

On the transport processes in wind-induced circulations with particular reference to chaotic advection

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Abstract

Advection-induced transport processes were investigated in a pyramidal test basin which is a reasonable representation of shallow lakes or bays. It is demonstrated that in periodical wind forcing the circulation pattern of the lake has chaotic advection characteristics while in steady-state cases it has not. The way these characteristics are in connection with periodicity and depth is also revealed. Based on leaking methods the parts of a lake in which contaminants' behaviour is predictable and non-predictable were also identified.

Introduction

Transport processes are one of the most important phenomena in nature. They determine the distribution of all drifting elements of the ecosystem from phytoplankton to suspended solids and various pollutants. It also determines which populations maintain an existence or consume in a specific habitat ([14]). The most decisive component of transport is advection the chaotic character of which was identified in several circumstances in environmental flows ([1]; [3]). Chaotic systems are best investigated in their phase space, from this point of view chaotic advection is a unique phenomenon of nature since phase space is the same as physical space. Nowadays chaotic advection is widely investigated from large scale like oceans ([13]) through shallow lakes ([7]) to micro-scale flows ([2]). Although mixing by chaotic advection is a frequently discussed topic there are only few investigations in wind-induced shallow basin or lake conditions ([11]; [4]).

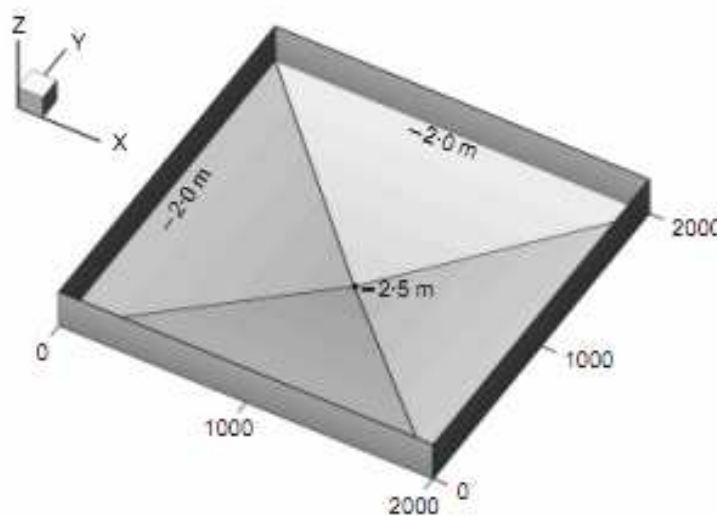


Fig. 1: Axonometric view of the pyramidal lake ([4])

In the latter they accomplished a two-dimensional investigation of chaotic mixing properties of a pyramidal lake (Figure 1). Strong and weak shearing sub-regions and a hyperbolic (saddle) point were identified and the chaotic behaviour of this time-dependent flow was proved. Further investigation ([15]) of circulations of pyramidal lake showed strong three-dimensional characteristics of this flow. Accordingly, the purpose of this paper is to reveal three-dimensional properties of chaotic advection in shallow lakes. As case study, the above mentioned pyramidal lake with horizontal dimensions 2 km x 2 km and depth 2 m all along the shore and growing linearly to 2.5 m toward the middle is used.

The hydrodynamic model

For numerical modelling the Panormus 3D ([9]) open-source hydrodynamic model has been used, which

solves the continuity and momentum equations using various turbulence approaches (RANS, LES) on a curvilinear non-orthogonal grid ([8]). The discretisation of the equations is accomplished through the finite-volume method using hexahedral cells. In our investigation a symmetrical grid distribution denser close to the boundaries has been applied.

The lake was forced by fetch-dependent wind ([5]) from SE and SW (Figure 2) directions. In steady-state the flow direction is aligned with the wind at the lake surface and opposite to it at the bottom. At all other depths the horizontal flow patterns consist of four gyres meeting at one point called hyperbolic point in the two-dimensional case but in three-dimensions it is a so-called critical point. The location of these critical point is different at every depth, nevertheless, they form a continuous curved line, the so-called critical curve (Figure 3).

Turning to time-dependent case, steady-state SE and SW wind speed fields alternating periodically were applied as a rough approximation of natural wind variability, however, appropriate enough to get acquainted with chaotic features of induced flows ([6]).

