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Quantitative Methods for Pairs Trading Risk Management

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1 Introduction

One of the most important statistical applications in trading is algorithmic trading, which is the use of computer programs to execute trades automatically based on predetermined rules. These rules may be based on technical indicators, statistical analysis, or a combination of both. Algorithmic trading is also known as automated trading, black box trading, or algo-trading.

A key benefit of algorithmic trading is that it allows traders to execute trades at high speeds and with a high degree of accuracy. This is because the computer program follows a strict set of rules, which eliminates the possibility of human error. Additionally, algorithmic trading allows traders to back-test their strategies on historical data to see how they would have performed under different market conditions. There are several types of algorithmic trading strategies, including:

1. Arbitrage: This involves taking advantage of price differences between different exchanges or markets.

2. High-frequency trading: This involves executing trades at extremely high speeds using sophisticated algorithms and powerful computers.
3. Statistical arbitrage: This involves using statistical analysis to identify mispriced assets and executing trades to profit from the price difference.

Algorithmic trading carries certain risks, such as the risk of technology failures or the risk of being front run by other market participants. Despite these risks, algorithmic trading has become increasingly popular in recent years, particularly among professional traders and institutional investors.

Pairs trading is a statistical arbitrage algorithmic trading strategy that involves simultaneously buying and selling two highly correlated assets in order to profit from the difference in their prices. The goal of pairs trading is to identify two assets that have a high historical correlation and are currently trading at an unusual price relationship, and then take a long position in the undervalued asset and a short position in the overvalued asset. Pairs trading can be implemented using a variety of assets, including stocks, futures, and currencies. It is often used as a hedging strategy to reduce the overall risk of a portfolio.

There are several steps involved in implementing a pairs trading strategy:

1. Identify a pair of assets that have a high historical correlation.
2. Use statistical analysis to determine if the current price relationship between the assets is unusual compared to the historical relationship.
3. Take a long position in the undervalued asset and a short position in the overvalued asset.
4. Set stop-loss orders to limit potential losses.
5. Close the positions when the price relationship between the assets returns to its historical average or when the trader's predefined risk tolerance is reached.

Pairs trading can be a complex and risky strategy, and it requires a certain level of technical expertise. It is important for traders to carefully consider the risks and limitations of pairs trading before implementing it in their own strategies.

The critical questions of the pairs trading strategy are about the profitability of the strategy and statistical methods to improve profitability. Most papers assess the profitability of the pairs trading strategy using empirical data from the stock markets. However, there is less emphasis on the profitability analysis of the strategy in a simulated environment. Furthermore, although risks are measured in most papers by the standard deviation and the maximum drawdown, due to the limitations of empirical data, the risks are usually not analyzed separately. This research aims to fill this gap by providing a simulated stock market environment, where stock prices have varying levels of co-movement. In this simulated environment, the research evaluates the three most common pairs trading strategies: the simple distance method, the Engle-Granger method, and the Johansen method. The first hypothesis of this research is that the simple distance method is less effective than the Engle-Granger and Johansen methods (H1). The second hypothesis is that the four major risks that are analyzed in this research all impact the profitability of the pairs trading strategy negatively (H2). Lastly, the third hypothesis is that the most severe risk of the pairs trading strategy in terms of profitability is the lost co-movement in the trading period (H3).

2 Literature Review

Pairs trading strategy has by this time become a well-known statistical arbitrage trading strategy among professionals and academics. The strategy had become known after statisticians had begun to use it in the trading group of the prestigious Morgan Stanley in the 1980s.

There have been many approaches developed for executing the pairs trading strategy before. One of the simplest pairs trading strategies, later known as "the simple pairs trading strategy", had been evaluated in 2006 (Gatev et al), in which the strategy's key pillar is to select those pairs with the lowest spreads in normalized prices, and trade when the difference is higher than two standard deviations of the spread between the prices. They found that the average excess returns for the strategies in some pairs were as high as 12 percent.

Ten years later, the strategy had been once again re-evaluated (Do, 2010), and it was found that the profitability of the simple pairs trading strategy was decreasing, but the decline is only partly attributable to increased efficiency among hedge funds. Surprisingly, 70% of the decrease was due to worsening arbitrage risk in various portfolios. Do and Faff introduced an enhancement to the previous simple strategy. They have used the minimization of the sum of the squared deviation of the normalized prices and paired the financial instruments from the same predefined industry groups for homogeneity of the instruments because equity from different industry groups might correlate for a reasonable amount of time, but specific industry related shocks will impact stocks in different industries in a different way, so this homogeneity filtering accounts for the deficiency of analyses limited by historical data, which do not necessarily predict future prices.

A zero-crossing measure was also implemented to detect how frequently the deviation returned to the trend, which indeed provided better profitability for the pairs trading strategy, although the speed of conversion was not measured which could have added further value from a profitability perspective, as the number of reversions can be attained by having several trigger options, the higher mean reversion speed will enable investors to realize their profits in smaller time which lead to higher annualized returns, a highly relevant measure for trading performance.

Trading transaction costs were not considered in 2010, so they revisited the pairs trading strategy after two years again (Do, 2012) analyzing the profitability after several transaction costs: commissions; market impact; and short-selling costs. They found that the baseline simple pairs trading approach (Gatev et al, 2006) was not profitable after the transaction costs and only a small proportion of the pairs can remain profitable: those pairs that are chosen from the same industry standards. Although profitability is decreasing, they conclude that institutional investors, who can trade with low costs, can still profit from liquid mispriced stocks in the U.S. market.

After the analyses of Do and Faff, there were more sophisticated pairs trading strategies investigated and the profitability and effectiveness of the three main statistical methods were compared as part of broad research on pairs trading strategies (Rad et al, 2016). The previously detailed simple distance method has been compared to a more sophisticated cointegration strategy and the highly complex copula method. The cointegration strategy is very similar to the simple distance method, but instead of analyzing the spread between the normalized prices, the cointegration strategy examines the cointegration relation between the prices. There are two main methods of testing for co-integration: Johansen's vector error correction model (Johansen, 1991) and the two-step Augmented Dickey-Fuller (ADF) method of Engle and Granger (Engle, 1987). The Engle-Granger method begins with the execution of an ordinary least squares (OLS) estimation on the two normalized prices and ends with a unit root test for the OLS residuals as proof of the long-lasting relationship between the prices. The most complex strategy is the copula method, which models multivariate

joint distributions and captures the dependence structure between the prices. The copula method enables the quantitative trader to estimate the probability of certain outcomes, so that triggers can be quantified on a higher level maximizing expected profits. One problem of the copula method is that it is complex to execute and requires stronger IT resources than the other two strategies. Surprisingly, even though the copula method is the most sophisticated, they had shown that the Sharpe ratio was lowest for the copula method, but it had been decreasing for all three strategies since the 1990s. Interestingly, it was found that the pairs trading strategies are market neutral, so it can be desirable for investors to hedge against market downturns utilizing pairs trading strategies, which generate higher returns under crisis. Although the copula method seemed promising for the authors, it requires further development, especially in the scope of filtering for convergent pairs. Information technology costs have not been calculated into the transaction costs, which can be highly relevant for decision-makers.

The previously detailed cointegration strategy had been further developed into a partial cointegration strategy (Clegg, 2017). This new strategy lets the cointegrating residual contain a random walk with a mean-reverting component. The innovation of the research is not only that it involves a more sophisticated trading approach, but the testing methodology of the strategy. They tested the profitability of the strategy on empirical data and on a simulated environment too, which highly mirrored the US equity market based on correlation and other parameters. Interestingly, after accounting for transaction costs, the partial cointegration strategy outperforms other pairs trading strategies based on Sharpe and Calmar ratios. Like other pairs trading strategies, the profitability of the partial cointegration strategy had also been decreasing. The research has emphasized the importance of pairs trading strategy testing on a simulated environment because historic prices are only one realization, so it may be a reasonable goal to create simulations to assess the success of strategies through expected profits, which is a highly relevant measure for strategy comparison.

It might not be sufficient to analyze historic equity prices for assessing the effectiveness of a trading strategy, because historic prices do not always represent the required data for the expected value calculation of the trading profitability. There are many approaches available for data generating processes, which simulate the prices of stocks instead of analyzing historic values of stocks for a limited period. One of the most important features of price simulation is the possibility of testing the effectiveness of cointegration tests on these data-generating processes. The main approach has been to create autoregressive time series, as had been done in 1993 to test for the finite sample bias of the cointegration tests (Cheung, 1993), where potential biases were indeed highlighted. The presence of conditional heteroskedasticity had been also analyzed as a continuation (Lee, 1996), where they found that conditional heteroskedasticity does decrease the performance of the cointegration tests, but it does not have a high impact on the tests. The cointegration tests were again evaluated on finite samples after more than 10 years from the analyses of Lee and Cheung, where they concluded (Toda, 2009) that at least 300 observations are required for good performance of the cointegration tests. The cointegration tests assumed Gaussian residuals for the processes, so after the performance analysis of Toda the impact of the violation of this assumption was examined in more detail (Huang, 2010). They found that the Engle-Granger test performs significantly better when the Gaussian assumption is violated than the Johansen test, although the Johansen test outperforms the Engle-Granger test with the Gaussian assumptions. The main questions of the current strategies are how to find those pairs that have a strong trend with a fast mean-reversion so that profit is highest in a specific time frame - which is one of the most prioritized attributes of trading strategies - and how those pairs can be filtered out of the pair list that would diverge after a historic temporary convergence trend. One idea for finding long-term convergence is to group asset classes based on common attributes such as industry or sector groupings for example.

3 Methodology

In this section, the methodology of this research is explained. Firstly, the financial mathematical models that were utilized to simulate a portfolio of 15 stocks are articulated. Afterward, the three pair selection methods that are chosen are described: the simple distance method, the Engle-Granger cointegration method, and the Johansen cointegration method. There is also a brief assessment of how the pairs trading strategy is executed in this research with a special emphasis on the included parameters. Lastly, the multiple conducted stress tests and their rationales are explained.

Several Monte Carlo (MC) simulations are run in the research to understand the distribution of risks and returns associated with the pairs trading strategy. The MC framework allows for the presentation of investment and risk metrics at the median, 10th, and 90th percentile levels. The name of the method comes from the famous casino in Monte Carlo, which builds on the foundation that the house always wins, because of the central limit theorem and the law of large numbers. In this research, this means that the risk and investment metrics can be inspected with various probabilities, and the trading risks can be better understood and exhibited.

As a result, not only the median paths of the simulation are presented, based on the pairs trading investment values, but also the 10th and 90th percentile paths to display the potential risks of interpreting the outcome of either a single successful or unsuccessful pairs trading strategy. More details are provided in the results section.

3.1 Stock Price Simulation

The equity market simulation begins with a single stock simulation that follows a geometric Brownian motion (GBM), which has not only been widely used by quantitative finance researchers and practitioners but has also been proven to be valid across various industries and markets (Marathe and Ryan, 2005).

The below (Equation 1) stochastic differential equation (SDE) articulates the GBM.

$$\Delta S_t = \mu S_t \Delta t + \sigma S_t W_t \sqrt{\Delta t} \quad (1)$$

S_t denotes the price of the stock at time $t \in [0, T] \subset \mathbb{R}$ in years, where T is the total period including the formation period and the trading period in years. The annual return of the stock is represented by μ , which is the drift coefficient in the SDE. $\Delta t := \frac{1}{\tau}$ is the change in time in years, where τ is the time points within a year in the simulation, which determines the level of discreteness and continuousness as if τ is approaching ∞ , then the price series is continuous. Furthermore, σ is the volatility of the annual return of the stock, and lastly, W_t is a normally distributed white noise with a mean of zero and variance of 1.

It can be proven based on Ito's Lemma, that the solution to the previous equation (Equation 1) is the below formula (Equation 2):

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t \sqrt{t}\right]. \quad (2)$$

Consequently, the first stock's price is simulated based on the following SDE (Equation 3), in line with the literature (Reddy and Clinton, 2016):

$$S_{t+\Delta t} = S_t \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma W_t \sqrt{\Delta t}\right] \quad (3)$$

To simulate cointegrated stock prices, the following SDEs are defined, which were obtained from Supakorn Mudchanatongsuk, James A. Primbs, and Wilfred Wong (2008). They assumed that the spread between the price of the first stock based on Equation 3 and the second stock that is cointegrated with the first stock follows the below Ornstein-Uhlenbeck process:

$$dX_t = k(\theta - X_t)dt + \eta dW_t. \quad (4)$$

As a result, the above process is estimated with the below equation:

$$\Delta X_t = k(\theta - X_t)\Delta t + \eta \Delta W_t. \quad (5)$$

Where k is the mean-reversion speed, θ is the equilibrium level of the spread between the first and the second stocks. η denotes the volatility factor of the spread. It is emphasized here that the Δ represents the discrete property of the process. Due to the fact that there are not only two stock price SDE-s simulated but a whole portfolio of stocks for the market, there had to be a methodology to simulate the other SDE-s. The same approach is followed for further stock prices as described in Equation 5, by simulating another Ornstein-Uhlenbeck spread process. As a general rule, the cointegrated processes have the below formula

$$S_{t,i} = S_{t,i-1} + X_{t,i} \quad (6)$$

Where $i \in [2, P] \cap \mathbb{N}$, because the case where $i = 1$ is the GBM process (Equation 3) and $X_{t,i}$ follows the process of X_t as exhibited in Equation 5. On the other hand, in order to factor in the changes in price dynamics in the market, the stock price SDE-s have been further modified by introducing another GBM process.

$$S_{t,i} = \rho(S_{t,i-1} + X_{t,i}) + (1 - \rho)\hat{S}_{t,i-1} \quad (7)$$

Where $\hat{S}_{t,i}$ follows the below GBM process with the white noise $\hat{W}_{t,i}$ process which is independent for $\forall t \in [0, T] \subset \mathbb{R}$ and $\forall i \in [2, P] \cap \mathbb{N}$. $P \subset \mathbb{N}$ is the number of stocks in the portfolio.

$$\hat{S}_{t+\Delta t,i} = \hat{S}_{t,i} \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \hat{W}_{t,i} \sqrt{\Delta t}\right] \quad (8)$$

The cointegration strength parameter is captured by $\rho \in [0, 1] \subset \mathbb{R}$. As it can be seen in Equation 7, the way the price process is formulated allows for a more sophisticated specification through the ρ parameter, as if the value of ρ is 0, then the price process is a GBM process, and when ρ is 1, then the price process $S_{t,i}$ is cointegrated to $S_{t,i-1}$. By setting the parameter ρ , it is possible to set the level of cointegration between two stock prices, which is highly important to be able to analyze pairs trading profitability under certain scenarios of cointegration strength.

