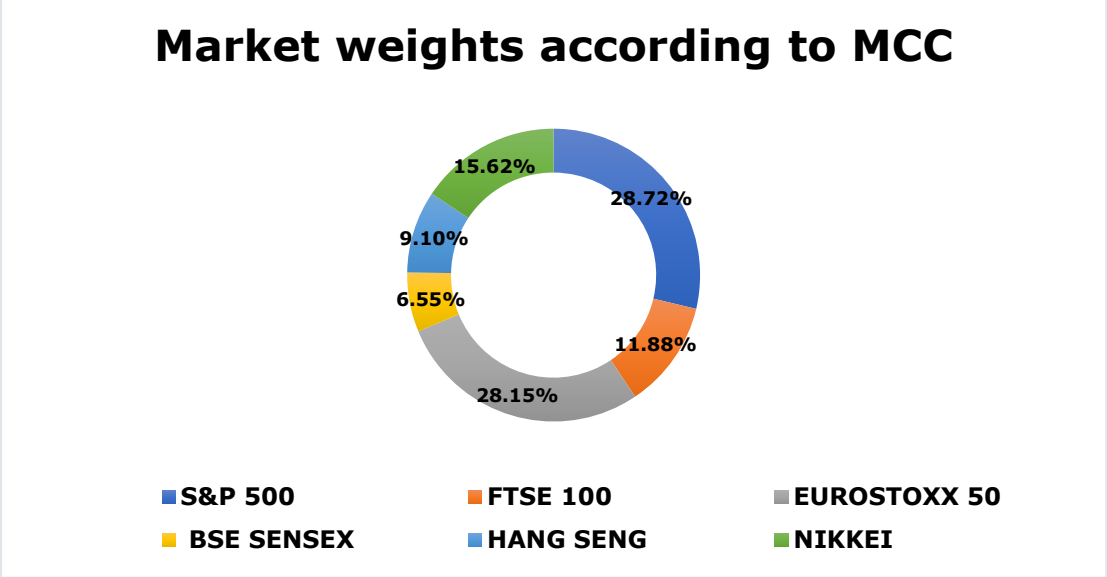


Figure 2.6: Market weights according to MCC portfolio

Source: own editing

This allocation resulted a "portfolio" $CVaR_{97.5}$ of 7.5%. The following table discusses the percentage CVaR contribution of each market to the "portfolio" CVaR.

Markets	Percentage contribution to CVaR
S&P 500	25%
FTSE 100	10%
EUROSTOXX 50	22%
BSE SENSEX	7%
HANG SENG	8%
NIKKEI	27%

Table 2.5: Percentage CVaR contribution of global markets

Source: own editing

As we can see in case of the S&P 500, EUROSTOXX 50, FTSE, BSE SENSEX and Hang Seng the risk contributions are similar to their weights, however NIKKEI has much bigger contribution to the portfolio CVaR than its portfolio weight.

2.5 Finding best performers

For robustness, I use monthly data of the constituents of stock market indices analyzed in previous sub-chapters, from 01/01/2013 to 01/12/2017 as training period, and from 01/01/2018 to 01/06/2022 as out-of-sample testing period. To overcome the discussion about the several different models and methods used to select best performing stocks, I developed an own method to select them. It relies only on market data, and less parameter sensitive than other popular models. It is based on a 12-month rolling time window fit of a simple linear regression while estimating popular measures such as alpha and beta of individual stocks. Additionally, for robustness it uses positional average measures (median) since no reasonable visual comparison can be made on several hundreds of stocks. After obtaining the median values we can make a comparison with respect to their median beta and alpha. Empirically, based on the S&P 500 index and its constituents, my threshold values are $\mathcal{A} = 0.034$ for alpha (bigger than) and $\mathfrak{B} = 0$ for beta (smaller than), each final portfolio element must meet both criteria.

	S_1	S_2	S_3	$\cdots$	S_{n-1}	S_n
$t = 13$	α_{13}^1	α_{13}^2	α_{13}^3	$\cdots$	α_{13}^{n-1}	α_{13}^n
$t = 14$	α_{14}^1	α_{14}^2	α_{14}^3	$\cdots$	α_{14}^{n-1}	α_{14}^n
$t = 15$	α_{15}^1	α_{15}^2	α_{15}^3	$\cdots$	α_{15}^{n-1}	α_{15}^n
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$t = i$	α_i^1	α_i^2	α_i^3	$\cdots$	α_i^{n-1}	α_i^n
$Median$	$\widetilde{\mathrm{A}^1}$	$\widetilde{\mathrm{A}^2}$	$\widetilde{\mathrm{A}^3}$	$\cdots$	$\widetilde{\mathrm{A}^{n-1}}$	$\widetilde{\mathrm{A}^n}$

	S_1	S_2	S_3	$\cdots$	S_{n-1}	S_n
$t = 13$	β_{13}^1	β_{13}^2	β_{13}^3	$\cdots$	β_{13}^{n-1}	β_{13}^n
$t = 14$	β_{14}^1	β_{14}^2	β_{14}^3	$\cdots$	β_{14}^{n-1}	β_{14}^n
$t = 15$	β_{15}^1	β_{15}^2	β_{15}^3	$\cdots$	β_{15}^{n-1}	β_{15}^n
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$t = i$	β_i^1	β_i^2	β_i^3	$\cdots$	β_i^{n-1}	β_i^n
$Median$	$\widetilde{\mathrm{B}^1}$	$\widetilde{\mathrm{B}^2}$	$\widetilde{\mathrm{B}^3}$	$\cdots$	$\widetilde{\mathrm{B}^{n-1}}$	$\widetilde{\mathrm{B}^n}$

Figure 2.7: Visualization of rolling estimates

Source: own editing

Where *S* means one of the constituents of a stock market index. After obtaining the median values of rolling time window estimates I checked the threshold values for each stock. Based on my model I chose 29 of the possibly best performing stocks (MYP) out of 834. As the tickers list which can be used for web-scraping is always the latest, the number 834 is the result of a previous filtering so that I use only those stocks which were constituents at our starting period.

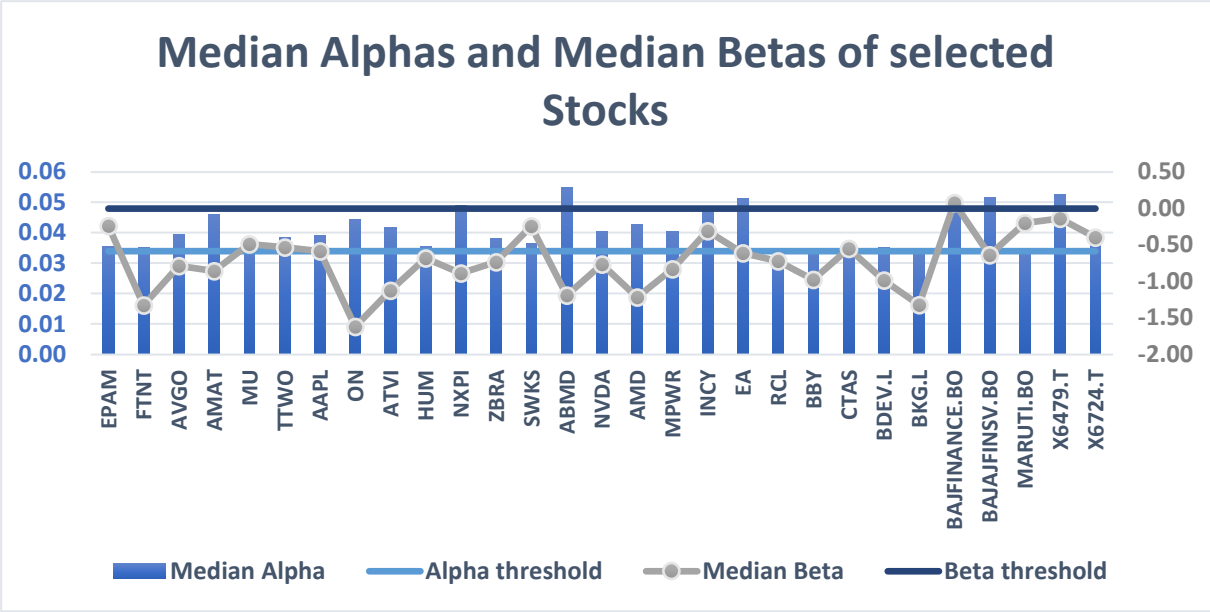


Figure 2.8: Result of the selection method

Source: own editing

Additionally, my model did not find worthy investments amongst the constituents of EUROSTOXX 50 and Hang Seng. The final allocation is shown in Figure 2.8. All the stocks are equally-weighted within their own market. The reallocation is based on the number of worthy stocks found in different markets, their characteristics to beat the S&P 500 and I reduced the ratio of NIKKEI with respect to its contribution to the total portfolio risk. The worthy stock found in the Indian market showed a high potential to do so, by overperforming most of the US stocks as well. On the other hand, UK stocks barely met the excess return criteria.

