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A Novel Method for Optimal Perforation Design

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Abstract

In case of hydrocarbon production optimization it is essential to know the relationship between the inflow rate and the performance of a perforated production well. It describes the flow rate at the bottom of the well in function of the applied drawdown pressure. The process is strongly influenced by the perforation parameters, formation characteristics and the properties of produced reservoir fluids. The correct calculation gives optimum perforation design and optimized fluid production.

In a cased and perforated well, the reservoir fluids flow through the perforation channels before entering into the well. Thus the parameters of the perforations like penetration depth, phase angle, length of perforated interval, entry hole diameter, average radius of perforation channels and the radius of the crushed zone, etc. have significant impact on the relationship between the flow rate and the well performance. The very first examinations on this phenomenon dates back to the mid 70's. Several equations have been developed to describe the pressure drop through the perforation. However, numerous assumptions have been applied in the derivations of these equations, which have resulted significant loss in accuracy. A new calculating method has been derived and presented in this paper by applying the most accepted methods, i.e. the equation of McLeod and the one of Tariq and Karakas.

The new complex method takes all of the perforation parameters and their effects into account to provide more accurate results. The effect of different perforation parameters on the productivity of oil and gas producing wells are also demonstrated.

As this method takes all the effects of perforation parameters into consideration, different perforation designs can easily be compared and optimized. The great advantage of this method is that it points out the difference between effects regarding the inflow performance of oil and gas wells.

Introduction

To calculate the pressure drop through the perforation channels several methods have been published since the seventies. The most widely used methods were proposed by McLeod (McLeod, O.H. Jr. 1983) and by Karakas and Tariq (Karakas, M., and Tariq, S. M. 1988). However, numerous assumptions have been applied in the derivations of these equations, which have resulted significant loss in accuracy.

McLeod assumed that perforations are small wells in the reservoir and used the Jones method to calculate the pressure drop across them. This method does not take the effect of perforation angle into consideration in the calculations. Karakas and Tariq presented a semi-analytical solution for the skin

calculation in perforated completions. They have quantified the wellbore and vertical-flow effects by accurate finite-element simulations. They have showed that the skin effect which is caused by perforations (s_p) is a combination of four different effects as follows: horizontal skin (s_H), vertical skin (s_V), wellbore skin (s_{wb}), and crushed zone-effects (s_c). This method includes all the perforation parameters, but not all the effects that can be caused by them. If turbulent flow forms it can cause significant pressure drop e.g. in case of gas flow. This additional pressure drop is not considered by the method of Karakas and Tariq.

We have developed a method which includes all the perforation parameters and considers all the effects caused by them. This novel method is based on the previously mentioned equations with some modifications which were applied mostly because of geometrical considerations. Its application provides an easy process for perforation design optimization to achieve sufficient inflow performance.

Statement of theory and definitions

We developed the method in two steps:

- Combination of the method of McLeod and the method of Karakas and Tariq and implication of the resulted equation into the inflow performance relationship (IPR) of Jones.
- Modification of the perforation tunnel radius and the crushed zone radius based on the perforation geometry and the anisotropy of the reservoir rock.

With the final equation the perforation design can be optimized with the use of the data from the core analysis and the well tests.

Development of the main body

The equations of McLeod:

For oil wells:

$$p_{wfs} - p_{wf} = a \left(\frac{q}{n_s \times h_p} \right)^2 + b \left(\frac{q}{n_s \times h_p} \right) \quad (1)$$

$$a = \frac{2.30 \times 10^{-14} \times \beta B_o^2 \rho \left(\frac{1}{r_p} - \frac{1}{r_c} \right)}{L_p^2} \quad (2)$$

$$b = \frac{\mu_o B_o \left[\ln \frac{r_c}{r_p} \right]}{7.08 \times 10^{-3} \times L_p k_p} \quad (3)$$

For gas wells:

$$p_{wfs}^2 - p_{wf}^2 = a \left(\frac{q}{n_s \times h_p} \right)^2 + b \left(\frac{q}{n_s \times h_p} \right) \quad (4)$$

$$a = \frac{3.16 \times 10^{-12} \times \beta \gamma_g T z \left(\frac{1}{r_p} - \frac{1}{r_c} \right)}{L_p^2} \quad (5)$$

$$b = \frac{1.424 \times 10^3 \mu_g T z \left[\ln \frac{r_c}{r_p} \right]}{L_p k_p} \quad (6)$$

The equations of Karakas and Tariq:

$$s_p = s_H + s_V + s_{wb} + s_c \quad (7)$$

$$s_H = \ln \left(\frac{r_w}{r_{we}} \right) \quad (8)$$

$$s_{wb}(\theta) = c_1(\theta)e^{c_2(\theta)r_{wD}} \quad (9)$$

$$s_V = 10^a h_D^{b-1} r_{pD}^b \quad (10)$$

$$s_c = \frac{\delta_p}{L_p} \left(\frac{k}{k_p} - 1 \right) \ln \left(\frac{r_c}{r_p} \right) \quad (11)$$

Table 1—Constants used by Karakas and Tariq

Θ	α_Θ	c_1	c_2	a_1	a_2	b_1	b_2
0° (360°)	0.250	1.6×10^{-1}	2.675	-2.091	0.0453	5.1313	1.8672
180°	2.500	2.6×10^{-2}	4.532	-2.025	0.0943	3.0373	1.8115
120°	0.648	6.6×10^{-3}	5.320	-2.018	0.0634	1.6136	1.7770
90°	0.726	1.9×10^{-3}	6.155	-1.905	0.1038	1.5674	1.6935
60°	0.813	3.0×10^{-4}	7.509	-1.898	0.1023	1.3654	1.6490
45°	0.860	4.6×10^{-5}	8.791	-1.788	0.2398	1.1915	1.6392

