

HIGH-FREQUENCY KINETIC INSTABILITIES DRIVEN BY ANISOTROPIC ELECTRON BEAMS

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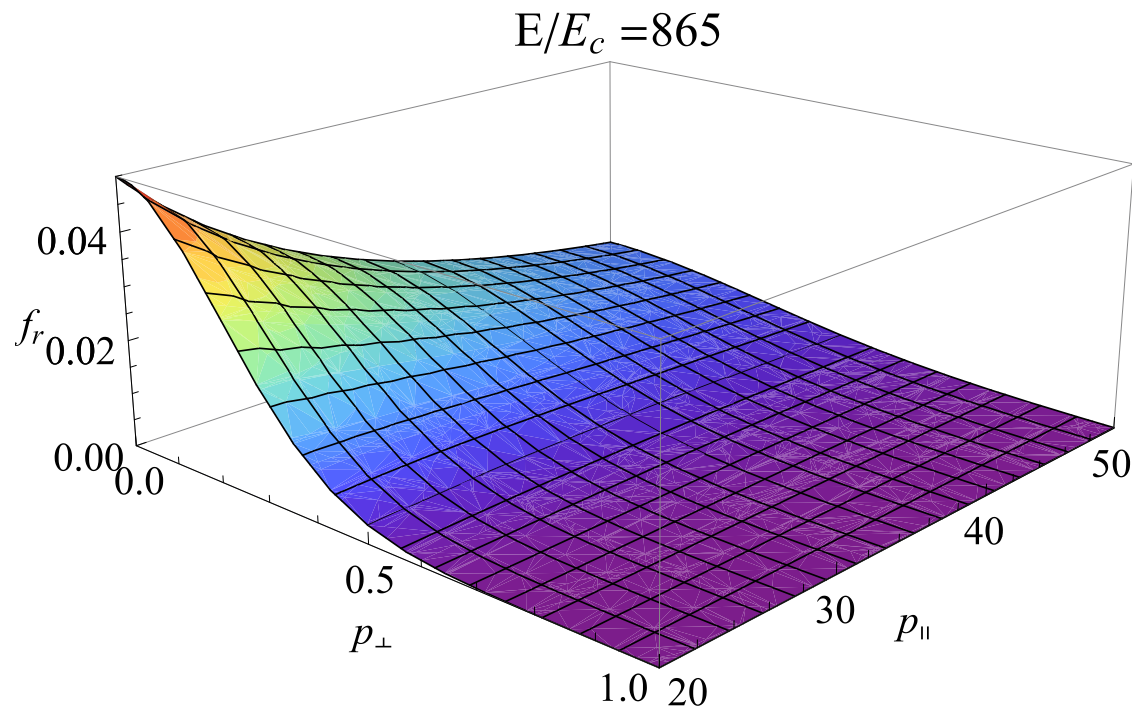
Outline

2/29

- Description of the instability
- Runaway electron distributions
- Wave dispersions
- Growth rate of the waves
- Critical runaway densities
- Plans, current problems

Runaway distribution

3/29



T. Fülöp, PoP **13**(062506), 2006

$p = \gamma v_e / c$
normalized momentum

Anisotropic $\rightarrow$ Instabilities

Particle-wave interaction

4/29

Dispersion of the plasma waves:

(homogeneous plasma)

$$\underline{\underline{\varepsilon}} = \underline{\underline{1}} + \underline{\underline{\chi}}^e + \underline{\underline{\chi}}^i$$

Dielectric tensor

$$\left(\varepsilon_{11} - \frac{k_{\parallel}^2 c^2}{\omega^2} \right) \cdot \left(\varepsilon_{22} - \frac{k^2 c^2}{\omega^2} \right) + \varepsilon_{12}^2 \left\{ \begin{array}{l} = 0 \\ = \chi_{11}^r \left(\frac{k^2 c^2}{\omega^2} - \varepsilon_{22} \right) + \chi_{22}^r \left(\frac{k_{\parallel}^2 c^2}{\omega^2} - \varepsilon_{11} \right) - \\ 2 \chi_{12}^r \cdot \varepsilon_{12} \end{array} \right.$$



$\delta\omega$ response



Growth rate

Perturbative
analysis

$$\text{Im } \delta\omega \ll \omega_0$$

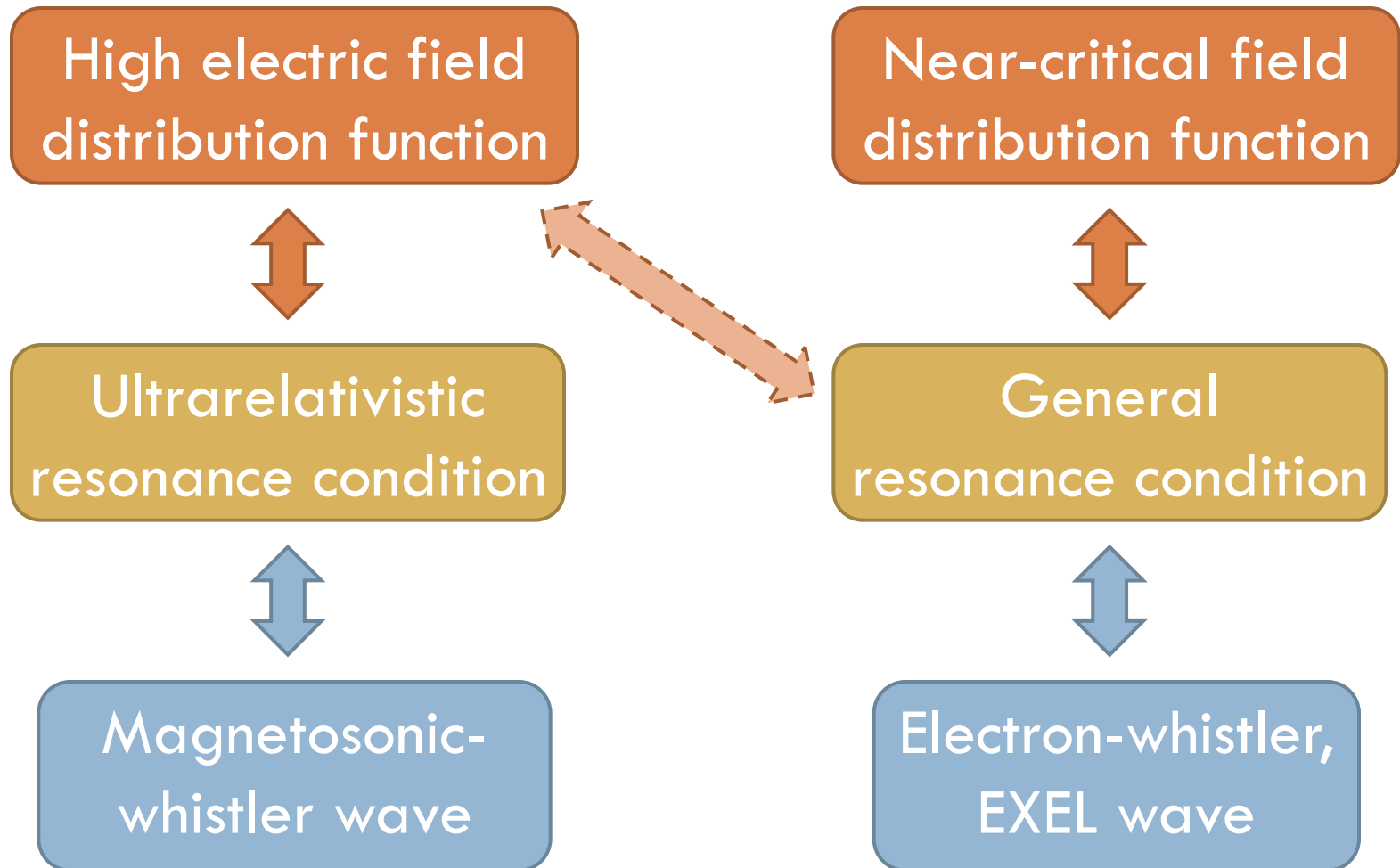
$$\text{Re } \delta\omega \ll \omega_0$$