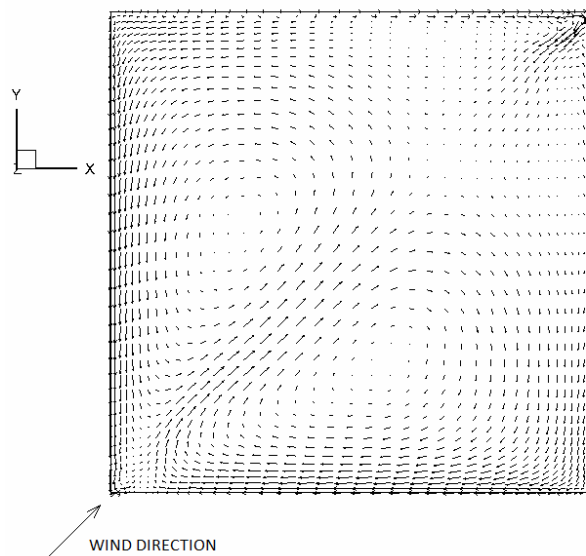


Fig. 2: Steady-state flow pattern in the pyramidal lake at $z=-1\text{m}$ plane, wind blows from SW.

Lagrangian method for the analysis of chaotic systems

The most appropriate method for the analysis of chaotic advection is the Lagrangian aspect, which is based on the motion of particles. In this study passive advection has been investigated ([10]) which means particles move like fluid elements without inertia thus solely the fluid velocity field determines particle trajectories governed by the following equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t) \quad (1)$$

Where $\mathbf{x}(t)$ is the spatial position of the particle at time t and $\mathbf{v}(\mathbf{x}, t)$ is the spatial flow velocity vector field. This equation was solved by a fourth-order Runge-Kutta scheme using tri-linear spatial interpolation. The Lagrangian method is appropriate to localize the critical curve (Figure 3) in the vicinity of which the scattering of fluid elements is strong.

Necessary but not sufficient conditions for chaotic dynamics to appear are the following ([16]):

1. The system is non-linear.
2. The system is described by at least three independent, first-order autonomous differential equations.

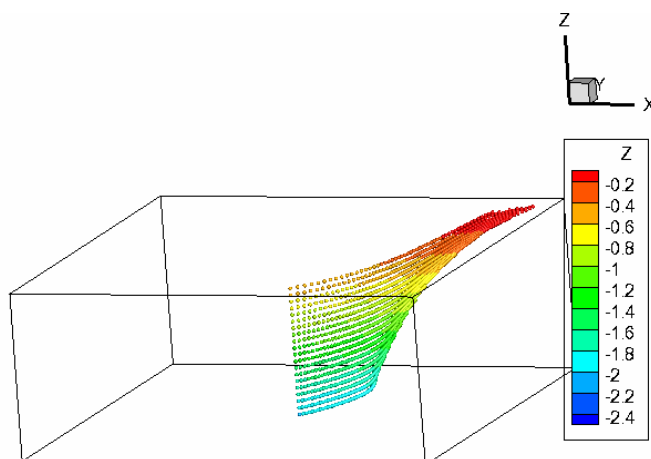


Fig. 3: So-called critical curve drawn by trajectories in pyramidal lake

Further properties of chaotic systems are that they show high sensitivity to initial conditions and even one single particle is able to travel over the whole domain. In the paper the cases the flow in the pyramidal lake presents chaotic characteristics will be demonstrated. The first and second condition is satisfied in every case since the flow velocity is a non-linear function of space and the flow is described by three independent, first-order autonomous differential equations (RANS equations). However, in the steady-state case particles are not able to travel over the whole domain, their trajectories are so-called quasi-periodic ones indicating that these particles do not return exactly to their initial position but close to them (Figure 4a), thus there is no high sensitivity to initial conditions consequently this flow cannot be chaotic.

In the next section periodic flow the degree of freedom of which is half more than in steady-state system is investigated. In this case particles are able to visit the entire domain (Figure 4a) and this system has high sensitivity to initial conditions (Figure 4b) thus the system is expected to be chaotic. Periodic flows will be investigated by means of different simple methods (e.g. stroboscopic mapping or leaking) usually applied to the analysis of dynamical systems in chaos theory.