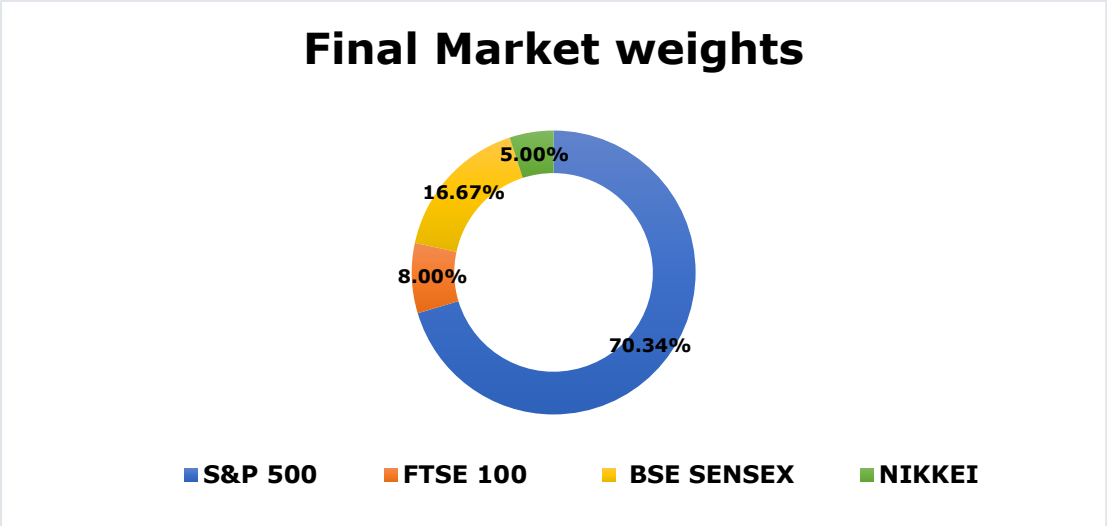


Figure 2.9: Final market weights

Source: own editing

The following plot shows the resulting wealth trajectories of my portfolio (MYP) compared to the S&P 500. The colored numbers indicate the highest first-day-of-the-month cumulative return during the holding period.

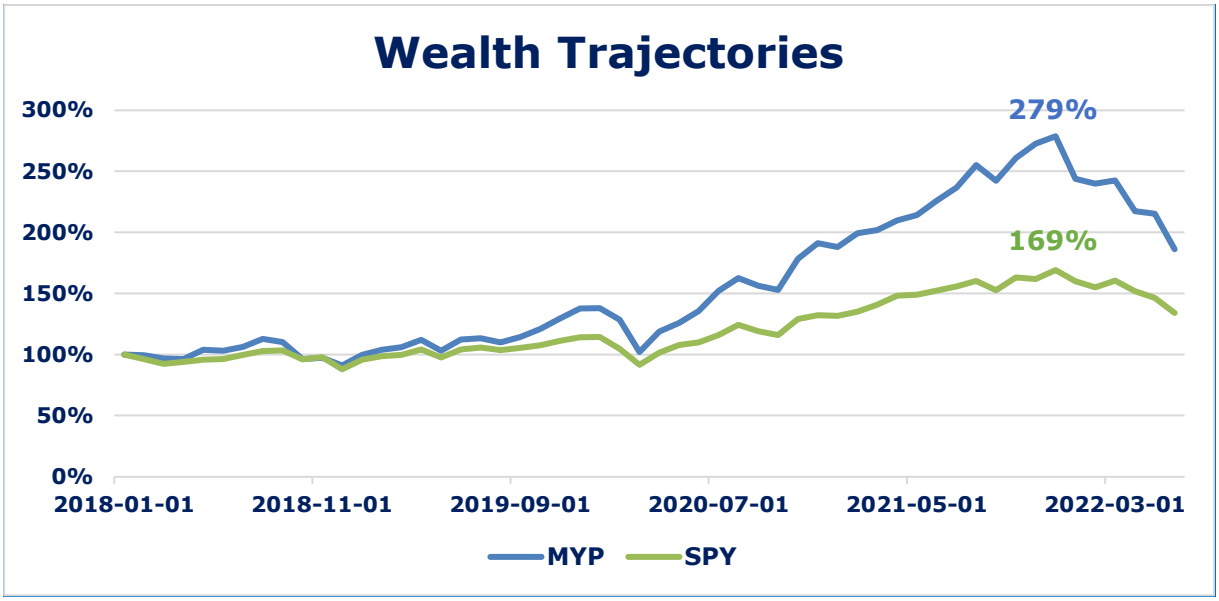


Figure 2.8: Wealth trajectories of my portfolio and the benchmark

Source: own editing

As can be seen, it would have been beneficial to close our positions at the end of December 2021, but it would have been hasty after the sudden crash at the end of March 2020, as there was a quick correction. So, the question arising is How to identify true regime switches and differentiate them from these sudden falls which are followed by sudden corrections?

Chapter 3

Asset management

Up to this point we have been discussing how to allocate our wealth into different geographical/economic areas. Although, MCC portfolio provides us a robust, downside risk allocation it is important to pay attention to exchange rate risk, as according to the MCC portfolio a significant part of our wealth is invested out of the United States.

The most challenging – and the most important – part is to select "winning" stocks. To do this we can see several methods like following the trend, DCF models to firm valuation, P/E ratio. However, none of them are truly effective, since if you follow the trend then you are right when everyone else is right and wrong when everyone else is wrong. Discounted Cash Flow models have some major drawbacks. Firstly, they are very sensitive to input parameters, and we must make a lot of assumptions. Secondly, all information is based on financial statements, which often fall victim to creative accounting practices. The same applies for P/E ratio.

3.1 Biases in investment management

The most important – and most difficult – part of managing investments is to be consistent and objective. As trading is much more than drawing arbitrary lines on a candlestick chart. In this section I would like to introduce some of the most common bias.

Confirmation bias means we, as humans, have a penchant to find support for our *preconceptions*. It has two common forms: we only pay attention to information that supports our opinion, or information that has already gone popular in public. (Gliner, 2014) So, you are wrong when everyone else is wrong, as well, and it gives some unreasonable relief.

Self-education and critical thinking are the best tools to overcome this bias. **Availability bias** is the fact that we tend to overestimate the probability of events based on how easily we can remember, we hear about those events. This can happen in case of very favorable or unfavorable events, or things that we can quickly look up on the Internet. Finally, I discuss **anchoring bias**, which means we all have an estimated or just simply desired value of what the future price should be. For example, the ones resulting from a discounted cash flow (DCF) model in case of stock investors.

3.2 Market bubbles

Global Stock Portfolios have always been risky investments, and they are only offered for long-term investment horizon for people who are willing to take a significant amount of risk. The self-developed method aims to choose a robust allocation which is supposed to outperform S&P500. That is why I do not change components over our holding period. Although, I use the data of approximately 4.5 years, I have only 927 observation due to the different holidays worldwide.

However, there might be times when the whole market starts to fall, and it is better to close all our opened (long) positions, and maybe start to consider opening short positions. These times are called regime switches, which are likely to occur when there is a so-called bubble. In the following sub-chapter, I am going to introduce bubble theory and its practical applications which are suitable for describing the past behavior of markets as well as for real-time monitoring. To monitor markets, I use daily data of holding period mentioned in sub-chapter 2.5, because the key word in case of monitoring portfolio assets is real-time.

