

State Space Control of Quadratic Boost Converter using LQR and LQG approaches

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Abstract—The systematic design of a Linear Quadratic Regulator (LQR) state-feedback controller with an integral action is presented for a quadratic DC/DC boost converter topology. The weighting factors are selected with respect to energy considerations. By using a Kalman Filter for both filtering and state estimation purpose a Linear Quadratic Gaussian (LQG) controller is obtained. For the proper operation of the LQG controller an average state-space model of the quadratic boost converter with non ideal elements is introduced. Detailed simulation study in the discrete time-domain is presented demonstrating the robustness of both LQR and LQG.

Index Terms—DC-DC power converters, Optimal control, Kalman filters

I. INTRODUCTION

Switch mode DC-DC converters are one of the mostly applied power electronic converters. They play important role in various devices, from low power home appliances via electric cars to the utilization of renewable energies, as they enable the efficient conversion of the voltage level. The novel application fields, like photovoltaic microinverters [1] or automotive applications [2], poses many challenges for the researchers to develop novel topologies and controllers. The quadratic converters received increasing attention in the last decade as they have the voltage conversion ratio of two cascaded converters, but with fewer switches. However, they are generally less efficient than other converter types, they can be useful for certain applications [1], [3], [4], [5].

Another intensive research field is the high performance control of DC-DC converters. In [6] a linear proportional-integral (PI) voltage controller are extended by using sensorless predictive current control with a self-correction observer. Three adaptive PI controllers applicable to a large class of switched power converters with unknown load resistance is introduced in [7]. Robust control methods also offer a proper solution for controlling DC-DC converters [5]. A consistent framework for robust Linear Quadratic Regulator (LQR) control of power converters is presented in [8]. Similarly to this paper a Linear Quadratic Gaussian (LQG) approach with an integral term for a boost converter is presented in [9]. In [10] both a PI based multi-loop and an LQG approach is used to control a buck converter. Nonlinear sliding mode control is applied for a quadratic boost converter in [1].

In this paper the design of an LQR and LQG control for a quadratic boost converter is presented.

II. QUADRATIC DC/DC CONVERTER

Figure 1 shows the conventional quadratic DC/DC boost converter with a single active switch where the elements are

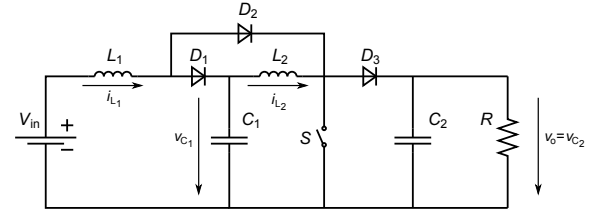


Fig. 1. Quadratic boost converter with ideal circuit elements

assumed to be ideal. It consists of two inductances L_1 , L_2 , two capacitors C_1 , C_2 , one active switch S and three diodes D_1 , D_2 , D_3 . The quadratic DC/DC boost converter is synthesized by cascading two DC/DC boost converters [3]. The operation of the converter is the following. During ON state of the switch S , diode D_2 is conducting, while diodes D_1 and D_3 are reverse biased and inductor L_1 is charged by the input voltage. During the OFF state of the switch S , the diodes D_1 and D_3 are conducting, while diode D_2 is reverse biased.

If the converter is assumed to operate in Continuous Conduction Mode (CCM), then the conversion ratio is given as

$$\frac{V_o}{V_{in}} = \frac{1}{(1-D)^2} \quad (1)$$

where $D = t_{on}/T_s$ is the duty ratio of the switch in steady-state, $T_s = t_{on} + t_{off}$ is the switching period, t_{on} and t_{off} is the duration of the ON state and OFF state, respectively [3].

The behaviour of the quadratic boost converter can be modelled with average state-space representation. State space averaging is a well-known method used in modelling switching converters [11]. For a system with a single switching device the average model may be developed by determining the state equations for each of the two switch states (ON and OFF state), then calculating a weighted average of the two sets of equations using ratio $t_{on}/T_s = d$ and $t_{off}/T_s = 1 - d$ as weighting factor, respectively. d is the actual duty ratio. The average state-space representation of the quadratic DC/DC boost converter is

$$\begin{aligned} \dot{\mathbf{v}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \quad (2)$$

where $\mathbf{x}^T = [i_{L1} \ i_{L2} \ v_{C1} \ v_{C2}]$ is the state vector, $\mathbf{v}$ is the velocity vector, $\mathbf{u} = [V_{in}]$ is the input vector, $\mathbf{C}^T = [0 \ 0 \ 0 \ 1]$ is the output vector and $y = v_{C2}$ is the output voltage of the converter. The $\mathbf{A}$ system and $\mathbf{B}$ input matrices

are

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -\frac{1-d}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-d}{L_2} \\ \frac{1-d}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-d}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In the case of DC/DC converters with voltage control loop the controller sets the duty ratio to ensure that, the output voltage be equal to the reference voltage. Therefore the duty ratio acts as an input signal. In the state space model derived $\mathbf{A}$ depends on the duty ratio d , and only the effect of the input voltage V_{in} is included in the input matrix. One way to design a voltage control loop and include the duty ratio in the input vector the model should be linearised around the working point. Thus, each variable should be written as the sum of a steady state, denoted by capital letter, and a small-signal component, denoted by $\tilde{\cdot}$, like $d = D + \tilde{d}$ or $v_{C2} = V_{C2} + \tilde{v}_{C2}$.