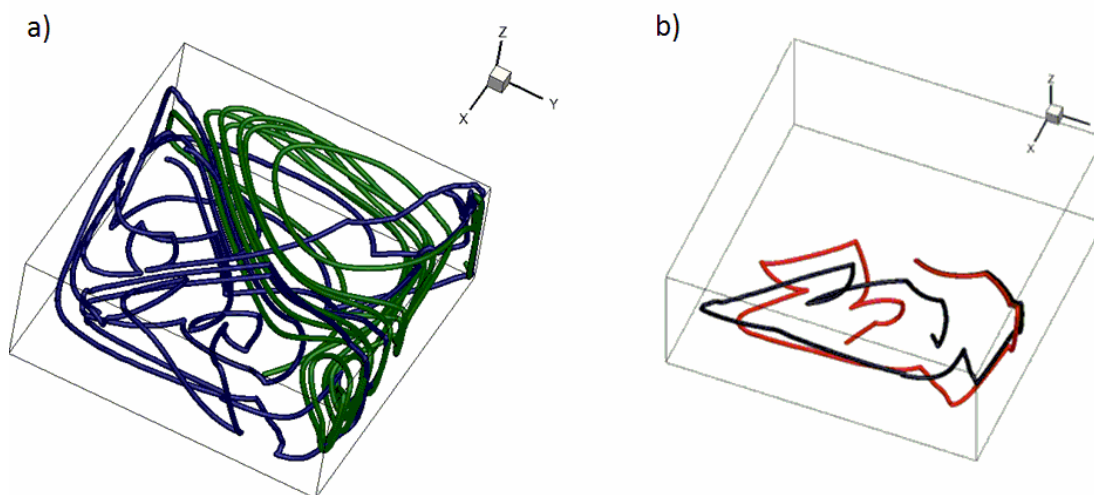


Fig. 4: a) Representative trajectories in steady-state (green) and periodically ($T=8h$) time-dependent (blue) flow; b) Path of two particles initially close to each other

Stroboscopic mapping

Stroboscopic mapping is a common method to investigate periodically driven dynamical systems. This method is appropriate to separate well- and poorly-mixed regions in the flow. In the course of stroboscopic mapping, the position of particle is registered and plotted at the end of every period. In our analysis the position registration was completed in every half period. The stroboscopic mapping transforms continuous

trajectories into a sequence of discrete points ([16]).

A typical chaotic system consists of regions which maintain strong chaotic properties and sub-regions which do not present chaotic characteristics called islands or tori. As to the stroboscopic mapping of a particle the starting point of which is in badly mixed or weak shearing sub-region (island), the mapped points cover a minor part of the entire domain.

In the first investigation the starting position of particle was in a badly mixed sub-region in 2 hour periodic time flow (5. figure). A torus is noticeable but it is not the same as known in nonlinear physics because it is not closed. There is outflow at the top and inflow at the bottom area, thus it is a so-called opened torus. Opened torus is very important in engineering point of view, since though particles will not be situated in torus forever but they will stay there for a longer period of time, which is in this example more than 200 hours.

The stroboscopic mapping of particles the starting positions of which are in well-mixed region covers the entire chaotic domain (Figure 6). If the area is observed and registered points are flashed subsequently it seems to be a random phenomenon, but it is also known that the phenomenon is governed by pure deterministic laws, all this being one of the most typical features of chaotic systems.

Marker method

This method is the numerical implementation of the well-known tracer technique used in field studies ([18]). The essential is to mark fluid elements and track them in the numerically given flow field thus investigating their behaviour. In our case 1.5 million markers were placed, initially distributed in a raster with 3 m horizontal cell size over the $(x, y)=0$ m to $(x, y)=2000$ m sub-domain, in three horizontal planes the depth of which was 1.89, 1.90 and 1.91 m. Using 8 hours time periodicity Figure 7a and 7b show the marked fluid elements' position after 60 hours. The pattern shows fractal structure which is indicative to the presence of specific attractor, the existence of a point or location where the marked fluid elements (representing neutrally buoyant pollutants) are accumulated. However, attractors would not exist in divergence-free flows (systems) but since here a part of the flow is investigated which is not divergence-free in itself, therefore the presence of attractors is not a paradox.