3.2 Pair Selection

The methodology of pair selection is a crucial element of the pairs trading strategy in the formation period, as during the trading period, the pair is usually unchanged. However, there could be some methods introduced to indicate serious deviation from the cointegration in the trading period in contrast to the formation period, but nevertheless, this aspect plays a major role in the effectiveness of the pairs trading strategy. Consequently, there is a large emphasis on the different methods, to eliminate the risk of choosing the wrong pairs selection

method. In this research, three methods of pair selection have been applied to analyze the difference in performance between pairs trading investment strategies utilizing these pair selection methods: the simple distance method, the Engle-Granger cointegration method, and the Johansen cointegration method.

The simple distance and Engle-Granger pairs trading selection methods begin with a regression in which the dependent variable is the price series of one stock in the formation period and the independent variable is the other stock from the pair, allowing also for a constant in the regression. The regression was run using the stats package of R using the `lm` function to fit the model with OLS estimation. Equation 9 exhibits the explained regression between price series $S_{t,i}$ and $S_{t,j}$, where $t \in [1, T_{form}] \subset \mathbb{R}$.

$$S_{t,i} = \alpha + \beta S_{t,j} + \epsilon_t \quad (9)$$

All possible combinations of the stock prices are assessed based on indicators that represent the co-movement strength of the pairs, and the best-ranking pair is selected in each of the three selection methods.

The selection indicator of the simple distance method is the sums of the squared residuals (ϵ_t). The pair with the smallest sums of the squared residuals is selected as the pair for the trading period. The rationale behind this indicator is that if the sum of the squared residuals is small, then it means that there is a strong explanatory power of the prices of one stock ($S_{t,j}$) on the other ($S_{t,i}$), because the error is the smallest.

In the case of the Engle-Granger method, the residuals are analyzed in a more sophisticated way. Cointegration of stock prices refers to a long-run equilibrium relationship between the two stock price series that are non-stationary time series variables, which means that they move together over time despite the presence of short-term fluctuations. The equilibrium series, which is the spread of the stock price series is stationary if the two stock prices are cointegrated.

The Engle-Granger method for cointegration includes a unit root test of the residuals after the aforementioned OLS regression (Equation 9). The `urca` package of R is used to conduct the ADF unit root test for this purpose. The null hypothesis of the ADF test is that the time series contains a unit root, and thus is not stationary. Equation 10 describes the AR(1) process as an example of an I(1) process with a unit root if β is significantly not zero. The significance of β is tested in the Dickey-Fuller test.

$$\epsilon_t = \alpha + \beta \epsilon_{t-\Delta t} + u_t \quad (10)$$

In Equation 10, the u_t process is the error term of the AR(1) process. The actual equation that is tested by the ADF unit root test is more complex than Equation 10, although it exhibits the fundamental idea behind the ADF unit root test in a simplified way. The ADF unit root test assesses the coefficients with several options such as including constant or trend. Neither a trend nor a constant are included in the model, meaning that the α and the β were not considered in the below equation:

$$\Delta \epsilon_t = \alpha + \beta t + \gamma y_{t-\Delta t} + \delta_1 \Delta y_{t-\Delta t} + \dots + \delta_{p-1} \Delta y_{t-p+\Delta t} \quad (11)$$

The unit root test is then carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$. The parameter p can be set manually or using information criteria such as the Akaike information criterion (AIC) or the Bayesian information criterion (BIC). The AIC is applied in this research. The best pair is selected based on the test statistic level of the ADF test, provided that it is accepted as significant on the 5 percent level.

The Johansen cointegration test is based on vector autoregressive (VAR) models. In the first step, the VAR model is estimated using the stock price time series. The lag length of the VAR could be determined

manually or using the AIC or BIC similar to the ADF test. In line with ADF, the AIC is employed to select the optimal lag length. Although there are also various testing methods for cointegration in the Johansen method, the trace test is used, which is suggested by the literature. The null hypothesis is that there are no cointegrating relationships among the stock price series. The trace test compares the sum of the eigenvalues of the estimated VAR model with critical values from a chi-squared distribution to get the number of cointegrating vectors.

$$< S_{t,i}, S_{t,j} > = \alpha + \Phi D_t + \Pi_p < S_{t-p,i}, S_{t-p,j} > + \dots + \Pi_1 < S_{t-\Delta t,i}, S_{t-\Delta t,j} > + e_t \quad (12)$$

Equation 12 describes the cointegration relationship tested by the Johansen method. The sum of the Π matrices is analyzed to assess cointegration by determining the rank of the sum of the Π matrices, where $< S_{t,i}, S_{t,j} >$ denotes a vector of the two stock prices at time t .

In the case of cointegration, the first step of the process rejects the null hypothesis that the sum of the Π matrices has a rank of 0. In the second step, the test checks the null hypothesis that the rank of the matrix is smaller or equal to 1. As in the first step it was already rejected that the matrix would have a rank of 0, this step essentially tests if the matrix has a rank of 1. If the matrix has a rank of 1 indeed, that means that there is cointegration between $S_{t,i}$ and $S_{t,j}$. The pair selection indicator in the case of the Johansen test is the sum of the difference between the two test statistics and the two critical values respectively. This rule is in line with the method of the Engle-Granger pair selection, although as a result of the two-step process of the Johansen test, two test statistics are considered.

3.3 Trading Strategy

To operate the simulated pairs trading strategy, it is necessary to determine the entry and exit points for the trades. The normalized pairs trading spread between the stock price time series $S_{t,i}$ and $S_{t,j}$ are calculated similarly to ϵ_t in Equation 9 by dividing ϵ_t with the standard deviation of ϵ_t , however, this case the time period is not the formation period but the trading period ($t \in [T_{form} + \Delta t, T] \subset \mathbb{R}$). The parameters of the ϵ_t standard deviation, α , and β are calculated from Equation 9 based on the formation period, as in the phase of the trading period, the traders have no information about future prices, and could not build a model on unavailable data. The normalized pairs trading spread in the trading period is referred to as $\hat{\epsilon}_t$ to avoid confusing it with the ϵ_t in the formation period.

$$\hat{\epsilon}_t = \frac{S_{t,i} - \alpha - \beta S_{t,j}}{SD(\epsilon)} \quad (13)$$

Where $t \in]T_{form}, T]$ and SD is the function for calculating standard deviation.

Let $\psi_{open} \subset \mathbb{R}$ and $\psi_{close} \subset \mathbb{R}$ be the opening and closing trigger values respectively. Let $\Psi_{t,ij}$ be the trading position time series, where the index ij exhibits that the trading position is for the pair including price series $S_{t,i}$ and $S_{t,j}$, where $i, j \in [1, P] \subset \mathbb{N}$, and $i \neq j$. $\Psi_{t,ij}$ have values of -1, 0, and 1. The value 1 denotes the trading position after the normalized pairs trading spread exceeds the ψ_{open} threshold, in which the position comprises a long position of stock i ($S_{t,i}$) and a short position of stock j ($S_{t,j}$). The value -1 denotes the opposite position, where stock j is in a long position and stock i is in a short position. The value 0 represents the closing of a previous -1 or 1 position if the normalized pairs trading spread crosses the predefined ψ_{close} threshold, and it can also exhibit that the normalized pairs trading spread is not high enough for a new trade to be triggered, so there is no active position at that time. The below formula explains these rules more precisely.

$$\Psi_{t,ij} = \begin{cases} 0, & \text{for } \Psi_{t-\Delta t,ij} = 0 \text{ and } |\hat{\epsilon}_t| \leq \psi_{open} \\ & \text{or } \Psi_{t-\Delta t,ij} = 1 \text{ and } \hat{\epsilon}_t \geq -\psi_{close} \\ & \text{or } \Psi_{t-\Delta t,ij} = -1 \text{ and } \hat{\epsilon}_t \leq \psi_{close} \\ 1, & \text{for } \Psi_{t-\Delta t,ij} = 0 \text{ and } \hat{\epsilon}_t \leq -\psi_{open} \\ & \text{or } \Psi_{t-\Delta t,ij} = 1 \text{ and } \hat{\epsilon}_t \leq \psi_{close} \\ -1, & \text{for } \Psi_{t-\Delta t,ij} = 0 \text{ and } \hat{\epsilon}_t \geq \psi_{open} \\ & \text{or } \Psi_{t-\Delta t,ij} = -1 \text{ and } \hat{\epsilon}_t \geq \psi_{close} \end{cases} \quad (14)$$

As it can be seen from Equation 14, there are 7 different scenarios that determine the value of $\Psi_{t,ij}$. The first scenario describes the scenario when there is no active trade position neither at time t nor at time $t - \Delta t$, because the normalized pairs trading spread $\hat{\epsilon}_t$ does not exceed the opening trigger either in the negative or the positive domain (thus the absolute value of the opening trigger is exhibited) to trigger the trade position 1 or -1. The strategy assumes that it is worth waiting for a better trade opportunity with a larger spread. Two transactions take place in the second scenario because the normalized pairs trading spread converges to its mean of zero - which must be zero because of the OLS estimation method - and it exceeds the closing trigger threshold, which does not have to be exactly zero. As in the second scenario the trade position turns to 0 from 1, the long position was in $S_{t,i}$ and the short position was in $S_{t,j}$, and as it can be seen in Equation 9, this means that $S_{t,i}$ was undervalued compared to $S_{t,j}$ and ϵ_t was negative. Consequently, to trigger the closing transaction, ϵ_t must be higher than the closing trigger, which is negative if not zero because the mean of the normalized pairs trading spread is zero. The third scenario is the opposite of the second scenario, so similar actions and rationales take place in that scenario. The fourth scenario is a trade position opening scenario, in which the investor takes a long position in the undervalued stock i and a short position in the overvalued stock j , because $\hat{\epsilon}_t$ exceeded the critical level of ψ_{open} , and funding was available for the transaction as there were no active positions as $\Psi_{t-\Delta t,ij}$ is zero. In the fifth scenario, there is an active long position in stock i and a short position in stock j , but the normalized pairs trading spread has not yet converged and exceeded the closing trigger in time t , so the position is held further expecting eventual convergence. The sixth and seventh scenarios are very similar to the fourth and fifth scenarios, but in these cases, the long position is in stock j , while the short position is in stock i .

The number of shares acquired for the long and short positions is determined by the transaction costs and the available funding for the investment. There are three types of transaction costs considered in the model: commission per trade, the price changes caused by the market impact of the transactions, and the cost of credit for the short position collateral. The commissions and the market impacts materialize at the moment of the transaction, but the cost of credit must be paid when the transactions are closed. As a result, after the transaction is triggered, the number of shares to be purchased is determined by the price of the stock in that the investor will take a long position. The number of shares is calculated in the following way:

$$Q_{t,i} = \frac{(1 - C_{comm} - C_{mkt})V_{t-\Delta t,ij}}{S_{t,i}} \quad (15)$$

The number of stock i shares in long position after the transaction at time t is denoted by $Q_{t,i}$. C_{comm} and C_{mkt} represent the commission fees and the market costs respectively as a proportion of the value invested, which latter is indicated by the $V_{t,ij}$ measure, which is a time series of the pairs trading strategy's investment value.

The number of stock j shares in the short position is determined from the number of stock i shares

because the pairs trading strategy builds on the hypothesis that there is a relationship (Equation 9) between stock i and j. Thus, the number of stock j shares has to be calculated in a way to end up with a symmetric hedged position, which after convergence has the equilibrium constant expected spread (α times the number of shares in stock i). The short position's number of stocks formula that achieves this goal is the following:

$$Q_{t,j} = -\beta Q_{t,i} \quad (16)$$

The β coefficient is the estimated relationship between stock i and j from Equation 9. This way the value of the position at the time of convergence before the transaction costs is $Q_{t_s,j}(\frac{\alpha}{\beta} + S_{t_s,j})$, as derived in Equation 17. The convergence time is denoted as t^* , while the position start time is denoted as t_s .

$$V_{t^*,ij} = Q_{t_s,i}S_{t^*,i} - Q_{t_s,j}(S_{t^*,j} - S_{t_s,j}) = \frac{Q_{t_s,j}}{\beta}(\alpha + \beta S_{t^*,j}) - Q_{t_s,j}(S_{t^*,j} - S_{t_s,j}) = Q_{t_s,j}(\frac{\alpha}{\beta} + S_{t_s,j}) \quad (17)$$

I have assumed that Equations 9 and 16 both hold and that ϵ_t is zero because that is its expected value. The investment value before the transaction costs is the difference between the value of the long position and the profit or loss of the short position, because of the definition of the pairs trading strategy.

