

STAFF MOBILITY REPORT*

1. Information on mission / secondment

Brief title of Research Topic: Quasi-linear interaction between runaway electrons and electromagnetic waves in tokamaks

Name of seconded person: Anna Kómár

Sending Institute: MTA Wigner RCP

Host Institute: Chalmers University of Technology

Dates of secondment / Mission: From 9th March 2013 to 30th March 2013

2. Work Plan / milestones

The background of the workplan: Association EURATOM-HAS and Chalmers University of Technology have a long history of collaboration on kinetic instabilities excited by runaway electrons. Our aim now is to investigate the quasi-linear evolution of high-frequency electromagnetic waves destabilized by runaway electrons.

The aim of the secondment: The aim of this secondment is to construct the diffusion equation governing the evolution of the extraordinary electron wave and provide the background to its numerical solution.

3. Report

The relevant distribution function

During my stay I reviewed the main features of CODE (COLLisional Distribution of Electrons, written in Matlab by Matt Landreman, MIT), which can calculate a quasi steady-state runaway distribution. This is done by solving the Fokker-Planck equation relevant for primary runaway generation:

$$\frac{\partial f}{\partial t} - eE\mathbf{e} \cdot \nabla_{\mathbf{p}} f = C\{f\} + S$$

where f is the distribution function, e is the electron charge, E is the electric field, $\mathbf{e}$ is the unit vector along it, $\nabla_{\mathbf{p}}$ is the gradient in the $\mathbf{p}$ relativistic momentum-space, $C\{f\}$ is the collision operator of the electrons and S is a source term. This equation is converted to pitch-angle coordinates, $\xi = \mathbf{p} \cdot \mathbf{e}/p$ and the model collision operator from Ref. [1] is introduced. It is then solved time-independently, i.e. $\partial f / \partial t = 0$. An example of the resulting distribution is plotted on Figure 1.

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Reasonable achievement of milestones ?	YES/NO
Can payment be authorised ?	YES/NO
Date : Name :	Signature :

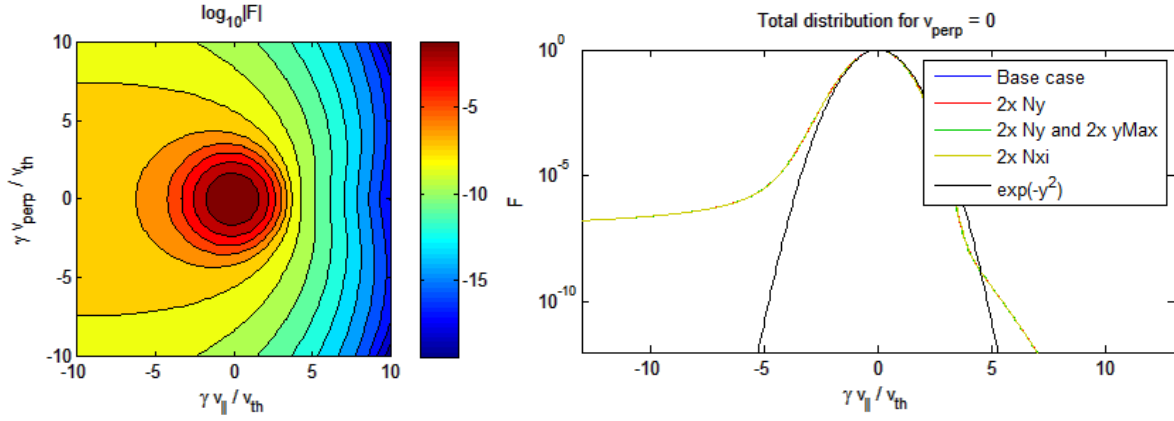


Fig. 1: Electron distribution provided by CODE for $T_e = 100$ eV bulk temperature, $E = 10$ V/m constant electric field, $Z = 1$ effective ion charge.

The normalization of the output F distribution is done the following way:

$$F = \frac{\pi^{3/2} m_e^3 v_{th}^3}{n_e} f$$

where m_e is the electron rest mass, v_{th} is the thermal velocity of electrons and f is the distribution function normalized to the n_e electron number density the following way:

$$n_e = \int d^3(\gamma m_e v_e) f$$

where $\gamma m_e v_e$ is the dimensional relativistic momentum of the electrons.