Imaginary
part

$$\gamma_i = \text{Im}(\delta\omega)$$

Generalizations

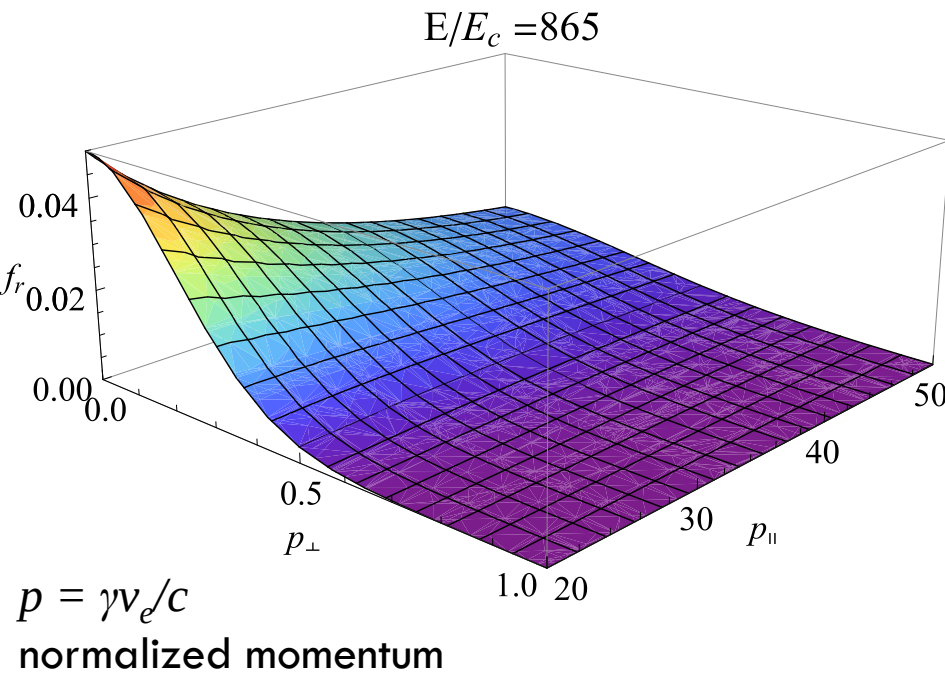
5/29



Distribution function

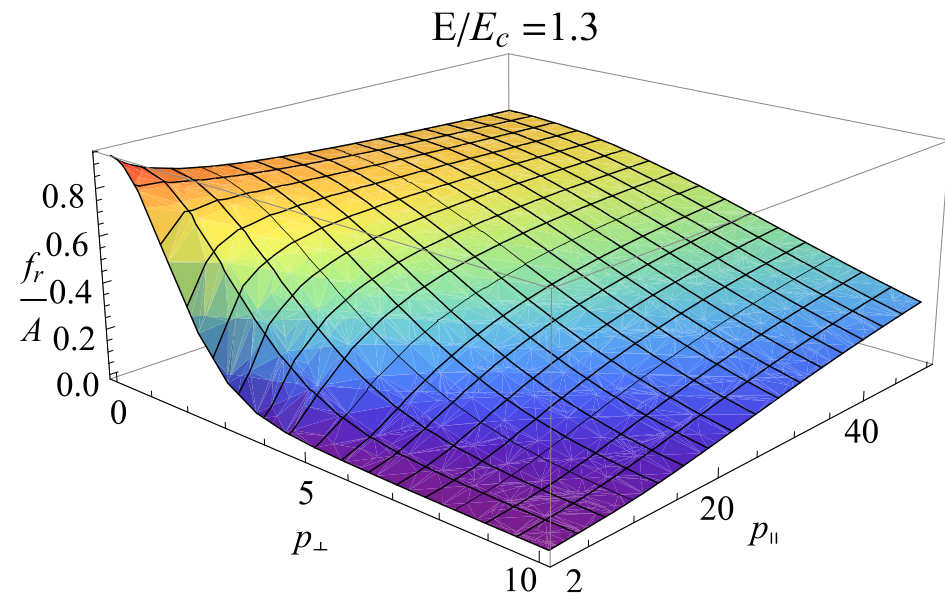
6/29

Avalanche distribution



T. Fülöp, PoP **13**(062506), 2006

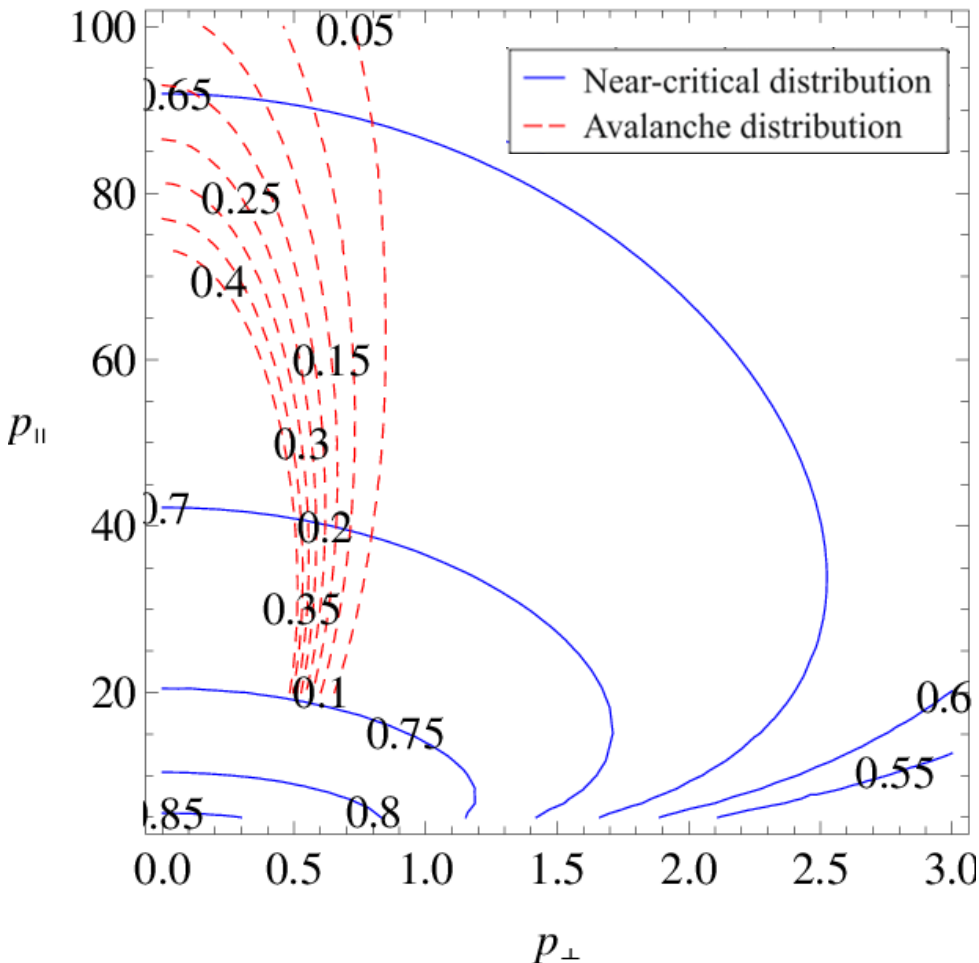
Near-critical distribution



P. Sandquist, PoP **13**(072108), 2006

Distribution function

7/29



- Qualitatively similar
- Lower electric field → less anisotropy
- Growth rate of the waves:
 - ▣ Ensured by the anisotropy
 - ▣ Not affected by the details

Electron plasma waves

8/29

- Relaxing the electromagnetic approximation:

$$\det \begin{pmatrix} \epsilon_{11} - \frac{k_{\parallel}^2 c^2}{\omega^2} & \epsilon_{12} & \frac{k_{\parallel} k_{\perp} c^2}{\omega^2} \\ -\epsilon_{12} & \epsilon_{22} - \frac{k^2 c^2}{\omega^2} & 0 \\ \frac{k_{\parallel} k_{\perp} c^2}{\omega^2} & 0 & \epsilon_{33} - \frac{k_{\perp}^2 c^2}{\omega^2} \end{pmatrix} = 0$$

T.H. Stix, Waves in Plasmas
(AIP, 1992)

- Approximations for the dielectric tensor:

$$\omega \gg \omega_{ce} \sqrt{m_e / m_i} \gg \omega_{ci}$$

Electron plasma waves

9/29

□ Dielectric tensor:

$$\epsilon_{11} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \quad \epsilon_{22} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \quad \epsilon_{33} = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