The linearised average state space model of the converter can be given as

$$\dot{\tilde{\mathbf{v}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\tilde{\mathbf{u}} \quad (3)$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & 0 & -\frac{1-D}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-D}{L_2} \\ \frac{1-D}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-D}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}$$

$$\tilde{\mathbf{B}} = \begin{bmatrix} \frac{V_{in}}{L_1(1-D)} & \frac{1}{L_1} \\ \frac{V_{in}}{L_2(1-D)^2} & 0 \\ -\frac{V_{in}}{RC_1(1-D)^4} & 0 \\ -\frac{V_{in}}{RC_2(1-D)^3} & 0 \end{bmatrix} \quad \tilde{\mathbf{u}} = \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

In this model $\tilde{\mathbf{A}}$ depends only on the steady-state value of the duty ratio D , and the small deviation of the duty ratio $\tilde{d}$ is included in the input vector.

III. STATE SPACE CONTROLLER DESIGN

For state space controller design let us consider the small signal state space model of the converter described in (3). From the two inputs of the converter only $\tilde{d}$ would function as controlling input, set by the controller. The voltage change $\tilde{v}_{in}$ appears only as a disturbance. Therefore, it is advisory to separate the input matrix into two parts

$$\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}_d\tilde{d} + \tilde{\mathbf{B}}_{v_{in}}\tilde{v}_{in} \quad (4)$$

where $\tilde{\mathbf{B}}_d$ and $\tilde{\mathbf{B}}_{v_{in}}$ are the first and the second column of $\tilde{\mathbf{B}}$, respectively.

In order to control the output voltage v_{C2} of the converter State Variable Feedback (SVFB) control is used. In this method the state variables of the system are fed back through a feedback gain vector denoted by $\tilde{\mathbf{K}}$. As a result the new system matrix is $(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}_d\tilde{\mathbf{K}})$. By properly set the values of $\tilde{\mathbf{K}}$, the poles of the new system matrix can be placed to obtain the desired dynamic performance. The drawback of this solution is that, as the quadratic boost converter is a type 0 system, the SVFB in closed loop application does not eliminate the steady state error. For this reason an integral term should be added [9]. Figure 2 shows the block diagram of the SVFB with an

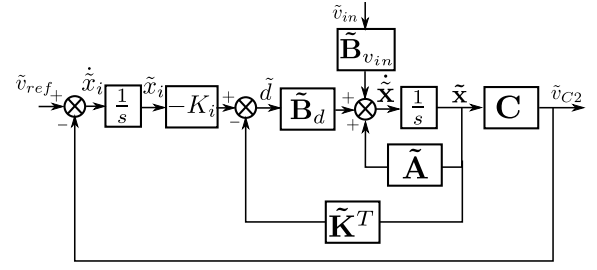


Fig. 2. State variable feedback extended by an integrator

integral term, where K_i is the gain of the additional integrator and $\mathbf{C}$ is the output vector (see (2)).

The integral term involves a new state variable $\tilde{x}_i = \tilde{v}_{ref} - \tilde{v}_{C2}$ which should be included into the state vector (Fig.2). Based on Fig.2 the state space model of the closed loop system using the extended state vector $\tilde{\mathbf{x}}_e$ and state matrices is the following

$$\dot{\tilde{\mathbf{x}}}_e = (\tilde{\mathbf{A}}_e - \tilde{\mathbf{B}}_{d,e}\mathbf{K}_e)\tilde{\mathbf{x}}_e + \mathbf{k}v_{ref} + \tilde{\mathbf{B}}_{v_{in},e}\tilde{v}_{in} \quad (5)$$

where $\tilde{\mathbf{x}}_e^T = [\tilde{\mathbf{x}}^T \quad \tilde{x}_i]$, $\mathbf{k}^T = [0 \ 0 \ 0 \ 0 \ 1]$, $\mathbf{K}_e^T = [\mathbf{K}^T \ K_i]$ and

$$\tilde{\mathbf{A}}_e = \begin{bmatrix} \tilde{\mathbf{A}} & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \tilde{\mathbf{B}}_{d,e} = \begin{bmatrix} \tilde{\mathbf{B}}_d \\ 0 \end{bmatrix} \quad \tilde{\mathbf{B}}_{v_{in},e} = \begin{bmatrix} \tilde{\mathbf{B}}_{v_{in}} \\ 0 \end{bmatrix}$$

There are several methods to calculate $\mathbf{K}_e$. For example by knowing the location of the desired poles the Ackermann's formula [12] can be used to calculate $\mathbf{K}_e$. In this paper a controller design method based on LQR is presented in the next section. It should be noted that a closed-loop system with LQR controller has a phase margin larger than 60° [8].