As for the calculation of the particle-wave interaction we need a distribution normalized to 1 in the $p = \gamma v_e/c$ dimensionless relativistic momentum, the function we need is

$$f_r = \frac{F}{\pi^{3/2} \frac{v_e}{c}}$$

where F is the output function of CODE.

Calculating the growth rates numerically

After obtaining the correctly normalized f_r distribution function, I proceeded to write a routine in Matlab, which can calculate the growth rates of the electron-whistler (EW) and the extraordinary electron (EXEL) waves from the anisotropic electron distribution provided by CODE as an initial seed.

This routine consists of the following parts:

- Calls CODE with the correct physical and input parameters;
- Calculates the growth rate within the defined region of k (wave number) and θ (propagation angle of the wave) on a grid with the defined step size:
 - Finds the resonant momentum corresponding to k and θ ;
 - Calculates the momentum-space integrals in the runaway susceptibilities using the runaway distribution from CODE;
 - Calculates the growth rate of the wave (EW or EXEL) based on the runaway susceptibilities within a first order perturbative analysis.
- Plots the growth rate on a contour plot.

The main approximations used are:

- Homogeneous, cylindrical plasma;
- Ultrarelativistic resonance condition: valid for high energy runaway electrons;

- Cold plasma approximation for the waves: valid for temperatures up to 1 keV;
- High frequency approximation for the waves: $\omega \gg \omega_{ce} \sqrt{m_e/m_i}$.

The physical input parameters of the routine are the following:

- Z , effective ion charge;
- B , magnetic field (in T);
- $\alpha = \frac{E}{E_c}$, the electric field to critical field ratio;
- n_e , thermal electron number density (in m^{-3});
- T_e , electron temperature (in eV).

The simulational input parameters are:

- Region of p_{par} (normalized momentum of runaways parallel to the static magnetic field) where the distribution function is calculated (the resonant momentum should be in this region);
- Region of p_{perp} (normalized momentum of runaways perpendicular to the static magnetic field) where the distribution function is calculated;
- Range of k (wave number) and step size;
- Range of θ (propagation angle) and step size where the growth rate is calculated.

Sample growth rates are shown on Figure 2 for the electron-whistler and the extraordinary electron wave.