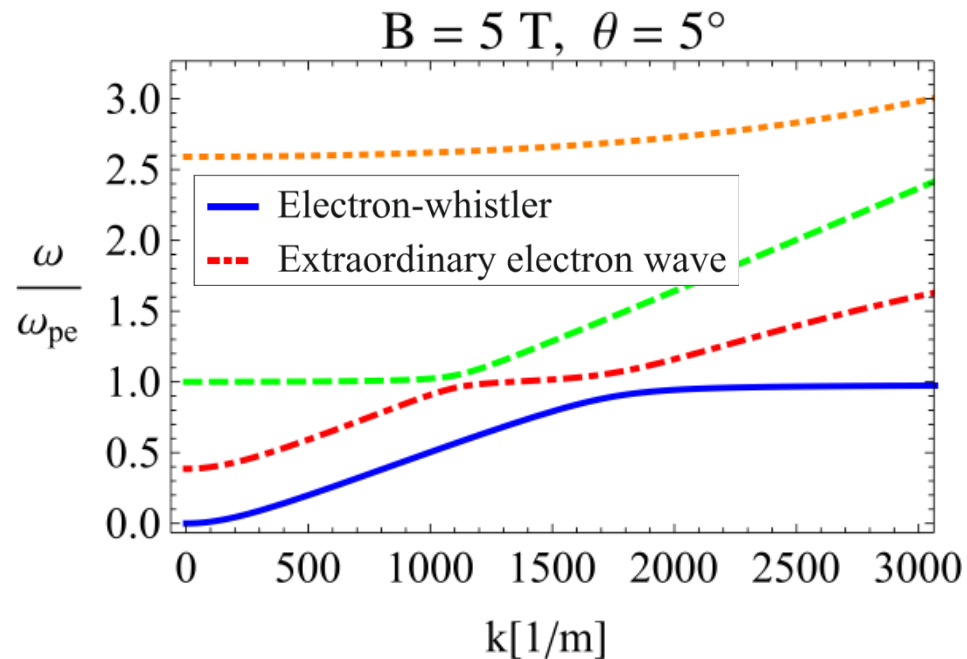
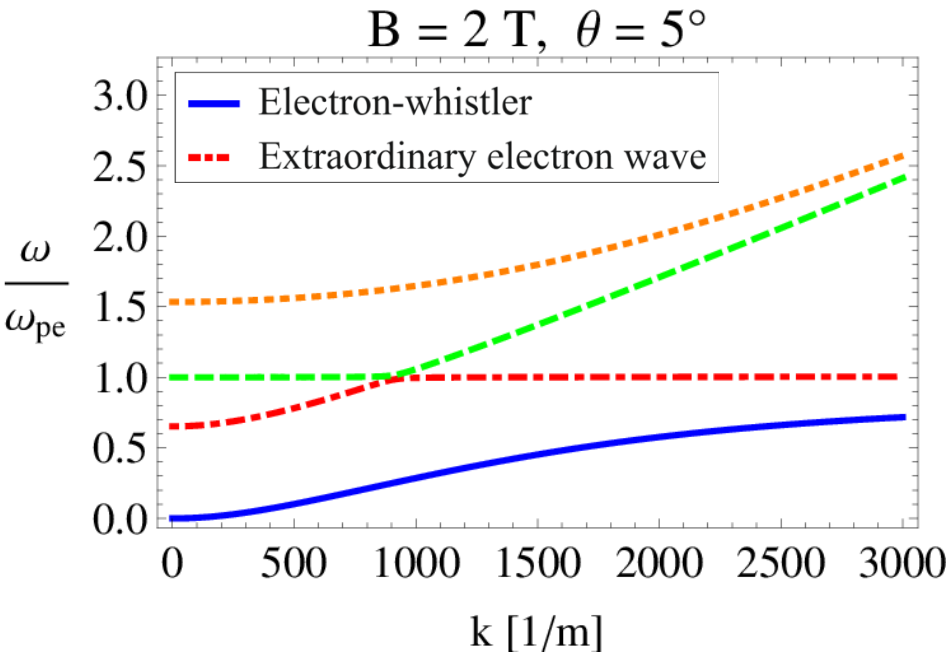
$$\epsilon_{12} = -i \frac{\omega_{pe}^2 \omega_{ce}}{\omega(\omega^2 - \omega_{ce}^2)}$$

□ Wave dispersion:

$$\left[\left(\epsilon_{11} - \frac{k_{\parallel}^2 c^2}{\omega^2} \right) \left(\epsilon_{22} - \frac{k^2 c^2}{\omega^2} \right) + \epsilon_{12}^2 \right] \left(\epsilon_{33} - \frac{k_{\perp}^2 c^2}{\omega^2} \right) - \frac{k_{\parallel}^2 k_{\perp}^2 c^4}{\omega^4} \left(\epsilon_{22} - \frac{k^2 c^2}{\omega^2} \right) = 0$$

Electron plasma waves

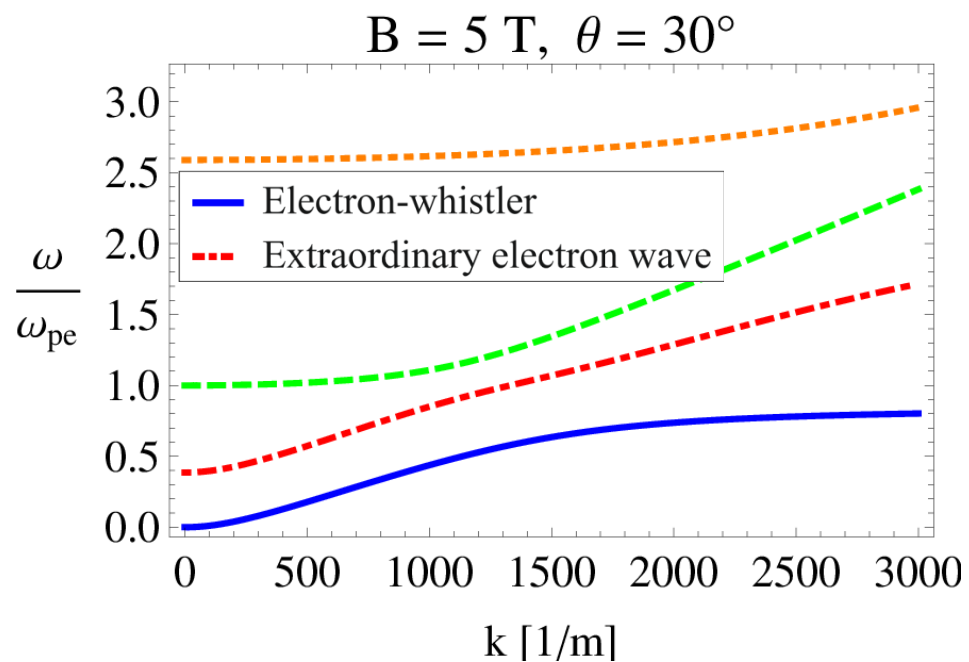
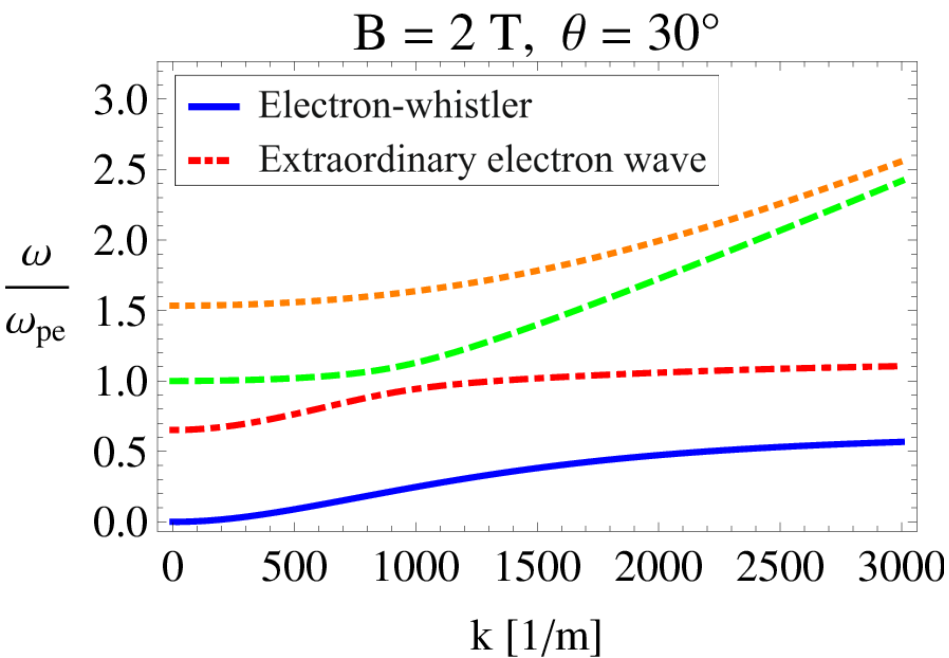
10/29