IV. LINEAR QUADRATIC REGULATOR (LQR) DESIGN

The LQR approach deals with the optimization of a cost function or performance index. Thus, the designer can weight which states and which inputs are more important in the control action to seek for appropriate transient and steady-state performances [8], [12].

The cost is described by a quadratic function [12]:

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{Q}_w \mathbf{x} + \mathbf{u}^T \mathbf{R}_w \mathbf{u}) dt \quad (6)$$

Here and later in point IV. we are applying the notations used in LQR technique.

For a system which has n state variables and m inputs $\mathbf{Q}_w$ matrix is an n by n positive semi-definite matrix which contains weights which penalize the deviations of the state variables from the desired state. The matrix $\mathbf{R}_w$ is an m by m matrix which contains weights that penalize the control effort. The $\mathbf{K}$ feedback gain vector which provides the solution to the problem can be calculated by [12]:

$$\mathbf{K} = \mathbf{R}_w^{-1} \mathbf{B}^T \mathbf{P} \quad (7)$$

where $\mathbf{B}$ is the input matrix of the plant and $\mathbf{P}$ is the solution of the algebraic Riccati equation

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T \mathbf{P} - \mathbf{P}\mathbf{B}\mathbf{R}_w^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q}_w = 0 \quad (8)$$

By knowing the state matrices $\mathbf{A}$ and $\mathbf{B}$ and properly selecting $\mathbf{Q}_w$ and R_w , the value of $\mathbf{K}$ can be obtained. One simple way can be for example to use the $lqr(sys, \mathbf{Q}, R, N)$ function in Matlab.

As the LQR method computes the feedback gain depending on the small signal state space model and the weighting matrix $\mathbf{Q}_w$, R_w , the dynamic behaviour of the resulting closed loop system will be affected by selection of the steady-state duty ratio D . Therefore the proper selection of D is crucial even if the converter will be operated in a wide range of D . The problem can be solved by dividing the operation range of the duty ratio into segments and calculating the proper feedback gain vector for each segment. In this paper the parameters of the LQR controller are calculated for $D = 0.65$. Naturally, the calculation procedure presented can be used for other duty ratio range as well.

As it was mentioned in the previous section the state space model of the controlled quadratic boost converter was extended by an integrator. Therefore it is a fifth order system so $\mathbf{Q}_w$ is a 5 by 5 matrix. As the state variables of the system $\tilde{i}_{L1}$, $\tilde{i}_{L2}$, $\tilde{v}_{C1}$ and $\tilde{v}_{C2}$ have well defined physical meanings and $\tilde{x}_i$ is the time-integral of the error signal $\tilde{v}_{ref} - \tilde{v}_{C2}$, it is straightforward to penalize the transients of these individually by placing non-zero elements only in the diagonal of $\mathbf{Q}_w$. The weighting factors were selected with respect to energy considerations. Similar method for an asymmetrical interleaved DC-DC converter is described in [13]. In steady state the energies stored in the inductors and capacitors of the circuit are:

$$E_{L_j} = \frac{1}{2} L_j I_{L_j}^2 \quad \text{and} \quad E_{C_j} = \frac{1}{2} C_j V_{C_j}^2 \quad (9)$$

where $j = 1, 2$ and I_{L1}, I_{L2}, V_{C1} and V_{C2} are the steady state values of the respective inductor currents and capacitor voltages.

As the $\mathbf{Q}_w$ matrix is related to energy costs the weighting matrices were set as

$$\mathbf{Q}_w = \frac{1}{C_2 V_{C2}^2} \begin{bmatrix} L_1 I_{L1}^2 & 0 & 0 & 0 & 0 \\ 0 & L_2 I_{L2}^2 & 0 & 0 & 0 \\ 0 & 0 & C_1 V_{C1}^2 & 0 & 0 \\ 0 & 0 & 0 & C_2 V_{C2}^2 & 0 \\ 0 & 0 & 0 & 0 & C_2 \cdot G \end{bmatrix}$$

and $R_w = 10^3$

The coefficients of $\tilde{i}_{L1}$, $\tilde{i}_{L2}$, $\tilde{v}_{C1}$ and $\tilde{v}_{C2}$ were set based on the average energies stored during steady state in the respective inductors or capacitors. In addition, matrix $\mathbf{Q}_w$ was normalized in such a way that the coefficient of the output voltage $\tilde{v}_{C2}$ be 1.

