

Control of a variable-geometry suspension as independent wheel steering

B. Németh, D. Fényes & P. Gáspár

Institute for Computer Science and Control, Hungarian Academy of Sciences, Hungary

ABSTRACT: The paper presents the modeling and control design of a variable-geometry suspension system, which is applied as independent wheel steering for in-wheel electric vehicles. The variable-geometry suspension modifies the tilting of the wheel, which has an effect on the steering of the wheels through the variation of the scrub radius. In the paper the model of the suspension-steering system and the relationship between the steering and the suspension geometry are formulated using nonlinear dynamical equations. The model is validated through a high-fidelity multi-body suspension model SimMechanics. In the paper the robust combined feedforward-feedback control design of the independent steering system is presented. Finally, the proposed control system is used in different vehicle scenarios.

1 INTRODUCTION AND MOTIVATION

The variable-geometry suspension is a novel intelligent actuator in road vehicle control. The purpose of the actuator is to modify the geometry of the vehicle suspension, by which the lateral and vertical dynamics of the vehicle are improved. Familiar applications of the variable-geometry suspension system are the rear-wheel steering with the modification of the toe-angles (Lee, Lee, Ha, & Han 1999, Goodarzia, Oloomia, & Esmailzadehb 2010), the improvement of vehicle handling (Lee & Han 2001) and trajectory tracking through wheel tilting (Németh & Gáspár 2013). The system is also used for the control of narrow tilting vehicles (Piyabongkarn, Keviczky, & Rajamani 2004). The advantages of the variable-geometry suspension are the simple structure, low energy consumption and low cost compared to other mechanical solutions such as an active front wheel steering, see (Evers, van der Knaap, Besselink, & Nijmeijer 2008, Lee, Sung, & Lee 2005).

The in-wheel electric drive offers new possibilities in the steering of the vehicle, such as independent steering. The goal of the independent steering concept is to improve the lateral dynamics of the vehicle using individually controlled wheels. The independent steering control for the rear wheels to modify the toe angle is presented by (Lee, Lee, Ha, & Han 1999). The analysis of the independent wheel-steering system for heavy vehicles is found in (Ronci, Artuso, & Bocci 2011). (Wang, Wang, Jin, & Song 2011) proposes an indirect power steering measure called differential drive torque assisted steering, and validates its feasibility. (Hu, Jing, Wang, Yan, & Chen 2015)

presents a fault-tolerant control approach for a four-wheel independently actuated electric vehicle in order to realize normal driving requirements when the active steering system is in complete failure. In this paper the variable-geometry suspension control is applied for independent steering control purposes. The advantage of this solution is that the controls of the variable-geometry suspensions are independent from each other. Thus, the steering of the left and right wheels can also be achieved independently. The contributions of the paper are the modeling and validation of the suspension system and the robust control design of the trajectory tracking control.

The paper is organized as follows. The modeling of the independent steering incorporating the variable-geometry suspension model in Section 2. Then the modeling of the lateral dynamics is presented in Section 3. The robust control design of the system in a hierarchical structure is found in Section 4. Section 5 demonstrates the efficiency of the control system. Finally, Section 6 summarizes the contribution of the paper.

2 MODELING OF THE VARIABLE-GEOMETRY SUSPENSION

In this section the modeling of independent steering is proposed. The variable-geometry suspension performs the modification of the wheel position and orientation. Thus, the wheel camber angle and the scrub radius of the suspension vary. In the following, the formulation of the lateral vehicle model with the consideration of the camber angle and scrub radius is presented.

The nonlinear variable-geometry suspension model is validated through the complex mechanical multi-body simulation system Matlab/SimMechanics. In SimMechanics the construction of the suspension has been built, and the modification of the wheel camber angle has been analyzed, see Figure 2. In the multi-body simulator the wheel and the arms are defined as body elements, whose mass and inertia values are calculated using a computer-aided design software. In the mechanical model several physical constraints using joint elements are involved, by which the connection points of the bodies are constructed. The tyre-ground contact is modeled as an elastic connection, which is able to guarantee the compression of the tyre. Furthermore, the lateral and longitudinal motions of the tyre are allowed using planar joint constraints.

