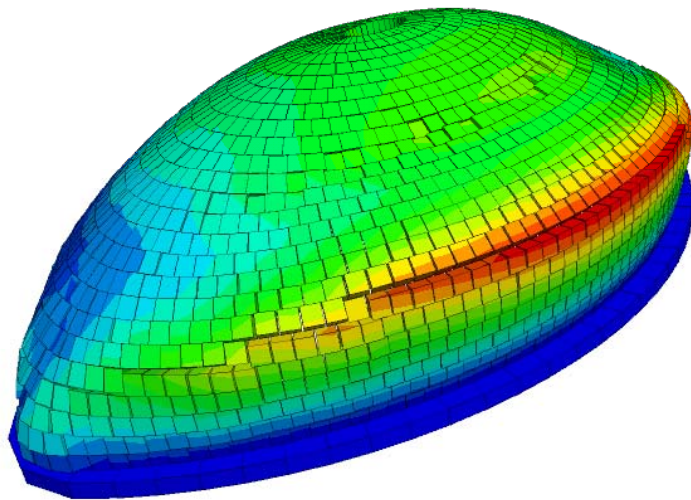


DEM analysis of masonry domes with oval plan



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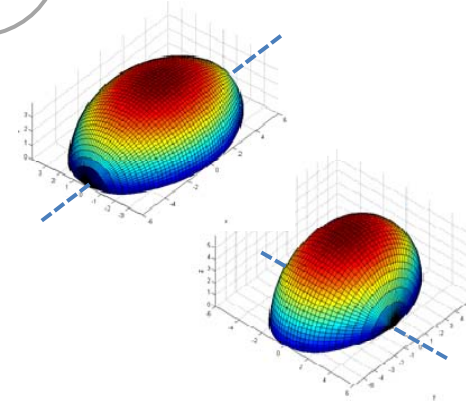
OVAL DOMES

widely applied from the Middle Ages;
very popular in the Baroque

Geometry:

- „oval”:
smooth, convex, symmetric
- Groundplan:
composed of circular arcs
- Rotation of the plan about:
the longer axis:
the shorter axis:

Mechanics: ???

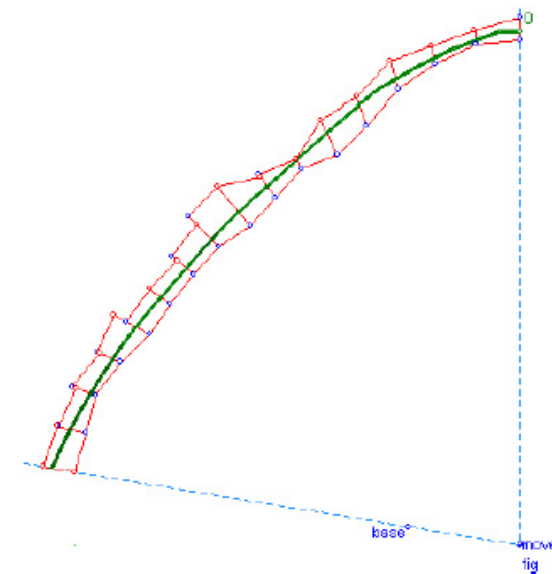
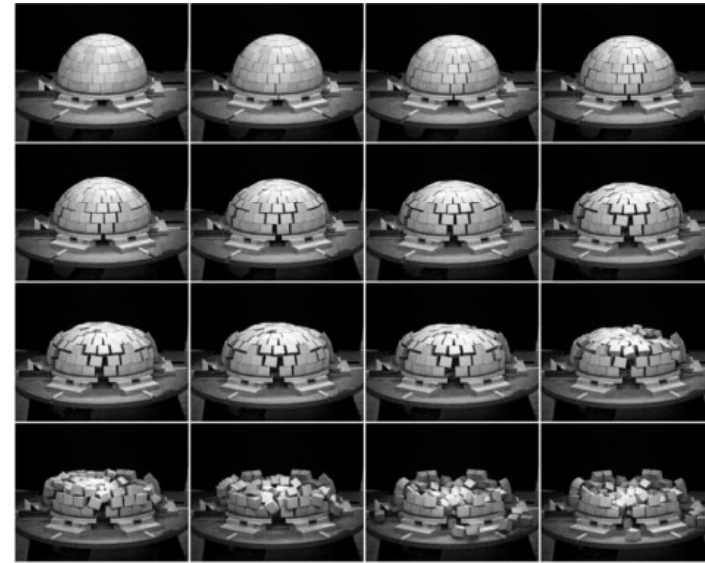


Failure of masonry domes:
problem with the **geometry**,
not with the material **strength**!

