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## **COMBINED ANALYSIS SYSTEM FOR PROPPANT SELECTION IN ASPECT OF FRACTURE CONDUCTIVITY**

### **INTRODUCTION**

The purpose of hydraulic fracturing is well stimulation by which the well performance can be significantly improved. To achieve this, high conductive fracture is created which is propped with a propping agent. The success of the treatment is highly influenced by the fracture conductivity which is the product of the apparent proppant pack permeability and the fracture width. The well productivity after hydraulic fracturing is definitely affected by the interaction between proppants and fracture surface under closure pressures, the proppants embed into the formations and deform due to the closure stress that can result a significant reduction in fracture conductivity. Proppant size, Young's modulus and Poisson's ratio of proppant as well as formation, closure pressure are factors which influence proppant embedment and the change of fracture aperture that actually can reduce the fracture conductivity from 20 to 300 % with a subsequent reduction in oil, gas or even in water production (geothermal wells). This effect in weakly consolidated formations is more significant. Therefore, it is of importance to investigate and understand the theoretical background of proppant embedment.

To examine the proppant embedment and their influence on fracture conductivity, a static force balance condition is considered to be the best way to investigate this phenomenon. The main benefit of an analytical analysis compared to empirical experiments is that it can be fitted to all the cases occurred in the practice. Empirical models may simulate the reality more accurately but these formulas can be used only in a narrow range, in which the equations were derived. These equations may contain fitting coefficients which can be determined only in experimental way, ergo it means parametric equations. Experimental way means mainly measurements which can be made on core samples for example, and it often requires an accurate measurement system with special apparatus. So an analysis system based on analytical model can be widely used, not to mention the case, when experimental data is available, the equations which were derived on the basis of analytical consideration can be also fitted with results gained by experiments.

### **Purpose of Research**

Companies and experts in the field, especially the project managers, are interested in the return of the projects. The pay-off of the hydraulic fracturing treatment is significantly affected by the productivity index (PI) of the well. In this paper, more models and considerations were integrated to develop a new analysis system by which different sensitivity tests (with deep consequences) and a kind of technical optimization of proppant selection can be executed. Ignoring any costs or revenue (CAPEX, OPEX, NPV, etc.) calculations, the outcome of this paper provides a method for proppant selection considering not only the factors that influence fracture conductivity but other optimization method, the so-called Unified Fracture Design (UFD) model as well.

In a practical point of view, the analysis system provides the opportunity to select the proper type of proppant (proppant size, elastic modulus, specific volume) and to determine

the amount of proppant (according to areal proppant concentration) which is needed to achieve the predetermined productivity index of the well (which could be based on net cash flow calculations and reservoir engineering considerations). In addition, one can compare the fracture conductivity properties of the proppant pack provided by the manufacturers to the results (computed conductivity values) got by performing the calculations studied in this paper. This makes one able to evaluate the manufacturer's data in an alternative way.

## THEORETICAL BACKGROUND

The analytical model, which was derived by Li' et al. (2015), approach the proppant embedment in theoretical aspect. In the followings, one can obtain the theoretical background of the analytical model since the whole detailed derivation process is described below. It is important to note that the model has some limitations. During the stress/strain analysis it was assumed that the proppant would deform and embed into the formation under closure stress, without considering the case of proppant crush. It means that linear elastic behavior was assumed in deriving the analytical model which entails that one can use the equations below the stress range which would cause proppant crush.

The derivation begins with a stress/strain formula of two mutually squeezing elastic spheres developed by Wu et al. (2001) and which is illustrated in Fig. 1. Spheres 1 and 2 deform under balanced load. The boundary of the contact area of the Spheres is a circle, and its radius ( $\alpha$ ) can be expressed as follows:

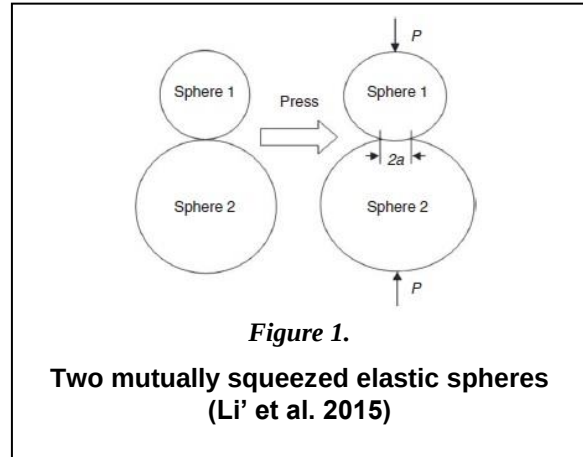


Figure 1.

Two mutually squeezed elastic spheres  
(Li' et al. 2015)

$$\alpha = \left( \frac{3}{4} P C_E \frac{R_1 R_2}{R_1 + R_2} \right)^{\frac{1}{3}}, \quad 1$$

where  $\alpha$  is the radius of the boundary of contact area, in mm;  $P$  is the external force on Spheres 1 and 2, in N;  $R_1$  is the radius of Sphere 1, in mm; and  $R_2$  is the radius of Sphere 2, in mm.  $C_E$  (in  $\text{MPa}^{-1}$ ) is defined as:

$$C_E = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \quad 2$$

where  $\nu_1$  is the Poisson's ratio of Sphere 1, dimensionless;  $\nu_2$  is the Poisson's ratio of Sphere 2, dimensionless;  $E_1$  is the elastic modulus of Sphere 1, in MPa;  $E_2$  is the Poisson's ratio of Sphere 2, in MPa.

Wu et al. (2001) proved that the change in distance between the centers of Spheres 1 and 2 can be described by the following relationship:

$$\alpha' = \frac{\frac{3}{4} P C_E}{\left( \frac{3}{4} P C_E \frac{R_1 R_2}{R_1 + R_2} \right)^{\frac{1}{3}}}, \quad 3$$

where  $\alpha'$  is the change in distance between the centers of Spheres, in mm.

