

Independent wheel steering control design based on variable-geometry suspension

Balázs Németh* Dániel Fényes* Péter Gáspár**

* *Systems and Control Laboratory, Institute for Computer Science and Control, Hungarian Academy of Sciences, Budapest, Hungary;*
E-mail: [balazs.nemeth,daniel.fenyas]@sztaki.mta.hu

** *Institute for Computer Science and Control, Hungarian Academy of Sciences and MTA-BME Control Engineering Research Group, Budapest, Hungary; E-mail: peter.gaspar@sztaki.mta.hu*

Abstract: The paper presents the control design of an independent wheel steering system for in-wheel electric vehicles. The steering of the wheels is based on the variable-geometry suspension control, which influences the camber angle and the scrub radius of the wheel. A nonlinear formulation of the variable-geometry suspension is derived, which is validated through a high-fidelity suspension model. Furthermore, the impact of the independent steering in the lateral model of the vehicle is incorporated. In the paper the robust control design of the independent steering system in a hierarchical structure is presented. The trajectory tracking of the vehicle is guaranteed by the robust $\mathcal{H}_\infty$ control, while the intervention of the variable-geometry suspension is designed based on the Linear Quadratic theorem.

© 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Variable-geometry suspension, Independent steering, Robust control, In-wheel electric vehicle

1. INTRODUCTION AND MOTIVATION

The electrification of the vehicle drivelines provides a new possibility to enhance the stability and safety of road vehicles. A novel solution of the electric drive is the application of in-wheel electric motors. It makes possible the distribution of the traction forces between the wheels, by which additional functionalities can be achieved, e.g. the torque vectoring of the vehicle.

The in-wheel electric drive offers new challenges in the steering of the vehicle, such as independent steering. The goal of the independent steering concept is to improve the lateral dynamics of the vehicle using individually controlled wheels. The independent steering control for the rear wheels to modify the toe angle is presented by Lee et al. [1999]. The analysis of the independent wheel-steering system for heavy vehicles is found in Ronci et al. [2011]. Wang et al. [2011] proposes an indirect power steering measure called differential drive torque assisted steering, and validates its feasibility. Hu et al. [2015] presents a fault-tolerant control approach for a four-wheel independently actuated electric vehicle.

* The research was supported by the National Research, Development and Innovation Fund through the project "SEPPAC: Safety and Economic Platform for Partially Automated Commercial vehicles" (VKSZ 14-1-2015-0125). This paper was partially supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

In the paper a new solution of independent steering is proposed, which is based on the variable-geometry suspension solution. The aim of the suspension control is the modification of the geometry, which results in a change in the camber or the toe angle. A rear-suspension active toe control for the enhancement of driving stability is proposed by Goodarzia et al. [2010]. The active tilt control system, which assists the driver in balancing the vehicle and performs tilting in the bend, is an essential part of a narrow vehicle system, see Piyabongkarn et al. [2004]. These vehicles require the design of innovative active wheel tilt and steer control strategies in order to perform steering similarly to a car on straight roads but in bends they tilt like motorcycles, see Suarez [2012]. The advantages of the variable-geometry suspension are the simple structure, low energy consumption and low cost compared to other mechanical solutions such as an active front wheel steering, see Evers et al. [2008], Lee et al. [2005].

In the paper the control design of an independent wheel steering system for the front wheels is proposed. The novelty of the paper is the application of the variable-geometry suspension in the steering solution. The contributions are the modeling and validation of the suspension system, the control design of the actuator and the robust trajectory tracking control of the vehicle. In the control solution the actuator and the tracking control are connected in a hierarchical structure.

The paper is organized as follows. The formulation of the independent steering model incorporating the suspension and lateral vehicle models is presented in Section 2. The robust control design of the system in a hierarchical structure is found in Section 3. Section 4 demonstrates the efficiency of the control system. Finally, Section 5 summarizes the contribution of the paper.

2. MODELING OF INDEPENDENT STEERING

In this section the modeling of independent steering is proposed. The variable-geometry suspension performs the modification of the wheel position and orientation. Thus, the wheel camber angle and the scrub radius of the suspension vary. In the following, the formulation of the lateral vehicle model with the consideration of the camber angle and scrub radius is presented.