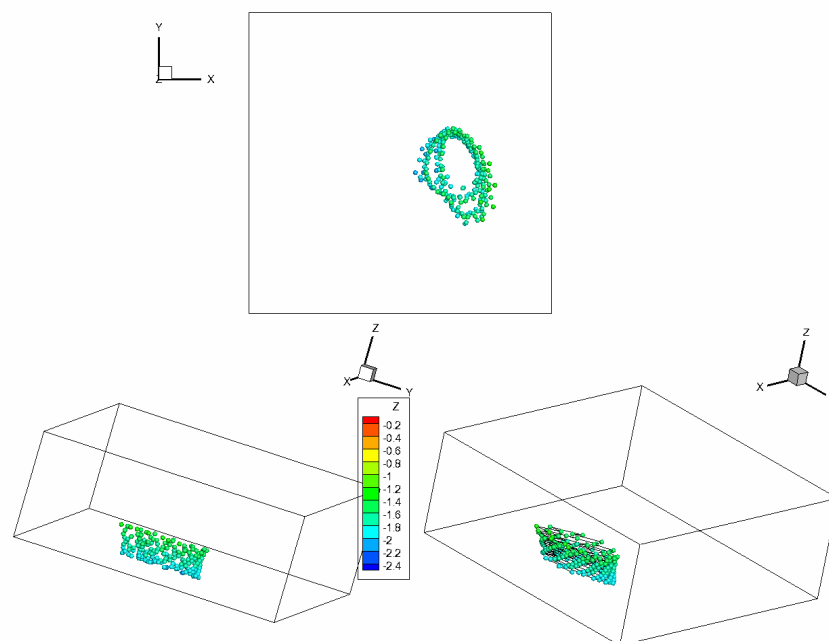


Fig. 5: Stroboscopic map of a particle the initial position of which was in badly-mixed region in periodically ($T=2h$) time-dependent flow, after 100 periods. Starting position of particle: $x=1500$ m, $y=1000$ m, $z=-1.9$ m

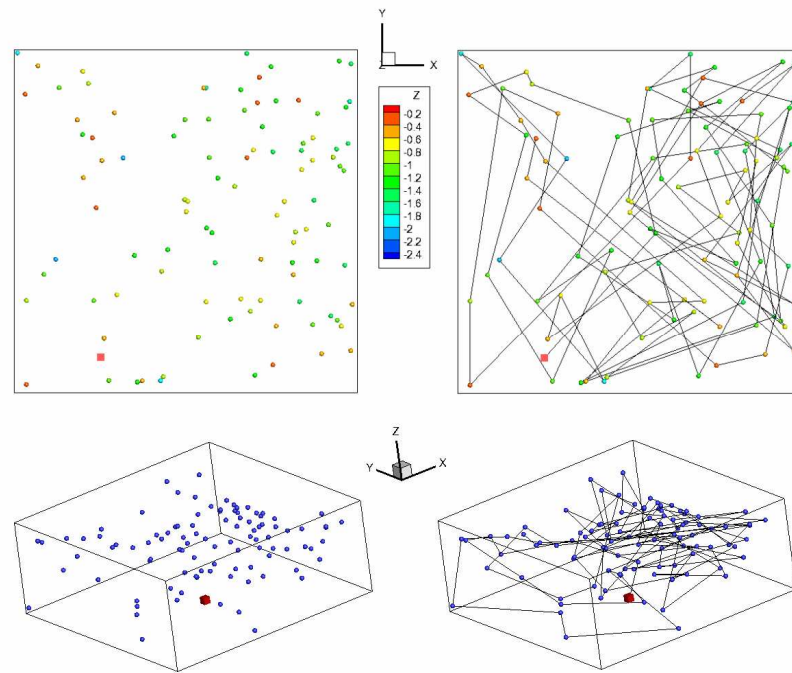


Fig. 6: Stroboscopic map of a particle whose initial position (red cube) was in well-mixed region in periodically ($T=8h$) time-dependent flow after 400 hours. Starting position of particle: $x=500$ m, $y=200$ m, $z=-1.0$ m