The opposite position value at convergence can be similarly derived, when the triggers lead to the 6th scenario in Equation 14, and the pairs trading strategy would comprise the short position of stock i and the long position of stock j. The i index in Equation 15 would be changed to the j index for the step of calculating the number of purchased shares in stock j, but Equation 16 would hold the same way for calculating the number of stock i shares to short.

The investment value after convergence can be derived from the below formula, which includes the aforementioned three transaction costs, of which two have already been revealed in Equation 15, but the C_{short} annual shorting fee is also included now.

$$V_{t_+^*,ij} = (1 - C_{comm} - C_{mkt})V_{t^*,ij} - C_{short}(t_+^* - t_s)Q_{t_s,j}S_{t_s,j} \quad (18)$$

To differentiate between the time before and after the transaction costs materialize, the time after transaction costs t_+^* is introduced too.

In general, the investment value in time t follows the value sum of the long and short positions and can be summarized by the below formula similar to Equation 14.

$$V_{t,ij} = \left\{ \begin{array}{ll} V_{t-\Delta t,ij} & \text{Case 1} \\ (1 - C_{comm} - C_{mkt})[Q_{t_s,i}S_{t,i} - Q_{t_s,j}(S_{t,j} - S_{t_s,j})] - C_{short}(t - t_s)Q_{t_s,j}S_{t_s,j} & \text{Case 2} \\ (1 - C_{comm} - C_{mkt})[Q_{t_s,j}S_{t,j} - Q_{t_s,i}(S_{t,i} - S_{t_s,i})] - C_{short}(t - t_s)Q_{t_s,i}S_{t_s,i} & \text{Case 3} \\ (1 - C_{comm} - C_{mkt})V_{t-\Delta t,ij} & \text{Case 4} \\ Q_{t_s,i}S_{t,i} - Q_{t_s,j}(S_{t,j} - S_{t_s,j}) & \text{Case 5} \\ (1 - C_{comm} - C_{mkt})V_{t-\Delta t,ij} & \text{Case 6} \\ Q_{t_s,j}S_{t,j} - Q_{t_s,i}(S_{t,i} - S_{t_s,i}) & \text{Case 7} \end{array} \right\} \quad (19)$$

The case numbers in Equation 19 refer to the cases in Equation 14, but are also detailed below.

Case 1: $\Psi_{t-\Delta t,ij} = 0$ and $|\hat{\epsilon}_t| \leq \psi_{open}$

Case 2: $\Psi_{t-\Delta t,ij} = 1$ and $\hat{\epsilon}_t \geq -\psi_{close}$

Case 3: $\Psi_{t-\Delta t, ij} = -1$ and $\hat{\epsilon}_t \leq \psi_{close}$

Case 4: $\Psi_{t-\Delta t, ij} = 0$ and $\hat{\epsilon}_t \leq -\psi_{open}$

Case 5: $\Psi_{t-\Delta t, ij} = 1$ and $\hat{\epsilon}_t \leq \psi_{close}$

Case 6: $\Psi_{t-\Delta t, ij} = 0$ and $\hat{\epsilon}_t \geq \psi_{open}$

Case 7: $\Psi_{t-\Delta t, ij} = -1$ and $\hat{\epsilon}_t \geq \psi_{close}$

3.4 Evaluation

Annual return is calculated in two steps. In the first step, the final investment value of the pairs trading strategy is divided by the initial funding amount. In the second step, the investment coefficient is converted to annual return following the method used for effective returns by raising it to the needed power as can be seen in the below equation:

$$r_{ann, ij} = \left(\frac{V_{T, ij}}{V_{T_{form} + \Delta t, ij}} \right)^{\frac{1}{T - T_{form}}} - 1 \quad (20)$$

The daily expected return of the pairs trading strategy is calculated as an average of the daily returns on the pairs trading strategy's investment value. Similarly, the standard deviation of the daily returns is calculated based on the pairs trading strategy's investment value. Daily return skewness and kurtosis are calculated as the skewness and kurtosis of the daily return based on the pairs trading strategy's investment value respectively.

$$r_{day, ij} = \sum_{t \in \tau} \frac{\frac{V_{t+\Delta t, ij}}{V_{t, ij}}}{N(T - T_{form})} \quad (21)$$

The daily Sharpe ratio is computed by dividing the average daily return by the standard deviation of the daily returns on the pairs trading strategy's investment value. The Sharpe ratio is useful to determine the risk-adjusted return, because of the general risk aversion of investors. This method can exhibit if one pairs trading strategy is only more profitable because of the higher risk level, or if it is more efficient.

$$sharpe_{ij} = \frac{r_{day, ij}}{sd_{day, ij}} \quad (22)$$

The research will exhibit another investment performance indicator: the highest monthly return. This measure demonstrates the potential positive deviation from the expected return. Similarly, the lowest monthly returns of the pairs trading strategies are also reported. The highest and lowest monthly returns are obtained by comparing the monthly returns for each month of the trading period and selecting the highest and lowest ones respectively.

To understand the distribution of the strategies' profitability, it is useful to compare the number of profitable months as a percentage of the total months in the trading period, so this measure is also included in the investment assessment.

Maximum drawdown is a great risk indicator, as it shows the highest possible loss in the trading period following the analyzed trading strategy. To calculate the maximum drawdown, the algorithm loops through the prices and compares the loss percentage between every time point, and it selects the highest loss. The idea behind the introduction of the maximum drawdown indicator is that the standard deviation is not the best method to quantify the level of risk, because investors do not see a positive deviation from the mean

as unattractive as a negative deviation. For instance, if there is an investment, which has extremely high returns in some cases, but has relatively low deviations apart from these outlier returns, then the standard deviation would exhibit a pessimistic high risk for the investment, which would not represent the general risk appetite of investors. The maximum drawdown on the other hand is not symmetric, as it only considers losses, not profits, which is in line with the literature on behavioral economics and investment (Kahneman, 2011).

The Calmar ratio is similar to the Sharpe ratio, but in the case of Calmar, the daily returns are not divided by the standard deviation of the daily returns, but by the maximum drawdown. The upgrade of the Calmar ratio is the better return normalization method, because the maximum drawdown is a better estimate of the investment's risk than the standard deviation, as described in the previous paragraph.

Lastly, the value at risk (VaR) indicates the amount of loss that may reasonably happen, because it is the fifth percentile of the daily returns. The interpretation of VaR is that there is only a 5 percent chance that the next day the investment will have a higher loss than the value of the VaR. The advantage of VaR over the maximum drawdown is that in VaR the probability of the risk is explicitly expressed, although it does not consider the loss amount with probabilities less than 5 percent, and the VaR only assesses daily losses, while the maximum drawdown does exhibit the largest potential loss and also considers longer time period returns than daily returns.

3.5 Stress Testing

The key attribute of this research is the stress testing analysis. The goal of the stress tests is to uncover certain risks associated with the pairs trading strategy, such as shocks to the pairs in terms of expected returns and their co-movement relationship. There are four distinct stress tests that exhibit these risks.

Firstly, the investment dynamics and metrics of the pairs trading strategies are assessed if the cointegration relationship is weaker in the trading period due to a shock to the pairs compared to the formation period. This way the investment loss implications are expected to be less remarkable compared to the lost relationship, but it is also more likely to happen in the actual stock market.

Secondly, the profitability of the pairs trading strategies is tested under the scenario of the lost cointegration relationship between the pairs in the trading period. Due to the lost relationship, the spreads between the pairs are not expected to converge, which may lead to serious investment losses, and could highlight severe challenges.

Thirdly, a single return shock to one element of the pair is also analyzed. The single-shock scenario includes a decrease in the expected return of a stock of the pair. It is possible that due to industry-specific or company-specific events, the expected return of a stock decreases such as a process failure in the operation, issues with the management, or even a loss in reputation due to unethical behavior.

Lastly, a volatility shock is applied to the spread process between the pairs of stocks. The shock is not only applied to Equation 5 in the spread process but also to Equation 8, which defines the process of $\hat{S}_{i,t}$. This shock allows for the simulation of the market with higher uncertainties and differences between the performance of similar companies, which result in higher temporary deviations from the equilibrium spread between cointegrating stock prices.

4 Results

This section exhibits the materialization of the simulated pairs trading strategy including the simulated price processes, and metrics that capture the level of risk and profitability. Afterward, the key aspect of this research is demonstrated: the impact of certain risks such as weakened or lost cointegration relationship between the pairs, shocks to one of the stocks in the pair, or increased volatility of the spread between the stocks.

4.1 Assessing the Simulated Pairs Trading Strategy

To understand the simulated pairs trading strategy that is discussed from a risk management perspective, it is essential to recognize the specific attributes of the pairs trading strategy, which are represented by the parameters of the equations presented in the methodology section. Due to the enormous amount of data that is created by the data-generating processes, the most suitable way to recognize the meaning of the parameters is to visualize the price processes, therefore some charts of the price processes are included and the cases of converging and diverging pairs are highlighted too. The pair selection methods are also compared visually based on certain selection performance indicators. Furthermore, a randomly selected path of the MC simulation is also showcased as an example to explain how the pairs trading strategy is changing the invested capital, before repeating the analysis also on the median, 10th and 90th percentiles of the MC simulation paths.

4.1.1 Parameters

The parameters of the pairs trading MC simulation are selected based on the literature mainly, but the author has also set a few parameters based on market experience (see Table 1).

The total period is selected to be 1.5 years because the formation and the trading periods are ideally one year and half year respectively. The choice of the formation period is critical, because if it is chosen to be too short, then there is a high risk, that the cointegration tests will be biased and not robust enough to select the best pairs, while if the formation period is too long, then it may indicate strong cointegration even if in the most recent months there has been a shock in the co-movement relationship, which also remain prevalent in the trading period. It was found in recent years, that the most suitable formation time period is one year (Huck and Afawubo, 2015; Fallenbüchel, 2021). Generally, the literature chooses the trading period to be 6 months, although there are some papers that use others such as 4 months (Caldeira and Moura, 2013). Nevertheless, the trading period is set to be 6 months in this research. Another important parameter is the time steps between the trade decisions, which is selected to be $\frac{1}{252}$ following the best practice of the literature, because there are 252 trading days in a year, so the strategy makes decisions once in every trading day essentially. The number of paths in the MC simulation is selected to be 300, in line with the suggestions of the literature (Toda, 2009).

The stock simulation parameters are selected to represent general stock prices in the market. The initial stock price is 100 to normalize the prices of stocks and make them easily comparable, which could be interpreted as percentage points. The expected return value is 5 percent, while the standard deviation of the return is 20 percent, which is in line with experience from stock prices in the market. The number of analyzed stocks is 15, which is not too much to constrain the interpretation from the charts, but at the same time large enough to allow for the distinction between converging and diverging pairs.

Type	Name	Value	Symbol	Equation
General	Total period (years)	1.5	T	20, 21
General	Time steps (years)	$\frac{1}{252}$	Δt	Nearly all
General	Formation period (years)	1	T_{form}	20, 21
General	Simulation paths	300	-	-
Stock simulation	Initial stock price	100	S_0	2
Stock simulation	Annual expected stock return	5%	μ	1, 2, 3, 8
Stock simulation	Annual stock return standard deviation	20%	σ	1, 2, 3, 8
Stock simulation	Number of stocks in the market	15	P	-
Cointegration	Spread equilibrium	0	θ	4, 5
Cointegration	Spread mean-reversion speed	70	k	4, 5
Cointegration	Spread white noise deviation	100%	η	4, 5
Cointegration	Highest cointegration strength	0.9	ρ	7
Cointegration	Lowest cointegration strength	0.2	ρ	7
Trading	Opening trigger	2	ψ_{open}	14, 19
Trading	Closing trigger	0	ψ_{close}	14, 19
Trading	Short fee per year	1.00%	C_{short}	18, 19
Trading	Commission fee per transaction	0.03%	C_{comm}	15, 18, 19
Trading	Market impact cost per transaction	0.20%	C_{mkt}	15, 18, 19
Trading	Initial fund for investment	100	V_{T-form}	-
Stress test	Zero cointegration strength	0	-	-
Stress test	Moderate cointegration strength	0.5	-	-
Stress test	Single stock shocked annual return	-3%	-	-
Stress test	Standard deviation shock	2	-	-

Table 1: These parameters are applied in the research for the MC simulation, stock price processes, cointegration, trading strategy, and stress tests.
Source: Author's own formulation

The parameters of the spread process in Equation 5 are chosen in a way to allow the cointegration tests to detect cointegration, but to also let the natural volatility of the spread remain between the stocks, which were checked in data visualizations, but only the final parameters are showcased in this research. As a result, the spread mean-reversion speed is set to be 70, while the standard deviation of the spread between the stock prices in Equation 5 is 100 percentage points, which is five times the standard deviation of the GBM in Equations 3 and 8. The spread equilibrium is set to be zero, because of the normalization method of the prices at the beginning of the process (at $t = 0$). The stock prices are formulated in chains, and in each link between the chain, the strength of the cointegration is determined by the ρ parameter as can be seen in Equation 7. The first stock price process is purely the GBM from Equation 3, but the stocks indexed between 2 and 15 are a result of the process exhibited in Equation 7. The only changing parameters between the price processes of the stocks between 2 and 15 are the ρ parameters, which are linearly decreasing from 0.9 to 0.2, and the spread process is always in relation to the stock with the preceding index as expressed in the methodology section by Equation 5.

