

Optimal control design of a variable-geometry suspension with electro-hydraulic actuator

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Abstract—The electrification of the vehicle drivelines provides a new possibility in the lateral control strategies of the vehicle. Through the independent control of the wheels the steering and driving capability of the vehicle can be improved. Although the enhanced solutions, such as torque vectoring and variable-geometry suspension result in additional functionalities, several new challenges must also be performed. In the paper the handling of the zero-crossing problems in the design of the variable-geometry suspension control is presented. In the algorithm a control rule in a hierarchical structure based on experimental results is formed, which is able to guarantee the trajectory tracking performance of the control system.

I. INTRODUCTION AND MOTIVATION

The research in variable-geometry suspension and torque vectoring control of the vehicles are novel trends in the automotive industry, which are able to enhance the stability and the agility of the road vehicles. The aim of the suspension control is the modification of the geometry, which results in a change in the camber or the toe angle [1]. A rear-suspension active toe control for the enhancement of driving stability is proposed by [2]. The active tilt control system, which assists the driver in balancing the vehicle and performs tilting in the bend, is an essential part of a narrow vehicle system, see [3].

In this paper a novel vehicle dynamic control system for the lateral motion control of the vehicle is presented. The modeling and control of the variable-geometry suspension, which uses electro-hydraulic actuator are proposed. Moreover, the solution of the zero-crossing problem of the actuator control system is also presented.

The paper is organized as follows. The formulation of the lateral vehicle model with the consideration of the variable-geometry suspension and the modeling of the hydraulic actuator are presented in Section II. The optimal control design of the system, incorporating the Linear Quadratic (LQ) based control design is found in Section III. Section IV demonstrates the efficiency of the control system through simulations. Finally, Section V summarizes the contributions of the paper.

II. MODELING OF THE SYSTEM

A. Modeling of the lateral dynamics

In the following, the modeling of the lateral dynamics of the vehicle concerning the variable-geometry suspension system is presented. The lateral dynamics of the vehicle is influenced in two ways: the wheel camber angle γ and the steering

angle have lateral force $F_{l,i}$ effects. Therefore, in the lateral dynamics both effects are considered. The modeling of the variable-geometry suspension is found in [1].

The lateral motion is described by the following equations, see [4]

$$J\ddot{\psi} = C_1 l_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) - C_2 l_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) + M_d \quad (1a)$$

$$mv(\dot{\psi} + \dot{\beta}) = C_1 \left(\delta - \beta - \frac{\dot{\psi} l_1}{v} \right) + C_2 \left(-\beta + \frac{\dot{\psi} l_2}{v} \right) \quad (1b)$$

where m is the vehicle mass, J is the yaw-inertia of the vehicle, l_1 and l_2 are geometric parameters, C_1 and C_2 are cornering stiffness values, ψ is the yaw angle of the vehicle, β is the side-slip angle and M_d is the generalized yaw moment.

The steering rotation of the wheel is formulated using a dynamical equation, such as

$$\ddot{\delta}_i = \frac{r_{\delta,i} F_{l,i}}{J_{\delta,i}} \quad (2)$$

where $r_{\delta,i}$ represents the scrub radius, and $J_{\delta,i}$ is the inertia of the steering system for one wheel. Due to the construction of the suspension, $r_{\delta,i}$ is in relation with the camber angle. This relationship depends on the construction parameters. Therefore, the function γ_i is determined through the investigation of the construction, e.g. using Matlab SimMechanics model. A simulation result is found in Figure 1.