Qualitatively different for $B > 2.6 \text{ T}$

Electron plasma waves

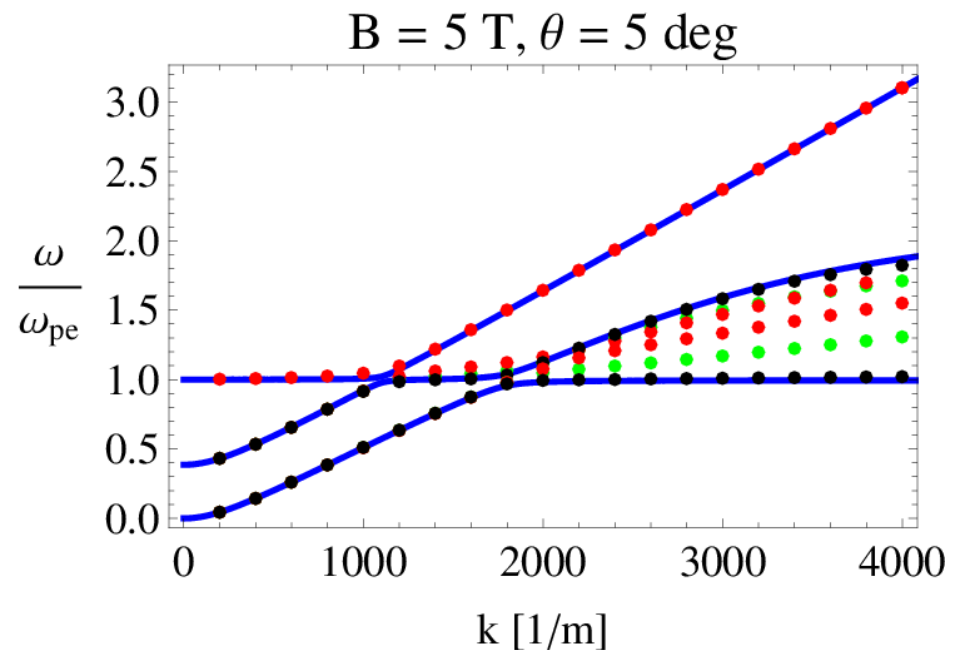
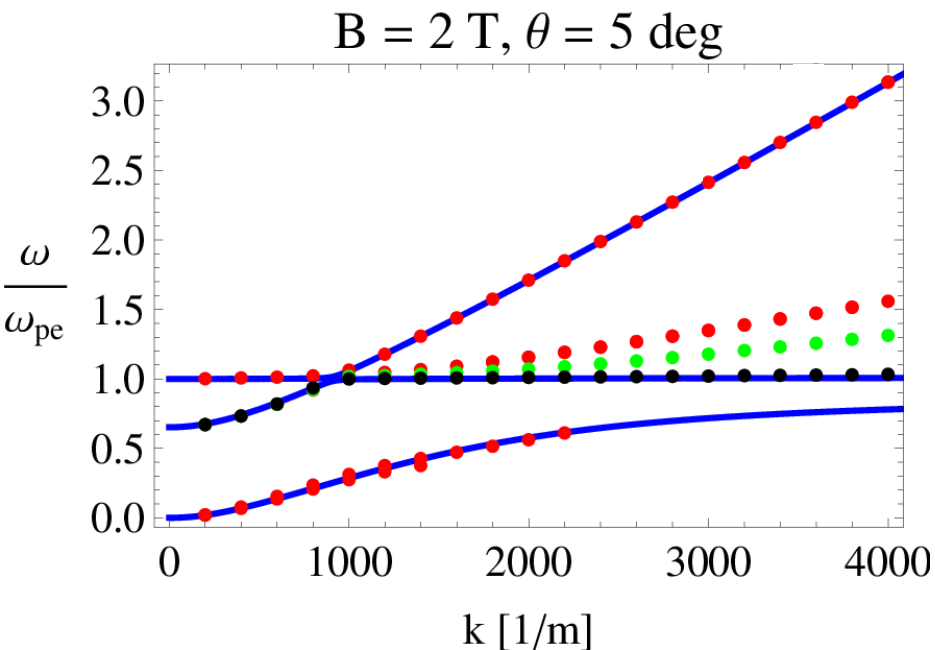
11/29



Electron plasma waves

12/29

- Cold plasma approximation is used
(th. motion $\ll$ gyro-motion)
- Validity: 20 keV, 10 keV, 1 keV



Calculating the growth rate of the waves

13/29

Unperturbed dispersion: $F(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}) = 0$

Perturbed dispersion: $F(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}) = G(\underline{\varepsilon}^{th}, \underline{\chi}^{runaway})$



The wave frequency changes:

$$\omega_0 \rightarrow \omega_0 + \delta\omega$$



$$\gamma_i = \text{Im}(\delta\omega)$$

Calculating the runaway susceptibilities:

$$\text{Im } \chi^r \sim \int d^3p \sum_n \frac{L(n)}{\gamma} \cdot \left(\frac{\partial f_r}{\partial p_\perp} n \omega_{ce} + \frac{\partial f_r}{\partial p_\parallel} k_\parallel c p_\perp \right) \cdot \delta(p_\parallel - p_{res})$$

$$n = -1, 0, 1$$

General resonance condition

14/29

□ Resonance condition: **implicit**

T.H. Stix, Waves in Plasmas
(AIP, 1992)

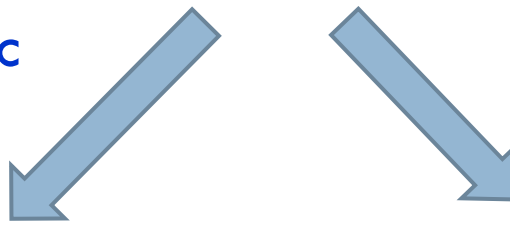
$$p_{par} + \frac{n\omega_{ce} - \omega_0 \cdot \gamma(p_{par}, p_{perp})}{k_{\parallel} c} = 0$$

Ultrarelativistic

$$\gamma \approx |p_{par}|$$

General case

$$\gamma = \sqrt{1 + p_{perp}^2 + p_{par}^2}$$



$$p_{res} = \frac{-n\omega_{ce}}{k_{\parallel} c - \omega_0}$$

$$p_{res} = \frac{-k_{\parallel} c n\omega_{ce} + \omega_0 \sqrt{(k_{\parallel}^2 c^2 - \omega_0^2)(1 + p_{\perp}^2)} + n^2 \omega_{ce}^2}{k_{\parallel}^2 c^2 - \omega_0^2}$$

Restrictions on the wave dispersion

15/29

Resonant momentum is physical if: $p_{res} \in \mathfrak{R}$ (1)

$$p_{res} > 0 \quad (2)$$

$$(1) \Rightarrow (k_{\parallel}^2 c^2 - \omega_0^2)(1 + p_{\perp}^2) + n^2 \omega_{ce}^2 \geq 0$$

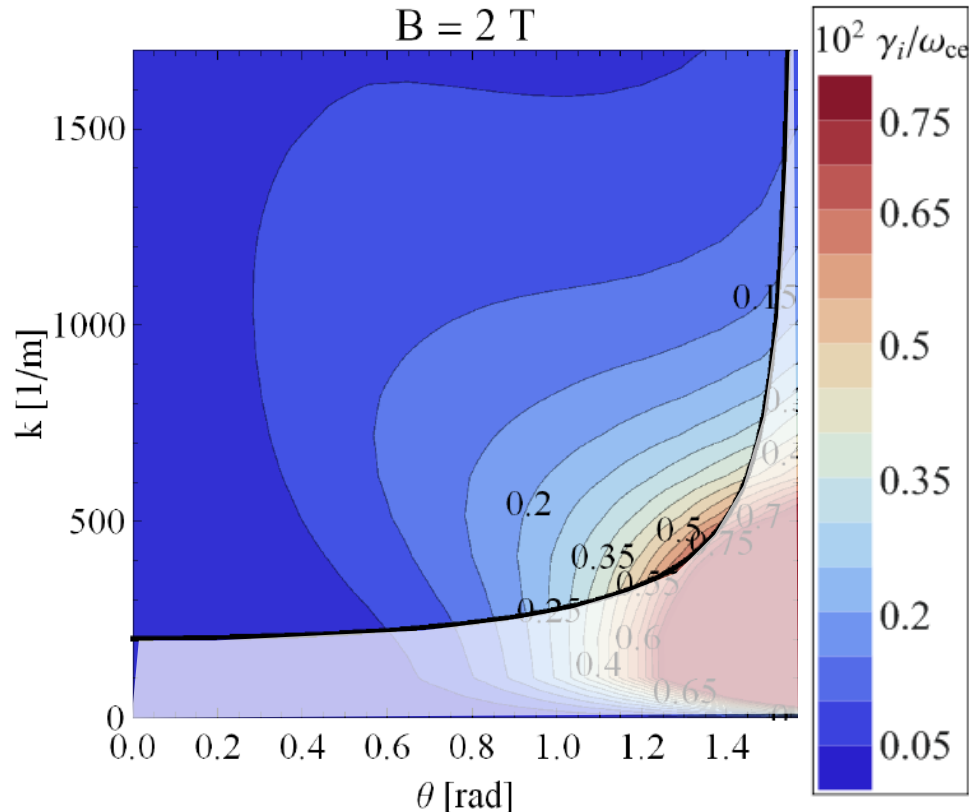
$$(2) \Rightarrow \boxed{k_{\parallel} c - \omega_0(k, \theta) > 0 \quad \text{and} \quad n \leq 0}$$

$$k_{\parallel} c - \omega_0(k, \theta) < 0 \quad \text{and} \quad n \geq 0$$

Whistler and high- k region of EXEL

Growth rate in near-critical field (*Whistler*)