The most crucial trading parameters are the opening and closing triggers and the transaction costs. The opening trigger is selected to be 2 standard deviations of the spread process and the closing trigger is chosen as 0, as the literature suggests (Gatev et al., 2006). The commission fee per transaction varied greatly in the past decades, which made it imperative to choose the most recent available data, which was at a 3 basis points level in 2020 (Fallenbüchel, 2021). Consequently, this research uses a 3 basis points commission fee

per transaction. There is another considerable transaction cost involved in the pairs trading strategy, which is the market impact of the transaction, because if a hedge fund makes a transaction with an outstanding ticket value, then it may affect the prices depending on the market tightness, and the order books of the stocks. The estimated value of the market impact implicit transaction cost is 20 basis points (Do and Faff, 2012). Furthermore, there is a special transaction cost for the pairs trading strategy, because it includes a short position. Therefore, the short fee is also assessed. The literature suggests a cautious constant loan fee of 1 percent annually during the position's life (Do and Faff, 2012), which acts as collateral for the short position, although the actual amount largely depends on the market, collateral requirements, and the specific security. Lastly, the initial fund available for the pairs trading investment is 100, to be able to interpret the capital fund as a percentage similar to the initial price of the stocks.

As for the stress tests, there are four distinct analyses of external shocks that affect the dynamics of the stock prices and the investment value. The first type of shock is the decrease in the strength of the co-movement relationship, which has two scenarios: the completely lost relationship, where the co-movement parameter ρ becomes 0, and a less radical scenario in which the co-movement parameter ρ is reduced to half of its initial value that the pair selection is based on. The levels of ρ stress were formulated this way to provide an easily comprehensible result for risk management. The co-movement is also stress tested from another perspective, which does not capture the strength of the relationship between the two stock prices, but its stability. The standard deviation of the spread between the two stock prices is doubled under the stress scenario, which is the η parameter in Equation 5. The single-shock stress test does not focus on the direct relationship between the stock price processes, but on one stock in the pair, which indirectly also affects the pair. Under this scenario, one of the stock price's annual return μ is decreased by 3 percent, which is a constant and immediate shock for the stock price, but the underlying dynamic is unchanged, except for the constant shock.

4.1.2 Stock Price Simulation Outcome

The results of the stock price simulations in the first path of the MC simulation show that the 15 price series are in line with expectations (see Figure 1).

The prices are following the trend of μ and have prices at the end of the year between the values of 160 and 60 approximately. All of the stock prices follow processes that exhibit signs of homogeneity, and the co-movement between the prices is prevalent but is also varying between stocks as intended. For instance, while stock prices indexed 1 and 2 are actually decreasing - which is not unexpected despite the positive expected value represented by the positive μ parameter - stock prices indexed 9 and 10 are on the rise and they end the period with a value above 100 too.

The prices with the strongest co-movement parameter, which are indexed by 1 and 2, exhibit a very strong co-movement indeed, as there are only minor deviations from each other. On the other hand, the prices with the weakest co-movement parameter do not seem to have a co-movement relationship, as the trend of stock 15 does not follow stock 1. In the case of moderate convergence, the relationship is observable, as prices 9 and 10 indeed move together, but after the first year, it can be seen that the prices have deviated and the reversion to the mean is not obvious. These results indicate that the formulation of Equation 7 indeed allows for determining the strength of cointegration between simulated stock prices by changing the ρ parameter. Although stock prices indexed 1 and 15 have the lowest co-movement relationship, the co-movement between stock prices indexed 14 and 15 is also not as strong as it is between stock prices indexed 1 and 2, as expected based on the methodology.

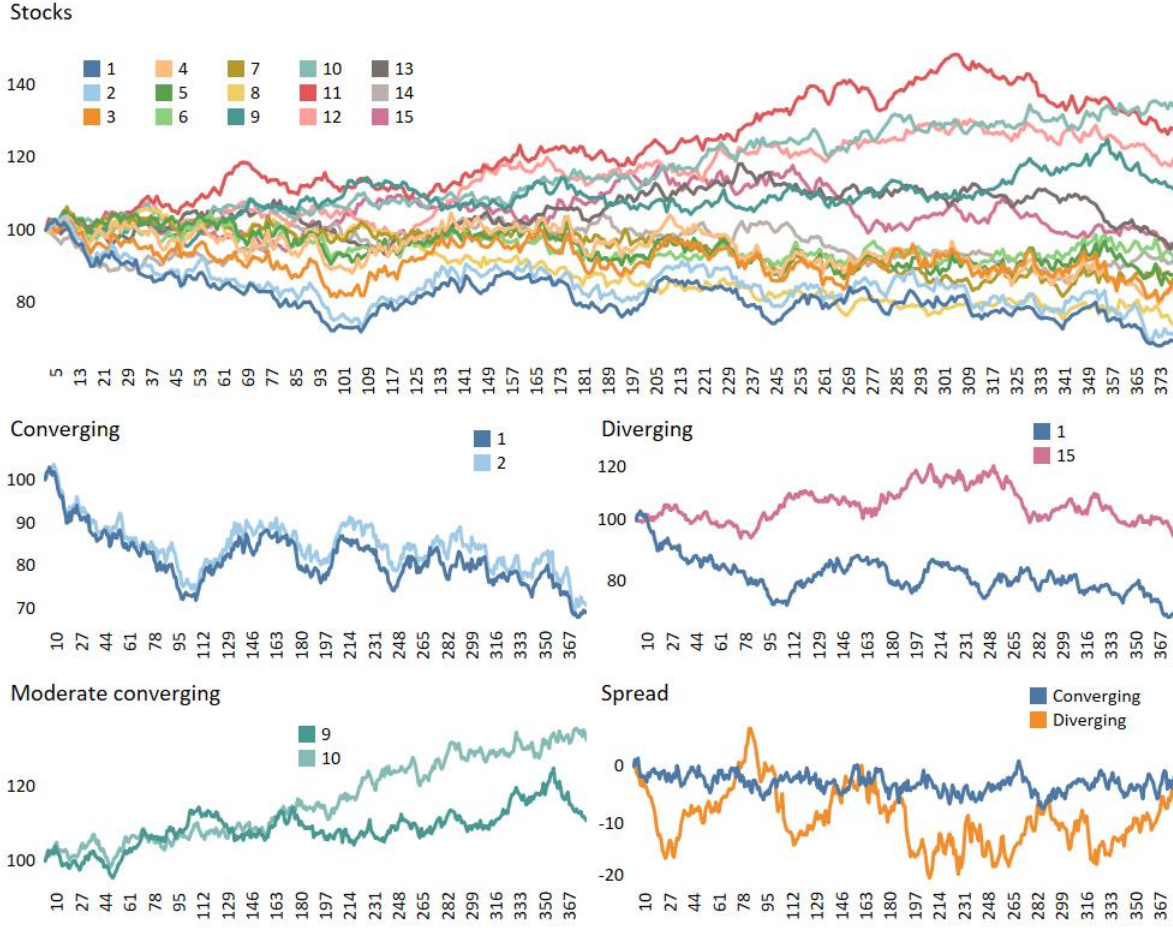


Figure 1: Simulation results of the 15 stock prices every day in the total period with different levels of cointegration. In the case of convergence, the ρ parameter is 0.9 (path=1).
Source: Author's own calculation

The spread of the converging pair is fluctuating around zero with relatively low volatility compared to the spread of the diverging pair, which may reach a level as low as -20. The mean reversion is not observable in the case of the diverging pair, while in the case of the converging pair, it can be seen that the sudden increase or decrease in the spread is followed by a gradual decrease or increase back to the zone around zero.

4.1.3 Pair Selection Performance

The three pair selection methods are compared in three metrics: the share of pairs including the best pairs (%), the share of pairs including the first stock (%), and the price index distance distribution (%).

The share of pairs including the best pairs is calculated by summing those paths, where the chosen pairs are the stocks indexed 1 and 2 by the respective pair selection method and dividing this sum by the total number of paths, which is 300. The rationale behind this performance indicator is that theoretically, the pair that has the strongest co-movement comprises the stocks indexed 1 and 2, thus the ultimate goal of the three methods is to select these pairs. However, there are barriers to this objective due to the prevalent uncertainty. The prices are stochastic, not deterministic, and the materialized prices may not indicate their fundamental relationship for the analytical pair selection methods. Consequently, it is important to test the

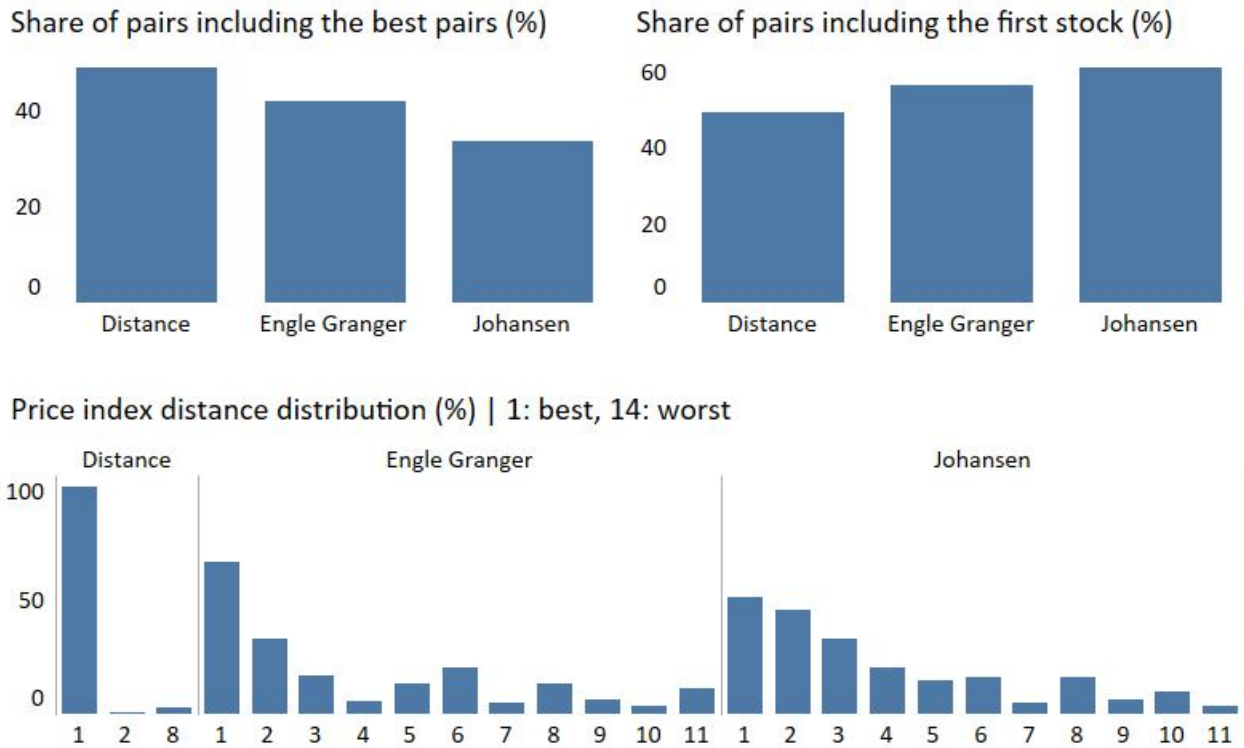


Figure 2: Power and size of the pair selection methods
Source: Author's own calculation

pair selection methods by the MC framework building on the law of large numbers.

Surprisingly, the simple distance method outperforms (49 percent) both the Engle-Granger (42 percent) and the Johansen (34 percent) cointegration methods in the selection of the best pairs. Interestingly, the Johansen test is the least effective in determining the best pairs.

On the other hand, the Johansen test includes the first stock in the selected pairs the most (60 percent), while the Engle-Granger (56 percent) and the simple distance (49 percent) methods slightly lag behind. This indicator is important because the first stock price process is the purest in the sense that the GBM process is widely used by researchers, while the other processes are less frequently used. As a result, this indicator exhibits how well the tests adapt to other rare processes.

The price index distance distribution exhibits the percentage of the differences between the index numbers of the stocks in the selected pair. For instance, the first column from the left shows that 99 percent of the pairs selected by the simple distance method selected pairs with stocks whose index numbers differed by only 1, in contrast to the Engle-Granger (66 percent) and Johansen (51 percent) methods, whose results are less outstanding. This indicator suggests the distance strategy is the superior pair selection method. Unusually, over 30 percent of the pairs that are selected by the Engle-Granger and Johansen tests have at least 8 index differences within the pairs, which is very high. On the other hand, due to the chain method of simulating the price processes, it is not obvious, how much the strength of cointegration is lower between the pairs with stock indices 3 and 4 and stocks indices 1 and 3. Nevertheless, the index distance distribution shows the difference between the cointegration-based co-movement tests and the simple distance method.

