

Novel Method for Quadcopter Controlling Using Nonlinear Adaptive Control Based on Robust Fixed Point Transformation Phenomena

Bence G. Czako

John von Neumann Faculty of Informatics
Óbuda University
Budapest, Hungary
Email: czako.bence@stud.uni-obuda.hu

Krisztián Kósi

Institute of Applied Mathematics
Óbuda University
Budapest, Hungary
Email: kosi.krisztiani@nik.uni-obuda.hu

Abstract—In the past few years quadcopters have become an integral part of life. There are many applications for these objects including entertaining purpose, agricultural monitoring, or gathering information from places what humans can not reach. An interesting problem is controlling these vehicles from remote locations in order to follow a desired trajectory. While many solutions exist to this task the vast majority of them heavily rely on linear control methods which are susceptible to parameter uncertainties or outer disturbances, which affect the tracking capability of the quadcopter adversely. In this paper a novel nonlinear approach, called *Robust Fixed Point Transformation based adaptive control (RFPT)* is presented which can provide remedy to the above mentioned disturbances.

Index Terms—Robust Fixed Point Transformation, RFPT, Quadcopter.

I. INTRODUCTION

In nonlinear control theory the well known “*Lyapunov’s 2nd method*” has been used widely, which he published in his doctoral thesis (in Russian language) in 1892 [1]. It was translated to English in 1966 [2] and since then the approach has remained popular among control engineers. While the method has some attractive features, the design process requires strong mathematical background. A novel approach called *Robust Fixed Point Transformation based adaptive control* is introduced by Tar et al. [3] - and its new variant [4] - which preserves many positive aspects of nonlinear control and offers an easy to design model based methodology. This new technique can be applied in order to design a trajectory tracking controller of a quadcopter. While there are many solutions exist which solve the tracking problem, the vast majority of them is based on linear techniques. Linear techniques have been used widely [5],[6],[7],[8],[9]. In this paper a tracking controller was created based on the RFPT method. The primary goal is to demonstrate that the RFPT technique is not just a theoretical method which can solve basic control tasks but an easy to use model based approach which works in practice.

II. THE MODEL OF THE SYSTEM

In order to create a controller based on the RFPT method one must derive the mathematical model of a quadcopter.

There are many ways to obtain a proper mathematical description [10],[11], but in this paper the model is based on the work of Raffo et al. [12] and T. Luukkonen [13]. Initially, two distinct coordinate systems must be defined; one is the base coordinate system, the other is the local inertial frame attached to the copter, denoted by $\mathcal{B} = \{\vec{x}_B, \vec{y}_B, \vec{z}_B\}$ and $\mathcal{L} = \{\vec{x}_L, \vec{y}_L, \vec{z}_L\}$ consecutively. The position of the quadcopter is given by the vector $\xi = [x, y, z]^T$ which links the center of both frames. The orientation can be described with Tait-Bryan angles. The rotation around x, y, z axes are called roll, pitch and yaw consecutively. It is denoted by the vector $\eta = [\phi, \theta, \psi]^T$ which is the orientation of the vehicle in the inertial frame. The angles are bounded in the following way: $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$; $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$; $-\pi < \psi < \pi$. With the help of the angles, the connection between the two frames can be defined by the **x-y-z** rotation matrix:

$$R = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \quad (1)$$

where $C_{(\cdot)} = \cos(\cdot)$ and $S_{(\cdot)} = \sin(\cdot)$. Based on the rotational matrix, the kinematic properties of the quadcopter can be represented by the equation $v_{\mathcal{B}} = Rv_{\mathcal{L}}$, where $v_{\mathcal{B}} = [u_0, v_0, w_0]^T$ is the velocity in the base frame and $v_{\mathcal{L}} = [u_L, v_L, w_L]^T$ is the local inertial frame. A sketch of the kinematic description is depicted in Fig. 1.

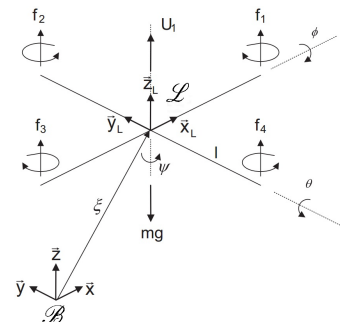


Fig. 1. Kinematic sketch of the quadcopter [12].

The orientation of the quadcopter is given by the following equation:

$$W\dot{\eta} = \omega \quad (2)$$

where $\omega = [p, q, r]^T$ is the angular velocities in the local inertial frame. W matrix describes the connection between angular velocities, and has the following structure:

$$W = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\theta & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \quad (3)$$

In order to obtain $\dot{\eta}$, one must multiply the equation with the inverse of matrix W . After multiplication the equation takes the form

$$\dot{\eta} = W^{-1}\omega \quad (4)$$

The inverse of matrix W exists if $\theta \neq (2k-1)\frac{\pi}{2}$, ($k \in \mathbb{Z}$). Then the inverse of W become

$$W^{-1} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi/C_\theta & C_\phi/C_\theta \end{bmatrix} \quad (5)$$

where $T(\cdot) = \tan(\cdot)$.