16/29



$$E/E_c = 1.3$$

$$B = 2 \text{ T}$$

$$n_e = 5 \cdot 10^{19} \text{ m}^{-3}$$

$$n_r = 3 \cdot 10^{17} \text{ m}^{-3}$$

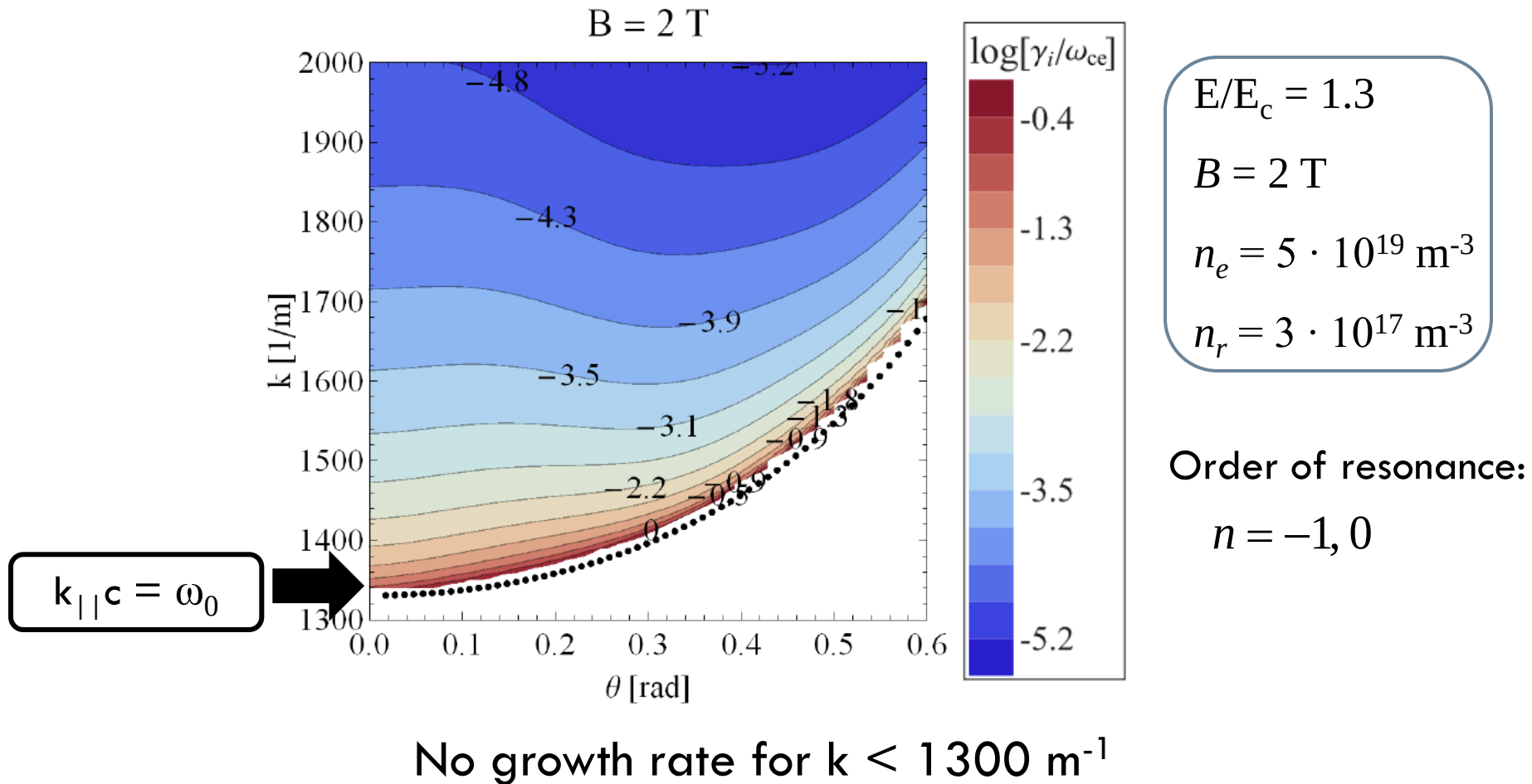
Order of resonance:

$$n = -1, 0$$

Maximum out of the EW region of validity

Growth rate in near-critical field (*EXEL*)

17/29

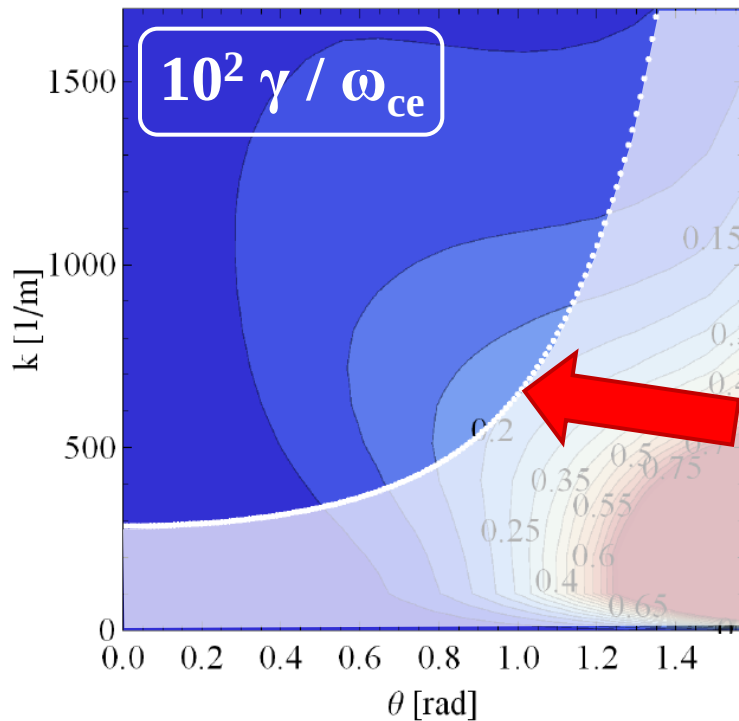


Growth rate in near-critical field

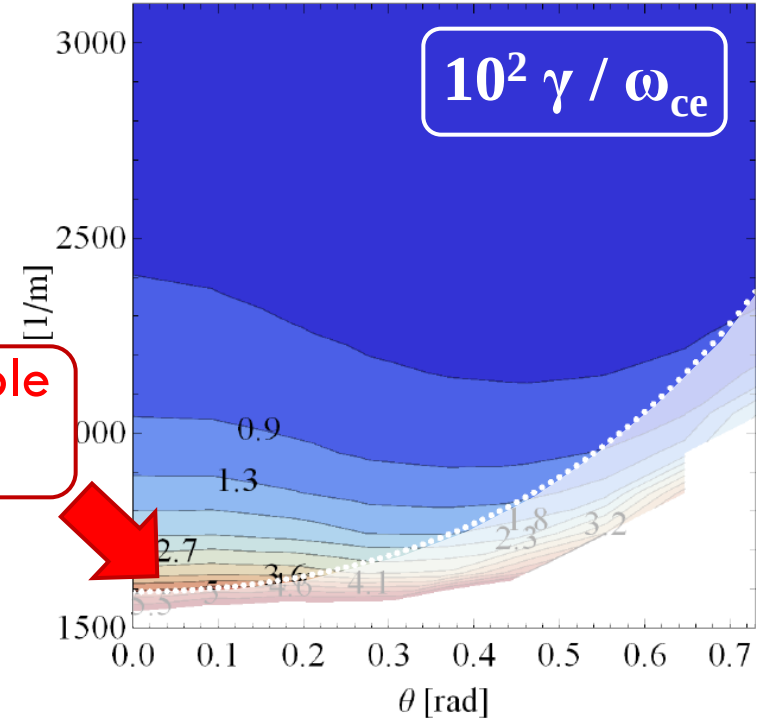
18/29

Near-critical case $\rightarrow$ max. energy (2.6 MeV, $p = 5$)

Electron-whistler wave



Extraordinary electron wave



$$E/E_c = 1.3, B = 2 \text{ T}, n_e = 5 \cdot 10^{19} \text{ m}^{-3}, n_r = 3 \cdot 10^{17} \text{ m}^{-3}$$

Damping rates of the wave, stability

19/29

□ Collisional damping:

M. Brambilla, PoP **2**(1094), 1995

$$\gamma_d = 1.5 \frac{n_i Z^2 e^4 \ln \Lambda}{3\pi^{3/2} m_{e0}^2 v_{Te}^3 \epsilon_0^2}$$

□ Convective damping:

- The runaway beam has a finite radius, L_r

$$\gamma_v = \frac{\partial \omega / \partial k_{\perp}}{4L_r}$$

G. Pokol, PPCF **50**(045003), 2008

Critical density (*Whistler*)

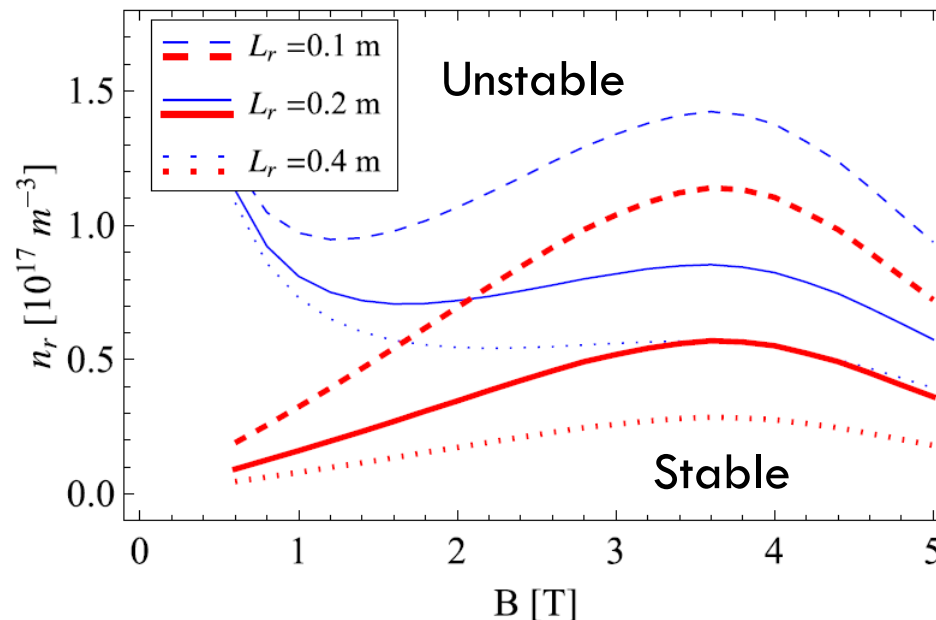
20/29

□ Stability:

Finding (**Growth rate – Damping rates**) = 0
(for the most unstable wave)



Critical runaway density



$T = 20 \text{ eV}$

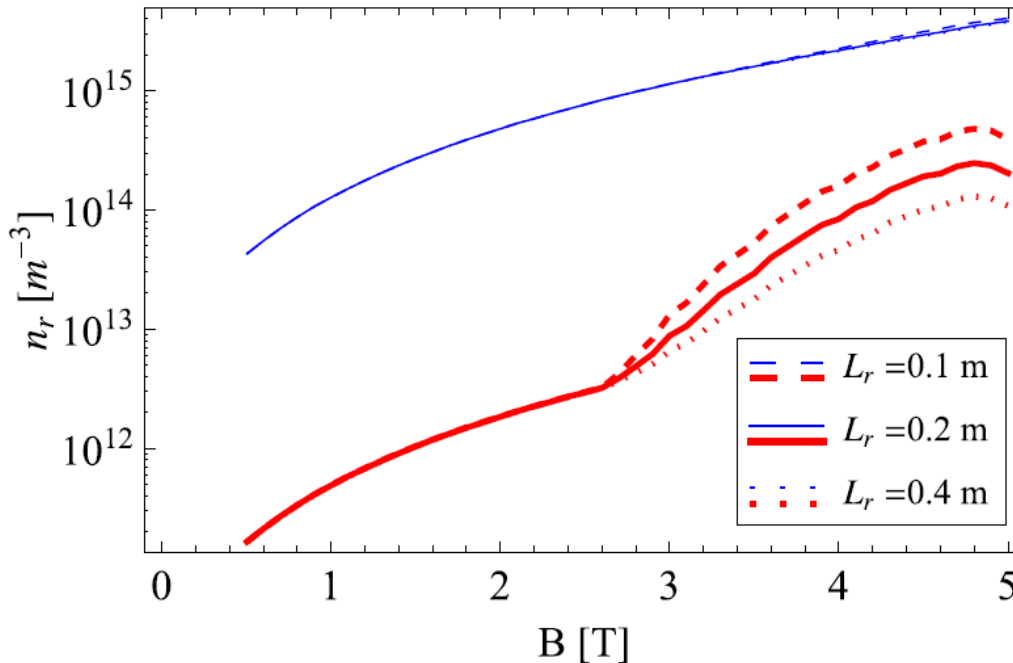
$T = 1000 \text{ eV}$

$E/E_c = 1.3$

$n_e = 5 \cdot 10^{19} \text{ m}^{-3}$

Critical density (*EXEL*)

21/29



$$E/E_c = 1.3$$

$$n_e = 5 \cdot 10^{19} \text{ m}^{-3}$$

$$T = 20 \text{ eV}$$

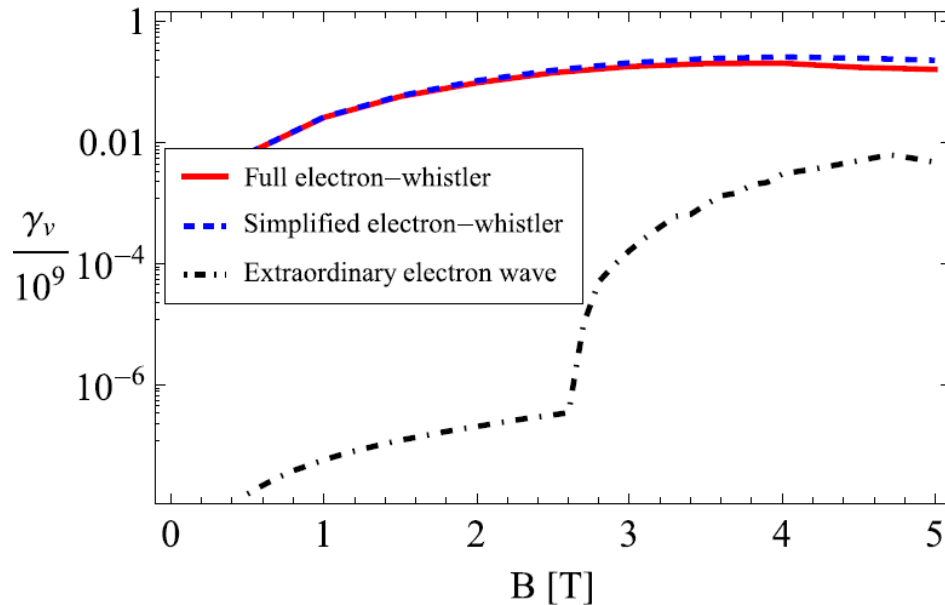
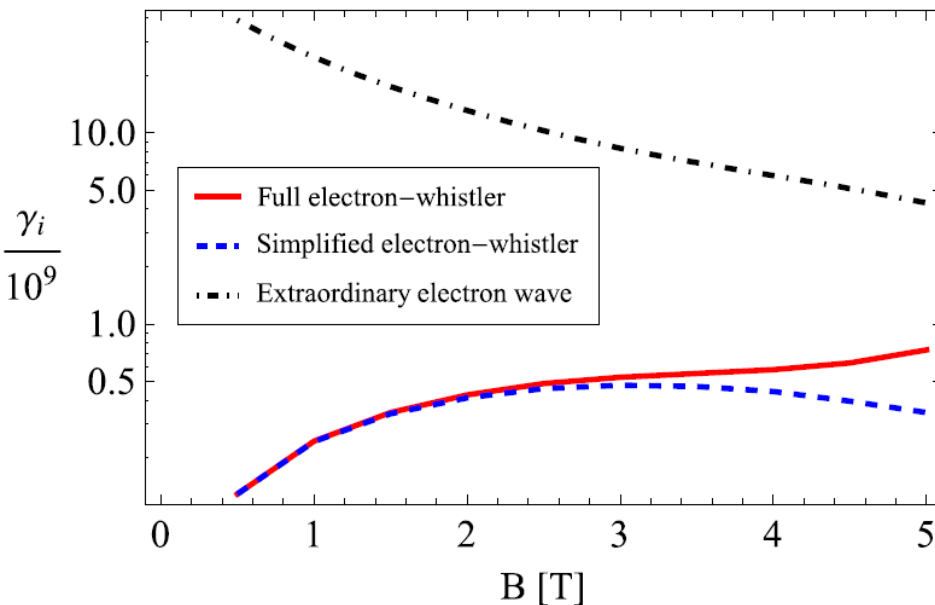
$$T = 1000 \text{ eV}$$

- Orders of magnitude lower critical density
- Break at $B \sim 2.6 \text{ T}$ (for high temperature)

What is different with *EXEL*?

22/29

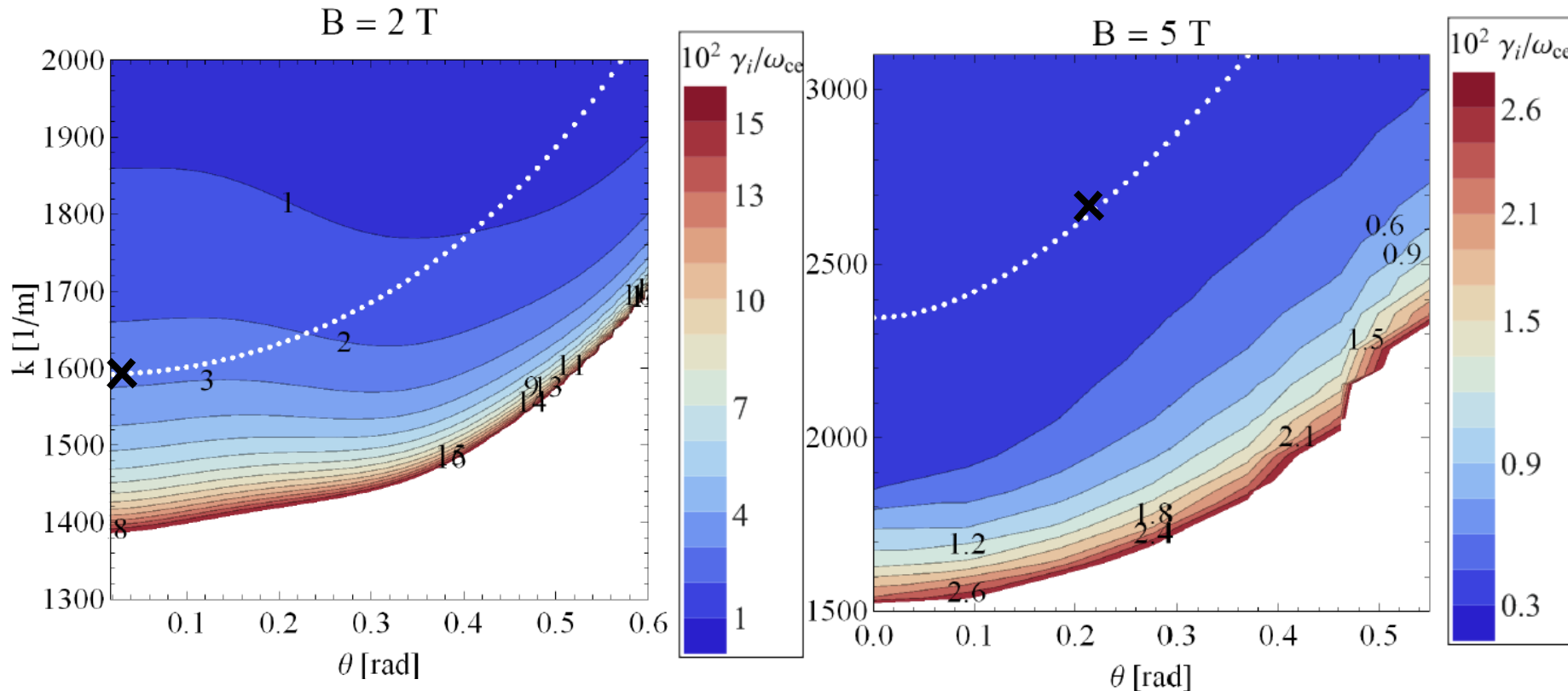
- Slightly **higher growth rate**
- Orders of magnitude **lower convective damping**
- Collisional damping smoothes this effect for low T



