

An improved method for estimating the domain of attraction of nonlinear systems containing rational functions

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Abstract. An optimization based method is proposed in this paper for the computation of Lyapunov functions and regions of attractions for nonlinear systems containing polynomial and rational terms. The Lyapunov function is given in a special quadratic form, and the negativity of its derivative is ensured using appropriate LMI conditions. The conservatism of the solution is reduced by utilizing Finsler's lemma. The number of monomial and rational terms in the computational problem is kept as low as possible using linear fractional transformation (LFT) and automatic model simplification steps. The operation of the method is illustrated on two examples taken from the literature.

1. Introduction

Finding or at least approximating the region of attraction of nonlinear dynamical systems is an important task in model analysis and controller design/evaluation, and numerous works have been devoted to this issue (see, e.g. [1]). An important early result in this field is the existence of so-called maximal Lyapunov functions for a wide class of nonlinear systems and the corresponding iterative procedure to approximate them [2]. In [3], maximal Lyapunov functions were defined and computed for hybrid (piecewise nonlinear) systems. At the same time, the use of linear matrix inequalities (LMI) and semidefinite programming (SDP) techniques for nonlinear systems has become very popular due to their advantageous properties and the availability of efficient numerical tools to solve LMI problems. These new techniques provide a powerful framework for stability analysis, robust control, and filtering problems. Ghaoui et.al. [4] used quadratic Lyapunov functions and linear fractional transformations (LFT) to represent a rational nonlinear system and defined convex conditions for stability analysis and state feedback design. The application of sum of squares (SOS) programming to maximize the estimate of the region of attraction can be found in [1, 5]. Stability conditions in both references are converted into LMIs using SOS relaxations and the generalization of the S-procedure. Topcu et.al. [6] utilized a further branch-and-bound type refinement in the parameter space to reduce the solution's conservatism. According to Trofino et.al. [7], LMI conditions can be obtained by using the Finsler's Lemma and the notion of annihilators. The newly introduced sufficient conditions