When  $R_2 \rightarrow \infty$ , the surface of Sphere 2 tends to be a plane, and the two objects become mutually squeezed sphere and plane. Fig. 2 demonstrates the interaction between the sphere and the plane. One can observe that  $\alpha'$  is affected by two factors: change in radius of Sphere 1 (which is called *deformation*) and embedment.

For the situation in Fig. 2, a modified formula can be derived on the basis of Eq. 3:

$$\alpha' = \frac{2(\frac{3}{8}PC_E D_1)^{\frac{2}{3}}}{D_1}, \quad 4$$

where  $D_1$  is the diameter of Sphere 1, in mm. At this step, Eq. 4 contains the two factors; these are the change in radius of Sphere 1 (deformation) and the embedment. To derive the deformation of Sphere 1 ( $\beta$ ) from Eq. 4, the other influencing factor (which is the embedment) must be separated. Therefore, assume that the elastic modulus of the plate ( $E_2$ ) is infinite, so Sphere 1 does not embed into the plate. In this case  $\alpha'$  is affected only by deformation. Thus, the formula of deformation of Sphere 1 ( $\beta$ ) can be expressed as a following relationship:

$$\beta = \frac{2(\frac{3}{8}PD_1^{\frac{1-\nu_1^2}{E_1}})^{\frac{2}{3}}}{D_1}. \quad 5$$

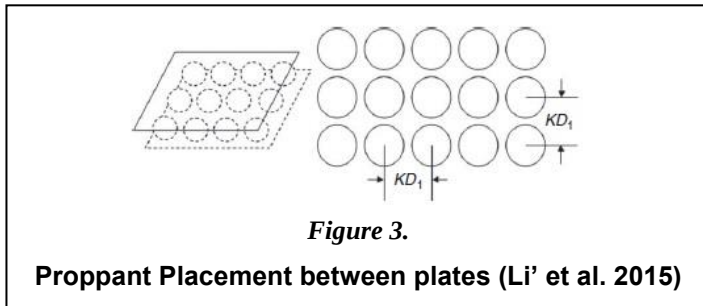
From Eq. 5, it can be stated that deformation is influenced only by the values of external force, the diameter, elastic modulus, and Poisson's ratio of Sphere 1, but is not affected by any parameters of the plate.

Since  $\alpha'$  was interpreted as the sum of the embedment and the deformation, combining Eq. 4 and 5, the formula of the embedment of Sphere 1 can be expressed as:

$$h = \alpha' - \beta = \frac{2(\frac{3}{8}PD_1)^{\frac{2}{3}}}{D_1} \left[ \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{\frac{2}{3}} - \left( \frac{1-\nu_1^2}{E_1} \right)^{\frac{2}{3}} \right], \quad (6)$$

where  $h$  is the value of embedment, in mm.

### Real Situation



To approximate the real situation of the fracture, the situation of spheres being squeezed by two horizontal plates must be described. The analytical model (Li' et al. 2015) assumes that the weight of spheres can be ignored, and the stress on spheres and contact areas of the upper and lower plate are the

same. To create the model more realistic, Sphere 1 is equivalent to the proppant and the

formation is equivalent to the upper and lower plates. It is also assumed that the proppants are evenly paved on the formation (monolayer pattern). Li' et al (2015) introduced a so-called distance coefficient  $K$  which helps to represent the distance between the centers of two adjacent proppants as:  $KD_1$ .  $K$  is equal to unity when proppants are touched by each other, and greater than unity if not. Fig. 3 represents the placement of proppants (single layer) in the fracture. In this paper, the distance coefficient  $K$  is disregarded because of simplification and since it does not represent the partial monolayer properties in connection with fracture conductivity.

Fig. 4 shows the stress analysis for a single proppant. Comparing Fig. 4 and 2, one can see that the unit of stress is MPa in Fig. 4, but the unit of load is N in Fig. 2. The forces between the plates and proppants are acting on the contact areas and the following relationship between the load and stress can be derived:

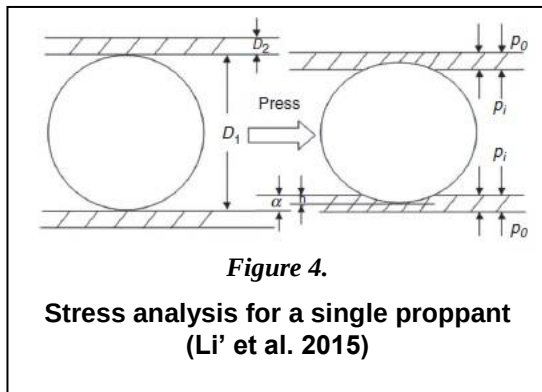
$$P = p(KD_1)^2, \quad (7)$$

where  $P$  is the load acting on the contact areas, in N;  $p$  is closure pressure, in MPa,  $K$  is distance coefficient, dimensionless; and  $D_1$  is the diameter of proppant, in mm.

Furthermore, a relationship can be derived for closure pressure which is the difference of overburden pressure and fracturing fluid pressure:

$$p = p_0 - p_i, \quad (8)$$

where  $p_0$  is overburden pressure, in MPa; and  $p_i$  is fluid pressure of fracturing fluid, in MPa. It should be noted when  $p_0 < p_i$ , which happens during the fracturing process (the pressure of



of the given rock, fracture aperture is initiated and fracture starts to propagate), the proppants neither deform nor embed into the formation. When  $p_0 > p_i$ , which always happens at the end of the fracturing treatment (pumps are switched off and due to this, there is no more extra pressure energy which would exceed the formation's breakdown pressure, so the fracking fluid leaks off and the pressure drops), the proppants deform and embed. Equations obtained in the followings

can be used only when  $p_0 > p_i$ , in the other case the equations have no sense.