The masonry:

- **voussoirs + joints**
- **very low tensional strength**
of the contacts
- **compression stresses:**
not comparable to the strength
- **dry contacts or old mortar:**
 $\approx$ Coulomb-friction

Thrust line / thrust surface: →
stable if thrust line/surface is inside

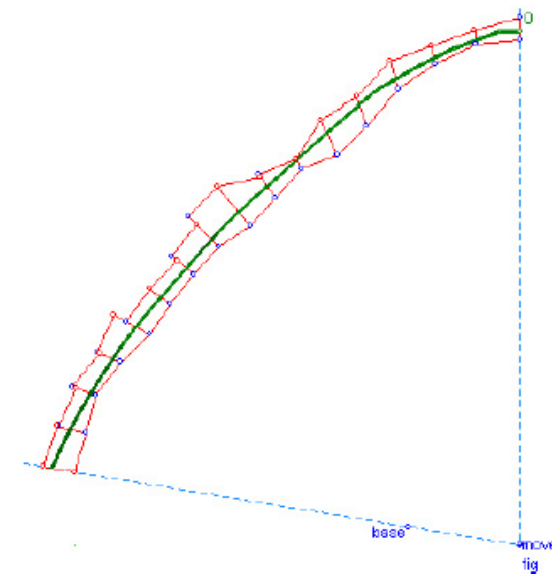
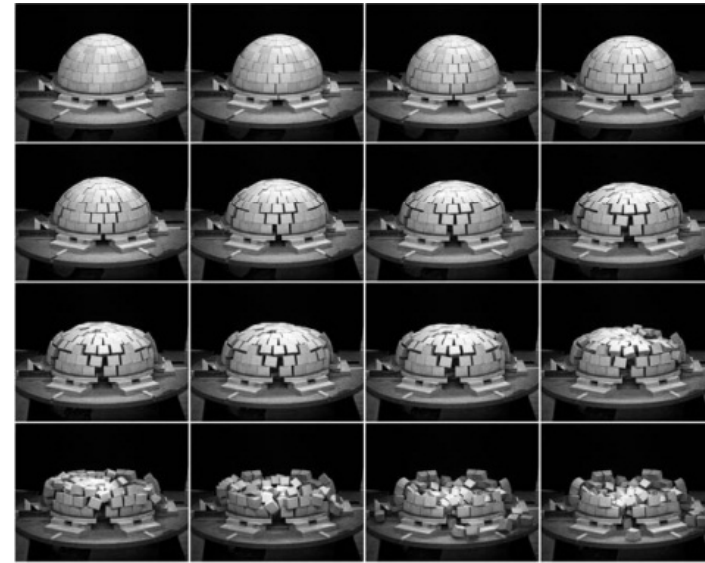


Failure of masonry domes:
problem with the **geometry**,
not with the material **strength**!

Classic theory based
on Limit State Analysis:

the masonry: zero tension strength;
infinite strength to compression;
frictional contacts – no sliding

Thrust line / thrust surface: →
stable if thrust line/surface is inside



Failure of masonry domes:
problem with the **geometry**,
not with the material **strength**!

