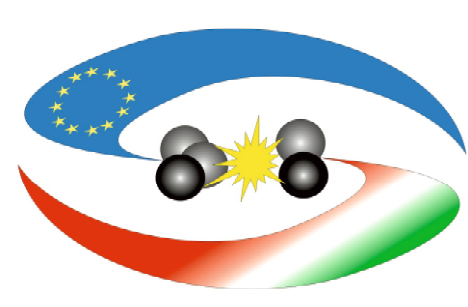
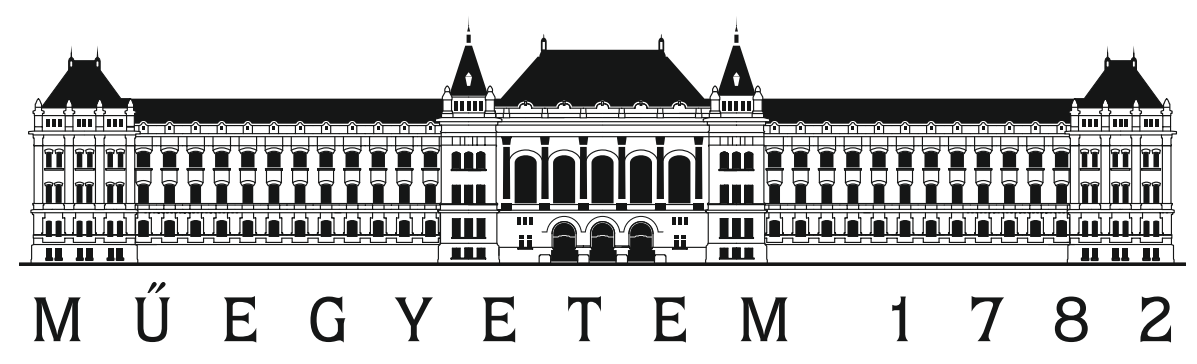




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High-frequency electromagnetic waves destabilized by runaway electrons in a near-critical electric field

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Introduction, motivation

Relativistic runaway electrons are produced in plasmas when the electric field is larger than a certain critical field (E_c), and the accelerating force overwhelms the retarding friction for high energy electrons. The **anisotropy of the runaway electron distribution can lead to destabilization of plasma waves** through wave-particle resonant interaction.

Recent work [1] has shown that **even in disruptions the electric field** in the core region of the plasma is only **slightly above the critical** electric field. This work considers whistler wave instability driven by an anisotropic electron distribution in near-critical field.

Dispersion relation of the waves

The dispersion relation of high frequency electromagnetic waves is

$$\left(\epsilon_{11} - \frac{k_{\parallel}^2 c^2}{\omega^2}\right) \left(\epsilon_{22} - \frac{k_{\perp}^2 c^2}{\omega^2}\right) + \epsilon_{12}^2 = 0$$

ω : wave frequency
 k : wave number
 ϵ : dielectric tensor

where the dielectric tensor is constituted of the ion, thermal electron and runaway electron susceptibilities:

$$\epsilon = \mathbf{1} + \chi^i + \chi^e + \chi^r$$

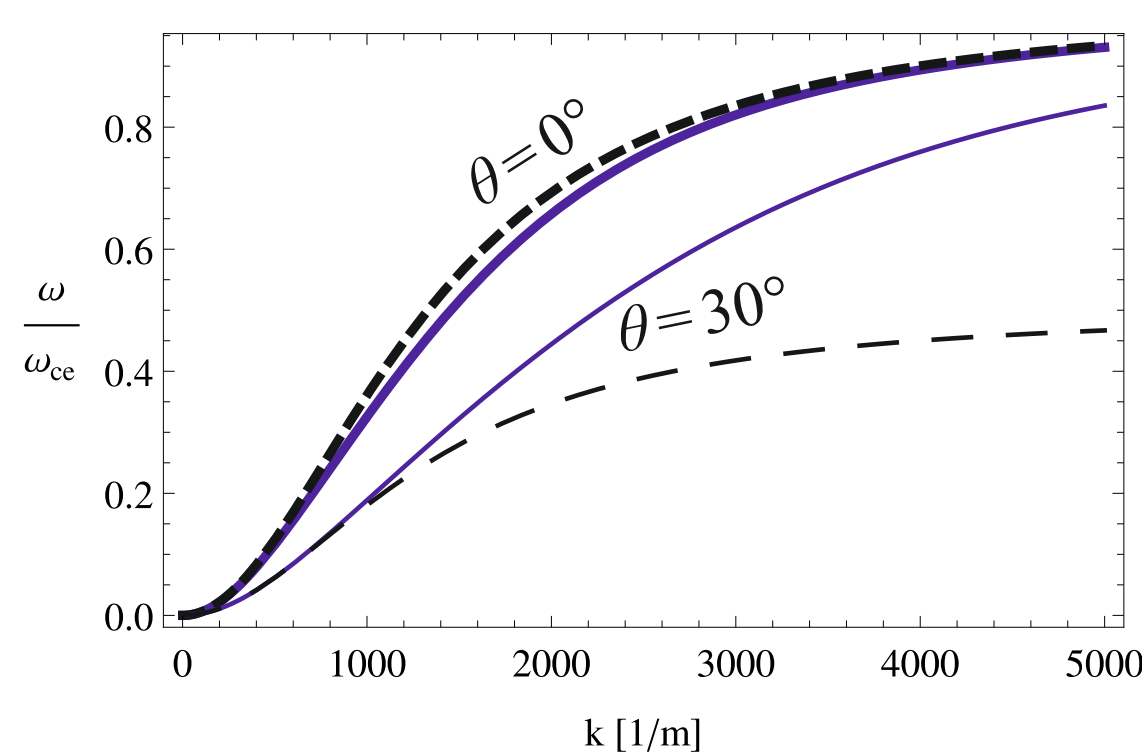
The contribution of the runaway population is expected to be small.