The thicknesses of the formation in upper and lower case are the same and denoted as  $D_2$ . The formation deforms under closure pressure and the deformation can be calculated according to the Hooke's law:

$$\Delta D_2 = D_2 \frac{p}{E_2}, \quad (9)$$

where  $D_2$  is the thickness of the formation, in mm; and  $\Delta D_2$  is the deformation of the formation, in mm. The deformation of the formation is reflected in the form of proppant embedment, because the thickness of the formation that does not contact with proppants is returned to the initial value. Substituting Eq. 7 and 9 into Eq. 4, one can obtain the following equation:

$$\alpha = 1.04D_1(K^2pC_E)^{\frac{2}{3}} + D_2 \frac{p}{E_2}, \quad (10)$$

where  $\alpha$  is the change in fracture aperture, in mm. Li' et al. (2015) noted that actually the value of  $\alpha$  is half of the change in fracture aperture, but for convenience it is referred to as change in fracture aperture. Later, in fracture conductivity calculations,  $\alpha$  is considered to be half of the change in fracture aperture of course.

Continuing the substitution (Eq. 7 into Eq. 5 and Eq. 7 and 9 into Eq. 6), one can obtain the following equations:

$$\beta = 1.04D_1(K^2p \frac{1-\nu_1^2}{E_1})^{\frac{2}{3}}, \quad 11$$

$$h = 1.04D_1(K^2p)^{\frac{2}{3}} \left[ \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{\frac{2}{3}} - \left( \frac{1-\nu_1^2}{E_1} \right)^{\frac{2}{3}} \right] + D_2 \frac{p}{E_2}, \quad 12$$

where  $\beta$  is the proppant deformation, in mm;  $h$  is the proppant embedment, in mm;  $E_1$  is the elastic modulus of proppant, in MPa; and  $E_2$  is the elastic modulus of the formation, in MPa.

## MULTILAYER PATTERN

### Interaction of Proppants

In the previous chapters, single-layer pattern was assumed to calculate proppant embedment, deformation and the change in fracture aperture. In practice, instead of monolayer pattern, multilayer pattern is created more often. During general fracturing treatments the proppants squeeze mutually in addition to the interaction between proppant and formation, therefore it is necessary to analyze the deformation and embedment between proppants as well.

By derivation of equations which can be used to calculate proppant embedment and deformation, Li' et al. (2015) assumed that the radius of Sphere 1 and 2 are equal to each other which is a good approximation because proppants produced by manufacturers have almost the same diameter. As it was mentioned before, the radii of Sphere 1 and 2 are the same, so  $R_1=R_2=R$ . In this case, Eq. 3 can be reduced to:

$$\alpha' = \frac{2(\frac{3}{8}PC_E R)^{\frac{2}{3}}}{R}, \quad 13$$

where  $\alpha'$  is the change in distance between the centers of Sphere 1 and 2, in mm; and  $R$  is the radius of Sphere 1 and 2, in mm. The deformation of Sphere 1 and 2 is expressed as:

$$\beta_1 = \frac{(\frac{3}{8}PR)^{\frac{2}{3}}(2\frac{1-\nu_1^2}{E_1})^{\frac{2}{3}}}{R}, \quad 14$$

$$\beta_2 = \frac{(\frac{3}{8}PR)^{\frac{2}{3}}(2\frac{1-\nu_2^2}{E_2})^{\frac{2}{3}}}{R}, \quad 15$$

where  $\beta_1$  and  $\beta_2$  are the deformations of Sphere 1 and 2, in mm. As deriving the embedment in case of Sphere and plate (deformation was separated from the change in fracture aperture), the embedment of the Spheres is calculated by combining Eq. 13, 14, and 15.

$$h = \alpha' - \beta_1 - \beta_2 = \frac{(\frac{3}{8}PR)^{\frac{2}{3}}}{R} \left[ 2\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{\frac{2}{3}} - \left(2\frac{1-\nu_1^2}{E_1}\right)^{\frac{2}{3}} - \left(2\frac{1-\nu_2^2}{E_2}\right)^{\frac{2}{3}} \right], \quad 16$$

where  $h$  is the embedment between Sphere 1 and 2, in mm.

Fig. 5 and Fig. 6 represent the results gained by using Eq. 13, 14, 15, and 16. The known conditions were taken to be  $\nu_1=\nu_2=0.2$ ,  $E_2=1500$  MPa,  $R_1=R_2=R=0.5$  mm,  $P=60$  N. Fig. 5 shows how the change between the centers of Spheres decreases as the elastic moduli of Sphere 1 increases, and how the deformation of Sphere 1 and 2 changes in function of  $E_1$ . One can observe that the change in distance between the centers of Spheres and deformation of Sphere 1 decreases with increasing elastic modulus of Sphere 1. However, deformation of Sphere 2 is constant because it is not dependent of elastic modulus of Sphere 1.