$$E/E_c = 1.3, n_e = 5 \cdot 10^{19} \text{ m}^{-3}, n_r = 3 \cdot 10^{17} \text{ m}^{-3}$$

What is different with *EXEL*?

23/29



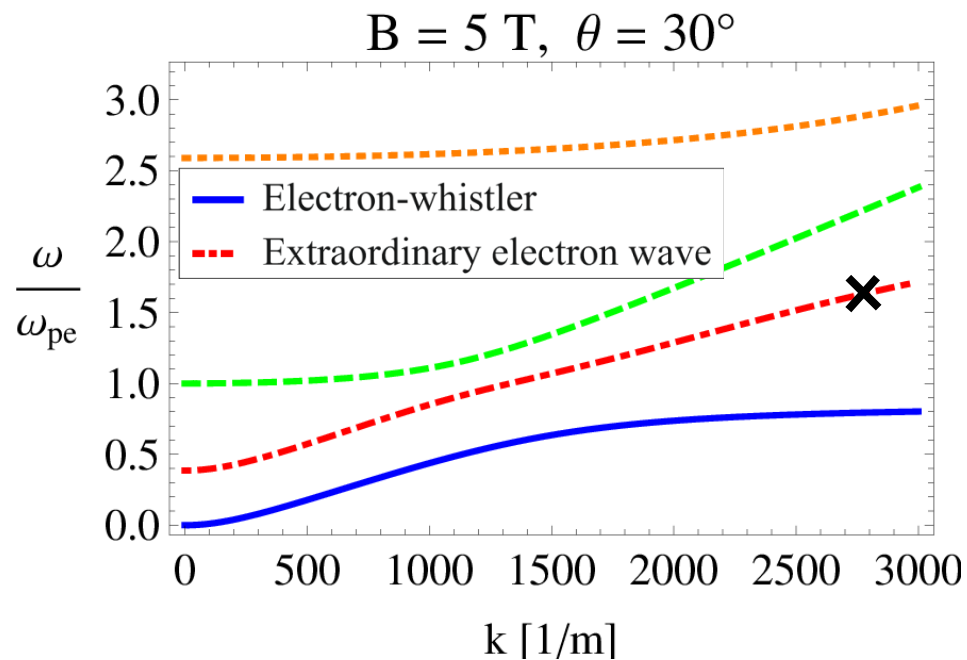
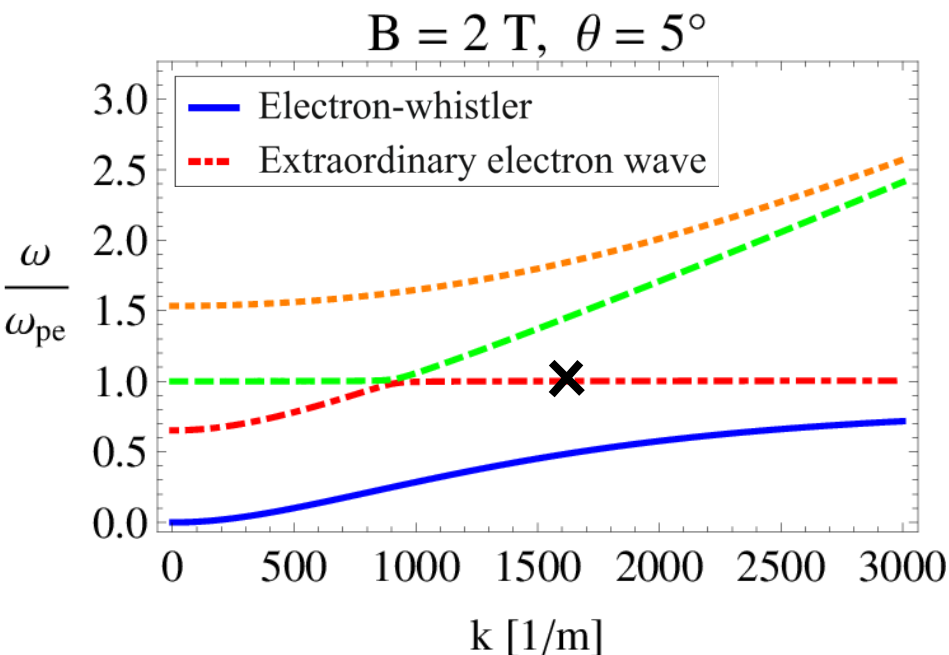
Parameters of the most unstable wave change from $\theta \sim 0$

$$E/E_c = 1.3, n_e = 5 \cdot 10^{19} \text{ m}^{-3}, n_r = 3 \cdot 10^{17} \text{ m}^{-3}$$

What is different with *EXEL*?

24/29

- For the parameters of the most unstable wave:



$$\left(\frac{\partial \omega}{\partial k} \right)_{2T} \ll \left(\frac{\partial \omega}{\partial k} \right)_{5T}$$

Conclusions

25/29

- Wave instability in near-critical electric field
- Generalization
 - ▣ Relaxing the electromagnetic approximation
 - ▣ General resonance condition
- Linear stability
 - ▣ The most unstable wave is dependent on the maximum runaway energy
 - ▣ **Stability threshold** is significantly **lower for the EXEL**

