

Handling of zero-crossing problems in the design of variable-geometry suspension control

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Abstract—The electrification of the vehicle drivelines provides a new possibility in the lateral control strategies of the vehicle. Through the independent control of the wheels the steering and driving capability of the vehicle can be improved. Although the enhanced solutions, such as torque vectoring and variable-geometry suspension result in additional functionalities, several new challenges must also be performed. In the paper the handling of the zero-crossing problems in the design of the variable-geometry suspension control is presented. In the algorithm a control rule in a hierarchical structure based on experimental results is formed, which is able to guarantee the trajectory tracking performance of the control system.

I. INTRODUCTION AND MOTIVATION

The research in variable-geometry suspension and torque vectoring control of the vehicles are novel trends in the automotive industry, which are able to enhance the stability and the agility of the road vehicles. The aim of the suspension control is the modification of the geometry, which results in a change in the camber or the toe angle. A rear-suspension active toe control for the enhancement of driving stability is proposed by [1]. The active tilt control system, which assists the driver in balancing the vehicle and performs tilting in the bend, is an essential part of a narrow vehicle system, see [2]. These vehicles require the design of innovative active wheel tilt and steer control strategies in order to perform steering similarly to a car on straight roads but in bends they tilt as motorcycles, see [3]. The advantages of the variable-geometry suspension are the simple structure, low energy consumption and low cost compared to other mechanical solutions such as an active front wheel steering, see [4], [5].

In our previous results in the field of the variable-geometry suspension a novel solution is presented [6], [7]. The aim of the method is the modification of the camber angle, the scrub radius is varied. Furthermore, the scrub radius and the longitudinal force on the wheels influence the steering angle of the wheel together. Through the control of the camber angle and the longitudinal forces on the wheels, the trajectory tracking of the vehicle can be influenced. In the design of the suspension control, the camber angle γ_i of the wheel is computed as

$$\gamma_i = \frac{u_i}{F_{l,i}} \quad (1)$$

where u_i is the output of the controller, $F_{l,i}$ is the longitudinal control force on the wheel and i index is related to the left

or real wheel. If $F_{l,i}$ is close to zero, then the tilting of the wheel is increased insufficiently, such as

$$\lim_{F_{l,i} \rightarrow 0} \frac{F_{l,i} \gamma_i}{F_{l,i}} \Rightarrow \infty \quad \gamma_i \Rightarrow \infty \quad (2)$$

The zero-crossing problem is illustrated in Figure 1. The increase in the camber angle has two consequences:

- the required camber angle cannot be guaranteed due to the physical limits of the wheel tilting,
- the sharp increase of the camber angle is disadvantageous in the motion and lateral stability of the vehicle.

In this paper the handling of the zero-crossing problem in the design of the suspension control is presented. The method of the solution is based on the tuning of control input limitation functions, which are built in the control architecture.

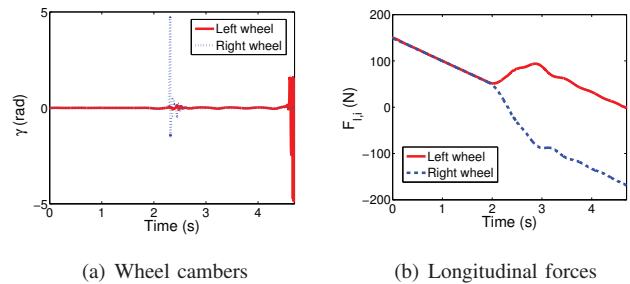


Figure 1. Problem formulation

The paper is organized as follows. The formulation of the lateral vehicle model with the consideration of the variable-geometry suspension is presented in Section II. The optimal control design of the system, incorporating the handling of the zero-crossing problem is found in Section III. Section IV demonstrates the efficiency of the control system through simulations. Finally, Section V summarizes the contribution of the paper.

II. MODELING OF THE LATERAL DYNAMICS

In this section the modeling of the lateral dynamics of the vehicle, considering the variable-geometry suspension is presented. The lateral dynamics of the vehicle is influenced in two ways: the wheel camber angle γ and the steering angle have lateral force $F_{l,i}$ effects. Therefore, in the lateral dynamics both effects are considered. However, in this paper the

impacts on the modeling of the variable-geometry suspension construction is not incorporated, it is found in [6].