Dispersion of high frequency electromagnetic waves **without the runaway term** and cold plasma approximation.

Electron-whistler wave

$$\text{Valid for } \frac{\omega_{ce}}{\sqrt{m_i/m_e}} \ll \omega \sim \omega_{ce}$$

Three different waves.
One of them is the electron-whistler wave, satisfying $\omega < k_{\parallel} c$.



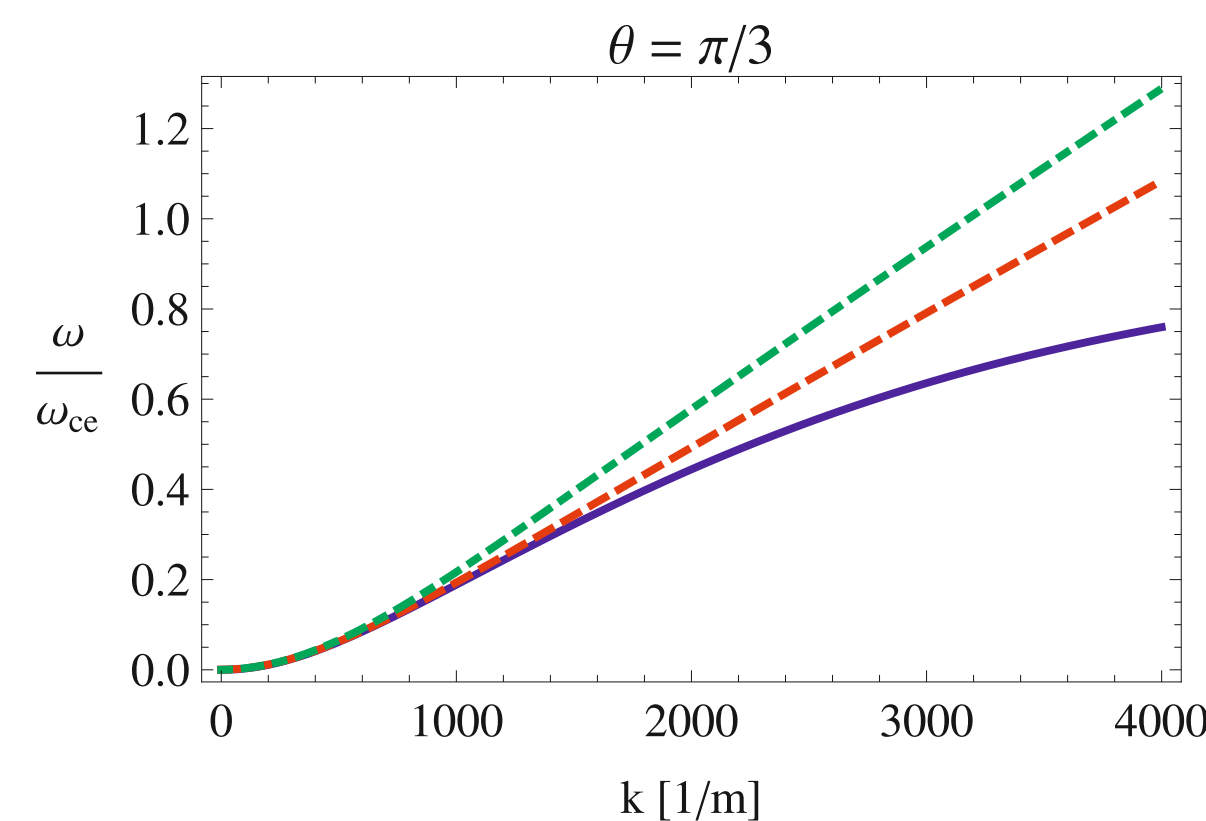
Blue solid is the electron-whistler; black dashed is the whistler wave defined by Sazhin in Ref. [2]. Thus the name electron-whistler.