These results show that the simple distance strategy is very effective in selecting the pairs with the highest

co-movement, and they might be less biased towards stocks following the GBM process. However, the Engle-Granger and the Johansen tests are also performing well in selecting the best pairs, despite selecting stocks that may have lower co-movement. Furthermore, the simple distance method does not have a statistic that would allow the strategy to set a minimum limitation to select any pairs at all, whereas in the cointegration strategies the 5 percent limit on the p-value set risk-limiting boundaries for the pairs trading strategy. The analysis confirms the effectiveness of all three pair selection methods, thus it is important to compare the profitability of the trading strategies based on these three pair selection methods, to make more conclusions.

4.1.4 A Trading Strategy Sample

The first path of the simulation is presented here as a sample of the MC simulations. The simple distance and the Johansen methods selected the same pairs (see Figure 3), while the Engle-Granger method selected stocks that have diverged.

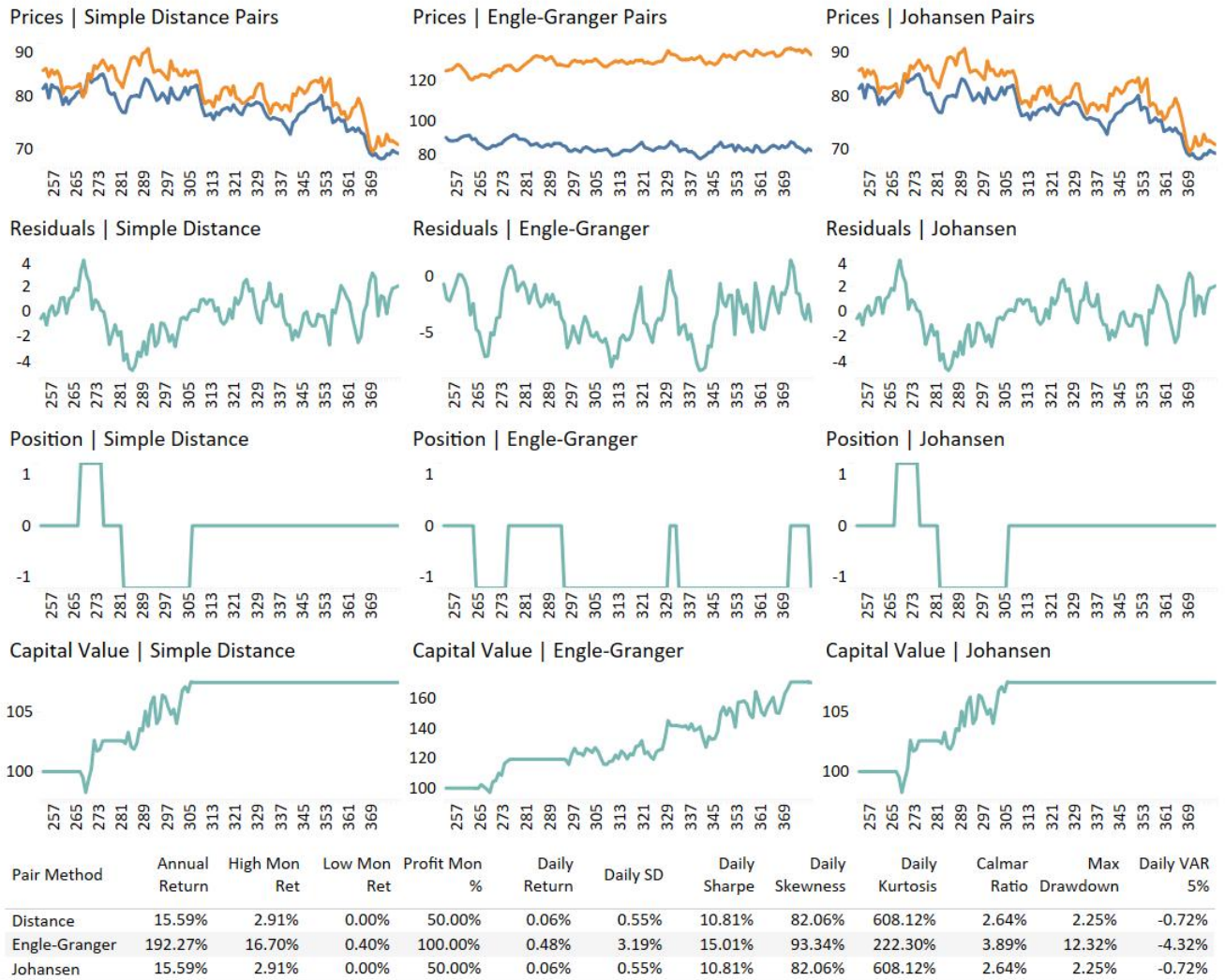


Figure 3: Pairs trading strategies' investment metrics of a randomly selected path ($path = 1$)
Source: Author's own calculation

The residuals of the simple distance and the Johansen pair selection deviate from the mean firstly in the

positive, then the negative area, which is mirrored in their trading position functions, which have values of 1 in the beginning, then after the first convergence they also have values of -1 before converging again. These positions translate into a slight profit for these strategies too, achieving over 15 percent annual returns.

Despite the convergence of the stocks in the pairs selected by the Engle-Granger method, it is much more profitable achieving an outstanding 192 percent annual return. However, the higher return is matched with a higher risk, but the risk-adjusted return is also higher in the Engle-Granger method in this path because the Calmar ratio is 3.9 percent compared to 2.6 percent in the other methods. The Engle-Granger method did not achieve this strategy because of the higher number of trades, but because of the higher level of volatility in the spread, which resulted in higher profits by each convergence, as the profit per convergence is determined by the opening and closing triggers.

4.1.5 Regular Pairs Trading

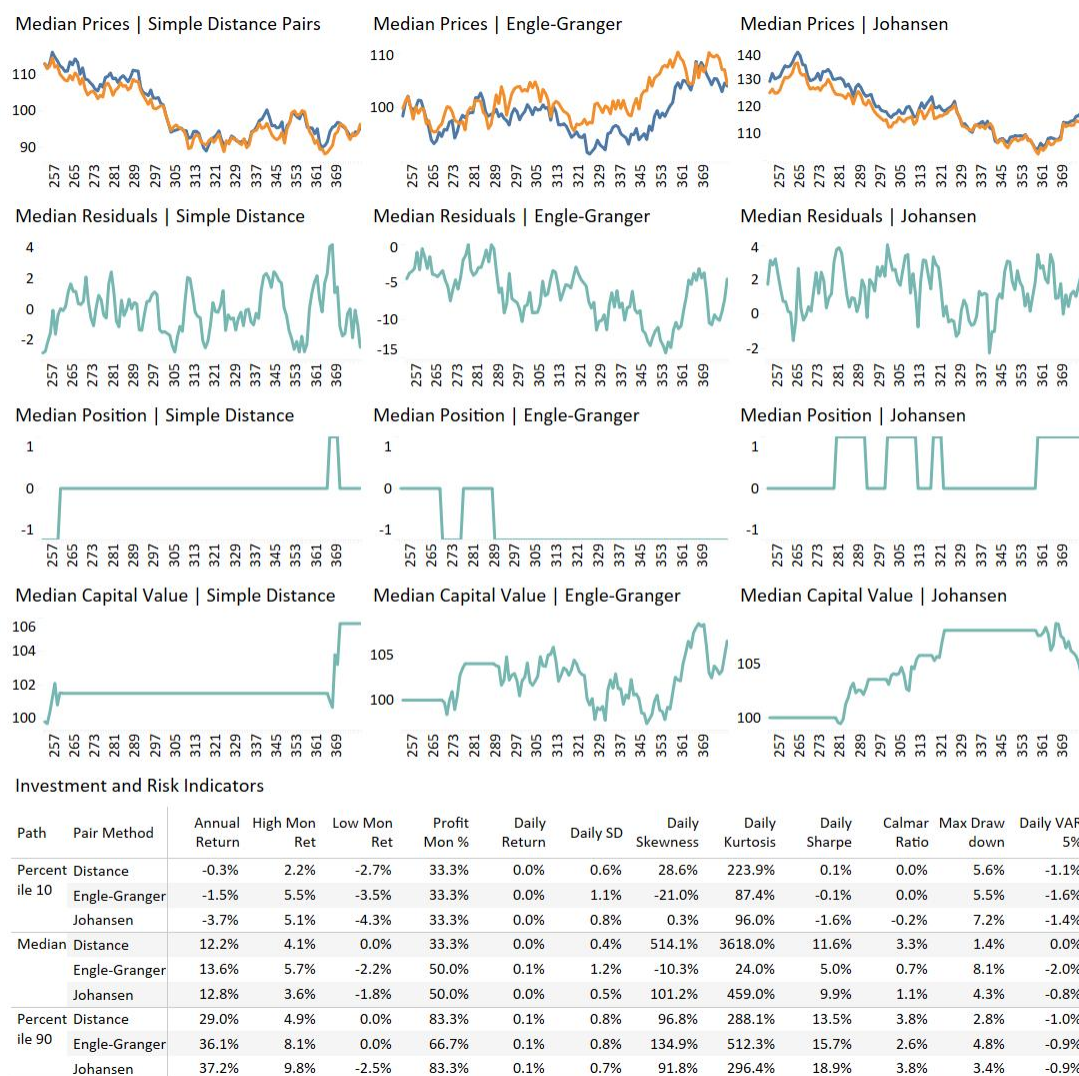


Figure 4: Pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value

Source: Author's own calculation

The median, 10th, and 90th percentile paths are selected for all three strategies based on the terminal investment value of the strategy. Although the annual return is the highest by the simple distance method in the 10th percentile path, it is actually the lowest in the median and 90th percentile paths (see Figure 4). The Sharpe ratio favors the simple distance method in the median and 10th percentile path, but in the 90th percentile path both the Engle-Granger and Johansen methods outperform the simple distance method. However, the Calmar ratio - which is arguably more relevant than the Sharpe ratio - favors the simple distance method in all the median, 10th, and 90th percentile paths.

The pairs in the median paths of each strategy are different. The residuals in the simple distance method fluctuate around zero frequently, but the opening trigger is not reached usually, and even when they do, the reversion to the mean is rapid. Consequently, the profit is relatively low due to the low number of trades, and could potentially increase if the opening trigger is lowered. The residuals in the Engle-Granger method are below zero during nearly the total trading period. There is only one successful convergence in the Engle-Granger method, which provides a high profit enough to outperform the simple distance method. The residuals of the Johansen method did not cross zero as many times as that of the simple distance method, but trade was opened four times, which contributed to the profitability of the Johansen method in the median path.

The annual returns, highest monthly returns, lowest monthly returns, and even the Sharpe and Calmar ratios are the highest in the 90th percentile and the lowest in the 10th percentile by comparing only the values of these measures with the ones of the respective pair selection method. These results are not surprising, because the investment value has strong relationships with the mentioned investment metrics. The risk metrics on the other hand do not follow the aforementioned rules. For instance, the standard deviation, maximum drawdown, and VaR of the daily returns express the largest risk in the median path of the Engle-Granger method. On the other hand in the simple distance method, the maximum drawdown is highest in the 10th percentile path. As a result, further analysis of the risk metrics may be needed to determine the different risk levels of the three pair selection methods.

4.2 Stress Testing Major Risks in Pairs Trading

There are multiple risks that can influence the profitability of pairs trading strategies. There are four specific risks discussed in this research that are used to stress the previously exhibited simulated pairs trading strategy. Changes in the strength of the co-movement, company-specific return shocks, and changes in uncertainty are the key risks that this research is focusing on.

4.2.1 Weakened Co-movement Strength

The weakened co-movement parameter interestingly lowers the annual returns in the median and 10th percentile paths mostly but in the 10th percentile paths, the annual returns of the three strategies remain at around 30 percent (see Figure 5). Although there are minor differences, all strategies materialize approximately the same losses due to the weakened co-movement strength. Unsurprisingly, the standard deviation of the Engle-Granger pairs trading returns is much higher than that of other strategies in the median paths, similar to the normal scenario before the stress.

The maximum drawdowns of the strategies have also increased significantly, especially in the median and the 10th percentile paths, while in the 90th percentile paths the Johansen method achieved a lower maximum drawdown in the stressed scenario than in the normal scenario. Generally, however, the stress

shifts not only the returns down but also the risks up.

