



PÁZMÁNY

Pázmány Péter Catholic University

Faculty of Information Technology and Bionics

Spin-wave based computing

MAUCHA, Levente

Info-Bionics Engineering MSc

Supervisors: Dr. PAPP, Adam, Dr. CSABA, György

2023

Abstract

During my work, I focused on designing micromagnetic structures to achieve specific functionalities. The primary goal was to experimentally design a structure capable of functioning as a wavelength demultiplexer. The input signal is delivered through the coplanar waveguide on the left side, which excites spin waves in a 100 nm thick Yttrium Iron Garnet (YIG) film. The output is measured as the spin-wave intensity at predefined positions on the film. The magnons interact with each other, forming an interference pattern within the YIG layer. To utilize this setup for computational tasks, we need to solve the inverse design problem, which involves determining the distribution of magnonic index of refraction that produces a specific input-output relationship through the formation of the interference pattern. Achieving the desired interference pattern is accomplished by locally modifying the magnonic index of refraction of YIG using Focused Ion Beam irradiation.

To perform the inverse design of the magnonic index of refraction, the custom micromagnetic solver, SpinTorch was utilized, which is built upon the machine-learning framework PyTorch. The SpinTorch is capable of a "design by specification" approach, where we specify the desired functionality of the device, and the algorithm discovers a structure that fulfills these requirements. The experimental validation of the design was carried out in collaboration with researchers from Markus Becherer's group at the Technical University of Munich.

In order to design the desired micromagnetic structure the SpinTorch solver was improved to be capable of modeling of interactions between the magnetic YIG thin film and the electromagnetic structures situated on top of the YIG layer. This expansion aims to generate more precise and realistic simulations, which can prove valuable for experiment design.

The parameters of the YIG films were determined from the time-resolved Magneto Optical Kerr Effect (tr-MOKE) microscopy measurements, employing the dispersion relations for the spin-waves. The experimental realization of our design was carried out in

collaboration with researchers from Markus Becherer's group at the Technical University of Munich. The inversely designed ion-dose pattern was irradiated by Focused Ion Beam in Munich, and the spin-wave propagation was measured using tr-MOKE microscopy.

The M_s was optimized for two frequencies during training; however, it demonstrates good performance across a range of frequencies. In the experiments, only the optimized frequencies were applied due to technical constraints. The experimental results show great agreement with the simulations, with a significant separation of wavelengths occurring in the sample.

Acknowledgements

I'm grateful to the researchers of Markus Becherer's group at the Technical University of Munich, especially to Martina Kiechle for providing the measurement results for the simulations and conducting the experiments for the experimental realization. The text was corrected using ChatGPT to improve its language.

Contents

Abstract	3
Acknowledgements	5
1 Introduction	7
1.1 Background	7
1.2 Outline	10
2 Improving the SpinTorch micromagnetic solver	11
2.1 Integration of the magnetic field of the coplanar waveguide source into SpinTorch	11
2.2 Calculation of the induced voltage in the pick-up antenna	14
3 Estimation of the parameters of the YIG film	19
3.1 Dispersion relation for spin-waves	19
3.2 Investigating the relationship between the FIB dose and the resulting change in saturation magnetization	21
4 The design and experimental demonstration of wavelength demulti- plexer	24
4.1 Designing wavelength demultiplexer on the YIG328 sample	24
4.2 Designing wavelength demultiplexer on the YIG331 sample	27
4.3 Experimental realization	29
5 Summary	34
5.1 Problem statement	34
5.2 Results	35
5.3 Conclusion	36
Bibliography	38

Chapter 1

Introduction

1.1. Background

In recent decades, there has been a surge in the popularity of alternative computing methods for two primary reasons. Firstly, despite significant increases in computational capacity, digital systems are not efficient in computing analog tasks. Moreover, MOS-transistor based devices have demonstrated remarkable stability in handling Boolean logic. This has fueled the research of emerging nanoelectric devices towards neuromorphic and analog problems. [1]

A notable challenge in the study of neuromorphic devices is that most computing models require highly interconnected systems, specifically artificial neurons with a large number of connections, often including all-to-all connections. Stand-alone neuronal units have limited utility; there must be an effective way to interconnect these devices to computing systems. Wave-based computing concepts offer a potential solution to this problem. If the computing device is realized in a wave-propagating substrate, interference patterns can create an all-to-all interconnection between points on the substrate. [1]

Several physical phenomena can be employed for this purpose, including optical systems or spin-wave (SW) based devices. [2] The advantage of SWs is that, unlike optical waves, they can display nonlinear wave propagation based on their amplitude. This characteristic makes them more effective in solving nonlinear computing problems. Figure 1.1a) illustrates an example in the linear case, demonstrating how a SW scatterer block can represent a perceptron layer. The functioning of a single perceptron layer can be described as a linear transformation followed by a nonlinear activation function: $\mathbf{y} = \sigma(\mathbf{W}\mathbf{x})$, where $\mathbf{x} \in \mathbb{R}^{1 \times n}$ represents the input, $\mathbf{W} \in \mathbb{R}^{n \times m}$ contains the optimizable

weights, $\mathbf{y} \in \mathbb{R}^{1 \times m}$ is the output vector, and σ is the nonlinear activation function [1]. In this way, one layer of perceptrons can implement m classifications. By utilizing sufficiently low amplitude excitation, the propagation of SWs can be described by a linear wave equation. Therefore, the linear transformation $\mathbf{W}$ can be achieved through a block of SW scatterers. The input signal excites SWs through the antennas, which propagate in the film and form an interference pattern reflecting the magnonic index of refraction of the film. Consequently, different SW intensities are observed at the outputs. However, since SW propagation is linear in this scenario, the application of the nonlinear activation function σ must be performed after the readout.

To create the desired interference pattern, we design the distribution of the magnonic index of refraction within the film. According to the dispersion of the SWs, there are various possibilities for achieving this. In the study conducted by Papp et al. (2021) [1] a non-uniform external magnetic field was applied. A grid of nanomagnets was utilized assuming they exhibit strong perpendicular magnetic anisotropy (PMA). The external field landscape was generated by adjusting the direction of the magnets, either up or down. The advantage of this method is its reprogrammability, as the direction of magnetization of the nanomagnets can be independently switched. However, the system's degrees of freedom are limited by the number of PMAs that can be placed on top of the film. Another method is described in the work of Kiechle et al. (2022) [3], where they were able to modify the saturation magnetization (M_s) of the film using Focused Ion Beam (FIB) irradiation. In this case, it becomes possible to create refractive index patterns in the film with nanometer resolution. We experimentally verified our designs via direct FIB irradiation of YIG films. Our results show a good agreement between simulation and experiment, and we hope that this demonstration could ignite interest in inverse magnonics as a possible route toward nanoscale signal processors and computing devices. [4]

The goal of this work is to design and experimentally demonstrate a micromagnetic structure capable of functioning as a frequency demultiplexer. The geometry of the physical system can be seen in Figure 1.1b. The input signal is delivered through the coplanar waveguide on the left side, which excites spin waves in a 100 nm thick Yttrium Iron Garnet (YIG) film. The output is measured as the spin-wave intensity at predefined positions on the film (O1-O3). The magnons interact with each other, forming an interference pattern within the YIG layer. To utilize this setup for computational tasks, we need to

solve the inverse design problem, which involves determining the distribution of Ms that produces a specific input-output relationship through the formation of the interference pattern [1].

To perform the inverse design of the Ms distribution, we utilize our custom micromagnetic solver, SpinTorch [5], which is built upon the machine-learning framework PyTorch [6]. With SpinTorch, we can achieve a "design by specification" approach, where we specify the desired functionality of the device, and the algorithm discovers a structure that fulfills these requirements [1].

The experimental demonstration of our design was conducted in collaboration with researchers from Markus Becherer's group at the Technical University of Munich [7] and researchers from János Volk's group at the Centre for Energy Research [8]. During the project, we designed the desired Ms distribution and the corresponding dose map using SpinTorch. Throughout the project, we designed the desired Ms distribution and the corresponding dose map using SpinTorch. Subsequently, the samples were irradiated by Focused Ion Beam (FIB) at both the Technical University of Munich and the Centre for Energy Research. Spin-wave propagation was measured at the Technical University of Munich in our laboratory at Pázmány Péter Catholic University using time-resolved Magneto-Optical Kerr Effect (trMOKE) microscopy.

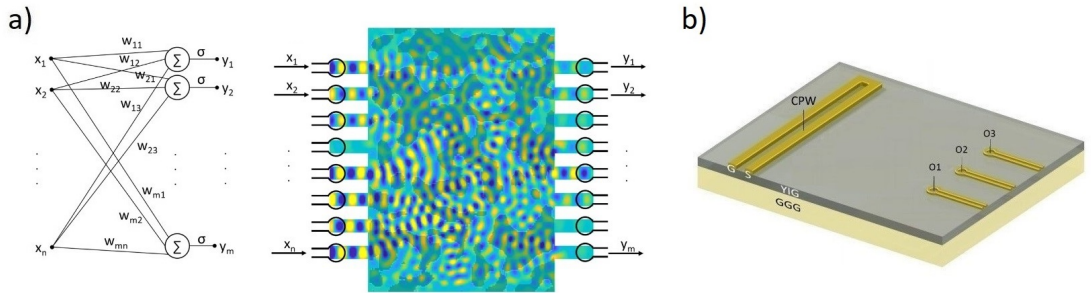


Figure 1.1: **a)** Representation of the linear transformation of a perceptron layer as a SW scatterer. *Source: [1]* **b)** The geometry of the physical system. The input arrives at the coplanar waveguide (CPW) on the left side and excites spin-waves in the ferrimagnetic Yttrium Iron Garnet (YIG) thinfilm, which is evaporated to the Gadolinium Gallium Garnet (GGG) layer. The output is the spin-wave intensity measured at predefined positions on the film (O1-O3). *Source: [1]*

1.2. Outline

In our work my tasks were to enhance SpinTorch to enable more realistic simulations for designing measurements. To determine the parameters of the YIG samples using data provided by the researchers at the Technical University of Munich. To design the micromagnetic structure using SpinTorch to achieve the desired SW interference pattern. Finally, to maintain communication with the researchers conducting the experiments, discuss the results, and address any upcoming issues.