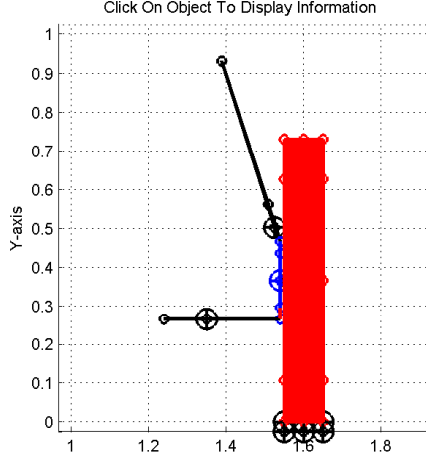


Figure 2: SimMechanics model of the suspension

Since the presented model formulates the motion of the suspension and the wheel, it can be used for control design purposes. Therefore, the nonlinear suspension model (3)-(5) is transformed into a linear control-oriented form. During the linearizing the following assumptions are made.

- In the formulation small wheel tilting angles are considered. As a result $\cos \gamma = 1$ and $\sin \gamma = \gamma$ approximations are placed into (3)...(5).
- Since γ values are small, the lateral tyre force F_y is approximated in a linear form: $F_y = C\alpha$, where α is the side-slip angle of the tyre. During the wheel tilting motion $\alpha = \tan\left(\frac{r_w \dot{\gamma}}{v}\right) \approx \frac{r_w \dot{\gamma}}{v}$ yields the lateral side-slip angle, which is the angle between the longitudinal and the lateral component of the velocity vector. Thus, the resulting lateral tyre force is

$$F_y = C \frac{r_w \dot{\gamma}}{v} \quad (8)$$

- The static compressions of the suspension and the tyre are neglected, see (1) and (2).

The approximations yield the following transformed model of equations (3)...(5)

$$m_{susp} \ddot{z}_w = \left(1 + \frac{l_{susp}}{l_{arm}} \frac{\sin \epsilon_2}{\sin \epsilon_1}\right) (-s_{susp} z_w - d_{susp} \dot{z}_w) + s_{tyre} (-l_{tyre} \gamma - z_w) + \frac{\sin \epsilon_2}{l_{arm}} M_{act} \quad (9)$$

$$J_w \ddot{\gamma} = M_{act} - s_{tyre} l_{tyre} (-\gamma l_{tyre} - z_w) - C \frac{r_w \dot{\gamma}}{v} (r_w - l_{tyre} \gamma) \quad (10)$$

It can be seen that in (10) a nonlinear term $C \frac{r_w \dot{\gamma}}{v} l_{tyre} \gamma$ is found. Since in the approximation the small

wheel camber angles are considered, the $\gamma \approx 0$ is used to avoid the nonlinearities. Thus, the following state-space representation of the variable-geometry suspension model is yielded

$$\dot{x}_{susp} = A_{susp} x_{susp} + B_{susp} u_{susp} \quad (11)$$

where the state vector is $x = [\dot{z}_w \ z_w \ \dot{\gamma} \ \gamma]^T$ and $u = M_{act}$.

3 MODELING OF THE LATERAL DYNAMICS

The lateral motion of the vehicle is influenced by the modification of the camber angle. Moreover, the camber angle influences the scrub radius of the wheel, which results in wheel steering. The lateral dynamics of the vehicle is influenced in two ways: the wheel camber angle and the steering angle have lateral force effects. Therefore, in the lateral dynamics both effects are considered.

The lateral motion is described by the following equations, see (Rajamani 2005)

$$J \ddot{\psi} = C_1 l_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) - C_2 l_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) \quad (12)$$

$$m v (\dot{\psi} + \dot{\beta}) = C_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) + C_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) \quad (13)$$

where m is the vehicle mass, J is the yaw-inertia of the vehicle, l_1 and l_2 are geometric parameters, C_1 and C_2 are cornering stiffness values, ψ is the yaw angle of the vehicle and β is the side-slip angle.

The steering rotation of the wheel is formulated using a dynamical equation, such as

$$\ddot{\delta}_i = \frac{r_{\delta,i} F_{l,i}}{J_{\delta,i}} \quad (14)$$

where i represents the left $i = l$ or the right $i = r$ wheel. $r_{\delta,i}$ is the scrub radius, $F_{l,i}$ is the longitudinal traction/braking force on the wheel and $J_{\delta,i}$ is the inertia of the steering system for one wheel.