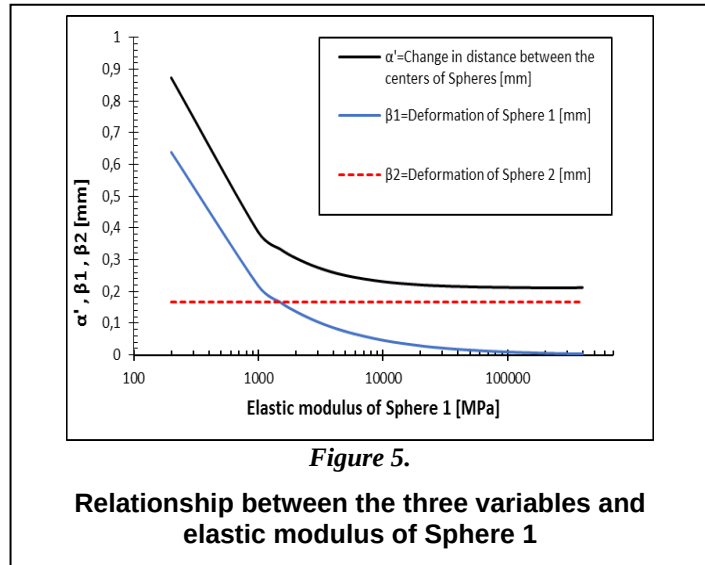
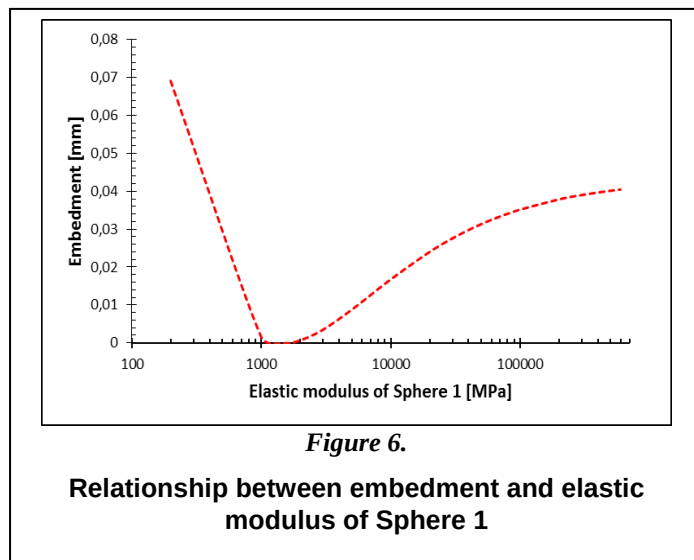
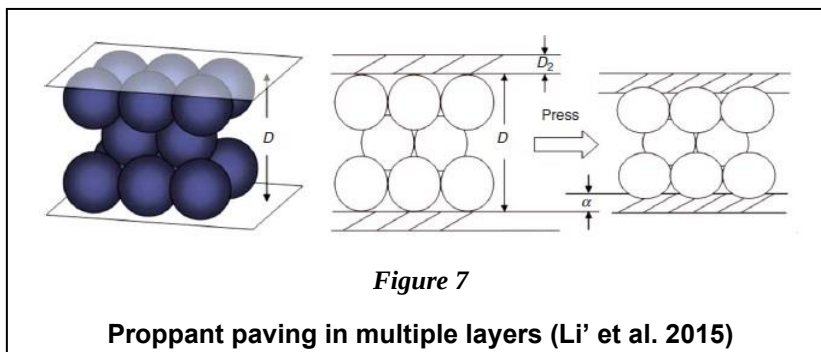


Fig. 6 demonstrates that the embedment (Sphere 2 embedding into Sphere 1) decreases as the elastic modulus of Sphere 1 increases until the elastic modulus of Sphere 2 is greater than elastic modulus of Sphere 1. When  $E_1=E_2$  the embedment is equal to zero and the contact area of the mutually squeezed spheres is a plane. When  $E_1$  gets greater than  $E_2$  the embedment (Sphere 1 embedding into Sphere 2) increases but not to as much value as before (since one of the elastic modulus is greater than earlier). Fig. 5 and 6 definitely demonstrates that the main cause of change in distance between the centers of spheres is the deformation and not the embedment.



### Embedment of Multilayer Pattern

Until this section, one could obtain the interaction between proppant and formation which was the single-layer situation, and the interaction between proppants as well. As it was already emphasized, the proppants are mostly paved in multiple layers in practice; therefore it is necessary to analyze the proppant embedment in case of multilayer pattern.



As it was shown in Fig. 6, there is no proppant embedment between proppants when the elastic modulus and the size of proppants are the same. Therefore, only proppant

deformation and embedment into the formation must be considered. In case of multilayer pattern, the deformation can be approximated as the value of superposition of each layer's proppant deformation. At the same time, the proppant embedment (into the formation and not into each other) is the same as in the case of a single-layer pattern because only the top and bottom layer is in direct contact with the formation. Based on the previous assumptions and the single layer pattern model, the corresponding equations can be expressed as follows:

$$\alpha = 1.04D(K^2p)^{\frac{2}{3}} \left\{ \left( \frac{1-\nu_1^2}{E_1} \right)^{\frac{2}{3}} + \frac{D_1}{D} \left[ \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{\frac{2}{3}} - \left( \frac{1-\nu_1^2}{E_1} \right)^{\frac{2}{3}} \right] \right\} + D_2 \frac{p}{E_2}, \quad 17$$

$$\beta = 1.04D(K^2p)^{\frac{1-\nu_1^2}{E_1}}^{\frac{2}{3}}, \quad 18$$

$$h = 1.04D_1(K^2p)^{\frac{2}{3}} \left[ \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{\frac{2}{3}} - \left( \frac{1-\nu_1^2}{E_1} \right)^{\frac{2}{3}} \right] + D_2 \frac{p}{E_2}, \quad 19$$

where  $D$  is the initial fracture aperture (as shown in Fig 7), in mm;  $\alpha$  is the change in fracture aperture of multilayer pattern, in mm;  $\beta$  is the proppant deformation of multilayer pattern, in mm; and  $h$  is the proppant embedment of multilayer pattern, in mm.

Li' et al. (2015) applied the equations 17, 18 and 19 to calculate proppant embedment in condition in which Lacy et al. (1998) conducted experiments in connection with proppant embedment. The results were compared to each other and they found that the results got by using the analytical model were close to the experimental results. However, the analytical model resulted smaller embedment values, and with increasing closure pressure the difference between the results gained by the two ways becomes greater. The reasons for the smaller values may be that the analytical model was derived from single-layer pattern with ideal assumptions, for instance even paving, uniform size distribution, perfect sphericity of proppant, homogeneous fracture surface were assumed. Inhomogeneous fracture surface and uneven paving might result in unequal stresses on each proppant which could mean uneven embedment. In addition, the analytical model was derived essentially on the basis of the linear elastic model. Therefore, conditions should be in the material linear elastic region in connection with stress, strain and deformation. It means that the model does not consider proppant crush at all, that is why the model is more accurate in low closure stress region. Li' et al. (2015) pointed out these values, they suggested closure pressure under 100 MPa and elastic modulus of proppant above 10 000 MPa to use this model. Considering this inaccuracy, fitting parameters were introduced in Eq. 19:

$$h = b_0 + b_1 \left\{ 1.04D_1(K^2p)^{\frac{2}{3}} \left[ \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{\frac{2}{3}} - \left( \frac{1-\nu_1^2}{E_1} \right)^{\frac{2}{3}} \right] + D_2 \frac{p}{E_2} \right\}. \quad 20$$