Chapter 2 Most common finite difference micromagnetic codes offer the option to use predefined external magnetic fields but often lack the capability to address electromagnetic problems. As part of my work, the SpinTorch solver was enhanced to facilitate the modeling of interactions between the magnetic YIG thin film and the electromagnetic structures situated on top of the YIG layer. This expansion aims to generate more precise and realistic simulations, which can prove valuable for experiment design. The first extension involved incorporating the magnetic field of the Coplanar Waveguide source into SpinTorch. The second extension aims to improve the algorithm for calculating the induced voltage in the pickup antenna based on the magnetization of the YIG film.

Chapter 3 To design a geometry on the YIG film for a specific purpose, it is essential to comprehend how the M_s of the YIG sample will evolve with the ion dose. For the experimental setup, two YIG samples, named YIG328 and YIG331, were prepared. This chapter describes our method for characterizing the samples and investigating the relationship between the FIB dose and the resulting change in saturation magnetization.

Chapter 4 The aim of this project was to design and experimentally realize a wavelength demultiplexer, directing waves below one wavelength to one output and waves above that wavelength to the other output. This chapter describes the progress in the design and experimental work.

Chapter 5 This chapter summarizes the results of the work and emphasizes its relevance.

Chapter 2

Improving the SpinTorch micromagnetic solver

2.1. Integration of the magnetic field of the coplanar waveguide source into SpinTorch

The block diagram of SpinTorch is illustrated in Figure 2.1. The micromagnetic simulator core, shown in the green box at the center, solves the Landau–Lifshitz–Gilbert equation by iteratively advancing through time using the 4th order Runge-Kutta method. The system reads the SW intensity at predefined areas (outputs), and evaluates the configuration’s performance using a loss function based on the power ratio between desired and non-desired outputs. In the subsequent step, the error is propagated backward and the training parameters are updated. An advantage of utilizing PyTorch is its integrated gradient calculation, making it sufficient to provide the appropriate parameters to the system, which then automatically backpropagates the error.[9]

The excitation field is introduced to the system via the external magnetic field (represented as B_{ext} in the figure). Previously, a line source was employed, resulting in excitation occurring solely along a single line within the geometry. However, in the experimental setup, coplanar waveguides (CPW) are utilized for exciting SWs. These waveguides possess a more intricate magnetic field profile. In order to achieve simulations that are more accurate and realistic, the magnetic field of the CPW was integrated into the SpinTorch.

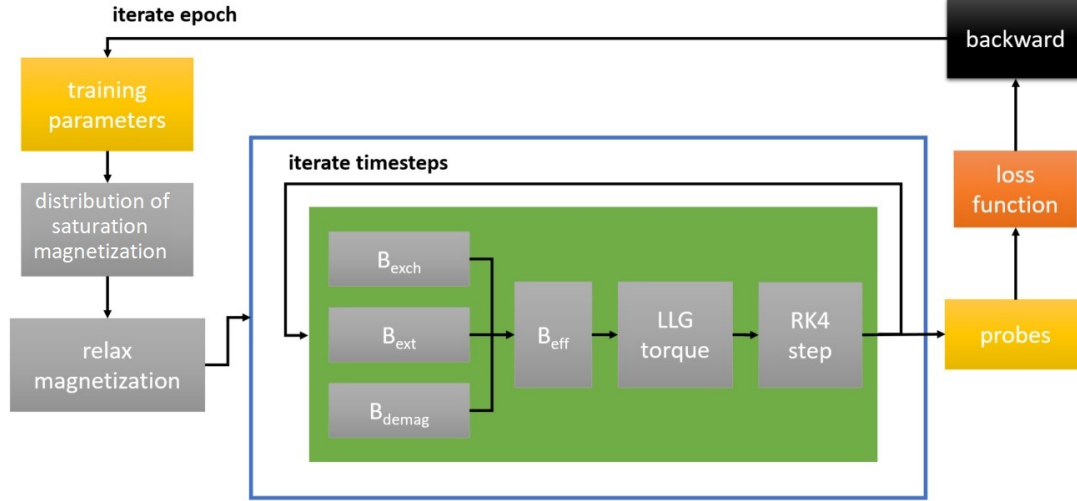


Figure 2.1: The block diagram of SpinTorch micromagnetic solver. *Figure from: [9]*

The magnetic field of the CPW was computed by Róbert Erdélyi, a member of our research group, utilizing the Finite Element Method Magnetism (FEMM) software. The CPW comprises 3 parallel copper striplines on the top of the YIG film, each with a thickness of $1\ \mu\text{m}$ and a width of $4\ \mu\text{m}$. The gaps between the lines measure $4\ \mu\text{m}$ in width. For the simulation, a current of $1\ \text{mA}$ flowed through the signal line (middle stripline), while the ground lines carried a current of $-0.5\ \text{mA}$. The excitation used was a sinusoidal signal at $3\ \text{GHz}$.

The outcome, depicted in Figure 2.2, where in Figure a), a slice of the magnetic field is displayed, while Figure b) presents the magnetic field components along the YIG film plane. Notably, in contrast to the previous line source scenario, we observe two non-zero components of the excitation field instead of just one.

The outcome of the FEMM simulation was saved to an OHF file. To enable the reading of this file, a new function called "OOMMF2torch" has been integrated into SpinTorch. This function is designed to read the OHF file and provide the magnetic field as a Torch tensor. In the case of the current geometry, it returns the CPW field within the YIG film plane as a tensor with dimensions of $n_x \times n_y \times 3$, where the third dimension corresponds to the x, y, and z components of the magnetic field. To use the CPW field as excitation, a new source class called "CPWSource" has been added to SpinTorch, which accepts the loaded magnetic field tensor and utilizes it as an excitation mask. In the simulation, this mask was multiplied by a sinusoidal signal.

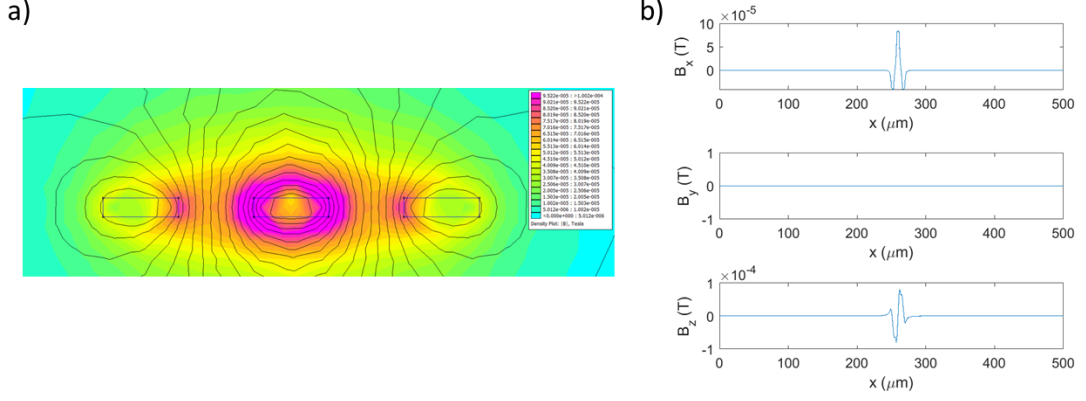


Figure 2.2: Magnetic field profile of the CPW. **a)** The slice of the magnetic field. **b)** The magnetic field components along the YIG film plane.

As a demonstration a training was conducted in SpinTorch on a $500 \mu\text{m} \times 400 \mu\text{m}$ area, which was bounded by high dose regions on the sides. The area was divided into 500×400 pixels with a pixel size of $1 \mu\text{m} \times 1 \mu\text{m} \times 540 \text{ nm}$. The excitation amplitude was set to 1 mT with a frequency of 3 GHz. The intrinsic M_s of the material was set to 140 kA/m, the maximum available M_s to 140.3 kA/m, the minimum M_s to 139.7 kA/m. The bias field was 278 mT. The damping coefficient was set to $5 \cdot 10^{-4}$. The task was to focus the SWs to a specified position in the geometry.

The simulation results are presented in Figure 2.3, where Figure a) displays the distribution of M_s optimized by SpinTorch, and Figure b) shows the steady-state waveform. The vertical lines on the left side demonstrate the coplanar waveguide, and the red circle on the right indicates the position where the SWs should be focused.

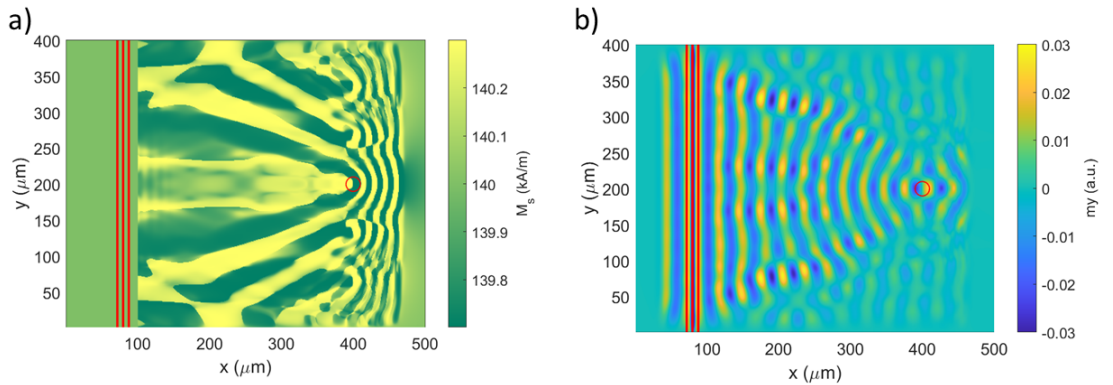


Figure 2.3: Demonstration of the method through spin-wave focusing lens. **a)** M_s distribution optimized by the SpinTorch micromagnetic solver. **b)** The steady-state waveform of the SpinTorch simulation.

2.2. Calculation of the induced voltage in the pick-up antenna

The other electromagnetic structure to be calculated is the pick-up antenna on the receiver side. The schematic of the geometry is shown in Figure 2.4a). The CPW exciting spin waves SWs in the YIG film is represented by the dark stripes on the left side of the film. The SWs are propagating towards the pick-up CPW antenna, denoted by the stripes on the right side.