2.1 Modeling of the variable-geometry suspension

The goal of the variable-geometry suspension is to perform the wheel camber angle and the scrub radius modification. The camber angle results in a lateral force on the tyre-ground contact. Since the longitudinal force has a rotatory effect on the wheel, the scrub radius of the wheel influences the steering dynamics of the wheel. Therefore, a lateral force from the wheel steering through the scrub radius modification results.

Since the variable-geometry suspension has one actuator in each wheel, it is necessary to find a suspension construction with which the lateral force generation of the camber and the scrub radius is in coordination. Thus, the forces from the wheel tilting and the steering from the scrub radius have the same effect on the vehicle dynamics. In the paper the McPherson construction is used to perform the motion of the wheel. The actuator is incorporated in the suspension between the wheel hub and the wheel. It is able to generate an active torque M_{act} around B to tilt the wheel. However, it also has a counter effect $-M_{act}$ on the hub. In the McPherson construction the suspension is able to rotate around the connection point A of the chassis. Moreover, the arm connects the hub D and the chassis C with joints, which are able to guarantee the rotation and the motion of the suspension.

The scheme of the variable-geometry suspension is shown in Figure 1. Several forces influence the motion of the suspension and the wheel. The force of the suspension compression and damping F_{susp} is formulated as

$$F_{susp} = s_{susp} \left(\frac{z_w + z_{w,0}}{\sin \epsilon_1} \right) + d_{susp} \frac{\dot{z}_w}{\sin \epsilon_1} \quad (1)$$

where s_{susp} and d_{susp} are the stiffness and damping coefficients, $z_{w,0}$ is the joint position, resulting from the static suspension compression.

The lateral force on the tyre is F_y . It is derived from the Magic Formula, see Pacejka [2004]. F_{tyre} is the force from the tyre compression, which has a direction to the wheel:

$$F_{tyre} = s_{tyre} \frac{(r_w \cos \gamma - l_{tyre} \sin \gamma - r_w - z_w) + z_{tyre,0}}{\cos \gamma} \quad (2)$$

where s_{tyre} is the tyre stiffness, r_w is the wheel radius and $z_{tyre,0}$ is the static compression of the tyre.

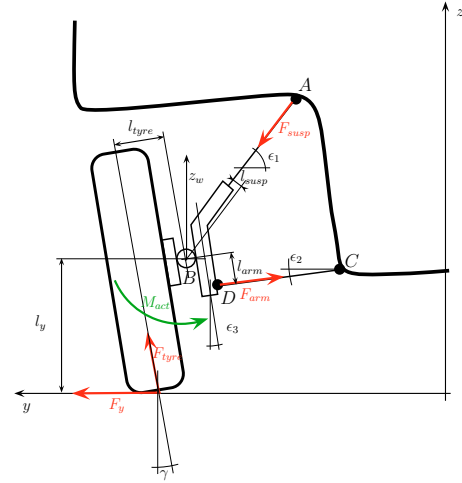


Fig. 1. Scheme of the suspension construction

The motion of the suspension is described by three dynamic equations. First, the vertical motion of the wheel hub z_w is formulated (3). Second, the torque on the suspension is described to derive the rotation of the wheel hub ϵ_3 , see (4). Third, the tilting of the wheel γ is formulated (5).

$$m_{susp} \ddot{z}_w = -F_{susp} \sin \epsilon_1 + F_{tyre} \cos \gamma + F_{arm} \sin \epsilon_2 \quad (3)$$

$$J_{susp} \ddot{\epsilon}_3 = F_{susp} l_{susp} + F_{arm} l_{arm} - M_{act} \quad (4)$$

$$J_w \ddot{\gamma} = M_{act} - F_{tyre} l_{tyre} - F_y l_y \quad (5)$$

The arm of the lateral tyre force is

$$l_y = r_w \cos \gamma - l_{tyre} \sin \gamma. \quad (6)$$

where r_w and l_{tyre} are construction parameters.