The dimension of R_w is 1 because the controller can manipulate only the duty ratio $\tilde{d}$. The coefficient G of $\tilde{x}_i$ and R_w were set by systematic simulation and recursive analytical calculations in such a way that the penalty on the control effort would result in controller output, that is, the duty ratio remains between 0 and 0.8 as it is necessary for properly controlling the switch. Over $D = 0.8$ the voltage amplification starts to deviate significantly from the theoretical one defined by (1) due to the non-ideal circuit elements [3]. Furthermore the efficiency of the converter is drastically decreases at high duty ratio [1]. At the end of the procedure the selected values

were $G = 10^5$, $R_w = 10^3$. For our case the resultant feedback gain vector $\mathbf{K}_e$ is

$$\mathbf{K}_e = [0.01238 \quad 0.02442 \quad -0.00185 \quad 0.037 \quad -51.041]$$

After the controller has been designed and simulated in continuous time assuming ideal elements the modelling was changed to operate the system in discrete time with sampling time $T_s = 2 \cdot 10^{-5}$ and the non-ideal circuit elements was also taken into consideration.

V. STATE ESTIMATION

The LQR approach requires the knowledge of all state variables. Even though the sensors required to measure voltage and current signals are rather cheap it can be worth with reducing the number of sensors by estimating the state variables. In this section the estimator design with state estimation is introduced. The goal was to create an estimator using only the input and output voltage (v_{in}, v_{C2}) measurements and which can run at the same sampling frequency as the LQR controller ($f_s = 50 \text{ kHz}$).

If a dynamic system is observable all state variables can be estimated from the measured inputs by knowing the initial conditions. Practically the observer is a mathematical model of the plant. A precise mathematical model is required to estimate the state variables accurately. Using the inputs and the averaged state space equations presented in (2) and substituting the actual duty ratio d , the estimated state variable values deviate from the real ones due to the non-ideal circuit elements. Therefore for the estimator an averaged state space model of the quadratic boost converter considering the non-ideal components should be derived.

Figure 3 shows the quadratic boost converter when the non-ideal circuit elements are taken into consideration. The non-ideal elements are the resistance of the inductors R_{L1} and R_{L2} , the equivalent series resistance (ESR) of the two capacitors R_{C1} and R_{C2} , the resistance of the diodes R_d , and the resistance of the semiconductor switch R_{SW} . The forward voltage drop of the diodes, denoted by V_F , are assumed to be constant and taken into consideration in the input vector $\mathbf{B}_{r,d}$. The average state space model of the converter with non-ideal circuit elements is

$$\mathbf{v} = \dot{\mathbf{x}} = \mathbf{A}_r \mathbf{x} + \mathbf{B}_{r,v_{in}} v_{in} + \mathbf{B}_{r,d} \quad (10)$$

where

$$\mathbf{B}_{r,v_{in}} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{B}_{r,d} = \begin{bmatrix} -\frac{V_F}{L_1} \\ -\frac{V_F(1-d)}{L_2} \\ 0 \\ 0 \end{bmatrix}$$

Here the subscript r denotes to "real" and $\mathbf{A}_r$ is given by (11) (see next page).

The estimator should be implemented in the discrete time domain. By using forward Euler method, (10) can be rewritten as

$$\frac{\mathbf{x}(k+1) - \mathbf{x}(k)}{T_s} = \mathbf{A}_r(k) \mathbf{x}(k) + \mathbf{B}_{r,v_{in}} v_{in}(k) + \mathbf{B}_{r,d} \quad (12)$$

By expressing $\mathbf{x}(k+1)$

$$\mathbf{x}(k+1) = \mathbf{x}(k) + T_s (\mathbf{A}_r(k) \mathbf{x}(k) + \mathbf{B}_{r,v_{in}} v_{in}(k) + \mathbf{B}_{r,d}) \quad (13)$$

$$\mathbf{A}_r = \begin{bmatrix} \frac{-RC_1(1-\tilde{d})-RL_1-R_d-R_{SW}\tilde{d}}{L_1} & \frac{RC_1(1-\tilde{d})-R_{SW}\tilde{d}}{L_1} & -\frac{1-\tilde{d}}{L_1} & 0 \\ \frac{RC_1(1-\tilde{d})}{L_2} - \frac{R_{SW}\tilde{d}}{L_2} & \frac{-(R_d + \frac{R}{R+R_{C2}})(1-\tilde{d})-R_{SW}\tilde{d}-RC_1-R_{L2}}{L_2} & \frac{1}{L_2} & \frac{-R(1-\tilde{d})}{L_2(R+R_{C2})} \\ \frac{1-\tilde{d}}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{R(1-\tilde{d})}{C_2(R+R_{C2})} & 0 & \frac{-1}{C_2(R+R_{C2})} \end{bmatrix} \quad (11)$$

