

About the Profile Constancy by Curved Teeth Cylindrical Gear's Cutter Head

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Abstract: Cylindrical gears with Archimedean spiral toothline represent a result of recent research. Similarly to the Öerlikon type bevel gears this type of gears cannot be grinded. This fact leads to the necessity of achieving best possible surface roughness while cutting using a specific cutter head. The paper presents the geometric model of the grinding technology applied for the main relief faces of the cutters and analyses the problems regarding the precision of the edge profile, its dependence with the technological setting parameters and the profile variation after the re-sharpenings. It concludes that the classic method of ground grinding allows a restricted number of re-sharpenings.

Keywords: rack angle, relief angle, re-sharpening, grinding profile, functional profile.

1. Basics of cylindrical gears with curved teeth

Regarding nowadays technological development there is no doubt about that gear industry is tasked to realize drives with increased load capacity, diminished noise level respecting also supportable manufacturing costs. Gear industry's main product, despite of the continuous diversification process of gears remains the external or internal cylindrical gear drive that is used more than 95% in a large variety of applications. There were existing a lot of actions to improve the contact geometry and the load capacity of this gear drives. Two excellent contributions are here to be mentioned. The first is the Novikov-Wildhaber gear pair originally patented in the early 50's. Here the load capacity was increased by contacting a convex and a concave tooth surface. It was for long time subject of research bringing basic modifications and improvements [1, 2]. Unfortunately the manufacturing costs (manufacturing requires two different

rack based cutters as demonstrated in general theory of gear meshing [3]) were appreciated as too high for the proposed goal.

Another nice approach was described in [4] where the gear's meshing rack tooth's profile is parabolic shaped. All these applications lead to the increasing of the load capacity, but the modifications are produced along the height of the teeth, the toothline remaining still linear or helical.

Another possibility to increase the load capacity lies in replacing the meshing rack's toothline with a plain curve. A particular solution was found by using an Archimedean spiral line. The meshing's geometrical elements are presented in figure 1. If the milling head rotates with angular velocity ω_s and it contains a number of identical cutting edges displayed on an Archimedean spiral, there appear in the axial plain a continuously moving rack profile that meshes both the pinion and its gear. Here the generating surface is attached to the milling head. While the milling head rotates the generating rack will mesh both elements of the gear pair respecting Olivier's first principle [3].

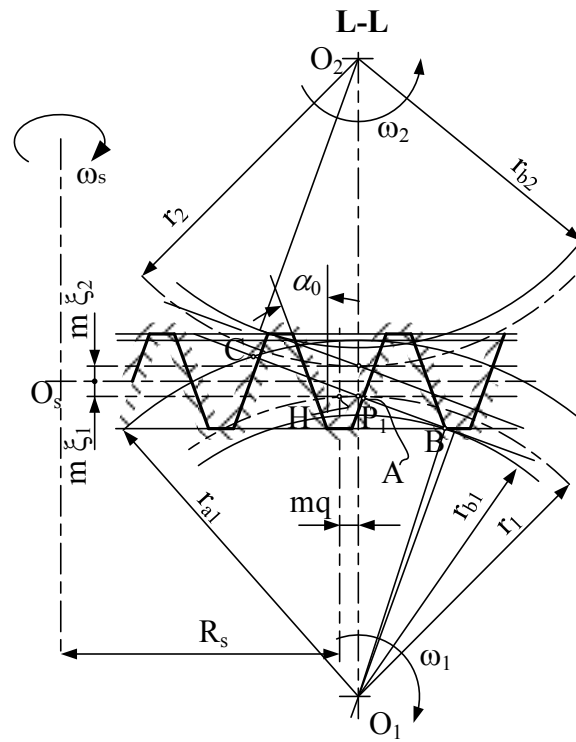


Figure 1: The meshing of cylindrical gears with Archimedean spiral toothline

If extending the plain model presented above a rack with curved toothline will continuously sliding along a radial direction related to the milling head's axis, maintaining its pitch plain. This rack's curved meshing surfaces allow the possibility of localizing the tooth contact somehow similarly to the bevel gears. There exist two variants of meshing regarding the tool motion: radial cutting where whole cutting depth is attained using a feed from up to down along tool's axis, and the tangential cutting where feed follows the pitchline. Detailed models were published in [5] respectively [6].

