

Analysis of Pressure Changes in Air Ducts

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Abstract

In air duct systems it is usually necessary to set up desired amount of air at every air terminal device, thus balancing is needed. In our paper we are going to show a sizing method which makes this balancing unnecessary. This is achieved by altering or keeping the desired static pressure along the length of the duct.

The needed static pressure can be achieved by changing the diameter and regaining the dynamic pressure. In our paper we distinguish between two different cases: the resistance of the diffusers along the air duct is constant or air is blown out through the permeable duct material (e.g. fabric.). We derived a dimensionless expression for the diameter's change along the duct's length. Numerically solving these expressions a function can be obtained which describes the diameter that enables uniform air distribution. In the paper we are going to examine the impact of dimensionless parameters, especially the Reynolds-number which is linked to the air flow rate. An interesting outcome of the investigations was that the diameter's function is sometimes nearly independent of the Reynolds-number.

Keywords – air duct sizing, air duct with permeable walls, uniform air distribution, regain of dynamic pressure

1. Objectives

Air duct systems have several different fields of application and system types. In our paper we are going to deal with supply air ducts, which are used as air terminal devices. Two different cases were found: air ducts with nozzles, grills, linear slot diffusers and air ducts made of permeable materials. In such ducts the evolving static pressure has a special importance (Fig. 1., [1]). The most important input parameter of air duct sizing is the air flow rate and the aim is to determine the cross-section. To do this a sizing method has to be used. The most common one is the velocity criteria but it is possible to size the ducts according to the equal friction method or static regain. These sizing methods cannot be applied to the investigated cases.

The goal of the paper is to calculate the diameters along the duct with continuously and equally distributed diffusers to maintain the deserved static pressure, which guarantees uniform air distribution. The air can be blown out

through diffusers e.g. nozzles or even through the own material of the ducts if it is permeable. (Fabric for instance.) It was assumed, that it is possible to create a continuously changing diameter, so it is possible to get the expected even airflow without balancing. This study was limited to circular cross-sections.

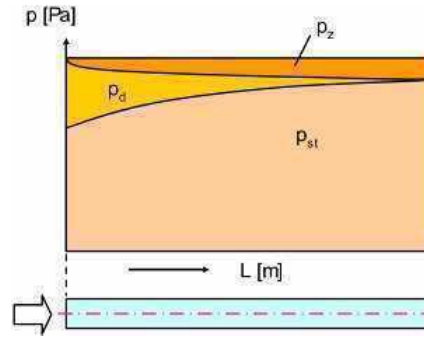


Fig. 1. Typical changes of pressure in textil air ducts, p_z : pressure losses due to friction, p_d : dynamic pressure, p_{st} : static pressure [1]

To understand the ideas, one has to think about the following: airflow is decreasing because of the air diffusion through the diffusers or the permeable duct wall. If the diameter does not change, the air velocity in the duct will decrease and so will the dynamic pressure. Because of that the static pressure will increase, so static pressure is regained (Fig. 1.). Because of that the amount of air blown out also increases. The maximum amount is blown out at the end of the duct. To avoid this, balancing dampers are used in practice. These dampers are used to decrease static pressure. It should be noted, that balancing raises initial costs.

2. Governing equations and boundary conditions

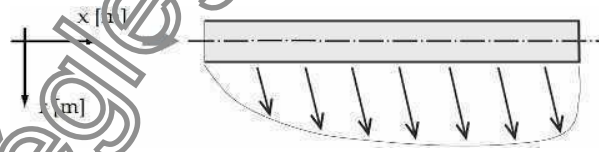


Fig. 2. The schematic draft of the modelled phenomenon

The draft of the phenomenon can be seen on Fig. 2. The flow was considered to be steady state, incompressible and isothermal. The simplified, 1D Navier-Stokes-equation can be used to describe the flow [2]:

$$w(x) \cdot \frac{dw(x)}{dx} = -\frac{\partial p_{stat}(x)}{\partial x} / \rho + \lambda(x) / 2D(x) \cdot w(x)^2, \quad (1)$$

where:

- w is the mean axial velocity, [m/s],
- ρ is the density of the fluid, [kg/m³],

- p_{stat} is the static pressure, [Pa],
- x is the axial coordinate, [m],
- λ is the Darcy friction factor, [-],
- D is the duct's diameter, [m].