The lateral motion is described by the following equations, see [8]

$$J\ddot{\psi} = C_1 l_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) - C_2 l_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) + M_d \quad (3a)$$

$$mv(\dot{\psi} + \dot{\beta}) = C_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) + C_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) \quad (3b)$$

where m is the vehicle mass, J is the yaw-inertia of the vehicle, l_1 and l_2 are geometric parameters, C_1 and C_2 are cornering stiffness values, ψ is the yaw angle of the vehicle, β is the side-slip angle and M_d is the generalized yaw moment.

The steering rotation of the wheel is formulated using a dynamical equation, such as

$$\ddot{\delta}_i = \frac{r_{\delta,i} F_{l,i}}{J_{\delta,i}} \quad (4)$$

where $r_{\delta,i}$ represents the scrub radius, and $J_{\delta,i}$ is the inertia of the steering system for one wheel. Due to the construction of the suspension, $r_{\delta,i}$ is in relation with the camber angle. This relationship depends on the construction parameters. Therefore, $\gamma_i = \{(\gamma_i)\}$ is determined through the investigation of the construction, e.g. using Matlab SimMechanics model. In this paper a McPherson suspension of a passenger car is selected, by which the relation is formed as

$$r_{\delta,i} = \varepsilon \gamma_i \quad (5)$$

where ε is a construction parameter. (4) is transformed as

$$\ddot{\delta}_i = \frac{\varepsilon F_{l,i}}{J_{\delta,i}} \gamma_i \quad (6)$$

However, in the single-track model (3) δ represents a common steering wheel, computed as the average of the left and right steering wheel angles:

$$\delta = \frac{\delta_l + \delta_r}{2} \quad (7)$$

In case of in-wheel vehicles, the control input M_d determines the longitudinal force on the wheels. It has impact on the lateral dynamics of the vehicle, and on the steering of the wheels, see (6). The longitudinal forces are determined through M_d and the required traction force F_0 of the speed controller, such as

$$F_{l,l} = F_0 - \frac{M_d}{b_f} \quad (8a)$$

$$F_{l,r} = F_0 + \frac{M_d}{b_f} \quad (8b)$$

where b_f is the lengths of the track on the front axis.

The lateral dynamics of the vehicle is formed using (3), (6), (7) and (8). The states of the model are defined as $x = [\dot{\psi} \ \beta \ \dot{\delta}_l \ \delta_l \ \dot{\delta}_r \ \delta_r]^T$. The state-space representation is

$$\dot{x} = Ax + Bu \quad (9)$$

where A and B are system matrices. The control input vector is defined in the following form

$$u = [F_{l,l} \gamma_l \ F_{l,r} \gamma_r \ M_d]^T. \quad (10)$$

III. OPTIMAL CONTROL DESIGN OF THE SYSTEM

The aim of the optimal control design is to guarantee the safety trajectory tracking of the vehicle through the modification of the camber angles and the longitudinal forces on the steered wheels. In this section a hierarchical structure of the control is proposed, which contains several layers.

A. Lateral control design

The goal of the lateral control design is to guarantee the trajectory tracking of the vehicle. It means that the vehicle must follow a reference yaw-rate signal $\dot{\psi}_{ref}$. The performance is defined as

$$z_1 = \dot{\psi}_{ref} - \dot{\psi} \quad |z_1| \rightarrow \min \quad (11)$$

where $\dot{\psi}_{ref}$ is computed using the velocity and the steering wheel angle of the driver, see [8]. Moreover, the performance z_1 must be reached using minimum control input $u_{lat} = [F_{l,l} \gamma_l \ F_{l,r} \gamma_r \ M_d]^T$:

$$z_2 = u_{lat} \quad |z_2| \rightarrow \min \quad (12)$$

The tracking criterion of the control system requires the reformulation of the state-space equation described in (10). The plant is augmented with an integrator on the performance z_1 to achieve zero steady-state error. The augmented system is written as follows:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{z}_1 \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -c & 0 \end{bmatrix} \begin{bmatrix} x \\ z_1 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{\psi}_{ref} \\ &= \tilde{A} \begin{bmatrix} x \\ z_1 \end{bmatrix} + \tilde{B} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \dot{\psi}_{ref} \end{aligned} \quad (13)$$

where $\tilde{A}$ and $\tilde{B}$ are the extended block matrices and c is resulted from $\dot{\psi} = [1 \ 0 \ 0 \ 0 \ 0 \ 0] x = cx$.

