

Pauli channel tomography with unknown channel directions

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EXTENDED ABSTRACT

The accurate description of different quantum phenomena is a key issue in their potential use in modern IT-technologies, for example communication, security, or quantum computers. The parameter estimation of quantum channels, which is commonly called quantum process tomography [1], plays a major role in quantum information processing.

In quantum mechanics, both dynamical changes and communication is treated using *quantum channels*. These are nothing else but trace preserving completely positive mappings $\mathcal{E}$ which transform the input state ρ given on the input of the channel to the output state $\mathcal{E}(\rho)$ appearing on the other side. It is a reasonable assumption that the channel belongs to a parametric family, i.e., for a fixed input state the output state belongs to the parametric family $\{\mathcal{E}_\theta(\rho)\}_{\theta \in \Theta}$, where Θ denotes the parameter space. Thus, the channel estimation problem can be traced back to parameter estimation problem [2].

Direct quantum process tomography is performed by sending known quantum systems into the channel, and then estimating the output state. In quantum mechanics the measurement has a probabilistic nature [3, 4], therefore many identical copies of the input quantum system are needed, and an estimator is constructed by using statistical considerations. For achieving efficient process tomography, experiment design is necessary that consists of selecting the optimal input state, optimal measurement of the output state, and an efficient estimator of the channel from the measured data.

The field of quantum process tomography is well-established, an exhaustive description of possible tomography methods can be found in [5]. The Pauli channels

form a relatively wide family of quantum channels. The tomography of Pauli channels has a huge literature, however, due to the level of difficulty of the topic, papers mostly deal with special cases, e.g., with the optimal parameter estimation of a depolarizing channel [6]. But there are some publications investigating the estimation of multi-parameter channels [7, 8], and the multidimensional case also appears [9, 10]. There are also some experimental results concerning the optimal estimation of the Pauli-channels [11, 12].

In contrast to the majority of the works in this area, we propose an extended problem statement: we investigate qubit Pauli channels with unknown channel directions. Despite of the novelty of the approach, there are a few papers that deal with optimally estimating qubit Pauli channels including their channel directions. In [13] the problem was examined using convex optimization methods, and a numerical method was provided for finding the optimal input - measurement pairs. In [14] we examined the optimality of the estimation problem using purely statistical considerations to achieve analytical results. However, analytical results could only be obtained for the case of known channel directions.

Therefore, the aim of this paper is to give an analytical description of the optimal estimation of qubit Pauli channels in the case of unknown channel directions, too.

We introduce the *channel matrix* that characterizes the Pauli channel, and give an estimation method of the channel matrix. This estimation scheme uses three input qubits and three von Neumann measurements for complete channel tomography. Computing the Jordan-decomposition of the estimated channel matrix one gets the estimations of the contraction parameters and the angle parameters that describe the channel directions. The efficiency of these estimations is measured here with three quantities: the mean squared error of the estimated contraction parameters and angle parameters, and the mean distance of the estimated and the real channel matrix are investigated.

The optimization of the different quantities needs dif-

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ferent mathematical techniques. The mean squared error of the estimated contraction parameters and the mean distance of the estimated and the real channel matrix is minimized analytically. The mean squared error of the estimated angle parameters has a complex form, therefore it is optimized with numerical methods in general. However, in an important special case this loss function can also be minimized analytically.

The main result of this work is that we determined the optimal measurement configurations for quantum bit

Pauli channels with respect to the most relevant loss functions. For optimally estimating the contraction parameters and the channel matrix we should have input qubits and measurements in the channel directions, however, for optimally estimating the channel directions, we should use different tomography conditions: from simulation investigations and analytical results in special cases we conjecture that using input and measurement directions that are complementary to the channel directions would give a nearly optimal result.

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