Firstly, a bubble is defined as the significant difference between the market value and the intrinsic value of an asset without supporting fundamentals. None of them are the same but they share similar characteristics, resulting from previously mentioned and other biases. There have been several books discussing this topic. I introduce the history of the biggest market bubbles since the 20th century, which is based on Harold L. Vogel's book: Financial Market Bubbles and Crashes (2018). The theoretical and practical introduction incorporates newer methods based on my arbitrary choice. These methods incorporate tools of unconventional risk management methods which are originated from different sciences like biology and physics.

3.2.1 History

The Great Depression was the first biggest bubble in the 20th century, when the Dow Jones Industrial Average (DJIA) was 381$ in the beginning of September in 1929 and fell under 50$ by the year of 1932. The year of 1929 is also known as the end of Roaring Twenties as the growth of DJIA significantly exceeded the level that would have been reasonable based on the growth of US GNP. (Vogel, 2018, Fig. 2.2)

The following great bubble in the history was the one in Japan, when the Japanese stock market crashed at the end of 1989. Based on the data from Noguchi (1994) and the Economic Planning Agency (1992), the Japanese stock market value was 650% from the beginning of 1981, the land price was 407%, while the GDP was only 154% of the starting value in the beginning of 1981. In this case the market also experienced a strong correction and the following years after 1989 are often called as *Ushinawareta Jūnen,* the Lost Decade. (Vogel, 2018)

One of the most known bubbles was the one in the US market, in October 19, 1987 which is known as the Black Monday. It is often cited as the first bubble of the modern history because it was the era when PCs appeared in trading, it contributed to record high trading volume and amplified by a type of portfolio insurance which was very popular at that time. Investors who were heavily invested in the quickly growing stock market tried to hedge themselves by selling short stock market index futures. In the following years the NASDAQ 100 which is known to be a "tech-heavy" index was 1200% of the starting value in 1995, and then it fell 200% of what the starting value was in 1995. This one was also caused by immense investments in tech companies that resulted unsustainable exponential growth. (Vogel, 2018)

Shortly after the recovery from the Dotcom bubble the US market experienced another market crash which had one of the highest social expenses amongst preceding crises. The US Congress passed the *American Dream Downpayment Act* in 2003 which was supposed to create the opportunity for people with fewer means, the "far-from-creditworthy" borrowers to buy their own houses. It was followed by the production of structured and derivative products like collateralized debt obligations (CDOs) and credit default swap (CDS) are probably the most

infamous of them. Collateralizations made these products – to seem – to be less risky than the underlying products in them. How is it possible?

These results were based on Value-at-Risk (VaR) calculations which had several unrealistic assumptions like normality of returns, constant correlation between assets, constant volatility, and small covariance of classes formed according to their credit risk, namely tranches. As a result of low-quality borrowers, payment delinquencies hit record high levels. Consequently, the stock market crashed, and the US Dollar weakened significantly. However, this crisis was somewhat different as it started in the housing market, so the duration, the consequences were much longer lasting than previous ones. (Vogel, 2018)

The following period from 2009 to 2017 is called Yield-chasing as systematically important firms like Lehman Brothers and AIG went bankrupt the Federal Reserve had to take steps to handle severe liquidity and social problem, as well. At first, they reduced short-term interest rates to near zero. Afterwards, they followed the so-called quantitative easing (QE) policy by purchasing long-term bonds to rectify liquidity problems. These QE policies had "inflated projected cash flow and anesthetized perceptions of risk." (Grant, 2016) They proved counterproductive as they are likely to increase the effect and unexpectedness of an imminent crash, by injecting unprecedented amounts of money into the economy without solving the underlying problem(s). (Vogel, 2018)

History tells us that bubbles tend to form when there is a high liquidity in the markets, since market participants tend to underestimate risk during those times. In summary, according to Harold L. Vogel, 2018, bubbles are not rare – as we can see they are getting more and more frequent – and not specified to time, place, or any asset class.

3.2.2 Methods

In the previous part we could see the history of the most influential bubbles of the last 100 years. Now I would like to focus on the current market situation and analyze the chosen markets searching for structural breaks, outliers, and critical dates – when a regime switch is due – during our holding period.

I start with Outlier analytics and Bayesian Change Point Detection. Afterwards, I implement the Generalized Supremum Augmented Dickey-Fuller (GSADF) test. Additionally, I introduce Log-Periodic Power Law Singularity (LPPLS) model which is based on dynamic systems. Furthermore, I analyze the non-stationarity of the markets with univariate Wavelet transform to find the most exuberant periods. Additionally, I also use Wavelets to analyze the co-movement of the markets with the US stock market as it is the most influential one and more than two thirds of our assets are allocated in US large cap companies.

Outliers in the PC space

The most basic method to monitor the stability of the markets is to search for the growing number and extent of outliers. I use the method which was presented by Filzmoser and Hron (2007), who had developed a robust version of the Mahalanobis distance to analyze extreme values in the principal component (PC) space. Principal components are a set of variables that maximize the variance in a given direction, which is often used as a measure of risk. They are in decreasing order based on how much of the total variance, they can explain. Extreme values have a significant impact on the total variance, so it is reasonable to analyze "outlyingness" of the observations in the PC space. (Würtz et al., 2011) The Mahalanobis distance looks like the following:

$$MD(x_i) = [(x_i - T)' C^{-1} (x_i - T)]^{1/2} \tag{3.1}$$

where x_i is an observation, T is the location estimator and C is the covariance estimator. Selecting T and C are the most essential to reach robustness, for which the method was presented by Filmoser and Hron, (2007).

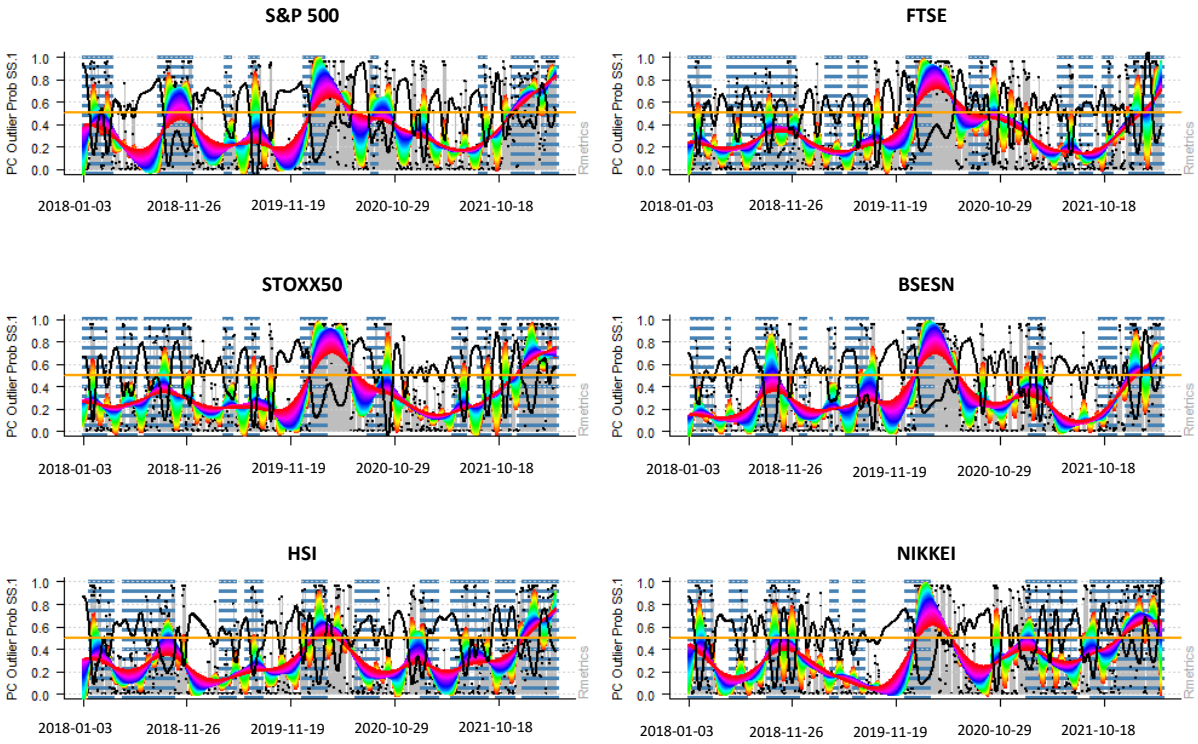


Figure 3.1: Outlier analytics in the PC space

Source: own editing, based on Würtz et al (2011)

In Figure 3.1, we can see the Outlier analytics in the PC space plots of all the markets that we initially started with. The rainbow-colored series are smoothing splines with smoothing parameters ranging from 0.3 to 0.8 by 0.01. The yellow line simply indicates the 0.5 posterior probability, and the black line is the difference between the strongest and the weakest smoothing splines, which depicts the stability of the markets. (Würtz et al., 2011) As we can see the period from the 1st of January 2018 to 1st of June 2022, was very turbulent in all markets. In my opinion, nowadays we must deal with exuberant periods in the financial markets more and more frequently, thus simply analyzing outliers themselves does not help us much in detecting market bubbles.

