

Girders with trapezoidally corrugated webs subjected by combined bending and shear

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Abstract

Girders with corrugated steel web is likely used in bridges as well as in building structures due to its numerous favourable properties. It is known from previous investigations that due to the web corrugation the normal and shear stress distribution in the flange and web plates of the corrugated web girders are different from the stress distribution of the conventional I-girders. The shear force in the corrugated web plate results in an additional transverse bending moment in the flanges, which indicates additional normal stresses. Previous investigations analysed this modified normal stress field and calculation methods for the maximum normal stresses were proposed. But there is a lack of investigations analysing the effect of the additional normal stresses on the bending resistance of the corrugated web girders. Different proposals can be found in the international literature to take this effect in the in-plane-bending resistance into account. Therefore this paper focuses on the determination of the transverse bending moment and its effect on the load carrying capacity of girders with trapezoidally corrugated webs.

Keywords: corrugated web girder; transverse bending moment; flange stress distribution; bending and shear interaction.

1. Introduction

In the last 30 years the corrugated steel plate has been increasingly used as a beam web. This structural layout has spread in the field of bridges as well as in the field of industrial buildings. Numerous studies detailed in Section 2 investigate the bending and shear resistance of the girders with corrugated webs. Different proposals were developed to determine the shear buckling resistance of the corrugated web considering the local and global buckling behaviour, and the interaction of the two buckling phenomena. Numerous previous investigations are also dealing with the determination of the bending resistance of the corrugated web girders. The main focus of these investigations is usually to determine the buckling resistance of the compressed flange plate under bending moment.

It is commonly accepted by the researchers and designers, that in case of corrugated web girders the main part of the bending moment is carried only by the flanges and the main part of the shear force by the corrugated web. It is also known from the previous investigations that the shear stress in the corrugated web indicates a transverse bending moment in the flanges, which results in additional normal stresses. Several previous investigations introduced in Section 2 analysed the additional normal stress distribution in the flange plates. Different proposal were developed to determine its maximum value, but there is a very small number of investigations focusing on the effect of the modified normal stress field on the bending resistance. Therefore the focus of this paper is the analysis of the bending resistance reduction of the girders with trapezoidally corrugated webs due to the transverse bending moment in the flanges.

There are several contradictory information in the international literature considering the influence of the additional normal stresses on the bending resistance. The design rule of EN1993-1-5 Annex D [1] gives a reduction factor for the bending resistance of the corrugated web girders, if the girder is also subjected by shear force. The bending resistance of the corrugated web girders according to [1] can be determined by the Eq. 1.

$$M_{Rd} = \min \left(\frac{b_{uf} \cdot t_{uf} \cdot f_{yf}}{\gamma_{M0}} \cdot \left(h_w + \frac{t_{uf} + t_{lf}}{2} \right); \frac{b_{lf} \cdot t_{lf} \cdot f_{yf}}{\gamma_{M0}} \cdot \left(h_w + \frac{t_{uf} + t_{lf}}{2} \right); \frac{b_{uf} \cdot t_{uf} \cdot \chi \cdot f_{yf}}{\gamma_{M0}} \cdot \left(h_w + \frac{t_{uf} + t_{lf}}{2} \right) \right) \quad (1)$$

If the girder is subjected also by shear force the bending resistance should be reduced by a reduction factor (f_T) which depends on the flange size and on the maximum normal stress coming from the transverse bending moment. The reduced bending resistance can be determined by Eqs. (2)-(3), if the value of χ is equal by 1,0.

$$M_{V,Rd} = f_T \cdot M_{Rd} \quad (2)$$

where:
$$f_T = 1 - 0,4 \cdot \sqrt{\frac{\sigma_x(M_z)}{f_{yf}}} \cdot \frac{f_{yf}}{\gamma_{M0}} \quad (3)$$

f_{yf} yield strength of the flange plate,

γ_{M0} partial factor for stress check,

$\sigma_x(M_z)$ maximum value of the additional normal stresses coming from transverse bending moment.

This reduction factor was developed by Lindner [2] and it was also implemented in the DAST-Richtlinie 015 [3]. However the same reduction factor is not considered in the Austrian and Swedish design rules, because it was stated that the effect of the additional normal stresses on the bending resistance is not too large. The commentary document of the EN1993-1-5 [1] also emphasizes that from theoretical point of view these bending moments are required for reasons of equilibrium, but it is questionable how important they actually are in real life [4]. Elgaaly et al. investigated the bending and shear resistance of the trapezoidally corrugated web girders in 1997-98 [5], [6] and [7]. Their conclusion was that there is no interaction between the bending and shear resistance of the trapezoidally corrugated web girders. It was proved by Pasternak and Hannebauer [8] in 2003 that in case of sinusoidal corrugation profiles the effect of the additional bending moment can be neglected, and the reduction factor can be set to 1.0. Against it Kuchta proposed in 2006 an M-V interaction equation for girders with sinusoidally corrugated webs [9], however it has to be mentioned that only a small interaction behaviour is observed during the investigations.

Due to the contradictory observations on the importance of the additional normal stresses on the bending resistance this paper focuses on the analysis of the structural behaviour of the trapezoidally corrugated web girders under combined bending and shear. The aim of the investigation is to analyse the effect of the transverse bending moment on the bending resistance and to harmonize the previous research results. The necessary consideration method of the transverse bending moment in the design is also discussed. The applied parameters and notations used in this paper are shown in Fig. 1.

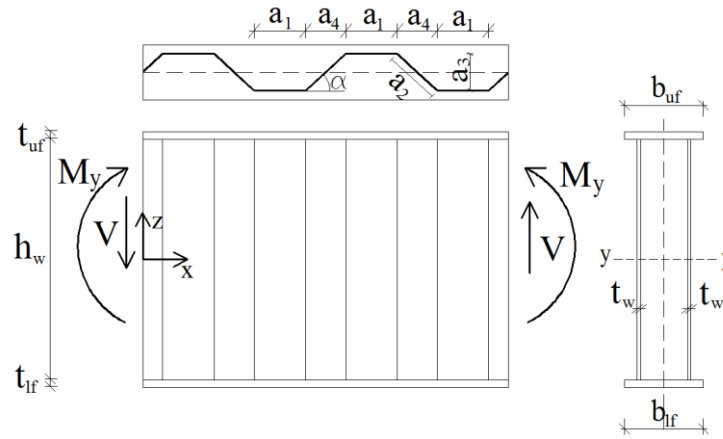


Fig. 1: Geometry of a girder with trapezoidally corrugated web.

2. Previous investigations and current research strategy

2.1. Previous studies on the stress distribution and on the bending resistance

There are many previous investigations dealing with the stress distribution in the web and flange plates of the corrugated web girders. The normal stress distribution in the web was investigated among others by Aschinger and Lindner [10] in 1997, Elgaaly et al. [5] in 1997, Huang et al. [11] in 2004, Mori et al. [12] in 2006, Hannebauer [13] in 2008 and Kövesdi et al. [14] in 2012. Based on the experimental and numerical investigations the conclusion was drawn that the corrugation profile has a significant effect on the axial stress distribution in the web. It was observed that the normal stress decreases rapidly under the flanges and its value is almost negligible in the largest part of the web. This phenomenon is called as “accordion effect”, which is typical for girders with corrugated webs. This is the reason, why the contribution of the web in the moment carrying capacity is neglected in the design process of the corrugated web girders.

Beside the special stress distribution in the web, corrugated web girders have also a special stress distribution in the flanges. It was observed by many researchers in the past, that the stress distribution in the flanges are not constant, there is a significant difference between the two flange edges and the difference has an alternating tendency along the girder length. This stress difference can be explained by two different effects. In case of girders subjected by bending moment without shear force, the stress difference between the two flange edges is relative small and it can be explained by the different interlock between the flange and web plates in the parallel and inclined folds. In the cross section of the parallel folds the interlock between the upper and lower flanges are better than in the cross section with an inclined fold. Therefore the web contribution in the bending resistance is more dominant in a cross section with a parallel fold than in a cross section with inclined folds.

The stress difference between the flange edges is more dominant if the girder is subjected by the combination of bending moment and shear force. A deeper overview on the previous studies on the stress distribution in the flanges for girders loaded by shear force can be found in [14]. This paper gives just a short summary on the conclusions of the previous investigations in this topic. The flange stress distribution and the bending resistance of the girders with corrugated web was investigated at first by Lindner [2] in 1992. Based on experimental investigation on large scale test specimens with corrugated webs it was proposed that the bending resistance should be calculated only from the contribution of the flanges taken the additional normal stresses coming from the shear force in the corrugated web into account. Design proposal was developed to determine the additional normal stresses.

The theoretical background of the calculation is that the additional normal stress comes from the shear flow in the corrugated web. The shear flow has different directions along the girder length in a parallel and in an inclined fold. Based on the theory of Lindner [2] the flange can be assumed as a girder loaded by additional lateral forces (F_y) and transverse bending moments (M_z), as shown in Fig 2.

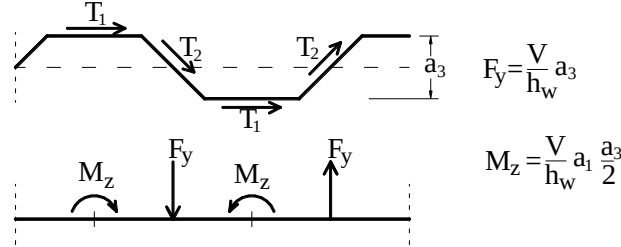


Fig. 2: Theoretical background of the transverse bending moment [2].

The theoretical background of the additional normal stress in the flanges was analyzed by many researchers in the past. The applicability of the mechanical model of Lindner [2] is proved by all the previous investigations.

Significant research activity is carried out by Abbas et al. [15], [16], [17] in 2006 and 2007 on this topic. The behaviour of the corrugated web girders under in-plane loading was investigated and the stress distribution in the flanges was analyzed. To determine the transverse bending moment a so called “fictitious load method” is developed. This load is introduced as a tool for analyzing flange transverse bending of corrugated web I-girders, what is similar to the mechanical model of Lindner.

