

On the absolute performance ratio of the First Fit algorithm

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In the classical one-dimensional bin-packing problem, we are given a sequence $L = (a_1, a_2, \dots, a_n)$ of items, each with a size in $(0, 1]$. We are required to pack them into a minimum number of unit-capacity bins.

The bin-packing problem was one of the earliest to use an approximation algorithm and worst case analysis. For a given list L and algorithm A , let $C^A(L)$ denote the number of bins used when A is applied to list L , and $C^*(L)$ denote the optimum number of bins for a packing of L .

The *absolute performance ratio* for A is defined as

$$R_A := \sup_{L \in \mathcal{D}} \frac{C^A(L)}{C^*(L)},$$

where $\mathcal{D}$ is the set of every possible lists L .

For simplicity, we use a_i to denote the size of item a_i . The *FF* algorithm can be described as follows: When we are packing a_i , we place it in the lowest indexed bin whose current content does not exceed $1 - a_i$. Otherwise, we start a new bin with a_i as its first item.

It is known that $1.7 \leq R_{FF}$, and we conjecture that it is exactly $\frac{17}{10}$. The most recent result regarding the upper bounds of the absolute performance ratio of *FF* is due to B. Xia and Z. Tan: They proved that $R_{FF} \leq \frac{12}{7}$.

In this talk we present an improved result regarding this bound. We are going to show that in the previous estimation the case of equality cannot hold, and thus we will be able to give a better estimation:

$$R_{FF} \leq \frac{101}{59} \approx 1.7119.$$

References

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