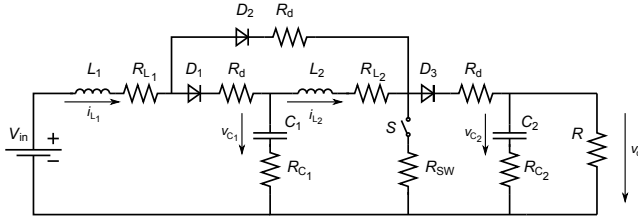


Fig. 3. Quadratic boost converter with non-ideal circuit elements

From (13) the resultant discrete system matrix is $(\mathbf{I} + T_s \mathbf{A}_r(k))$ where $\mathbf{I}$ is the identity matrix. To obtain the state observer's equations the error of the model's output $\hat{\mathbf{y}}_{err} = \hat{\mathbf{y}}_{meas} - \hat{\mathbf{y}}_{est} = \hat{v}_{C2} - \hat{v}_{C2}$ has to be fed back through a gain vector $\mathbf{L}$ to the state equation

$$\hat{\mathbf{x}}(k+1) = (\mathbf{I} + T_s \mathbf{A}_r(k) - T_s \mathbf{C}_r \mathbf{L}) \hat{\mathbf{x}}(k) + \quad (14)$$

$$T_s \mathbf{B}_{r,v_{in}} v_{in}(k) + T_s \mathbf{B}_{r,d} d + T_s \mathbf{L} y_{meas}(k) \quad (15)$$

$$\hat{\mathbf{y}}(k) = \mathbf{C}_r \hat{\mathbf{x}}(k)$$

where $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ denote that they are estimated values and $\mathbf{C}_r^T = [0 \ 0 \ 0 \ 1]$.

In the observer-based controller implementation the duty ratio $d(k-1)$ which was the controller output in the $(k-1)^{th}$ sampling is used to re-calculate the system matrix $\mathbf{A}_r(k)$ in the k^{th} sampling.

VI. KALMAN FILTER

Kalman Filter are used for both filtering and state estimation purpose. In the paper due to feasibility aspects a steady-state Kalman Filter is used applying a constant gain matrix $\mathbf{L}$ as this solution requires much less computational time than the time-varying one, which updates the gain in every sampling period [14]. Consider the state space equations with process ω and measurement noises w included to the system description. The state space equations with noises in their usual form are:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \omega \quad (16)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + w$$

For the optimal design of the observer gain matrix $\mathbf{L}$, the covariance matrix of the process noises $\mathbf{Q}_o$ and the covariance matrix of the measurement noises $\mathbf{R}_o$ have to be set. $\mathbf{L}$ then is determined using a cost functional similar to that in the case of the LQR. The resultant Kalman Filter is also called as a Linear-Quadratic Estimator (LQE). The combination of LQE and LQR approaches is called Linear Quadratic Gaussian (LQG) controller. The continuous time solution to the optimal observer problem (similarly to the LQR) is [12]

$$\mathbf{L} = \mathbf{P}_o \mathbf{C}^T \mathbf{R}_o^{-1} \quad (17)$$

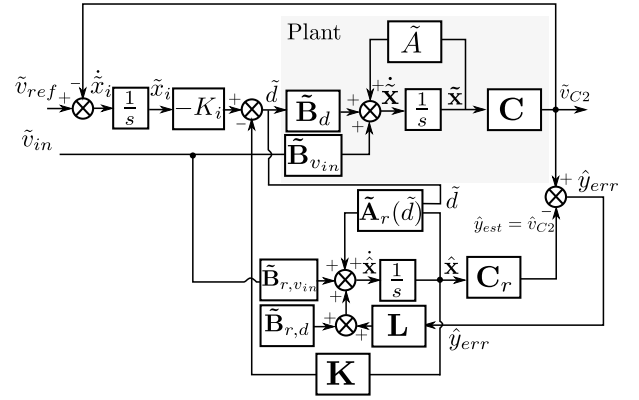


Fig. 4. State variable feedback extended by an integrator applying observer (LQG control). Continuous model.

where $\mathbf{P}_o$ is the solution of the algebraic Riccati equation:

$$\mathbf{A} \mathbf{P}_o + \mathbf{P}_o \mathbf{A}^T - \mathbf{P}_o \mathbf{C}^T \mathbf{R}_o^{-1} \mathbf{C} \mathbf{P}_o + \mathbf{Q}_o = \mathbf{0} \quad (18)$$

The calculation was executed in Matlab using the `kalman(sys,Qn,Rn,Nn)` function. The function returns the discrete observer gain vector $\mathbf{L}$ if the system *sys* is in discrete time. The LQG problem was solved for $\mathbf{L}$ using the resultant discrete system matrix of the advanced equations $(\mathbf{I} + T_s \mathbf{A}_r)$ with nominal duty ratio $D = 0.65$ and the output matrix $\mathbf{C}_r$. $\mathbf{Q}_o, \mathbf{R}_o$ were determined by estimating the noise:

$$\mathbf{Q}_o = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

and $R_o = 1$. $R_o = 1$ means that, the voltage measurements will contain less or equal to $\sqrt{1} = 1V$ measurement noise with $\sim 68\%$ probability. Finally the resulting gain vector is:

$$\mathbf{L}^T = [-0.282 \quad 0.747 \quad -2.666 \quad 0.6158]$$

The block diagram of the complete LQG controller can be seen on Fig. 4. For sake of simplicity the blokk diagram shows the operation in continuous time domain.