Proposal to determine the maximal transverse bending moment can be also found in the „Commentary and worked examples to DIN-EN-1993-1-5 Plated Structural Elements” [4]. According to [4] the maximum transverse bending moment $M_{z,max}$ occurs where the inclined part of the web intersects the centerline of the flange and its value can be calculated by Eq. (4).

$$M_{z,max} = \frac{V \cdot a_3}{4 \cdot h_w} \cdot (2 \cdot a_1 + a_4) \quad (4)$$

The theoretical investigations of Balaz [18] proved also the applicability of this theory and it is proposed to implement the Eq. (4) into the EN1999-1-1 [19].

Significant numerical investigation has been carried out by Kövesdi et al [14] in 2012 to analyze the flange stress distribution of the corrugated web girders. The additional normal stresses are determined by different girder geometries, corrugation profiles and loading and support conditions. The effect of the geometrical parameters and the loading conditions are determined on the additional normal stress distribution. Based on the numerical calculations the following conclusions were drawn:

1. Analyzing the most unfavorable geometrical situation from point of view the additional normal stresses the value of the maximal transverse bending moment can be determined by Eq. (5). The internal forces coming from the shear force and the maximum possible transverse bending moment diagram are presented in Fig. 3.

$$M_{z,max} = \frac{V \cdot a_3}{2 \cdot h_w} \cdot (2 \cdot a_1 + a_4) \quad (5)$$

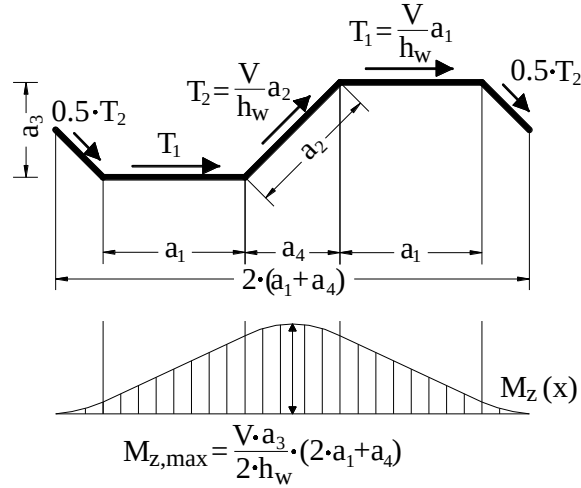


Fig. 3: Proposal for the maximum possible transverse bending moment calculation [14].

2. The numerical calculations showed that the proposal of Lindner and Aschinger [10] gives a good approximation of the maximum additional transverse bending moment, if the shear force diagram along the girder length is constant. The design proposal of Lindner and Aschinger gives smaller values than the numerical calculations in the analyzed parameter range, if the shear force diagram is not constant, or the fold lengths (a_1 and a_2) of the corrugation profile are not equal.
3. The numerical calculations showed that the value of the maximum transverse bending moment can be also determined according to the proposed closed form solution by Abbas et al. [15] - [17], if the girder is loaded by uniformly distributed load. But the application range of this method is relative narrow, because there are no closed form solution for different loading conditions. The values according to the proposal of Abbas et al. resulted exactly the same transverse bending moment as calculated by Eq. (5).

It is clear from the previous investigations, that the shear force in the web results in an additional transverse bending moment in the flange. The mechanical background of it is introduced by Lindner and proved by several researchers. The calculation method of the transverse bending moment is also clarified in the previous studies. But there is a lack of investigations dealing with the effect of this transverse bending moment on the bending resistance of the corrugated web girders.

2.2. Previous studies on the shear buckling resistance

There are numerous studies, which are dealing with the determination of the shear buckling resistance of the corrugated web girders. Research in this topic was started by Easley and McFarland [20] in 1969, they proposed a global shear buckling equation for corrugated webs by treating the corrugated web as an orthotropic flat web. Since then numerous significant experimental, numerical and theoretical investigations are executed in this topic. The shear buckling resistance of the corrugated web girders was analyzed by Lindner and Aschinger [21], [10] in 1988 and 1992. Experimental investigations were completed by Elgaaly et al. [7], Yamazaki [22], Bergfelt et al. [23], Driver et al. [24], Scheer [25], and Moon et al. [26] focusing on the determination of the shear buckling resistance of corrugated web girders.

Significant numerical investigation was executed by Luo and Edlund [27] and by Yi et al. [28] where the different geometric parameters (web depth (h_w), span (L), web thickness (t_w), corrugation depth (a_3), corrugation angle (α) and the fold length (a_1 and a_2) were analyzed. The research was continued by Pasternak and Hannebauer [29], Moon et al. [30], Eldib [31]

and Hassanein and Kharoob [32]. In these research studies the shear buckling behaviour of the corrugated web girders were studied in detail. Numerous experiments were executed and different numerical models were developed. Based on these investigations different design proposals were derived to determine all the three type of shear buckling failure modes, namely the local, global and interactive shear buckling resistances.

2.3. Previous studies on the bending and shear interaction

Despite there are many papers available dealing with the bending and shear buckling resistance of corrugated web girders, there is only a small number of previous investigations dealing with bending and shear interaction.

Both the bending and shear buckling resistance of the corrugated web girders was investigated by Elgaaly et al. in 1997 and 1998 [5]-[7]. Based on the experimental investigations a numerical model was developed and the stress distribution in the flange and in the corrugated web was studied in detail. According to the experimental and numerical results the conclusion was drawn that the contribution of the web to the ultimate moment capacity of the girder is negligible, the bending resistance can be calculated only from the contribution of the flanges. It was also demonstrated that the shear force is carried only by the corrugated web. Based on their investigations they also stated that there is no interaction between the shear and bending resistance of the trapezoidally corrugated web girders.

The EN1993-1-5 [1] gives proposal to take the effect of the shear force on the bending resistance into account by the reduction factor (f_T) (see Eqs. (2)-(4)). As far as the authors know, this reduction factor was calibrated based on experimental results on specimens tested by combined bending and shear, where the failure mode was lateral torsional buckling of the specimens. Based on the authors opinion, it is an important task, because the current paper focuses only on the bending and shear resistance of the corrugated web girders and the lateral torsional buckling resistance determination is not topic of this paper.

More intensive research activity has been made on this research field in case of sinusoidally corrugated web girders. At first Pasternak and Hannebauer [8] investigated the effect of the shear force on the bending resistance of the sinusoidally corrugated web girders. The result of their investigations was that the effect of the additional normal stress coming from the shear force is relative small in case of the usual girders with sinusoidal corrugations used in the praxis. The corrugation depth compared to the usually used flange widths are relative small, therefore the reduction of the bending resistance due to the transverse bending moment can be neglected. An additional note is also introduced in the EN1993-1-5 Annex D [1] with the statement, that the reduction factor (f_T) can be equal by 1,0 in case of girders with sinusoidal corrugation profile.

Kuchta [9] investigated also the structural behaviour of the girders with sinusoidally corrugated webs under the combined bending and shear in 2006. Based on the executed experimental research program a numerical model was developed. The interaction behaviour and the stress distribution under the different loading situation was studied. The result of the investigation showed that there is a small reduction in the load carrying capacity under the combined loading situation and therefore an interaction equation is proposed for girders with sinusoidally corrugated webs. But the bending resistance model used in the proposed interaction equation does not take the effect of the transverse bending moment into account, and the magnitude of the additional normal stresses was not studied in frame of this investigation. Furthermore the shear resistance of the girders are determined by the plastic shear resistance of the web and the bending resistance is also determined by the plastic bending resistance of the flanges neglecting the possible shear buckling or flange buckling

effects. The maximum reduction due to the M-V interaction is 1/12 (8,33%) of the bending or the shear resistance according to the proposal of Kuchta [9].

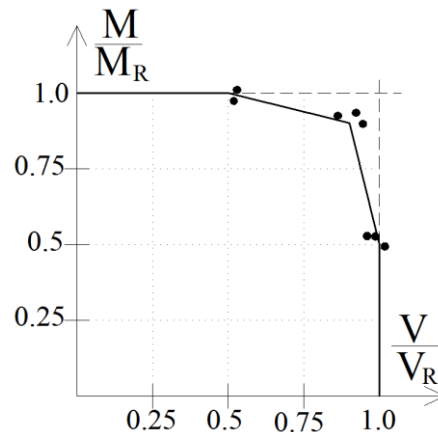


Fig. 4: M-V interaction diagram for sinusoidally corrugated web girders [9].

2.4. Evaluation of the previous research activities and current research aims

It can be seen from the previous investigations, that the effect of the transverse bending moment on the in-plane-bending resistance of the corrugated web girders is not a clarified topic till now. There are different research results which should be harmonized. The importance in the practical design of the reduction factor (f_T) is stated questionable by several researchers in the past in [4] and [14], therefore it needs further investigation to clarify the structural behavior of the corrugated web girders under combined bending and shear.

The questions to be answered in this paper are the followings.

1. How large is the additional normal stress in case of typical girders used in the praxis?
2. Which parameters have influence on the additional normal stresses?
3. How large can be the effect of the shear force on the bending resistance?
4. Is the consideration of the transverse bending moment necessary in the practical design, and how can be considered in design equations?

To answer these questions, the executed research strategy is the following. At the first step the existing experimental, analytical and numerical investigations on this research field are analysed, evaluated and compared. The effect of the M-V interaction equation taking the reduction factor of the EN1993-1-5 [1] is analysed in case of typical girder geometries. More than 1900 girder geometries found in the international literature are collected and the bending moment reduction factors are calculated and compared. This comparison gives an overview about the importance of the reduction factor in case of typical girders used in the praxis or investigated by researchers in the past.

Numerical investigations showed, that the support conditions have a large influence on the additional normal stresses in the flanges. In case of real structural elements, the transverse bending moment is influenced by the support conditions and its value can be significantly smaller than the theoretically determined maximum value. To give overview about the effect of the lateral and longitudinal support conditions on the normal stress distribution, girders with different support conditions are investigated and the normal stress distributions are compared.

At the third step of the research the structural behaviour of the corrugated web girders are analyzed under combined bending and shear using numerical simulations (GMNIA). Numerical model is developed and verified based on test results found in the literature. On the verified numerical model the M-V interaction behaviour is studied, and the effect of the shear

force on the bending resistance is determined on various girder geometries. The results of the numerical calculations are compared to the previous design equations and proposal is developed to handle the M-V interaction for girders with trapezoidally corrugated webs for the practical design.

3. Effect of the additional normal stresses for typical girder geometries

This Section analyses the value of the transverse bending moment and the bending resistance reduction factor in case of typical girder geometries used in the praxis. Considering the reduction factor (f_T) of EN1993-1-5 [1] its value depends on the applied shear force, therefore this equation represents actually an M-V interaction curve, as shown in Fig. 5.