The flow chart of computing the induced voltage in the output antenna is shown in Figure 2.4b). To calculate the induced voltage, the vector potential was first computed from the magnetization as follows:

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\nabla' \times \mathbf{M}(\mathbf{x})}{|\mathbf{x} - \mathbf{x}'|} dV' + \frac{\mu_0}{4\pi} \oint_S \frac{\mathbf{M}(\mathbf{x}) \times \mathbf{n}'}{|\mathbf{x} - \mathbf{x}'|} da'$$

The first term of the sum is a volume integral, where $\mathbf{A}(\mathbf{x})$ represents the vector potential, $\mathbf{M}(\mathbf{x})$ represents the magnetization at the point $\mathbf{x}$, $|\mathbf{x} - \mathbf{x}'|$ represents the distance between the point of interest $\mathbf{x}'$ and the specified position. The second term of the sum is a surface integral, where $\mathbf{n}'$ denotes the normal vector of the surface. [10] From the magnetic vector potential the electric field can be calculated as follows:

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

Here, $\mathbf{E}$ represents the electric field, and ϕ represents the electric potential, which is currently set to 0.[10] In the final step, the voltage is calculated from the electric field using the following formula:

$$\Delta V_{AB} = - \int_p \mathbf{E} \cdot d\mathbf{l}$$

Where V_{AB} represents the voltage.[10]

To implement the voltage calculation, a new modul was created in spintorch called *PickupCPW*, which obtains the magnetization for a predefined number of timesteps. In the code, the curl of the magnetization is computed implemented as a function called *curl*. Since PyTorch does not have a built-in curl calculation, the *torch.gradient()* function was employed for this purpose, and the curl was calculated as follows:

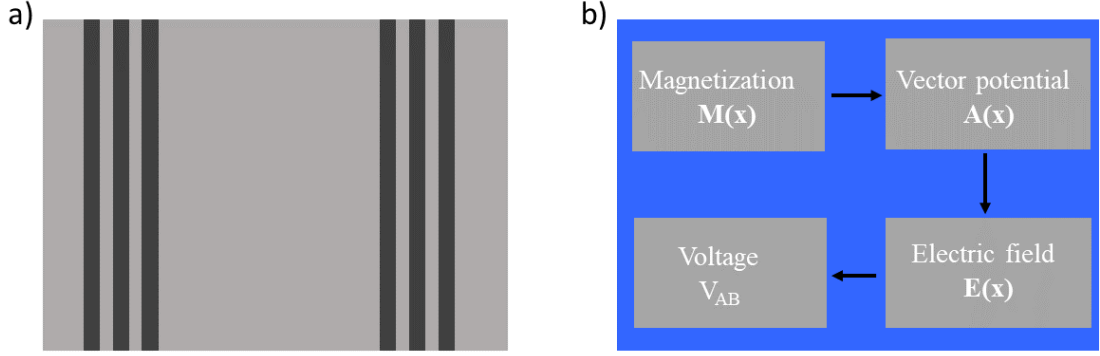


Figure 2.4: **a)** Schematic of the geometry. The input coplanar waveguide is represented by the dark stripes on the left side of the film. The SWs are propagating towards the pick-up CPW antenna, denoted by the stripes on the right side. **b)** The flow chart depicts the computation of the induced voltage in the output antenna. Starting with the magnetization provided by the micromagnetic solver, the vector potential is calculated. Using this information, the electric field is then determined. Finally, the induced voltage is computed based on the electric field.

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{\mathbf{i}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \hat{\mathbf{j}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{\mathbf{k}}$$

The challenge in this task arises from the fact that the *torch.gradient()* function can only handle $R^3 \rightarrow R^3$ vector fields, whereas in this case, we have $R^2 \rightarrow R^3$. To address this issue, for the computation, the magnetic layer (matrix) is duplicated in the z-direction. After the computation, one layer is removed from the result.

The *curl* function acquires the magnetization tensor at each timestep and yields the curl of the tensor. With the result of the curl, the code proceeds to calculate the vector potential from the magnetization, the electric field from the vector potential, and ultimately the voltage from the electric field, employing the aforementioned formulas. To reduce both runtime and memory consumption, the vector potential, electric field, and voltage are computed solely at the position of the pickup antenna.

To test the algorithm, a simulation was created in SpinTorch using the same magnetic parameters and CPW source as in Section 2.1. The difference is that here, only plane waves were simulated without optimizing the geometry for a specific task. The SW interference pattern is shown in Figure 2.5a). In this simulation the position of the pick-

up antenna ranges from 350 to 362 μm in the x-direction, running parallel to the CPW, represented as the red stripe on the right side of the figure. The z-coordinate of the antenna is 420 nm.

The results are illustrated in in Figure 2.5b), where the voltage was computed along the 12 lines one by one in the region of the output antenna, presenting the $V(t)$ curve for the first filament. As this is the first simulator capable of computing voltage from SWs, the results cannot be directly compared to a reference. Robert Erdélyi, from our research group, is currently working on enhancing our model and validating it using experimental results.

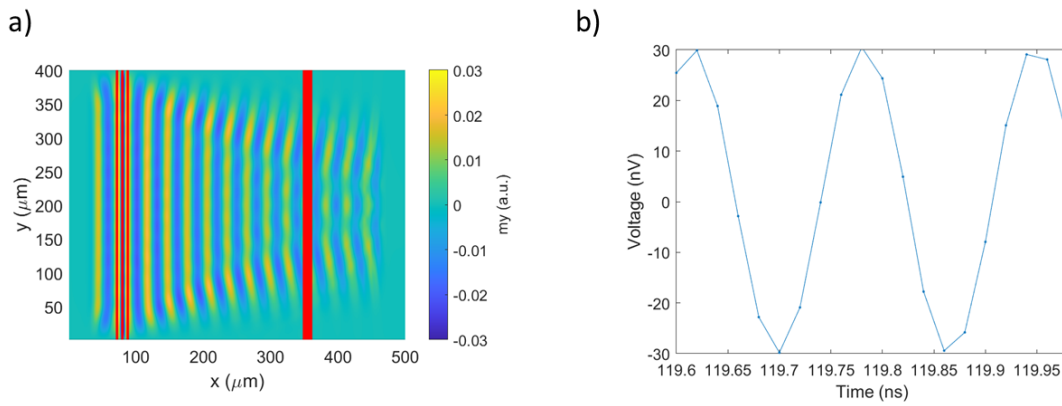


Figure 2.5: **a)** Wave snapshot from the SpinTorch simulation. **b)** Voltage calculated in the first filament.

Here, an example is also provided where the electromagnetic module was employed to enhance the coupling between the CPW source antenna and the magnetic film. The basic concept is that, through FIB irradiation, it is possible to modify the saturation magnetization (M_s) of the magnetic material. The objective is to enhance the efficiency of SW excitation by designing the M_s pattern beneath the source antenna.

Therefore, a simulation was created over a $250 \mu\text{m} \times 200 \mu\text{m}$ area, which was divided into 250×200 pixels with a pixel size of $1 \mu\text{m} \times 1 \mu\text{m} \times 540 \text{ nm}$. The frequency of the excitation was set to 3 GHz. The intrinsic M_s of the material was specified as 140 kA/m, the maximum available M_s as 140.5 kA/m, the minimum M_s as 139.5 kA/m. The bias field was 274 mT, and the damping coefficient was set to $5 \cdot 10^{-4}$. The time step was specified as 20 ps and the simulation time was 120 ns.

The system's geometry is illustrated in Figure 2.6a). The input CPW is represented

by the gray stripes on the left side at $x = 50 \mu\text{m}$, while the output CPW is on the right side at $x = 185 \mu\text{m}$. A $40 \mu\text{m} \times 200 \mu\text{m}$ area was defined around the input waveguide to optimize the M_s distribution, denoted by the red box. In this case, the 12 vertical lines where voltage was computed did not form a connected area; instead, 4 lines were calculated within each stripe. The overall voltage was determined as the sum of the three stripes, with the voltage of the middle stripe being subtracted from the voltage of the other stripes.

The optimization was carried out for 40 epochs using a custom loss function $L = -V_{\text{output}} * 1e7$, where V_{output} represents the amplitude of the sinusoidal signal at the output. During the training, the built-in Adam optimizer was employed with a learning rate set to 0.5.

The results are presented in Figure 2.6. In Figure a), the M_s distribution after optimization is depicted. Figure b) illustrates the SW interference pattern generated by SpinTorch. Figure c) shows a slice of the magnetization at the $y = 100 \mu\text{m}$ line. It's noticeable that the difference in SW amplitude is only a few percent when compared to the reference simulation with homogeneous M_s . On the other hand, the increase in amplitude is significantly higher in the output signal, as shown in Figure d).

The physics governing the optimization operation is not fully understood at this stage. The results indicate that modifying the M_s pattern can increase the amplitude of the excited spin waves (SW) by enhancing the coupling between the waveguide and the magnetic material. The exact reason for the increased output amplitude has not been determined yet. On one hand, it is evident that a higher SW amplitude leads to a greater output signal. However, the increase in the output is more substantial than what the increase in the SW amplitude alone would account for. It is possible that the optimized pattern alters the waveform in a manner that strengthens the induced voltage through the emerging interference pattern beneath the output antenna.