Bayesian Change Point Detection

The Bayesian Change Point Detection methods searches for change points in the mean and variance of the time series. It is originated from Barry and Hartigan's (1992) Bayesian Product Partition Model and published the model for Bayesian Change Point analysis in 1993. The introduction is also based on Erdman and Emerson 2008. In their model ρ is an unknown partition. It assumes independent observations as $N(\mu_i, \sigma^2)$, and the probability of a change point is p at time (a.k.a. position) i. However, they weakened the original assumption of independent observations in their research work in 2013. Since with this original assumption their model was likely not to detect change points (in mean) if they were present only for a short period, and the mean of the block ending at j, just as it was the case during our holding period. They developed the following independent priors for $\mu_0, p, \sigma^2$and $w = \sigma^2/(\sigma^2 + \sigma_0^2)$, where w is the "ratio of signal to error variance". (Erdman and Emerson, 2008)

$$f(\mu_0) = 1, -\infty \leq \mu_0 \leq \infty, \tag{3.2}$$

$$f(\sigma^2) = \frac{1}{\sigma^2}, 0 \leq \sigma^2 \leq \infty, \tag{3.3}$$

$$f(p) = \frac{1}{p_0}, 0 \leq p \leq p_0, \tag{3.4}$$

$$f(w) = \frac{1}{w_0}, 0 \leq w \leq w_0, \tag{3.5}$$

and finally, we can identify partitions as

$$f(\rho) = \frac{1}{p_0}\left[\int_0^{p_0} p^{b-1}(1-p)^{n-b}dp\right] \tag{3.6}$$

where n is the sample size and b is the number of blocks in the partition, $\rho = (U_{1,}U_2, \ldots U_n)$, and U_i is initially 0 and $U_i = 1$ means a change point at time $i + 1$, from the conditional distribution of the Markov Chain Monte Carlo (MCMC) implementation. The probability of a change point is, where **X** is the data: (Erdman and Emerson, 2008)

$$\frac{p_i}{1-p_i} = \frac{P(U_i = 1|\boldsymbol{X}, U_j, j \neq i)}{P(U_i = 0|\boldsymbol{X}, U_j, j \neq i)} \tag{3.7}$$

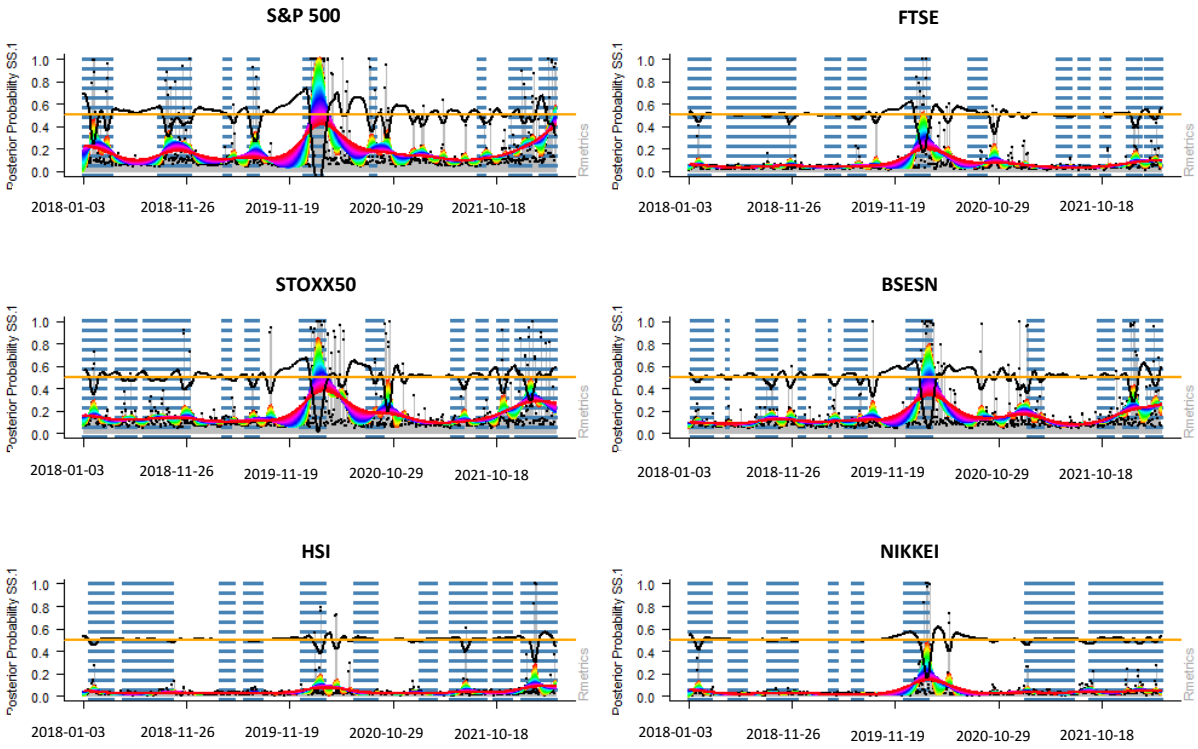


Figure 3.2: Bayesian Change Point analytics of the markets

Source: own editing, based on Würtz et al (2011)

In Figure 10, the interpretation of the plot elements is analogous to the interpretation on Figure 9. We can see that Bayesian Change Point analytics is a significant improvement compared to Outlier analytics. As can be seen the US, European and Indian markets are more sensitive and similar to each other. We can observe change points in European markets more frequently, but the US market is more sensitive than any other, thus it provides the best opportunities. The Japanese market also shares the characteristics of the abovementioned markets regarding the occurrence of change points; however, it seems to be less sensitive. Finally, we could see the advantageous position of the Chinese market with respect to volatility shocks arriving from other markets in Chapter 2, where it was mostly neutral or a net receiver other than a sharp peak. Now, the change point analytics confirms the continuity of this stability of the Chinese market in the holding period.

Generalized Supremum Augmented Dickey-Fuller (GSADF) test

The mathematical introduction of the method is heavily based on Vasilopoulos, Pavlidis and Martínez-García, (2020). It is originated from the standard ADF test, which is carried out on a full sample and our result is merely to keep or reject the null hypothesis. The standard ADF_0^1 test, with $r_1 = 0$ and $r_2 = 1$, the limiting distribution of the test to approximate the real distribution is:

$$\frac{\int_0^1 W dW}{\left(\int_0^1 W^2\right)^{1/2}} \tag{3.8}$$

where W denotes a Wiener process. The null hypothesis is that the time series is stationary, so its statistical properties do not change over time, whilst the alternative hypothesis refers to exuberant behavior.

As a result of the limitation of the standard ADF test, Phillips et al. (2011) presented another approach to use this test. They applied an expanding time window estimation of the standard ADF test on the sample where the starting point r_1 is fixed but the end point r_2 is expanding forward, and r_0 is the minimum window size. The supremum of the expanding time window estimates of the test statistics is called supremum ADF (SADF) test:

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{3.9}$$

with a limit distribution of

$$\sup_{r_2 \in [r_0, 1]} \frac{\int_0^{r_2} W dW}{\left(\int_0^{r_2} W^2\right)^{1/2}} \tag{3.10}$$

It provides us a more detailed picture about the extreme characteristics of the time series; however, it still gives us only limited information about the start of the bubble. As it is based on an expanding window, it is more and more affected by previous observations which can result false acceptance of the null hypothesis and a “too-late” bubble identification issue.