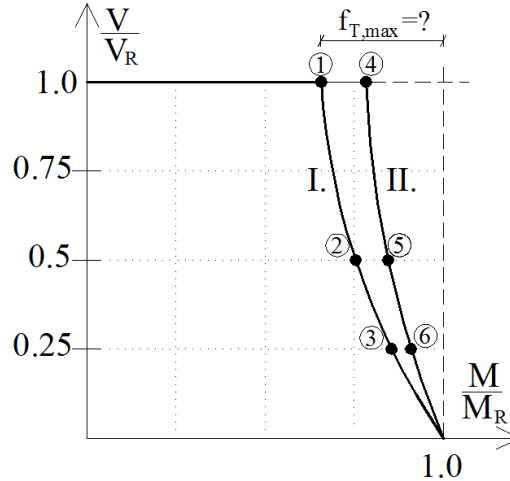


Fig. 5: M-V interaction diagrams according to EN1993-1-5 [1] and evaluation strategy.

It can be seen from the diagram, that this interaction equation has a different character than the usual M-V interaction equations used for steel structures. The maximum possible value ($f_{T,max}$) is reached by a shear force equal by the shear resistance of the analyzed girder. This maximum value characterizes the interaction diagram exactly, therefore it is the basis of the current comparison (point 1 in Fig. 5). More than 1900 girder geometries are collected from the international literature and the maximum values of the bending resistance reduction factors ($f_{T,max}$) are determined. For all the analyzed girders the shear resistance is calculated according to the rules of EN1993-1-5 [1]. The maximum reduction factor ($f_{T,max}$) is determined using a shear force equal by the shear resistance of the analyzed girder. Thus this maximum value occurs rarely or never in case of real structures, the same evaluation is also made at shear force levels of 25% and 50% of the shear resistance (points 2-3, in Fig. 5), which can usually occur in case of real structures. The transverse bending moments are calculated on two different ways, using Eq. 4 and Eq. 5, representing line I. and II in Fig. 5. The expression according to Eq. 4 gives the maximum possible average value of the additional normal stresses (line II.). Equation 5 gives the possible maximum value (line I.). It means, that for each analyzed geometries 6 different values are calculated representing the bending reduction factors at three different shear force levels using the average or the maximum value of the transverse bending moment (points 1-6 in Fig. 5). It has to be mentioned, that the reduction factor of Lindner [2] is calibrated by using the maximum value of the additional normal stresses. The theoretically derived equation consisted a factor of 0.816, but its value has been reduced to 0.4 in Eq. 3 for design purposes. The background of it is that the value of the transverse bending moment has large changes along the girder length, even its sign can be changed.

Beside the determination of the reduction factors the maximum additional normal stresses in the flanges are also determined and compared to the yield strength of the analyzed girder. This comparison shows the stress increasing effect of the transverse bending moment on the normal stress field in the flanges. It means that if the girder is designed based on the allowable elastic stresses, and its bending utilization ratio for in-plane-bending is 1.0, the effect of the transverse bending moment can be characterized by the ratio of σ_x/f_y . This ratio represents the overstress ratio in case of elastic design, if the additional normal stresses are neglected from the design and the bases of the design is the yield strength. This comparison is just a fiction, gives not real utilization values, but it characterizes the importance of the transverse bending moment in the practical design.

Typical girder geometries with trapezoidally and sinusoidally corrugated webs are also collected and evaluated in the frame of this study. In case of sinusoidal web corrugation an equivalent trapeze profile is created to be able to evaluate the geometrical properties and to determine the additional normal stresses coming from shear. The typical girder geometries are collected from the international literature. The evaluation is separated for the trapezoidally and sinusoidally corrugated web girders. For both girder types, the available girder geometries of the previous experimental and numerical calculations are also collected.

115 different girder geometries are found in the international literature for trapezoidally corrugated web girders which were tested experimentally and 292 different girder geometries analyzed by numerical parametric studies.

12 different girder geometries are collected from existing bridge structures with corrugated web girders. These bridges are the Altwipfergrund bridge [33] [34], Shinkai bridge [30], Matsunoki bridge [30], Hondani bridge [30], Cognac bridge [30], Maupre bridge [30], Dole bridge [30], Maetani bridge [35], Ilse bridge [36] and the Móra Ferenc bridge [37]. In case of several bridges only the geometry of the corrugation profile and the web depths were available from the literature. In these cases fictitious flange sizes are assumed, which are usually used for these kind of bridges.

The typical geometries of the sinusoidally corrugated girders are also collected. A total of 1519 different girder geometries can be found in the product description of the company Zeman [35]. The most research activities analyzing the sinusoidally corrugated web girders are using the same geometries which can be found in this product catalog, but slightly different girders are analyzed by Hannebauer [13] and Kuchta [9].

A total of 1941 different girder geometries are collected and the effect of the transverse bending moments are evaluated for each girders. It is observed that girders with sinusoidally corrugated webs have usually small corrugation depths (40 – 43 mm) compared to the flange width, and the fold lengths of the equivalent trapeze profiles are also relative small (30 - 60 mm). At the same time the web thickness is also smaller (1,5 – 6 mm). These structures are usually used in building structures as shown in Fig. 6.



Fig. 6: Corrugated web girders used in industrial buildings, from [35].

Girders with trapezoidal corrugation has usually totally different corrugation sizes. The applied fold lengths are usually between 150 – 400 mm, what is 5-7 times larger than for sinusoidal corrugation and the used flanges are usually also larger. It comes from the fact, that the trapezoidally corrugated girders are usually used in bridge structures. One example can be seen in Fig. 7.



Fig. 7: Corrugated web girders used in bridges, from [36].

The differences in the typical girder geometries between the sinusoidal and trapezoidal corrugations results in significantly different values in the transverse bending moment and in the reduction factor (f_T) of the bending resistance.

Table 1 summarizes the typical girder geometries containing the minimum, the maximum and the average values of the corrugation angle and the fold lengths of the analyzed girders. The values for the sinusoidal and trapezoidal corrugations and the existing bridges are separately evaluated. The minimum and maximum values have a much larger difference in case of trapezoidally corrugated girders. The reason of it is that this girder type was extensively investigated by numerical parametric studies by researchers in the past in a wide range of the applied parameters. The average values give a good comparison possibilities. It can be seen, that the average corrugation angles are almost the same for all the three groups. The average fold lengths are much smaller in case of girders with sinusoidal corrugations and much larger for girders with trapezoidal corrugations, including the existing bridges as well.

Table 1: Girder geometries collected from international literature.

	corrugation angle (α) [°]			fold length (a_1) [mm]			fold length (a_2) [mm]		
	Min.	Max.	Average	Min.	Max.	Average	Min.	Max.	Average
Sinusoidal corrugation	20	38	37,9	26	100	26,5	65	220	66,1
Trapezoidal corrugation	10	90	39,4	20	750	245,1	14	707	237,1
Existing bridges	25	39	33,3	250	430	334,8	250	430	340,3

Table 2 summarizes the results of the calculated additional normal stresses in the flanges and the bending resistance reduction factors using the maximum possible transverse bending moment (points 1-2-3 in Fig. 5).

The average of the maximal normal stresses coming from the shear force is 9,18 MPa in case of typical girders with sinusoidal corrugated webs. The same value for girders with trapezoidal corrugation and existing bridges are 107,9 MPa and 159,6 MPa, respectively. It shows, that the effect of the transverse bending moment for girders with trapezoidally corrugated webs is much larger, that for sinusoidal.

Table 2: Calculated additional normal stresses and reduction factors using the maximum transverse bending moment (Eq. 5).

Evaluation of Line I. using the maximum transverse bending moment		additional normal stress $\sigma_{x,(M_z)}$ [MPa]			bending resistance reduction factor f_T [-]		
		Min.	Max.	Average	Min.	Max.	Average
Point #1 (100% shear force)	Sinusoidal corrugation	0,34	65,68	9,18	0,79	0,98	0,93
	Trapezoidal corrugation	0,8	488,9	107,9	0,58	0,99	0,81
	Existing bridges	60,4	651,7	159,6	0,46	0,84	0,75
Point #2 (50% shear force)	Sinusoidal corrugation	0,2	32,8	4,6	0,85	0,99	0,95
	Trapezoidal corrugation	0,4	244,4	54,1	0,71	0,99	0,87
	Existing bridges	30,2	325,9	79,8	0,62	0,88	0,82
Point #3 (25% shear force)	Sinusoidal corrugation	0,1	16,4	2,3	0,89	0,99	0,97
	Trapezoidal corrugation	0,2	122,2	27,2	0,79	0,99	0,90
	Existing bridges	15,1	162,9	39,9	0,73	0,92	0,88

The author opinion is that this large difference comes not from the shape of the corrugation (sinusoidal or trapezoidal), but on the typical geometries used for the different girder types. The average of the maximum bending resistance reduction factors are 0,93, 0,81 and 0,75 for the three groups. It means that the average value of the maximal bending resistance reduction for the sinusoidally corrugated web girders is 7%, but the same values are 19% and 25% for the trapezoidally corrugated web girders and existing bridges. The investigations show that the effect of the additional normal stresses can be significant in case of girders with trapezoidally corrugated webs. In extreme cases the maximum possible resistance reduction can reach 50% of the original bending resistance without accompanying shear force.

The above mentioned reduction factors are the maximum values of the M-V interaction diagram, shown in Fig. 5. If the applied shear force acting on the girder is only 50% or 25% of the shear resistance the additional normal stresses and the reduction factors are significantly smaller. The value of the additional normal stress is in linear relationship with the applied shear force, therefore if the shear force is only 25% or 50% of its total value the normal stress will be also 25% or 50% of its maximum. The results show, that the additional normal stresses due to shear force what is 25% of the shear resistance of the girder results is in average 2,3 MPa for girders with sinusoidal corrugation. The same values for girders with

typical trapezoidal corrugations are 27,2 or 39,9 MPa, which can have a dominant effect on the structural behavior of the bending resistance.

The same tendencies can be observed in the reduction factors. The average reduction factor for girders with typical sinusoidal corrugation by 25% shear force is 0,97 what represent 3% reduction in the bending resistance. The maximum of the bending resistance reduction factor is not larger than 10%. The same values for girders with trapeze profiles are 0,88 and 0,73 what represents in average 12% and maximum 27% reduction in the bending resistance. These reduction factors can be evaluated as significant reductions for typical girders with trapezoidally corrugated webs.