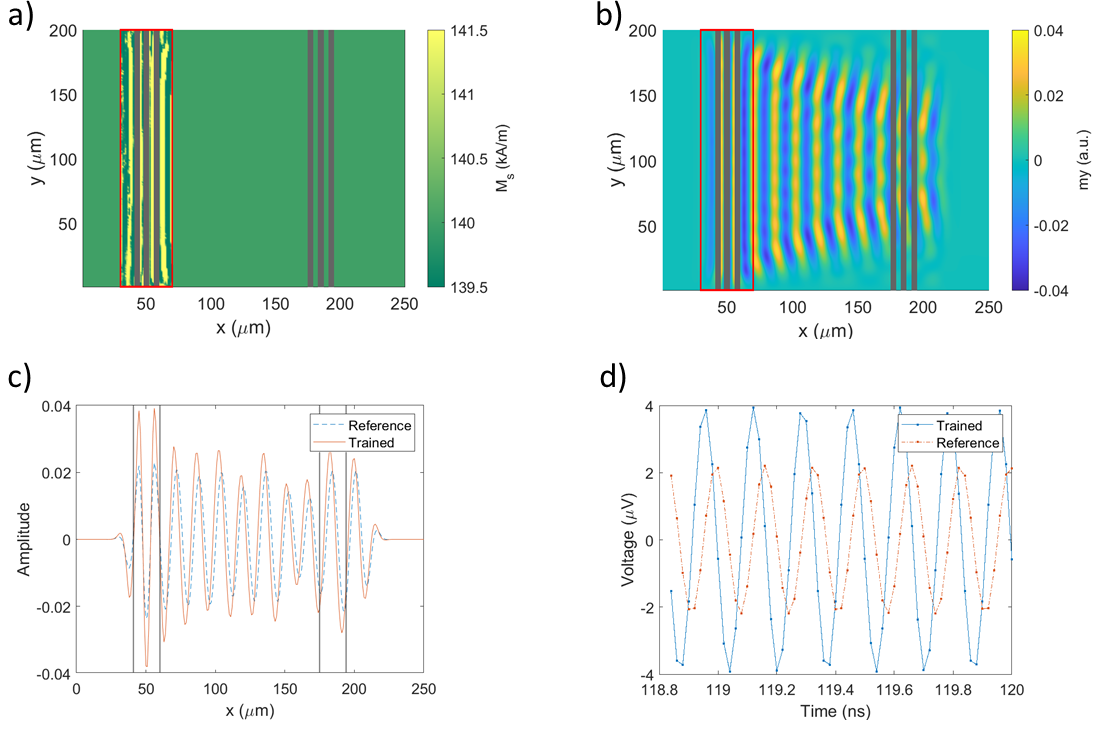


Figure 2.6: **a)** The M_s distribution optimized by SpinTorch to maximize the signal on the output waveguide. **b)** Spin-wave interference pattern generated by SpinTorch using the optimized M_s distribution. **c)** A slice of the magnetization at the $y = 100 \mu\text{m}$ line compared to the reference simulation with homogeneous M_s . **d)** The voltage on the output antenna computed by the *PickupCPW* modul.

Chapter 3

Estimation of the parameters of the YIG film

3.1. Dispersion relation for spin-waves

In physics, the dispersion curve illustrates the relationship between the wave number ($k = 2\pi / \lambda$) and frequency. In the case of spin waves, this interdependence is highly complex due to their propagation being influenced by multiple interactions.

On one hand, the magnetic dipole-dipole interaction determines the shape of the dispersion curve at longer wavelengths. Another influential factor is the exchange interaction, which operates at shorter distances and is approximated as acting between adjacent spins. However here, it is significantly stronger than the dipole-dipole interaction. As the wavelength decreases, the alignment of adjacent spins deviates increasingly from being parallel. Consequently, for sufficiently small wavelength ($\lambda < 100$ nm), the exchange interaction becomes the dominant factor. [11][12]

For SWs, the dispersion relation can be calculated as follows. The geometry of the magnetic material is depicted in Figure 3.1a). The waves propagate within a homogeneous magnetic layer with a thickness of d . The effective magnetic field (H_{eff0}) and the magnetization (M_s), which is parallel to it, form an angle θ with the normal vector of the magnetic field. The wave vector (k), which lies in the plane of the film, makes an angle φ with its projection perpendicular to the plane of the film, H_{eff0} . The circular frequency ω can be approximated by the following formula:

$$\omega^2 = \omega_H(\omega_H + \omega_M F(\varphi, \theta)), \quad (3.1)$$

where

$$\omega_H = \gamma\mu_0 \left(H_{eff0} + \frac{2A}{\mu_0 M_s} k^2 \right) \quad (3.2)$$

includes the influence of the exchange interaction, where k is the wave number, $\omega_M = \gamma\mu_0 M_s$. The magnetic dipole-dipole interaction is expressed by the function $F(\varphi, \theta)$:

$$F(\varphi, \theta) = P + \sin^2\theta \left[1 - P(1 + \cos^2\varphi) + \omega_M \frac{P(1 - P)\sin^2\varphi}{\omega_H} \right], \quad (3.3)$$

where

$$P = 1 - \frac{1 - e^{-kd}}{kd}. \quad (3.4)$$

Hence the frequency $f = \frac{\omega}{2\pi}$. [13]

The condition for using the 3.1 equation is, that the magnetic material on which the spin waves propagate can be approximated with an almost 2-dimensional arrangement. In the case of a thicker geometry, the spins in the interior of the material perform a precessional movement with a larger amplitude than near the surface, but according to the current assumption, the waves on the surface behave in the same way as in the interior of the material. On the other hand the excitation amplitude must be the same everywhere along the thickness of the film.[13]

The equation 3.1 demonstrates a highly complex relationship between frequency and wave vector, in contrast to, for example, the linear dispersion curve of electromagnetic waves. This characteristic of spin waves offers various advantages, which are elaborated in detail in the article [11]. From an experimental design perspective, it's crucial to note that, as depicted in Figure 3.1b), the wavelength changes with variations in the external magnetic field at a given frequency. In other words, the external field value can be used to set the desired working wavelength. However, precise control of the external space is necessary for the measurements to function effectively.

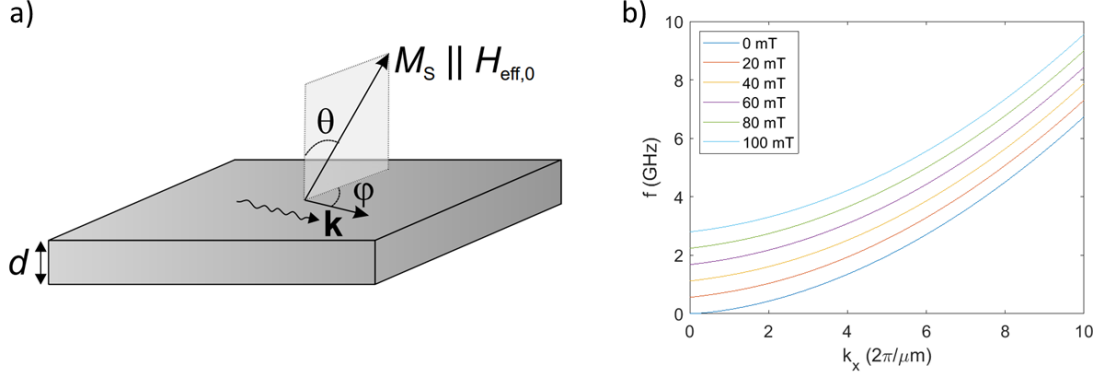


Figure 3.1: **a)** The geometry of the magnetic material, on which the dispersion relation can be calculated with the formula 3.1. The waves propagate within a homogeneous magnetic layer with a thickness of d . The effective magnetic field (H_{eff0}) and the magnetization (M_s), which is parallel to it, form an angle θ with the normal vector of the magnetic field. The wave vector (k), which lies in the plane of the film, makes an angle φ with its projection perpendicular to the plane of the film, H_{eff0} . *Figure from [13]* **b)** Dependence of the dispersion curve on the external magnetic field. The curves shown above were calculated with out-of-plane magnetization. To set the spins parallel to the normal of the surface, an external field of at least $\mu_0 M_s$ is required, which includes the curve marked 0 mT in the figure. In the other cases, the value indicated in the figure was added to this field. *Figure from [11]*

3.2. Investigating the relationship between the FIB dose and the resulting change in saturation magnetization

The theory regarding the effect of FIB irradiation on magnetic materials is still not fully clarified. Experimental results demonstrate that irradiation leads to a change in the wavelength of spin waves, which can be modeled by modifying M_s . However, this is a highly complex statistical process that likely affects several material parameters. Nonetheless, as the following chapters will demonstrate, for designing micromagnetic structures, it is sufficient to model the effect through M_s modification.

To quantify this effect, an experiment called "dosemap" was conducted by Martina Kiechle from the Technical University of Munich. In this measurement, various ion dose values ranging from 0 to 18×10^{12} ions/cm² were applied to the YIG film at different locations, and the propagation of SWs was measured using tr-MOKE microscopy. The SW

pattern is depicted in Figure 3.2a). As the figure illustrates, the wavelength decreases from 0 to approximately 7×10^{12} ions/cm², and then increases again above this dose value. In this project, only the decreasing region was utilized. However, the behavior of the high dose regions is currently under active research.

From the experimental results, the wavelength corresponding to each dose value was approximated using spatial Fourier transform. The results are presented in Figure 3.2b). Knowing that the thickness of the film is 99 nm, the excitation frequency was 2.05 GHz, and the external magnetic field was 214.1 mT perpendicular to the plane of the film, the M_s of the YIG can be estimated utilizing the dispersion relation of SWs described in Section 3.1. Consequently, dose- M_s data points were calculated, on which a 4th order polynomial curve was fitted illustrated in Figure 3.2c). It is evident that for the experiments, the available M_s values range from 116.23 kA/m to 118.72 kA/m.

Another parameter to be estimated is the damping coefficient (α) of the film. To determine this, simulations were performed using the same parameters as in the experiment, with several different values of α . The value that best matches the experimental results was found to be 5×10^{-4} .

The same steps were repeated for the YIG331 sample. In this case, the sample had a thickness of 100 nm, an excitation frequency of 2.1 GHz, and an external magnetic field of 215 mT with the same orientation. The measured SW pattern is shown in Figure 3.3a), and the estimated dose- M_s curve is presented in Figure 3.3b). It's evident that in this case, the available M_s range falls between 115.4 kA/m and 117.39 kA/m. The damping coefficient was found to be 17×10^{-4} in this case.