Since the inertia and the stiffness of the suspension through F_{susp} and F_{arm} are high, the effect of M_{act} on the suspension motion is relatively small. Thus, in practice the angle ϵ_3 is constant, thus $\ddot{\epsilon}_3 = 0$. Therefore, F_{arm} is computed as:

$$F_{arm} = \frac{M_{act} - F_{susp} l_{susp}}{l_{arm}} \quad (7)$$

Thus, it is assumed that ϵ_1, ϵ_2 and l_{arm}, l_{susp} can be handled as constant suspension parameters.

The nonlinear variable-geometry suspension model is validated through the complex mechanical simulation system Matlab/SimMechanics. In SimMechanics the construction of the suspension has been built, and the modification of the wheel camber angle has been analyzed. Figure 2 shows a simulation example, in which the suspension model (3)...(5) and the SimMechanics model are compared. In the simulations the same chirp input signals M_{act} are realized, see Figure 2(a). The camber angles of the model are presented in Figure 2(b). It shows that the resulting camber angles are very close to each other in a high operation range.

Figure 3 shows another example for the validation of the nonlinear suspension model. The step signal of the control input M_{act} is shown in Figure 3(a). As an effect of the torque modification, the camber angle also varies, see Figure 3(b). The results show that the steady-state error of the camber angle is below 2%. It means that the proposed nonlinear suspension model fits the SimMechanics model well.

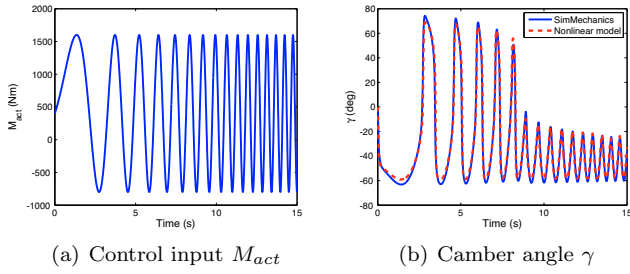


Fig. 2. Model validation - chirp signal

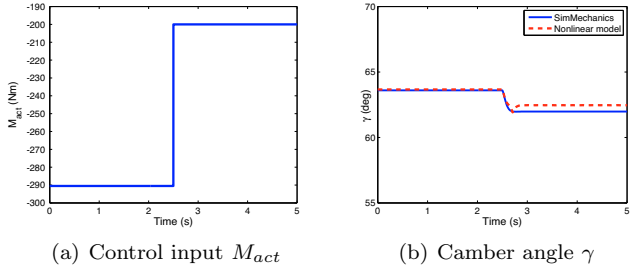


Fig. 3. Model validation - step signal

Since the presented model formulates the motion of the suspension and the wheel, it can be used for control design purposes. Therefore, the nonlinear suspension model (3)...(5) is transformed into a linear control-oriented form. During the linearizing the following assumptions are made.

- In the formulation small wheel tilting angles are considered. As a result $\cos \gamma = 1$ and $\sin \gamma = \gamma$ approximations are placed into (3)...(5).
- Since γ values are small, the lateral tyre force F_y is approximated in a linear form: $F_y = C\alpha$, where α is the side-slip angle of the tyre. During the wheel tilting motion $\alpha = \tan\left(\frac{r_w \dot{\gamma}}{v}\right) \approx \frac{r_w \dot{\gamma}}{v}$ results in the lateral side-slip angle, which is the angle between the longitudinal and the lateral component of the velocity vector. Thus, the resulting lateral tyre force is

$$F_y = C \frac{r_w \dot{\gamma}}{v} \quad (8)$$

- The static compressions of the suspension and the tyre are neglected, see (1) and (2).

The approximations yield the following transformed model of equations (3)...(5)

$$m_{susp} \ddot{z}_w = \left(1 + \frac{l_{susp}}{l_{arm}} \frac{\sin \epsilon_2}{\sin \epsilon_1}\right) (-s_{susp} z_w - d_{susp} \dot{z}_w) + s_{tyre} (-l_{tyre} \gamma - z_w) + \frac{\sin \epsilon_2}{l_{arm}} M_{act} \quad (9)$$

$$J_w \ddot{\gamma} = M_{act} - s_{tyre} l_{tyre} (-\gamma l_{tyre} - z_w) - C \frac{r_w \dot{\gamma}}{v} (r_w - l_{tyre} \gamma) \quad (10)$$