The same evaluation process is made using the average transverse bending moment (line II. in Fig. 5). Table 3 summarizes the results of this evaluation procedure. It can be seen from the results, that the average value of the maximum reduction factor for girders with sinusoidal corrugation is 0,95. For trapezoidally corrugated web girders and existing bridges these values are 0,87 and 0,82. The reduction factors by 25% shear force are in average between 0,91 – 0,97.

Table 3: Calculated additional normal stresses and reduction factors using the average transverse bending moment.

Evaluation of Line II. using the average transverse bending moment		additional normal stress $\sigma_{x,(Mz)}$ [MPa]			bending resistance reduction factor f_T [-]		
		Min.	Max.	Average	Min.	Max.	Average
Point #4 (100% shear force)	Sinusoidal corrugation	0,2	32,8	4,6	0,85	0,99	0,95
	Trapezoidal corrugation	0,4	244,4	54,1	0,71	0,99	0,87
	Existing bridges	30,2	325,9	79,8	0,62	0,88	0,82
Point #5 (50% shear force)	Sinusoidal corrugation	0,1	16,4	2,3	0,89	0,99	0,97
	Trapezoidal corrugation	0,2	122,2	27,2	0,79	0,99	0,90
	Existing bridges	15,1	162,9	39,9	0,73	0,92	0,88
Point #6 (25% shear force)	Sinusoidal corrugation	0,1	16,6	1,2	0,92	0,99	0,98
	Trapezoidal corrugation	0,1	98,1	13,7	0,80	1,00	0,93
	Existing bridges	7,5	81,5	20,0	0,81	0,94	0,91

For the analyzed cases with 25% and 50% shear forces the additional normal stresses coming from the transverse bending moment are compared to the yield strength of the flange plate. This comparison represents the effect of the additional transverse bending moment on the bending resistance, if elastic design method is used with maximum allowable normal stress. The ratios in Table 4 represents the difference in the bending utilization, if only the in-plane-bending effect is taken into consideration (utilization ratio equal by 1) or the additional normal stresses coming from the transverse bending moment are also considered. The results show, that the average effect of the additional normal stresses in case of girders with sinusoidal corrugation is between 1,8-3,6%, but in case of trapezoidal corrugation this ratio can be between 10-20%.

Table 4: Differences between bending utilization ratios considering or neglecting the effect of the transverse bending moment.

Evaluation of Line I. using the maximum transverse bending moment		$\sigma_{x,(Mz)} / f_y$ [%]		
		Min.	Max.	Average
Point #2 (50% shear force)	Sinusoidal corrugation	0,35	17,47	3,61
	Trapezoidal corrugation	0,06	54,04	14,18
	Existing bridges	6,89	91,79	19,80
Point #3 (25% shear force)	Sinusoidal corrugation	0,2	8,7	1,8
	Trapezoidal corrugation	0,0	27,0	7,1
	Existing bridges	3,4	45,9	9,9

4. Effect of support conditions on the additional normal stresses

4.1. Numerical model development and verification

The aim of the numerical investigations is to study the effect of the transverse bending moment on the flange stress distribution under different support conditions. Numerical model is developed with the finite element program Ansys 14.5 [37]. The effect of the corrugation profile, the geometrical properties, the corrugation geometry at the beam end and the loading conditions are extensively studied by Kövesdi et al. in 2012 [14]. These investigations showed, that the maximum possible value of the additional normal stresses ($\Delta\sigma$) can be determined by Eq. 5 with adequate safety. It is also observed that this maximum value can be significantly influenced by the lateral and longitudinal support conditions. The current investigations are supplementary investigations to analyze the additional normal stress distribution, if the girders have different support conditions. The analyzed girders are modeled using eight node thin shell elements. The developed numerical model, the applied finite element mesh, the loading and support conditions can be seen in Fig. 8.

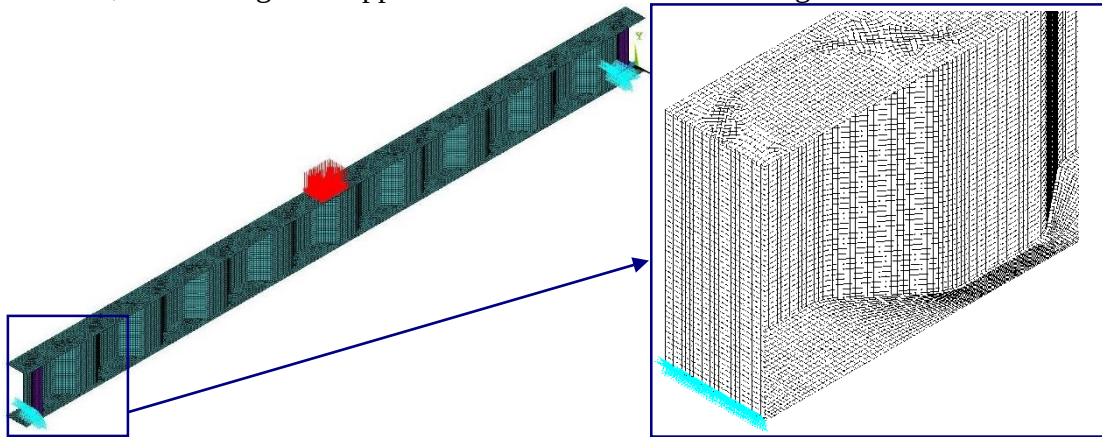


Fig. 8: Developed numerical model.

The stresses in the numerical model are calculated by a linear analysis using linear elastic material model ($E=210000$ MPa; $\nu=0.3$). Convergence study is executed to ensure the model reliability. As a principle it is concluded that one fold should be modeled by minimum of 8-10 elements along the fold length to determine the stresses with adequate accuracy. It means, that in case of the analyzed girders the applied finite element size varies between 2 - 30 mm. The detailed description of the numerical model, and the model verification based on own test results are described in [14]. It has to be mentioned, that the investigations in [14] showed that the most unfavorable situation from point of view of the additional normal stresses is if the supports and load introduction places are located at the middle of an inclined fold and a full number of corrugation waves are between these two points as shown in Fig. 9. Because this corrugation layout is the most unfavorable from point of view the transverse bending moment, all the calculations are made on this geometry to get results always on the safe side.

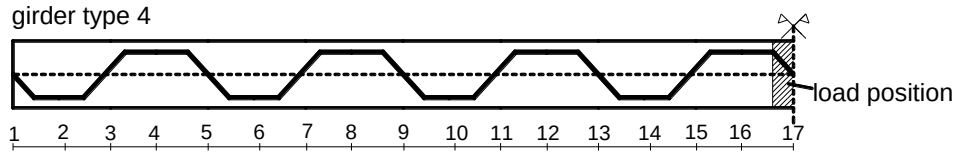


Fig. 9: Analyzed corrugation layout, support and load introduction locations.

4.2. Results of the numerical calculations

Girders with typical bridge geometries and with typical geometries used in industrial buildings are also analysed. The same tendencies are observed for both girder types, therefore only the tendencies observed for typical bridge girders are presented here. The analysed girder geometry is near to the average dimensions of the existing bridges made with corrugated webs. The flange width is 400 mm with thickness of 30 mm, the web plate has a depth of 2500 mm and thickness of 10 mm. The fold lengths ($a_1=a_2$) are 350 mm and the corrugation angle is set to 39° . The manually determined maximum value of the transverse bending moment, the maximum value of the additional normal stress and the reduction factor of the bending resistance can be determined by Eqs. 6-8. The applied shear force is 1000 kN, what is about 25% of the shear resistance of the analysed girder.

$$M_{z,\max} = \frac{V \cdot a_3}{2 \cdot h_w} \cdot (2 \cdot a_1 + a_4) = \frac{1000 \text{ kN} \cdot 220 \text{ mm}}{2 \cdot 2500 \text{ mm}} \cdot (2 \cdot 35 \text{ cm} + 272 \text{ cm}) = 4281,9 \text{ kNcm} \quad (6)$$

$$\sigma_x(M_z) = \frac{M_{z,\max}}{I_{fz}} \cdot \frac{b_f}{2} = \frac{4281,9 \text{ kNcm}}{\frac{(40 \text{ cm})^3 \cdot 3 \text{ cm}}{12}} \cdot 20 \text{ cm} = 53,5 \text{ MPa} \quad (7)$$

$$f_T = 1 - 0,4 \cdot \sqrt{\frac{\sigma_x(M_z)}{f_y}} = 1 - 0,4 \cdot \sqrt{\frac{53,5 \text{ MPa}}{355 \text{ MPa}}} = 0,845 \quad (8)$$

The real additional normal stresses coming from the transverse bending moment determined by numerical calculations are presented here using different support conditions. The descriptions of the analysed support conditions are the following:

- model I: simply supported girder with lateral supports at the two ends (modeling the worst case as reference),
- model II: simply supported girder with 5 lateral supports, where the lateral restrains do not give accompanying rotational supports (modeling the effect of transverse girders or purlins),
- model III: simply supported girder with 5 lateral supports, where the lateral restrains give also rotational supports (modeling the effect of transverse girders with fixed connections),
- model IV: simply supported girder with 7 lateral supports, where the lateral restrains do not give accompanying rotational supports,
- model V: simply supported girder with continuous lateral supports (modeling the effect of concrete slab, what is usually used in bridges with corrugated webs),
- model VI: fixed supports at both ends without lateral restrains (modeling the effect of the longitudinal support conditions).

The results of the numerical calculations are shown in Figs. 10-15 for each model. The horizontal axis of the diagrams show the number of the folds as introduced in Fig. xx. The

vertical axis represent the value of the calculated additional normal stresses in each fold along the girder length. The red horizontal lines represent the manually calculated maximum values of the additional normal stress. The additional normal stress distribution of the reference model with simply support conditions can be seen in Fig. 10.