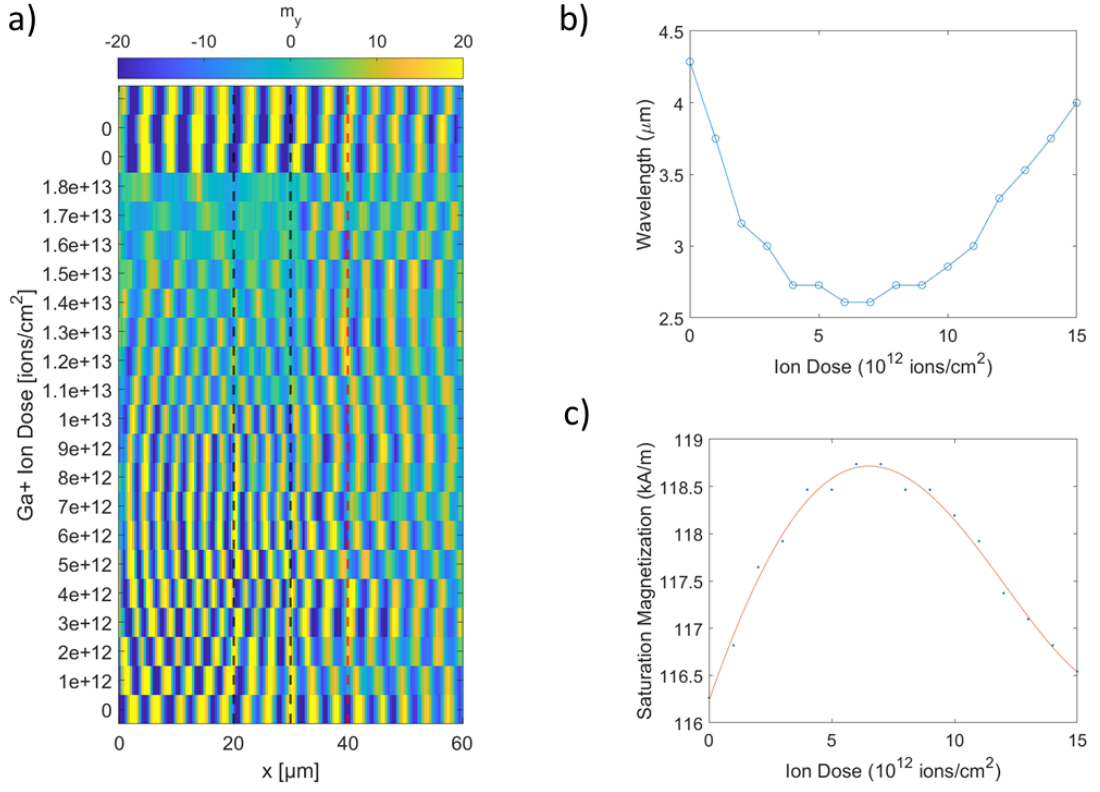


Figure 3.2: **a)** Result of the dosemap experiment on the YIG328 sample measured by tr-MOKE microscope. **b)** The relationship between the ion dose of FIB irradiation and the wavelength of the SWs. **c)** The relationship between the ion dose of FIB irradiation and the saturation magnetization in the model.

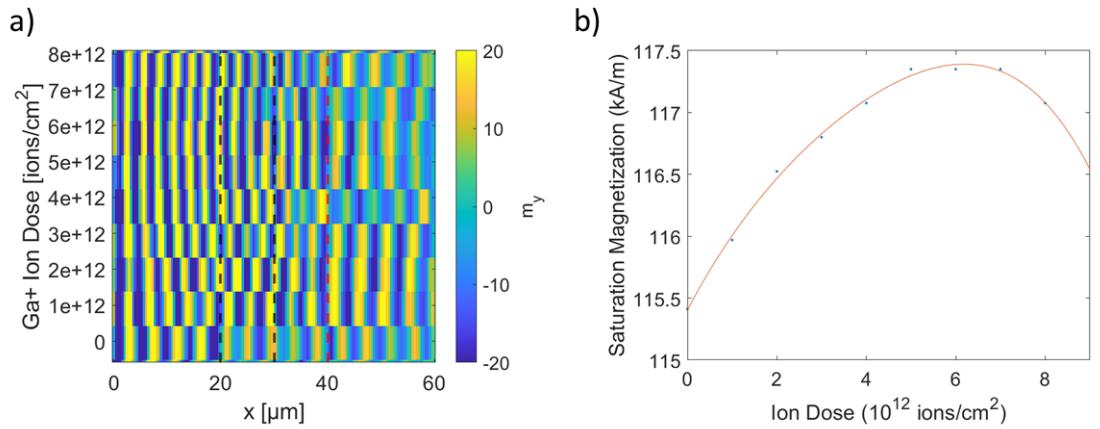


Figure 3.3: **a)** Result of the dosemap experiment on the YIG331 sample measured by tr-MOKE microscope. **b)** The relationship between the ion dose of FIB irradiation and the saturation magnetization in the model.

Chapter 4

The design and experimental demonstration of wavelength demultiplexer

Modern microprocessors incorporate tens to hundreds of cores on a single die to enhance performance. To optimize their functionality, a substantial bandwidth is required for the cores to access data stored in both local and distant caches, as well as in off-chip main memory [14]. Achieving the necessary bandwidth for efficient communication among the cores and off-chip memory remains a challenging task. An effective solution involves leveraging advanced multiplexing technologies, such as wavelength-division multiplexing (WDM), a crucial technology in contemporary optical communications[14][15]. Wavelength multiplexers and demultiplexers play a pivotal role in combining and separating multi-wavelength lights, respectively, serving as key components in WDM systems[15]. In optics, various solutions have emerged for wavelength separation[16][17]. The concept of multiplexing is also intriguing for new approaches to data processing, such as magnonics, where wave-based logic can be realized by utilizing interference effects of spin waves[18]. Here, we present a solution for demultiplexing waves based on wavelength, utilizing a spin-wave-based demultiplexer.

4.1. Designing wavelength demultiplexer on the YIG328 sample

For the optimization our custom micromagnetic solver SpinTorch was utilised. During the training a $70\text{ }\mu\text{m} \times 74$ geometry was simulated, which was divided into 700×740

cells, with a cell size of $100 \text{ nm} \times 100 \text{ nm} \times 99 \text{ nm}$. Within this geometry, a $40 \text{ }\mu\text{m} \times 50 \text{ }\mu\text{m}$ area was dedicated for optimization, represented by the red rectangle in Figure 4.1a). The output regions were placed from the edge of the trained region, as illustrated by the red circles in the figure. Additionally $2 \text{ }\mu\text{m}$ wide stripes were designed on the sides of the trained region. In these stripes, the M_s value was set to be 112 kA/m , significantly lower than the intrinsic M_s of the sample. As a result, no propagating SWs were simulated in these regions. The purpose of these walls is to separate the trained area from other parts of the film. Previous experiments have shown that interactions with these regions can have a negative impact on performance.

The parameters were set based on measurements. The intrinsic M_s of the material was set to 116.23 kA/m , with the maximum available M_s set at 118.72 kA/m . The temporal resolution was set to be 20 ps , the simulated time was 144 ns . The temporal resolution was 20 ps , and the simulated time was 144 ns . The applied bias field was 214.1 mT , with an excitation amplitude of 1 mT . Two different wavelengths were excited by two frequencies. The longer wavelength was $4.6 \text{ }\mu\text{m}$, corresponding to 2.05 GHz , while the shorter wavelength was $3.2 \text{ }\mu\text{m}$, corresponding to 2.1 GHz .

The result of the optimization of the M_s distribution is shown in Figure 4.1a). As the figure illustrates, the trained pattern is almost binary in this case. To validate the result of the SpinTorch simulation, SW propagation on the trained pattern was computed using another micromagnetic solver named Mumax3, utilizing the design frequencies and wavelengths.

In the Mumax3 simulation, only two M_s values were used, the intrinsic (116.23 kA/m) and the maximal (118.72 kA/m) in the trained region. The size of the simulated area was the same, but the spatial resolution was twice as fine as in the SpinTorch simulation, and periodic boundary conditions were defined on the sides to make the simulation more realistic.

The interference pattern in the Mumax3 simulation is shown in Figure 4.1b) for the 2.05 GHz excitation and in Figure 4.1c) for the 2.1 GHz excitation frequency. As the figures illustrates, the wavelength splitting is working well at the simulation level. Figure 4.1d) shows a case where the two excitation frequencies were applied at the same time. In practice, the excitation signal was the sum of two sinusoids with the two frequencies. As the figure represents, the geometry splits the two wavelengths.

During the training, only two frequencies were used as inputs, so the system was optimized for two wavelengths. To assess its performance with other wavelengths, a series of simulations were conducted with input frequencies ranging from 2 to 2.15 GHz, and the SW intensity at the outputs was integrated. The results are shown in Figure 4.2. As visible, the separation works effectively for frequencies that are close to the the trained frequencies, but the performance is weaker for wavelengths that are far for the wavelengths which was used during the training. The reason for using only two frequencies is that the primary aim of the project was experimental realization, and we can effectively excite these frequencies on this sample. Therefore, we optimized for the measurements. A next step in the simulations will be to design a geometry that can achieve frequency splitting for a wide frequency range.