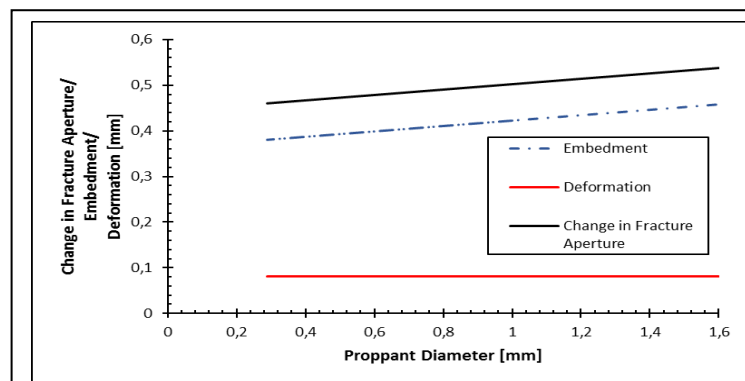
The fitting coefficients  $b_0$  and  $b_1$  can be determined by fitting with experimental data. This new equation was referred to as a revised new model (Li' et al. 2015). By introducing the fitting coefficient, experimental measurement is required to be able to use it, so the revised new model is a semi-empirical one. It was emphasized in the section of intuition of this research that the benefit of the analytical model is that one can use it on a wide range (actually for all cases), therefore the semi-empirical interest is out of this paper.

## Multilayer Pattern Results

In the followings, two examples are introduced to demonstrate what the earlier derived equations (Eq. 17, 18, 19) resulted.

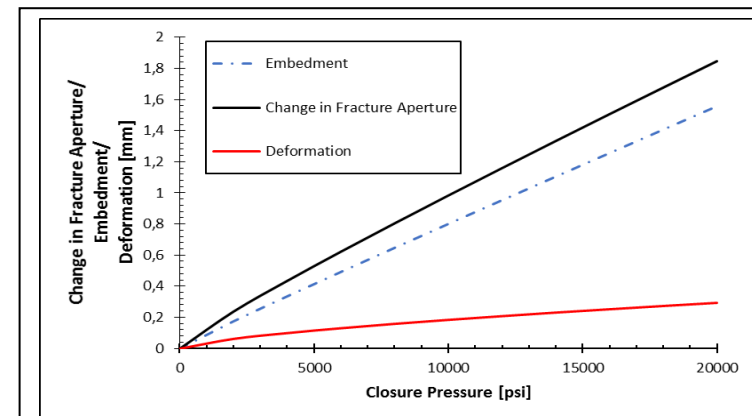
Fig. 8 demonstrates how the proppant embedment, deformation, and the change in fracture aperture changes with increasing proppant size. The known parameters were taken to be  $\nu_1=\nu_2=0.2$ ,  $E_1= 10000$  MPa,  $E_2= 1100$  MPa,  $p= 20$  MPa,  $D= 5$ mm,  $D_2=20$  mm and the proppant diameter ranged from 0.2 to 1.6 mm. Substituting these values into Eq. 17, 18, and 19, one can observe that deformation is a constant line while proppant embedment and the change in fracture aperture increases as the proppant diameter increases. So, the deformation is not affected by the proppant diameter, and as it was expected from the equations, the change in fracture aperture is the sum of proppant embedment and deformation.

Fig. 9 demonstrates how the proppant embedment, deformation, and the change in fracture aperture changes with closure pressure. The known parameters were taken to be  $\nu_1=\nu_2=0.2$ ,  $E_1= 10000$  MPa,  $E_2= 2000$  MPa,  $D= 5$ mm,  $D_1=1.328$  mm (12/18 mesh size),  $D_2=20$  mm and the closure pressure ranged from 0 to 20 000 psi. Substituting these values into Eq. 17, 18, and 19, one can observe that each three variable increases as the closure pressure increases, as it was expected. In addition, since the elastic modulus of proppant is relatively high compared to the elastic modulus of formation which is relatively low, the main cause of the change in fracture aperture is proppant embedment rather than proppant deformation.



**Figure 8.**

**Relationship between the three variables and proppant diameter**



**Figure 9**

**Relationship between the three variables and closure pressure**

## FRACTURE CONDUCTIVITY

### Derivation

The main interest of this research is the influence of several factors on fracture conductivity which is the essence of hydraulic fracturing. The fracture conductivity is the product of the permeability and the fracture width. The permeability of a propped fracture is related with its porosity, radius of pore throat, and pore tortuosity. These parameters are influenced by the factors (proppant embedment, proppant deformation, and the sum of these: the change in fracture aperture) presented in the previous chapters.

Yang and Wei (2004) derived a commonly used formula for permeability calculation and it was expressed as:

$$k = \frac{\Phi r^2}{8\tau^2}, \quad (21)$$

where  $k$  is permeability, in  $\mu\text{m}^2$  (equal to 1 Darcy);  $\Phi$  is the porosity, dimensionless;  $r$  is the radius of pore throat, in  $\mu\text{m}$ ; and  $\tau$  is the pore tortuosity, dimensionless.

For the proppant pack shown in Fig. 7, ignoring the pore-distribution differences in the layers, the equations of porosity, radius of pore throat, and pore tortuosity can be easily derived using only mathematical considerations for the case when closure pressure is equal to zero. The equations are:

$$\phi_0 = \frac{4\sqrt{2}R_1^3 - \frac{4}{3}\pi R_1^3}{4\sqrt{2}R_1^3} = 0,25951, \quad (22)$$

$$r_0 = 1000 \frac{2\sqrt{3}-3}{3} R_1, \quad (23)$$

$$\tau_0 = \frac{\sqrt{\left(\frac{2}{3}R_1\right)^2 + \left(\frac{\sqrt{2}}{3}R_1\right)^2}}{\frac{2}{3}R_1} = \frac{\sqrt{6}}{2}, \quad (24)$$

where  $\phi_0$  is the porosity of the proppant pack when closure pressure is equal to zero, dimensionless;  $r_0$  is the radius of pore throat when closure pressure is equal to zero, in  $\mu\text{m}$ ;  $\tau_0$  is the pore tortuosity when closure pressure is equal to zero, dimensionless;  $R_1$  is the radius of proppant, in mm.