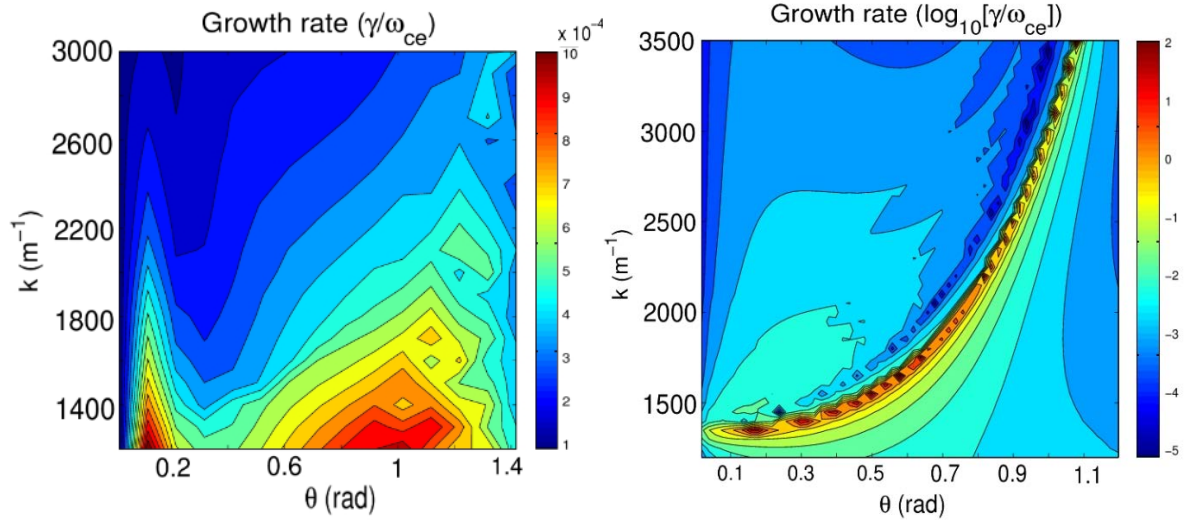


Fig. 2: Growth rates for the electron-whistler and the EXEL wave for $T_e = 10$ eV, $E/E_{\text{crit}} = 1.3$, $n_e = 5 \cdot 10^{19} m^{-3}$; the EXEL growth rate is only valid for $k_{\parallel} c > \omega_0$, i.e. for wave numbers higher than approx. $1300 m^{-1}$.

Polarization of the waves

The numerical calculation of the growth rates were needed in order to proceed to calculate the quasi-linear part of this resonant wave-particle interaction. The first step toward this calculation is to determine the polarization of the waves, as it is needed for the quasi-linear diffusion equation.

In order to calculate the polarization of electromagnetic waves, we start from their wave dispersion

$$\begin{pmatrix} \epsilon_{11} - n^2 \cos^2 \theta & \epsilon_{12} & n^2 \cos \theta \sin \theta \\ -\epsilon_{12} & \epsilon_{22} - n^2 & 0 \\ n^2 \cos \theta \sin \theta & 0 & \epsilon_{33} - n^2 \sin^2 \theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

where ϵ_{ij} are the dielectric tensor elements consisting of thermal electron and ion susceptibilities, $n = kc/\omega_0$ is the index of refraction, k is the wave number, ω_0 is the wave frequency, c is the speed of light, θ is the propagation angle of the wave and E_x , E_y and E_z are the electric field elements corresponding to the wave.

As the polarization vector is defined as

$$\begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} = \begin{pmatrix} 1 \\ E_y/E_x \\ E_z/E_x \end{pmatrix}$$

we can use the previous equation to determine the vector elements.
By using the second row

$$-\epsilon_{12}E_x + (\epsilon_{22} - n^2)E_y = 0$$

we can construct the e_y element:

$$\frac{E_x}{E_y} = \frac{\epsilon_{22} - n^2}{\epsilon_{12}}$$

with $\epsilon_{22} = 1 - \frac{\omega_{pe}^2}{\omega_0^2 - \omega_{ce}^2}$ and $\epsilon_{12} = -i \frac{\omega_{pe}^2 \omega_{ce}}{\omega_0(\omega_0^2 - \omega_{ce}^2)}$ where ω_{pe} and ω_{ce} are the electron plasma- and cyclotron frequencies respectively.
Similarly, by using the third row

$$\frac{k_{\parallel} k_{\perp} c^2}{\omega_0^2} E_x + \left(\epsilon_{33} - \frac{k_{\perp}^2 c^2}{\omega_0^2} \right) E_z = 0$$

with $\epsilon_{33} = 1 - \frac{\omega_{pe}^2}{\omega_0^2}$ we can construct the e_z element:

$$\frac{E_x}{E_z} = \frac{\omega_{pe}^2 + k_{\perp}^2 c^2 - \omega_0^2}{k_{\parallel} k_{\perp} c^2}$$

Therefore, the polarization of the EW and the EXEL wave is

$$\begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} = \begin{pmatrix} 1 \\ -i \frac{\omega_{pe}^2 \frac{\omega_{ce}}{\omega_0}}{\omega_0^2 - k_{\perp}^2 c^2 - \omega_{ce}^2 - \omega_{pe}^2 + k_{\perp}^2 c^2 \frac{\omega_{ce}^2}{\omega_0^2}} \\ \frac{k_{\parallel} k_{\perp} c^2}{\omega_{pe}^2 + k_{\perp}^2 c^2 - \omega_0^2} \end{pmatrix}$$

where $\omega_0 = \omega_{EW/EXEL}(k, \theta)$ is the wave dispersion for the EW/EXEL wave, and can be treated as a simple function of k and θ , as we know its analytical form.

References

- [1] G. Papp, M. Drevlak, T. Fülöp, P. Helander. *Nuclear Fusion*, **51**, 043004 (2011).