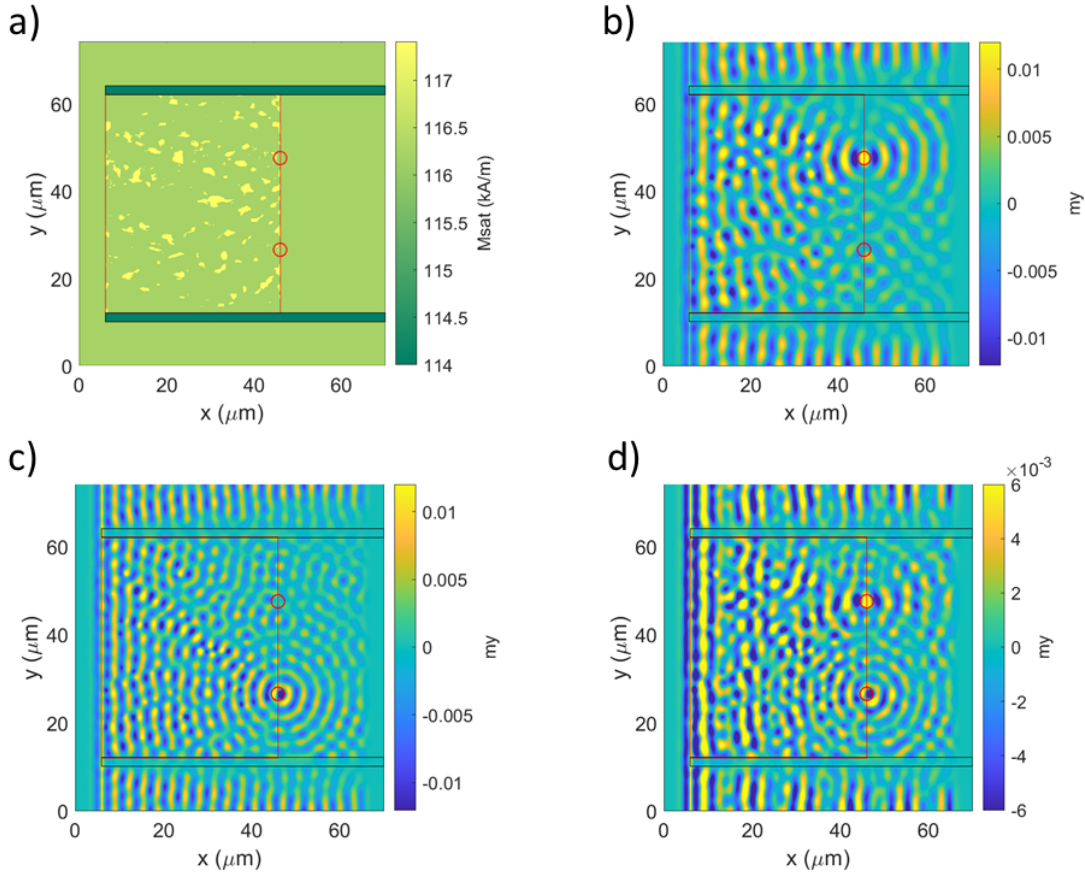


Figure 4.1: **a)** The M_s distribution optimized by the SpinTorch for the YIG328 sample. **b)** The SW interference pattern computed by the Mumax3 simulator applying 2.05 GHz excitation frequency. **c)** The SW interference pattern computed by the Mumax3 simulator applying 2.1 GHz excitation frequency. **d)** The SW interference pattern computed by the Mumax3 simulator applying and the 2.05 GHz excitation in the same time.

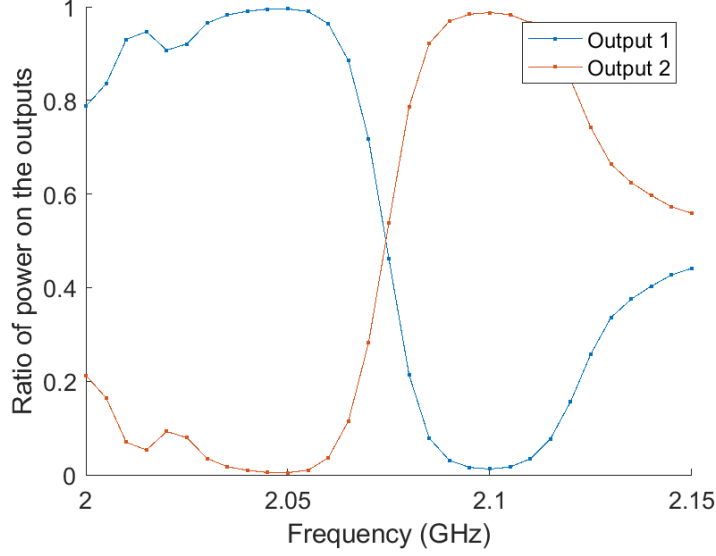


Figure 4.2: To assess its performance with other wavelengths, a series of simulations were conducted with input frequencies ranging from 2 to 2.15 GHz, and the SW intensity at the outputs was integrated. The figure shows the normalized output intensity on the to outputs for the simulated frequencies.

4.2. Designing wavelength demultiplexer on the YIG331 sample

Another geometry was optimized for the YIG331 sample. In this case, the size of the geometry was the same as described in Section 4.1, except for the thickness of the YIG film, which is 100 nm. The parameters were set based on measurements. The external field was set to be 215 mT with an excitation amplitude of 1 mT. The intrinsic M_s of the magnetic material was defined as 115.4 kA/m, with the maximal available M_s value set at 117.39 kA/m. The temporal resolution was 20 ps, and the simulated time was 144 ns, the same as in Section 4.1. The damping coefficient was set to be $17e-4$. This time, the two frequencies for excitation were 2.1 GHz and 2.15 GHz. The reason for these parameters is that experiments have shown they are a better choice for effectively exciting SWs on this sample.

The M_s distribution after the optimization is shown in Figure 4.3a). As illustrated, SpinTorch trained an almost discrete distribution in this case as well. In the Mumax3 simulation, only the intrinsic and maximal M_s values were used, and the spatial resolution was doubled. The results of the Mumax3 simulation are shown in Figure 4.3b) for the 2.1

GHz excitation and in Figure 4.3c) for the 2.15 GHz excitation frequency. As the figures illustrate, the wavelength splitting works effectively in the simulation once again. Figure 4.3d) shows a case where the two excitation frequencies were applied simultaneously. As the figure represents, the geometry also splits the two wavelengths in this case.

The performance of the SW splitting was assisted by a series of simulations with frequency ranging from 2.05 to 2.2 GHz and the SW intensity in the outputs were computed. The outcome is shown in Figure 4.2. As visible, the separation works effectively for frequencies that are close to the the trained frequencies, but the performance is weaker for wavelengths that are far for the wavelengths which was used during the training.

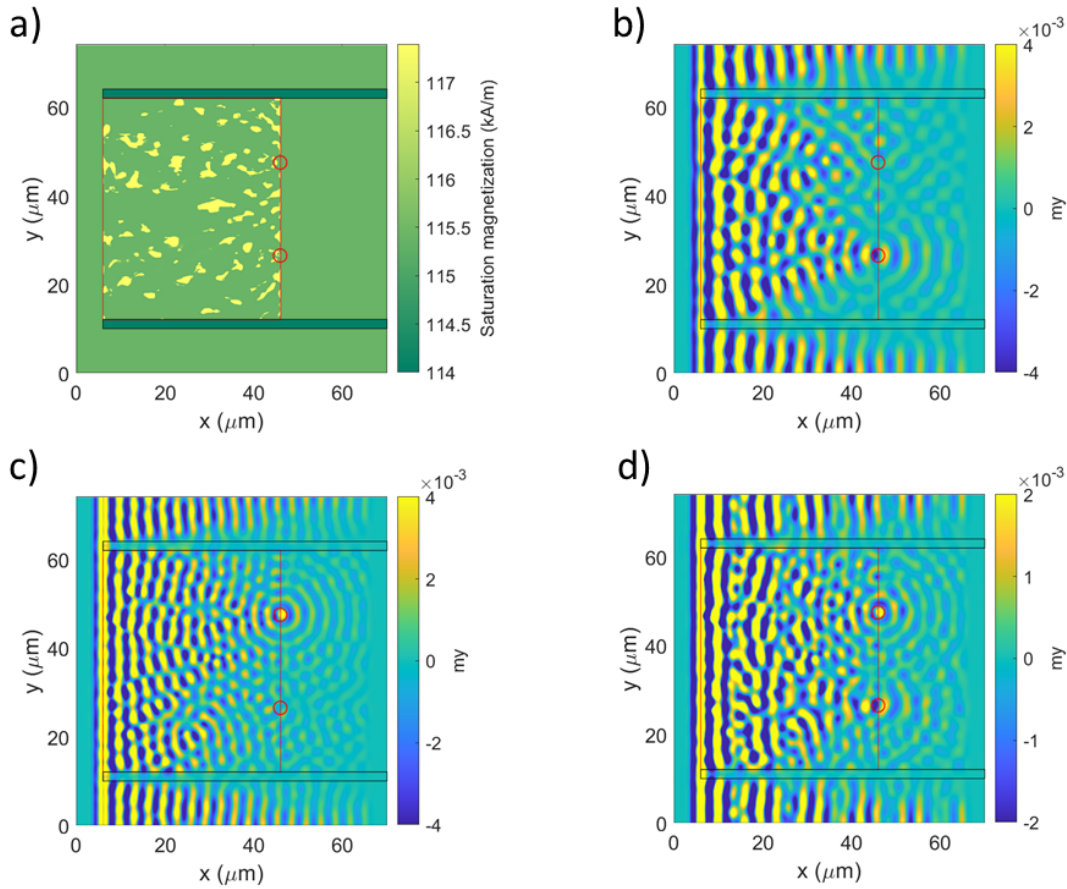


Figure 4.3: **a)** The M_s distribution optimized by the SpinTorch for the YIG331 sample. **b)** The SW interference pattern computed by the Mumax3 simulator applying 2.1 GHz excitation frequency. **c)** The SW interference pattern computed by the Mumax3 simulator applying 2.15 GHz excitation frequency. **d)** The SW interference pattern computed by the Mumax3 simulator applying and the 2.05 GHz excitation in the same time.

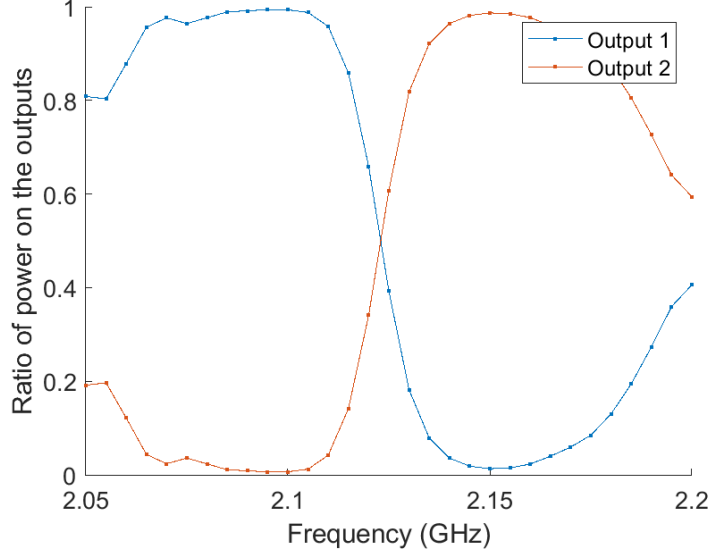


Figure 4.4: To assess its performance with other wavelengths, a series of simulations were conducted with input frequencies ranging from 2.05 to 2.2 GHz, and the SW intensity at the outputs was integrated. The figure shows the normalized output intensity on the to outputs for the simulated frequencies.