A balance between the performances in the control design is reached through the Linear Quadratic (LQ) method. The LQ controller design is based on the minimization of a cost function, which incorporates the performances z_1 and z_2 , such as:

$$J = \frac{1}{2} \int_0^\infty [z_1^T Q z_1 + z_2^T R z_2] dt \rightarrow \min. \quad (14)$$

The weights Q and R guarantee a balance between the performances. The minimization problem leads to a continuous time algebraic Riccati equation (CARE) [9]:

$$V\tilde{A} + \tilde{A}^T V - V\tilde{B}R^{-1}\tilde{B}^T V + Q = 0 \quad (15)$$

where V is the solution of CARE. The optimal state feedback LQ controller K is derived from V , such as

$$K = R^{-1}\tilde{B}^T V. \quad (16)$$

B. Improvement of the lateral control

The designed K controller results in the control inputs $F_{l,l}\gamma_l, F_{l,r}\gamma_r, M_d$. The torque M_d is distributed using the relationship (8), which leads to the computation of $F_{l,i}$.

In case of the computation of the γ_i , a division must be performed:

$$\gamma_i = \frac{u_i}{F_{l,i}}. \quad (17)$$

In the introduction of the paper the zero-crossing problem is illustrated in an example. In the following the handling of this problem based on an experimental tuning method is presented.

First, $F_{up} > 0$ upper and $F_{low} < 0$ lower limits are defined using previous experimental results. In the region $F_{l,i} \in [F_{low}, F_{up}]$ the zero-crossing problem is appeared, since the sign of $F_{l,i}$ is changed, e.g. at a braking maneuver. Between the limits, the longitudinal forces are modified based on the following rules:

$$\begin{aligned} \text{if } 0 \leq F_{l,i} < F_{up}, \text{ then } F_{l,mod,i} &= F_{up} \\ \text{if } F_{low} < F_{l,i} < 0, \text{ then } F_{l,mod,i} &= F_{low} \end{aligned} \quad (18)$$

Since this rule modifies the result of the torque distribution M_d , it must be compensated on the rear wheels:

$$F_{r,i} = F_{l,i} - F_j \quad (19)$$

where $F_{r,i}$ represents the longitudinal control force on the rear wheels, and

$$\begin{aligned} \text{if } 0 \leq F_{l,i} < F_{up}, \text{ then } j &= up \\ \text{if } F_{low} < F_{l,i} < 0, \text{ then } j &= low \end{aligned} \quad (20)$$

Using this algorithm the zero-crossing is avoided. However, $F_{l,mod,i}$ changes rapidly between the limits F_{low} and F_{up} , which also results in the rapid variation of γ_i . It is handled in the next step.

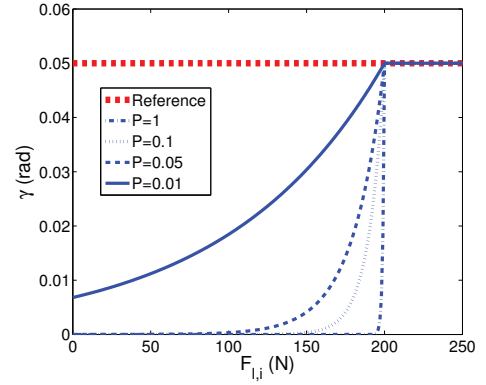
Second, the camber angle of the wheel must be also modified. In the range $F_{l,i} \in [F_{low}, F_{up}]$, the following rule on γ_i is applied:

$$\gamma_{mod,i} = e^{-|F_j - F_{l,i}|P} \gamma_i \quad (21)$$

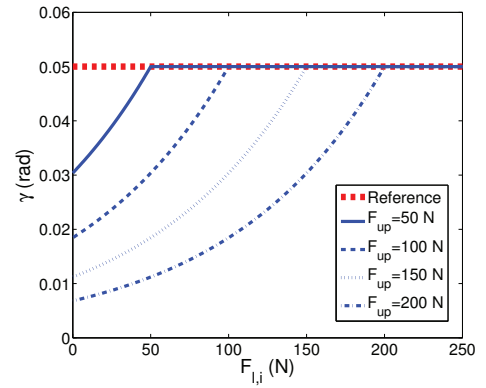
where P is a tuning parameter. The form of the function for different scenarios are found in the illustrative example Figure 2. The selected exponential function is suitable for the problem, because it guarantees the smooth variation of γ . If $F_{l,i} \rightarrow 0$, then

$$\lim_{F_{l,i} \rightarrow 0} \gamma_{mod,i} = \lim_{F_{l,i} \rightarrow 0} e^{-|F_j - F_{l,i}|P} \gamma_i \rightarrow 0. \quad (22)$$