The success of the meshing and the fair localization of the contact depends mainly on the precision of the cutters mounted in the milling head. The cutting edges must fulfill very severe geometrical requirements regarding the form and the form fidelity after re-sharpenings.

2. The construction of the milling head

The milling head includes three groups of cutters, each of them consisting of a roughing cutter, two external finishing and two internal finishing cutters. The roughing cutter has two profile edges. As a consequence it materialize both profile lines of the rack tooth. The finishing cutters works each only on one side of the cut gap: external cutter's main edge cuts the external (right) side of the gap, meshing the concave surface the internal cutter works on the opposite side of the gap meshing the convex surface. Present paper deals only with the finishing cutters. The model of the milling head and the cutters are presented in figure 2. The cutter shaft is set and fixed in the slot of the head using a cylindrical shaft screw. The cutter's edge must be located as much as possible in an axial plane of the milling head, maintaining the required distance from the axis.

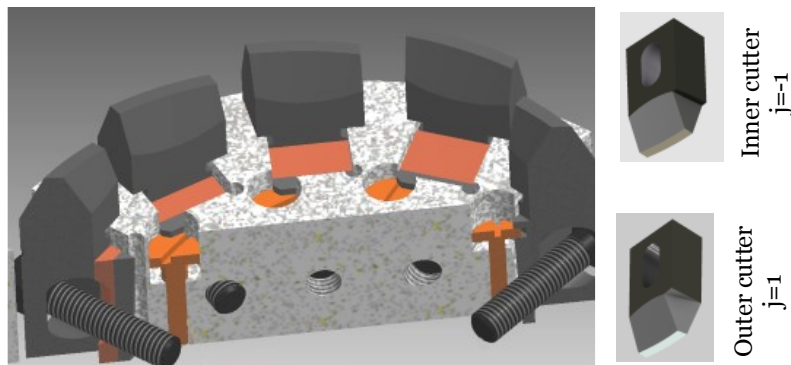


Figure 2: The milling head and the cutters

The cutting part of the cutter is correlated to the rake profile's frame (x_h, y_h, z_h) . The cutting wedge is delimited by a common plain rake face and two relief faces. The main cutting edge results as the intersection of the rake face and the side relief face.

The position of the rake face results by rotating the cutter's reference plain (x_h, z_h) about the virtual profile's line by the angle γ_s as it results from figure 3.

The side relief face is obtained simultaneously for more cutters fixed in a grinding device. The realized relief face is a conical surface whose axis coincides with the axis of revolution of the device mentioned above. The grinding scheme is represented in figure 4.

3. The model of the relieving process

The cutter's $O_h z_h$ axis is positioned slanted to the rotation axis $\Delta_1 \equiv Oy_0$ of the grinding device. This is necessary to ensure the tip relief angle. Generatrix AD and BC will draw the external and internal conical relief surfaces. Here the cylindrical grinding wheels rotate about their axes of revolution Δ_{kk} and Δ_{kb} while cutter's positioning device rotates about its own axis Δ_1 . When the cutter is re-positioned in the milling head, its axis became parallel to the milling head's axis. Thus the conical surface stands slanted and a positive side relief angle appear on the main cutting edge.

Rake face is grinded using the same device. It is important to state that re-sharpening will be done only on the rake face. The cutters will be fixed in a same sharpening device that will be rotated over two perpendicular axis till the rake face become parallel to the grinding plane. The grinding depth will be adjusted only through rotating the device about its own axis.