Magnetosonic-whistler wave

$$\text{Valid for } \omega_{ci} \ll \omega \ll \omega_{ce}$$

A perpendicularly propagating limit before in Refs. [3-5].

In this work a more general dispersion was derived.



Red dashed is the general magnetosonic-whistler; green dashed is the limit introduced in Ref. [4] in Eq. (13).

Runaway contribution

Runaway electron population: taken into account with their susceptibility added to the dielectric tensor. Thus, the frequency of the wave changes with a small $\delta\omega$ term and a **first order perturbative analysis** is done. The imaginary part of the resulting $\delta\omega$ is the growth rate of the selected wave.

$$\text{Growth rate } \gamma_i = \Im(\delta\omega)$$

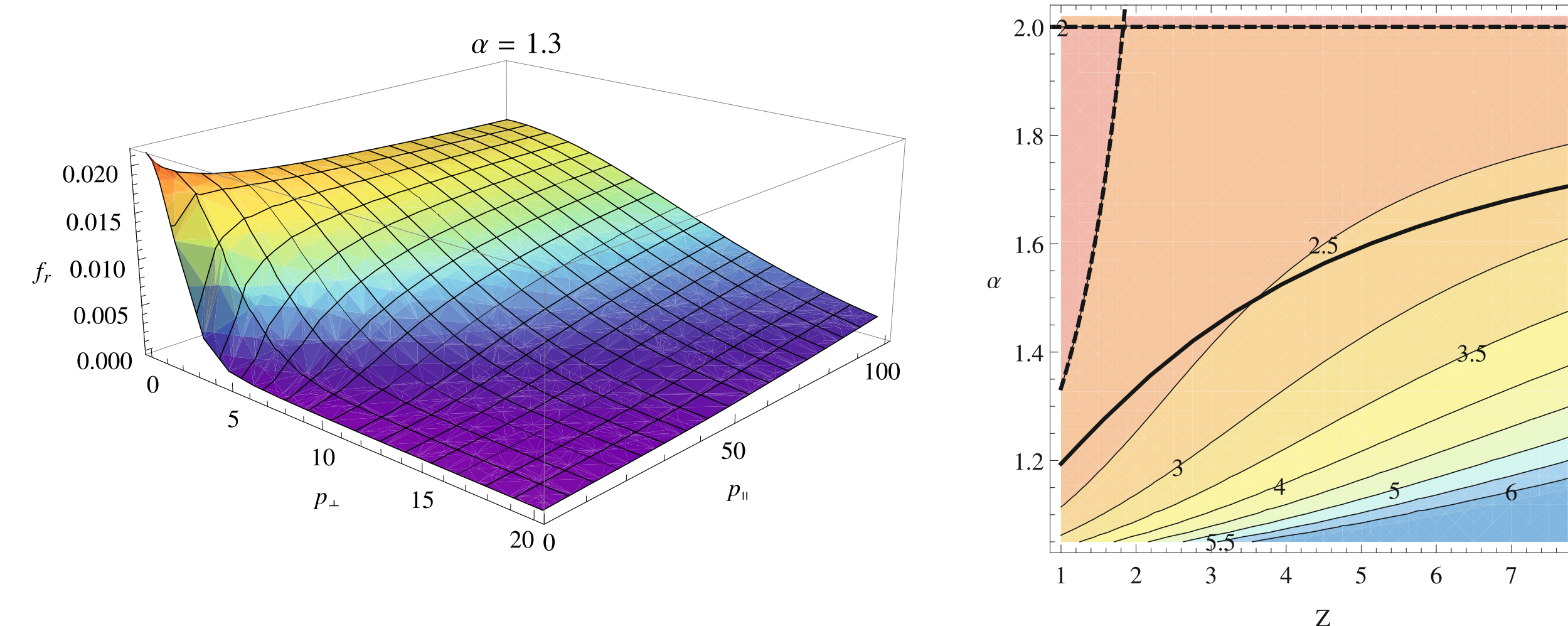
To calculate the growth rate, one has to know the runaway susceptibilities. These are given by Stix [6] for an arbitrary population with the distribution f .

