

Computation of hadron jet cross sections at high accuracy

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Hadron jets

In e^-e^+ collisions the most frequent event is the hadron jets. From an initial quark-antiquark pair showers of parton appear which contain quarks and gluons (see Figure 1.). At low energy the partons form hadrons and we get hadron jets.

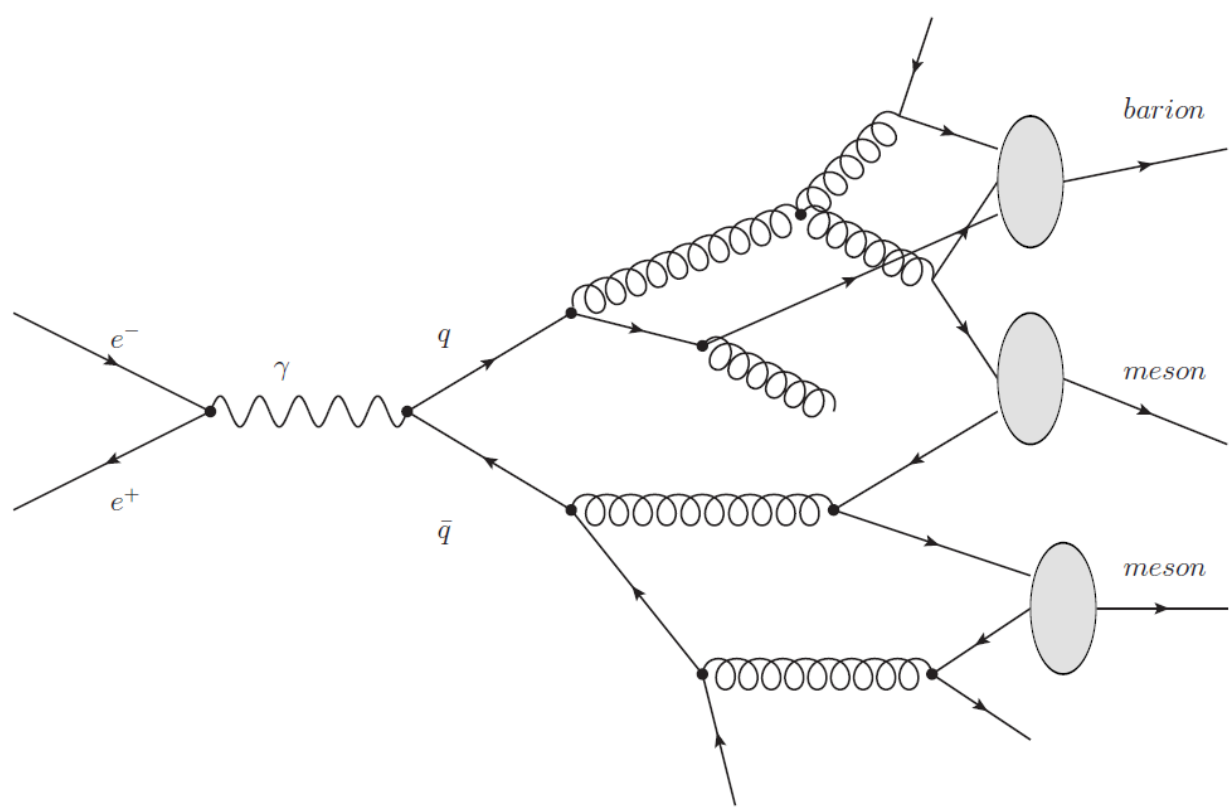


Figure 1.: parton branching and hadronization

Cross sections

To predict the hadron jet reactions we should know the probability of them. For this purpose physicist use the quantity of cross sections which is related to the probability of the reaction. By hadron jets the computation of the cross sections is based on the perturbation theory of quantum chromodynamics. These calculations could lead on very complicated formulas. Now we are working on the computation of the higher order radiation correction of the cross section which could be written in the term of different integrals which contain operators. These operators could be constructed as linear combination of so called „master integrals”. The computation of these master integrals takes a lot of work and time even with a numerical integration.

For example the integration could go on 5-10 minutes only in one point so in 1000 points (which is quite usual in Monte Carlo methods) it would take a half-one week. This makes necessary to develop an efficient approximation method which is fast enough and has a proper accuracy.