To overcome this issue Phillips et al. (2015a, b) further developed their method, to cover several subsamples with varying r_1 and r_2 parameters. This extension is called the generalized supremum augmented Dickey-Fuller test (GSADF), which is a significant improvement compared to its predecessors. Another uniqueness of the test is that it attempts to identify ending points of bubbles, too. The GSADF test looks like the following:

$$GSADF(r_0) = \sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{3.11}$$

with a limit distribution of

$$\sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r) dr [W(r_2) - W(r_1)]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r_2} W(r)^2 dr - \left[\int_{r_1}^{r_2} W(r) dr \right]^2 \right\}} \right\} \tag{3.12}$$

where r_w is the expanding window size, which is defined as $r_2 - r_1$. To calculate the minimum window size, I use the "rule of thumb" suggested by Phillips et al., where it is $r_0 = 0.01 + 1.8/\sqrt{T}$, where T is the sample size. The following plot shows the GSADF tests on markets where there is a change point with the given level of significance indicated on the plot.

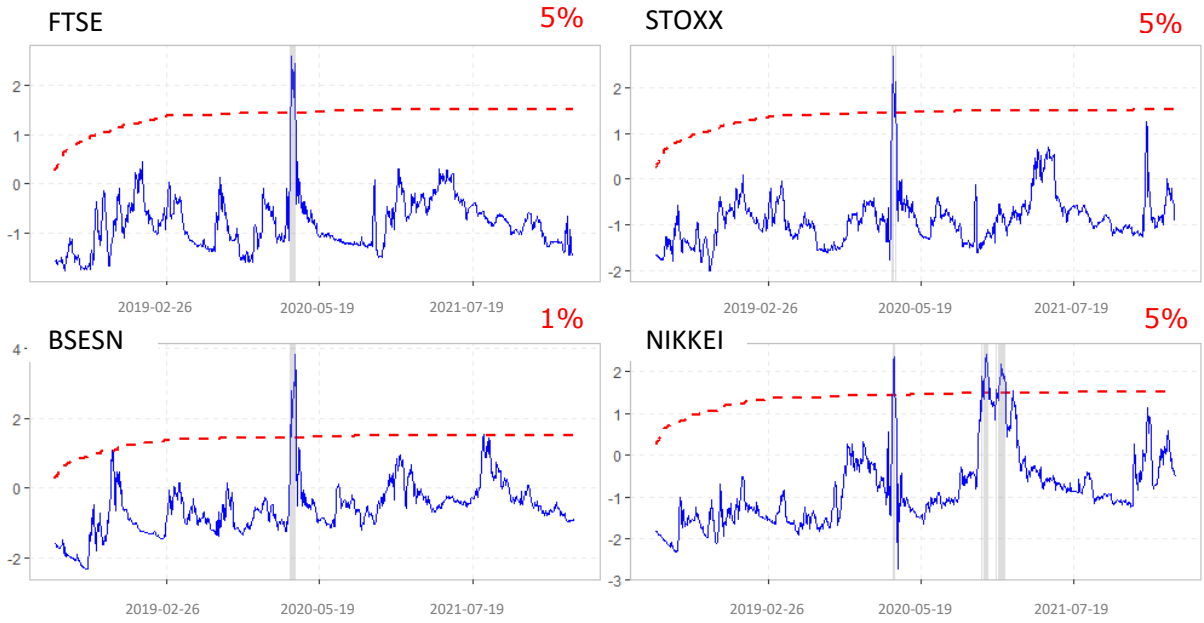


Figure 3.3: GSADF test results

Source: own editing, based on Vasilopoulos et al. (2020)

Only four markets are plotted because, in other markets we cannot reject the null hypothesis. The dashed red line shows the significance level indicated by the red numbers. As can be seen this method also focuses on the increase of volatility like the previous Bayesian Change Point analytics. It indicated a bubble in March 2020, which was followed by a quick and strong correction in all the markets, but it could not detect the start of the actual bubble at the end of 2021. In my opinion, it is still far from useful real-life application, at least in stock markets with high frequency data (e.g. daily).

As I see based on this – and previous methods – the problem is the attempt to find only the peak of a bubble, without providing any detail to the user about the characteristics and evolution of the bubble itself. It is a very sophisticated technique that uses a simple test statistic, which consequently results a simple "Yes" or "No"-like answer by "Rejecting" or "Retaining" the null hypothesis. We need more detailed information about how the bubble is forming up to the critical time. We need to be critical about these results. To do so, we need to be able to be critical about the results concerning such an important change, so to find a model which provides us enough details. Which means monitoring not just the markets but also the bubble-like characteristics.

Log-Periodic Power-Law Singularity (LPPLS) model

The first introduction of the basic Log-Periodic Power-Law (LPPL) model was introduced by Johansen, Sornette and Ledoit (1999). However, over the years several developments have been made on the model, some of them for computational purposes, some of them for accuracy. This model gives us detailed information about the evolution of the bubble as well as an estimation for the critical time / start of the crash. According to the rational bubble theory the price of an asset can be written as (Demos, 2017) (Blanchard and Watson, 1982)

$$P(t) = P(t)_{fundamental} + P(t)_{bubble} \tag{3.13}$$

The foundation of the LPPLS model is a stochastic jump-diffusion model, which can be seen often in finance, in the option pricing theory. (Ide and Sornette, 2002) Where the dynamics of the logarithm of the asset price looks like the following:

$$\frac{dp}{p} = \mu(t)dt + \sigma(t)dW - \kappa dj \tag{3.14}$$

where $\mu(t)$ is the expected return, $\sigma(t)$ means the volatility with dW, the infinitesimal increment of the standard Wiener process. Additionally, dt is the difference between the start date (t_1) and the end date (t_2) in decimal date format, dj is the discontinuous jump, where $j = n$ before the crash, critical time t_c, and $j = n + 1$ after the crash. The parameter κ denotes the amplitude of the projected crash. (Gerlach, Demos and Sornette, 2019) Furthermore, they attempt to quantify the aggregated effect of noise trader who further contribute to destabilization of the "complex system". This measure is called the crash hazard rate:

$$h(t) = \alpha(t_c - t)^{m-1}(1 + \beta\cos(\omega\, ln(t_c - t) - \phi')) \tag{3.15}$$

where α, β, ω and ϕ' are parameters and $(t_c - t)^{m-1}$ denotes the power-law part. In 2003 Sornette and Zhou added $\cos(\omega ln(t_c - t) - \phi')$ which proved to be more efficient than log-periodicity alone. While $1 + \beta\cos(\omega ln(t_c - t) - \phi')$ means the oscillation caused by panic, resulting in variations in the bubble-like behavior and interrupting its way. As a combination

of the log-price dynamics and the crash hazard rate dynamics we obtain the Log-Periodic Power-Law Singularity (LPPLS) model, with no arbitrage condition and the precondition that a crash has not occurred yet $dj = 0$, as the following:

$$\begin{aligned} LPPLS(t) &\equiv \mathbb{E}[\ln p(t)] \\ &= A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega\, ln(t_c - t) - \phi') \end{aligned} \tag{3.16}$$

where $B = -\kappa\alpha/m$ and $C = -\kappa\alpha\beta/\sqrt{m^2 + m^2}$, A means what the log-price would be if the bubble last until the projected t_c, B is the power-law acceleration. If $B < 0$, it refers to upward movement of the log price – "positive bubble" – while values $B > 0$ refers to downward movement of the log price, thus indicating a "negative bubble". (Gerlach, 2021) Furthermore, C means the amplitude of oscillations around the exponential growth, and t means any time before the critical time. The parameter m is simply the exponent of the power-law, ω is the frequency of oscillations and ϕ is the time scale of the oscillations. However, the optimization procedure is very challenging in case of the original function in longer time horizons, with respect to its quasi-periodic structure. In 2013, Filimonov and Sornette presented a better approach which shows computational efficiency and improved accuracy over the original model. They divided the most challenging part C into two sections:

$$C_1 = C\cos\phi\,, \qquad C_2 = C\sin\phi \tag{3.17}$$

The new formula looks like the following:

$$LPPLS(t) := \mathrm{E}[\ln P(t)] = A + B(f) + C_1(g) + C_2(h) \tag{3.18}$$

The $P(t)$ is the vector of the logarithm of the price and (Demos, 2017)

$$f \equiv (t_c - t)^m \tag{3.19}$$

$$g \equiv (t_c - t)^m \cos(\omega\, ln(t_c - t)) \tag{3.20}$$

$$h \equiv (t_c - t)^m \sin(\omega\, ln(t_c - t)) \tag{3.21}$$

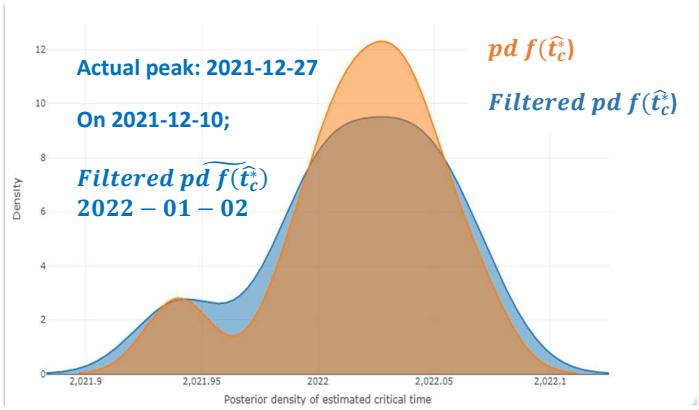
Despite this improvement the LPPLS model still suffered from calibration issues, false bubble identification. To overcome this issue there have been several research works concerning the evolution of the parameters during significant historical bubbles. They introduce so-called filtering conditions on some parameters of the model and on the acceptance of predicted critical times. It means we only accept a fit if these conditions are met.

Filtering conditions

The introduction of filtering conditions is based on Sornette and Filimonov, 2013 and Demos, 2017. The exponent m should be between 1 and 0 according to Sornette and Filimonov, 2013, however based on Demos, 2017, I also use stricter conditions and I only qualify a fit where $0.1 < m < 0.9$. The following is ω which is called angular log-periodic oscillation, and it should be between 6 and 13. Too big values – too fast – would result fitting the model to the random component of the data, while too small – too slow – values would result fitting the data to the trend, which would be a significant shortcoming of a model that attempts to predict the end of the trend. (Sornette and Filimonov, 2013) Furthermore the search interval for the critical time should be $[t_2 - 0.05dt,\ t_2 + 0.1dt]$. Finally, there are additional conditions to accept the estimated critical time t_c parameter. The "Damping" condition is conditioned as $D > 1$, which is $D := \frac{m|B|}{\omega|C|}$, where $C = \sqrt{C_1^2 + C_2^2}$ from the new model. This condition ensures that the magnitude (C) and frequency (ω) oscillation should not be too large compared to the acceleration of the price. The next to validate our estimations for t_c we must pay attention to the number of oscillations must be $O > 2.5$, where $O := \frac{\omega}{2\pi} \ln\left(\frac{t_c - t_1}{t_c - t_2}\right)$. It is needed to differentiate log-periodicity from pure noise. An additional condition is that the residuals should follow an autoregressive process $r \sim AR(1)$, which can be tested by ADF or Phillips-Perron test, but I do not use this filtering condition with respect to the weaknesses of these types of tests, as mentioned earlier.

In the following section I present the result of the fitted LPPLS model to the S&P500 index from the end of the market crash in March 2020 to 10th December 2021. The reason to use this time frame is because in case of LPPLS model there is no universal sample size to analyze. It is suggested to fit it to the period which shows exponential growth. I only present it to the S&P500 index because during our holding period it is the one where we can observe exponential growth. In this case the actual peak was 27th December 2021, and I would like to

see the density of the rolling time window estimates of t_c. This way I have 437 observations and I chose a 325 rolling time window to the estimations. It is important because we might experience false convergence if the actual t_c is far from our final observation t_2, and when $t_c - t_2$ is proportionally big compared to $t_2 - t_1$ it amplifies false convergence. (Demos, 2017) The following plot shows the density of estimated critical time parameters, both filtered and unfiltered ones, and the filtering conditions.



$$O := \frac{\omega}{2\pi} \ln\left(\frac{t_c - t_1}{t_c - t_2}\right) > 2.5$$

$$D := \frac{m|B|}{\omega|C|} > 1$$

Figure 3.4: Posterior density of estimated critical times and filtering conditions

Source: own editing

As can be seen, the posterior density of estimated critical times is close to the actual peak, however the unfiltered version indicates the end of the bubble close to the actual starting date. Each version depicts the immense growth of the posterior density as we are approaching the actual peak. However, by applying filtering conditions, we can see that it is neither the actual nor the end of the bubble just the start of an even bigger one. To sum up, the filtered version provides us more consistent results which also incorporate information about the evolution of the bubble. With respect to the previous sudden market loss in March 2020 the indicated only two values for the t_c parameter when we tried to predict it approximately 2.5-3 week ahead. There is no plot about them since there would be no point in depicting the density of only two values.

3.3 Co-movement of global markets and the US economy

After we successfully closed all our positions and retained most of the wealth produced by our asset allocation methodology the next important step is to determine when the effect of a financial crash dies away. I would like to find out when markets return to their "general" structure of co-movement after the crash. To do this, the co-movement can be analyzed by the Continuous Wavelet Transforms (CWT) of the time series. It has several different versions; in this thesis the following "mother" Morlet wavelet is used:

$$\psi(t) = \pi^{-1/4} e^{i\omega t} e^{-t^2/2} \tag{3.22}$$

where ω is the angular frequency, which means the rotation in radians per time unit. I use the most common value (6) in the literature as it provides beneficial properties. The Wavelet transform of a time series x_t looks like the following:

$$W(\tau, s) = \sum_t x_t \frac{1}{\sqrt{s}} \psi^* \left(\frac{t - \tau}{s} \right) \tag{3.23}$$

where τ is the time shift parameter, and s is the shift in scale. In its initial version Wavelet transform was "biased", which means it used to tend to underestimate higher frequencies (short periods), to overcome this issue Liu et al., in 2007, presented an "unbiased" version.

The bivariate extension measures so-called Wavelet Coherence to identify co-movement of the continuous wavelet transforms of the time series, like the following:

$$R_{x,y}(\tau, s) = \frac{\left| S\left(W_{x,y}(\tau, s) \right) \right|}{\left(S[|W_x(\tau, s)|^2] S\left[\left| W_y(\tau, s) \right|^2 \right] \right)^{1/2}} \tag{3.24}$$

Where S is a smoothing window in time and frequency space, I use Hanning window as it has some beneficial properties compared to other possibilities. However, in the literature there is no consensus which one to use. Although higher values indicate stronger co-movement, smaller values indicate weaker co-movement the color of the power spectrum does not indicate

if the variables move in the same or the opposite direction. To obtain this information we need to measure the phase difference in the following way:

$$\Phi_{x,y}(\tau, s) = arctan\left(\frac{\Im\left(S\left(W_{x,y}(\tau, s)\right)\right)}{\Re\left(S\left(W_{x,y}(\tau, s)\right)\right)}\right) \tag{3.25}$$

The value of $\Phi_{x,y}$ can be $-\pi \leq \Phi_{x,y} \leq \pi$. The following plot shows the visual representation of it, where *in phase* means positive connection and *out of phase* means negative connection.