In the actuation of the variable-geometry suspension the scrub radius $r_{\delta,i}$ through the camber angle γ is modified. The relationship between γ and $r_{\delta,i}$ is determined by the suspension geometry and the positions of the suspension arms. Thus, in the formulation of the relationship between γ and $r_{\delta,i}$ the chassis roll angle is considered. The relation of γ and $r_{\delta,i}$ is examined using SimMechanics simulations of the suspension construction, the results are shown in Figure 3.

The analysis presents that the relationship between γ and $r_{\delta,i}$ can be approximated by a linear function, and the roll angle can be handled as a nonlinear offset in the formulation. Thus, the following model is formed:

$$r_{\delta,i} = \varepsilon_1 \gamma_i + \varepsilon_2 \varphi^2 + \varepsilon_3 \varphi + \varepsilon_4 \quad (15)$$

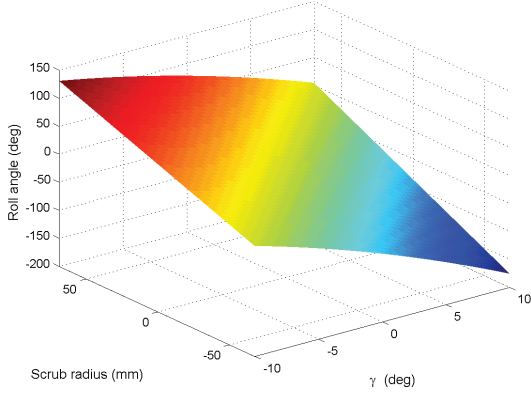


Figure 3: Relation between the scrub radius and the camber angle

where $\varepsilon_{1...4}$ are construction parameters and φ is the chassis roll angle. Thus, (14) is transformed as

$$\ddot{\delta}_i = \frac{\varepsilon_1 F_{l,i}}{J_{\delta,i}} \gamma_i + \frac{(\varepsilon_2 \varphi^2 + \varepsilon_3 \varphi + \varepsilon_4) F_{l,i}}{J_{\delta,i}}. \quad (16)$$

In case of a single-track model (12) δ represents a common steering wheel, computed as the average of the left and right steering wheel angles:

$$\delta = \frac{\delta_l + \delta_r}{2} \quad (17)$$

The lateral dynamics of the vehicle is formed using (12), (16) and (17). The states of the model are defined as $x_{lat} = [\dot{\psi} \ \beta \ \dot{\delta}_l \ \delta_l \ \dot{\delta}_r \ \delta_r]^T$. The chassis roll angle is handled as a disturbance of the system, such as $w_{lat} = [\varepsilon_2 \varphi^2 + \varepsilon_3 \varphi + \varepsilon_4]$. The state-space representation is

$$\dot{x}_{lat} = A_{lat} x_{lat} + B_{lat,1} w_{lat} + B_{lat,2} u_{lat} \quad (18)$$

where the control input vector is $u_{lat} = [\gamma_l \ \gamma_r]^T$.

4 CONTROL DESIGN OF THE SUSPENSION SYSTEM

In the followings the control design of the independent steering system is presented. The goal of the control is to guarantee the trajectory tracking of the vehicle through the modification of the front wheel steering angle. The control strategy covers two layers:

- the high-level control computes the required steering angle δ_{ref} , with which the predefined trajectory is computed,
- the torque actuation is calculated by the low-level control, which realizes the steering angle command of the high-level layer. It means that the low-level control must guarantee the tracking of δ_{ref} . For the design of the control the model (11) is used, see (Németh, Fényes, & Gáspár 2016).

In the rest of the paper the design of the high-level control using a combined robust feedback-feedforward strategy is presented. The control is based on the control-oriented lateral vehicle model (18). It contains a disturbance, which is the roll angle of the chassis. Since the

chassis roll angle is considered to be measured, w_{lat} is a measured disturbance of the system. In the control strategy the input u_{lat} contains two elements, such as

$$u_{lat} = \begin{bmatrix} \gamma_{l,ff} + \gamma_{l,fb} \\ \gamma_{r,ff} + \gamma_{r,fb} \end{bmatrix} \quad (19)$$

where the index ff is related to the feedforward compensation of the disturbance, while fb is denoted to the feedback control input.