The approximation method

The basic idea is to fit the formula of the operator with a simple function. First we compute the integral in few points numerically then fit it with a function F . We developed the fitting method and the form of the fitting function F on the master integrals. We have tested the form of the F function on several type of integrals and we have found a simple formula which is easy to calculate and programme.

$$\mathcal{F}(x_1, x_2, \dots, x_s) = \sum_{p_i, l_i} C_{m; p_1, p_2, \dots, p_s; l_1, l_2, \dots, l_s} (x_1^{p_1} x_2^{p_2} \dots x_s^{p_s}) \times (\log^{l_1}(x_1) \log^{l_2}(x_2) \dots \log^{l_s}(x_s)).$$

$$l_1 + l_2 + \dots + l_s \leq n$$

$$p_1 + p_2 + \dots + p_s \leq m$$

Figure 2.: the form of the F fitting function, where „n” and „m” means the highest degree of the logarithm and the polynomial respectively

$$\mathcal{I}_{3C,1}(x_{\overline{t}t}, \epsilon; a_0, d_0; k_1, k_2, k_3, l) = 2^{-4\epsilon} \frac{\Gamma^2(1-\epsilon)}{\pi^2(1-2\epsilon)} \int_0^{a_0} d\alpha \int_0^1 dt_{ts} d\tau_{ts} dw_s \alpha^{-1-2\epsilon} \times (1-\alpha)^{2d_0-3+2\epsilon} (\alpha + (1-\alpha)x_{\overline{t}t})^{-1-2\epsilon} (2\alpha + (1-\alpha)x_{\overline{t}t})^{-l} t_{ts}^{k_1+k_2+1-2\epsilon} \times (1-t_{ts})^{k_1+k_2+1-2\epsilon} \tau_{ts}^{k_3-1-2\epsilon} (1-\tau_{ts})^{k_3-1-2\epsilon} w_s^{-1/2-\epsilon} (1-w_s)^{-1/2-\epsilon} \times (1-\tau_{ts} + \tau_{ts} t_{ts})^{-k_1-k_2-2+2\epsilon} (\alpha + (1-\alpha)x_{\overline{t}t} w_s)^l.$$

$$\mathcal{F}_{3C,1}(E) = 4.81 - 28.45x + 31.94x^2 - 3.77 \log(x) - 18.35x \log(x) - 17.31x^2 \log(x) + 1.33 \log(x)^2 - 0.53x \log(x)^2 + 3.21x^2 \log(x)^2. \quad (4.3)$$

Figure 3.: „before – after”

Results

With this fitting method and with the F function we could reach a 10^{-4} relative accuracy which is enough for us. In some cases we know the asymptothical expression of the integral analytically and it shows a 10^{-6} relative accuracy compared to the fitted function. The next two figures show examples for the fitting of an integral with one variable and an integral with two variables.

