

Mobility report

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Framework: EURATOM Mobility

The background of the workplan: Association EURATOM-HAS and Chalmers University of Technology have a long history of collaboration on kinetic instabilities excited by runaway electrons. Our new aim is to extend the study of such interactions from disruption runaways to steady state, near-critical runaway populations that may be easier to compare to experiments.

The aim of the secondment: Aim of this secondment is to identify the runaway electron distribution function relevant for the near-critical case.

Milestones:

- Identify the runaway electron distribution function relevant for the near-critical case.

Report

During the visit it was found that the electron distribution functions valid for different parameter regions have been studied by P. Sandquist in [1]. Therefore the aim was to understand the different distribution functions deduced in [1] and find out the region of relevancy for each. After a conclusion was made as to which function(s) should be used as the runaway electron distribution function in the near-critical case.

1. Parameters and main regions

While finding the different solutions of the Fokker-Planck equation with different approximations, and later in the distribution functions the following parameters were used:

- $\alpha = \frac{E}{E_C}$, the electric field normalized to the critical electric field;
- $\varepsilon = \frac{E_C}{E_D}$, the fraction of the critical and Dreicer electric field, assumed to be $\ll 1$;
- $\mu = \cos \Theta = \frac{p_{\parallel}}{p}$, characterizing the angle of observation, p is the momentum, $p_{\parallel}$ is the momentum parallel to the electric field;
- $q = \frac{p}{m_e c}$, the normalized momentum, m_e is the electron rest mass;

- Z , the ion effective charge;
- n_e , the thermal electron density.

The electron distribution functions were discussed in the following regions:

- $\alpha < 1$, subcritical region;
- $\alpha \sim 1$, near-critical region;
- $\alpha > 1$, supercritical region.

Cases of non relativistic, weakly relativistic and relativistic momentum were discussed.

2. Relativistic distribution function in the near critical limit

After examining the different distribution functions and discussing their region of validity with Tünde Fülöp and Gergő Pokol we came to the conclusion that there are two candidates for the runaway distribution function for the near critical case, one of them is the function valid in the relativistic and near critical region. This function can be written as

$$f_{III} = \exp(F_0 + (\alpha - 1)F_1),$$

where F_0 is the following:

$$F_0 = \varepsilon^{-1}F_0^0 + \varepsilon^{-1/2}F_0^1 + F_0^2 + C_{III,0},$$

where

$$F_0^0 = \frac{1}{\sqrt{1+q^2}},$$

$$F_0^1 = 2(\sqrt{1+\mu} - \sqrt{2}) \cdot \sqrt{\frac{2}{1+Z}} \cdot \frac{q^2}{1+q^2} - \sqrt{1+Z} \cdot \sqrt{1+q^2},$$

$$F_0^2 = \frac{1+Z}{4} \cdot \left[\frac{5+2q^2}{3} \cdot \sqrt{1+q^2} + \ln\left(\frac{q}{\sqrt{1+q^2}}\right) \right] + \frac{1}{2}q^2 + \ln\left(\frac{q}{\sqrt{1+q^2}}\right) + \frac{q^2}{1+Z} \cdot \frac{\mu-1}{\sqrt{1+q^2}} + \frac{1}{4} \cdot \ln(1+\mu) - \int_{\mu}^1 \frac{1}{1-m^2} \left\{ m + \frac{2}{1+Z} \cdot (1+(m-1)q^2) \cdot \left[(1+m-\sqrt{2(1+m)}) \cdot \frac{2}{(1-q^2)^{3/2}} - \frac{1+Z}{2} \cdot \frac{\sqrt{1+m}}{\sqrt{2}} \right] \right\} dm,$$

$$C_{III,0} = C_I + b(\alpha, Z) - \frac{1}{\varepsilon} + \sqrt{\frac{1+Z}{\varepsilon}} - \frac{5+Z}{16} \cdot \ln \varepsilon - \frac{1+Z}{4} \cdot \frac{5}{3} + \frac{1+Z}{4} \cdot \ln 2.$$

Here $b(\alpha, Z)$ is a constant acquired by numerical techniques and for the relevant parameters it is $b \approx 0.69$.