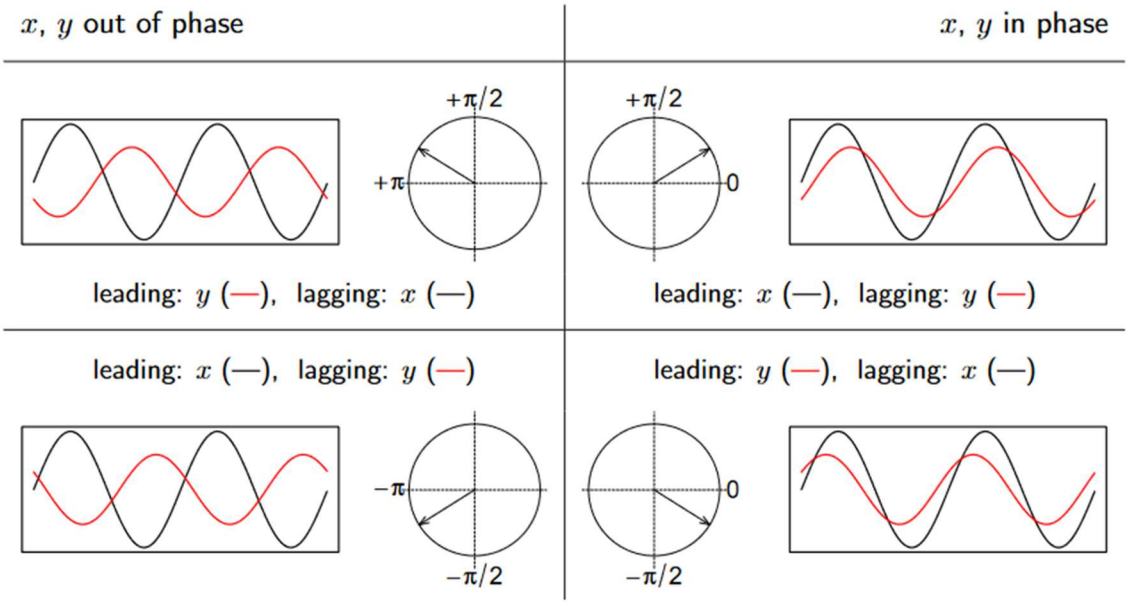


Figure 3.5: Interpretation of phase differences

Source: Schmidbauer and Roesch, 2018

The following plot shows the co-movement of global market with the US economy. I focus on this because it is the most influential one, and more than two thirds of our wealth is invested in different US large cap companies. The arrows indicate phase differences as shown above and they are only plotted on encircled areas which means 95% confidence level. The S&P 500 index is always time series x and one of the global market indices are time series y. When the arrows are completely vertical or horizontal then there is no "leading-lagging" relationship between the time series. The white shaded parts indicate the cone of influence.

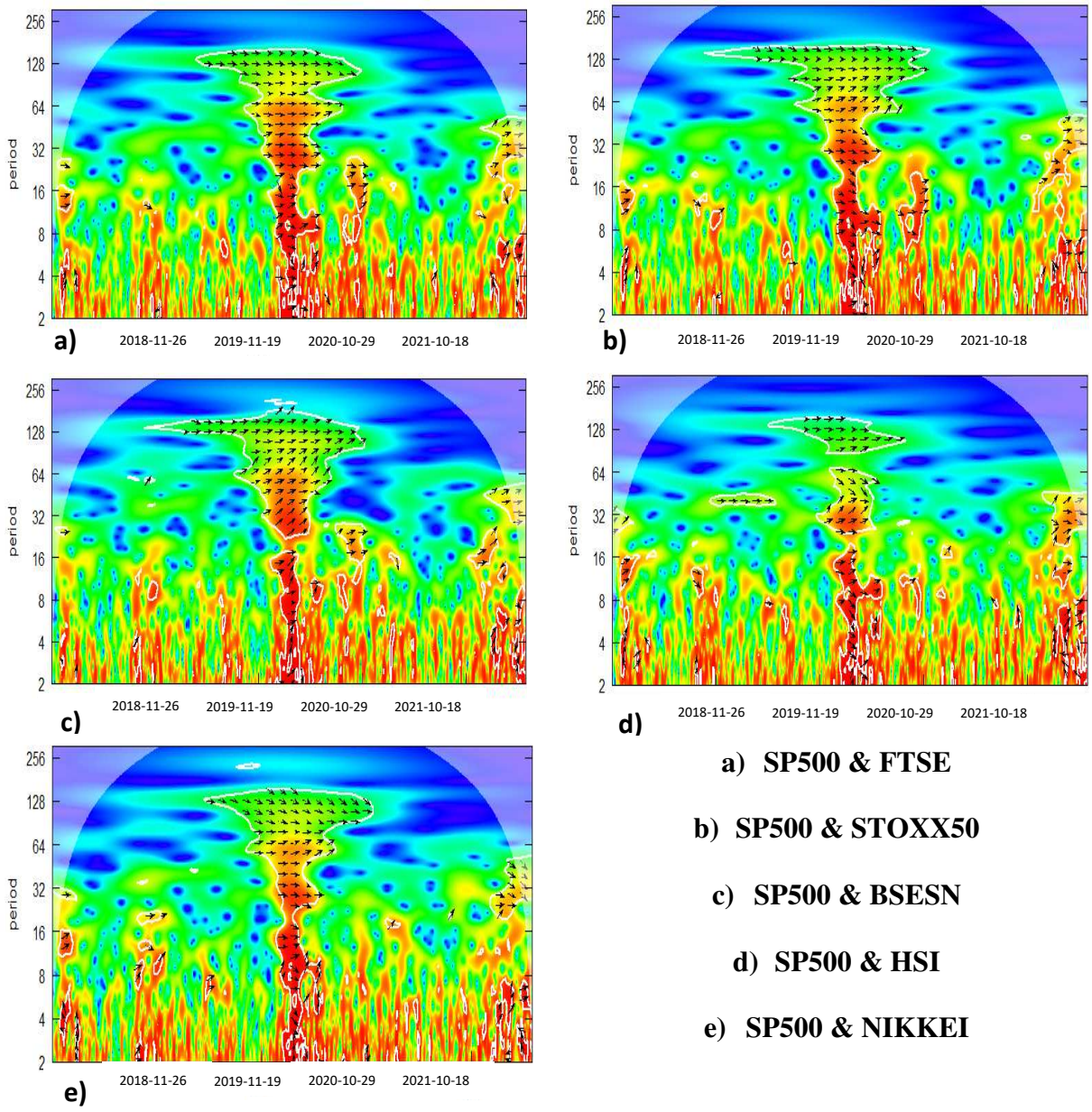


Figure 3.6: Wavelet coherence of the S&P500 and global market indices

Source: own editing

We can see that in dollar terms all indices show similar co-movements with the S&P 500 Index. It confirms our exit strategy, that we close our positions in times of regime-switches because there is no point in trying to time the market every time, we experience increased volatility. However, this method cannot differentiate regime-switches from sudden crashes followed by quick corrections. We can see no obvious leading-lagging relationship in the long term. However, we should be critical of the "very beginning" and the "very end" at higher frequencies, as indicated by the cone of influence, as relationships and information concerning co-movements are the most insecure at these periods.

Chapter 4

Conclusions

In this thesis I had three main focuses. First, I intended to get a general overview of the markets that I am interested in. Second, I wanted to find the possibly best performing stocks from their stock market indices' constituents. Third, I endeavored to present the current best-practice to identify regime-switches in financial markets.

At first, it showed that stock market characteristics are far from being normally distributed and they show different correlations across different frequencies and in different times. As we are dealing with a global portfolio it is crucial to know their possible reaction to extreme events. To examine this, I presented a method based on Diebold and Yilmaz (2015). It showed that Eastern markets are more stable in general, especially India. It is often close to neutral or a net receiver. Amongst USA, UK and Europe, Europe proved to be the most sensitive to the aftermaths of the 2007-2009 crisis, followed by the UK and then US. The Chinese market gave very modest reactions which is in accordance with. Afterwards the Minimum CVaR Concentration portfolio weights served as guidelines to further considerations concerning the allocation. However, the presented selection methods provide us the above-mentioned robust allocation which could outperform the S&P 500 index, did not find worthy investments in some markets of our initial interest. As a result of it, I did some changes based on my own selection criteria in the market weights in accordance with the number of worthy investments found in different markets and their ability to beat the Index.

Once my model successfully outperformed the S&P 500 index, what my focus was is to prevent that significant and long-lasting losses. These points are called regime-switches and as we could see by simply observing outliers, we cannot detect them. The Bayesian change point detection method is simple to implement and understand; and can outperform more complex methods like the GSADF model.