Distribution function

The runaway distribution for near-critical electric field is [7]:

$$f_r(p_{\parallel}, p_{\perp}) = \frac{A}{p_{\parallel}^{(C_s-2)/(\alpha-1)}} \exp\left(-\frac{(\alpha+1)p_{\perp}^2}{2(1+Z)p_{\parallel}}\right) {}_1F_1\left(1 - \frac{C_s}{\alpha+1}, 1; \frac{(\alpha+1)p_{\perp}^2}{2(1+Z)p_{\parallel}}\right)$$

where $\alpha = E/E_c$, Z is the effective ion charge and $C_s = \alpha - \frac{(1+Z)(\alpha-2)}{4} \sqrt{\frac{\alpha}{\alpha-1}}$



To have a positive distribution: $C_s < \alpha + 1$.
Furthermore, the condition $f_r \rightarrow 0$ as $p_{\parallel} \rightarrow \infty$ requires that $C_s > 2$.

This gives a narrow region in the α - Z space where the distribution function is useful.

Resonance condition

In the runaway susceptibilities a resonance condition appears:

$$p_{\parallel} = \frac{\omega_0 \gamma - n \omega_{ce}}{k_{\parallel} c}$$

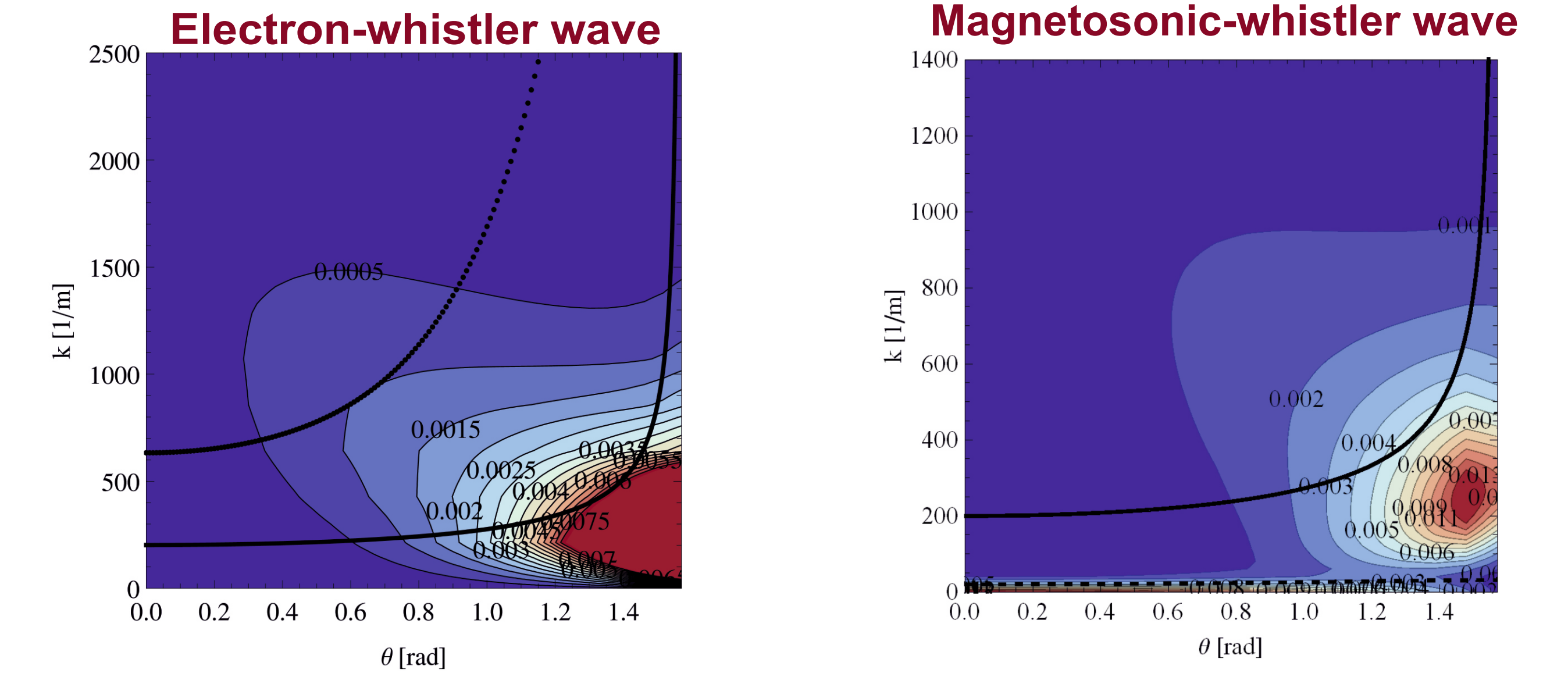
ω_0 : dispersion of the wave
 γ : relativistic factor
 ω_{ce} : electron cyclotron frequency
 c : speed of light

This is an **implicit condition** for the resonant momentum.
 n is a parameter describing the resonance: $n = 1$ Doppler, $n = -1$ anomalous Doppler, $n = 0$ Cherenkov resonance.