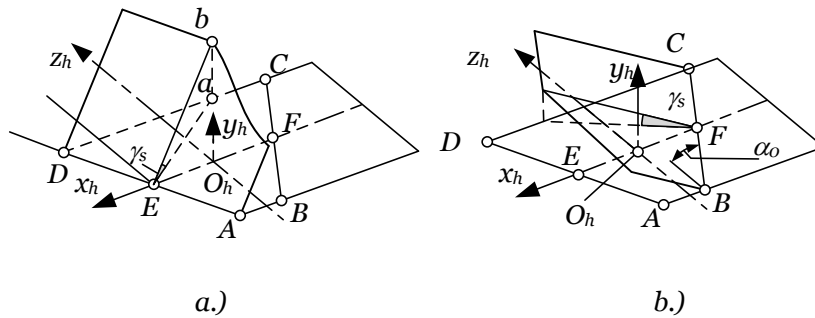


Figure 3: The rake faces for the outer (a) and the inner (b) cutter

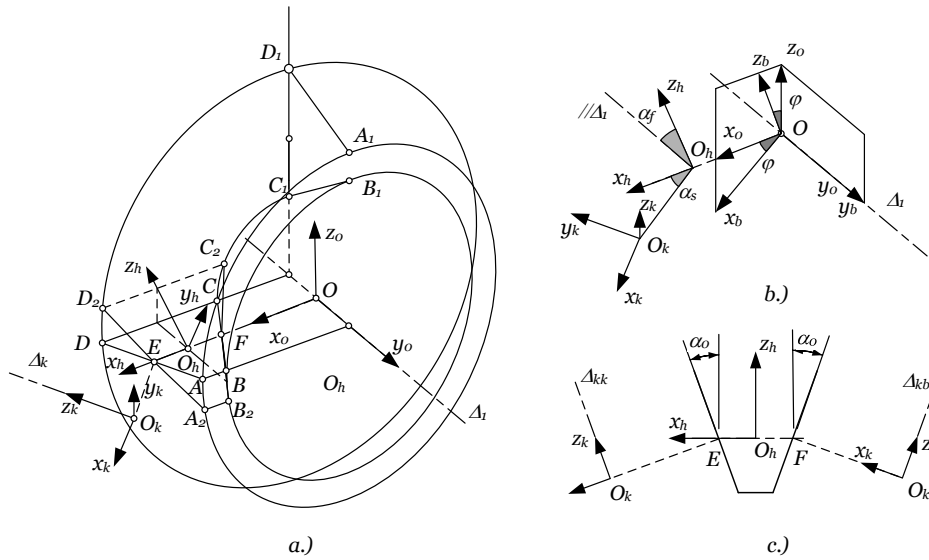


Figure 4: The grinding scheme (a), the applied frames (b) and the grinding wheel's setting(c)

This procedure will ensure the form constancy by all cutting edges that will appear after the re-sharpening process.

A peculiarity of the maintenance of the cutters consists in simultaneous re-sharpening. Applying simultaneous re-grinding this all cutters of the same type reach the same re-sharpening rate.

The analysis of the cutting edge and the generating profile requires the equations of the geometric entities involved in the present study. In order to obtain this, following frames are used (figures 4a and 4b):

- a fixed frame $(x_0y_0z_0)$ denoted (F_0) ;
- a grinding device attached frame $(x_by_bz_b)$ denoted (F_b) ;
- a cutter shank attached frame $(x_hy_hz_h)$ denoted (F_h) ;
- a grinding wheel attached frame $(x_ky_kz_k)$ denoted (F_k) .

The grinding wheel's (F_k) frame become useful when relative motion of the workpiece to the tool becomes more complicated e.g. axial or slanted relieving processes. These are not discussed in the present paper.

The relative positions and motions encountered by the frames above are related to the (F_0) fixed frame. Initially (F_b) overlaps (F_0) . When grinding, (F_b) rotates about axis $\Delta_1 \equiv Oy_0$. Cutter's frame (F_h) is attached to (F_b) . Its position is given by the distance R_k and the angle α_f meaning a rotation about the common axes $x_b \equiv x_h$. Here is to emphasize the importance of the basic rack

profile $ABCD$: its center O_h middles the tooth thickness EF on the pitch line. Setting of the cutter requires the distance to the rotation axis of the device that will be given by the parameter R_k and the half of the tooth thickness denoted in the followings with a . Frame (F_k) remains in the analyzed case fixed while grinding process is going on. The equations of edges and emphasized surfaces were deduced in a general form that is applicable for both inner and outer cutter.

The generating lines AD and BC are slanted to the symmetry plane of the theoretical profile with angle $\alpha_s \neq \alpha_0$. The parametric generalized equations are

$$\begin{cases} x_0(u, j) = R_k + j(a - u \operatorname{tg} \alpha_{sj}) \\ y_0(u, j) = u \\ z_0(u, j) = 0 \end{cases} \quad j \in \{-1; 1\} \quad (1)$$

The j parameter lower value sets the equations (1) for the inner, and the higher value for the outer edge.