The feedforward compensation of the disturbance is computed as follows. (16) is rewritten using the sum $u_{lat,ff} + u_{lat,fb}$, such as

$$\ddot{\delta}_i = \frac{\varepsilon_1 F_{l,i}}{J_{\delta,i}} \gamma_{i,ff} + \frac{\varepsilon_1 F_{l,i}}{J_{\delta,i}} \gamma_{i,fb} + \frac{(\varepsilon_2 \varphi^2 + \varepsilon_3 \varphi + \varepsilon_4) F_{l,i}}{J_{\delta,i}} \quad (20)$$

The role of $\gamma_{i,ff}$ is to compensate the measured disturbance φ . Thus, the feedforward control input is derived from the equation

$$\frac{\varepsilon_1 F_{l,i}}{J_{\delta,i}} \gamma_{i,ff} + \frac{(\varepsilon_2 \varphi^2 + \varepsilon_3 \varphi + \varepsilon_4) F_{l,i}}{J_{\delta,i}} = 0 \quad (21)$$

which results that

$$\gamma_{i,ff} = -\frac{\varepsilon_2 \varphi^2 + \varepsilon_3 \varphi + \varepsilon_4}{\varepsilon_1}. \quad (22)$$

However, in practice the measurement of φ is imprecise. It means that the compensation of the measured disturbance incorporates an error φ_e . The state-space representation of the compensated system is

$$\dot{x}_{lat} = A_{lat} x_{lat} + B_{lat,1} \varphi_e + B_{lat,2} u_{lat,fb} \quad (23)$$

The value of the disturbance, which is resulted by φ_e , is in the range of the sensor noise. Thus, the feedforward compensation reduces the disturbance significantly. The rest of the disturbance is compensated through the robust feedback controller.

The feedback is designed based on the robust $\mathcal{H}_\infty$ control method. The role of the feedback is to guarantee the trajectory tracking of the vehicle. It means that the vehicle must follow a reference yaw-rate signal $\dot{\psi}_{ref}$. The performance is defined as

$$z_{lat,1} = \dot{\psi}_{ref} - \dot{\psi} \quad |z_{lat,1}| \rightarrow \min \quad (24)$$

where $\dot{\psi}_{ref}$ is computed using the velocity and the steering wheel angle of the driver, see (Rajamani 2005). Moreover, the performance $z_{lat,1}$ must be reached using minimum control input $u_{lat} = [\gamma_l \ \gamma_r]^T$:

$$z_{lat,2} = u_{lat,fb} \quad |z_{lat,2}| \rightarrow \min \quad (25)$$

The performances are compressed in a vector $z_{lat} = [z_{lat,1} \ z_{lat,2}]^T$. The state-space representation of the system for the control design which incorporates the performances and the measurement is the following:

$$\dot{x}_{lat} = A_{lat} x_{lat} + B_{lat,1} \varphi_e + B_{lat,2} u_{lat,fb} \quad (26a)$$

$$z_{lat} = C_{lat,1} x_{lat} + D_{lat} u_{lat,fb} \quad (26b)$$

$$y_{lat} = C_{lat,2} x_{lat} \quad (26c)$$

5 SIMULATION EXAMPLE

where the measured signal is $y = \dot{\psi}$. The augmented plant for the $\mathcal{H}_\infty$ design is illustrated in Figure 4. In the robust control design weighting functions are considered, which are formed as transfer functions, e.g. $W_{p,1}$, $W_{p,2}$ are the weights on the performance signals and W_{ref} scales the reference signal. In the robust control design the disturbance of the system is also handled through the block W_{dist} . This transfer function represents the amplitude and frequency range of the chassis roll measurement error.

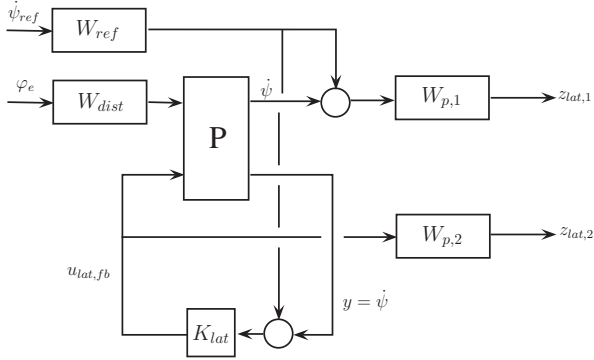


Figure 4: Closed-loop interconnection structure

The objective of $\mathcal{H}_\infty$ control is to minimize the $\mathcal{H}_\infty$ -norm of the cosensitivity function $T_{z_\infty w}$. More precisely, the problem can be stated as follows (Scherer & Weiland 2000, Boyd, Ghaoui, Feron, & Balakrishnan 1997). The LMI problem of $\mathcal{H}_\infty$ performance is formulated as follows: the closed-loop RMS gain from w to z_∞ does not exceed Γ if and only if there exists a symmetric and definite positive matrix X_∞ such that