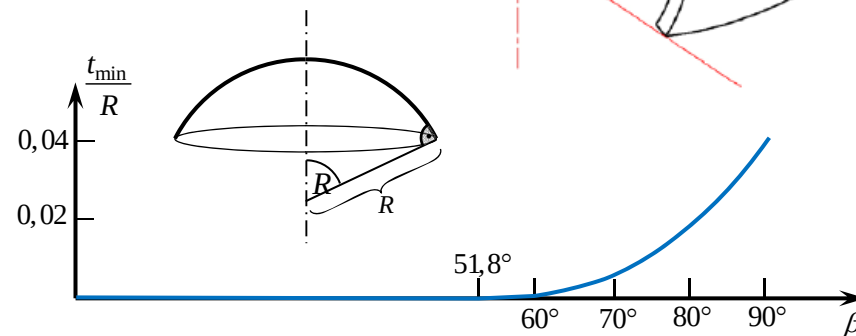
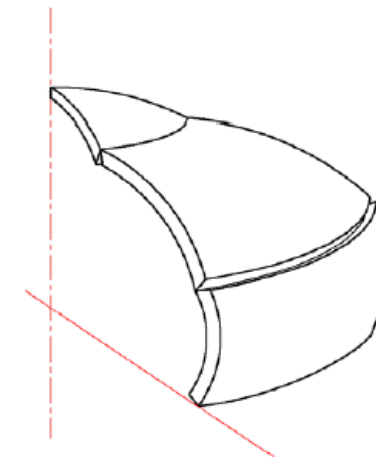
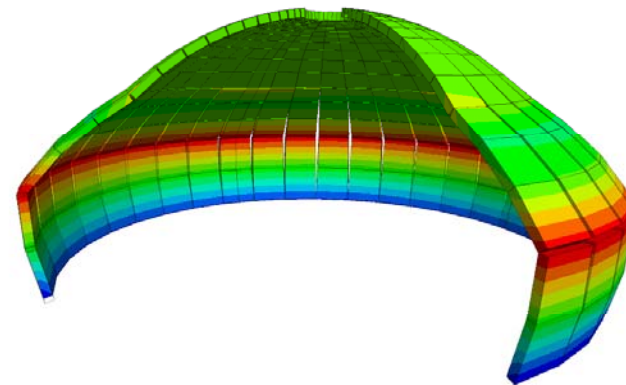
Classic theory based
on Limit State Analysis:

the masonry: zero tension strength;
infinite strength to compression;
frictional contacts – no sliding

e.g. Spherical domes:

Heyman (1966):

$$t_{\min} = 0,042 R$$



OVAL DOMES

Mechanics:

- ? generalization of Heyman's **minimal wall thickness**
- ? behavior under **support displacements**
- ? **stresses** and **reactions** under self weight / live loads
- ? **failure mechanisms** under self weight / live loads

Possible methods of the analysis:

- continuum methods:
 - membrane theory; FEM etc. ← continuous!
- Limit State Analysis of plasticity ← no sliding or detachment!
- FEM with contact elements ← complicated, time consuming!
- Discrete Element Method (DEM)

THE DISCRETE ELEMENT METHOD

- **separate** elements having their independent degrees of freedom
- finite (**large**) **displacements**, as a series of small increments
- contact **creation** and deletion automatically followed