It can be seen that in (10) a nonlinear term $C \frac{r_w \dot{\gamma}}{v} \cdot l_{tyre} \gamma$ is found. Since in the approximation the small wheel camber angles are considered, the $\gamma \approx 0$ is used to avoid the nonlinearities. Thus, the following state-space representation of the variable-geometry suspension model is yielded

$$\dot{x}_{susp} = A_{susp} x_{susp} + B_{susp} u_{susp} \quad (11)$$

where the state vector is $x = [\dot{z}_w \ z_w \ \dot{\gamma} \ \gamma]^T$ and $u = M_{act}$. Moreover, A_{susp} and B_{susp} matrices are

$$A_{susp} = \begin{bmatrix} \frac{d_{susp} \eta}{m_{susp}} & \frac{s_{tyre} + s_{susp} \eta}{m_{susp}} & 0 & -\frac{s_{tyre} l_{tyre}}{m_{susp}} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{s_{tyre} l_{tyre}}{J_w} & \frac{C r_w^2}{J_w v} & \frac{s_{tyre} l_{tyre}^2}{J_w} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_{susp} = \begin{bmatrix} \frac{\sin \epsilon_2}{m_{susp} l_{arm}} \\ 0 \\ \frac{1}{J_w} \\ 0 \end{bmatrix}$$

$$\text{where } \eta = \left(1 + \frac{l_{susp}}{l_{arm}} \frac{\sin \epsilon_2}{\sin \epsilon_1}\right).$$

2.2 Lateral vehicle model

The lateral motion of the vehicle is influenced by the modification of the camber angle. Moreover, the camber angle influences the scrub radius of the wheel, which results in wheel steering. The lateral dynamics of the vehicle is influenced in two ways: the wheel camber angle and the steering angle have lateral force effects. Therefore, in the lateral dynamics both effects are considered.

The lateral motion is described by the following equations, see Rajamani [2005]

$$J \ddot{\psi} = C_1 l_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) - C_2 l_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) \quad (12)$$

$$m v (\dot{\psi} + \dot{\beta}) = C_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) + C_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) \quad (13)$$

where m is the vehicle mass, J is the yaw-inertia of the vehicle, l_1 and l_2 are geometric parameters, C_1 and C_2 are cornering stiffness values, ψ is the yaw angle of the vehicle and β is the side-slip angle.

The steering rotation of the wheel is formulated using a dynamical equation, such as

$$\ddot{\delta}_i = \frac{r_{\delta,i} F_{l,i}}{J_{\delta,i}} \quad (14)$$

where i represents the left $i = l$ or the right $i = r$ wheel. $r_{\delta,i}$ is the scrub radius, $F_{l,i}$ is the longitudinal traction/braking force on the wheel and $J_{\delta,i}$ is the inertia of the steering system for one wheel. Moreover, $r_{\delta,i}$ in the function of γ_i based on the SimMechanics simulations is expressed, see Figure 4.

$$r_{\delta,i} = \varepsilon \gamma_i \quad (15)$$

where ε is a construction parameter. (14) is transformed as

$$\ddot{\delta}_i = \frac{\varepsilon F_{l,i}}{J_{\delta,i}} \gamma_i \quad (16)$$

However, in the single-track model (12) δ represents a common steering wheel, computed as the average of the left and right steering wheel angles:

$$\delta = \frac{\delta_l + \delta_r}{2} \quad (17)$$

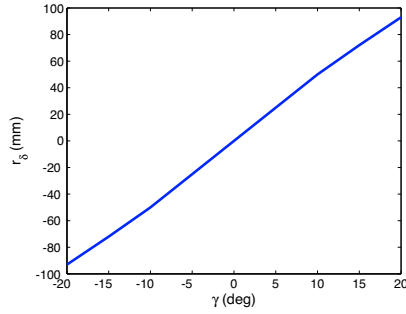


Fig. 4. Relation between the scrub radius and the camber angle

The lateral dynamics of the vehicle is formed using (12), (16) and (17). The states of the model are defined as $x_{lat} = [\psi \ \beta \ \dot{\delta}_l \ \delta_l \ \dot{\delta}_r \ \delta_r]^T$. The state-space representation is