$$\begin{bmatrix} A_{cl}X_\infty + X_\infty A_{cl}^T & X_\infty B_{cl} & C_{cl}^T \\ B_{cl}^T X_\infty & -\Gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\Gamma I \end{bmatrix} < 0 \quad (27)$$

with $\Gamma > 0$. It means that it is necessary to find an admissible control K_{lat} for the open-loop plant P , which satisfies the following design criteria:

- the closed-loop system must be asymptotically stable,
- the closed-loop transfer function from w to z_∞ satisfies the constraint:

$$\|T_{z_\infty w}\|_\infty < \Gamma, \quad (28)$$

for a given real positive value Γ ,

where the index cl is related to the matrices of the closed-loop system. The resulted robust control design guarantees the trajectory tracking of the vehicle through the computation of the appropriate $\gamma_{l,fb}$, $\gamma_{r,fb}$.

Finally, the control inputs of the high-level layer are computed as

$$\gamma_i = \gamma_{i,ff} + \gamma_{i,fb} \quad (29)$$

The resulted γ_l, γ_r signals are used as the reference of the low-level layer, see (Német, Fényes, & Gáspár 2016).

Finally, a simulation example is presented, in which the efficiency and the operation of the suspension system is demonstrated. In the simulation the vehicle must perform a double-lane change maneuver, which is formulated as a trajectory tracking control task. The selected vehicle is a full-size passenger car, whose mass is 1690 kg, the front suspensions are McPherson types. The simulation scenario is performed in CarSim, the control algorithm is implemented in Matlab/Simulink.

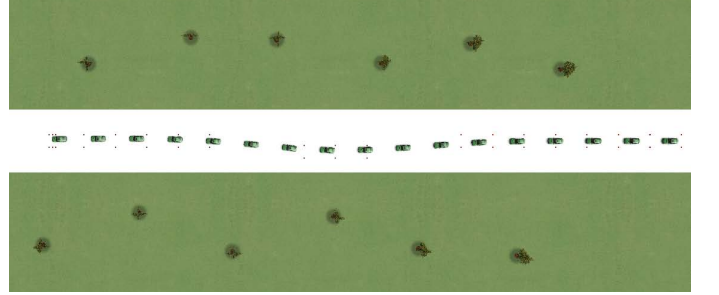


Figure 5: Vehicle trajectory

The course and the trajectory of the vehicle are illustrated in Figure 5. Since the vehicle avoids lane departure during the maneuvers, the designed control is successful, the steering of the front wheels is adequately actuated. In this simulation the velocity of the vehicle is 130 km/h. The details of the simulation are shown in Figure 6.

The yaw-rate tracking performance is shown in Figure 6(a). It can be seen that the tracking of the vehicle is performed with only a small error. This is the reason why lane departure is avoided. The camber angle control input of the front left wheel is presented in Figure 6(b). The feed-forward input γ_{ff} compensates for the effect of the chassis roll, see Figure 6(c). Moreover, the feedback signal γ_{fb} guarantees the yaw-rate tracking. The total camber angle,

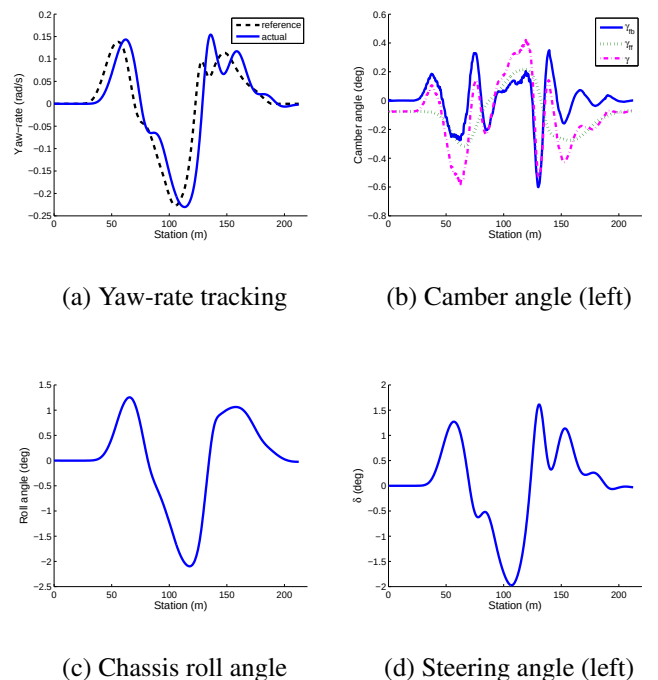


Figure 6: Simulation results

which is the sum of γ_{ff} and γ_{fb} yields the steering angle, see Figure 6(d).