The second governing equation was the equation of continuity, applied for the studied case:

$$w(0) \cdot D(0)^2 \cdot \pi / 4 - w(x) \cdot D(x)^2 \cdot \pi / 4 - \int_{A_{\text{out}}(x)} w_s(\zeta) dA = 0, \quad (2)$$

where:

- $w_s(\zeta)$ is the seepage velocity, it depends on the location, if the duct's material is permeable, [m/s],
- D is the diameter in the particular location, [m].
- A_{out} is the seepage area, it depends on the location, if the duct's material is permeable, [m²].

From that point it is necessary to speak about two different problems. The first is (A), when the seepage area is equal in every location, so it does not depend on the diameter. This is the case of the air duct with frequently installed grills, etc. (Fig. 3.)



Fig. 3. Air duct with grills, case A

The case is (B), when the seepage area depends on the diameter, e.g. the wall is permeable (e.g. textile airducts). The change of the air flow can be considered as continuous just in this instance, but we assume that it is linear in case A too. In the first case (A), so if e.g. nozzles are installed on the air duct with equal resistance, our aim must be to keep constant static pressure. In the second case (B), if the duct's wall is permeable, the diameter is changing and so is the seepage area. Because of this, static pressure should change too.

A connection is needed between the static pressure and the seepage velocity. The Forcheimer-Dupuit-equation can be derived from Darcy's law. The application of Darcy's law is limited; it can be used if the seepage velocity depends on the pressure gradient linearly [3]. After a big enough (material dependent) seepage velocity linearity is breaking down due to the fact that the form drag due to solid obstacles gets comparable with the surface drag due to friction. The Forcheimer-Dupuit-equation in 1D form:

$$\partial p_{\text{stat}} / \partial r = -\mu \cdot w_s / K_r - c_F \cdot \rho \cdot w_s \cdot |w_s| / \sqrt{K_r}, \quad (3)$$

where:

- c_F is the dimensionless form-drag constant, [-],
- μ is the dynamic viscosity of the fluid, [Pa·s],
- K_r is the radial specific permeability of the duct, [m²].

The investigated materials are thin, so the difference between the concentric inner and outer cylinder surfaces can be neglected. With this simplification, seepage velocity is a constant function of the radius. After variable separation, integration and rearrangement:

$$p_{\text{st}} = s \cdot \mu \cdot w_s / K_r + s \cdot c_F \cdot \rho \cdot w_s \cdot |w_s| / \sqrt{K_r}, \quad (4)$$

where:

- s is the investigated material thickness, [m],
- p_{st} is the static gauge pressure in the duct, [Pa].

We have to calculate the Darcy friction factor to complete the model. For turbulent flow (Reynolds-number is bigger than 2320) the Colebrook-White equation can be used, although its validity has not been checked in ducts with wall suction. Another difficulty is the absolute roughness of the fabric materials, which is usually not given by the manufacturers. Approximate values can be found in the literature [4]. For laminar flow Hagen-Poiseuille-equation can be used.

3. Boundary conditions

The boundary conditions are summarized in the next table which is based on the above-mentioned on the above-mentioned.

Table 1. Summary of the boundary conditions.

	$x = x_0 (= 0)$	$x \in (0; L)$	$x = L$
Case A.	The input data is known. (Airflow rate etc.)	The static pressure is constant: $p_{\text{st}} = \text{all.}$	The end of the duct is closed: $w(L) = 0$. The airflow rate in the duct is zero: $\dot{V}(L) = 0$.
Case B.		The airflow rate is known as a function of the location: $\dot{V}(x) = \dot{V}(x_0) \cdot \frac{1-x}{L}$ The connection between the seepage velocity and the static pressure is known: $p_{\text{st}} = \frac{s \cdot \mu}{K_r} \cdot w_s + \frac{s \cdot c_F}{\sqrt{K_r}} \cdot \rho \cdot w_s \cdot w_s $ and: $w_s(x) = \frac{\dot{V}(x_0)}{L \cdot \pi \cdot D(x)}$	

4. Solution for air ducts with diffusers (Case A)

From previous equations the following can be derived:

$$D'(x) = [\lambda(x)/2 - \delta/(1-\xi) \cdot D(x_0)/L]/2, \quad (5)$$

where:

- $D'(x)$ is the derivative of the diameter,
- $D(x_0)$ is the diameter of the duct at the initial location, [m],
- $\xi=x/L$ is the dimensionless location, [-],
- $\delta=D(x)/D(x_0)$ is the dimensionless diameter, [-].