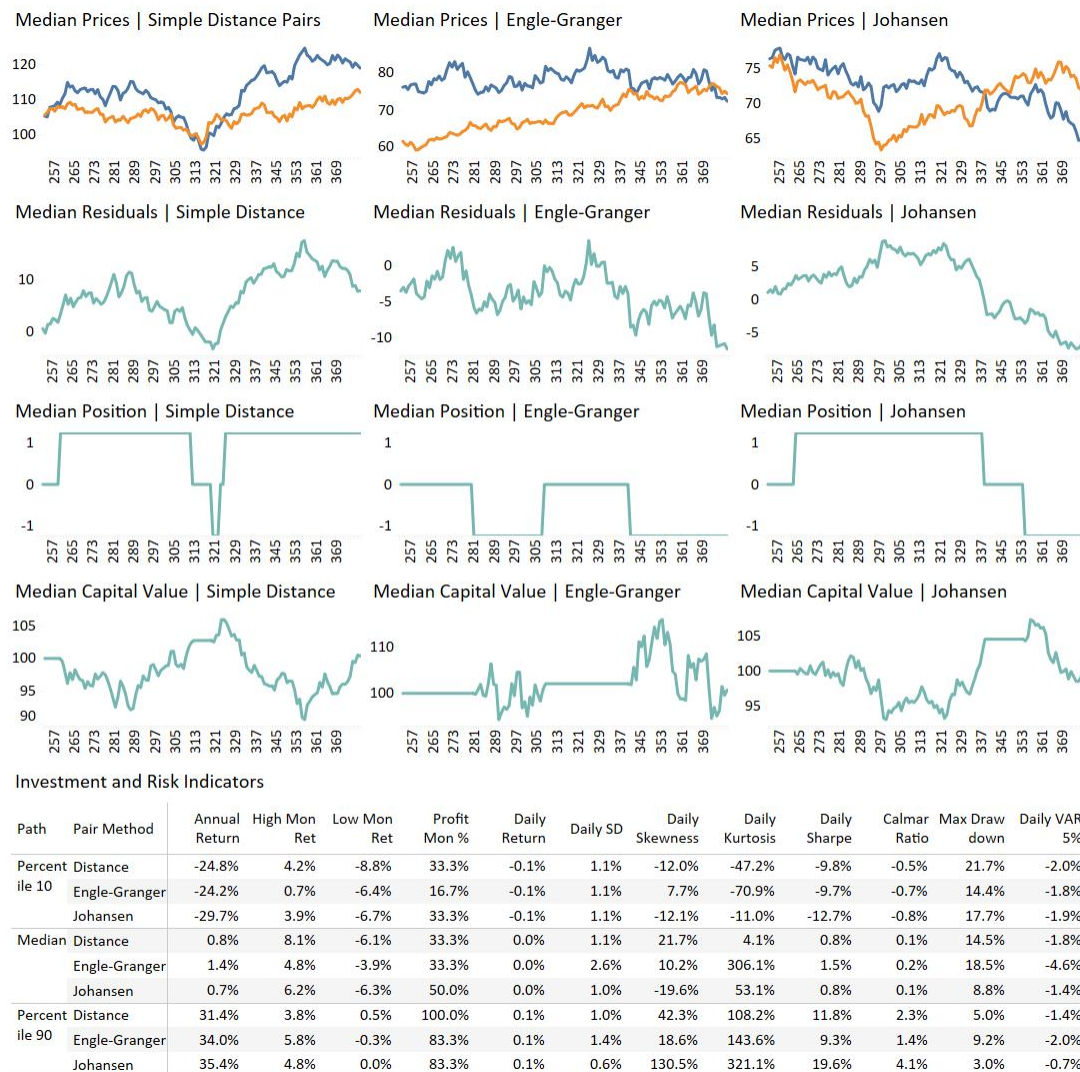


Figure 5: Weakened co-movement stressed pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value
Source: Author's own calculation

There are slight differences in the Sharpe ratio of the distinct pair selection methods. In the 10th percentile, the Engle-Granger method has the highest Sharpe ratio (-9.7 percent), followed by the simple distance (-9.8 percent), and the Johansen method is lagging in the last position (-12.7 percent). The Engle-Granger method is still leading slightly in terms of the Sharpe ratio in the median paths (0.2 percent), followed closely by the simple distance (0.1 percent) and the Johansen (0.1 percent) methods. However, the Johansen method performs the best (19.6 percent) in the 90th percentile paths, followed by the simple distance method (11.8 percent), and the Engle-Granger method (9.3 percent). As a result, the Engle-Granger pairs trading strategy is less risky toward negative returns in case of a co-movement shock, but the simple distance and Johansen strategies have a higher chance to perform outstandingly despite the co-movement shock.

As a result of the shock, the capital value of the simple distance pairs trading strategy in the median

path varies highly due to the large residual movements away from zero, despite the low number of position openings. In the median path of the Engle-Granger strategy, the residuals are not diverging until the last 2 months of the trading period, which reverts the capital value of the pairs trading strategy. As for the Johansen method, the residuals in the median path increase in the first half of the trading period triggering a 1 position, and eventually the residuals cross zero after several months, but the residual decreases further and triggers another trade, which does not converge and causes the investment value to drop toward the initial capital value.

4.2.2 Completely Lost Co-movement

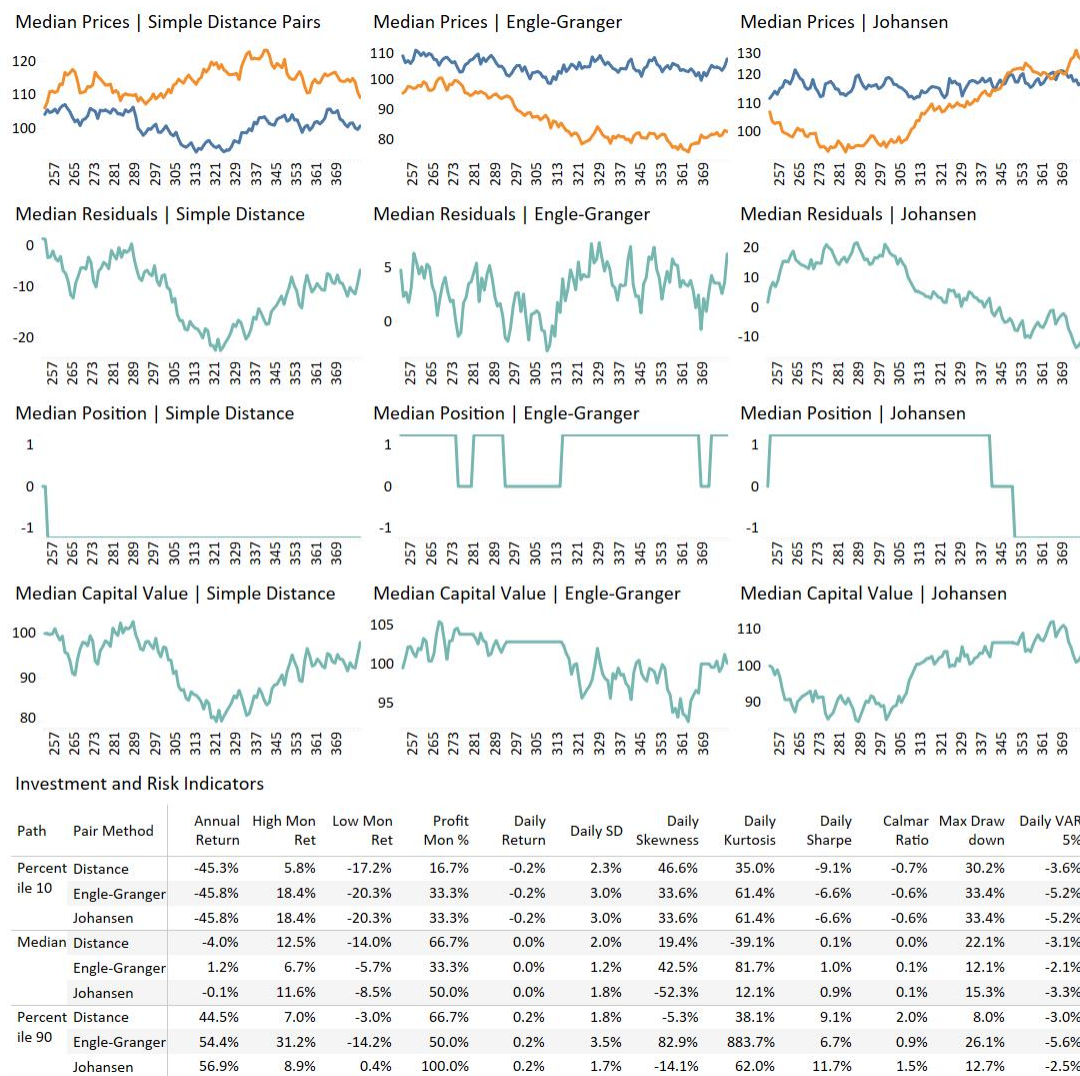


Figure 6: Lost co-movement stressed pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value
Source: Author's own calculation

A similar but more severe scenario is when the co-movement is completely absent between the chosen stocks in the pair of the pairs trading strategy. Interestingly, the median annual returns have decreased only

slightly by the Engle-Granger and Johansen methods, although it did decrease more in the simple distance method. The annual returns are below -45 percent in the 10th percentile paths, which is extremely low (see Figure 6). Surprisingly, however, the annual returns in the 90th percentile paths are over 40 percent for all three pairs trading strategies. These findings indicate that the lost co-movement does not necessarily eliminate profit potential, but severely increases the risks of the pairs trading strategy.

The increased risk is demonstrated indeed by the increased standard deviations and maximum drawdowns, although the risk metrics of the Engle-Granger strategy in the median path have not increased outstandingly. The changes in the risk metrics are different, which may not necessarily mean that for instance, the VaR of the Engle-Granger is generally higher in the 90th percentile and lowest in the median paths. The results indicate that there may be high variance in the risk indicators even in the top 10 paths based on terminal investment value in the case of a lost co-movement shock.

Although the Engle-Granger strategy residuals in the median path exhibit mean reversion, but this could be explained by the random character of the process. The simple distance and the Johansen strategies do not display any strong trend or mean reversion attribute, but there is a low level of mean reversion due to the fact that the path is chosen to show the residual trend of a pairs trading strategy with only moderate capital losses.

4.2.3 Shock Impacting a Single Stock

There are various shocks that may impact only a single company negatively such as management, industry shocks, supply chain disruptions, political events, or natural disasters. The single stock shock decreases the price of one of the stocks by a certain amount to assess its impact on pairs trading profitability.

The annual returns decrease remarkably as a result of the single stock shock (see Figure 7). The Johansen pairs trading strategy yielded a -16.4 percent annual return in the 10th percentile path, although it also achieved a 37.1 percent annual return in the 90th percentile path.

The key result of the single stock shock is that it shifts the residuals up or down based on the stock that is affected. This causes the position to be stuck in one position because the shifted residuals are essentially blocked from converging to the closing trigger as they converge to a new equilibrium after the shock which is not captured by the pairs trading strategy. However, the losses remain moderate, because the co-movement relationship remains, and so the movements of the stocks of the pair are hedged by the relationship. The losses and gains depend on the performance of the stocks relative to each other driven by the volatility of the spread.

The Sharpe ratios decreased in the 10th percentile path from between -0.2 and 0 percent in the normal scenario to between -3 and -5 percent in the single shock scenario by the three pairs trading strategies. In the median path, the simple distance takes the highest performance loss measured by the Sharpe ratio, as its value is 11.6 percent in the normal scenario, but it drops to 1 percent in the single shock scenario stress test. On the other hand, the impact of the single shock is less severe in the Engle-Granger method, as the Sharpe ratio decreased from 5 percent in the normal scenario to 1.5 percent in the single shock scenario in the median path.

The shock's impact on the maximum drawdown of the pairs trading strategies varies by the paths and the pair selection methods. The Johansen strategy scores a maximum drawdown of 3.4 percent in the 90th percentile path, which quadruples as a result of the stress test attaining a maximum drawdown of 14.7 percent. However, the maximum drawdown of the Engle-Granger strategy in the median path and the normal scenario is 8.1 percent, which is higher than that in the single stock shock scenario (4.6 percent).

This unexpected result is explained by the Calmar ratios, because the Calmar ratio of the Engle-Granger method in the median path and normal scenario is indeed higher than that of the stressed scenario.