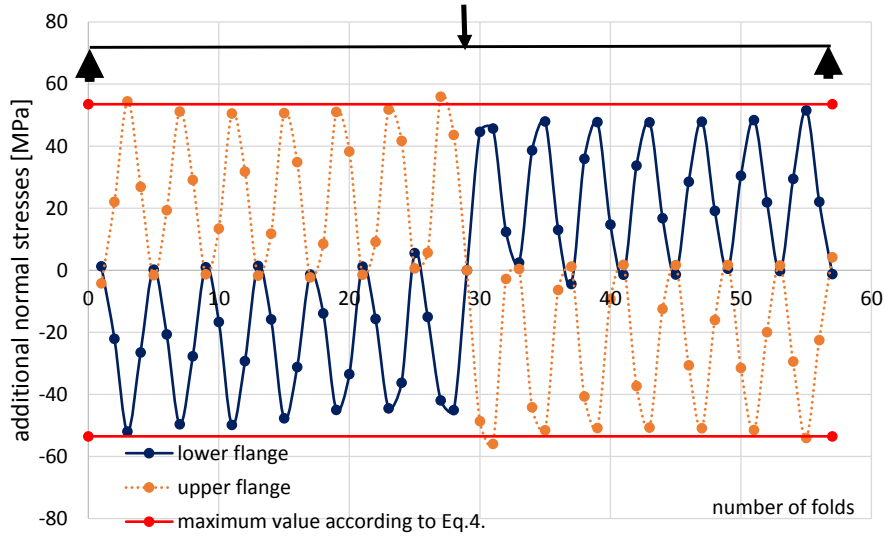


Fig. 10: Additional normal stresses ($\Delta\sigma$) for model I.

The value of $\Delta\sigma$ has an alternating tendency along the girder length, as observed also in the previous investigations. It can be seen from the results that the maximal values of the additional normal stresses are equal in the upper and lower flanges and the calculated values by the FE model shows a very good agreement with the hand calculations according to the proposed model of Kövesdi et al [14]. This model is handled as reference to analyse the effect of the following support conditions.

Figure 11 shows the effect of the lateral supports on the normal stress distribution. The dashed line shows the simply supported girder without intermediate lateral supports and the continuous line represents the normal stresses using 5 lateral supports.

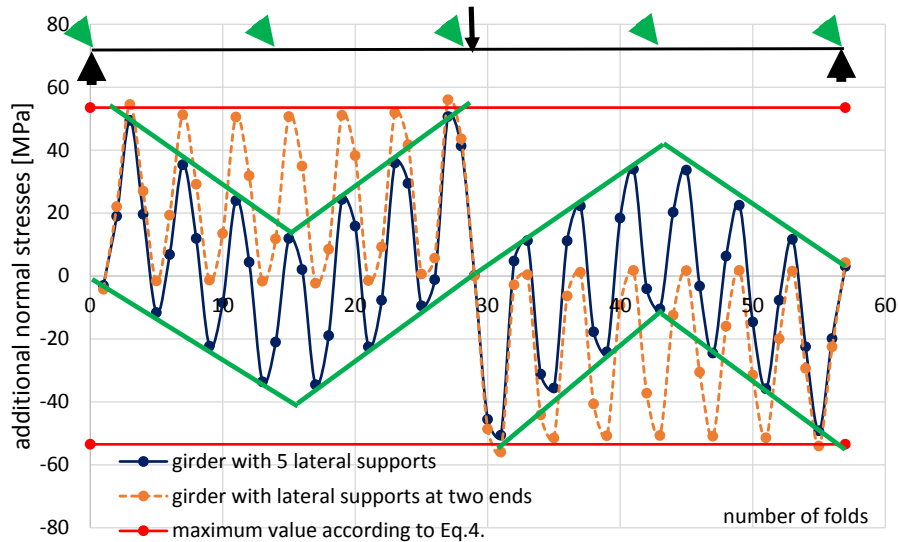


Fig. 11: Additional normal stresses ($\Delta\sigma$) for model II.

It can be seen from the results that the lateral supports have a significant effect on the stress distribution. The reason of it is the following. The transverse bending moment comes from lateral forces and eccentric longitudinal forces according to the mechanical model of Lindner

[2]. The lateral forces are supported by the lateral supports and the transverse bending can be developed only from the eccentric longitudinal forces. Therefore the transverse bending moment and the additional normal stresses in the flange plates reduce significantly near to the supports. This effect can be clearly observed in Fig. 11. The green continuous lines show the modified tendency of the normal stress distribution. The maximum value of the additional normal stress in the region of the lateral supports is reduced by 30% due to the lateral restrains. If the lateral supports are located in cross sections where the effect of the lateral forces are negligible (at the two end and at midpan cross sections) the lateral supports have no influence on the normal stress distribution.

If the lateral restrains can also give support against rotation, its effect will be larger because the transverse bending moment coming from the eccentric longitudinal forces can be also reduced in the region of the lateral supports, as shown in Fig. 12.

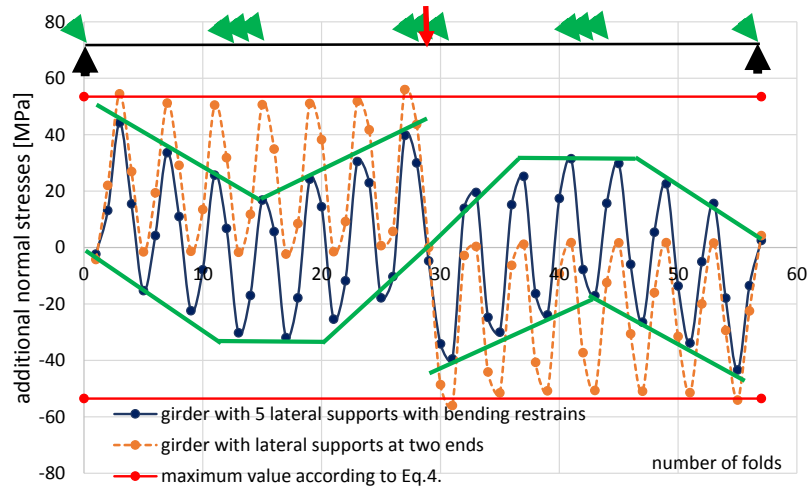


Fig. 12: Additional normal stresses ($\Delta\sigma$) for model III.

The same tendency as shown in Fig. 12 can be observed if more lateral supports are applied along the girder length. Figure 13 shows the additional normal stress distribution for a girder, where 7 lateral support are applied. The results shows, that the additional normal stresses are significantly reduced (~40%) by both support conditions.

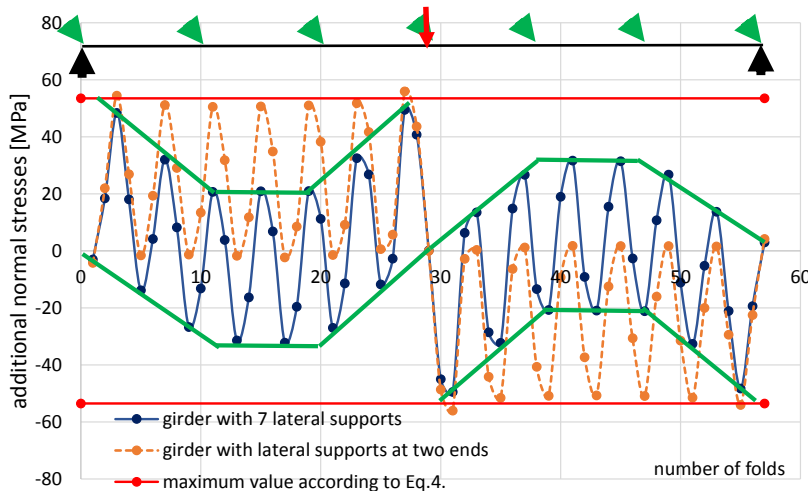


Fig. 13: Additional normal stresses ($\Delta\sigma$) for model IV.

In practical cases by hybrid girders, where both flanges are made from concrete and the web is usually made from steel with corrugated profile, the concrete slab gives continuous lateral support to the steel web. Therefore a model with continuous lateral supports are also investigated in the present study. The comparison of the additional normal stresses for a model without and with continuous lateral supports is presented in Fig. 14. It can be seen from the results that the tendency of the additional normal stresses are the same for the two models, but the maximum values reduced by about 75% of the original values without lateral restrains.

These investigation show, that the effect of the lateral supports on the additional normal stress are significant. Transverse girders, or purlins, which gives lateral support to the girders can reduce the additional normal stresses in the region of the lateral supports by 35-40% in case of the analyzed girder. Continuous lateral support reduced the additional normal stresses by 75%. These reductions depend on the ratio of the parallel and inclined fold lengths and the corrugation angle. But anyway it can be concluded that the effect of the lateral support is significant on the additional normal stresses.

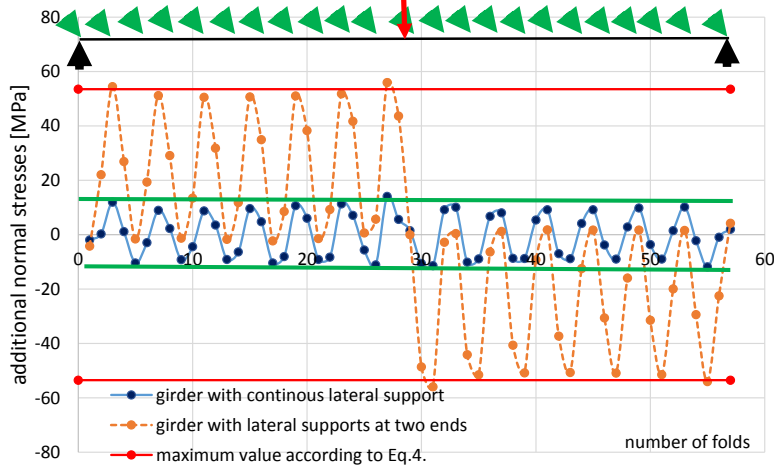


Fig. 14: Additional normal stresses ($\Delta\sigma$) for model V.

During the numerical calculations it was also observed, that the longitudinal supports have also influence on the additional normal stresses. The comparison the normal stress distribution along the girder length calculated on a simply supported girder (model I.), and on a girder with fixed boundary conditions are shown in Fig. 15.