Transformation matrices regarding the involved frames are

$$\mathbf{M}_{hb} = \begin{pmatrix} 1 & 0 & 0 & -R_k \\ 0 & \sin \alpha_f & \cos \alpha_f & 0 \\ 0 & -\cos \alpha_f & \sin \alpha_f & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \mathbf{M}_{b0} = \begin{pmatrix} \cos \varphi & 0 & -\sin \varphi & 0 \\ 0 & 1 & 0 & 0 \\ \sin \varphi & 0 & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

Using equations (1) and (2), the generalized relief face equations attached matrix format is

$$(A_{\alpha}^{(k,b)}): \underline{\mathbf{r}}_h = \mathbf{M}_{hb} \mathbf{M}_{b0} \underline{\mathbf{r}}_0 \quad (3)$$

For the analysis the equations of the rake faces are required too, that will be written in generalized form too. The direction of the normal vectors can be easily deduced from figure 3. Considering points E and respectively F the discussed equation can be written as

$$(A_{\gamma}): (x_h - ja) \operatorname{tg} \gamma_s + j y_h - j z_h \operatorname{tg} \gamma_s \sin \alpha_0 = 0 \quad (4)$$

The cutting edges result as intersection between the rake face and the relief face. It can be written through the matrix format (3) by replacing the second independent variable of the surfaces with the required parameter linking functions $\varphi^{(k,b)} = \varphi^{(k,b)}(u)$. While the goal of the paper is to study the edge constancy and the linearity of the generating profile after the re-sharpenings the mathematical model must be written for a general case, when the rake face is

arbitrary rotated about the grinding device axis with angle θ . In order to find the general rake face position let's consider the auxiliary frames F_g that overlaps the cutter's frame F_h and a carrying frame F_c overlapping F_b for the zero value of the rotation parameter θ . This frames are joined. They are not represented in figure 4 because they are identical to S_h and S_b if the cutting edge is not re-sharpened. It can be concluded that the equation of the re-sharpened rake face in system F_g formally fits equation (4) rewritten in variables x_g, y_g, z_g . To find the expression in frame F_h where the relief face is written, a transformation between the rotated F_g and the immobile F_h is necessary. The matrix form of the required transformation is:

$$\underline{\mathbf{r}}_g = \mathbf{M}_{gc} \mathbf{M}_{cb}(\theta) \mathbf{M}_{bh} \underline{\mathbf{r}}_h \quad (5)$$

It is to be observed that matrix $\mathbf{M}_{gc}$ coincides with $\mathbf{M}_{hb}$. Rewriting equation (4) in variables x_g, y_g, z_g and applying the back transformation (5) a condensed form of the generalized rake face equation results as follows:

$$\begin{pmatrix} j \sin \gamma_s \cos \alpha_0 \\ \cos \gamma_s \\ -\sin \gamma_s \sin \alpha_0 \\ -a \sin \gamma_s \cos \alpha_0 \end{pmatrix} \cdot \mathbf{M}_{hb} \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{M}_{bh} \begin{pmatrix} x_h \\ y_h \\ z_h \\ 1 \end{pmatrix} = 0 \quad (6)$$

Substituting the parametric coordinates of the relief surface given by (3) in the generalized equation (6) of the rake face leads to very complicated formulae. The first solution for $\theta = 0$ has the following expressions:

$$\begin{aligned} \varphi^{(k,b)}(u) &= j \arcsin \frac{C(u)}{\sqrt{A^2(u) + B^2(u)}} - \arctg \frac{B(u)}{A(u)} \\ E(u) &= R_k + j(a - u \operatorname{tg} \alpha_s) \\ A(u) &= E(u) \cos(\alpha_f + \gamma_l) \\ B(u) &= E(u) \sin \gamma_l \cot \alpha_0 \\ C(u) &= B(u) + j u (\operatorname{tg} \alpha_s \cot \alpha_0 \sin \gamma_l - \sin(\alpha_f + \gamma_l)) \\ \operatorname{tg} \gamma_l &= \operatorname{tg} \gamma_s \sin \alpha_0 \end{aligned} \quad (7)$$

For different values of θ corresponding to different re-sharpening state the dependence between the parameters was numerically evaluated.