$$\dot{x}_{lat} = A_{lat}x_{lat} + B_{lat}u_{lat} \quad (18)$$

where the control input vector is $u_{lat} = [\gamma_l \ \gamma_r]^T$ and the matrices are

$$A_{lat} = \begin{bmatrix} \frac{-C_1 l_1^2 + C_2 l_2^2}{mv^2} & \frac{-C_1 l_1 + C_2 l_2}{mv} & 0 & \frac{C_1 l_1}{2m} & 0 & \frac{C_1 l_1}{2m} \\ \frac{-C_1 l_1 + C_2 l_2}{mv^2} & \frac{-C_1 + C_2}{mv} & 0 & \frac{2J}{C_1} & 0 & \frac{2J}{C_1} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 & 0 & 0 & \frac{\varepsilon F_{l,l}}{J_{\delta,l}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\varepsilon F_{l,r}}{J_{\delta,r}} \end{bmatrix}^T$$

3. ROBUST CONTROL DESIGN OF THE SYSTEM

The goal of the independent steering is to control the lateral motion of the vehicle through the wheel camber angle modification. It requires the solutions of two control problems. First, the lateral performance of the vehicle by appropriate wheel camber angles must be guaranteed. Second, the camber angles of the wheels through the variable-geometry suspension actuation must be realized. Thus, in the following the solutions of these control problems in a hierarchical structure are proposed.

3.1 Lateral control design

The goal of the lateral control design is to guarantee the trajectory tracking of the vehicle. It means that the vehicle must follow a reference yaw-rate signal $\dot{\psi}_{ref}$. The performance is defined as

$$z_{lat,1} = \dot{\psi}_{ref} - \dot{\psi} \quad |z_{lat,1}| \rightarrow \min \quad (19)$$

where $\dot{\psi}_{ref}$ is computed using the velocity and the steering wheel angle of the driver, see Rajamani [2005]. Moreover, the performance $z_{lat,1}$ must be reached using minimum control input $u_{lat} = [\gamma_l \ \gamma_r]^T$:

$$z_{lat,2} = u_{lat} \quad |z_{lat,2}| \rightarrow \min \quad (20)$$

The performances are compressed in a vector $z_{lat} = [z_{lat,1} \ z_{lat,2}]^T$. The state-space representation of the sys-

tem for the control design which incorporates the performances and the measurement is the following:

$$\dot{x}_{lat} = A_{lat}x_{lat} + B_{lat}u_{lat} \quad (21)$$

$$z_{lat} = C_{lat,1}x_{lat} + D_{lat}u_{lat} \quad (22)$$

$$y_{lat} = C_{lat,2}x_{lat} \quad (23)$$

where the measured signal is $y = \dot{\psi}$. The augmented plant for the $\mathcal{H}_\infty$ design is illustrated in Figure 5. In the robust control design weighting functions are considered, which are formed as transfer functions. The weighting functions of $z_{lat,1}$ and $z_{lat,2}$ are formulated as $W_{p,i} = (b_{1,i}s + b_{0,i})/(a_{1,i}s + a_{0,i})$ where $b_{1,i}, b_{0,i}, a_{1,i}, a_{0,i}$ are design parameters. The selection of these parameters has significant relevance. First, the parameters determine to accuracy of the yaw-rate tracking and the limitation of the actuator intervention. Second, the overshoot of the signal $\dot{\psi}$ is also determined by the parameters.

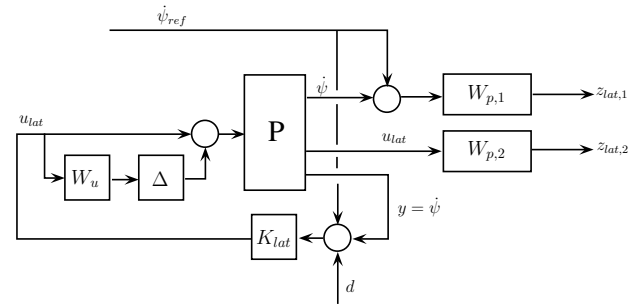


Fig. 5. Closed-loop interconnection structure

In the robust control design the uncertainties and the noises are also handled. It means that the performances of the lateral dynamics must be guaranteed under various circumstances. Since the steering of the wheels is realized through the camber angle actuation, it has an inaccuracy. Moreover, the wheel tilting intervention has its own dynamics, which is not considered in the lateral dynamics. These effects are considered in an input multiplicative uncertainty modeling approach. The uncertainty of the input u_{lat} is represented with Δ and its amplitude is scaled by the weighting function W_u , see Figure 5.