5. Solution for air ducts made of permeable material (Case B)

We have to define more variables. It can be shown, that the seepage velocity depends on the location and it can be expressed with the air flow rate in the initial location. From this, particular parts of the Forcheimer-Dupuit-equation can be written in the following forms:

$$p_{stD}(x) = s \cdot \mu / K_r \cdot \dot{V}(x_0) / [L \cdot \pi \cdot D(x)] = p_{stD}(x_0) / \delta, \quad (6)$$

where:

- p_{stD} is the static pressure loss due to friction losses during the seepage through permeable material, [Pa],
- $\dot{V}(x_0)$ is the air flow rate in the initial x_0 location, [m³/h].

The second part of the equation:

$$p_{stF}(x) = s \cdot c_F / \sqrt{K_r} \cdot \rho \cdot (\dot{V}(x_0) / [L \cdot \pi \cdot D(x)])^2 = p_{stF}(x_0) / \delta^2, \quad (7)$$

p_{stF} is the static pressure loss due to solid obstacles during the seepage through permeable material. The expressions are only valid for a permeable duct with circular diameter. A dimensionless ratio can be introduced, which expresses ratio of the friction losses and the form drag:

$$\varepsilon_{FD}(x) = p_{stF}(x) / p_{stD}(x) = \varepsilon_{FD}(x_0) / \delta. \quad (8)$$

Another dimensionless number can be introduced, which is the Euler-number

$$Eu(x) = [p_{stD}(x) + 2 \cdot p_{stF}(x)] / [2 \cdot p_{din}(x)] = \quad (9)$$

$$= p_{stD}(x_0) / [2 \cdot p_{din}(x_0)] \cdot [\delta^3 / (1-\xi)^2 + 2 \cdot \varepsilon_{FD}(x_0) \cdot \delta^2 / (1-\xi)^2] . \quad (10)$$

With these numbers the derivative of the diameter can be written as:

$$D'(x) = \frac{\lambda(x)/2 - \delta/(1-\xi) \cdot D(x_0)/L}{Eu(x) + 2}. \quad (11)$$

The derived equation is a differential equation. This equation cannot be solved analytically, so a numerical method is needed. The fourth order Runge-Kutta-method was used. The error produced by the method is proportional to the fifth power of the step size that is why it is called as fourth order Runge-Kutta-method (it is accurate until the fourth order).

6. Results for air ducts with diffusers (Case A)

The calculations have been carried out with the program Matlab [5]. The differential equation to be solved was (5). The following constants have an influence on the results:

- $D(x_0)/L$, we are going to use the reciprocal value, $L/D(x_0)$, which we called as length-diameter ratio.

The dimensionless diameter, the dimensionless location also appeared. The friction factor can also be found in the equation. It depends on:

- the $Re(x)$ Reynolds-number. It can be shown, that this number depends on its initial value: $Re(x) = Re(x_0) \cdot (1-\xi)/\delta$.
- the $k/D(x)$ relative roughness, which depends on its initial value too: $k/D(x) = k/D(x_0)/\delta$.

Based on the foregoing it can be seen, that the results depend on these two other constants. Before we started the calculation we had to define the limits of the dimensionless numbers. The limiting values of the length-diameter ratio ($L/D(x_0)$), the Reynolds-number ($Re(x_0)$) and the relative roughness are based on the practical experiences. In the next two figures (Fig. 5., Fig. 6.) one can see the impact of the Reynolds-numbers.