The IPR equations of Jones (Jones, L.G., Blount, E.M., and Glaze, O.H. 1967):
For oil wells:

$$p_r - p_{wf} = aq^2 + bq \quad (12)$$

$$a = 5.359 \times 10^{-4} \frac{B_o^2 \rho}{h_p^2 k^{1.201} r_w} \quad (13)$$

$$B = 141.24 \times \frac{\mu_o B_o \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + s \right]}{kh} \quad (14)$$

For gas wells:

$$p_r^2 - p_{wf}^2 = aq^2 + bq \quad (15)$$

$$A = 7.3628 \times 10^{-2} \frac{\gamma_g T z}{h_p^2 k^{1.201} r_w} \quad (16)$$

$$B = 141.24 \times \frac{\mu_g T z \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + s \right]}{kh} \quad (17)$$

Main body of the novel equations:

For oil wells:

$$p_r - p_{wf} = Aq^2 + Bq \quad (18)$$

$$A = 5.359 \times 10^{-4} \frac{B_o^2 \rho (1 + \lambda)}{h_p^2 k^{1.201} r_w} \quad (19)$$

$$B = 141.24 \times \frac{\mu_o B_o \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s + s_{pt} \right]}{kh} \quad (20)$$

For gas wells:

$$p_r^2 - p_{wf}^2 = Aq^2 + Bq \quad (21)$$

$$A = 7.3628 \times 10^{-2} \frac{\gamma_g T_z (1+\lambda)}{h_p^2 k^{1.201} r_w}, \quad (22)$$

$$B = 141.24 \times \frac{\mu_g T_z \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s + s_{pt} \right]}{kh} \quad (23)$$

In both cases:

$$\lambda = \frac{\frac{r_w}{r_p} - \frac{r_w}{r_c}}{L_p^2 n_s^2 \alpha^{1.201}}, \quad (24)$$

$$s_{pt} = s_{perf} + s_V + s_H + s_{wb} \quad (25)$$

The detailed deduction of the main body can be found in [Appendix A](#).

Modification of the perforation tunnel – and crushed zone radius

In the presented equations the shape of perforations are assumed to be cylindrical, but in most cases the perforations are conical. Therefore, we correct the radius of the penetration and the crushed zone.

Correction of the perforation radius r_p :

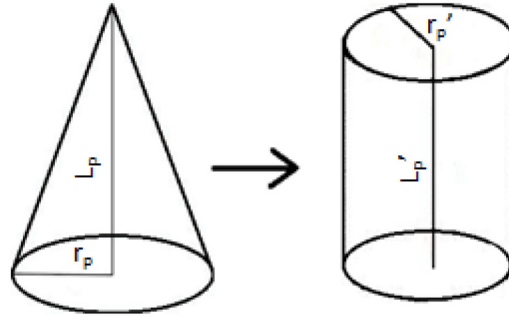


Figure 1—Correction of the perforation radius r_p

The surface area of the cone using the uncorrected radius and the surface area of the cylinder applying the corrected radius must be equal. The modified perforation tunnel radius r'_p is given by:

$$r'_p = \frac{r_p \sqrt{r_p^2 + L_p^2}}{2L_p}. \quad (26)$$

We assume that the length of the conical zone L_p is much larger than the uncorrected radius (r_p) and therefore simplify Eq 26. to:

$$r'_p \cong \frac{r_p}{2}. \quad (27)$$

Modification of the crushed zone radius r_c :

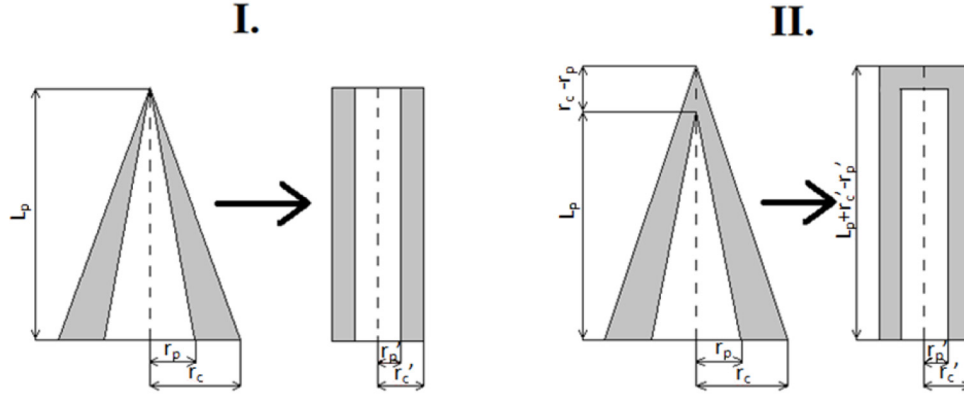


Figure 2—Modification of crushed-zone radius

The crushed zone radius is corrected in such a way that the volume of the crushed zone is kept constant (shaded regions in Figure 2). We consider two cases: (1) when the crushed zone length and the perforation tunnel length are equal, and (2) when the crushed zone length is longer by $r_c - r_p$. In both cases we get:

$$r'_c = \sqrt{\frac{r_c^2 - r_p^2}{3}} + r_p'^2. \quad (28)$$

Karakas and Tariq pointed out that the perforation tunnel radius must be modified with considering the anisotropic effect. Muskat argued that the solution of the isotropic formation can also be used for the anisotropic formations, providing that the dimensionless height and perforation radius are modified accordingly (Muskat, M. 1943). In anisotropic formations, the dimensionless spacing is modified by the square root of the anisotropy ratio. In addition, the flow into perforations in the vertical plane is elliptical in anisotropic formations. Therefore the effective perforation radius in an anisotropic formation is (Muskat, M. 1949; Kuchuk, F. and Brigham, W.E. 1979):

$$r_{pe} = \left(\frac{r_p}{2}\right) \left(1 + \sqrt{\frac{k_V}{k_H}}\right). \quad (28)$$

Including all of the corrections yields the following equations for the perforation and crushed zone radius:

$$r_{pe}' = \left(\frac{r_p'}{2}\right) \left(1 + \sqrt{\frac{k_V}{k_H}}\right), \quad (29)$$

$$r'_{ce} = r'_{pe} + r'_c - r_p' \quad (30)$$

The detailed deductions of the modifications are presented in [Appendix B](#).