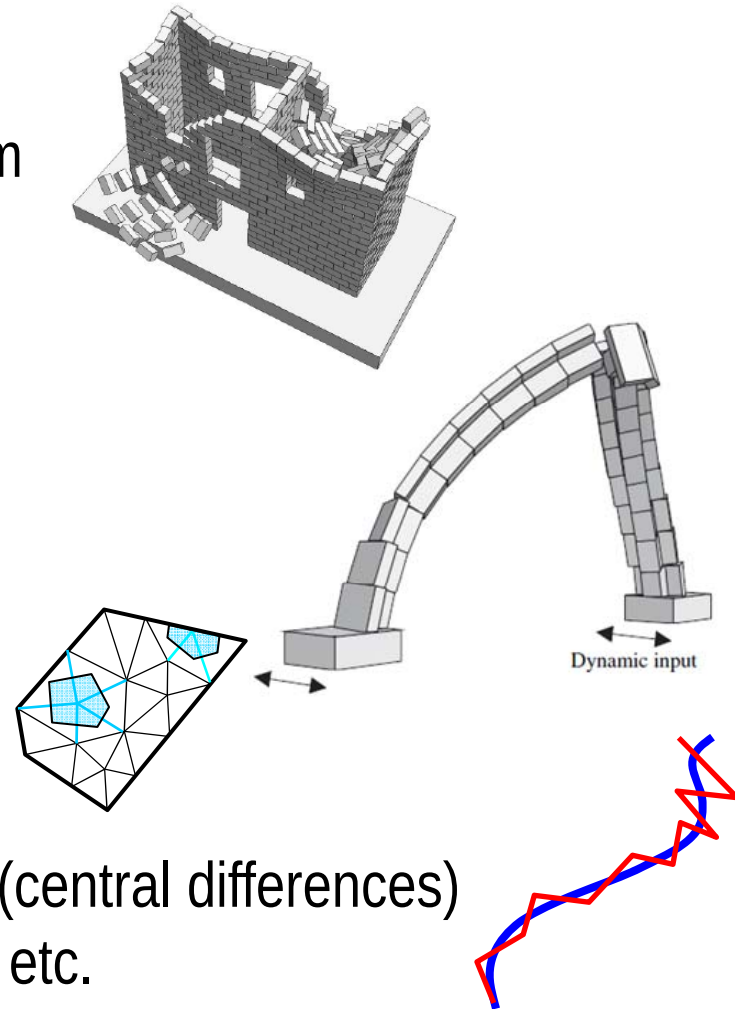
3DEC:

polyhedral elements, subdivided
into uniform-strain tetrahedra

Newtonian equations of motion

time integration: explicit scheme (central differences)

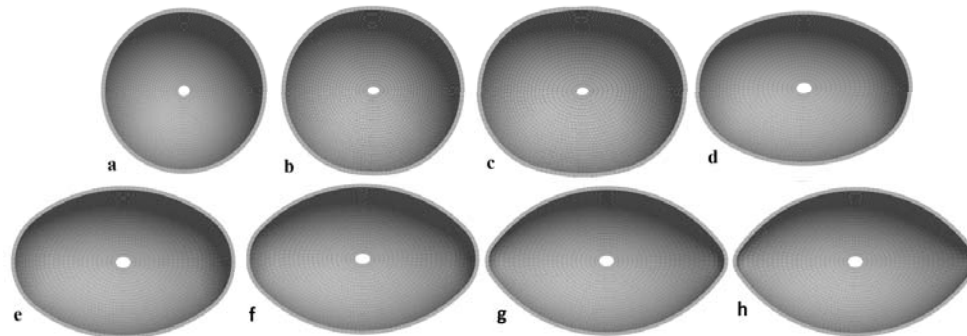
contacts: e.g. frictional; cohesive etc.



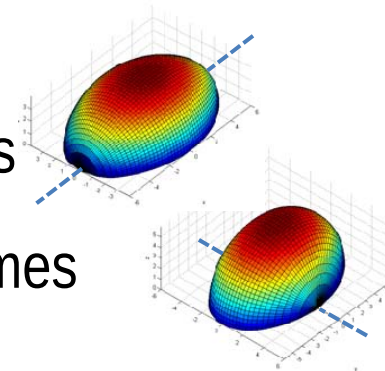
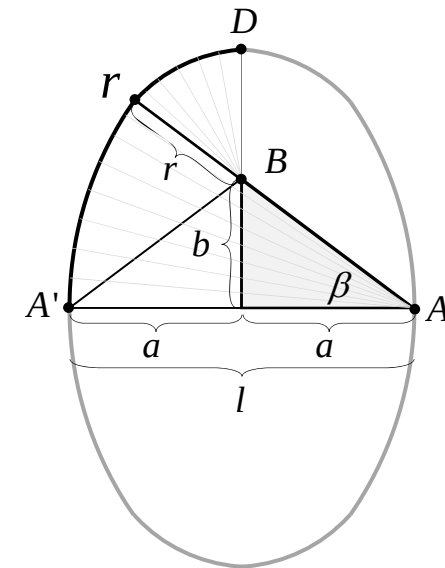
OUR INVESTIGATIONS