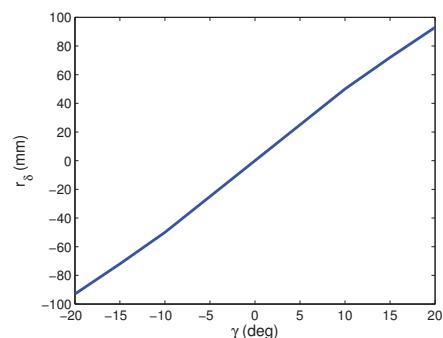


Figure 1. Hydraulic actuator

In this paper a McPherson suspension of a passenger car is selected, by which the relation is formed as

$$r_{\delta,i} = \varepsilon \gamma_i \quad (3)$$

where ε is a construction parameter. (2) is transformed as

$$\ddot{\delta}_i = \frac{\varepsilon F_{l,i}}{J_{\delta,i}} \gamma_i \quad (4)$$

However, in the single-track model (1) δ represents a common steering wheel, computed as the single of the left and right steering wheel angles:

$$\delta = \frac{\delta_l + \delta_r}{2} \quad (5)$$

In case of in-wheel vehicles, the control input M_d determines the longitudinal force on the wheels. It has impact on the lateral dynamics of the vehicle, and on the steering of the wheels, see (4). The longitudinal forces are determined through M_d and the required traction force F_0 of the speed controller, such as

$$F_{l,l} = F_0 - \frac{M_d}{b_f} \quad (6a)$$

$$F_{l,r} = F_0 + \frac{M_d}{b_f} \quad (6b)$$

where b_f is the lengths of the track on the front axis.

The lateral dynamics of the vehicle is formed using (1), (4), (5) and (6). The states of the model are defined as $x = [\psi \ \beta \ \dot{\delta}_l \ \delta_l \ \dot{\delta}_r \ \delta_r]^T$. The state-space representation is

$$\dot{x} = Ax + Bu \quad (7)$$

where A and B are system matrices. The control input vector is defined in the following form

$$u = [F_{l,l} \gamma_l \ F_{l,r} \gamma_r \ M_d]^T. \quad (8)$$

In the paper the focus is on the realization possibilities of the computed control input u . Therefore, in the following the modeling of the electro-hydraulic system is presented.

B. Modeling of the hydraulic actuator

In this section the modeling of the hydraulic actuator is presented. The aim of the actuator is to realize the desired camber angle (γ). The control input of the system is the position of the valve $A(x_v)$ and the output of the system is the displacement of the piston. The chosen actuator is linear and asymmetric.

The actuation of the variable-geometry suspension is a crucial problem of the vehicle control systems. Nowadays, there are two different constructions, such as pure electronic and electro-hydraulic solutions. The advantage of the electronic solution is the simple construction, which requires only electric power source. However, the electric motor is able to actuate only limited control torque, and it requires a large space in the vehicle. The other actuator type, the electro-hydraulic construction is capable to guarantee an increased control intervention, and the hydraulic cylinder can be built-in

small places at the suspension. Moreover, the static position of the wheel can be guaranteed without continuous energy inflow in the system. As a disadvantage, it requires external hydraulic power sources and further elements to realize the pressure supply. In this paper the electro-hydraulic solution is preferred, of which scheme is found in Figure 2.

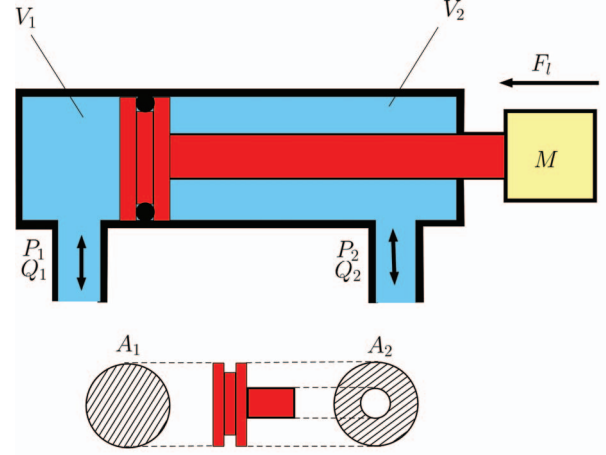


Figure 2. Hydraulic actuator

The nonlinear model of the hydraulic actuator is derived from four hydrodynamic equations and a motion equation [5]. The chosen actuator is a double-acting actuator, which means that the actuator can be supplied by both side. Thus, the nonlinear model is written for two cases (Extending and Retracting).