When closure pressure is greater than zero, the porosity, radius of pore throat, and pore tortuosity approximately satisfy the following relationships (Li' et al. 2015):

$$\phi = \frac{D\phi_0 - 2\beta}{D - 2\beta}, \quad (25)$$

$$r = \left(\frac{D - 2\beta}{D}\right) r_0, \quad (26)$$

$$\tau = \sqrt{\left(\frac{D - 2\beta}{D}\right)^2 (\tau_0^2 - 1) + 1}, \quad (27)$$

where  $\beta$  is the proppant deformation calculated by Eq. 18, in mm, and  $D$  is the initial fracture aperture, in mm.

According to the concept of fracture conductivity and combining Eq. 21, 25, 26, and 27 the following equation can be obtained:

$$F_{CD} = kw = \frac{(D\phi_0 - 2\beta)(D - 2\beta)r_0^2}{80\left[\left(\frac{D - 2\beta}{D}\right)^2 (\tau_0^2 - 1) + 1\right]D^2} (D - 2\alpha), \quad (28)$$

where  $F_{CD}$  is the fracture conductivity, in  $\mu\text{m} \cdot \text{cm}$ ;  $w$  is the fracture aperture under closure pressure, in cm; and  $\alpha$  is the half of the change in fracture aperture calculated by Eq. 17, in mm.

## SENSITIVITY TEST

In this chapter, the most important sensitivity tests are presented in connection with fracture conductivity and analyzing these tests relevant consequences can be drawn. The main influencing factors which have impact on fracture conductivity are the closure pressure, proppant size, elastic modulus of proppant and formation, Poisson's ratio of proppant and formation, initial fracture aperture. As it was mentioned before, the analytical model does not consider proppant crush and formation compatibility, and some additional assumptions were also introduced by derivation of the mathematical equations.

In addition, one should note that the conditions were taken to conduct sensitivity tests are theoretical ones (generated by the author), but at the same time these can be fitted in real situation as well.

### Effect of Proppant Size

Fig. 10 shows the results according to the parameters:  $\nu_1=\nu_2=0.2$ ,  $E_2=2000$  MPa,  $D=5$  mm,  $D_2=20$  mm,  $p=20$  MPa,  $E_1$  ranged from 2000 to 100 000 MPa, and the proppant diameter ranged from 0.3 to 1.6 mm.

Investigating Fig. 14, one can observe that greater proppant diameter results greater fracture conductivity. However, in the previous examples it was shown that increasing proppant size resulted greater embedment, deformation and change in fracture aperture. This is the result of proppant permeability, since greater proppant size means higher permeability. In Eq. 28 one can see that the fracture conductivity depends on the square of pore throat calculated at zero closure pressure, and the value of pore throat depends on the proppant diameter directly, so the proppant size modifies the permeability much better than the proppant embedment and deformation. The dependence of the fracture conductivity on elastic modulus of proppant can be also observed. The greater the elastic modulus of proppant the higher the value of fracture conductivity, since the proppant deformation is less effective in this case.

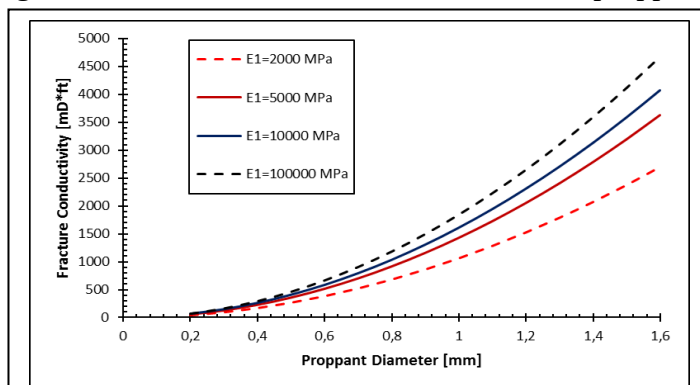


Figure 10.

Fracture conductivity as a function of proppant diameter and elastic modulus of proppant

### Effect of Closure Pressure

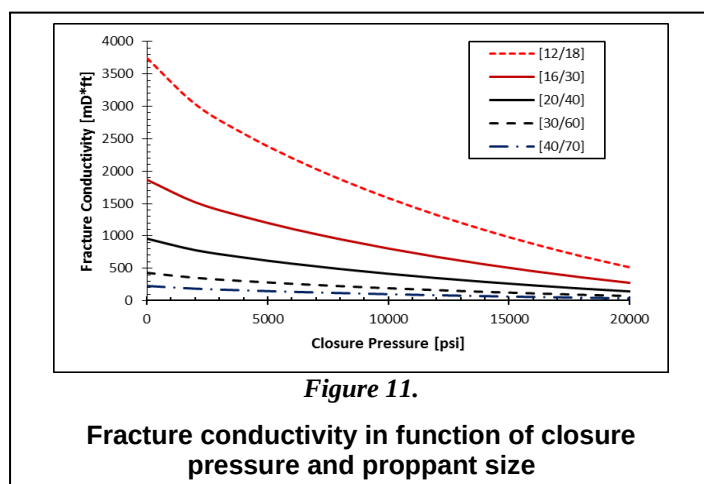


Figure 11.

Fracture conductivity in function of closure pressure and proppant size

The conditions demonstrated in Fig. 11 were taken to be  $\nu_1=\nu_2=0.2$ ,  $E_1=10\,000$  MPa,  $E_2=2000$  MPa,  $D=5$  mm,  $D_2=20$  mm, proppant size ranged from 12/18 mesh to 40/70 mesh and  $p$  ranged from 0 to 20 000 MPa. Fracture conductivity decreases when closure pressure increases and it can be also observed that with greater proppant size much higher fracture conductivity values can be achieved. However, when closure pressure increases the

differences between fracture conductivity values related to different proppant sizes are much less.