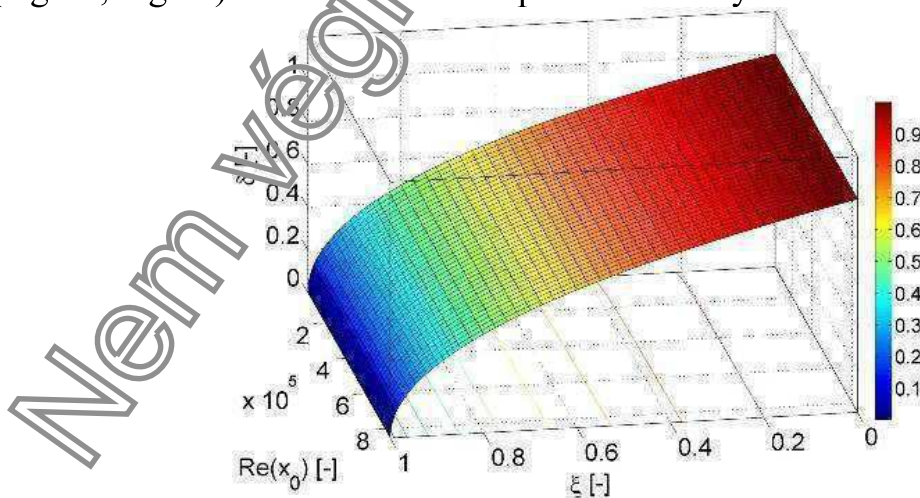


Fig. 5. The changes of the dimensionless diameter , $L/D(x_0)=10$, $k/D(x_0)=0$

It can also be seen that if the $L/D(x_0)$ is small (Fig. 5.) the solution is nearly independent of the Reynolds-number. If the $L/D(x_0)$ is bigger (Fig. 6.), we cannot say that, but the dependency is only notable in a narrow range. We can say according to these calculations, that in most cases the determined optimal diameters remain optimal diameters even beside variable flow rates. On Fig. 7. it can be seen, that as the relative roughness is increasing, these slight differences which appeared before (Fig. 6.) are disappearing. To explain that the Moody chart have to be recalled. As the relative roughness is increasing, the differences between the friction factors calculated with different Reynolds-numbers are decreasing. It is also possible, to have a constant friction factor after a big enough Reynolds-number. (See the limit curve on the Moody chart.)

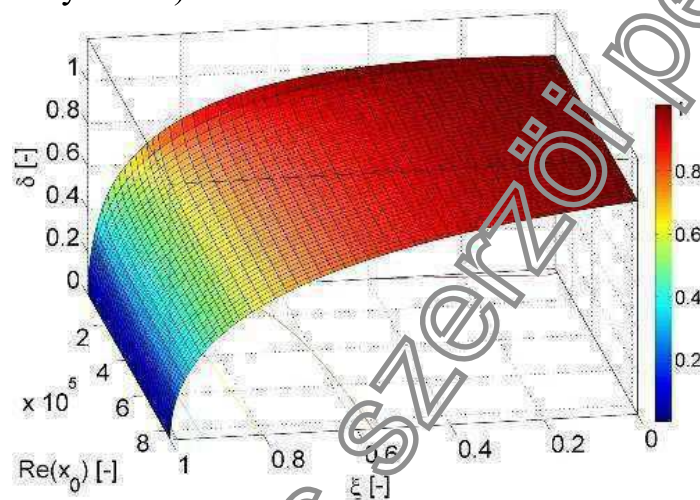


Fig. 6. The changes of the dimensionless diameter, $L/D(x_0)=100$, $k/D(x_0)=0$

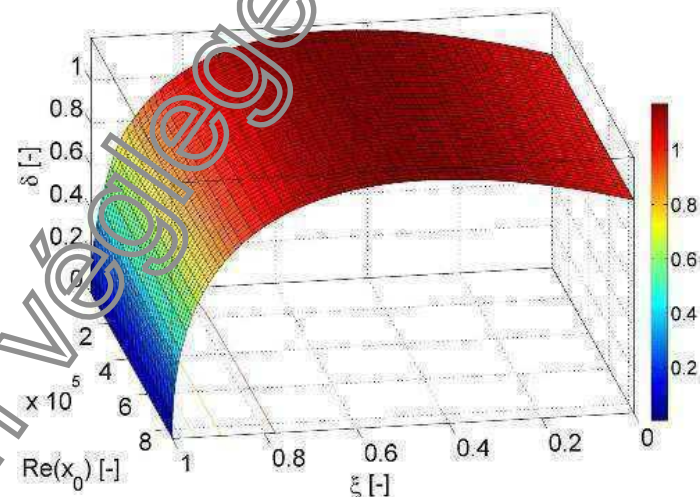


Fig. 7 The changes of the dimensionless diameter, $L/D(x_0)=100$, $k/D(x_0)=0.015$

As it can be seen, $L/D(x_0)$ the length-diameter ratio has significant impact on the optimal shape.