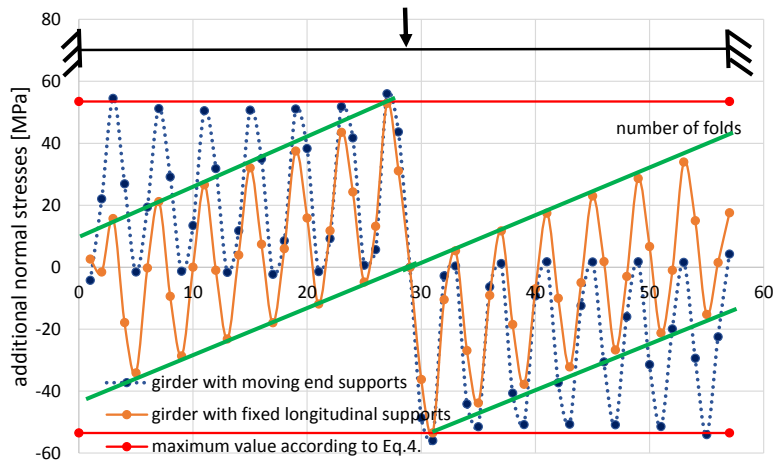


Fig. 15: Additional normal stresses ($\Delta\sigma$) for model VI.

The results show that the longitudinal support has also a significant effect on the additional normal stress distribution. The values can be significantly reduced in the region of the longitudinal support, which are placed at the two ends in case of the analyzed girder.

Based on the numerical calculations analyzing the effect of the different support conditions on the corrugated web girders, the conclusion can be drawn that both type of supports (longitudinal and lateral) can reduce the maximum value of the additional normal stresses. Therefore in case of real structures with real support conditions that maximum value of the additional normal stresses can be significantly smaller than the theoretically determined maximum. Thus the maximum value of the normal stress in the flange plate depends on the corrugation geometry, the flange plate size, the corrugation layout and on the lateral and longitudinal support conditions, the determination of the maximum value is quite hard by analytical methods. The maximum possible value, or the maximum from the average values can be determined by simple analytical solution proposed by Kövesdi et al. in [14]. But if the determination of a more accurate value of the transverse bending moment is necessary, the authors propose to use finite element software, modeling the real boundary conditions and girder geometries.

5. Effect of the transverse bending moment on the in-plane-bending resistance

5.1. Aim of the investigations, research strategy

The results presented in Sections 3 and 4 showed that the effect of the transverse bending moment on the flange stress distribution can be significant, especially in case of typical bridge girders. But the effect of the modified stress distribution on the bending resistance needs further investigations. This Section has a focus on the bending resistance determination of corrugated web girders with typical geometries used in the praxis. The resistances of the analyzed girders are determined under pure bending and under combined bending and shear. The resistance reductions in function of the applied shear force are determined and compared to the proposed reduction factor of EN1993-1-5 [1]. The resistance of the analyzed girders are determined by geometrical and material nonlinear analysis using equivalent geometric imperfections (GMNIA).

The numerical models are verified based on test result found in the literature. Based on the verified numerical model a numerical parametric study is executed to analyze the effect of the shear force on the bending resistance. The previous calculations showed that girders with typical geometries used in bridges and in industrial buildings can have different structural behaviour under combined shear and bending, therefore girders with both typical geometries are investigated. The effect of the parameters which have influence on the transverse bending moment (fold length, corrugation angle, web depth) are varied in the numerical parametric study. For all the analyzed girders the bending and shear buckling resistances are determined as a first step and after it the M-V interaction behaviour is analyzed by 8 calculations for each specimens.

5.2. Numerical model development and verification

Applied analysis type

To analyze the structural behaviour of the corrugated web girders under pure bending and under the combined loading situation, a numerical model is developed and verified. The analyzed numerical models are developed using the finite element software Ansys 14.5 [37]. The numerical modelling is based on a full shell model using four node thin shell elements. The ultimate loads are determined by geometrical and material nonlinear analysis using

equivalent geometric imperfections (GMNIA). The Newton-Raphson approach is used in the nonlinear analysis.

Analyzed test specimens

Model verification is executed by recalculations of experimental results found in the literature. The primary aim of the determination is to determine the bending resistance and its reduction due to the accompanying shear force, therefore the numerical model is verified based on bending test results. Elgaaly et al. [5] tested 6 large scale test specimens under 4-point-bending. The failure mode occurred in all cases in the “pure” bending zone (middle part of the test specimens) by the failure of the flange plates. The aim of the investigations of Elgaaly et al. [5] was to determine the bending resistance of girders with trapezoidally corrugated webs, therefore these test results are optimal to the numerical model verification. The numerical model of all the 6 test specimens are developed and the bending resistances are recalculated.

Thus the aim of the current investigation is the study of the M-V interaction, the determination of the shear resistance is also an important task. Therefore the numerical model is also verified by test results where the observed failure mode was shear buckling of the corrugated web. Test specimens taken from two independent previous research programs are selected and the model verification is made for both test results. Experimental research program on corrugated web girders was made by Moon et al. in 2009 [30] on three large scale test specimens. From the three specimens one specimen is selected (M14) to the model verification. Independent experimental research program was also executed by Hannebauer in 2008 [13]. One girder with trapezoidally corrugated web was investigated in this research program. The numerical model of these test specimens are developed and the shear buckling resistances are determined by the numerical simulations.

Thus the model verification is based on three different research programs in the current paper, the detailed description of the test layouts and test specimens are not given here. All the necessary information can be found in the relevant papers [5], [13] and [30]. But all the geometrical and material properties of the analysed specimens, which are necessary to the model development and recalculation of the test results are listed in Table 5.

The applied numerical models, boundary conditions and typical failure modes can be seen in Fig. 16. The boundary conditions applied for the specimens of Elgaaly et al. [5] are simply supports loaded by bending moments at the two end of the girder. Only the middle part of the test specimen is modelled in the current investigation and the two sides of the girder are supplemented by the applied internal forces. In case of the test specimens of Moon et al. [30] only the half of the girder is modelled. This simplification could be made, because the test specimens were also symmetrical. In case of these models symmetry conditions are defined at the left end of the girder and simply support at the right end.

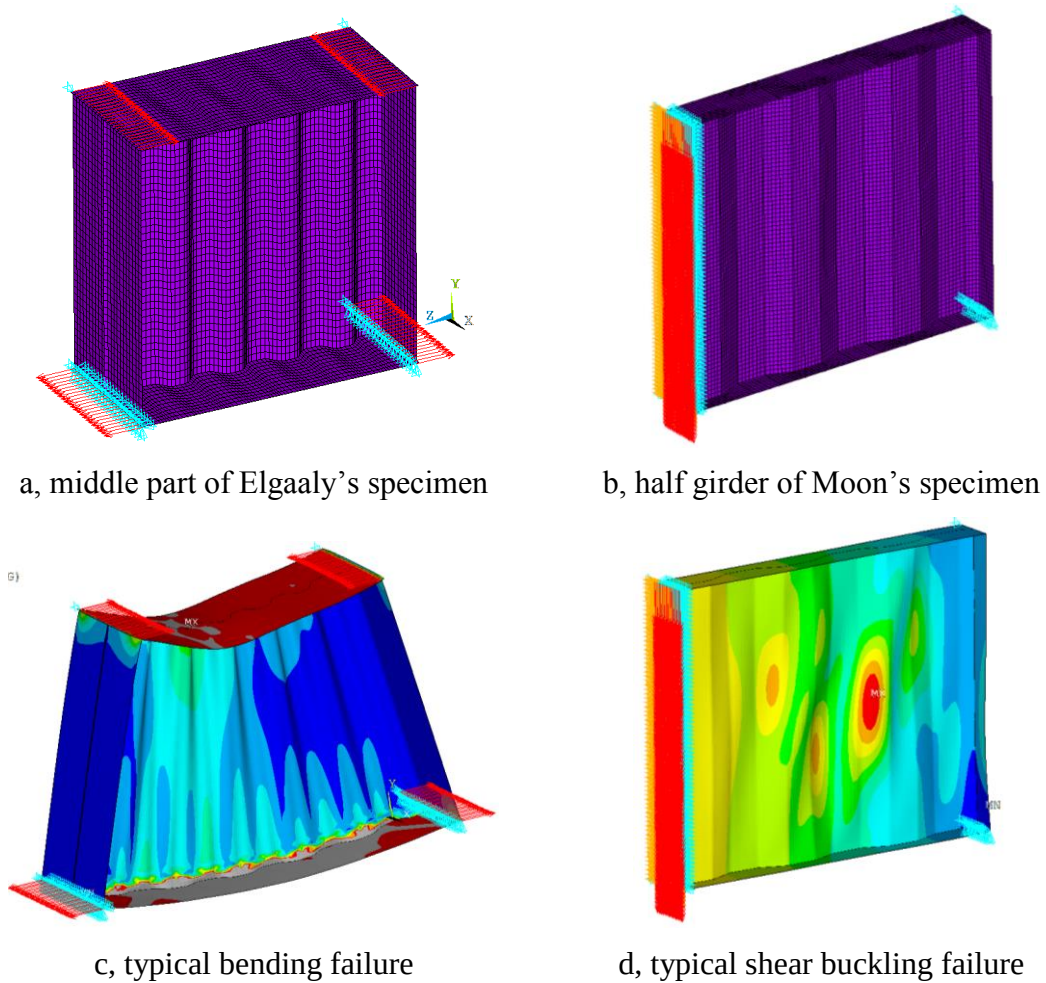


Fig. 16: Applied numerical models and typical failure modes.

Applied material models

Two different material models are used. The first model is used to the numerical model verification and the second one to the numerical parametric study. The first material model follows the measure material properties observed in the tests, to ensure the best agreement of the numerical calculations to the measured resistances. In the numerical parametric study the characteristic value of the yield strength is used in the calculations instead of the measured values. Only girders made from S355 steel material are analysed in frame of the calculations, where the yield strength (f_y) is equal by 355 MPa and the ultimate strength (f_u) is equal by 510 MPa. The character of the applied material model is a linear elastic - plastic material model using von Mises yield criterion. This material model has in the plastic domain an isotropic hardening behaviour. The material is assumed to behave linearly elastic and to obey Hook's law with a Young's modulus equal by 210000 MPa up to the yield stress. Thereafter and until it reaches the ultimate stress, the material is assumed to behave linearly with a reduced modulus. The ultimate strength is defined by $\epsilon=10\%$, which usually fits to the material tests. The material is assumed to be perfectly plastic when it reaches the ultimate stress.