→ groundplans: „generalized Egyptian ovals”

measure of eccentricity: $e = \frac{a}{r} - 1$



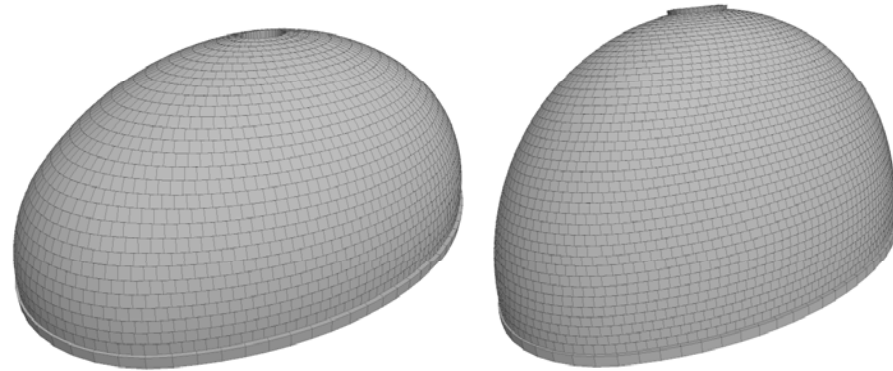
→ intrados: Rotation of the plan about
the longer axis: Type 1 („flat”) domes
the shorter axis: Type 2 („high”) domes
→ thickness: uniform



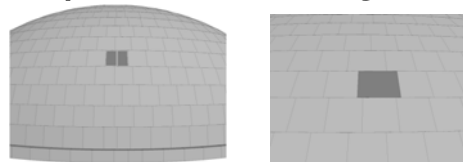
OUR INVESTIGATIONS

→ the complete dome:

[horizontal rings]



special problems: e.g.



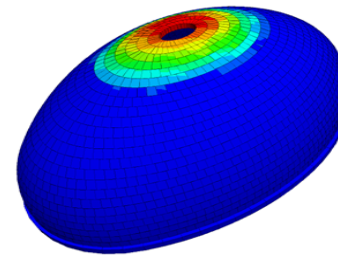
→ material properties:

elements: rigid →← clay brick

contacts: Coulomb-frictional

→ equilibrating under self weight:
simulation of de-centring

3DEC DP 4.10
©2007 Kinetics Consulting Group, Inc.
Step 7501
9/25/2011 5:44:39 PM
Displacement magnitude
0.0000E+00
1.0000E-04
2.0000E-04
3.0000E-04
4.0000E-04
5.0000E-04
6.0000E-04
7.0000E-04
8.0000E-04
9.0000E-04
1.0000E-03
1.1000E-03