The Archimedean generating surface meshed by the cutting edges has the same axial profile with the revolved surface swept by an arbitrary cutting edge while it rotates about the milling head's axis. The milling head's frame F_m and the assembled cutter's frame are related by a simple translation along their common axes $x_m \equiv x_h$ with the distance L that depends on the standard basic radius of the Archimedean rack R_s and the index of the cutter in the group [7]. According to this and denoting the general parameter dependence with $\varphi(u, \theta)$ the equations of the cutting edge in the frame of the milling head becomes:

$$\begin{cases} x_m(u, j) = [R_k + j(a - u \tan \alpha_s)] \cos \varphi(u, \theta, j) - R_k + L \\ y_m(u, j) = [R_k + j(a - u \tan \alpha_s)] \cos \alpha_f \sin \varphi(u, \theta, j) + u \sin \alpha_f, \quad j \in \{-1; 1\} \\ z_m(u, j) = [R_k + j(a - u \tan \alpha_s)] \sin \alpha_f \sin \varphi(u, \theta, j) - u \cos \alpha_f \end{cases} \quad (8)$$

Using this, profile of the requested revolved surface can be computed using following equations:

$$\begin{cases} x_m^{(pr)}(u, j) = \sqrt{(x_m(u, j))^2 + (y_m(u, j))^2} \\ y_m^{(pr)}(u, j) = 0 \\ x_m^{(pr)}(u, j) = z_m(u, j) \end{cases}, \quad j \in \{-1; 1\} \quad (9)$$

4. Numerical investigation

The numerical investigation was performed for a cutter head of normal module $m_n = 5\text{mm}$, basic radius $R_s = 80\text{mm}$ and rack profile angle $\alpha_0 = 20^\circ$. Tip relief angle is admitted in the interval $\alpha_s \in [5^\circ, 10^\circ]$ while γ_s was set at 10° . Four different re-sharpening states were considered adopting the values $\theta \in \{-1^\circ, 0, 1^\circ, 2^\circ\}$ for the rotation of the rake face about the axis of the grinding device. This allows a re-sharpening reserve among 4.5 mm. The negative value is set to study the possibility of diminishing the errors similarly with the calculus of the gear hob [8]. The cutting edge and the profile coordinates were computed in $N_p = 30$ points. Over the profile was fit a spline function of form $x_m = S(z_m)$. The characteristic point of the profile is given by its intersection with the pitch plane (x_m, y_m) . Its coordinates are $(S(0), 0, 0)$.

The profile error is defined as the distance between the spline computed profile point and the theoretical profile line passing through the characteristic point, measured on the x axis direction:

$$\delta(z_m) = S(z_m) - (S(0) + jz_m \operatorname{tg} \alpha_0), j \in \{-1; 1\} \quad (10)$$

The best estimation for the profile angle of the real generating rack can be reached by computing the best profile fitting line that passes through the reference point of the profile. Here the last squares approximation was used.

4.1. The precision of the generating profile

The precision was previously defined as the deviation of the real profile from the theoretical rack profile. Numerical investigation shows that grinding profile angle α_s influences significantly the deviation. Figure 5 shows the deviations in dependence with α_s whose basic value was set on α_0 , and starting from this it was increased step-by step with an increment of $6'$. The negative value of the deviations means that real profile point is nearer to the milling head's axis that the theoretical profile's point situated on the same height. The study shows that deviations present a parabolic evolution. As a consequence always exists an optimal value for α_s that leads to minimum deviation values. Example shown in figure 5 demonstrates an approximately $2\mu\text{m}$ maximum deviation that is considered full acceptable value. It is also to remark that deviation curves are symmetric for the inner and the outer cutter.

Once optimal α_s value is set, new cutter's edge will generate a revolved surface with minimal profile deviation. Investigation shows that re-sharpenings lead to increase of the profile errors. Figure 6 shows the deviation curves obtained for the four set values of the re-sharpening's state characteristic θ angles. It is to conclude that errors present the same distribution for the inner and the outer edge.

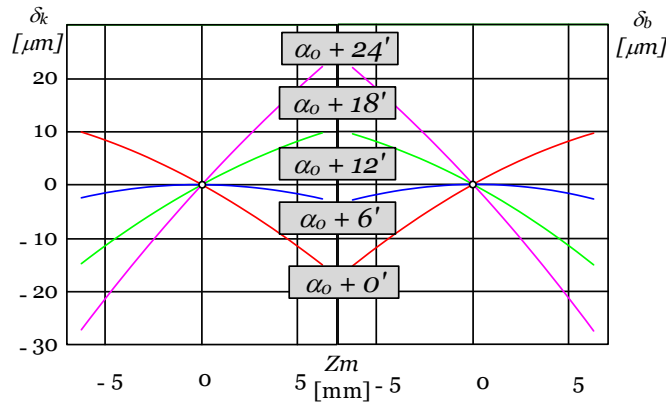


Figure 5: The dependence of the profile deviation with angle α_s .