However, when it detects change a point it might be too late to act resulting from its nature. The GSADF model did not perform well in this case as it lacks consistency, it had sharp peaks in the value of the test statistics and also mistakes sudden crashes for true . Furthermore, I implemented the Log-Periodic Power-Law Singularity model which is a relatively new and promising tool in the field of bubble theory. I followed the filtering conditions introduced by Sornette and Filimonov, 2013 and the somewhat stricter one introduced by Demos, 2017 with respect to the exponent. This model truly performed better than others, as it provides information about the evolution of the bubble as well. Although it did not indicate the sudden crash in March 2020, I consider it as a proof of the model's usefulness since that crash was not a bubble since there was a sudden correction. Filtering conditions also proved to be helpful to improve the consistency of the model concerning the critical time. Finally, I implemented the Wavelet coherence analysis of the global markets and the US economy. They also indicated the start of times of exuberant periods in other markets as well, that I identify as increased Wavelet power in longer periods but did not show obvious "leading-lagging" relationship. However, the role of wavelets is also rather confirmatory and not a tool of forecasting.

The final allocation resulted the following characteristics in comparison with the S&P 500 index. The managed version's close is based on the median of the filtered posterior density of estimated critical times. On 10th December 2021, the above-mentioned value was 2nd January 2022. Assuming some time is necessary to close all the positions, and these cannot be executed on the best price I present the values on 1st February 2022.

	S&P 500	MYP - USD Adj.	Holding period in years
Avg. Annual Ret. Unmanaged (2022.06.01)	6.86%	15.10%	4.42
Avg. Annual Ret. Managed (2022.02.01)	14.38%	29.91%	3.92

Table 4.1: Performance of managed and unmanaged portfolio in comparison with the S&P 500 index

Source: own editing

Bibliography

[1] Ardia D., Boudt K., Carl P., Mullen K., and Peterson B. 2011b Differential Evolution (DEoptim) for non-convex portfolio optimization. The R Journal 3(1), 27–34.

[2] Barry, D., & Hartigan, J. A. (1992). Product Partition Models for Change Point Problems. The Annals of Statistics, 20(1), 260–279.

[3] Barry, D., & Hartigan, J. A. (1993). A Bayesian Analysis for Change Point Problems. Journal of the American Statistical Association, 88(421), 309–319

[4] Bernhard Pfaff, Financial Risk Modelling and Portfolio Optimization with R (2nd ed.), 2016. Wiley.

[5] Blanchard, Olivier J. and Mark W. Watson. Bubbles, Rational Expectations and Financial Markets." Crises in the Economic and Financial Structure, Paul Wachtel, editor, pp. 295-316. Lexington, MA: D.C. Heathand Company, (1982).

[6] Boudt K., Carl P., and Peterson B. 2010 Portfolio optimization with CVaR budgets. Presentation at R/Finance Conference, Katholieke Universteit Leuven and Lessius, Chicago, IL.

[7] Boudt K., Carl P., and Peterson B. 2011 Asset allocation with conditional value-at-risk budgets.

[8] Boudt K., Carl P., and Peterson B.: Journal of Risk, March 2013 Journal of Risk 15(3):39-68, Asset allocation with conditional value-at-risk budgets

[9] Choueifaty Y., Froidure T., and Reynier J. 2011 Properties of the most diversified portfolio. Working paper, TOBAM, Paris.

[10] Choueifaty, Y. and Coignard, Y., 2008 Toward Maximum Diversification. The Journal of Portfolio Management, 35, 40-51.

[11] Demos GDL, Dissertation. ETH Zurich NO. 24574, Information Geometry and the Dynamic Detection of Financial Bubbles and Crashes (2017)

[12] Denault, M. Journal of Risk, September 2011, Coherent allocation of risk capital

[13] Diebold, F. X. and Yilmaz, K. (2015), Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring, Oxford University Press, April 2015

[14] Diebold, F. X., Yilmaz, K. (2012) Better to Give Than to Receive: Predictive Directional Measurement of Volatility Spillovers

[15] Diethelm Würtz, Haiko Bailer, Yohan Chalabi, Fiona Grimson, Tobias Setz: Long Term Statistical Analysis of US Asset Classes, 2011

[16] Enrique Martinez-Garcia & Efthymios Pavlidis & Kostas Vasilopoulos, 2020. "exuber: Recursive Right-Tailed Unit Root Testing with R," Globalization Institute Working Papers 383, Federal Reserve Bank of Dallas, revised 19 Oct 2021.

[17] Erdman C, Emerson JW. A fast Bayesian change point analysis for the segmentation of microarray data. Bioinformatics. 2008 Oct 1;24(19):2143-8.

[18] Filimonov, V. and D. Sornette . 2013. A stable and robust calibration scheme of the log-periodic power law model. Physica A: Statistical Mechanics and its Applications, 392(17):3698–3707.

[19] Filzmoser, P., Hron, K., Outlier Detection for Compositional Data Using Robust Methods, April 2007, Mathematical Geosciences 40(3):233-248

[20] Francis Haeuck In, Sangbae Kim: An Introduction to Wavelet Theory in Finance: A Wavelet Multiscale Approach, 2013

[21] Gary P. Brinson, L. Randolph Hood, Gilbert L. Beebower: Financial Analysts Journal Vol. 42, No. 4 (Jul. - Aug., 1986), pp. 39-44, Determinants of Portfolio Performance

[22] Gerlach J C, Demos G, Sornette D.: Dissection of Bitcoin's multiscale bubble history from January 2012 to February 2018, (2019)

[23] Gerlach, Jan-Christian, Doctoral Thesis: Modelling of Speculative Bubbles and Forecasting of Market Crashes, (2021)

[24] Grant, J. (2016, September 10) Hostage to a Bull Market, *Wall Street Journal*

[25] Greg Gliner, Bloomberg Financial Series: Global Macro Trading: Profiting in a New World Economy, 2014

[26] Harald Schmidbauer, Angi Roesch: WaveletComp 1.1: A guided tour through the R package (2018)

[27] Harold L. Vogel: Financial Market Bubbles and Crashes, Second Edition: Features, Causes, and Effects, 2018

[28] Harry Markowitz, The Journal of Finance, Volume 7, Issue 1, pp. 77-91, Portfolio Selection, 1952

[29] Ide, K. & Sornette, D., 2002. Oscillatory Finite-Time Singularities in Finance, Population and Rupture, Physica A, 307((1-2)), pp. 63-106.

[30] Jozef Baruník & Tomáš Křehlík, 2018. "Measuring the Frequency Dynamics of Financial Connectedness and Systemic Risk," Journal of Financial Econometrics, Society for Financial Econometrics, vol. 16(2), pages 271-296.

[31] Marta Bańbura, Domenico Giannone, Lucrezia Reichlin: Journal of Applied Econometrics: Volume25, Issue1, Special Issue: Model Uncertainty and Macroeconomics – Large Bayesian vector auto regressions, 2010

[32] Phillips PCB, Shi S, Yu J (2015a). "Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500." International Economic Review, 56(4), 1043–1078.

[33] Phillips PCB, Shi S, Yu J (2015b). "Testing for Multiple Bubbles: Limit Theory of Real-Time Detectors." International Economic Review, 56(4), 1079–1134.

[34] Phillips PCB, Wu Y, Yu J (2011). "Explosive Behavior in the 1990s NASDAQ: When Did Exuberance Escalate Asset Values?" International Economic Review, 52(1), 201–226.

[35] Victor DeMiguel, Lorenzo Garlappi, Raman Uppal: The Review of Financial Studies, Volume 22, Issue 5, May 2009, Pages 1915–1953, Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy

[36] Wei-Xing Zhou and Didier Sornette. of the 2000-2002 anti-bubble in the US S&P 500 index: Explanation of the hierarchy of 5 crashes and Prediction. Physica A: Statistical Mechanics and its Applications, 330:584–604, 2003.

[37] William Kinlaw, Mark P. Kritzman, David Turkington: A Practitioner's Guide to Asset Allocation (Wiley Finance) 1st Edition, 2017

[38] William Nicholson, David S. Matteson, Jacob Bien: VARX-L: Structured Regularization for Large Vector Autoregressions with Exogenous Variables, 2017b

[39] Xiaoyong Xiao & Jing Huang, 2018. "Dynamic Connectedness of International Crude Oil Prices: The Diebold–Yilmaz Approach," Sustainability, MDPI, vol. 10(9), pages 1-16, September.

[40] Yonggang Liu, X. San Liang, and Robert H. Weisberg: Rectification of the Bias in the Wavelet Power Spectrum, Journal of Atmospheric and Oceanic Technology, Volume 24: Issue 12, 2007