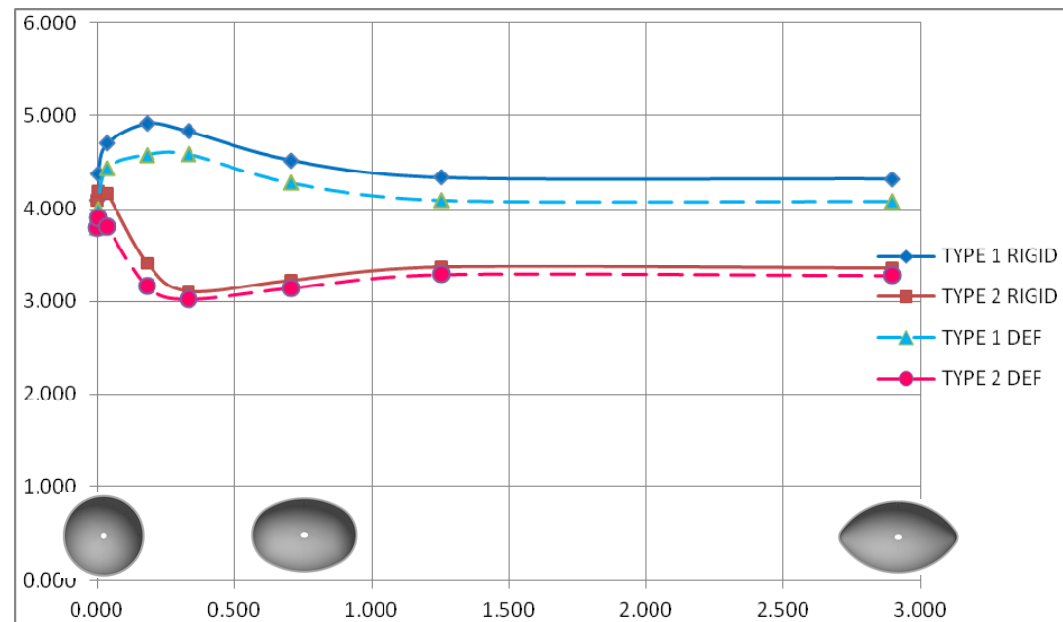


OUR INVESTIGATIONS

- ?1: generalization of Heyman's **minimal wall thickness**
- ?2: **stresses** and **reactions** under self weight / live loads
- ?3: failure mechanisms to **support settlements**
- ?4: **failure mechanisms** under self weight / live loads

OUR INVESTIGATIONS

1a. Minimal wall thickness

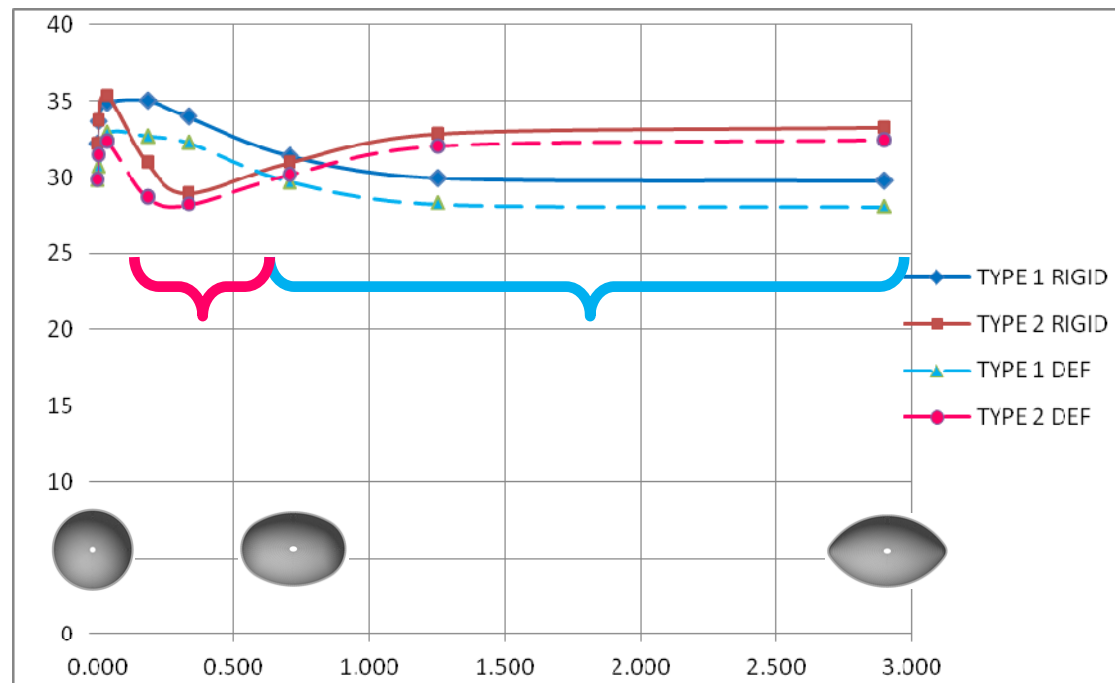


⇒ rigid elements instead of deformable: SAFE approximation!