### Initial Fracture Aperture

For Fig. 12 the same conditions were taken to be as in the previous example. This test resulted in a very important conclusion; the fracture conductivity is highly dependent on the initial fracture aperture which is actually the initial fracture width. This conclusion is a key parameter in proppant selection discussed in more detail later.

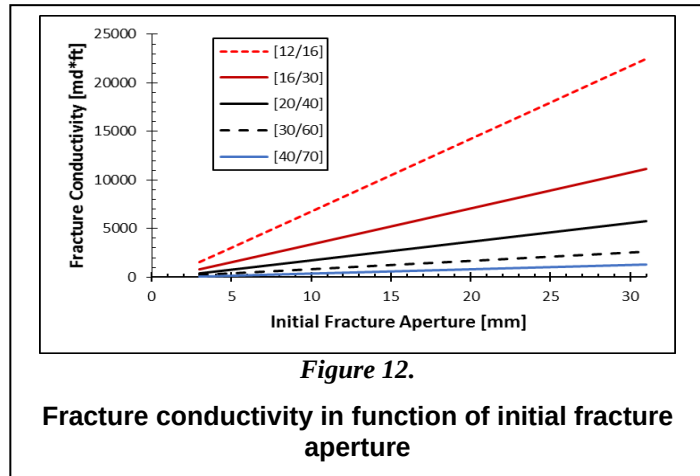


Figure 12.

Fracture conductivity in function of initial fracture aperture

### PROPPANT SELECTION

The purpose of this research was to develop an analysis system by which a deep investigation of proppant embedment can be performed, and based on this analysis system, a technical optimization of proppant selection can be established. The analytical model (Li' et al. 2015) was described and examined in the previous chapters provides a deep understanding of proppant embedment. Combining the model with some other mathematical consideration and the Unified Fracture Design (Economides et al. 2002), a new method was developed for proppant selection.

### Determining Initial Fracture Width

The analytical model introduced before, requires several input parameters to calculate the actual proppant embedment, deformation, change in fracture aperture and finally the fracture conductivity which is one of the most important parameters in connection with hydraulic fracturing.

Most of the parameters can be easily determined or approximated by some assumptions, but the initial fracture aperture is a critical value as shown in Fig. 20, and it must be calculated precisely. To solve this problem, the relationship between the porosity and proppant concentration was used which can be expressed as (Rivers et al. 2012):

$$\Phi = 1 - \frac{C_p}{w\rho_p} \quad , \quad 29$$

where  $\Phi$  is the porosity, dimensionless;  $C_p$  is the proppant areal concentration, in lb/ft<sup>2</sup>,  $w$  is the fracture width at zero closure pressure, in ft; and  $\rho_p$  is the proppant density, in lb/ft<sup>3</sup>. Rearranged the Eq. 29 one can obtain the following expression for initial fracture width calculation:

$$w = \frac{C_p}{(1-\Phi)\rho_p} \quad . \quad 30$$

Proppant pack porosity at zero closure pressure can be easily calculated by Eq. 22 (or it can be measured as well), proppant density is always given by the manufacturers, and the proppant concentration is the input parameter which also can be determined by UFD model.

### Unified Fracture Design

Valkó and Economides (2002) introduced a physical optimization method to maximize the productivity index of a hydraulically fractured well. It was referred to Unified Fracture

Design (UFD, Economides et al. 2002). They introduced the concept of the dimensionless Proppant Number,  $N_{prop}$ , given by:

$$N_{prop} = \frac{2k_f V_p}{k V_r}, \quad 31$$

where  $k_f$  is the proppant pack permeability, in mD;  $V_p$  is the volume of proppant in the pay (the total volume injected times the ratio of the net height to the fracture height), in  $m^3$ ;  $k$  is the reservoir permeability, in mD; and  $V_r$  is the reservoir drainage volume, in  $m^3$ . The concept of the dimensionless Proppant Number is the appropriate way to express the relative size of a given treatment, and it can be defined as the ratio of the propped volume of the fracture in the pay to the reservoir drainage volume, weighted by twice the related permeability ratio. It was established that for a given value of  $N_{prop}$  there is an optimal dimensionless fracture conductivity at which the productivity index is maximized.

The dimensionless fracture conductivity ( $C_{fD}$ ) can be interpreted as the ratio of proppant pack permeability times the fracture width to the reservoir permeability times the fracture length, and it can be expressed as:

$$C_{fD} = \frac{k_f w}{k X_f}. \quad 32$$

Valkó and Economides also presented correlations for the maximum achievable dimensionless productivity index as a function of the dimensionless Proppant Number. Similarly, correlations were presented for the optimal dimensionless fracture conductivity for the entire range of Proppant Numbers. These correlations are expressed as follows:

$$C_{fD,opt}(N_{prop}) = \begin{cases} 1.6 & \text{if } N_{prop} < 0.1 \\ 1.6 + \exp\left(\frac{-0.583 + 1.48 \ln(N_{prop})}{1 + 0.142 \ln(N_{prop})}\right) & \text{if } 0.1 \leq N_{prop} \leq 10 \\ N_{prop} & \text{if } N_{prop} > 10 \end{cases} \quad 33$$

$$I_{Dmax}(N_{prop}) = \begin{cases} \frac{1}{0.990 - 0.5 \ln(N_{prop})}, & N_{prop} \leq 0.1 \\ \frac{6}{\pi} - \exp\left[\frac{0.423 - 0.311 N_{prop} - 0.089 (N_{prop})^2}{1 + 0.667 N_{prop} + 0.015 (N_{prop})^2}\right], & N_{prop} > 0.1 \end{cases} \quad 34$$