Final form of the novel equation

For oil wells:

$$p_r - p_{wf} = Aq^2 + Bq, \quad (31)$$

$$A = 5.359 \times 10^{-4} \frac{B_o^2 \rho (1 + \lambda)}{h_p^2 k^{1.201} r_w}, \quad (32)$$

$$B = 141.24 \times \frac{\mu_o B_o \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s + s_{pt} \right]}{kh}. \quad (33)$$

For gas wells:

$$p_r^2 - p_{wf}^2 = Aq^2 + Bq, \quad (34)$$

$$A = 7.3628 \times 10^{-2} \frac{\gamma_g T z (1+\lambda)}{h_p^2 k^{1.201} r_w}, \quad (35)$$

$$B = 141.24 \times \frac{\mu_g T z \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s + s_{pt} \right]}{k h}. \quad (36)$$

In both cases:

$$\lambda = \frac{\frac{r_w}{r'_{pe}} - \frac{r_w}{r'_{ec}}}{L_p^2 n_s^2 \alpha^{1.201}}, \quad (37)$$

$$s_{pt} = s_{perf} + s_V + s_H + s_{wb}. \quad (38)$$

Description and Application of Equipment and Processes

The absolute open flow potential (AOFP) of a well can be calculated by applying parameters “a” and “b”. The productivity of two wells can be compared by their AOFPs. This can be used for investigating the effects of perforation parameters on the productivity.

First, the well's AOFP is calculated in case of open hole completion (assuming that the skin is zero) and second perforated completion using different perforation parameters. In the next step, the ratio of the open hole and the perforated completion is plotted in function of the variable parameter. 20 values for each parameter were used in the investigation. Table 2. and Table 3 show the base parameters in case of oil and gas wells and Table 4 shows the first and last values used in the sensitivity tests:

Table 2—Base parameters in case of oil wells

Completion	Open hole	Perforated	Open hole	Perforated
k [mD]	10.00	10.00	500.00	500.00
r _e [ft]	1000.00	1000.00	1000.00	1000.00
P _r [psi]	3000.00	3000.00	3000.00	3000.00
k _H [mD]	10.00	10.00	500.00	500.00
k _V [mD]	1.00	1.00	50.00	50.00
API density	45.38	45.38	45.38	45.38
μ _o [cP]	0.75	0.75	0.75	0.75
B _o	1.16	1.16	1.16	1.16
r _w [ft]	0.29	0.29	0.29	0.29
h [ft]	25.00	25.00	25.00	25.00
L _p [ft]		1.00		1.00
ns [spf]		5.00		5.00
α		0.30		0.30
ε		1.00		1.00
θ [°]		0.00		0.00
r _p [in]		0.18		0.18
r _c [in]		0.68		0.68
AOFP [bpd]	821.45	593.84	37782.81	22126.73

Table 3—Base parameters in case of gas wells

Completion	Open hole	Perforated	Open hole	Perforated
P_r [psi]	3000.00	3000.00	3000.00	3000.00
k [mD]	10.00	10.00	500.00	500.00
r_e [ft]	1000.00	1000.00	1000.00	1000.00
k_H [mD]	10.00	10.00	500.00	500.00
k_V [mD]	1.00	1.00	50.00	50.00
T [R°]	630.00	630.00	630.00	630.00
z	0.89	0.89	0.89	0.89
μ_o [cP]	0.02	0.02	0.02	0.02
γ_g	0.64	0.64	0.64	0.64
r_w [ft]	0.29	0.29	0.29	0.29
h [ft]	25.00	25.00	25.00	25.00
L_p [ft]		1.00		1.00
ns [spf]		5.00		5.00
α		0.30		0.30
ε		1.00		1.00
θ [°]		0.00		0.00
r_p [in]		0.18		0.18
r_c [in]		0.68		0.68
AOFP [cft/d]	29041.81	9904.52	323840.76	106682.62

Table 4—Variable parameters of the perforations

Variable	Starting value	End value
Shot density [spf]	2	8
α	0.1	0.6
Perforation length [ft]	0.15	2.85
Perforation tunnel radius [in]	0.09	0.36

The ratio of the crushed zone permeability and the permeability of the surrounding rock was specified by previous authors, and its value is between 0.1 and 0.4.

0.4 is for underbalanced perforated wells (King, G. E., Anderson, A., & Bingham, M. 1968; Pucknell, J. K., & Behrmann, L. A. 1991). Greater values can only be achieved by stimulation, which is indicated with a dashed line in the plots.

Results

The results of the sensitivity tests are presented in [Appendix C](#). Note that the results represent the percentage of the open hole completed well's absolute open flow potential. Its primary importance is summarized as follows:

- The AOFP percentage can be more than 100% if the perforated well has a better productivity than the open hole completed well. Figure E.3 represents this case, which indicates that the performance of wells with sufficiently long perforation channels is better than the one of wells with open hole completion.

- Low permeability yields better results than high permeability. This finding can be explained by the open hole completed well having lower AOFP.