⇒ Type 2 („high”) domes: SMALLER wall thickness may be enough

OUR INVESTIGATIONS

1b. Minimal masonry volume



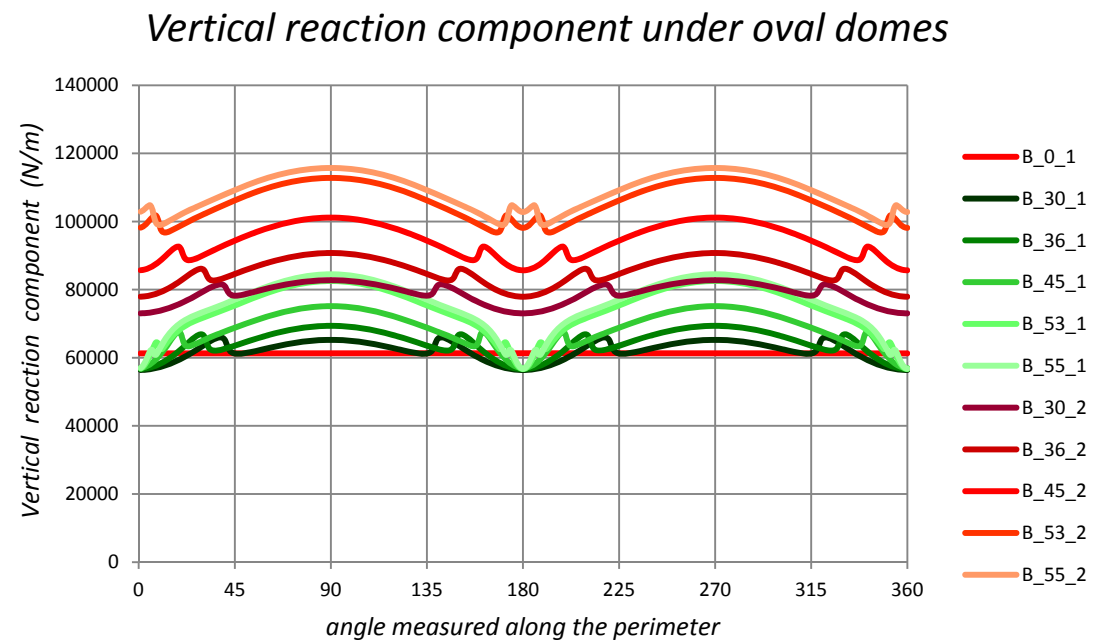
⇒ oval groundplans may require LESS MATERIAL

OUR INVESTIGATIONS

2. Stresses and reactions under self weight

Continuum solution:

[ANSYS]



⇒ higher at the long edges than at the sharp edges

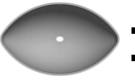
⇒ perturbation at the curvature change

OUR INVESTIGATIONS

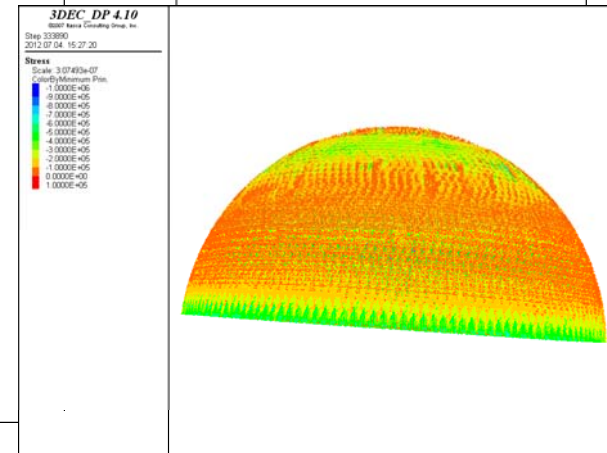
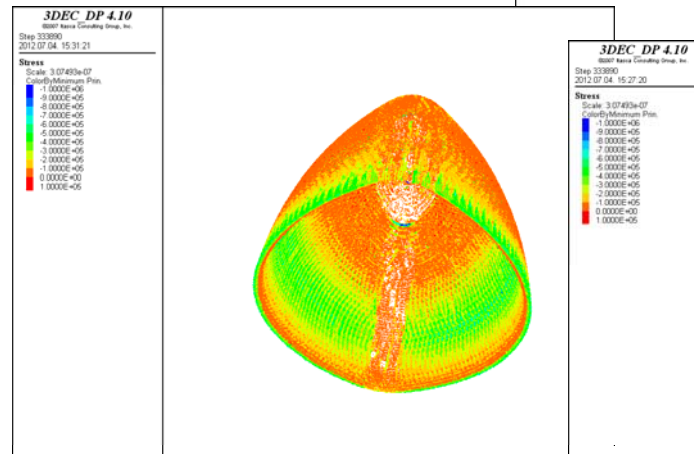
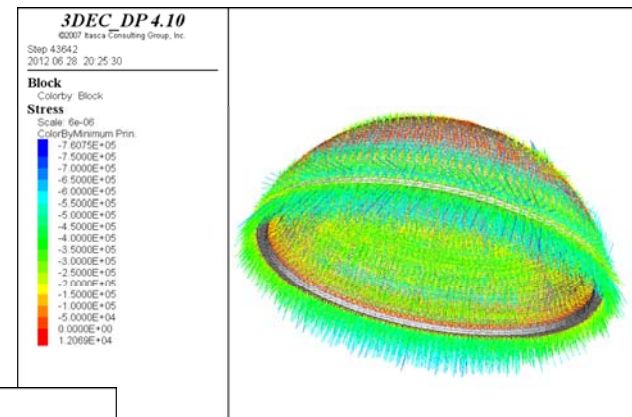
2. Stresses and reactions under self weight