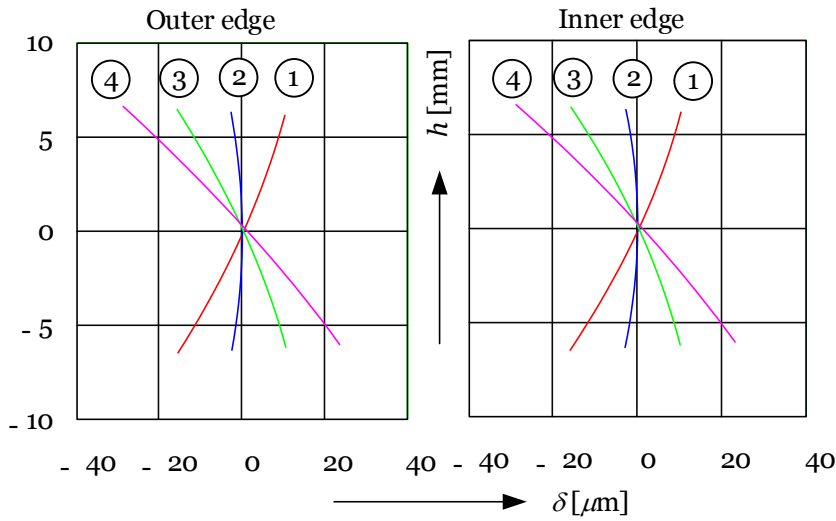
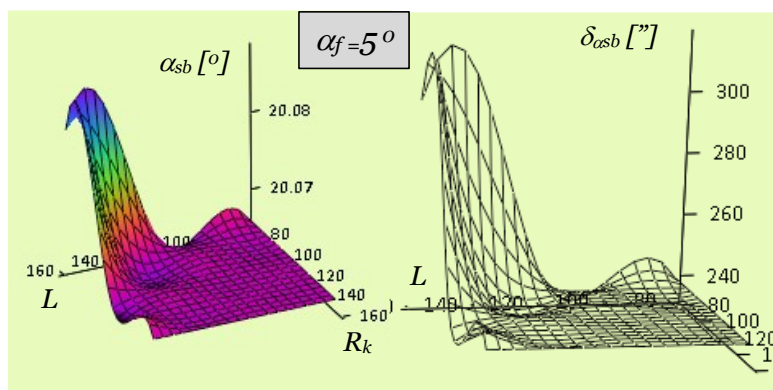


Figure 6: The evolution of deviations with the state of re-sharpening

It is also to remark that deviation curves rotate about their reference point while θ increases. For the first value of θ deviation is negative on the tip of the edge and positive on its root. When θ reaches its set extreme value of 3° the absolute value and the sign of the deviation changes but the curvature of the deviation lines remain approximately constant. That allows us to conclude that deviation from the straight line is negligible, but modification of the profile angle is consistent.

The value of the grinding profile angle α_s depends on tip relief angle α_f , the cutter's setting distance L and the basic radius R_s . The numerical investigation's results are represented in graphical form shown in figure 7



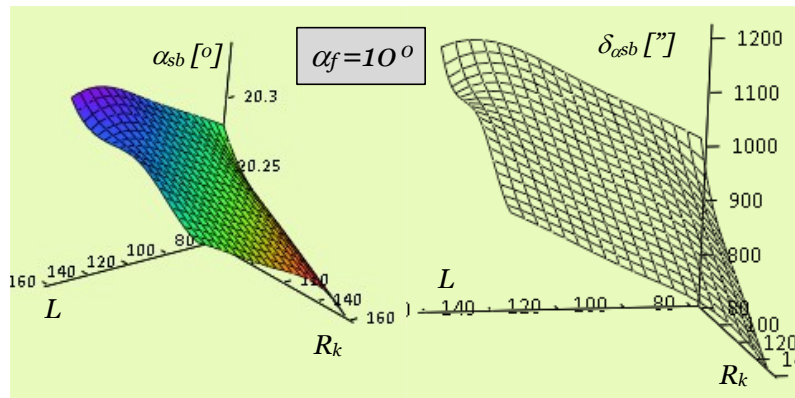


Figure 7: The dependence of grinding profile angle on parameters L and R_s for different values of tip relief angle α_r

The variation of the grinding relief angle is significantly different for the extreme considered values of the tip relief angle. It is also to remark that the major influence of the discussed angle value is exerted by the cutters setting distance. It is also proven that influences are similarly both for external and internal edge.