7. Results for air ducts with permeable wall (Case B)

If the duct wall is permeable, two other dimensionless numbers must be taken into account. The calculations have been carried out with the program Matlab [5], as in the chapter before. The equation to be solved was (11). The two dimensionless numbers are:

- the Euler-number. The definition was:

$$Eu(x_0) = (p_{st0D} + 2 \cdot p_{st0F}) / (\rho \cdot w_0^2) = p_{st0D} / (\rho \cdot w_0^2) \cdot (1 + 2\varepsilon_{FD}(x_0)).$$
- the $\varepsilon_{FD}(x_0)$ pressure loss ratio, the definition was:

$$\varepsilon_{FD}(x_0) = p_{stD}(x_0) / p_{stF}(x_0).$$

The second number, $\varepsilon_{FD}(x_0)$ is equal to zero, if the dimensionless form-drag constant, c_F is zero. It is not true for several fabrics, but due to space reasons only this case was investigated. If the $\varepsilon_{FD}(x_0)$ is zero, the Euler-number is equal to the ratio of the static pressure and twice the dynamic pressure in the duct. To get the limiting values of the static pressure catalogues of manufacturers were used. One of them recommends to keep the static pressure between 60 and 120 Pascals. Another says, that the minimal value depends on the weight of the material because the pressure has to be able to hold it. 30 Pascals were chosen as the min and 120 Pascals as the max value, so the min of the Euler-number is 0.39, and the max is 25.

On the next figures (Fig. 8. and Fig. 9.) it can be seen, that if the Euler-number is near to its minimum value, the optimal diameter is considerably changing. As it was mentioned, the maximum value of the Euler-number can be up to 25. In spite of this, on the charts the max value was 4. We used 4 instead of 25 because the major changes happen before this number.

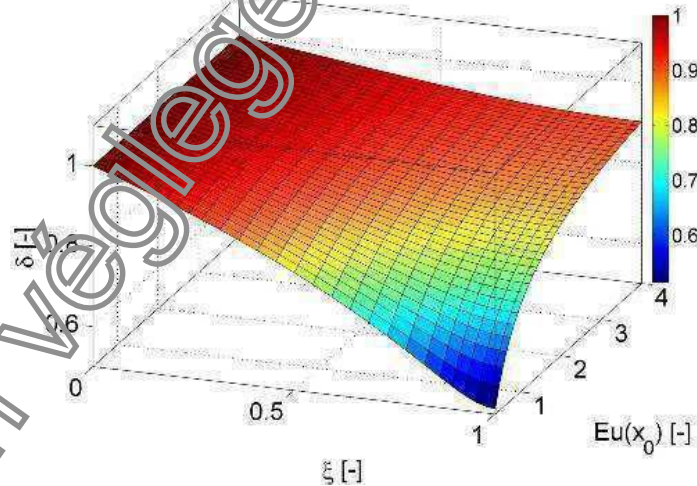


Fig. 8. The changes of the dimensionless diameter,
 $L/D(x_0)=100$, $Re(x_0)=100000$, $\varepsilon_{FD}(x_0)=0$, $k/D(x_0)=0.0075$

To explain that graphically, the Euler-number depends on the static pressure. If the Euler-number is big, it means that the static pressure is big compared to the dynamic one, so it is not significantly changing, it is nearly constant.

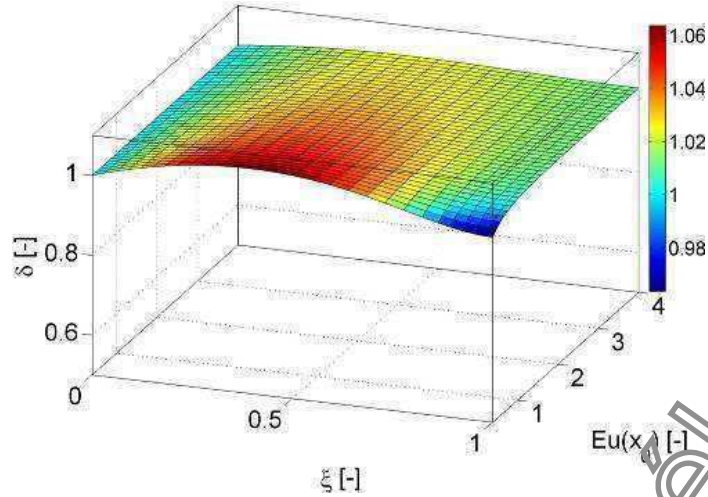


Fig. 9. The changes of the dimensionless diameter,
 $L/D(x_0)=100$, $Re(x_0)=100000$, $\varepsilon_{FD}(x_0)=0$, $k/D(x_0)=0.0075$

The $L/D(x_0)$ variable has a deep impact on the shape of the function. If it is 10 (Fig. 8.), the dimensionless diameter is always less than one. If it is 100 (Fig. 9.), the diameter can be bigger than one. The reason for this is the friction. If the length-diameter ratio is bigger we have to regain the static pressure.