C_I is the constant characterizing the Maxwellian distribution:

$$n_e = \int_0^{\infty} \exp\left(C_I - \frac{q^2}{2\varepsilon}\right) dq.$$

F_1 is the following:

$$F_1 = \varepsilon^{-1}F_1^0 + \varepsilon^{-1/2}F_1^1 + C_{III,1},$$

where

$$F_1^0 = \frac{1}{\sqrt{1+q^2}} + \sqrt{1+q^2},$$

$$F_1^1 = \sqrt{\frac{2}{1+Z}} \cdot \frac{q^2(1-q^2)}{1+q^2} \cdot \sqrt{1+\mu} + \frac{2}{\sqrt{1+Z}} \cdot \frac{3+2q^2+q^4}{1+q^2} - \frac{\sqrt{1+Z}}{6} \cdot (1+q^2)^{3/2},$$

$$C_{III,1} = -\frac{2}{\varepsilon} + \frac{\sqrt{1+Z}}{6\sqrt{\varepsilon}}.$$

The function is valid for $0 < \alpha - 1 \ll 1$ and in the region, where the normalized momentum $q^2 \sim O(1)$, and for $q < q_c$. Here $q_c = \frac{1}{\sqrt{\alpha - 1}}$ is a critical value for the normalized momentum where the distribution function has a singularity.

Plotting this function together with the non relativistic Maxwellian distribution function in the parallel and perpendicular directions we can see that, unlike in the Maxwellian, in the relativistic function anisotropy appears (Figures 1-4). This anisotropy of the runaway distribution function might destabilize certain waves.

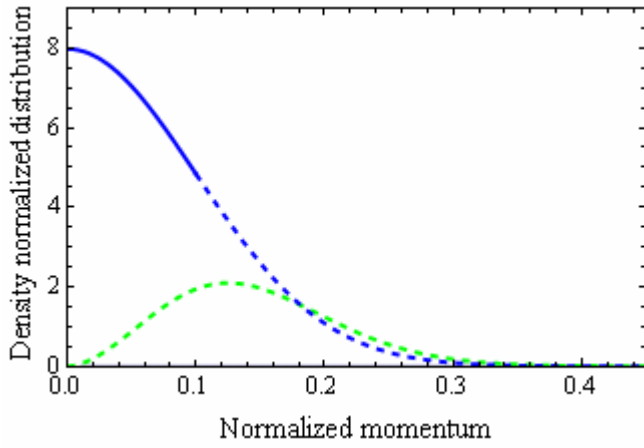


Figure 1. Relativistic distribution function (green) and Maxwellian (blue) in the parallel direction, $\mu = 1$. $n_e = 1$, $\varepsilon = 0.01$, $Z = 2.1$, $\alpha = 1.01$.

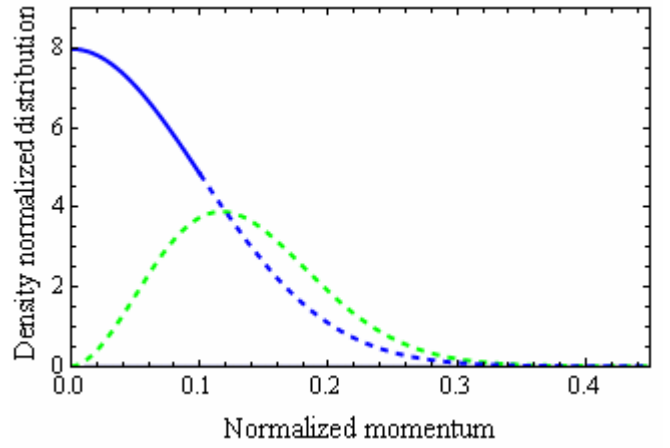


Figure 2. Relativistic distribution function (green) and Maxwellian (blue) in the perpendicular direction, $\mu = 0$. $n_e = 1$, $\varepsilon = 0.01$, $Z = 2.1$, $\alpha = 1.01$.

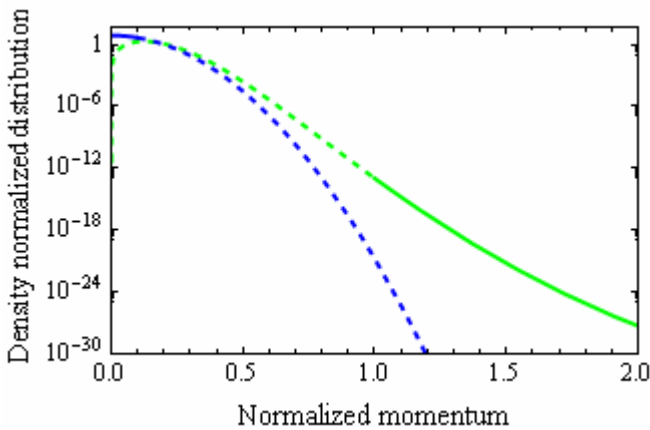


Figure 3. Relativistic distribution function (green) and Maxwellian (blue) in the parallel direction, $\mu = 1$, logarithmic scale. $n_e = 1$, $\varepsilon = 0.01$, $Z = 2.1$, $\alpha = 1.01$.