Applied imperfections

The aim of this research work is the determination of the bending and shear buckling resistances of the analyzed girders. In case of both failure modes buckling can govern the structural behaviour, therefore the application of the imperfections is a key point of the

calculation. There are different possibilities to define the equivalent geometric imperfection according to EN1993-1-5 [1]. Thus in the current research work different failure modes are studied separately and also on a combined way (M-V interaction), the best and always a safe side solution is ensured by using eigenmode imperfections. This imperfection type can handle the change of the failure mode in the interaction domain. Maybe the usage of the first eigenmode imperfection shape leads to a conservative design, but it is sure, that all the calculations are on the safe side. Furthermore in the current investigations not the exact values of the resistances are studied, only the resistance reductions. Therefore all the numerical parametric studies are executed using the first eigenmode imperfection shape scaled to the proposed amplitude of EN1993-1-5 [1].

Calculation results

All the geometrical and material properties, the results of the tests and the numerical calculations are summarized in Table 5.

Table 5: Comparison of the experimental and numerical results.

	tf [mm]	bf [mm]	hw [mm]	tw [mm]	L [mm]	alpha [°]	a1 [mm]	a2 [mm]	a4 [mm]	a3 [mm]	f _{yt} [MPa]	f _{yw} [MPa]	V _{test} [kN]	M _{test} [kN]	V _{num} [kN]	M _{num} [kN]	V _{test} / V _{num}	M _{test} / M _{num}
Bending tests of Elgaaly et al. []																		
1	12,7	152,4	304,8	0,607	304,8	50	19,8	18,53	12	14	289	682	-	180,9	-	178,9	-	1,01
2	12,7	152,4	304,8	0,607	304,8	45	38,1	35,92	25	25	289	682	-	193,3	-	181,4	-	1,07
3	12,7	152,4	304,8	0,607	304,8	55	41,9	40,7	23	33	289	682	-	175	-	178,8	-	0,98
4	12,7	152,4	304,8	0,607	304,8	62,5	49,8	57,25	26	51	289	682	-	175,2	-	178,7	-	0,98
5	12,7	152,4	304,8	0,76	304,8	55	41,9	40,7	23	33	376	682	-	237,8	-	231,1	-	1,03
6	12,7	152,4	304,8	0,76	304,8	62,5	49,8	57,25	26	51	376	682	-	223,2	-	229,4	-	0,97
Shear test of Moon et al. []																		
7	30	300	2000	4	3110	22,6	220	195	180	75	296	296	1052,8	-	1003	-	1,05	-
Shear test of Hannebauer []																		
8	15	200	500	2,5	1000	40	30	62	47	40	270	270	206,5	-	195,6	-	1,06	-

The results show, that there is a good agreement between the test results and the numerical calculations. In case of the bending tests the difference is between $\pm 3\%$. The observed failure modes in the tests and in the numerical models are the same, yielding of the flanges and the plastic buckling of the compressed flange.

A slightly larger difference (5-6%) between the test results and the numerical calculations can be observed in case of the shear buckling tests. The observed failure modes are the same, global shear buckling of the whole web panel, but the shear buckling resistance calculated by the numerical simulations are 5-6% smaller than the test results. The reason of it is the application of the first eigenmode imperfection with standard magnitudes, which results in safe solution for the shear buckling resistance. The test specimen could have smaller imperfections, what resulted the slightly larger resistance.

5.3. Numerical research program

The verification of the numerical model proved its applicability to determine the bending and shear buckling resistance of the analyzed girders. Based on the developed model a numerical parametric study is executed to analyze the structural behaviour of the corrugated web girders under combined bending and shear.

In frame of the numerical calculations girders with typical geometries used in bridges and in industrial buildings are also analyzed. A total of 20 specimens are investigated, 11 with typical bridge geometries and 9 with typical geometries of building structures. The analyzed geometries and material properties are summarized in Table 6.

Table 6: Analyzed girder geometries in the numerical parametric study.

	bf [mm]	tf [mm]	hw [mm]	tw [mm]	L/2 [mm]	alpha [°]	a1 [mm]	a2 [mm]	f _{yf} [MPa]	f _{yw} [MPa]
Typical bridge geometries										
1	300	30	2000	4	3200	23	220	200	296	460
2	300	20	1000	4	1000	45	150	150	355	510
3	400	40	1500	6	1365	45	200	200	355	510
4	400	40	1000	10	2000	45	200	200	355	510
5	450	40	2500	9	2440	32	330	330	355	510
6	450	40	2500	9	2250	45	330	330	355	510
7	450	40	2500	9	1960	60	330	330	355	510
8	300	20	2500	9	2440	32	330	330	355	510
9	300	30	2500	9	2440	32	330	330	355	510
10	350	30	2000	8	2440	32	330	330	355	510
11	350	30	1200	9	1800	36,8	250	250	355	510
Typical building geometries										
12	200	15	500	2,5	922	40	30	62	270	430
13	300	20	1000	4	1000	45	50	50	355	510
14	250	16	750	5	887	45	65	65	355	510
15	250	16	750	5	780	60	65	65	355	510
16	250	16	750	5	960	30	65	65	355	510
17	250	16	750	5	820	45	120	120	355	510
18	250	16	750	5	1365	45	200	200	355	510
19	250	16	750	5	546	45	40	40	355	510
20	220	12	1000	1,5	760	45	140	70	270	360

In the numerical calculations only the half of a simply supported girder is modelled. Symmetry conditions are defined at the left end of the model and simply support at the right end. There are two loads applied, which ratios are varying in the parametric study. The shear force ($V_{E,num}$) is introduced at the midspan of the girder and the bending moment ($M_{E,num}$) is supplemented by two normal forces applied in the center of gravities of the upper and lower flanges. The shear force is constant along the girder length, therefore the shear resistance can be determined on a definite way. The in-plane-moment diagram has a linear character along the girder length, which has the minimum value by the simply support and the maximum value at the midspan. The analyzed girder and the support and loading conditions can be seen in Fig. 17.

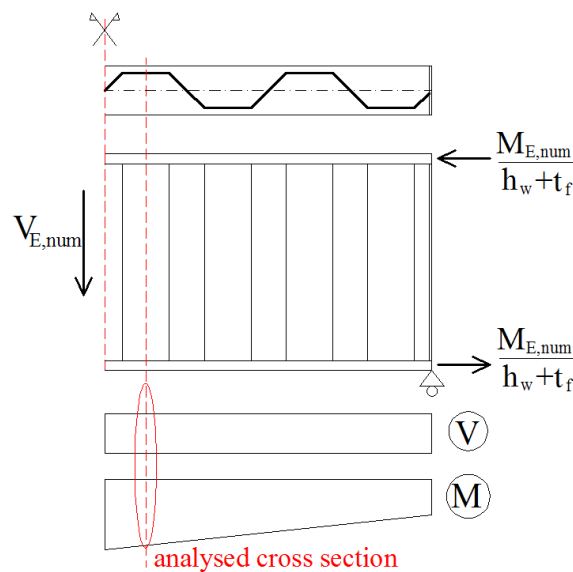
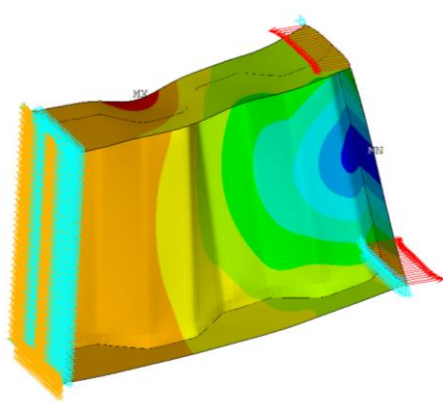


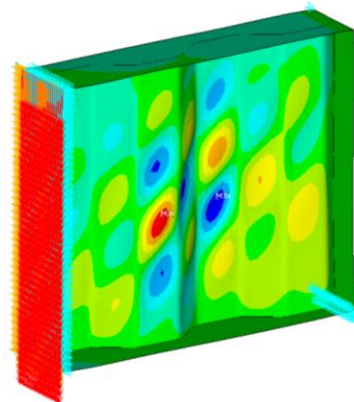
Fig. 17: Support and loading conditions of the analyzed model.

It is observed in the numerical calculations, that the bending failure of the corrugated web girders is mainly localized in a cross section in the middle of a parallel fold, where the flange outstand is the largest and at the same time the bending moment is also relative large. Therefore the M-V interaction is checked at the cross section marked by red in Fig. 17. The bending resistance of the analyzed girders are calculated in this cross section by the accompanying shear force. Four typical failure modes can be seen in Fig. 18. The first one (Fig.18.a.) is a typical bending failure under in-plane-bending. Two typical shear buckling failure modes can be seen in Fig. 18. For typical bridge geometries the dominant shear failure mode is usually the local buckling of the sub-panels. However for girders with geometries used in building structures are usually global buckling of the whole panel. Figure 18. d shows a typical failure mode under the combined shear and bending. It can be seen from the combined failure mode that the flange buckling occurs in the cross section at the middle of the first parallel fold. This cross section is the weakest from point of view the M-V resistance. Therefore the bending resistance is calculated to this cross section in the numerical parametric study.

The linear calculations and the evaluation of the stress distributions showed, that the geometry and the layout of the corrugation profile has a significant effect on the transverse bending moment. To analyze the worst situation from point of view the transverse bending moment to get the largest reduction in the bending resistance due to the shear force, the corrugation layout is modelled according to Figs. 8-9. In the numerical parametric study the bending and shear buckling resistances of all the analyzed girders are determined at first. In case of all girders 8 calculations are executed to analyze the M-V interaction domain by varying the values of V and M. A total of 160 numerical calculations are carried out.



a, bending failure



b, shear buckling failure (local buckling)

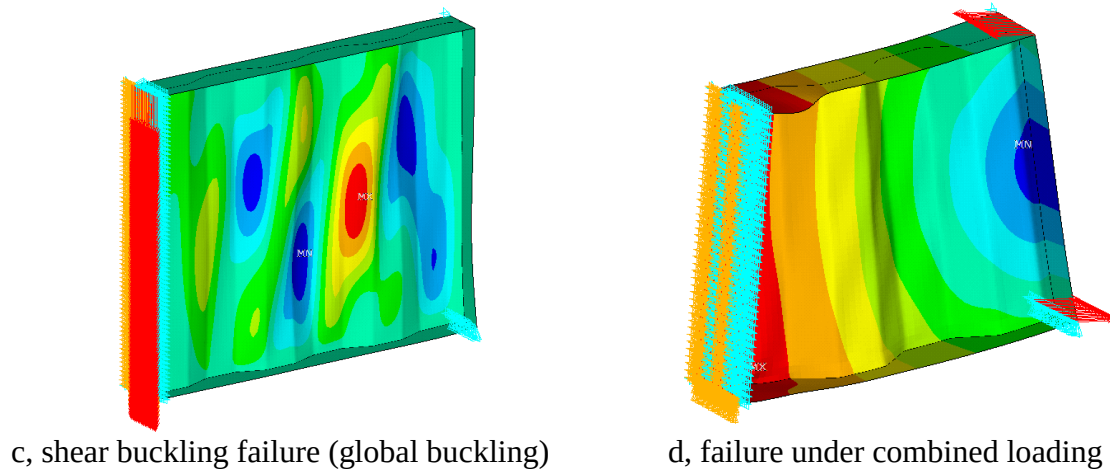


Fig. 18: Observed typical failure modes.