Another robustness aspect in the control design is the rejection of the external disturbances. In Figure 5 the sensor noise d on the yaw-rate measurement is considered as a disturbance, which must be rejected by the robust controller.

The state space representation of the closed-loop control system is formulated in the following way:

$$\dot{x}_{cl} = A_{cl}x_{cl} + B_{cl}w \quad (24)$$

$$z = C_{cl}x_{cl} + D_{cl}w \quad (25)$$

where w compresses d and the disturbance effect from the multiplicative uncertainty.

The objective of $\mathcal{H}_\infty$ control is to minimize the $\mathcal{H}_\infty$ -norm of the cosensitivity function $T_{z_\infty w}$. More precisely, the problem can be stated as follows [Scherer and Weiland, 2000, Boyd et al., 1997]. The LMI problem of $\mathcal{H}_\infty$ performance is formulated: The closed-loop RMS gain from w to z_∞ does not exceed Γ if and only if there exists a symmetric and definite positive matrix X_∞ such that

$$\begin{bmatrix} A_{cl}X_{\infty} + X_{\infty}A_{cl}^T & X_{\infty}B_{cl} & C_{cl}^T \\ B_{cl}^T X_{\infty} & -\Gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\Gamma I \end{bmatrix} < 0 \quad (26)$$

with $\Gamma > 0$. It means that it is necessary to find an admissible control K_{lat} for the open-loop plan P , which satisfies the following design criteria:

- the closed-loop system must be asymptotically stable,
- the closed-loop transfer function from w to z_{∞} satisfies the constraint:

$$\|T_{z_{\infty}w}\|_{\infty} < \Gamma, \quad (27)$$

for a given real positive value Γ .

The robust control design guarantees the trajectory tracking of the vehicle through the computation of the appropriate γ_l, γ_r .

3.2 Variable-geometry suspension control

The objective of the variable-geometry suspension control is to realize the camber angle, which is yielded by the robust controller. The controlled system must guarantee the precise tracking of γ_i without high overshoots and time delays. Therefore, a Linear Quadratic (LQ) control for the wheel tilting actuation is designed.

The performance of the suspension control is the minimization of the error in the wheel camber angle, such as

$$z_{susp} = \gamma_{ref,i} - \gamma_i \quad |z_{susp}| \rightarrow \min \quad (28)$$

where $\gamma_{ref,i}$ is computed by the lateral dynamic controller K_{lat} in u_{lat} . In the LQ control design the aim is to minimize a quadratic cost function, which is formulated as

$$J = \frac{1}{2} \int_0^{\infty} [z_{susp}^T Q z_{susp} + M_{act}^T R M_{act}] dt \quad (29)$$

where Q and R are the weights, where Q improves the tracking performance (28) and R has an effect on the minimization of the control input M_{act} . The goal of the weights is to find a balance between the performance and the control input.

In the LQ control design problem it is necessary minimize the cost (29), which results in the controller K_{susp} . The minimization leads to continuous-time control algebraic Riccati equation, Boyd et al. [1997]

$$P_{susp} A_{susp} + A_{susp}^T P_{susp} - \quad (30)$$

$$- P_{susp} B_{susp} R^{-1} B_{susp}^T P_{susp} + Q = 0 \quad (31)$$

where P_{susp} is the solution of (30) and A_{susp} , B_{susp} represent the dynamics of the variable-geometry suspension, see (11). The optimal state feedback LQ controller K_{susp} is derived from P_{susp} .

3.3 Architecture of the control systems

In Figure 6 the architecture of the lateral control and the variable-geometry suspension control are presented. It illustrates the cooperation of the different controllers. The lateral dynamic control computes the required left and right wheel camber angles using the yaw-rate error. The variable-geometry suspension control generates the actuation torques $M_{act,l}$ and $M_{act,r}$, by which the steering

of the wheels is realized. Thus, the lateral motion of the vehicle is improved. The reference trajectory is designed through the velocity of the vehicle and the cornering intention of the driver, estimated from the steering wheel angle δ_{driver} .