It means that the insufficiently high camber angles can be avoided through this algorithm. As a result of the improvement, the modified control inputs $F_{l,mod,i}$ and $\gamma_{mod,i}$ are actuated in the variable geometry suspension instead of $F_{l,i}$ and γ_i .



(a) Parameter: P



(b) Parameter: F_{up}

Figure 2. Illustration of the modification function

C. Architecture of the control systems

In Figure 3 the architecture variable-geometry suspension control is presented. It illustrates the cooperation of the different controllers, such as the actuator control, the lateral control, the distribution of the wheel forces and the handling strategy of the zero-crossing problem. The lateral dynamic control computes the required left and right wheel camber angles, together with the yaw moment using the yaw-rate error. Moreover, the longitudinal forces are distributed, the force and the camber angle signals are modified based on the previous rules. The variable-geometry suspension control generates the actuation torques $M_{act,l}$ and $M_{act,r}$, by which the steering of the wheels is realized. The reference trajectory is designed through the velocity of the vehicle and the cornering intention of the driver, estimated from the steering wheel angle δ_{driver} .

The controllers are ordered in a hierarchical structure with several subsystems. The stability and performance problems of the hierarchical control design, and further solutions are found in [10], [11].

IV. SIMULATION EXAMPLE

In this section, the efficiency of the proposed control system is demonstrated. In the scenario a D-class passenger car

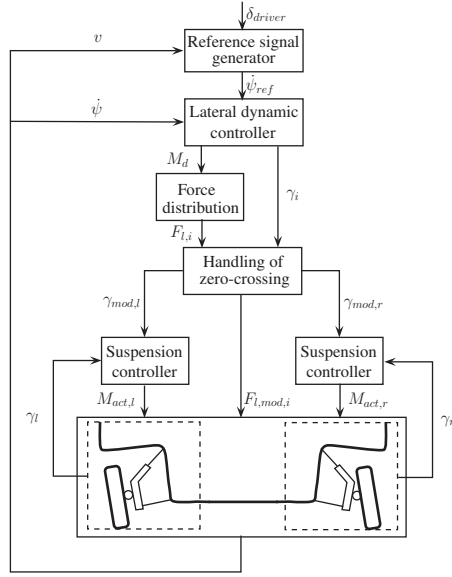
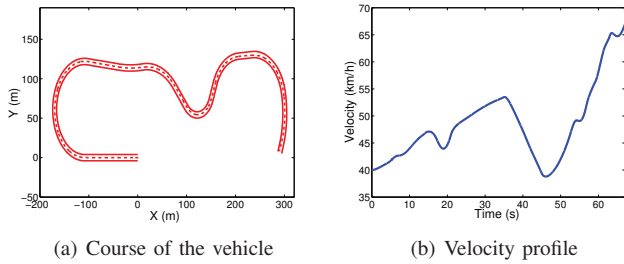


Figure 3. Architecture of the control systems

is used, whose total mass is $m = 1690 \text{ kg}$. The camber angles and the active torques of the first wheels are controlled by the presented control system. Therefore, the aim of the simulation is to show the trajectory tracking capability of the entire system. The simulations are performed through the high-fidelity CarSim software, while the control algorithm is designed in Matlab/Simulink.