The hydrodynamic equations can be divided into two types:

- 1) The equations of the flows (9b), (9d), (10b), (10d) depend on the pressure difference ΔP and the surface of the valve $A(x_v)$.
- 2) The equations of the pressures (9a), (9c), (10a), (10c) depend on the flows Q_i and on the position (x_p) and the velocity ($\dot{x}_p$) of the piston.

The notations are the following:

V_i : initial volumes, B_f : Bulk modulus, α : internal cross-port leakage coefficient, l : max. displacement of the piston, C_f : leakage coefficient, b : viscous damping coefficient, P_s : supply pressure, P_0 : environmental pressure, $A(x_v)$: the surface of the valve, ρ : density of the fluid, m : the mass of the piston x_p : the position of the piston.

a) Extending: [5]

$$\dot{P}_1 = \frac{B_f}{A_1 x_p + V_1} (Q_1 - \alpha(P_1 - P_2) - A_1 \dot{x}_p) \quad (9a)$$

$$Q_1 = C_f |A(x_v)| \sqrt{\frac{2|P_s - P_1|}{\rho}} \quad (9b)$$

$$\dot{P}_2 = \frac{B_f}{A_2(l - x_p) + V_2} (-Q_2 + \alpha(P_1 - P_2) + A_2 \dot{x}_p) \quad (9c)$$

$$Q_2 = C_f |A(x_v)| \sqrt{\frac{2|P_2 - P_0|}{\rho}} \quad (9d)$$

$$m \ddot{x}_p = F_l - b \dot{x}_p + A_1 P_1 - A_1 P_2 \quad (9e)$$

b) Retracting: [5]

$$\dot{P}_1 = \frac{B_f}{A_1 x_p + V_1} (-Q_1 + \alpha(P_2 - P_1) + A_1 \dot{x}_p) \quad (10a)$$

$$Q_1 = C_f |A(x_v)| \sqrt{\frac{2|P_1 - P_0|}{\rho}} \quad (10b)$$

$$\dot{P}_2 = \frac{B_f}{A_2(l - x_p) + V_2} (Q_2 - \alpha(P_2 - P_1) - A_2 \dot{x}_p) \quad (10c)$$

$$Q_2 = C_f |A(x_v)| \sqrt{\frac{2|P_s - P_2|}{\rho}} \quad (10d)$$

$$m\ddot{x}_p = F_l - b\dot{x}_p + A_1 P_1 - A_1 P_2 \quad (10e)$$

The states of the model are: $x_{hyd} = [P_1 \ P_2 \ \dot{x}_p \ x_p]^T$. The state-space representation is derived from (9) and (10):

$$\dot{x}_{hyd} = A_{hyd} x_{hyd} + B_{hyd} u_{hyd} \quad (11)$$

The control input of the system is $u_{hyd} = A(x_v)$.

III. OPTIMAL CONTROL DESIGN OF THE SYSTEM

The aim of the optimal control design is to guarantee the safety trajectory tracking of the vehicle through the modification of the camber angles and the longitudinal forces on the steered wheels. The optimal control of the vehicle dynamics is found in [1]. In the following the design of the hydraulic actuator controller is presented. Moreover, this controller is taken part in a hierarchical structure, which is also illustrated.

The goal of the control design is to guarantee the desired position of the piston. It means that the actuator must follow a reference position signal $x_{p,ref}$. The performance is defined as

$$z_1 = x_{p,ref} - x_p \quad |z_1| \rightarrow \min \quad (12)$$

where $x_{p,ref}$ is computed using the reference camber angle γ_{ref} . Moreover, the performance z_1 must be reached using minimum control input $u_{act} = A(x_v)$:

$$z_2 = u_{act} \quad |z_2| \rightarrow \min \quad (13)$$

The tracking criterion of the control system requires the reformulation of the state-space equation described in (8). The plant is augmented with an integrator on the performance z_1 to achieve zero steady-state error. The augmented system is written as follows:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{hyd} \\ \dot{z}_1 \end{bmatrix} &= \begin{bmatrix} A_{hyd} & 0 \\ -c & 0 \end{bmatrix} \begin{bmatrix} x_{hyd} \\ z_1 \end{bmatrix} + \begin{bmatrix} B_{hyd} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_{p,ref} \\ &= \tilde{A}_{hyd} \begin{bmatrix} x_{hyd} \\ z_1 \end{bmatrix} + \tilde{B}_{hyd} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_{p,ref} \end{aligned} \quad (14)$$

where $\tilde{A}_{hyd}$ and $\tilde{B}_{hyd}$ are the extended block matrices and c is resulted from $x_{p,ref} = [0 \ 0 \ 0 \ 1 \ 0 \ 0] x_{hyd} = c_{hyd} x_{hyd}$.

A balance between the performances in the control design is reached through the Linear Quadratic (LQ) method. The LQ controller design is based on the minimization of a cost

function, which incorporates the performances z_1 and z_2 , such as:

$$J = \frac{1}{2} \int_0^\infty [z_1^T Q z_1 + z_2^T R z_2] dt \rightarrow \min. \quad (15)$$

The weights Q and R guarantee a balance between the performances. The minimization problem leads to a continuous time algebraic Riccati equation (CARE) [6]:

$$V \tilde{A}_{hyd} + \tilde{A}_{hyd}^T V - V \tilde{B}_{hyd} R^{-1} \tilde{B}_{hyd}^T V + Q = 0 \quad (16)$$

where V is the solution of CARE.

The optimal state feedback LQ controller K is derived from V , such as

$$K = R^{-1} \tilde{B}_{hyd}^T V. \quad (17)$$

As a result, $u_{act} = -K x_{hyd}$ is a continuous signal. However, in the physical realization the hydraulic valve has three discrete positions (left, right and closed). Thus, the computed control input u_{act} is modified based on the following rules:

$$\begin{aligned} \text{if } u_{act} > u_{lim}, & \text{ then } u_{act,mod} = \text{left position} \\ \text{if } -u_{lim} > u_{act}, & \text{ then } u_{act,mod} = \text{right position} \\ \text{if } -u_{lim} < u_{act} < u_{lim}, & \text{ then } u_{act,mod} = \text{closed} \end{aligned} \quad (18)$$

where u_{lim} is a design parameter, which is close to the maximum valve surface. The transformation of the control input signal is illustrated in Figure 3.

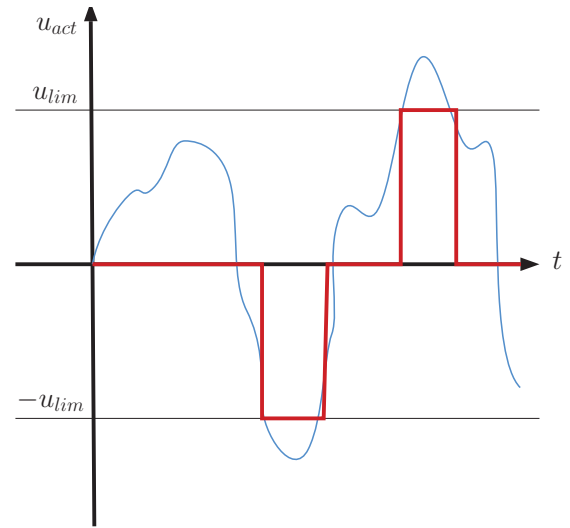


Figure 3. Illustration of the control input transformation

In Figure 4 the architecture variable-geometry suspension control in a hierarchical form [7], [8] is presented. It illustrates the cooperation of the different controllers, such as the actuator control, the lateral control and the control input generators. The lateral dynamic control computes the required left and right wheel camber angles, together with the yaw moment using the yaw-rate error. Moreover, the longitudinal forces are distributed and the Reference γ_i generator provides the reference chamber angles. The Reference $x_{p,i}$ generators

compute the reference position of the pistons from $\gamma_{ref,i}$ for the actuators. The actuator controllers generate the position of the valves, by which the variation of the suspension geometry is realized. The reference trajectory is designed through the velocity of the vehicle and the cornering intention of the driver, estimated from the steering wheel angle δ_{driver} .