e.g.  :

⇒ perturbations are smoothed out

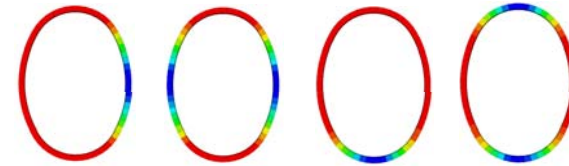
e.g.  :

⇒ increasing e :
the two halves
separate

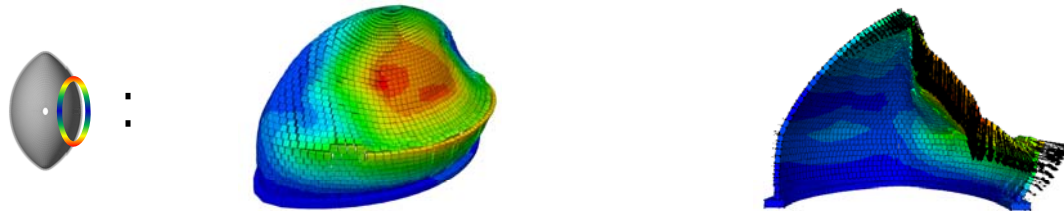


OUR INVESTIGATIONS

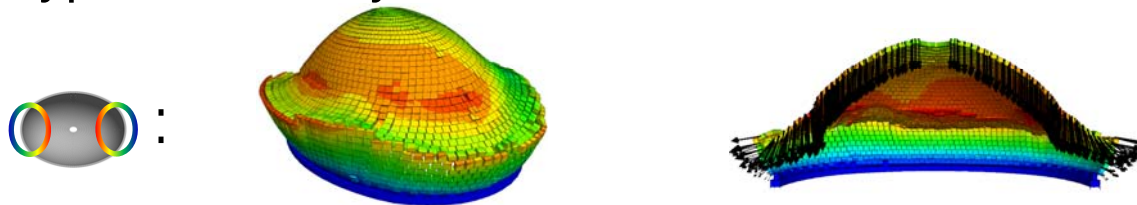
3. Support settlements

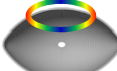


e.g. Type 2 dome, non-symmetric on a long edge:



e.g. Type 2 dome, symmetric at the narrow ends:



⇒ most dangerous:  least dangerous: 

⇒ In general: Type 1 („flat”) domes are more vulnerable,
Type 2 („high”) domes are less vulnerable for settlements

OUR INVESTIGATIONS

Future research

⇒ simulation of strengthening techniques

⇒ other types of vaults

⇒ multispans arches