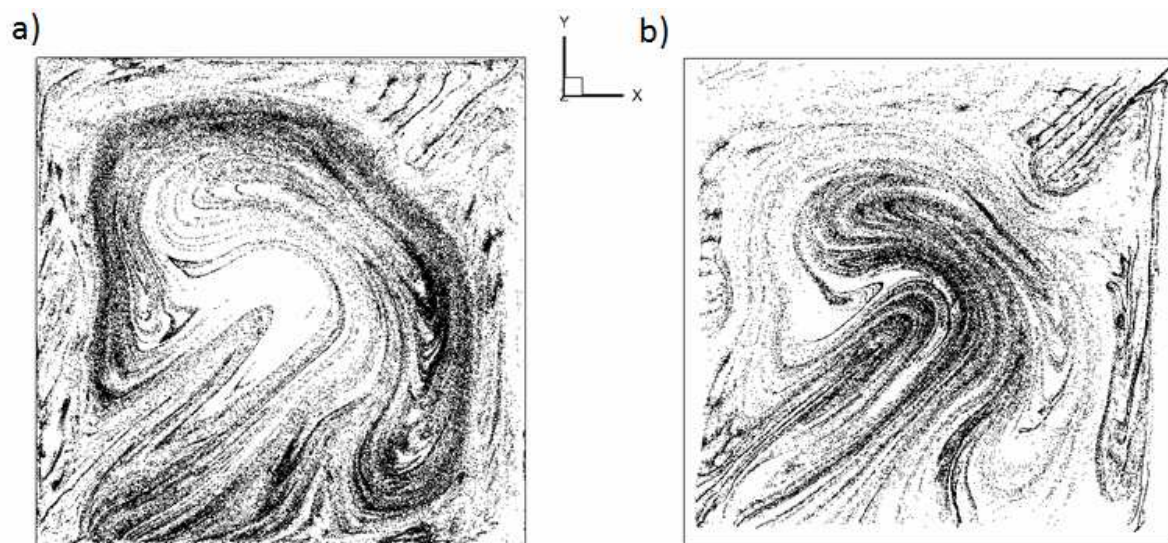


Fig. 7: Top view of positions of marked particles between a) $z=-1$ m and $z=-1.3$ m; b) $z=-2$ m and $z=-2.5$ m in periodically ($T=8h$) time-dependent flow, after 60 hours. Initial positions $z=1.89-1.91$ m, $(x, y)=0$ and 2000m domain. The number of particles is 1.5 million

Leaking advection

The leaking method ([12]; [17]) is based on monitoring particles which enter or not a pre-selected sub-domain called leak. If the behaviour of advection is chaotic then the pattern of monitored particles has fractal structure. Leaking is a good example for how method developed for the analysis of chaotic systems can be very useful in engineering sciences since it is suitable to classify separate starting positions of pollutants according to which shoreline segment is primarily affected by them. In this investigation four leaks were placed in a 50m wide coastal zone all around the lakeshore and the horizontal distribution of the particles' starting points are the same as in the previous section. In periodically time-dependent case there is a fractal structure in contrary to the steady-state case. If the applied alternating time period is short, the pattern is almost symmetric and the chaos is weak; as the time period is increased asymmetry and chaos is enhanced

(Figure 8). In fact, the effect of firstly applied wind direction and related flow velocity field becomes more significant due to the longer existence, which creates then asymmetry because the particles' positions are asymmetric when switching on the subsequent velocity field.