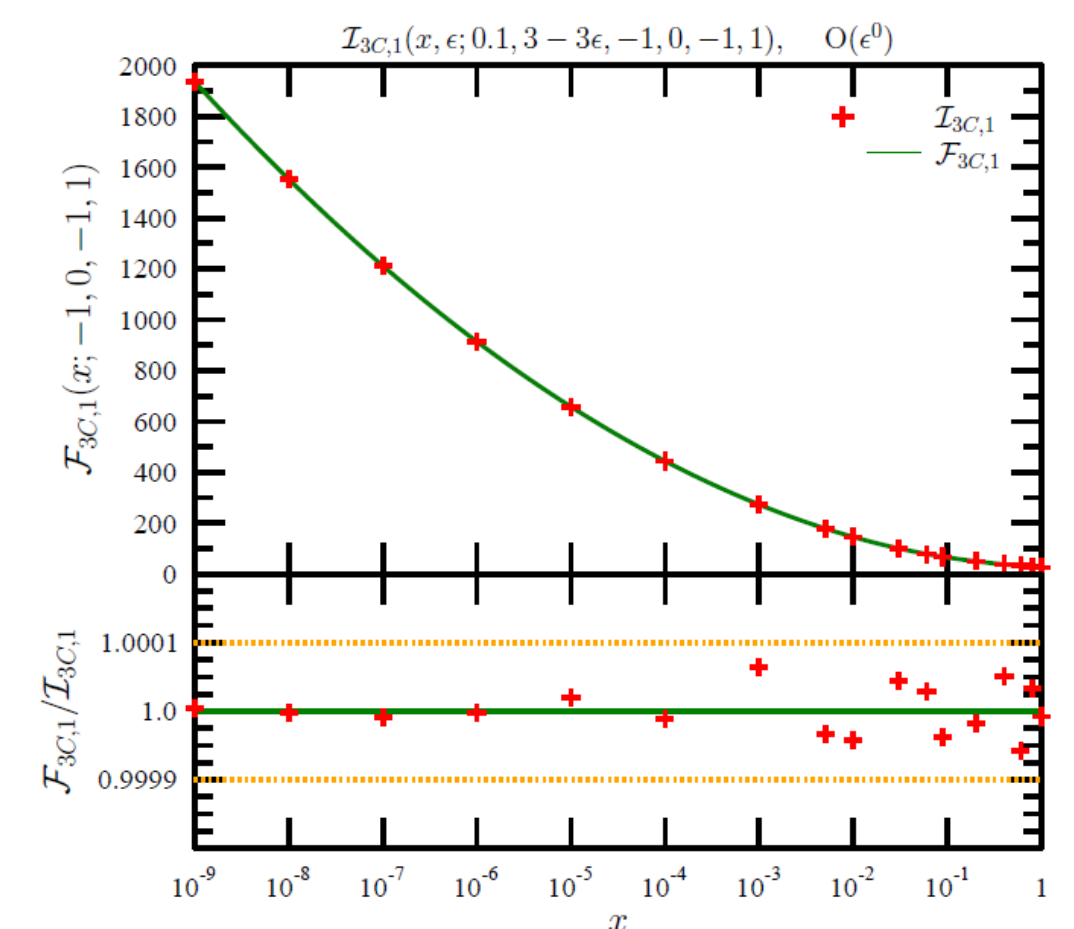


Figure 4.: example for a fitting of an integral with one variable

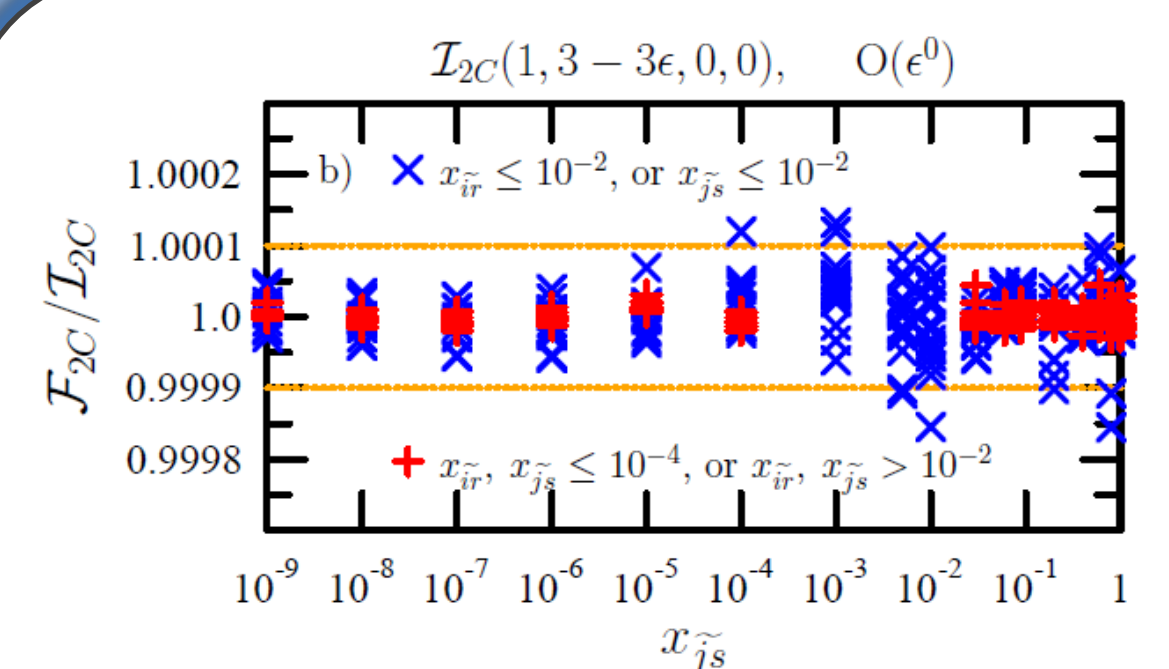


Figure 5.: example for a fitting of an integral with two variables

Present and future

At the moment we are working on the construction of the operators and test their validity. After that we will fit the operators. If it has the proper accuracy we will be able to compute the NNLO cross-sections with Monte Carlo integration.