VII. SIMULATION RESULTS

The operation of the quadratic boost converter using both LQR and LQG controller were investigated by simulation. A complete converter model was developed in MATLAB/Simulink environment with the help of the SimPower Systems toolbox. In the simulation model non-ideal circuit elements were used. To make the simulation more realistic the algorithms of the discrete controllers and the estimator with sampling time $T_s = 2 \cdot 10^{-5} s$ were implemented using the *Embedded Matlab Function (EMF)* block. The calculation

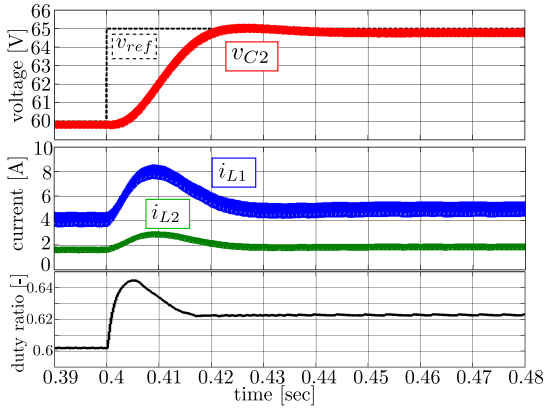


Fig. 5. Simulated response of the LQR voltage control for sudden change in the reference signal $v_{ref} = 60 \rightarrow 65$ V

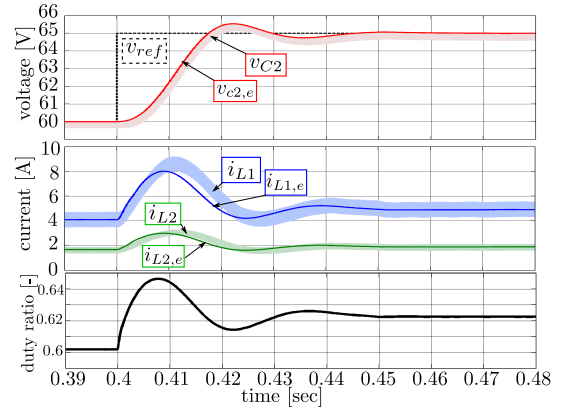


Fig. 6. Simulated response of the LQG voltage control for sudden change in the reference signal $v_{ref} = 60 \rightarrow 65$ V

time is modelled by one sampling time long delay. The switching period was the same as the controller period.

A low power (≈ 100 W) quadratic boost converter was designed and built in the laboratory. Its parameters were used during the simulation analysis presented in this paper. The parameters can be found in Table I.

TABLE I
PARAMETERS OF THE QUADRATIC BOOST CONVERTER

L_1	L_1 inductance	0.1 [mH]
L_2	L_2 inductance	0.56 [mH]
C_1	C_1 capacitance	22 [μF]
C_2	C_2 capacitance	100 [μF]
R	loading resistance	100 [Ω]
R_{L1}	L_1 resistance	0.047 [Ω]
R_{L2}	L_2 resistance	0.087 [Ω]
R_{C1}	C_1 ESR	0.063 [Ω]
R_{C2}	C_2 ESR	0.2 [Ω]
V_F	diodes voltage drop	0.7 [V] Ω
R_d	diodes on-resistance	0.035 [Ω]
R_{SW}	MOSFET on-resistance	0.12 [Ω]

In this section the simulated dynamic performance of the LQR and LQG control methods are presented for sudden change in the voltage reference signal v_{ref} , in the input voltage v_{in} and in the loading resistance R at $t = 0.4$ sec.

A. Sudden change in the reference voltage

v_{ref} changes stepwise from 60 V to 65 V. Figure 5 and 6 show the time function of the output voltage v_{C2} , the currents i_{L1} and i_{L2} and the duty ratio d applying LQR and LQG control approach, respectively. For LQG the estimated values of the state variables, denoted by subscript "e" are also plotted (Fig.6). It can be seen that, the output voltage v_{C2} in both cases follows the reference signal. The response by using LQG controller is more oscillatory and the overshoot is greater comparing with LQR. In steady-state the estimated and the actual value practically are the same, when LQG control approach was used. During transient the estimated current state variables $i_{L1,e}$ and $i_{L2,e}$ slightly deviate from the actual i_{L1} and i_{L2} . Nevertheless, the controller works properly.