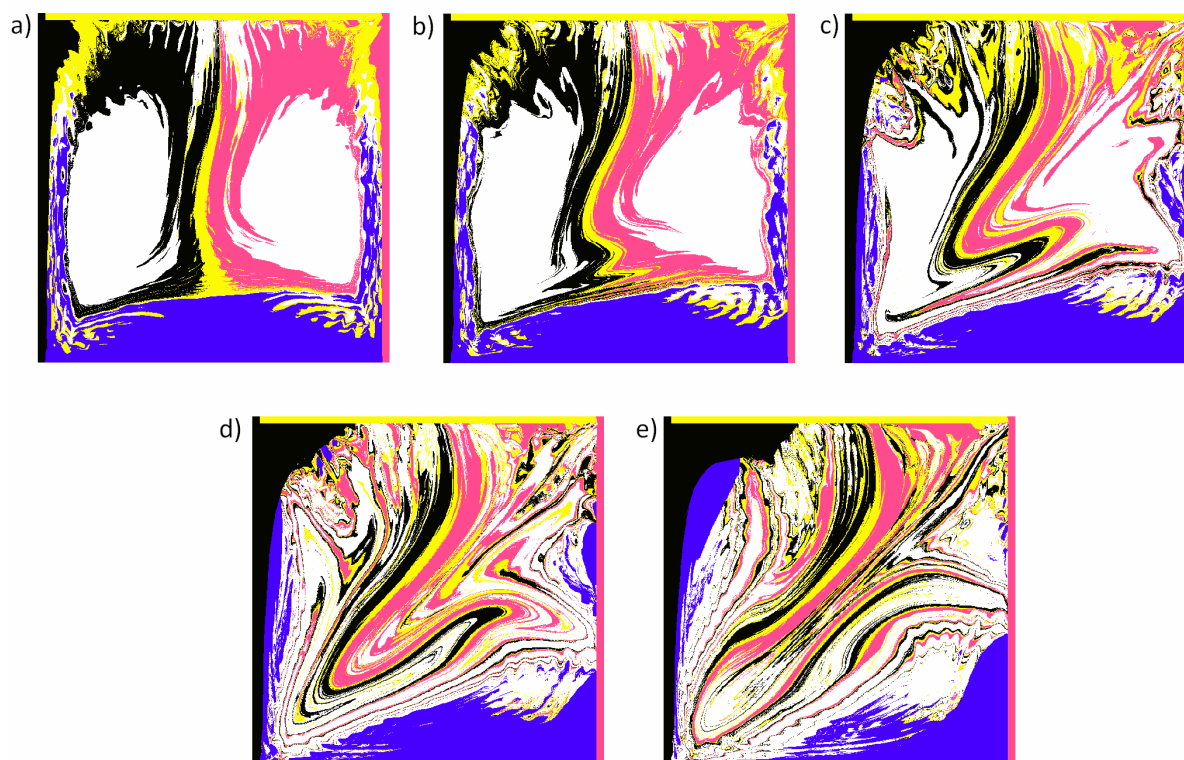


Fig. 8: Results of the leaking method in periodically time-dependent flow (a) $T=2h$; b) $T=4h$; c) $T=6h$; d) $T=8h$ and e) $T=12h$ at ($T=8h$)), after 60 hours. Initial positions $(x, y) = 0$ and 2000m domain, at $z=-1.3m$ horizontal plane. The number of particles is 500,000.

Consequently, as the T time period increases the dependence of initial conditions is enhanced and badly mixed regions from which pollutants do not affect the shore disappear. In practical aspect, raised time period requires higher accuracy to determine the location of pollutants to predict which shore is affected.

Chaotic characteristics are depth-dependent as shown Figure 9. The deeper we are, the finer filamental horizontal pattern is found, thus dependence on the initial condition increases as well. The accuracy of this investigation can be increased by reducing the horizontal spacing of particles. Increasing the number of particles results in revealing more refined filament pattern. At a given resolution, filaments are distinguishable according to the fact they are fractal or not. If it decomposes to additional filaments due to increasing number of particles it is fractal. Accordingly, the analysis was extended to a sub-region with ~ 40 times finer resolution. The distribution of the time to first reach the shore was investigated (Figure 10) in this sub-region. Supposing particles do not move after “falling” to a leak for the first time, this system can be assumed transient chaotic system due to the fact that the duration of chaotic behaviour strongly depends on particles' starting position. The distribution of the time to reach the near-shore zones has the same characteristics as the known fractal lifetime in transient chaos.

Conclusions

In this study a previous two dimensional investigation ([11]; [4]) was extended to three dimensional analysis. Though steady-state three-dimensional flows could be in principle chaotic, results have shown that only periodically time-dependent flow has strong sensitivity to initial conditions, which is a clear indication of chaotic behaviour. Investigating part of the flow with marker method the presence of specific attractor has been showed. By using the leaking method it has been demonstrated that as the time period increases the dependence on the initial conditions and thus also the chaotic features are enhanced, furthermore, filamental structure becomes finer with depth. Finally, the distribution of the time to reach the lakeshore was

manifested in a sub-region and showed that the time to reach the lakeshore has the same characteristics as the fractal lifetime known in transient chaos.