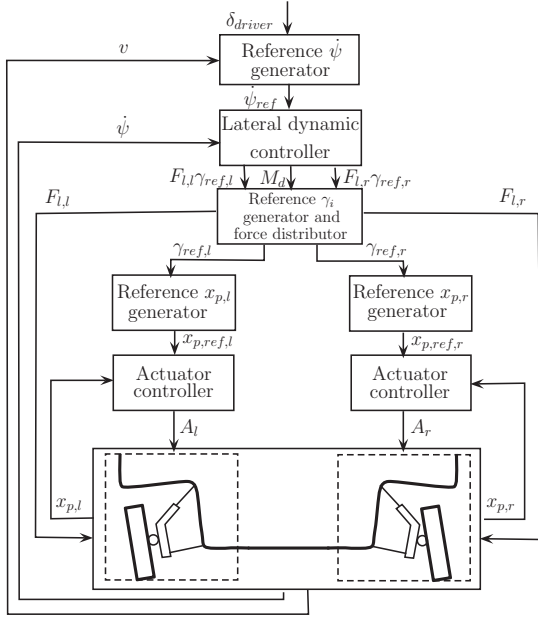


Figure 4. Architecture of the control systems

IV. SIMULATION EXAMPLE

In this section, the efficiency of the proposed control system is demonstrated. In the scenario a D-class passenger car is used, whose total mass is $m = 1690 \text{ kg}$. The camber angles and the active torques of the first wheels are controlled by the presented control system. Therefore, the aim of the simulation is to show the trajectory tracking capability of the entire system. In the example the maximum surface of the valve is 100 mm^2 . Therefore, the limit for the control input signal transformation is chosen as $u_{lim} = 99 \text{ mm}^2$. The simulations are performed through the high-fidelity CarSim software, while the control algorithm is designed in Matlab/Simulink.

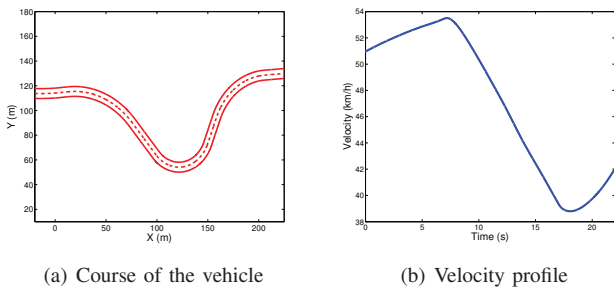


Figure 5. Track of the vehicle

Figure 5(a) shows the course and the velocity trajectory of the vehicle. The track contains a sharp bend, which is

a suitable maneuver to show the efficiency of the proposed control system. The velocity of the vehicle varies during the simulation, as shown in Figure 5(b).