As a consequence, the resulted control strategy is able to guarantee the trajectory tracking of the vehicle through the modification of the camber and the steering angles.

6 CONCLUSIONS

In the paper the modeling and control of independent steering based on the variable-geometry suspension have been proposed. In the presented solution the variation of the camber angle and the scrub radius of the wheels are actuated. The nonlinear formulation and the control-oriented linearizing of the suspension system, together with the modeling of the lateral dynamics have been presented. The multi-body analysis of the suspension system showed that the chassis roll angle has an important role in the positioning of the wheels. In the paper a robust combined feedforward-feedback control strategy is proposed, which is able to guarantee the trajectory tracking of the vehicle and to compensate for the disturbance effect of the chassis roll.

ACKNOWLEDGEMENT

The research was supported by the National Research, Development and Innovation Fund through the project "SEP-PAC: Safety and Economic Platform for Partially Automated Commercial vehicles" (VKSZ 14-1-2015-0125).

This paper was partially supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

REFERENCES

Boyd, S., L. E. Ghaoui, E. Feron, & V. Balakrishnan (1997). *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: Society for Industrial and Applied Mathematics.

Evers, W., A. van der Knaap, I. Besselink, & H. Nijmeijer (2008). Analysis of a variable geometry active suspension. *International Symposium on Advanced Vehicle Control, Kobe, Japan*.

Goodarzia, A., E. Oloomia, & E. Esmailzadehb (2010). Design and analysis of an intelligent controller for active geometry suspension systems. *Vehicle System Dynamics* 49(1), 333–359.

Hu, C., H. Jing, R. Wang, F. Yan, & N. Chen (2015). Fault-tolerant control of FWIA electric ground vehicles with differential drive assisted steering. *9th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, Paris, France* 48(21), 1180–1185.

Lee, H., U. Lee, S. Ha, & C. Han (1999). Four-wheel independent steering (4wis) system for vehicle handling improvement by active rear toe control. *JSME International Journal Series C* 42(4), 947–956.

Lee, S., H. Sung, & U. Lee (2005). A study to the enhancement of vehicle stability by active geometry control suspension (AGCS) system. *13th International Pacific Conference on Automotive Engineering, Gyeongju, Korea*, 1–6.

Lee, U. K. & C. S. Han (2001). A suspension system with a variable roll centre for the improvement of vehicle handling characteristics. *Proceedings of the IMechE, Part D: Journal of Automobile Engineering* 215(6), 677–696.

Németh, B., D. Fényes, & P. Gáspár (2016). Independent wheel steering control design based on variable-geometry suspension. *8th IFAC Advances in Automotive Control, Kolmarden, Sweden*.

Németh, B. & P. Gáspár (2013). Control design of variable-geometry suspension considering the construction system. *IEEE Transactions on Vehicular Technology* 62(8), 4104–4109.

Pacejka, H. B. (2004). *Tyre and vehicle dynamics*. Oxford: Elsevier Butterworth-Heinemann.

Piyabongkarn, D., T. Keviczky, & R. Rajamani (2004). Active direct tilt control for stability enhancement of a narrow commuter vehicle. *Int. J. Automotive Technology* 5(2), 77–88.

Rajamani, R. (2005). *Vehicle dynamics and control*. Springer.

Ronci, M., P. Artuso, & E. Bocci (2011). Four independent wheels steering system analysis. *SAE Technical Paper*.

Scherer, C. & S. Weiland (2000). *Lecture Notes DISC Course on Linear Matrix Inequalities in Control*. Delft, Netherlands: Delft University of Technology.

Wang, J., Q. Wang, L. Jin, & C. Song (2011). Independent wheel torque control of 4WD electric vehicle for differential drive assisted steering. *Mechatronics* 21, 63–76.