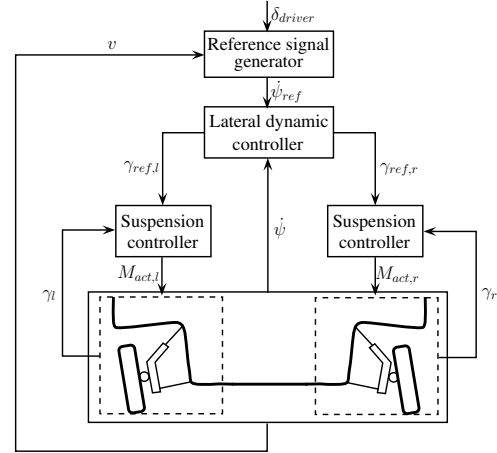


Fig. 6. Architecture of the control systems

The controllers are ordered in a hierarchical structure. It has two layers, such as the high-level and the low-level controls. In this structure the high level involves the lateral dynamic controller, which gives camber angle commands for the low level. Since the left and the right side of the front axle are controlled, the low level contains two variable-geometry suspension controllers, which actuate independently from each other. The stability and performance problems of the hierarchical control design, and further solutions are found in Németh et al. [2015], Sename et al. [2013].

4. SIMULATION EXAMPLE

In this section the actuation of the control system is proposed. The purpose of the simulation is to show the efficiency of the variable-geometry suspension system in trajectory tracking. In the simulation a D-Class passenger car is used, whose mass is $m = 1690 \text{ kg}$. The camber angles of the front wheels are controlled by the proposed control system. Thus, the steering of the wheels and trajectory tracking are guaranteed. The simulations are performed through the high-fidelity CarSim software. Moreover, the complex multi-body SimMechanics model of the variable-geometry suspension is applied during the simulation scenario.

The vehicle is driven along a section of the Michigan Waterford Hill course, see Figure 7(a). Since the track contains several bends, the velocity of the vehicle also varies, as shown in Figure 7(b).

The yaw-rate tracking of the vehicle is shown in Figure 8(a). It can be seen that the vehicle is able to guarantee the precise tracking of the ψ_{ref} signal. It results from the appropriate coordination of the lateral dynamic controller and the variable-geometry suspension control. Figure 8(b) illustrates the required and the real steering angle of the front right wheel. The simulation shows that the variable-geometry suspension is able to realize the required steering

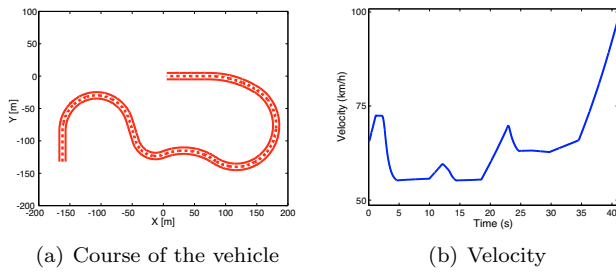


Fig. 7. Track of the vehicle

angle, which is achieved by the camber angle modification, see Figure 8(c). The camber angle illustrates that the low-level suspension control is sufficiently accurate to guarantee the required actuation which is commanded by the high-level controller. The M_{act} intervention on the right wheel is presented in Figure 7(d).