5.4. Results of the numerical calculations

For each analyzed girder the calculated resistances and the proposed interaction equation according to EN1993-1-5 [1] are compared. Two comparisons are shown in Fig. 19 for girders with geometries used in typical bridge and building structures. The horizontal axis shows the bending resistance, the vertical axis represents the shear resistance of the analyzed girders. The points show the results of the numerical calculations. The proposed interaction equation of the EN1993-1-5 [1] is also presented in both diagrams.

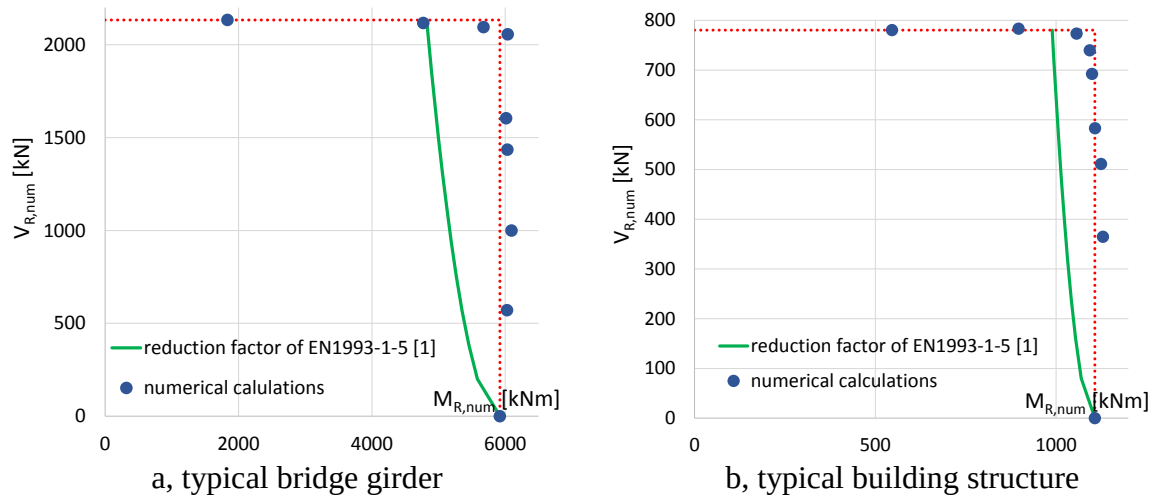


Fig. 19: Typical M-V interaction behaviour.

Results of the GMNI analysis show that there is just a very small resistance reduction due to the combined loading situation. All the blue points follow the dotted lines, which shows the “pure” bending and shear buckling resistances. It can be also observed, that the interaction equation on EN1993-1-5 [1] does not results in a large resistance reduction in case of typical building structure, but in case of bridges the reduction can be significant. The evaluation of the numerical calculations showed, that such a large resistance reduction is not needed for any girder geometries, and the current proposal of EN1993-1-5 [1] results in a conservative design.

The results of all the numerical calculations are presented in one diagram (Fig. 20).

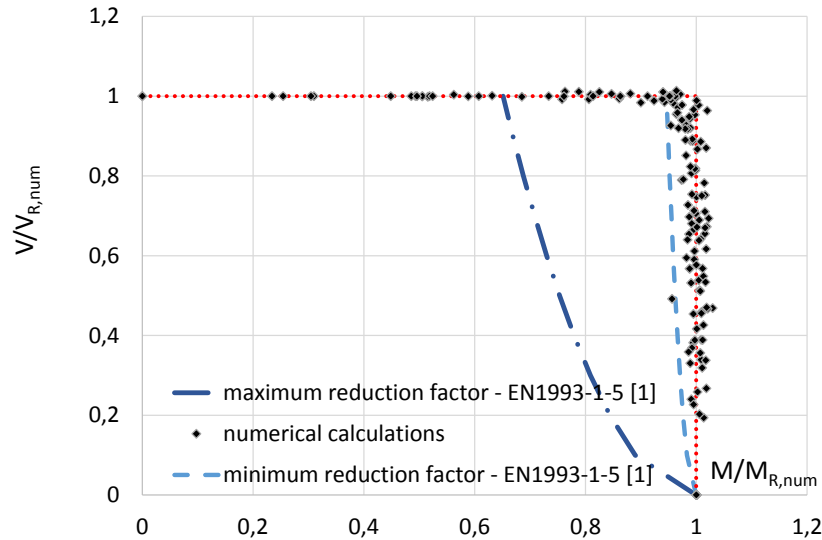


Fig. 20: Observed M-V interaction behaviour.

The results show, that the maximum reduction on the bending resistance due to the combined M-V loading situation is 4,7% in case of the analyzed 20 girders. These calculations prove the experimental results of Kuchta [9] who observed also just a small resistance reduction due to the combined loading situation, and proves the establishment of Elgally [5], that the M-V interaction for girders with trapezoidally corrugated webs could be neglected.

The observed resistance reduction is not larger than the web contribution in the bending resistance or the flange contribution in the shear buckling resistance. But these contributions are neglected in the design of the corrugated web girders.

Another important observation is also made based on the numerical calculations, that the M-V interaction behaviour of the corrugated web girders does not depend on the corrugation profile, however the maximum value of the transverse bending moment depends on the corrugation profile. The executed numerical simulations do not show relationship between the transverse bending moment and the bending resistance reduction, if plastic design is used to determine the bending resistance of the girder.

6. Conclusions

It was known from previous investigations that the shear force in the corrugated web results in an additional transverse bending moment in the flanges, which indicates additional normal stresses. Different design proposals can be found in the international literature to take this effect in the in-plane-bending resistance calculation of the corrugated web girders into account. The importance of the bending reduction factor (f_T) due to the transverse bending moment in the practical design is stated questionable by several researchers in the past, therefore further investigations was needed to clarify the structural behavior of the corrugated web girders under combined bending and shear.

Based on the evaluation of the previously analyzed girder geometries, or the geometries of existing real structures with corrugated webs, it is observed that girders with sinusoidally corrugated webs have usually small corrugation depths compared to the flange width, and the fold lengths of the equivalent trapeze profiles are also relative small. Girders with trapezoidal corrugation has usually different corrugation profiles. The applied fold lengths are usually 5-7 times larger than for sinusoidal corrugations and the used web thicknesses are also larger. These differences indicate, that the value of the transverse bending moment can be

significantly larger for girders with trapezoidal web corrugation, than for girders with sinusoidal corrugation. But this difference comes not from the shape of the corrugation profile, but from the usually used girder dimensions.

Analyzing the reduction factor (f_T) of EN1993-1-5 [1] it is observed, thus its value depends on the applied shear force, therefore this equation represents actually an M-V interaction curve. But this interaction equation has a different character than the usual M-V interaction equations used for steel structures, however the effect of the transverse bending moment is also unique. The evaluation of the calculated reduction factors showed that it can significantly reduce the bending resistance of the trapezoidally corrugated web girders with typical geometries used in the praxis. The average reduction in the bending resistance can be about 5% for sinusoidally and 13-18% for trapezoidally corrugated web girders by shear forces equal by 50% of the shear resistance if the most unfavorable girder layout is used. The analysis proves that the effect of the transverse bending moment can be significant in practical cases, therefore its investigation is necessary.

The additional normal stresses and the normal stresses coming from the in-plane-bending effect are compared and evaluated in this paper. The calculations showed, that the difference between the maximum value of the flange normal stresses can be significantly by considering or neglecting the effect of the transverse bending moment. If the limit state of the design is the maximum allowed stress, the effect of the transverse bending moment could have importance.

Previous investigations by Kövesdi et al. [14] stated, that the support conditions have a large influence on the maximum value of the transverse bending moment. The numerical calculations showed that in case of real structures the transverse bending moment can be influenced by the support conditions and its value can be significantly smaller than the theoretically determined maximum value. Based on the numerical calculations it can be concluded that both type of supports (longitudinal and lateral) can significantly reduce the maximum value of the additional normal stresses within a region of the support location. It means that in case of real structures with real support conditions the maximum value of the additional normal stresses can be significantly smaller than its theoretically determined maximum. Due to the large number of parameters which have influence on the transverse bending moment, its determination is quite hard by analytical methods. The maximum possible value, or the maximum from the average values can be determined by simple analytical solution proposed by Kövesdi et al. in [14]. But if the determination of a more accurate value of the transverse bending moment is necessary, the authors propose to use finite element software, modeling the real boundary conditions and girder geometries.

To analyze the effect of the transverse bending moment on the in-plane bending resistance of the corrugated web girders, the bending resistance and the M-V interaction behaviour is analyzed by numerical simulations. Results of the GMNI analysis show that there is just a very small resistance reduction due to the combined loading situation. The evaluation of the numerical calculations showed, that the current proposal of EN1993-1-5 [1] results in a conservative design.

The results show, that the maximum reduction on the bending resistance due to the combined M-V loading situation is 4,7% in case of the analyzed 20 girders. These calculations prove the experimental results of Kuchta [9] who observed also just a small resistance reduction due to the combined loading situation, and proves the establishment of Elgally [5], that the M-V interaction for girders with trapezoidally corrugated webs could be neglected. The observed resistance reduction is not larger than the web contribution in the bending resistance or the flange contribution in the shear buckling resistance. But these contributions are neglected in the design of the corrugated web girders.

Based on the numerical calculations it is also observed that the M-V interaction behaviour of the corrugated web girders does not depend on the corrugation profile, however the maximum value of the transverse bending moment depends on it. The executed numerical simulations do not show relationship between the transverse bending moment and the bending resistance reduction, if plastic design is applied.

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