Previously [3-5] the ultrarelativistic limit was used. In this work a general formula was introduced for lower energies.

We concluded that only waves satisfying the $\omega < k_{\parallel} c$ condition can interact with runaway electrons and only with the **anomalous Doppler or Cherenkov resonance**.

Most unstable wave

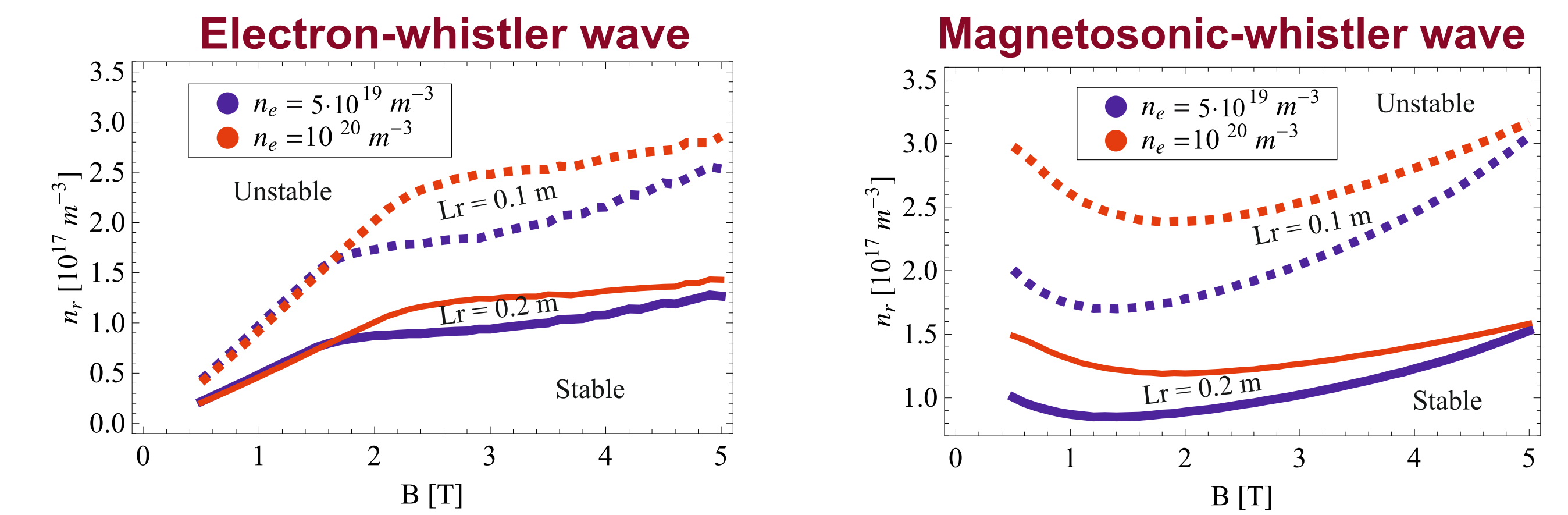


The growth rate for the magnetosonic-whistler wave has a maximum. The resonance condition cannot be satisfied in the region of the maximum (only for energies ~ 20 MeV), thus, the **electron-whistler is the most unstable wave**. The parameters depend on the maximum runaway energy (1 MeV on the figure, dotted).

Stability: growth rate compared to the damping rates.

For temperatures higher than 200 eV (with runaway beam width $L_r \sim 0.2$ m):

$$\text{Collisional damping: } \gamma_d = 1.5 \tau_{ei}^{-1} \ll \text{Convective damping: } \gamma_v = \frac{\partial \omega / \partial k_{\perp}}{4 L_r}$$



Conclusions

By studying the linear growth rate of the electron-whistler branch we found that the frequency and wave number of the **most unstable wave depends strongly on the maximum runaway energy**. By comparing the growth rate to the collisional and convective damping rates we find that for temperatures higher than 200 eV the number density of runaways that is required to destabilize the waves increases with increasing magnetic field, while it is only weakly dependent on the density of the background plasma.

References

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