To demonstrate the extent of the different perforation parameters' effects on the productivity, the following tables represent the calculated AOFP % using both starting and end parameter values in case of $\Theta=90^\circ$

Table 5—Extent of the perforation parameters' effect on productivity (oil wells)

Oil wells k= 500 mD			
Variable	AOFP %		
	At starting value	At end value	Increment
Shot density	31.5	77.9	46.4
α	39.5	72.9	33.4
Perforation length	13.9	100.4	86.5
Perforation tunnel radius	48.5	74.3	25.8
Oil wells k= 10 mD			
Variable	AOFP %		
	At starting value	At end value	Increment
Shot density	49.3	90.2	40.9
α	59.9	83.5	23.6
Perforation length	28.4	110.4	82.0
Perforation tunnel radius	69.2	85.7	16.5

Table 6—Extent of the perforation parameters' effect on productivity (gas wells)

Gas wells k= 500 mD			
Variable	AOFP %		
	At starting value	At end value	Increment
Shot density	13.9	48.7	34.9
α	17.8	46.7	28.9
Perforation length	5.3	70.6	65.3
Perforation tunnel radius	23.0	46.9	23.8
Gas wells k= 10 mD			
Variable	AOFP %		
	At starting value	At end value	Increment
Shot density	14.5	50.2	35.7
α	18.6	48.1	29.5
Perforation length	5.5	72.2	66.7
Perforation tunnel radius	24.0	48.3	24.2

It can be seen that the most important parameter of the perforation is the length of perforation channels.

Conclusions

The examination of the effects of perforation parameters on the productivity of hydrocarbon wells results in the following conclusions:

- In case of oil wells best productivity can be achieved with the application of 90° phase angle.

- In case of gas wells, the effect of phase angle on productivity is negligible.
- The perforation length has the greatest effect on the productivity; furthermore the radius of the perforation tunnel has the smallest one. The consequence of the mechanics of the perforation process is that they are not independent; therefore long perforations are more preferred than wide ones.
- In case of oil wells, better productivity can be achieved by proper perforation design than by open hole completion.
- It is more difficult to achieve a good productivity in gas wells by proper perforation design because the large energy loss of transient flows. Thus, it is very important to achieve a low λ value (high $L_p^2 \times n_s^2$).

Nomenclature

a	= Turbulent term
A	= Turbulent term for perforation
b	= Darcy flow term
B	= Darcy flow term for perforation
B_o	= Formation volume factor of oil [bbl/stb]
h	= Net thickness of the formation [ft]
h_D	= Dimensionless perforation spacing [-]
h_p	= Perforated interval [ft]
k_o	= Permeability of oil [mD]
k_p	= Permeability of crushed zone [mD]
L_p	= Perforation tunnel length [ft]
ns	= Shot density [shot/ft]
p_r	= Average reservoir pressure [psia]
p_{wif}	= Flowing bottomhole pressure in wellbore [psia]
p_{wfs}	= Bottomhole flowing pressure at sandface [psia]
q_g	= Gas flow rate into the well [Mcf/d]
q_o	= Oil flow rate into the well [stb/d]
r_c	= Radius of crushed zone [in]
r_e	= Radius of drainage [ft]
r_p	= Radius of perforation tunnel [in]
r_{pD}	= Dimensionless perforation radius [-]
r_w	= Wellbore radius [ft]
r_{wD}	= Dimensionless well radius [-]
r_{we}	= Effective wellbore radius [ft]
s_c	= Crushed-zone effects
s_H	= Horizontal skin caused by perforations
s_p	= Skin effect caused by perforations
s_t	= Total skin [-]
s_V	= Vertical skin caused by perforations
s_{wb}	= Wellbore skin caused by perforations
T	= Temperature [°R]
z	= Supercompressibility [-]
α	= k_p/k [-]
β	= Biott coefficient [ft-1]
γ_g	= Gas specific gravity [-]

δ_d	= Spacing between perforations [ft]
ε	= h_p/h [-]
Θ	= Phase angle [°]
λ	= Perforation's impact on turbulent term
μ_g	= Viscosity of gas [cP]
μ_o	= Viscosity of oil [cP]
ρ	= Fluid density [lb/cf]

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Appendix A

Incorporation of McLeod's equation into the Jones method

For oil wells:

$$p_r - p_{wf} = \left(a_1 + \frac{a_2}{(n_s \times h_p)^2} \right) q^2 + \left(b_1 + \frac{b_2}{n_s \times h_p} \right) q \quad (A.1.)$$

$$p_r - p_{wf} = Aq^2 + Bq \quad (A.2.)$$

$$A = a_1 + \frac{a_2}{(n_s \times h_p)^2} \quad (A.3.)$$

$$B = b_1 + \frac{b_2}{n_s \times h_p} \quad (A.4.)$$

For gas wells the left part of Eq. A.1. changes to $p_r^2 - p_{wf}^2$.

$$A = 5.359 \times 10^{-4} \frac{B_o^2 \rho}{h_p^2 k^{1.201}} \left[\frac{1}{r_w} + \frac{\frac{1}{r_p} \frac{1}{r_c}}{L_p^2 n_s^2 \alpha^{1.201}} \right] \quad (A.5.)$$

Let's define λ as the impact of the perforation on the turbulent term!

$$\lambda = \frac{\frac{r_w}{r_p} \frac{r_w}{r_c}}{L_p^2 n_s^2 \alpha^{1.201}}. \quad (A.6.)$$

With this the A become:

$$A = 5.359 \times 10^{-4} \frac{B_o^2 \rho}{h_p^2 k^{1.201}} \left[\frac{1}{r_w} + \frac{\lambda}{r_w} \right] \quad (A.7.)$$

$$A = 5.359 \times 10^{-4} \frac{B_o^2 \rho (1 + \lambda)}{h_p^2 k^{1.201} r_w} \quad (A.8.)$$

Let's define Δs as $s_t - s_{perf}$!

$$B = \frac{\mu_o B_o \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s \right]}{7.08 \times 10^{-3} \times kh} + \frac{\frac{\mu_o B_o \left[\ln \frac{r_c}{r_p} \right]}{7.08 \times 10^{-3} \times L_p k p}}{n_s \times h_p} \quad (A.9.)$$

$$B = \frac{\mu_o B_o \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + s_t \right]}{7.08 \times 10^{-3} \times kh} \quad (A.10.)$$

Solving Eq.A.10. For s_t :

$$s_t = \frac{B \times 7.08 \times 10^{-3} \times kh}{\mu_o B_o} - \ln \left(0.472 \left(\frac{r_e}{r_w} \right) \right) \quad (A.11.)$$