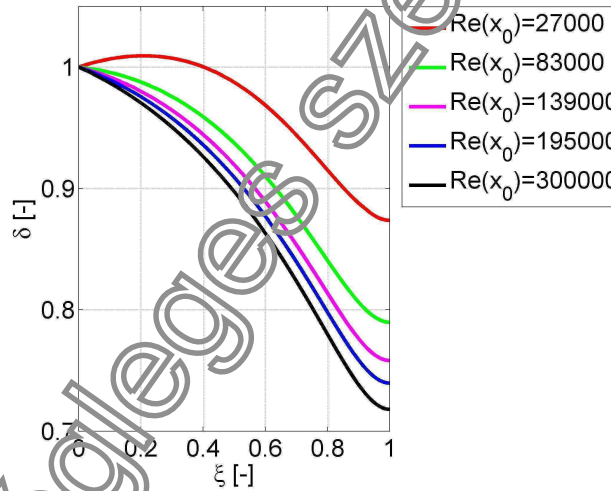


Fig. 10. The changes of the dimensionless diameter,
 $Eu(x_0)=0.39$, $L/D(x_0)=100$, $\varepsilon_{FD}(x_0)=0$, $k/D(x_0)=0$.

The effect of the Reynolds-number is quite similar to the examples we have seen before. It can be stated, that if the Euler-number is small (so the axial velocity is big), the effect of the friction is big, so the change of the Reynolds-number has an influence on the calculated diameters. If the Reynolds-number is big enough, according to our investigations this impact will be less important. Its influence will also be smaller with bigger relative roughness. On the last diagram (Fig. 11.) it is shown, that if the Euler-number gets bigger, the effect of the friction will be smaller. The difference between the calculated diameters will be less with the different Reynolds-numbers.

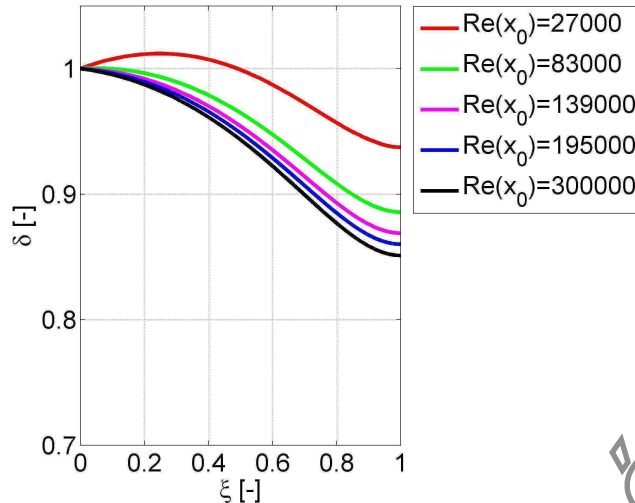


Fig. 11. The changes of the dimensionless diameter, $Eu(x_0)=0.8$, $L/D(x_0)=100$, $\varepsilon_{FD}(x_0)=0$, $k/D(x_0)=0.0005$

According to our study the impact of the Reynolds-number can be neglected in many cases, so we can say it is possible to create air ducts, which will keep the optimal even air diffusion in every point along the length in VAV systems as well.

Summary

Our paper was about airducts in which air flow rate is continuously, linearly decreasing due to the out-blown amount of air through diffusers or the permeable duct wall. With some simplifications the diameters of these ducts along the length can be determined. It was assumed, that with these continuously changing diameters it is possible to maintain uniform air distribution. Dimensionless numbers were introduced to make the results more applicable. This way number of variables was also decreased.

In the future it is needed to test these results with measurements and CFD calculations. As the developed model had many simplifications this validation is needed. Another interesting question is the production of these ducts. The shape may have to be simplified, so a method is needed to determine how to simplify it while the optimal specifications of the ducts are maintained.

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