

DEM Analysis of Masonry Domes with Oval Plan

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ABSTRACT

1. Aims

The term “oval” in architecture usually denotes a closed convex curve with two axes of symmetry. (An ellipse is, for instance, a special oval; a wide variety of oval shapes can be constructed using circular arcs of different radii.) Though most of the existing domes in our architectural heritage have a circular plan, domes with oval plan are more common than usually thought. Already in the Middle Ages, hundreds of Romanesque churches were built with an oval plan; then in the Renaissance and later in Baroque the idea became very popular and quickly spread all over Europe. An excellent overview was given by Huerta [1] on the history of oval shapes in architecture.

Our study focused on the following two issues:

- Generalization of Heyman’s minimal wall thickness (derived for spherical domes of uniform wall thickness, see [2]) for domes with oval plans;
- Analysis of the effect of support displacements.

2. The simulation technique

In our investigations we used the three-dimensional discrete element code 3DEC. A discrete element model considers the structure to be a collection of separate blocks, “discrete elements”, each of which is able to move and – in most software – to deform independently of each other. These blocks may come into contact with each other, where forces can be transmitted from one block to another, causing stresses and deformations in them. There are several techniques for calculating the motions of the elements caused by the forces (external loads and contact forces) acting on them.

The elements in 3DEC may have any polyhedral shape so they can mimic bricks or stone voussoirs. The elements can either be perfectly rigid or deformable, with deformable, frictional or tension-resisting contacts between them in both cases. The state-changing processes of the analyzed system of elements are followed along the series of small finite

time-steps, according to Newton's force-acceleration law. An explicit time integration scheme (based on the central difference technique) is applied.

The domes in our simulations consisted of bricks of mostly equal size, though special attention was devoted to problems like e.g. to avoid having interstices just under one another. Fig. 1. shows an example.

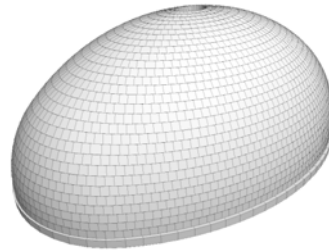


Figure 1: Discrete element model of a dome with oval plan

The ovals consisted of four circular arcs, and the domes were constructed by rotating the oval about either the shorter or the longer axis of symmetry. Mechanical parameters of the elements and of the contacts were based on laboratory experiments.

3. Results

The necessary thickness to ensure equilibrium under self-weight was determined for several different geometries. (The results show that oval domes may require less building material than spherical domes to cover the same area of plan.) The distributions of the reaction forces were also analyzed; they usually take their maximal value at the “edges”, i.e. where the boundary of the plan is roughly parallel to the longer axis of symmetry. The domes behaved rather sensitively to support displacements, and interesting failure modes were found (see Fig.2).

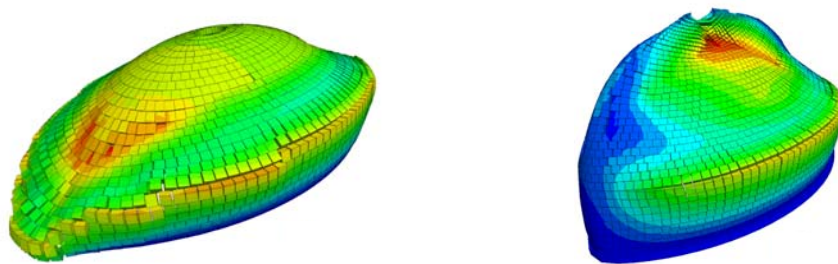


Figure 2: Two failure modes

References

- [1] Huerta, S.: Oval domes: History, Geometry and Mechanics. Nexus Network Journal 9: 211-248, 2007
- [2] Heyman, J.: The Stone Skeleton. Cambridge University Press, Cambridge, UK, 1995