Substituting Eq. A.9. In Eq. A.11.:

$$s_t = \Delta s + \frac{\ln \left(\frac{r_c}{r_p} \right) \frac{kh}{L_p k p}}{n_s \times h_p} \quad (A.12.)$$

Let s_{perf} be the skin component caused by the perforation!

$$s_{perf} = s_t - \Delta s \quad (A.13.)$$

$$s_{perf} = \frac{\ln \left(\frac{r_c}{r_p} \right)}{n_s L_p \alpha \varepsilon}. \quad (A.14.)$$

$$B = 141.24 \times \frac{\mu_o B_o \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s + s_{perf} \right]}{kh}. \quad (A.15.)$$

For gas wells:

$$A = 7.3628 \times 10^{-2} \frac{\gamma_g T z (1 + \lambda)}{h_p^2 k^{1.201} r_w} \quad (\text{A.16.})$$

$$B = 141.24 \times \frac{\mu_g T z \left[\ln \left(0.472 \left\{ \frac{r_e}{r_w} \right\} \right) + \Delta s + s_{perf} \right]}{k h} \quad (\text{A.17.})$$

Appendix B

Modification of perforation tunnel – and crushed zone radius

Modification of r_p :

$$A_{cone} = \pi \times r_p \times \sqrt{r_p^2 + L_p^2} \quad (B.1.)$$

$$A_{cylinder} = 2r'_p L_p \pi \quad (B.2.)$$

$$A_{cone} = A_{cylinder} \quad (B.3.)$$

$$\pi \times r_p \times \sqrt{r_p^2 + L_p^2} = 2r'_p L_p \pi \quad (B.4.)$$

$$r'_p = \frac{r_p \sqrt{r_p^2 + L_p^2}}{2L_p} \quad (B.5.)$$

As L_p is at least ten times bigger than r_p from Eq 5.:

$$r'_p \cong \frac{r_p}{2} \quad (B.6.)$$

Modification of r_c :

First case:

$$\frac{r_c^2 \pi L_p}{3} - \frac{r_p^2 \pi L_p}{3} = r_c'^2 \pi L_p - r_p'^2 \pi L_p \quad (B.7.)$$

$$r'_c = \sqrt{\frac{r_c^2 - r_p^2}{3} + r_p'^2} \quad (B.8.)$$

Second case:

$$\frac{r_c^2 \pi}{3} (L_p + r_c - r_p) - \frac{r_p^2 \pi}{3} L_p = r_c'^2 \pi (L_p + r'_p - r'_c) - r_p'^2 \pi L_p \quad (B.9.)$$

$$\frac{r_c^2}{3} L_p + \frac{r_c^3}{3} - \frac{r_p r_c^2}{3} - \frac{r_p^2 L_p}{3} + r_p'^2 L_p = r_c'^3 + r_c'^2 L_p - r_c'^2 r_p' \quad (B.10.)$$

$$\frac{r_c^2}{3} + \frac{r_c^3}{3L_p} - \frac{r_p r_c^2}{3L_p} - \frac{r_p^2}{3} + r_p'^2 = \frac{r_c'^3}{L_p} + r_c'^2 - \frac{r_c'^2 r_p'}{L_p} \quad (B.11.)$$

$$\frac{r_c^3}{3L_p}, \frac{r_p r_c^2}{3L_p}, \frac{r_c'^3}{L_p}, \frac{r_c'^2 r_p'}{L_p} \cong 0 \text{ because } L_p \gg r_n^3 \quad (B.12.)$$