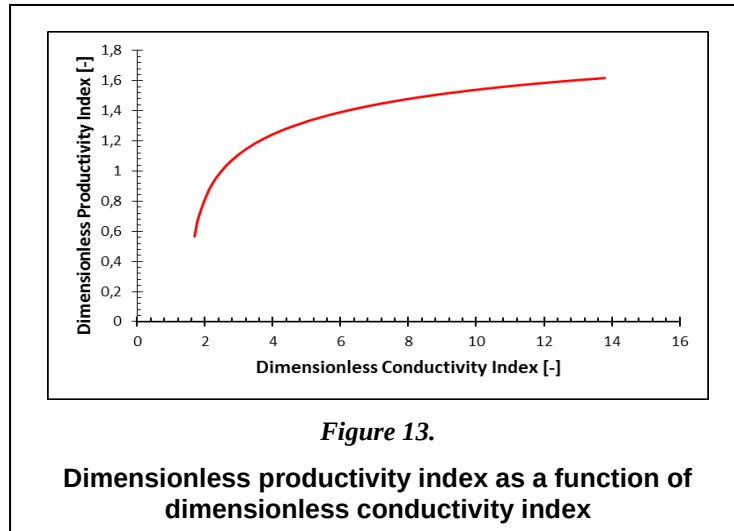
One should note that the absolute maximum dimensionless productivity index is  $J_D = 6/\pi = 1.909$  which is the productivity index for a perfect linear flow in a square reservoir.

### Method for Proppant Selection

Combining the UFD model with the previous mathematical considerations and the analytical model which was chosen to investigate proppant embedment, a simple way was developed to select the proper proppant size which could provide the sufficient fracture conductivity to achieve the predetermined productivity index.

The input parameters of the analytical model are: Poisson's ratios of proppant and formation (measured or estimated), elastic modulus of proppant and formation (measured or estimated), proppant size (manufacturer data determined by sieve analysis), closure pressure (can be calculated and measured as well), layer thickness (estimated value), initial fracture aperture which can be calculated as a function of proppant concentration (Eq. 30).

The input parameters of the UFD model are: injected proppant mass, formation height, reservoir drainage area, reservoir permeability. These are the main parameters required to compute a simplified productivity index. To optimize the treatment and calculate a more accurate productivity index (considering turbulence effect and other skin factors) more complex models were published (Economides et al. 2007, Lengyel and Pusztai 2014), however, it is not necessarily required to demonstrate the results of this research.



The main input parameter of the combined method for proppant selection is the dimensionless productivity index which is predetermined by NPV calculations and what is required to be achieved by the stimulation treatment. The operation of this proppant selection algorithm is the following: several relative treatment sizes are generated which means different dimensionless Proppant Numbers ( $N_{prop}$ ) gained by generating injected proppant masses.

According to this process, dimensionless productivity index – dimensionless fracture conductivity curve is performed, as shown in Fig. 13. According to this curve, the dimensionless conductivity index can be determined from which the required fracture conductivity can be calculated (Eq. 32) using the optimal fracture length determined by the UFD model as well. This conductivity value can be compared to the value computed by the analytical model, and the decision can be made whether the selected proppant (which was the input parameter of the proppant embedment – fracture conductivity analysis) is sufficient or not.

## FIELD EXAMPLE

### Fracture Design Example

Service companies perform hydraulic fracturing designs with fracturing software. This kind of programs are very sophisticated ones, and they provide with high accuracy, since a lot of experience are integrated to develop them. A comparison was made between data from a fracturing design report and calculated values provided the analytical model.

A fracturing treatment was designed with 16/30 sintered bauxite proppant with a mean diameter of 1035 microns. The formation was considered to be a well consolidated eocen sandstone and conglomerate with an elastic modulus of 10 000 MPa. The Young's modulus of proppant was equal to 7 270 000 psi which is approximately 49 500 MPa. The specific gravity of the proppant was 3.1. Poisson's ratios were not given, but 0.24 could be a good approximation for that.

This kind of software simulates the whole process step by step, for the given pumping rates and proppant concentrations, it computes the actual fracture width, conductivity etc. An average proppant concentration value of 10.8 kg/m<sup>2</sup> was calculated by the software, and the average propped fracture width was found as 5.9 mm (the same fracture width value was got from the Eq. 30). The simulation resulted 1312 mD\*m fracture conductivity.

Substituting the input parameters into the analysis system, the fracture conductivity was found as 1252 mD\*m. The difference is only 60 mD\*m which is more than sufficient.

## CONCLUSION

The analytical model, which was described in this paper, can determine the proppant embedment, proppant deformation, and the sum of these: the change in fracture aperture. As a final result of the model, the fracture conductivity can be also computed. The model is based on mathematical and physical considerations, and it can be used for all cases. The model has some limitations according to the assumptions, one substantial assumption is that only elastic behavior of proppant is considered, therefore, it cannot be used when proppant crush happens.

Using the analytical model, sensitivity tests could be made to investigate the influence of factors. One can conclude from these tests that proppant embedment and proppant deformation is highly influenced by the closure pressure. The impact of elastic modulus of proppant and formation was also tested, and the results showed that the Young's modulus of proppant has a much greater effect on the change in fracture aperture than the Young's modulus of the formation.

Tests were also conducted to investigate the influencing factors of the fracture conductivity. As a conclusion, it can be noted that the fracture conductivity increases with proppant size, initial fracture aperture, and elastic modulus of formation and elastic modulus of proppant and it decreases with the increase in closure pressure.

Combined this model with the Unified Fracture Design model (Economides et al. 2002), a proppant selection method was performed in connection with well productivity. For a given productivity index of well, one can choose the proppant which satisfy the requirements in connection with the predetermined productivity index.

## SUMMARY

Considering well stimulations, the task of petroleum engineers is improving well productivity. In case of hydraulic fracturing, one of the most important parameters is the fracture conductivity which determines directly the productivity of the well.

The fracture conductivity is highly influenced by factors such as elastic modulus of proppant, elastic modulus of formation, proppant diameter, closure pressure etc. An analytical model was chosen to gain deep understanding in this topic. As a result of the model, the proppant embedment, proppant deformation, change in fracture aperture, and fracture conductivity can be calculated.

Several sensitivity tests were carried out in this paper to investigate influencing parameters in connection with fracture conductivity. Analyzing the results of the tests a number of conclusion could be drawn, which made possible to perform an analysis system for proppant selection.

The analytical model was combined with other consideration and with the optimization method referred to Unified Fracture Design. The combination makes one able to optimize the proppant selection technically in function of the productivity index.

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