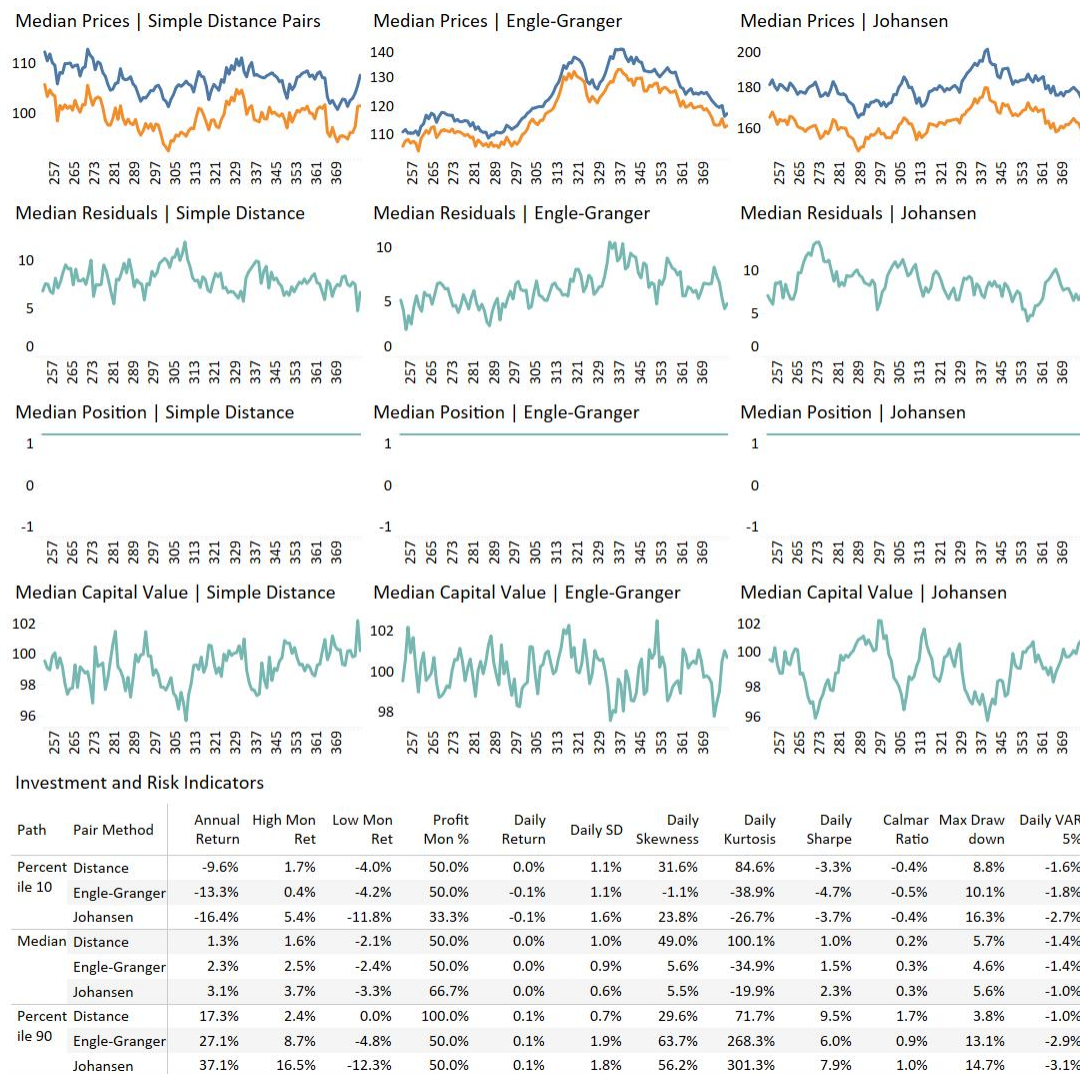


Figure 7: Single stock shock stressed pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value
Source: Author's own calculation

4.2.4 Increased Spread Volatility

The shock to the spread process's volatility impacts the annual returns in the median paths slightly negatively, in the 90th percentile paths slightly positively, and in the 10th percentile paths remarkably negatively (see Figure 8).

For instance, the Johansen method pairs trading strategy in the 10th percentile paths yielded -3.7 percent in the normal scenario, but in the spread volatility stress scenario, the annual return is only -13.1 percent.

The higher volatility of the spread process does not impact the standard deviation of the returns outstandingly, except for the Engle-Granger pairs trading strategy in the 90th percentile path, which is 22.2

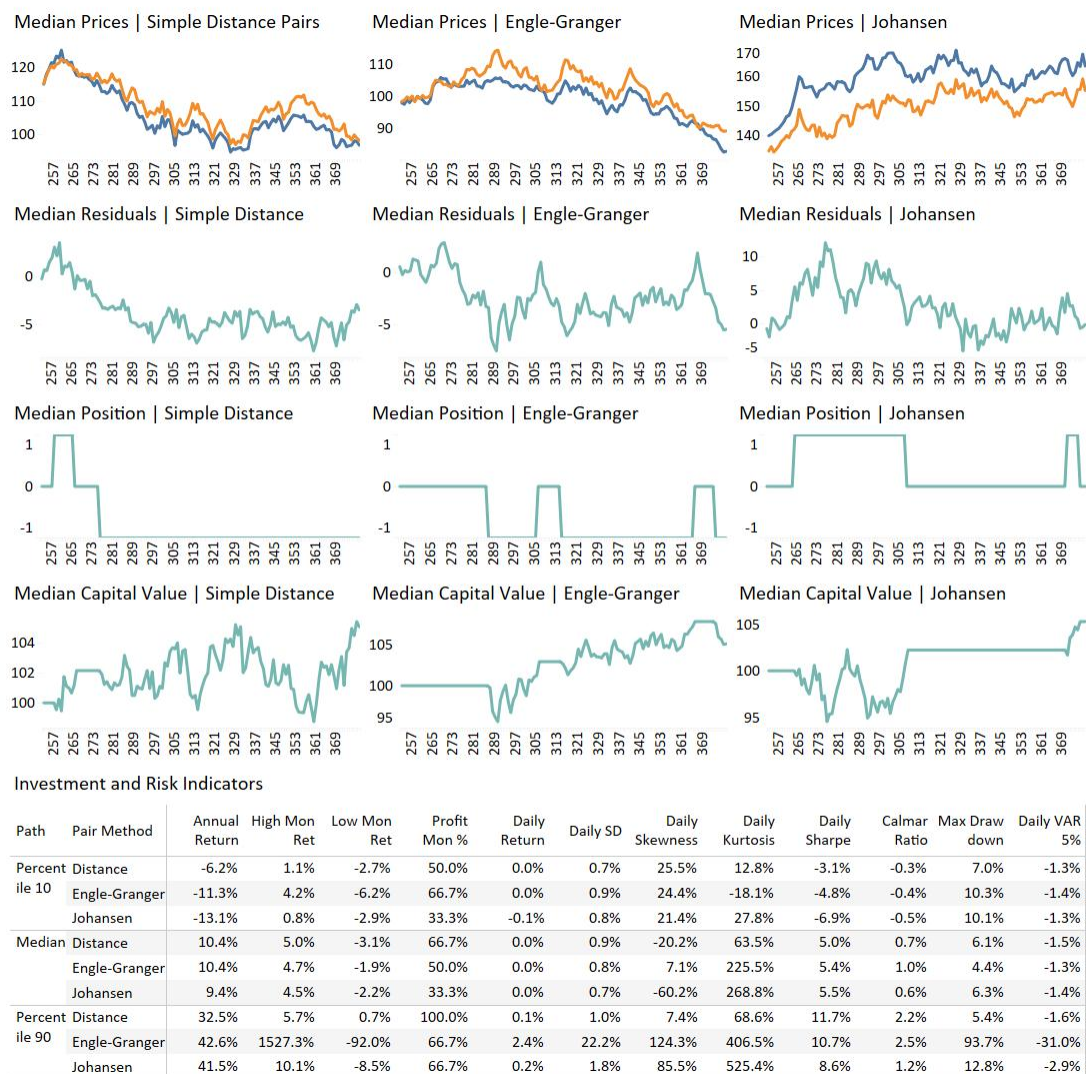


Figure 8: Spread volatility stressed pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value
Source: Author's own calculation

percent in the spread volatility stressed scenario. For instance, the standard deviation of returns of the Johansen pairs trading strategy in the 10th percentile path of the normal scenario is 0.8 percent, which equals that of the stressed scenario.

The maximum drawdowns, on the other hand, have increased remarkably. In the 10th percentile path, the Engle-Granger strategy's maximum drawdown is 10.3 percent in the stressed scenario compared to 5.5 percent in the normal scenario. The Johansen strategy is also exceptionally impacted because its maximum drawdown in the 90th percentile path is 3.4 percent in the normal scenario, but it jumps up to 12.8 percent in the volatility-stressed scenario.

The Sharpe ratios are notably lower in the volatility-stressed scenario than in the normal scenario. The Sharpe ratios of the Johansen strategy for instance decreased from 18.9 percent in the 90th percentile path, 9.9 percent in the median path, and -1.6 percent in the normal scenario to 8.6 percent, 5.5 percent, and -6.9 percent in the stressed scenario respectively. These changes exhibit the impact of the volatility stress. The

Calmar ratios are also impacted by the stress slightly, but the amount of change is not as extreme as in the case of the Sharpe ratios.

Based on the profit results and the residuals of the median paths, it does not seem that the higher volatility would cause more trade in general too because even though it is possible that the trade opening trigger is reached more quickly, the convergence to the closing trigger might be slower, which could offset the benefit of faster trade opening.

4.2.5 Stress Test Comparison

To summarize the stress tests, the completely lost co-movement affects the annual returns, Sharpe-, and Calmar ratios most in the median paths by all selection methods (see Table 2). The second most impactful stress is the weakened co-movement shock, which is followed by the single stock shock, while the volatility shock influenced the annual returns the least.

Investment and Risk Metrics Summary

Path	Pair Method	Metrics	Normal	Moderate Cointegration	No Cointegration	Single Return Shock	Volatility Shock
Percentile 10	Distance	Annual Return	-0.3%	-24.8%	-45.3%	-9.6%	-6.2%
		Daily SD	0.6%	1.1%	2.3%	1.1%	0.7%
		Daily Sharpe	0.1%	-9.8%	-9.1%	-3.3%	-3.1%
		Calmar Ratio	0.0%	-0.5%	-0.7%	-0.4%	-0.3%
		Max Drawdown	5.6%	21.7%	30.2%	8.8%	7.0%
		Daily VAR 5%	-1.1%	-2.0%	-3.6%	-1.6%	-1.3%
	Engle-Granger	Annual Return	-1.5%	-24.2%	-45.8%	-13.3%	-11.3%
		Daily SD	1.1%	1.1%	3.0%	1.1%	0.9%
		Daily Sharpe	-0.1%	-9.7%	-6.6%	-4.7%	-4.8%
		Calmar Ratio	0.0%	-0.7%	-0.6%	-0.5%	-0.4%
		Max Drawdown	5.5%	14.4%	33.4%	10.1%	10.3%
		Daily VAR 5%	-1.6%	-1.8%	-5.2%	-1.8%	-1.4%
	Johansen	Annual Return	-3.7%	-29.7%	-45.8%	-16.4%	-13.1%
		Daily SD	0.8%	1.1%	3.0%	1.6%	0.8%
		Daily Sharpe	-1.6%	-12.7%	-6.6%	-3.7%	-6.9%
		Calmar Ratio	-0.2%	-0.8%	-0.6%	-0.4%	-0.5%
		Max Drawdown	7.2%	17.7%	33.4%	16.3%	10.1%
		Daily VAR 5%	-1.4%	-1.9%	-5.2%	-2.7%	-1.3%
Median	Distance	Annual Return	12.2%	0.8%	-4.0%	1.3%	10.4%
		Daily SD	0.4%	1.1%	2.0%	1.0%	0.9%
		Daily Sharpe	11.6%	0.8%	0.1%	1.0%	5.0%
		Calmar Ratio	3.3%	0.1%	0.0%	0.2%	0.7%
		Max Drawdown	1.4%	14.5%	22.1%	5.7%	6.1%
		Daily VAR 5%	0.0%	-1.8%	-3.1%	-1.4%	-1.5%
	Engle-Granger	Annual Return	13.6%	1.4%	1.2%	2.3%	10.4%
		Daily SD	1.2%	2.6%	1.2%	0.9%	0.8%
		Daily Sharpe	5.0%	1.5%	1.0%	1.5%	5.4%
		Calmar Ratio	0.7%	0.2%	0.1%	0.3%	1.0%
		Max Drawdown	8.1%	18.5%	12.1%	4.6%	4.4%
		Daily VAR 5%	-2.0%	-4.6%	-2.1%	-1.4%	-1.3%
	Johansen	Annual Return	12.8%	0.7%	-0.1%	3.1%	9.4%
		Daily SD	0.5%	1.0%	1.8%	0.6%	0.7%
		Daily Sharpe	9.9%	0.8%	0.9%	2.3%	5.5%
		Calmar Ratio	1.1%	0.1%	0.1%	0.3%	0.6%
		Max Drawdown	4.3%	8.8%	15.3%	5.6%	6.3%
		Daily VAR 5%	-0.8%	-1.4%	-3.3%	-1.0%	-1.4%
Percentile 90	Distance	Annual Return	29.0%	31.4%	44.5%	17.3%	32.5%
		Daily SD	0.8%	1.0%	1.8%	0.7%	1.0%
		Daily Sharpe	13.5%	11.8%	9.1%	9.5%	11.7%
		Calmar Ratio	3.8%	2.3%	2.0%	1.7%	2.2%
		Max Drawdown	2.8%	5.0%	8.0%	3.8%	5.4%
		Daily VAR 5%	-1.0%	-1.4%	-3.0%	-1.0%	-1.6%
	Engle-Granger	Annual Return	36.1%	34.0%	54.4%	27.1%	42.6%
		Daily SD	0.8%	1.4%	3.5%	1.9%	22.2%
		Daily Sharpe	15.7%	9.3%	6.7%	6.0%	10.7%
		Calmar Ratio	2.6%	1.4%	0.9%	0.9%	2.5%
		Max Drawdown	4.8%	9.2%	26.1%	13.1%	93.7%
		Daily VAR 5%	-0.9%	-2.0%	-5.6%	-2.9%	-31.0%
	Johansen	Annual Return	37.2%	35.4%	56.9%	37.1%	41.5%
		Daily SD	0.7%	0.6%	1.7%	1.8%	1.8%
		Daily Sharpe	18.9%	19.6%	11.7%	7.9%	8.6%
		Calmar Ratio	3.8%	4.1%	1.5%	1.0%	1.2%
		Max Drawdown	3.4%	3.0%	12.7%	14.7%	12.8%
		Daily VAR 5%	-0.9%	-0.7%	-2.5%	-3.1%	-2.9%

Table 2: Summary of the stressed pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value
Source: Author's own calculation

Interestingly, the annual returns increase after most stress tests by all selection methods. This phenomenon implies that the assessed stress tests do not affect the high end of the investment value distribution, but mainly the low end and the middle area.