$$r'_c = \sqrt{\frac{r_c^2 - r_p^2}{3} + r_p'^2}$$

Appendix C

Results of sensitivity tests

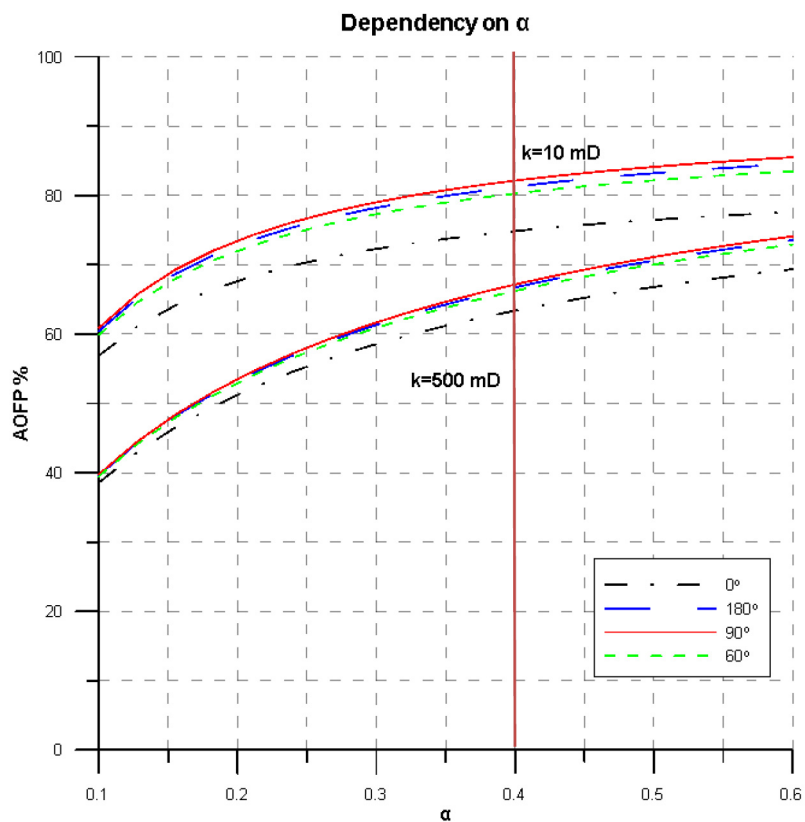


Figure C.1—Oil well's productivity dependency on α

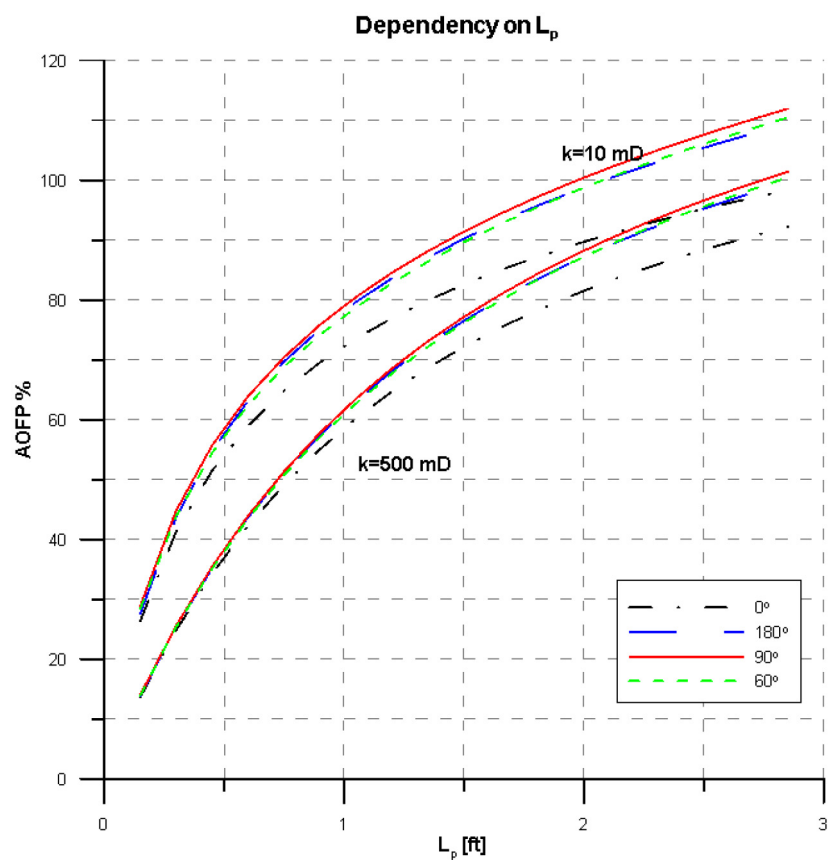


Figure C.2—Oil well's productivity dependency on perforation length