Figure 4(a) shows the course and the velocity trajectory of the vehicle. The track contains several bends, which results in the velocity variation of the vehicle during the simulations, as shown in Figure 4(b)



(a) Course of the vehicle (b) Velocity profile

Figure 4. Track of the vehicle

Figure 5(a) illustrates the yaw-rate tracking, together with the reference and the actual signal. It can be seen that the difference between the two signals is small, thus the designed control guarantees the performance z_1 . The control inputs of the system, such as the camber angles and the generalized yaw moment M_d are shown in Figures 5(b)...(d). The results of the lateral dynamical control are transformed to the suspension actuator torque M_{act} , see Figures 5 (e),(f). The maximum of the torques is around 320 Nm , which is a reasonable value for the suspension actuator. Figures 5(g),(h) present the original $F_{l,i}$ and the modified $F_{l,mod,i}$ longitudinal forces, respectively. In this simulation the low and the upper limits are selected

symmetrically, as $\pm 200 \text{ N}$. Finally, the steering angles of the wheels are illustrated in Figure 5(i). The steering angle of the wheels are the results of the camber angles and the longitudinal forces. Although the γ_{mod} and the $F_{l,mod,i}$ values are different in the wheels, the same steering angles are resulted on the left and the right sides.

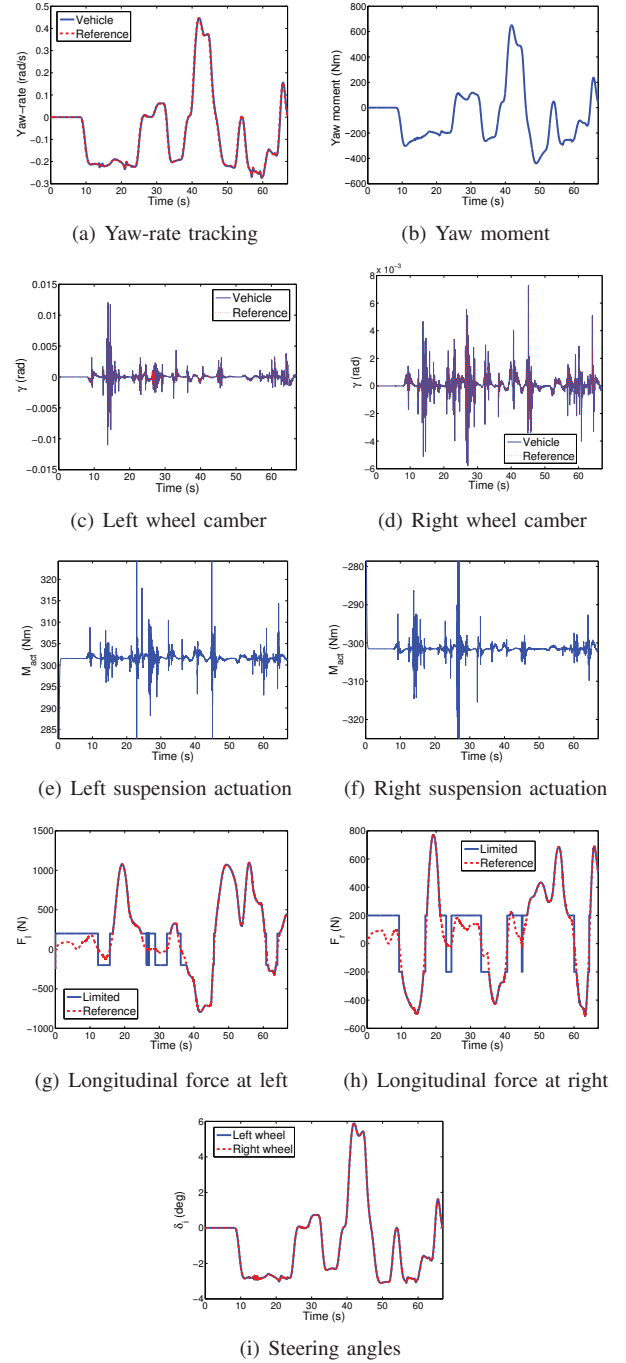


Figure 5. Simulation results

The presented simulation shows that the variable-geometry suspension control together with the torque vectoring is able to guarantee the trajectory tracking of the vehicle. Although the

longitudinal force on the wheels varies during the simulation significantly, the disadvantageous effects of the zero-crossing problem is successfully handled.

V. CONCLUSION

In the paper the handling of the zero-crossing problems in the design of the variable-geometry suspension control is presented. An algorithm is proposed, which is able to guarantee the tracking of a predefined trajectory using the suspension based steering system and the torque distribution. The main contribution of the paper is the improvement of the previous solutions, thus the zero-crossing of the longitudinal control force is handled in a hierarchical control architecture. Finally, high-fidelity simulations have illustrated the efficiency of entire vehicle dynamic control system.

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