B. Sudden change in the input voltage

v_{in} changes stepwise from 12 V to 10 V while keeping the reference voltage constant ($v_{ref} = 60$ V). Figure 7 and 8 show the response for LQR and LQG controller, respectively. In both cases after a small transient provoked by the change in the

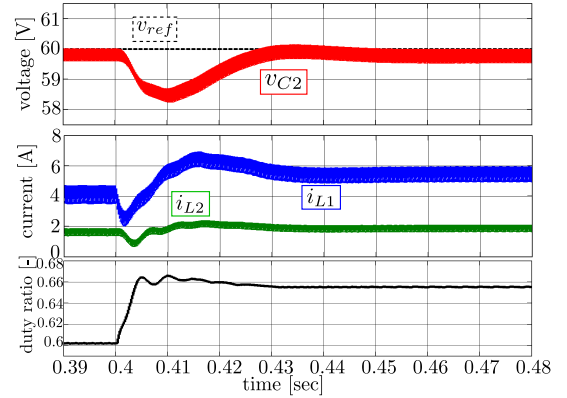


Fig. 7. Simulated response of the LQR voltage control for sudden change in the input voltage $v_{in} = 12 \rightarrow 10$ V

input voltage, the output voltage v_{C2} settles down and follows the reference signal. Again the response is more oscillatory for LQG. Similarly to the previous case the estimated variables $i_{L1,e}$ and $i_{L2,e}$ differ from the real values i_{L1} and i_{L2} only during transient (Fig.8).

C. Sudden change in the load resistance

R changes stepwise from 100 Ω to 90 Ω while keeping the reference voltage $v_{ref} = 60$ V and the input voltage $v_{in} = 12$ V constant. Figure 9 and 10 show the response for LQR and

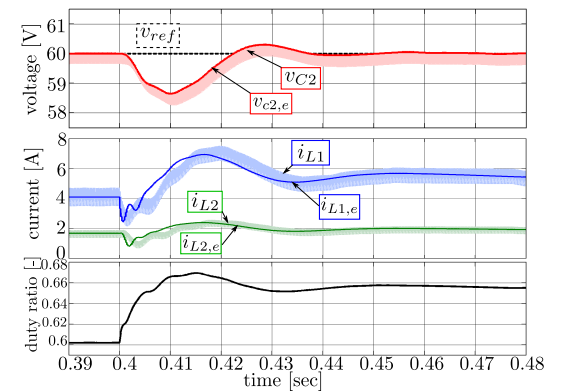


Fig. 8. Simulated response of the LQG voltage control for sudden change in the input voltage $v_{in} = 12 \rightarrow 10$ V

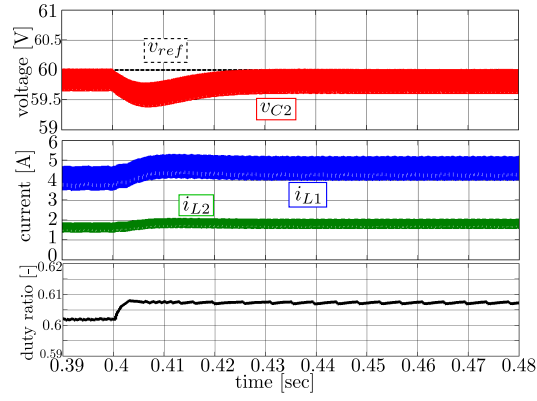


Fig. 9. Simulated response of the LQR voltage control for sudden change in the load $R = 100 \rightarrow 90 \Omega$

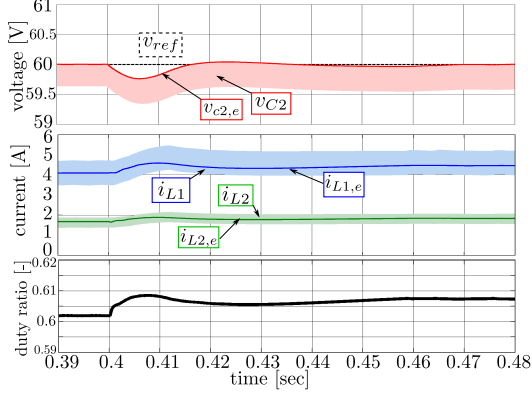


Fig. 10. Simulated response of the LQE voltage control for sudden change in the load $R = 100 \rightarrow 90 \Omega$

LQG, respectively. The two responses are the same for the two controllers. The estimated values of the state variables, when LQG control approach is applied, practically follows the time function of the real current even in transient.

By applying a greater change in the load the estimator fails to calculate the actual value of the state variables. In Fig.11 at $t = 0.4$ sec R changes stepwise from 100Ω to 50Ω . However, the output voltage tracks the reference voltage after an oscillating transient response the estimated current values considerable differ from the actual one.