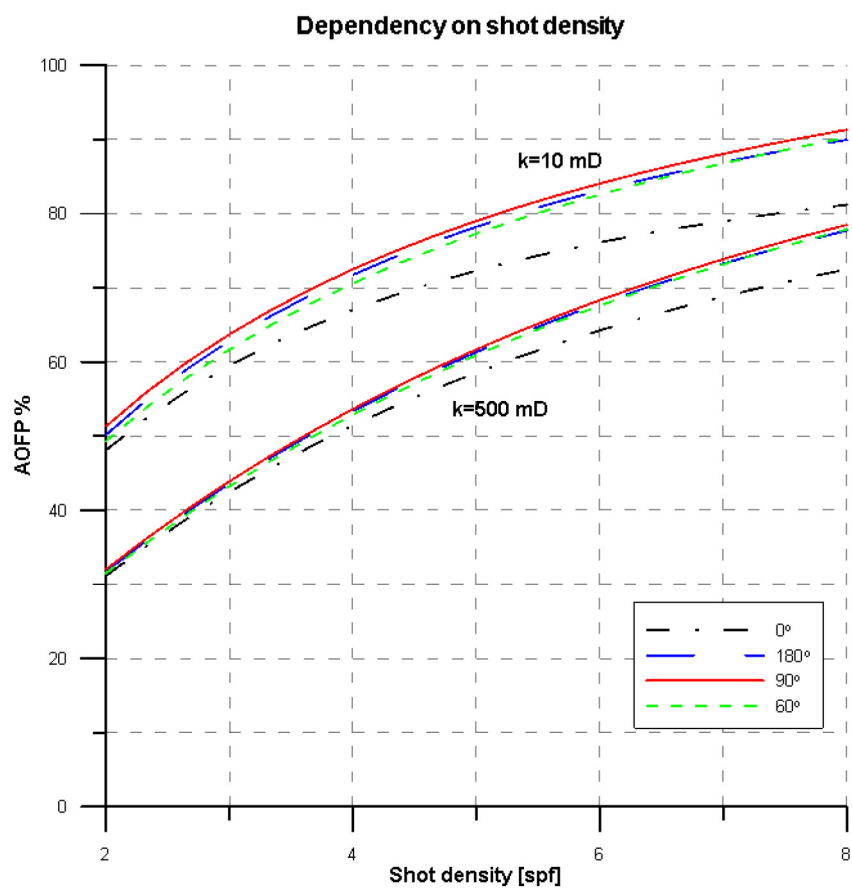


Figure C.3—Oil well's productivity dependency on shot density

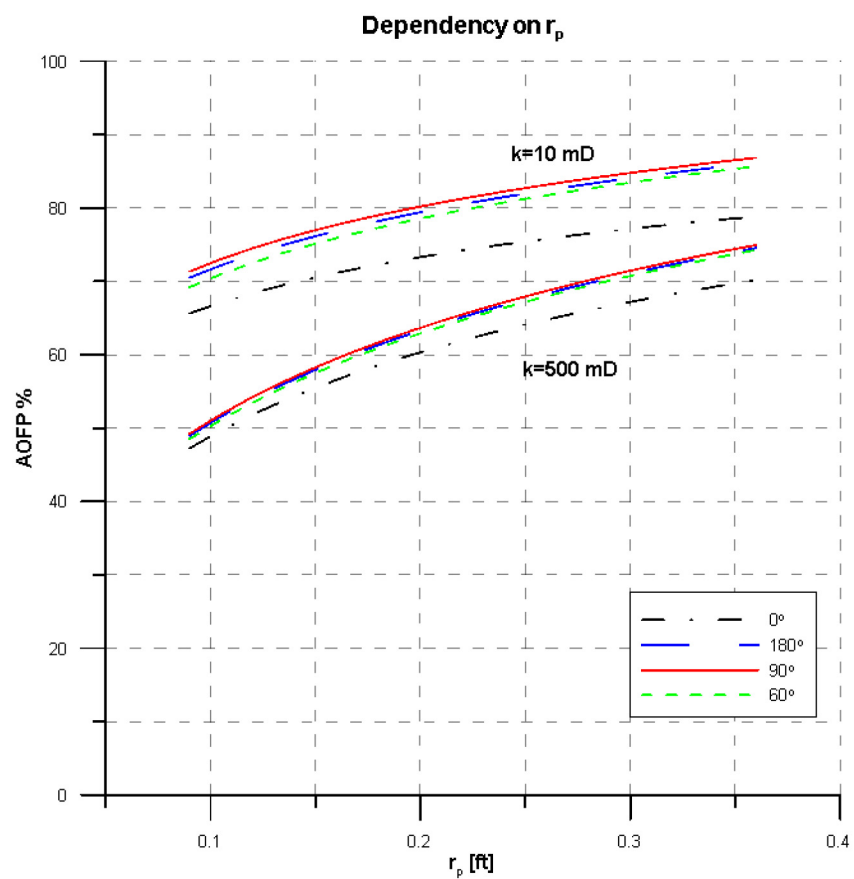


Figure C.4—Oil well's productivity dependency on perforation tunnel radius

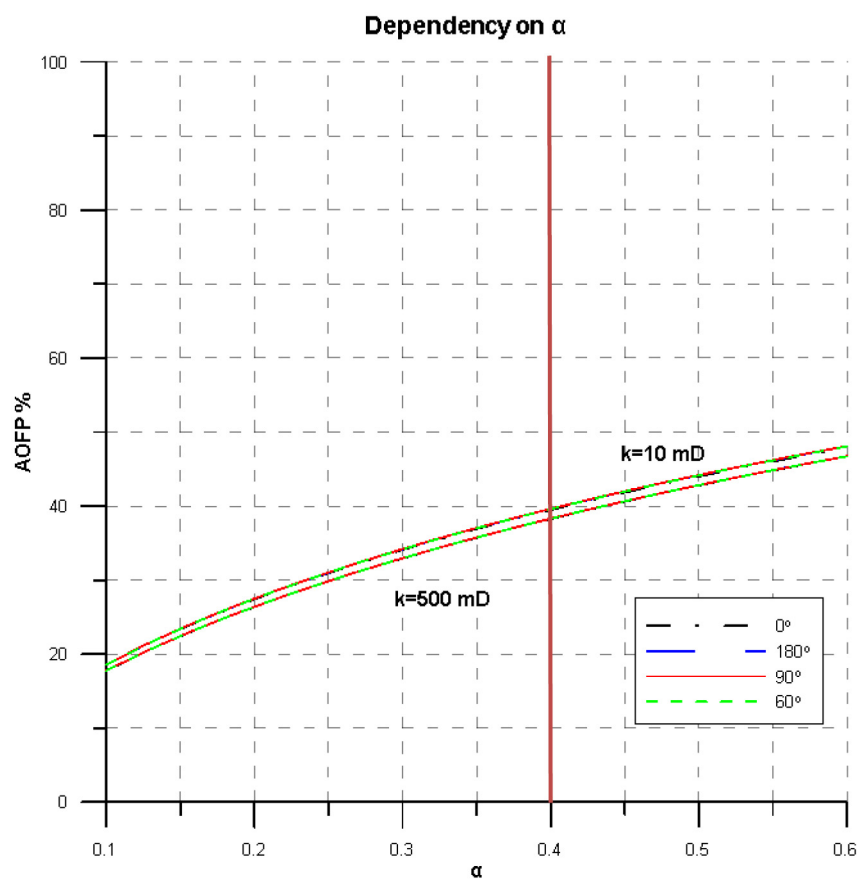


Figure C.5—Gas well's productivity dependency on α