For instance, in the median paths, the Sharpe ratios in the normal scenario of the simple distance strategy, the Engle-Granger strategy, and the Johansen strategy is 11.6 percent, 5.0 percent, and 9.9 percent respectively, while in the lost co-movement stress scenario they drop down to 0.1 percent, 1.0 percent, and 0.9 percent respectively, which exhibit the impact.

The 10th percentile paths are also heavily impacted by stress scenarios. The single-stock stress scenario raises the maximum drawdown up from 5.6 percent in the simple distance method, 5.5 percent in the Engle-Granger method, and 7.2 percent in the Johansen method to 8.8 percent, 10.1 percent, and 16.3 percent respectively. These risk increases are supplemented with negative returns, thus making the strategy extremely risky. However, the probability of the 10th percentile materializing and its maximum drawdown to happen for the investor is low.

In the median paths, the maximum drawdown is not affected highly by the single stock return stress test and the volatility shock, which may build confidence in the implementation of the pairs trading strategy from a risk management perspective too. The key risk of the pairs trading strategy is the uncontrolled loss of co-movement relationship between the stocks of the chosen pair.

5 Conclusion

It has been shown that the parameters used in the model for stock price simulation generate realistic stock price processes. The results of comparing the performance of pair selection methods suggest that the simple distance method is the most effective method for selecting the pairs with the strongest co-movement. However, the difference in effectiveness is not extreme, so it is worth considering all three strategies for the profitability analysis. This finding rejects the hypothesis (H1) that the simple distance method would not be as effective as the cointegration methods.

The pairs trading strategies' profitability in the normal scenario exhibited that there are only slight differences between the annual returns of the different pairs trading strategies: the simple distance method, the Engle-Granger method, and the Johansen method. Although some metrics such as the maximum drawdown indicate that the simple distance method has the lowest risk but the Sharpe and Calmar ratios do not highlight a clear best performer in terms of risk-adjusted returns. Consequently, the simple distance, the Engle-Granger, and the Johansen cointegration method may all be considered for pairs trading strategy traders.

The weakened and the lost co-movement shocks cause the median paths to decrease remarkably, but in the case of the 10th percentile paths, the impacts are extreme, which are demonstrated not only by the annual returns, but also by the Sharpe, and Calmar ratios, and the maximum drawdown too. The single stock return shocks shift the residuals, denying the residuals to cross the closing trigger, which decreases the return, but due to the existing co-movement relationship, the risks of the strategies do not change drastically. The volatility shock does not affect the median paths remarkably, but in the 10th and 90th percentile paths the pairs trading strategies' annual returns decrease and increase respectively, as expected due to the higher uncertainty. Overall, the most impactful risk is the lost or weakened co-movement relationship in the trading period after the formation period. These results confirm the other two hypotheses (H2 and H3) of this research.

This research highlights that algorithmic traders may consider generating a simulated environment for testing the profitability of their pairs trading strategies in general, as it provides several advantages compared to empirical analysis, due to the limitation of data, which is affected by macroeconomic and industry shocks making it difficult to focus on one specific risk separately because the shocks are presented simultaneously in the real market. Algorithmic traders could determine the parameters of their asset markets to simulate them and analyze certain shocks. These separated shocks could allow investors to better understand the specific risks that their investment possesses, which may result in a higher likelihood of capital gained by the algorithmic trading fund.

The specific risks analyzed in this research could be managed by different techniques. The weakened or lost co-movement strength could be tracked during the trading period, and if the indicator that is determining that the strength of the co-movement has dropped significantly, then the risk trigger could close the position and remove the pairs from the trading strategy to avoid further losses. For instance, if there is a change in the sum of the squared difference of the simple distance method or the p-values of the cointegration methods, then the risk trigger could be activated. Similarly, the immediate shocks to the stocks in the pairs and a shock in the volatility of the pairs could be determined using statistical methods, and closing triggers may be implemented based on the risk appetite of investors.

Further research could be conducted on the process of acquiring the parameters of historical data for the simulation environment and testing the effectiveness of such a transformation. There might be a need for exploring more stock price and spread processes apart from the geometric Brownian motion and the Ornstein-Uhlenbeck processes, to allow the transformation of market data to the simulated environment more precisely. Although the scope of this research is limited to four pairs trading risks, there are several more risks involved which could be explored and analyzed similarly.

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7 Appendix

All Investment and Risk Metrics Summary

Metrics	Risk	Percentile 10			Median			Percentile 90		
		Distance	Engle-Granger	Johansen	Distance	Engle-Granger	Johansen	Distance	Engle-Granger	Johansen
Annual Return	Normal	-0.3%	-1.5%	-3.7%	12.2%	13.6%	12.8%	29.0%	36.1%	37.2%
	Moderate Cointegrat..	-24.8%	-24.2%	-29.7%	0.8%	1.4%	0.7%	31.4%	34.0%	35.4%
	No Cointegration	-45.3%	-45.8%	-45.8%	-4.0%	1.2%	-0.1%	44.5%	54.4%	56.9%
	Single Return Shock	-9.6%	-13.3%	-16.4%	1.3%	2.3%	3.1%	17.3%	27.1%	37.1%
	Volatility Shock	-6.2%	-11.3%	-13.1%	10.4%	10.4%	9.4%	32.5%	42.6%	41.5%
High Mon Ret	Normal	2.2%	5.5%	5.1%	4.1%	5.7%	3.6%	4.9%	8.1%	9.8%
	Moderate Cointegrat..	4.2%	0.7%	3.9%	8.1%	4.8%	6.2%	3.8%	5.8%	4.8%
	No Cointegration	5.8%	18.4%	18.4%	12.5%	6.7%	11.6%	7.0%	31.2%	8.9%
	Single Return Shock	1.7%	0.4%	5.4%	1.6%	2.5%	3.7%	2.4%	8.7%	16.5%
	Volatility Shock	1.1%	4.2%	0.8%	5.0%	4.7%	4.5%	5.7%	1527.3%	10.1%
Low Mon Ret	Normal	-2.7%	-3.5%	-4.3%	0.0%	-2.2%	-1.8%	0.0%	0.0%	-2.5%
	Moderate Cointegrat..	-8.8%	-6.4%	-6.7%	-6.1%	-3.9%	-6.3%	0.5%	-0.3%	0.0%
	No Cointegration	-17.2%	-20.3%	-20.3%	-14.0%	-5.7%	-8.5%	-3.0%	-14.2%	0.4%
	Single Return Shock	-4.0%	-4.2%	-11.8%	-2.1%	-2.4%	-3.3%	0.0%	-4.8%	-12.3%
	Volatility Shock	-2.7%	-6.2%	-2.9%	-3.1%	-1.9%	-2.2%	0.7%	-92.0%	-8.5%
Profit Mon %	Normal	33.3%	33.3%	33.3%	33.3%	50.0%	50.0%	83.3%	66.7%	83.3%
	Moderate Cointegrat..	33.3%	16.7%	33.3%	33.3%	33.3%	50.0%	100.0%	83.3%	83.3%
	No Cointegration	16.7%	33.3%	33.3%	66.7%	33.3%	50.0%	66.7%	50.0%	100.0%
	Single Return Shock	50.0%	50.0%	33.3%	50.0%	50.0%	66.7%	100.0%	50.0%	50.0%
	Volatility Shock	50.0%	66.7%	33.3%	66.7%	50.0%	33.3%	100.0%	66.7%	66.7%
Daily Return	Normal	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	0.1%	0.1%	0.1%
	Moderate Cointegrat..	-0.1%	-0.1%	-0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%
	No Cointegration	-0.2%	-0.2%	-0.2%	0.0%	0.0%	0.0%	0.2%	0.2%	0.2%
	Single Return Shock	0.0%	-0.1%	-0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%
	Volatility Shock	0.0%	0.0%	-0.1%	0.0%	0.0%	0.0%	0.1%	2.4%	0.2%
Daily SD	Normal	0.6%	1.1%	0.8%	0.4%	1.2%	0.5%	0.8%	0.8%	0.7%
	Moderate Cointegrat..	1.1%	1.1%	1.1%	1.1%	2.6%	1.0%	1.0%	1.4%	0.6%
	No Cointegration	2.3%	3.0%	3.0%	2.0%	1.2%	1.8%	1.8%	3.5%	1.7%
	Single Return Shock	1.1%	1.1%	1.6%	1.0%	0.9%	0.6%	0.7%	1.9%	1.8%
	Volatility Shock	0.7%	0.9%	0.8%	0.9%	0.8%	0.7%	1.0%	22.2%	1.8%
Daily Skewness	Normal	28.6%	-21.0%	0.3%	514.1%	-10.3%	101.2%	96.8%	134.9%	91.8%
	Moderate Cointegrat..	-12.0%	7.7%	-12.1%	21.7%	10.2%	-19.6%	42.3%	18.6%	130.5%
	No Cointegration	46.6%	33.6%	33.6%	19.4%	42.5%	-52.3%	-5.3%	82.9%	-14.1%
	Single Return Shock	31.6%	-1.1%	23.8%	49.0%	5.6%	5.5%	29.6%	63.7%	56.2%
	Volatility Shock	25.5%	24.4%	21.4%	-20.2%	7.1%	-60.2%	7.4%	124.3%	85.5%
Daily Kurtosis	Normal	223.9%	87.4%	96.0%	3618.0%	24.0%	459.0%	288.1%	512.3%	296.4%
	Moderate Cointegrat..	-47.2%	-70.9%	-11.0%	4.1%	306.1%	53.1%	108.2%	143.6%	321.1%
	No Cointegration	35.0%	61.4%	61.4%	-39.1%	81.7%	12.1%	38.1%	883.7%	62.0%
	Single Return Shock	84.6%	-38.9%	-26.7%	100.1%	-34.9%	-19.9%	71.7%	268.3%	301.3%
	Volatility Shock	12.8%	-18.1%	27.8%	63.5%	225.5%	268.8%	68.6%	406.5%	525.4%
Daily Sharpe	Normal	0.1%	-0.1%	-1.6%	11.6%	5.0%	9.9%	13.5%	15.7%	18.9%
	Moderate Cointegrat..	-9.8%	-9.7%	-12.7%	0.8%	1.5%	0.8%	11.8%	9.3%	19.6%
	No Cointegration	-9.1%	-6.6%	-6.6%	0.1%	1.0%	0.9%	9.1%	6.7%	11.7%
	Single Return Shock	-3.3%	-4.7%	-3.7%	1.0%	1.5%	2.3%	9.5%	6.0%	7.9%
	Volatility Shock	-3.1%	-4.8%	-6.9%	5.0%	5.4%	5.5%	11.7%	10.7%	8.6%
Calmar Ratio	Normal	0.0%	0.0%	-0.2%	3.3%	0.7%	1.1%	3.8%	2.6%	3.8%
	Moderate Cointegrat..	-0.5%	-0.7%	-0.8%	0.1%	0.2%	0.1%	2.3%	1.4%	4.1%
	No Cointegration	-0.7%	-0.6%	-0.6%	0.0%	0.1%	0.1%	2.0%	0.9%	1.5%
	Single Return Shock	-0.4%	-0.5%	-0.4%	0.2%	0.3%	0.3%	1.7%	0.9%	1.0%
	Volatility Shock	-0.3%	-0.4%	-0.5%	0.7%	1.0%	0.6%	2.2%	2.5%	1.2%
Max Drawdown	Normal	5.6%	5.5%	7.2%	1.4%	8.1%	4.3%	2.8%	4.8%	3.4%
	Moderate Cointegrat..	21.7%	14.4%	17.7%	14.5%	18.5%	8.8%	5.0%	9.2%	3.0%
	No Cointegration	30.2%	33.4%	33.4%	22.1%	12.1%	15.3%	8.0%	26.1%	12.7%
	Single Return Shock	8.8%	10.1%	16.3%	5.7%	4.6%	5.6%	3.8%	13.1%	14.7%
	Volatility Shock	7.0%	10.3%	10.1%	6.1%	4.4%	6.3%	5.4%	93.7%	12.8%
Daily VAR 5%	Normal	-1.1%	-1.6%	-1.4%	0.0%	-2.0%	-0.8%	-1.0%	-0.9%	-0.9%
	Moderate Cointegrat..	-2.0%	-1.8%	-1.9%	-1.8%	-4.6%	-1.4%	-1.4%	-2.0%	-0.7%
	No Cointegration	-3.6%	-5.2%	-5.2%	-3.1%	-2.1%	-3.3%	-3.0%	-5.6%	-2.5%
	Single Return Shock	-1.6%	-1.8%	-2.7%	-1.4%	-1.4%	-1.0%	-1.0%	-2.9%	-3.1%
	Volatility Shock	-1.3%	-1.4%	-1.3%	-1.5%	-1.3%	-1.4%	-1.6%	-31.0%	-2.9%

Table 3: Detailed summary of the stressed pairs trading strategies' investment metrics of the median, 10th, and 90th percentile paths based on the terminal investment value

Source: Author's own calculation