4.3. Experimental realization

The experiments were conducted by Martina Kiechle from the Technical University of Munich, who also fabricated the samples. The trained pattern was exposed using FIB (Focused Ion Beam) irradiation. With this technique, it is possible to image, analyze, or pattern materials on a sub-10 nm scale.[19] The FIB machine used at the Technical University of Munich generates a focused beam of Ga^+ ions applying 50 keV acceleration voltage, enabling it to scan the sample with a resolution on the order of 10 nm.

To measure the arising SW interference pattern time-resolved Magneto-Optical Kerr Effect (trMOKE) microscopy was used. A well known phenomenon is that the light reflected from a magnetized surface may change its polarization and intensity, called Magneto-Optical Kerr Effect. If we illuminate the magnetic material by polarized light and measure the the difference of the polarization of the reflected light, it is possible to measure the magnetization of the material. However in case of spin-waves the direction of the magnetization precessing with GHz frequency, thus a picosecond laser is utilized to measure the spin-wave dynamics. Although the repetition rate of the laser is 10 MHz, which is 2 orders of magnitude lower than the frequency of the excited spin-waves,

using stroboscopic measurement method, it is possible to undersample the signal on this frequency. The sample is illuminated by 405 nm polarized laser light, which will become circularly polarized after the reflection due to the polar MOKE effect[20]. The reflected light is split by a Wollaston prism[21] and directed to the two inputs of a balanced photodetector, that measures the difference of the two beam intensity. To improve the signal to noise ratio a Lock-In Amplifier is used[22].

In order to expose the desired M_s pattern using the FIB machine, it is necessary to transform the optimized M_s distribution into an ion dose distribution suitable for exposure. To perform this conversion, the estimated dose- M_s curves described in Chapter 3 were inverted. Using these inverted curves, the M_s values within the M_s matrix were converted into dose values, resulting in an ion dose distribution matrix. The FIB machine reads the dose pattern as a binary xbm image. Given that the pattern is nearly binary in this case, a thresholding process was applied to the trained region. The binary dose map was then converted back to a binary M_s map and simulated in Mumax3 to verify if the discretization of the pattern maintains the desired functionality.

The dose distribution exposed on the YIG328 sample is depicted in Figure 4.5a). For the FIB process, two binary xbm images were created. The first one corresponds to the highest available M_s in the trained region with a value of 6.4×10^{12} ions/cm². The other image corresponds to the walls on the sides of the trained region with a dose value of 20×10^{12} ions/cm².

The wave patterns from the tr-MOKE measurements on the YIG328 sample are shown in Figure 4.5b)-d). Figure b) illustrates the resulting SWs for the 2.05 GHz excitation, while Figure c) shows the SW pattern for the 2.1 GHz excitation frequency. As depicted, the experimental results exhibit good agreement with the simulations. However, when the two frequencies are applied simultaneously, as shown in Figure d), the separation is only recognizable to some extent, as the spikes due to frequency beating distort the image.

In the case of the YIG331 sample, the dose values for the xbm images were 6 ions/cm² for the trained geometry and 16×10^{12} ions/cm² for the separating walls. The tr-MOKE measurements reveal wave patterns under a bias field of around 214 mT, as displayed in Figures 4.6b)-d). Specifically, Figure b) showcases the resulting spin waves (SWs) for the 2.05 GHz excitation, while Figure c) displays the SW pattern for the 2.1 GHz excitation

frequency. Once again, the experimental results closely align with the simulated outcomes, demonstrating consistent agreement. However, when both frequencies are applied simultaneously, as illustrated in Figure d), the separation is only discernible to a certain extent, mirroring the observations made with the other sample.

The reason for choosing 2.09 GHz instead of 2.1 GHz is that this frequency provided a better match for the wavelength in the simulation. One possible reason for this could be that in the TUM tr-MOKE setup, the strength of the bias field cannot be precisely adjusted since it uses a permanent magnet to create the external field, making it not perfectly homogeneous.

In conclusion, the experimental results exhibit great agreement with the simulations when the input frequencies are applied separately. However, if they are excited simultaneously, the separation is not observable due to frequency beating. An apparent solution to enhance the visibility of the low-intensity parts is to increase the excitation; however, this would result in nonlinearity in the wave propagation. Equation 3.2 indicates that the wavelength of the SWs depends on the external magnetic field. If the excitation amplitude is high enough, the magnetic field of the SWs can influence their wavelength, resulting in a changing wavelength along the propagation. Since the geometry was designed for linear wave propagation, this would disrupt the function. Due to this phenomenon, better results have not been measured yet in the case of simultaneous excitation.

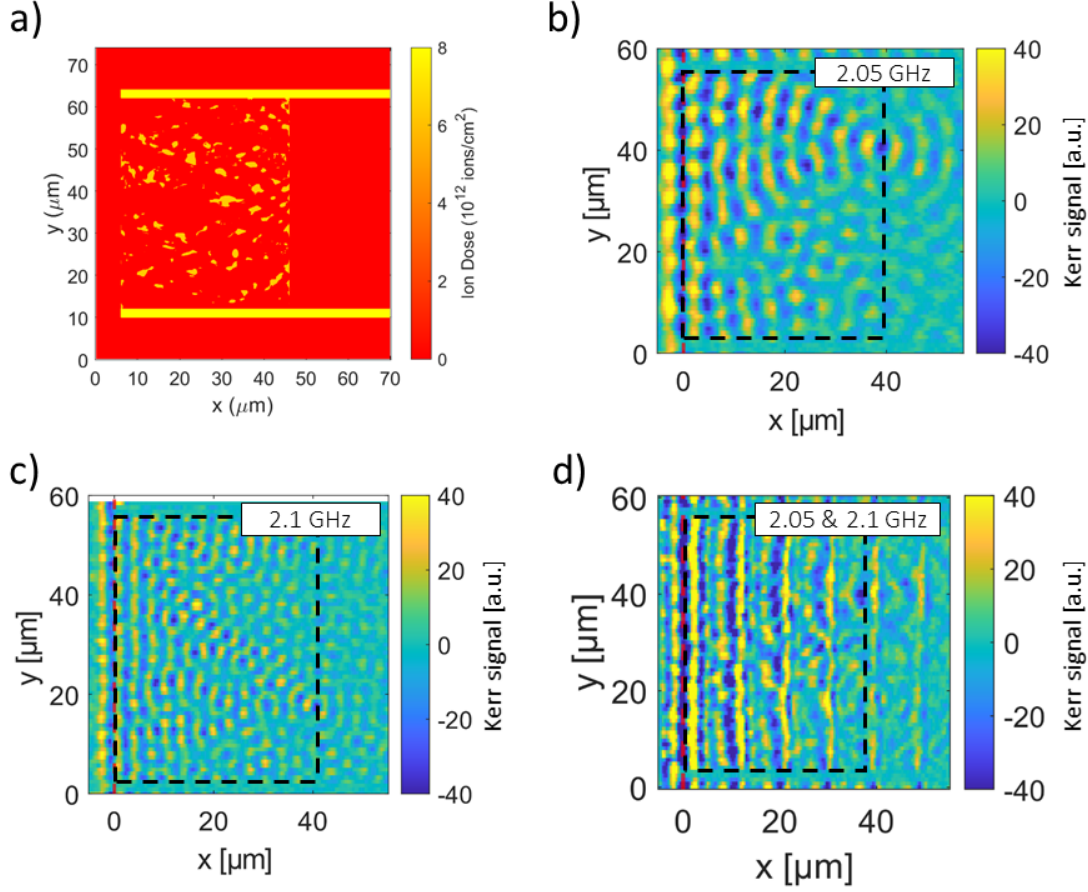


Figure 4.5: Experimental realization of the wavelength demultiplexer on the YIG328 sample. **a)** The ion dose pattern exposed by FIB. **b)** Time-resolved MOKE measurement applying 2.05 GHz excitation frequency. The wavelength of approximately $5 \mu\text{m}$ is focused to the higher output. **c)** Time-resolved MOKE measurement applying 2.1 GHz excitation frequency. The wavelength of approximately $3 \mu\text{m}$ is directed to the lower output. **d)** Time-resolved MOKE measurement applying the 2.05 GHz and 2.1 GHz excitation simultaneously result in a frequency beating effect.

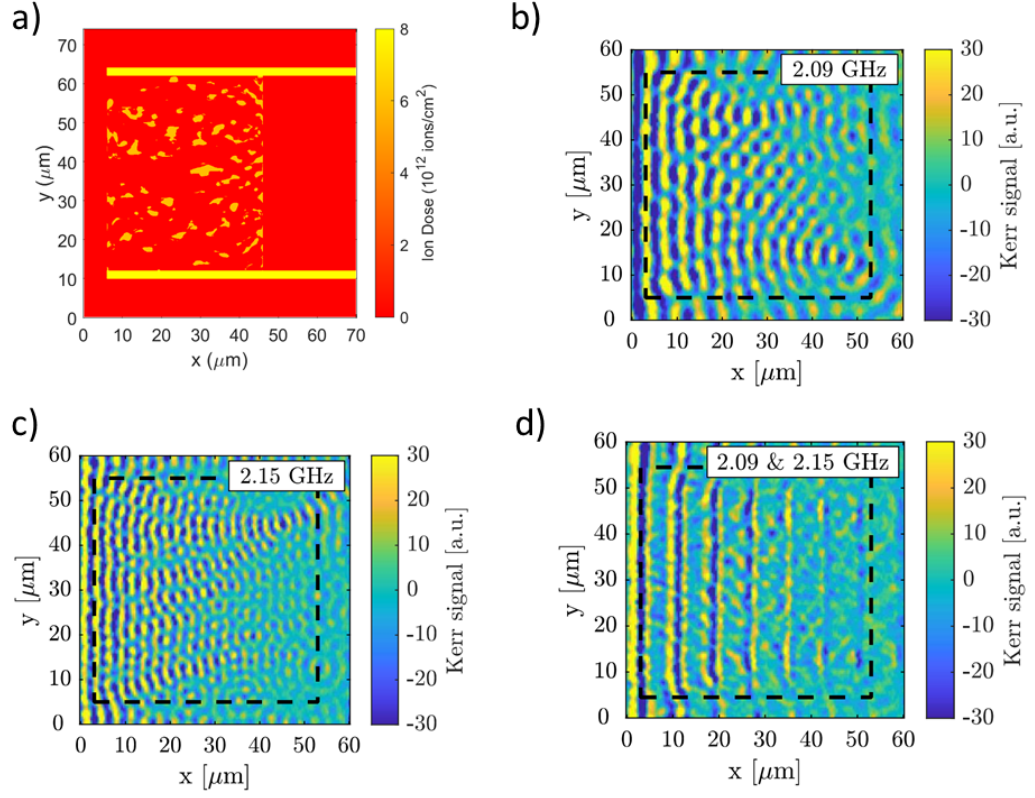


Figure 4.6: Experimental realization of the wavelength demultiplexer on the YIG331 sample. **a)** The ion dose pattern exposed by FIB. **b)** Time-resolved MOKE measurement applying 2.05 GHz excitation frequency. The wavelength of approximately $5 \mu\text{m}$ is focused to the lower output. **c)** Time-resolved MOKE measurement applying 2.1 GHz excitation frequency. The wavelength of approximately $3 \mu\text{m}$ is directed to the higher output. **d)** Time-resolved MOKE measurement applying the 2.05 GHz and 2.1 GHz excitation simultaneously result in a frequency beating effect.