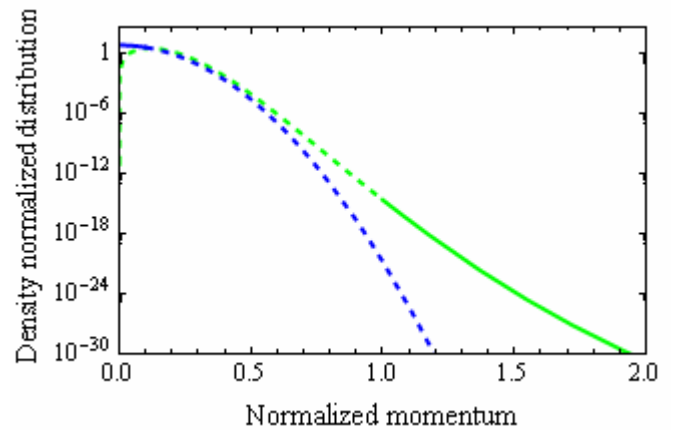


Figure 4. Relativistic distribution function (green) and Maxwellian (blue) in the perpendicular direction, $\mu = 0$, logarithmic scale. $n_e = 1$, $\varepsilon = 0.01$, $Z = 2.1$, $\alpha = 1.01$.

The condition $q < q_c$ means this function describes electrons only in the suprathermal region, therefore we needed to examine the $q > q_c$ runaway region as well.

3. Runaway region in the near critical limit

The other function presented in [1] which might be used as the runaway distribution function for the near critical case is the following:

$$f_V = \frac{A}{q_{\parallel}^{(C_S-2)/(\alpha-1)}} \cdot \exp\left(-\frac{(\alpha+1)q_p^2}{2(1+Z)q_{\parallel}}\right) \cdot {}_1F_1\left(1-\frac{C_S}{\alpha+1}, 1; \frac{(\alpha+1)q_p^2}{2(1+Z)q_{\parallel}}\right),$$

where ${}_1F_1$ is the confluent hypergeometric function, $q_{\parallel}$ is the normalized momentum in the parallel direction, q_p in the perpendicular direction, and C_S and A are constants:

$$C_S = \alpha - \frac{1+Z}{4} \cdot (\alpha-2) \sqrt{\frac{\alpha}{\alpha-1}},$$

$$A = \exp\left(C_{IV} + \frac{\alpha-2}{3(\alpha-1)} \cdot \left(1 - \frac{1+Z}{4} \sqrt{\frac{\alpha}{\alpha-1}}\right) \cdot \ln \varepsilon\right) \cdot \left(\frac{\alpha}{\alpha-1}\right)^{\frac{\alpha-3}{2(\alpha-1)} - \frac{1+Z}{4} \cdot \frac{\alpha-2}{\alpha-1} \sqrt{\frac{\alpha}{\alpha-1}}}$$

$$\cdot \left(\frac{1}{\alpha-1}\right)^{\frac{\alpha-2}{2(\alpha-1)} \left(1 - \frac{1+Z}{4} \sqrt{\frac{\alpha}{\alpha-1}}\right)} \cdot \left(\frac{1}{2}\right)^{\frac{\alpha-2}{\alpha-1} \left(1 - \frac{1+Z}{4} \sqrt{\frac{\alpha}{\alpha-1}}\right)},$$

where C_{IV} is a constant:

$$C_{IV} = \frac{2}{\varepsilon} \left(\sqrt{\alpha(\alpha-1)} - \alpha + \frac{1}{2} \right) - \frac{1}{2} \sqrt{\frac{\alpha(1+Z)}{\varepsilon(\alpha-1)}} \cdot \left[\frac{\pi}{2} - \arcsin\left(1 - \frac{2}{\alpha}\right) \right]$$

$$- \frac{\ln \varepsilon}{48(\alpha-1)} \cdot \left\{ \alpha(3Z+19) - 11 - 3Z + 2(1+Z)(\alpha-2) \sqrt{\frac{\alpha}{\alpha-1}} \right\} + C_I + b(\alpha, Z) - \frac{1+Z}{4} \cdot \frac{\alpha}{\alpha-1}$$