In summary, it can be concluded that the LQG control

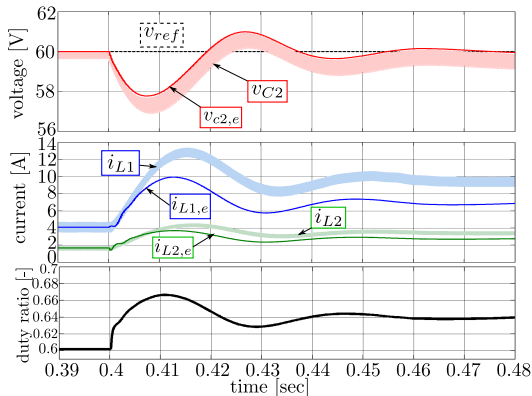


Fig. 11. Simulated response of the LQE voltage control for sudden change in the load $R = 100 \rightarrow 50 \Omega$

approach has very similar performance as the LQR while using only two voltage sensors to measure the input and output voltages. The drawback of the LQG approach is that, when the load resistance R deviates from the nominal one, the state estimations accuracy weakens (Fig.11). Another advantage of LQG besides the less amount of required measurements is that, the measurement-noise suppression can be controlled directly by R_o .

VIII. CONCLUSIONS

The paper provides a systematic method to design state space controller of a quadratic boost converter using LQR and LQG approaches. The energy consideration were used in the selection of the parametes of LQR and were calculated for duty ratio $D = 0.65$. The duty ratio remains in the range from 0.6 to 0.67 even in transient. The simulation results verified the robust performance of the controllers and the estimator.

REFERENCES

- [1] O. Lopez-Santos, L. Martinez-Salamero, G. Garcia, H. Valderrama-Blavi, and T. Sierra-Polanco, "Comparison of quadratic boost topologies operating under sliding-mode control," in *Power Electronics Conference (COBEP), 2013 Brazilian*, Oct 2013, pp. 66–71.
- [2] O. Pelan, O. Cornea, N. Muntean, and F. Blaabjerg, "Comparative efficiency evaluation of buck and hybrid buck dc-dc converters for automotive applications," in *14th International Conference on Optimization of Electrical and Electronic Equipment*, 2014, pp. 582–587.
- [3] D. S. Wijeratne and G. Moschopoulos, "Quadratic power conversion for power electronics: principles and circuits," *Circuits and Systems I: Regular Papers, IEEE Transactions on*, vol. 59, no. 2, pp. 426–438, 2012.
- [4] P. Yang, J. Xu, G. Zhou, and S. Zhang, "A new quadratic boost converter with high voltage step-up ratio and reduced voltage stress," in *Power Electronics and Motion Control Conference (IPEMC), 2012 7th International*, vol. 2. IEEE, 2012, pp. 1164–1168.
- [5] F. Alonge, R. Rabbeni, M. Pucci, and G. Vitale, "Identification and robust control of a quadratic dc/dc boost converter by hammerstein model," in *Energy Conversion Congress and Exposition (ECCE), 2014 IEEE*. IEEE, 2014, pp. 3355–3362.
- [6] Q. Zhang, R. Min, Q. Tong, X. Zou, Z. Liu, and A. Shen, "Sensorless predictive current controlled dc-dc converter with a self-correction differential current observer," *Industrial Electronics, IEEE Transactions on*, vol. 61, no. 12, pp. 6747–6757, Dec 2014.
- [7] M. Hernandez-Gomez, R. Ortega, F. Lamnabhi-Lagarigue, and G. Escobar, "Adaptive pi stabilization of switched power converters," *Control Systems Technology, IEEE Transactions on*, vol. 18, no. 3, pp. 688–698, 2010.
- [8] C. Olalla, R. Leyva, A. El Aroudi, and I. Queinnec, "Robust lqr control for pwm converters: an lmi approach," *Industrial Electronics, IEEE Transactions on*, vol. 56, no. 7, pp. 2548–2558, 2009.
- [9] L. Camacho-Solorio and A. Sarinana-Toledo, "I-lqg control of dc-dc boost converters," in *Electrical Engineering, Computing Science and Automatic Control (CCE), 2014 11th International Conference on*, Sept 2014, pp. 1–6.
- [10] R. Priewasser, M. Agostinelli, C. Unterrieder, S. Marsili, and M. Huemer, "Modeling, control, and implementation of dc-dc converters for variable frequency operation," *Power Electronics, IEEE Transactions on*, vol. 29, no. 1, pp. 287–301, 2014.
- [11] J. G. Kassakian, M. F. Schlecht, and G. C. Verghese, *Principles of power electronics*. Addison-Wesley Reading, USA, 1991.
- [12] R. Tymerski, A. Chuinard, and F. Rytönen, "Applied classical and modern control system design," Lecture Notes, ECE451, Portland State University.
- [13] E. Arango, C. A. Ramos-Paja, J. Calvente, R. Giral, and S. I. Serna-Garces, "Asymmetrical interleaved dc/dc switching converters for photovoltaic and fuel cell applicationspart 2: Control-oriented models," *Energies*, vol. 6, no. 10, pp. 5570–5596, 2013. [Online]. Available: <http://www.mdpi.com/1996-1073/6/10/5570>
- [14] M. S. Grewal and A. P. Andrews, *Kalman filtering: theory and practice using MATLAB*. John Wiley & Sons, 2011.