Chapter 5

Summary

5.1. Problem statement

The goal of my work is to design and experimentally demonstrate a micromagnetic structure capable of functioning as a frequency demultiplexer. The input signal is a radio frequency wave, delivered through a coplanar waveguide, which excites spin waves in a 100 nm thick Yttrium Iron Garnet (YIG) film. The output is measured as the spin-wave intensity at predefined positions on the film. The magnons interact with each other, forming an interference pattern within the YIG layer. To utilize this setup for computational tasks, we need to solve the inverse design problem, which involves determining the distribution of M_s that produces a specific input-output relationship through the formation of the interference pattern.

Achieving the desired interference pattern is accomplished by locally modifying the magnonic index of refraction of YIG using Focused Ion Beam irradiation. The effect of the FIB exposure was modelled as a change in the saturation magnetization of the material. To perform the inverse design of the M_s distribution, we utilize our custom micromagnetic solver, SpinTorch, which is built upon the machine-learning framework PyTorch. With SpinTorch, we can achieve a "design by specification" approach, where we specify the desired functionality of the device, and the algorithm discovers a structure that fulfills these requirements.

My tasks were to enhance SpinTorch to enable more realistic simulations for designing measurements. To determine the parameters of the YIG samples using data provided by the researchers at the Technical University of Munich. To design the micromagnetic structure using SpinTorch to achieve the desired SW interference pattern. Finally, to

maintain communication with the researchers conducting the experiments, discuss the results, and address any upcoming issues.

5.2. Results

The improvements to the SpinTorch simulator are discussed in Chapter 2. To enhance the realism of simulations for measurements, a new module was incorporated into the system. This module can load the magnetic field of the wave source and use it as an excitation mask to stimulate the spin-waves. Additionally, another module was added to SpinTorch, capable of computing the voltage on the output antenna excited by the spin-waves. The operation was demonstrated through an example in which SpinTorch was utilized to design an M_s pattern beneath the excitation antenna to maximize the voltage on the output.

The process for characterizing the YIG samples used in the experiments is detailed in Chapter 3. Two samples, referred to as YIG328 and YIG331, were employed in this project. Using the time-resolved MOKE measurements provided by the researchers at the Technical University of Munich and the dispersion relations for spin-waves, the relationship between the FIB ion dose and the saturation magnetization was determined. The damping coefficient of the films was defined by fitting simulations to the measured results.

Our approach to designing and implementing a wavelength demultiplexer is detailed in Chapter 4. Using the SpinTorch micromagnetic solver, the parameters of the YIG samples were inverse-designed to create an M_s pattern capable of separating different frequencies based on the excited spin-wave wavelength. The optimized M_s distribution was then converted into an ion dose map for irradiation by the FIB machine. The experimental realization was conducted in collaboration with the researchers at the Technical University of Munich. The propagation of spin-waves in the irradiated samples was measured using time-resolved MOKE microscopy. The experimental results demonstrate great agreement with the simulations. However, simultaneous excitation reveals that the separation is not observable due to frequency beating.

5.3. Conclusion

The relevance of my work in improving the SpinTorch simulator lies in the fact that while most common finite difference micromagnetic codes offer the possibility to utilize predefined external magnetic fields, they often lack the capability to address electromagnetic problems. During my internship, I improved a micromagnetic solver to enable the modeling of electromagnetic structures on top of magnetic materials. Based on my work, SpinTorch will have the capability to model the behavior of electromagnetic components on top of the YIG film. Consequently, it can offer more reliable simulations for experiments. Additionally, it will have the potential to facilitate inverse design of antenna geometries, enhancing energy transfer between the electric and magnetic domains, and optimizing phase shifters and other RF devices.

The significance of our work in designing and experimentally realizing a structure capable of separating different frequencies lies in its importance as a crucial milestone toward creating complex spin-wave devices through inverse design. This would mark the first functional device designed by inverse design. In our previous work, we demonstrated how Focused Ion Beam irradiation can locally modify the refractive index of ferrimagnetic YIG thin films.[3] Additionally, on a simulation level, we showed that it is possible to design micromagnetic structures through inverse design to perform high-level computational tasks. [1] However, there is still a lack of experiments where the simulated functionality has been realized experimentally. As a first step, we demonstrated that our method is capable of designing and showcasing a spin-wave focusing lens. [4] In this work, we took a step further and demonstrated that our approach can provide a non-trivial micromagnetic structure capable of separating different frequencies based on the excited wavelength. [23]

Bibliography

- [1] Á. Papp, W. Porod, and G. Csaba, “Nanoscale neural network using non-linear spin-wave interference,” *Nature communications*, vol. 12, no. 1, p. 6422, 2021.
- [2] D. Marković, A. Mizrahi, D. Querlioz, and J. Grollier, “Physics for neuromorphic computing,” *Nature Reviews Physics*, vol. 2, no. 9, pp. 499–510, 2020.
- [3] M. Kiechle, A. Papp, S. Mendisch, *et al.*, “Spin-wave optics in yig realized by ion-beam irradiation,” *Small*, p. 2207293, 2023.
- [4] M. Kiechle, L. Maucha, V. Ahrens, *et al.*, “Experimental demonstration of a spin-wave lens designed with machine learning,” *IEEE Magnetism Letters*, vol. 13, pp. 1–5, 2022.
- [5] A-Papp, *A-papp/spintorch: Micromagnetic simulator with machine-learning capabilities*. [Online]. Available: <https://github.com/a-papp/SpinTorch>.
- [6] [Online]. Available: <https://pytorch.org/>.
- [7] Oct. 2023. [Online]. Available: <https://www.ee.cit.tum.de/nan/home/>.
- [8] Jun. 2022. [Online]. Available: <https://www.ek.hun-ren.hu/nanoerzekelok-laboratorium/>.
- [9] A. Papp, G. Csaba, and W. Porod, “Neuromorphic computing using spin-wave scatterers,” in *Joint European Magnetic Symposia*. 2020.
- [10] J. D. Jackson, *Classical electrodynamics*, 1999.
- [11] G. Csaba, Á. Papp, and W. Porod, “Perspectives of using spin waves for computing and signal processing,” *Physics Letters A*, vol. 381, no. 17, pp. 1471–1476, 2017. DOI: 10.1016/j.physleta.2017.02.042.
- [12] A. Chumak, *Introduction to spin waves*, 2021. [Online]. Available: <https://www.youtube.com/watch?v=V5b48utfpMQ&t=2213s>.

- [13] J. W. Klos and M. Krawczyk, “Magnonic crystals: From simple models toward applications,” in *Magnetic Structures of 2D and 3D Nanoparticles*, J.-C. S. Levy, Ed., 2016, ch. 8, pp. 283–312.
- [14] D. Dai and J. E. Bowers, “Silicon-based on-chip multiplexing technologies and devices for peta-bit optical interconnects,” *Nanophotonics*, vol. 3, no. 4-5, pp. 283–311, 2014.
- [15] C. Li, B. Xiong, and T. Chu, “Polarization-independent wavelength demultiplexer based on a single etched diffraction grating device,” *Communications Engineering*, vol. 2, no. 1, p. 4, 2023.
- [16] S. Xiao and A. M. Weiner, “2-d wavelength demultiplexer with potential for 1000 channels in the c-band,” *Optics Express*, vol. 12, no. 13, pp. 2895–2902, 2004.
- [17] A. Y. Piggott, J. Lu, K. G. Lagoudakis, J. Petykiewicz, T. M. Babinec, and J. Vučković, “Inverse design and demonstration of a compact and broadband on-chip wavelength demultiplexer,” *Nature Photonics*, vol. 9, no. 6, pp. 374–377, 2015.
- [18] F. Heussner, M. Nabinger, T. Fischer, *et al.*, “Frequency-division multiplexing in magnonic logic networks based on caustic-like spin-wave beams,” *physica status solidi (RRL)–Rapid Research Letters*, vol. 12, no. 12, p. 1800409, 2018.
- [19] J. Orloff, “Focused ion beam characterization techniques,” *Encyclopedia of Materials: Science and Technology*, pp. 3226–3231, 2001.
- [20] S. Yamamoto and I. Matsuda, “Measurement of the resonant magneto-optical kerr effect using a free electron laser,” *Applied Sciences*, vol. 7, no. 7, p. 662, 2017.
- [21] [Online]. Available: https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=917.
- [22] Dec. 2019. [Online]. Available: <https://www.zhinst.com/en/resources/principles-of-lock-in-detection>.
- [23] M. Kiechle, A. Papp, L. Maucha, *et al.*, “Frequency demultiplexing of spin waves by inverse-designed magnetization patterns, experimentally realized by fib irradiation,” in *2023 IEEE International Magnetic Conference-Short Papers (INTERMAG Short Papers)*, IEEE, 2023, pp. 1–2.