4.2. The calculus of the deviation from the prescribed rack profile angle value

Figure 6 shows that profile deviation increases with the number of re-sharpenings. Applying the least squares regression for a line that passes through the characteristic profile point it can be concluded that profile angle present a considerable variation with the increase of re-sharpenings. The results of the numerical simulation is presented in table 1.

Table 1. The variation of the realized rack profile angle with the re-sharpenings

$\theta [^\circ]$	-1°	0°	1°	2°
$\alpha_b [^\circ]$	$19^\circ 54' 58''$	$19^\circ 59' 24''$	$20^\circ 6' 58''$	$20^\circ 11' 2''$
$\alpha_k [^\circ]$	$20^\circ 6' 58''$	$20^\circ 58''$	$19^\circ 54' 58''$	$19^\circ 46' 1''$

5. Conclusions

Numerical investigation shows that the realized rack profile angle and the deviation from the theoretical profile line increases significantly if round grinding technology is applied. That limits the number of possible re-sharpenings thus the active part of the cutter is recommended to be dimensioned as small as possible. For increasing the number of possible re-sharpenings a relieving technology must be adopted.

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References

- [1] Litvin, F.L., Pin-Hao, F. Lagutin, S.A. "Computerized Generation and Simulation of Meshing and Contact of New Type of Novikov-Wildhaber Helical Gears,"*R—2000-209415*" [Online]. Available: http://gearexpert.free.fr/fichiers_pdf/engrenage_Novikov_Wildhaber_NASA_report.pdf [Accessed: 30-Jun-2012]
- [2] Nacy, S.M., Abdullah, M.Q., Mohammed, M.N. "Generation of Crowned Parabolic Novikov gears," *Adept Scientific Knowledge Base*. Available: <http://www.adeptsience.co.uk/kb/articleprint.php?noteid=6E9E>. [Accessed: 19-Mar-2012].
- [3] Litvin, F.L. Theory of gearing. National Aeronautics and Space Administration, Scientific and Technical Information Division, 1989.
- [4] Litvin, F.L., Liam, Q., Kapelevich, A.L. "Asymmetric modified spur gear drives: reduction of noise, localization of contact, simulation of meshing and stress analysis". EMI improvements using the switching frequency modulation in a resonant inverter for domestic induction heating appliances", in *PComputer Methods in Applied Mechanics and Engineering* 188 (2000), pp. 363-390.
Available: http://www.akgears.com/pdf/paper2_litvin.pdf, [Accessed: 31-Dec-2014]
- [5] Máté, M., Hollanda, D. "The Cutting of Cylindrical Gears Having Archimedean Spiral Shaped Tooth Line", in *13th International Conference on Tools, 27-28 March 2012, Miskolc, ISBN 978-963-9988-35-4*, pp. 357-362.I.
Available: http://www.akgears.com/pdf/paper2_litvin.pdf, [Accessed: 31-Dec-2014]
- [6] Máté, M., Hollanda, D., Tolvaly-Rosca, F., Popa-Müller, I."The localization of the contact patch by cylindrical gear having an Archimedean toothline using the method of setting the tangential displacement", in *XXI-th International Conference of Mechanical Engineers, Arad, 25-28 apr. 2013, Conference Proceedings, ISSN 2068-1267*, pp.265-268.
- [7] Máté, M.The micro-geometric Model of the Toothflanks of a cylindrical Gear With Archimedean Spiral Shaped Toothline. 6th International Scientific and Expert Conference TEAM 2014-Technique, Education, Agriculture &Management, Kecskemét, November10-11, 2014. *PROCEEDINGS OF TEAM 2014, ISBN 978-615-5192-22-7, Volume VI, Number 1, Year 2014*, pp. 348-356
- [8] Dudas, I., Banyai, K., Varga, G. "Simulation of meshing of worm gearing", *American Society of Mechanical Engineers, Design Engineering Division (Publication) DE* 88, pp. 141-146,1996.