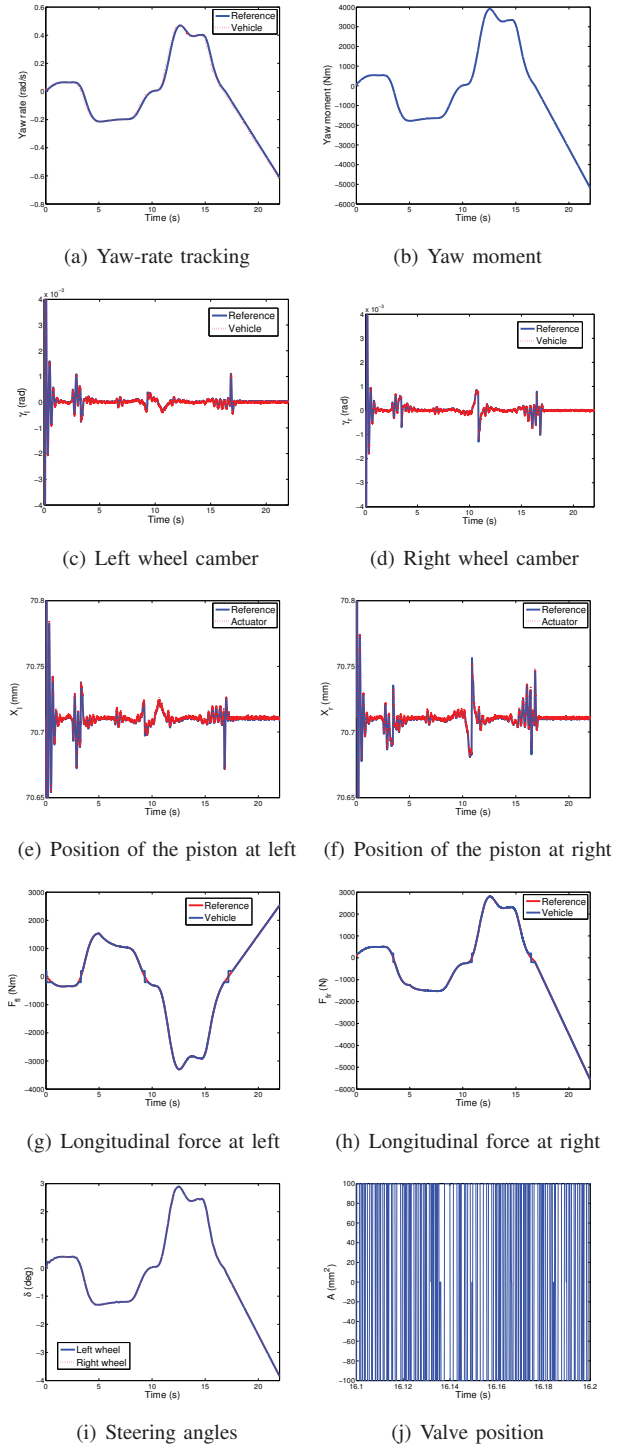


Figure 6. Simulation results

Figure 6(a) illustrates the yaw-rate tracking, together with the reference and the actual signal. It can be seen that the two signals are close to each other, therefore the proposed control system guarantees the performance z_1 . The control inputs of

the system, such as the camber angles and the generalized yaw moment M_d are shown in Figures 6(b)...(d). Figures 6(e),(f) show the displacement of the pistons. It can be seen that the maximum of the displacement is below $0.2mm$, which is a reasonable value for the electro-hydraulic actuator. Figures 6(g),(h) present the original $F_{l,i}$ and the modified $F_{l,mod,i}$ longitudinal forces, respectively. The steering angles of the wheels are illustrated in Figure 6(i). The steering angle of the wheels are the results of the camber angles and the longitudinal forces. Although γ_{mod} and $F_{l,mod,i}$ values are different in the wheels, the same steering angles are resulted in the left and the right sides. Finally, Figure 6(j) shows a section of the valve position. It can be seen that the valve has only three different positions. However, the tracking control of the entire system is precise enough.

V. CONCLUSION

In the paper the trajectory tracking of the vehicle based on the variable-geometry suspension system with the electro-hydraulic actuator has been presented. An algorithm is also proposed for the variable-geometry suspension-based steering and the torque distribution. The main contribution of the paper is the control design of the electro-hydraulic actuator, which is based on the optimal LQR method.

Finally, the efficiency of entire vehicle dynamic control system is illustrated through simulation examples. They show that the variable-geometry suspension control together with the torque vectoring is able to guarantee the trajectory tracking of the vehicle. Although the longitudinal force on the wheels varies significantly, the control system is able to realize the required steering angles.

ACKNOWLEDGMENT

The research was supported by the National Research, Development and Innovation Fund through the project "SEP-PAC: Safety and Economic Platform for Partially Automated Commercial vehicles" (VKSZ 14-1-2015-0125).

This paper was partially supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

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