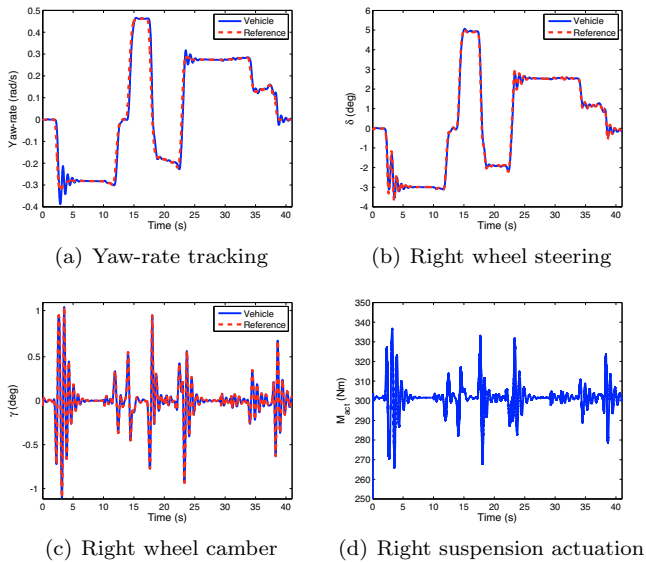


Fig. 8. Simulation results

The simulation results show that the proposed control architecture is able to guarantee the trajectory tracking of the in-wheel electric vehicle. The variable-geometry suspension-based independent steering yields good performances of the vehicle.

5. CONCLUSION

In the paper the modeling and control of independent steering for an in-wheel electric vehicle have been proposed. The presented solution is based on the variable-geometry suspension, by which the camber angle and the scrub radius of the wheels are modified. The nonlinear formulation and the control-oriented linearizing of the suspension system, together with the modeling of the lateral dynamics have been presented. The main contribution of the paper is a hierarchical control architecture, which incorporates the robust $\mathcal{H}_\infty$ control of the trajectory tracking and the LQ optimal design of the wheel tilt positioning. Finally, high-fidelity simulations have illustrated the efficiency of the novel independent steering control.

The paper has proposed that the variable-geometry suspension can be an alternative way to independent steering control. Since in-wheel electric vehicles are likely to have a significant impact in the future, further research on the variable-geometry suspension control is reasonable.

REFERENCES

- S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics, Philadelphia, 1997.
- W.J. Evers, A. van der Knaap, I. Besselink, and H. Nijmeijer. Analysis of a variable geometry active suspension. *International Symposium on Advanced Vehicle Control, Kobe, Japan*, 2008.
- Avesta Goodarzia, Ehsan Oloomia, and Ebrahim Esmailzadehb. Design and analysis of an intelligent controller for active geometry suspension systems. *Vehicle System Dynamics*, 49(1):333–359, 2010.
- C. Hu, H. Jing, R. Wang, F. Yan, and N. Chen. Fault-tolerant control of fvia electric ground vehicles with differential drive assisted steering. *9th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, Paris, France*, 48(21):1180–1185, 2015.
- H.S. Lee, U.K. Lee, S.K. Ha, and C.S. Han. Four-wheel independent steering (4wis) system for vehicle handling improvement by active rear toe control. *JSME International Journal Series C*, 42(4):947–956, 1999.
- S. Lee, H. Sung, and U. Lee. A study to the enhancement of vehicle stability by active geometry control suspension (agcs) system. *13th International Pacific Conference on Automotive Engineering, Gyeongju, Korea*, pages 1–6, 2005.
- B. Németh, B. Varga, and P. Gáspár. Hierarchical design of an electro-hydraulic actuator based on robust LPV methods. *International Journal of Control*, 88(8):1429–1440, 2015.
- H. B. Pacejka. *Tyre and vehicle dynamics*. Elsevier Butterworth-Heinemann, Oxford, 2004.
- D. Piyabongkarn, T. Keviczky, and R. Rajamani. Active direct tilt control for stability enhancement of a narrow commuter vehicle. *Int. J. Automotive Technology*, 5(2): 77–88, 2004.
- R. Rajamani. *Vehicle dynamics and control*. Springer, 2005.
- M. Ronci, P. Artuso, and E. Bocci. Four independent wheels steering system analysis. *SAE Technical Paper*, 2011. doi: 10.4271/2011-01-0241.
- C. Scherer and S. Weiland. *Lecture Notes DISC Course on Linear Matrix Inequalities in Control*. Delft University of Technology, Delft, Netherlands, 2000.
- O. Sename, P. Gáspár, and J. Bokor. *Robust Control and Linear Parameter Varying Approaches*. Springer Verlag, Berlin, 2013.
- L. Suarez. *Active Tilt and Steer Control for a Narrow Tilting Vehicle: Control design and implementation*. Lambert Academic Publishing, 2012.
- J. Wang, Q. Wang, L. Jin, and C. Song. Independent wheel torque control of 4WD electric vehicle for differential drive assisted steering. *Mechatronics*, 21:63–76, 2011.