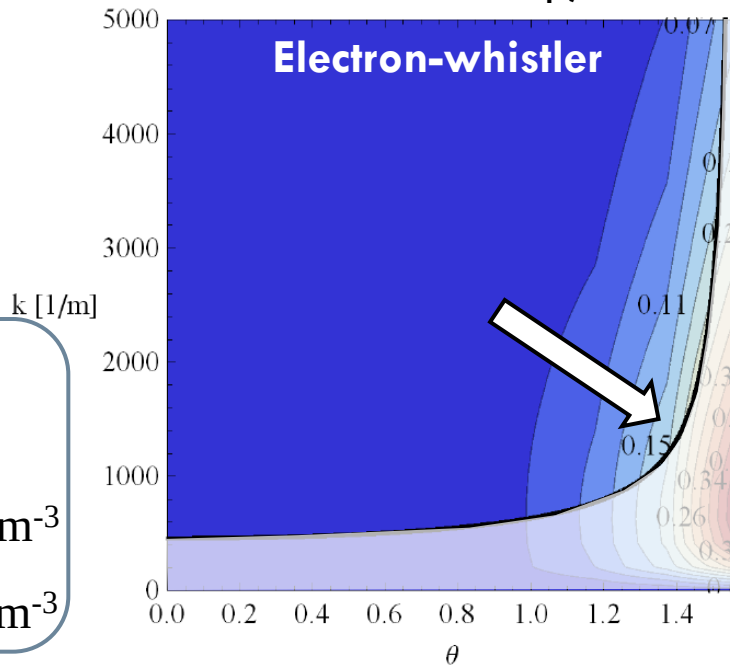
Plans, current problems

26/29

□ Growth rates for high electric field

- ▣ Avalanche runaway distribution
- ▣ No (or much higher) maximum energy

□ Whistler wave T. Fülöp, PoP **13**(062506), 2006

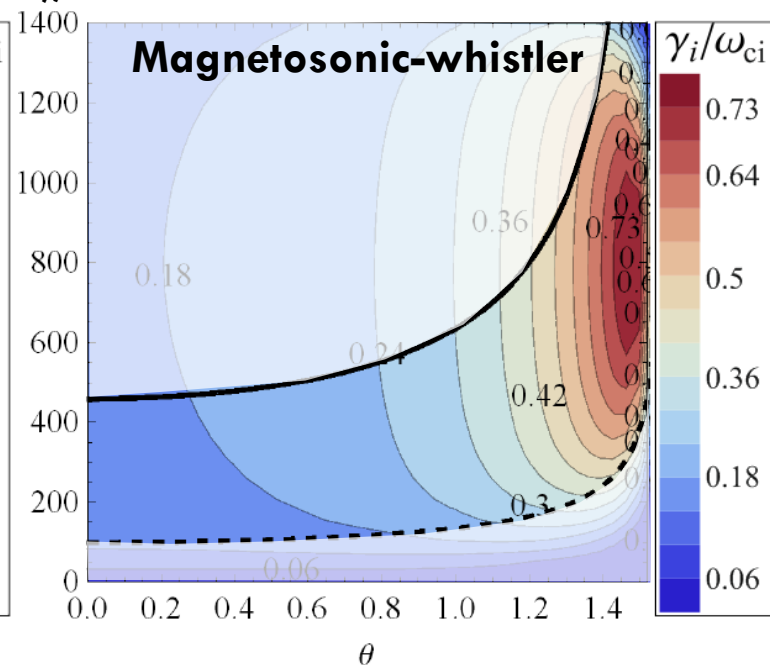


$$E/E_c = 865$$

$$B = 2 \text{ T}$$

$$n_e = 5 \cdot 10^{19} \text{ m}^{-3}$$

$$n_r = 3 \cdot 10^{17} \text{ m}^{-3}$$

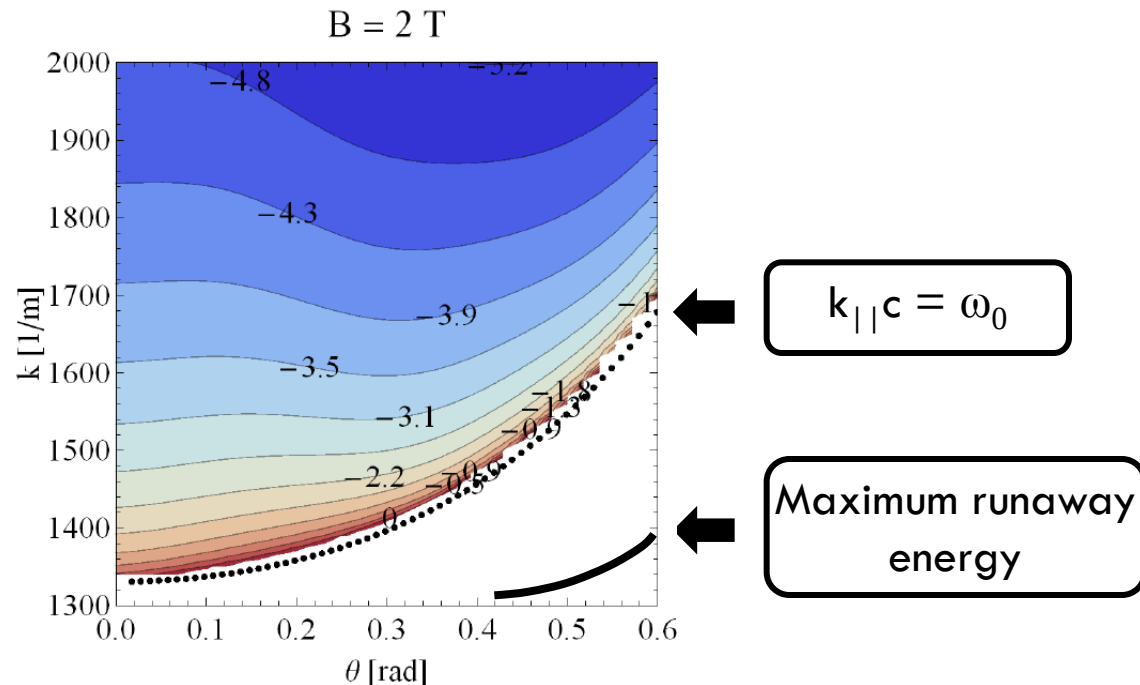


Plans, current problems

27/29

- Extraordinary electron wave
 - ▣ Most unstable wave would be in the region of no-growth rate (if there is a maximum runaway energy)

This is not a problem!



Plans, current problems

28/29

□ For $k_{||}c > \omega_0$:

□ $n \leq 0$

$$\square p_{res} = \frac{-k_{||}c n \omega_{ce} + \omega_0 \sqrt{(k_{||}^2 c^2 - \omega_0^2)(1 + p_{\perp}^2)} + n^2 \omega_{ce}^2}{k_{||}^2 c^2 - \omega_0^2}$$

□ For $k_{||}c < \omega_0$:

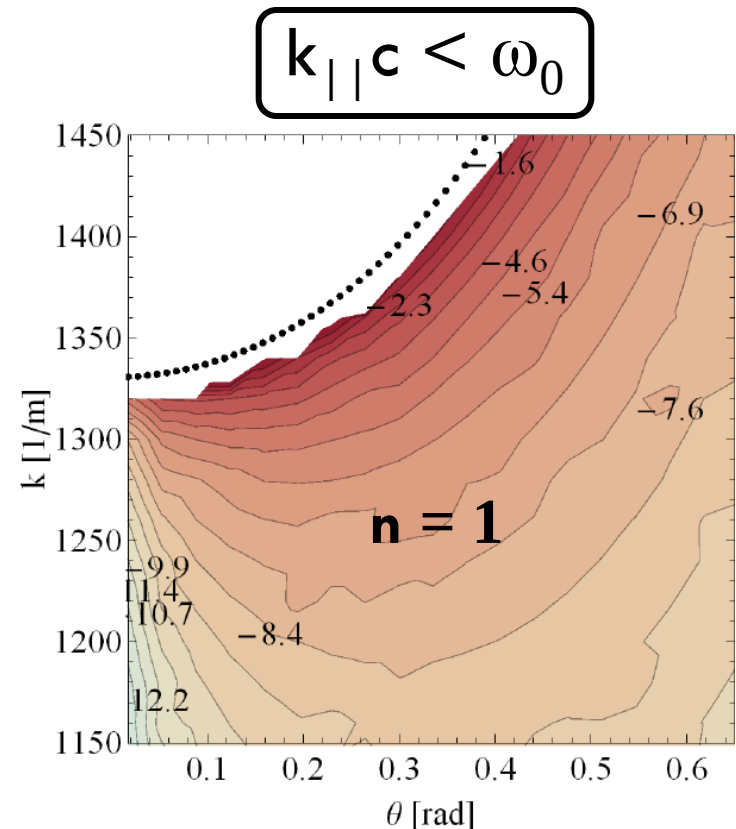
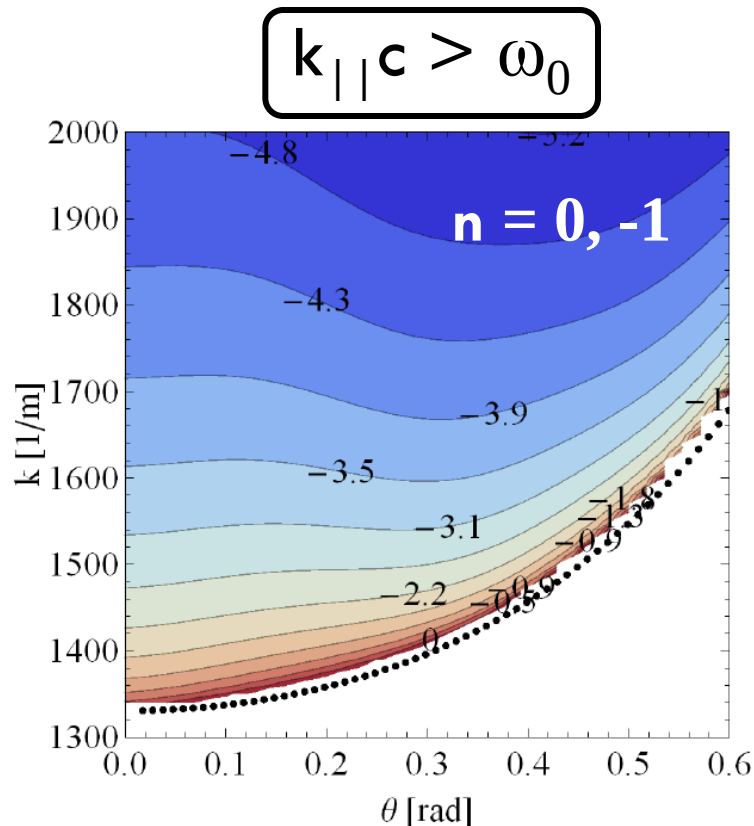
□ $n > 0$

$$\square 1 + p_{\perp}^2 < \frac{n^2 \omega_{ce}^2}{\omega_0^2 - k_{||}^2 c^2}$$

$$\square p_{res} = \frac{-k_{||}c n \omega_{ce} - \omega_0 \sqrt{(k_{||}^2 c^2 - \omega_0^2)(1 + p_{\perp}^2)} + n^2 \omega_{ce}^2}{k_{||}^2 c^2 - \omega_0^2}$$

Plans, current problems

29/29



□ Maximum at $k_{||}c = \omega_0$: with $n \neq 0$