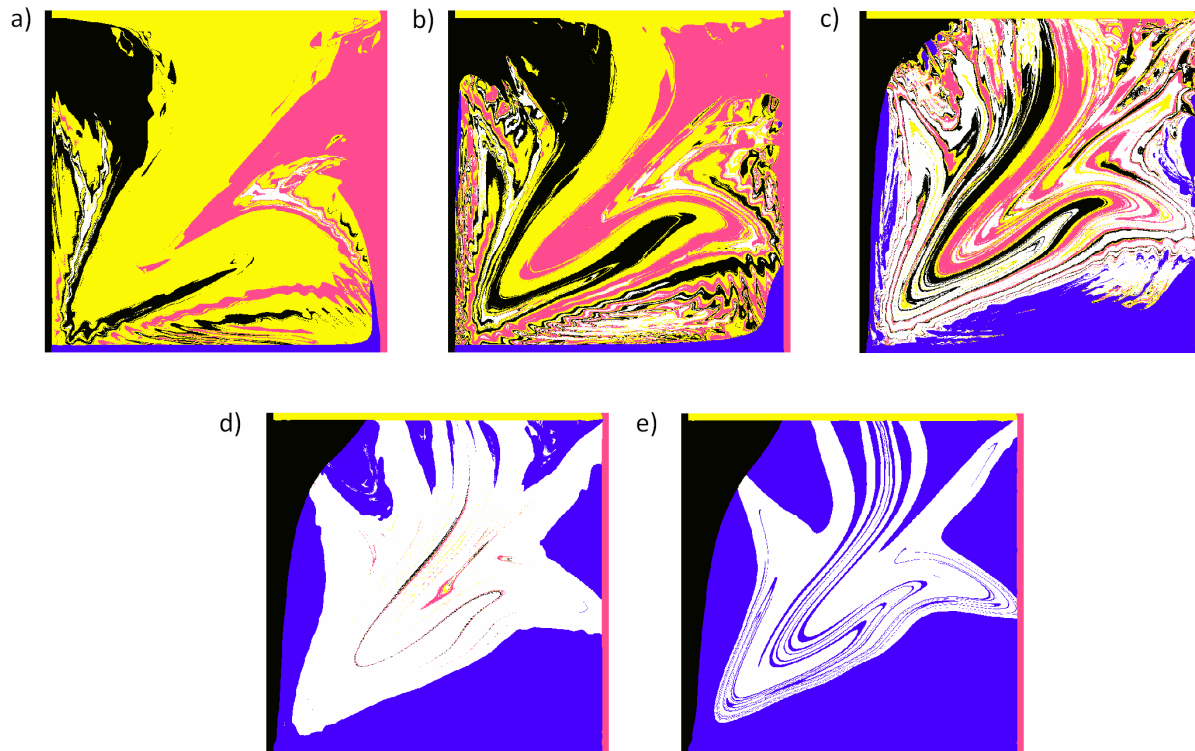


Fig. 9: Results of leaking method in periodically ($T=8h$) time-dependent flow, after 60 hours. Initial positions $(x, y)=0$ and 2000m domain at a) $z=-0.7m$; b) $z=-1.0m$; c) $z=-1.3m$; d) $z=-1.6m$ and e) $z=-1.9m$ horizontal plane. The number of particles is 500,000 at each plane.

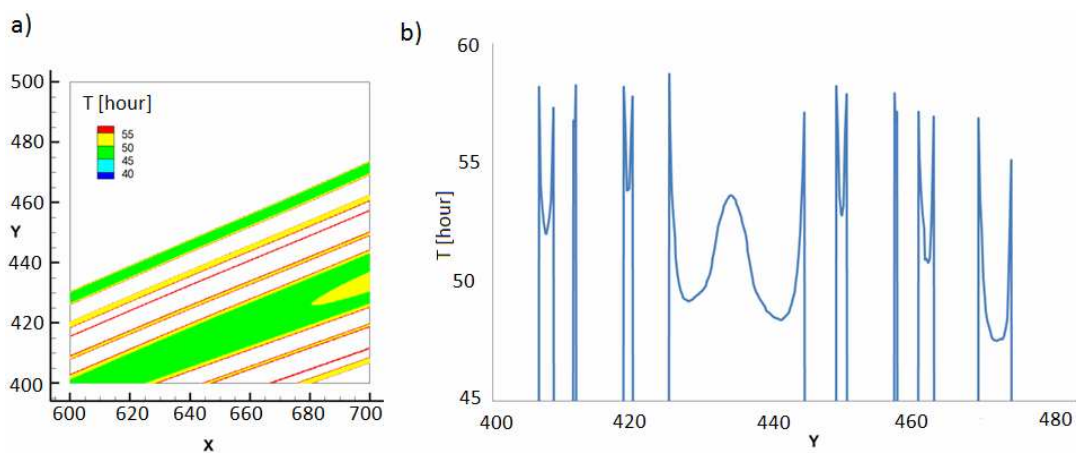


Fig. 10: a) Fractal lifetime (time to reach the leak (lakeshore)) distribution in periodically ($T=8h$) time-dependent flow. Initial positions $x=600-700$ m, $y=500-600$ domain. Number of particles: 40,000 b) Fractal lifetime distribution along the $x=700$ line.

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