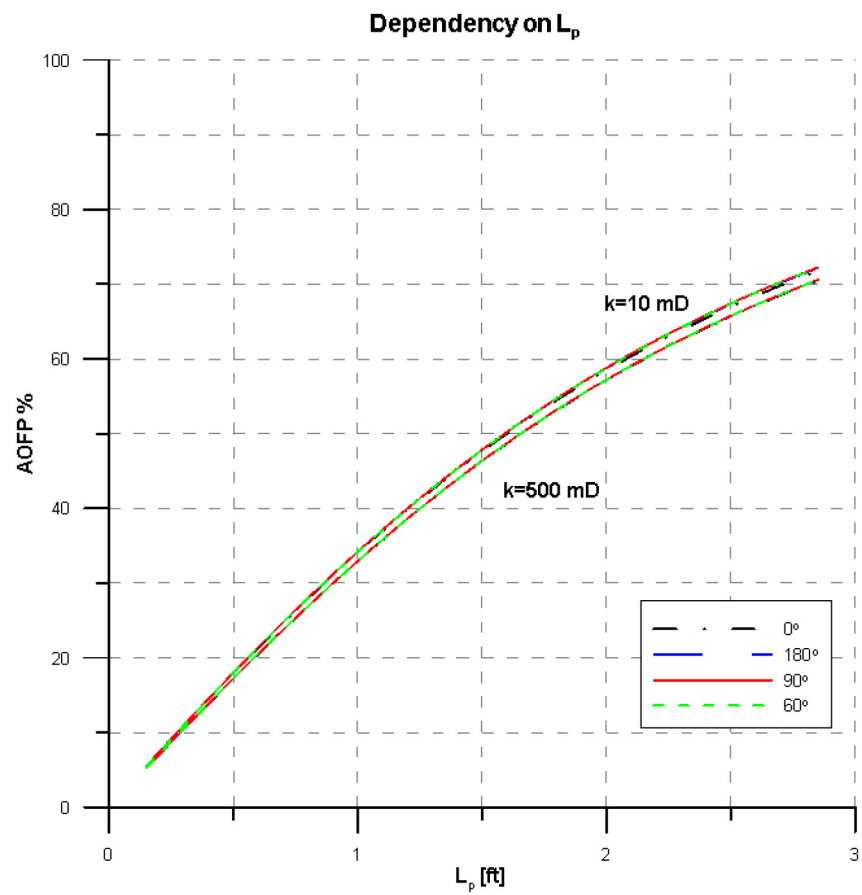


Figure C.6—Gas well's productivity dependency on perforation length

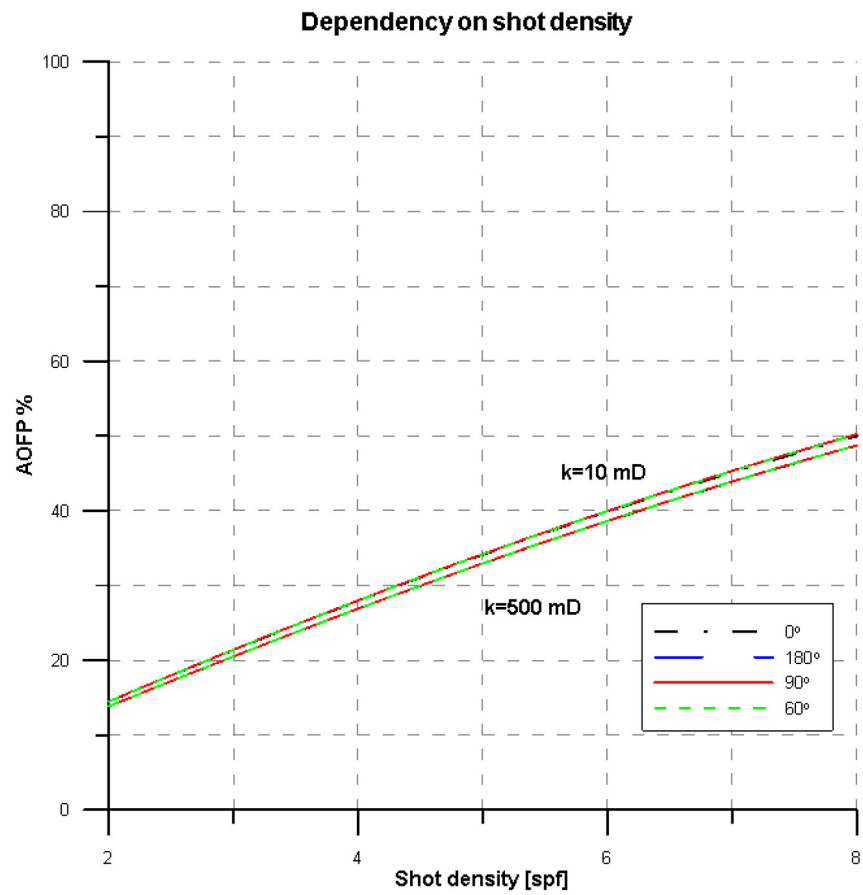


Figure C.7—Gas well's productivity dependency on shot density

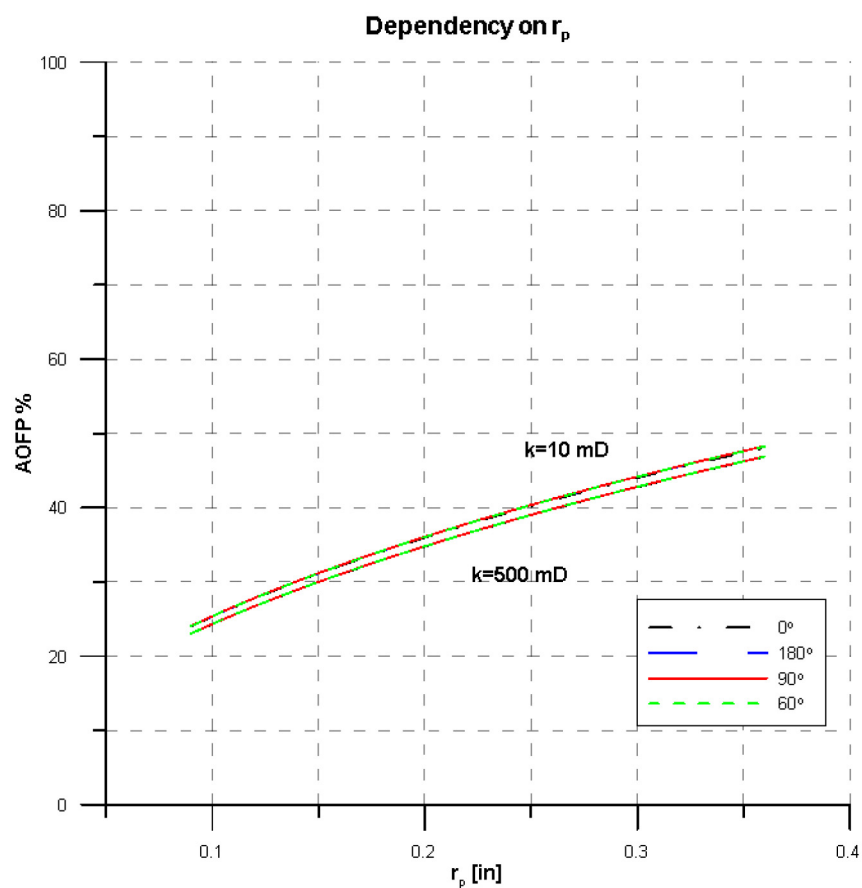


Figure C.8—Gas well's productivity dependency on perforation tunnel radius