$$+ \frac{1+Z}{4} \cdot \ln 2 + \frac{1}{4} \left(\frac{\alpha-3}{\alpha-1} - \frac{1+Z}{2} \cdot \frac{\sqrt{\alpha}(\alpha-2)}{(\alpha-1)^{3/2}} \right) \cdot \ln(1+Z) - \frac{1+Z}{4} \cdot \ln\left(1 + \sqrt{\frac{\alpha-1}{\alpha}}\right)$$

$$+ \frac{1}{4} \left(3 + (1+Z) \frac{\sqrt{\alpha}(\alpha-2)}{(\alpha-1)^{3/2}} \right) \cdot \ln 2 + \frac{1+Z}{8} \cdot \frac{\sqrt{\alpha}(3-\alpha)}{(\alpha-1)^{3/2}} - \frac{1}{4} \left(2 \frac{\alpha+1}{\alpha-1} + \frac{1+Z}{2} \right) \cdot \ln \alpha$$

$$- \frac{1+Z}{4} \cdot \frac{\sqrt{\alpha}(\alpha-2)}{(\alpha-1)^{3/2}} \cdot \ln(\sqrt{\alpha} + \sqrt{\alpha-1}).$$

The function is valid for $0 < \alpha - 1 \ll 1$ and in the region, where the normalized momentum $q > q_c \gg 1$.

This function gives a beam-like solution peaked around $\mu = 1$, as we can see on Figures 5-6, where the same plot regions are used for the purpose of comparing the two directions.

On Figures 5-6 it is visible that the number of electrons possessing momentum in the perpendicular direction as well is decreasing remarkably with increasing momentum, compared to the function in the parallel direction. This means the solution truly is beam-like around $\mu = 1$.

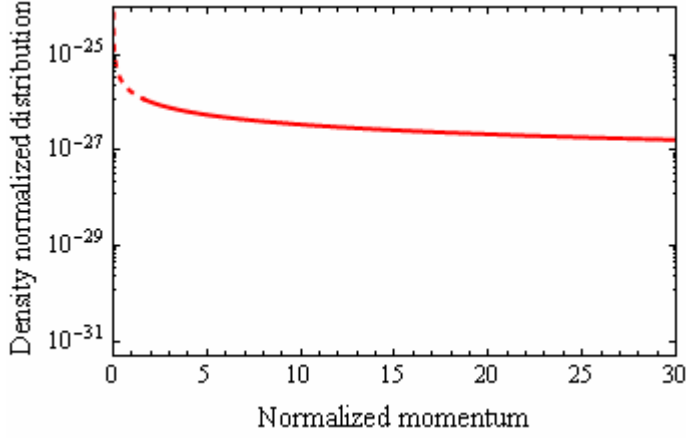


Figure 5. Distribution function for the runaway region in the parallel direction, $q_p = 0$, logarithmic scale. $n_e = 1$, $\varepsilon = 0.01$, $Z = 2.1$, $\alpha = 1.4$.

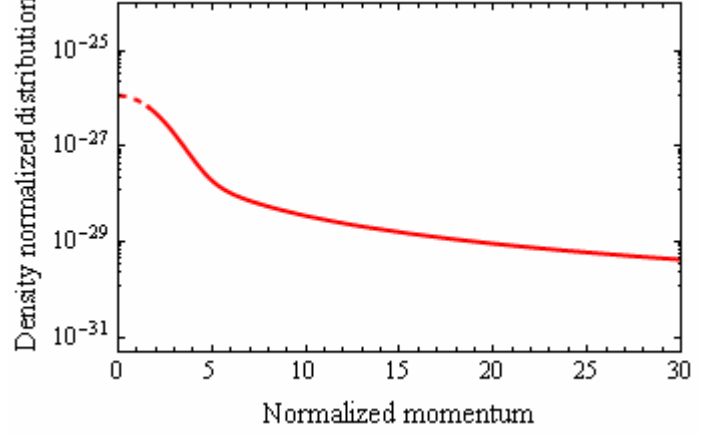


Figure 6. Distribution function for the runaway region in the direction, $q_{||} = 1.7$, logarithmic scale. $n_e = 1$, $\varepsilon = 0.01$, $Z = 2.1$, $\alpha = 1.4$.

Conclusions

In order to calculate the term in the susceptibility tensor related to the runaway electrons in a near critical case we are in need of the runaway distribution function relevant to this case. Finding the two functions discussed above was the first step in studying the possible electron-wave interactions in a near critical electric field.

3rd February 2011

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References

- [1] P. Sandquist, S. E. Sharapov, P. Helander and M. Lisak, „Relativistic electron distribution function of a plasma in a near-critical electric field”, *Physics of Plasmas* **13**, 072108 (2006).