The dynamical equations can be obtained by using the Euler-Lagrange formalism:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = \begin{bmatrix} F_\xi \\ \tau \end{bmatrix} \quad (6)$$

$$\mathcal{L} = T - U \quad (7)$$

In (6) and (7), $\mathcal{L}$ is the Lagrangian, q is the vector of generalized coordinates and $q = [x, y, z, \phi, \theta, \psi]^T$, F_ξ is the thrust, $\tau = [\tau_\phi, \tau_\theta, \tau_\psi]^T$ is the vector of torques, T is the kinetic energy, and U is the potential energy. By solving the expression one could obtain the following equations:

$$\ddot{\xi} = \frac{F_\xi}{m} \begin{bmatrix} C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} - \frac{1}{m} \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix} \dot{\xi} \quad (8)$$

$$\ddot{\eta} = J^{-1}(\tau - C(\eta, \dot{\eta})\dot{\eta}) \quad (9)$$

Equation (8) describes the relationship between the accelerations and the thrust combined with the orientation of the vehicle. The last term on the right side of the equation is intended to model the aerodynamical effects, thus A_x, A_y, A_z are wind coefficients. One can get F_ξ if summarizes all the thrust that are generated by the individual propellers. To obtain the thrust of a single propeller (f_i), one must multiply the angular velocity of the propeller with a constant k called the lift constant of the motor.

$$f = k\omega^2 \quad (10)$$

In the possession of the individual thrusts, F_ξ can be calculated by a simple summation:

$$F_\xi = \sum_{i=1}^4 f_i = k \sum_{i=1}^4 \omega_i^2 \quad (11)$$

The other dynamic equation (9) forms a connection between the angles and torques. Matrix J contains the elements of the inertial properties of the quadcopter, and $C(\eta, \dot{\eta})$ is the Coriolis matrix. By assuming that the center of mass coincides the center of the frame of the quadcopter, the generalized torques can be obtained by the following expression:

$$\tau = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \\ lk(\omega_2^2 - \omega_4^2) \\ lk(\omega_1^2 - \omega_3^2) \end{bmatrix} \quad (12)$$

where b is the drag coefficient of the motors', and l is the distance between the center of mass and an individual motor. Because of the above stated assumption, the inertial matrix only contains elements in its diagonal:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (13)$$

By performing the multiplication $W^T I W$, J can be expressed as the following matrix:

$$J = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} \quad (14)$$

which elements are:

$$\begin{aligned} j_{11} &= I_{xx} \\ j_{12} &= 0 \\ j_{13} &= -I_{xx} S_\theta \\ j_{21} &= 0 \\ j_{22} &= I_{yy} C_\phi^2 + I_{zz} S_\phi^2 \\ j_{23} &= (I_{yy} - I_{zz}) C_\phi S_\phi C_\theta \\ j_{31} &= -I_{xx} S_\theta \\ j_{32} &= (I_{yy} - I_{zz}) C_\phi S_\phi C_\theta \\ j_{33} &= I_{xx} S_\theta^2 + I_{yy} S_\phi^2 C_\theta^2 + I_{zz} C_\phi^2 C_\theta^2 \end{aligned} \quad (15)$$

According to *Raffo et al.* [12], the Coriolis matrix is defined by:

$$C(\eta, \dot{\eta}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad (16)$$

where

$$\begin{aligned}
c_{11} &= 0 \\
c_{12} &= (I_{yy} - I_{zz})(\dot{\theta}C_\phi S_\phi + \dot{\psi}S_\phi^2 C_\theta) + (I_{zz} - I_{yy})\dot{\psi}C_\phi^2 C_\theta - I_{xx}\dot{\psi}C_\theta \\
c_{13} &= (I_{zz} - I_{yy})\dot{\psi}C_\phi S_\phi C_\theta^2 \\
c_{21} &= (I_{zz} - I_{yy})(\dot{\theta}C_\phi S_\phi + \dot{\psi}S_\phi^2 C_\theta) + (I_{yy} - I_{zz})(\dot{\psi}C_\phi^2 C_\theta + I_{xx}\dot{\psi}C_\theta) \\
c_{22} &= (I_{zz} - I_{yy})\dot{\phi}C_\phi S_\phi \\
c_{23} &= -I_{xx}\dot{\psi}S_\theta C_\theta + I_{yy}\dot{\psi}S_\phi^2 C_\theta S_\theta + I_{zz}\dot{\phi}C_\phi^2 S_\theta C_\theta \\
c_{31} &= (I_{yy} - I_{zz})\dot{\psi}C_\phi^2 S_\phi C_\phi - I_{xx}\dot{\theta}C_\theta \\
c_{32} &= (I_{zz} - I_{yy})(\dot{\theta}C_\phi S_\phi S_\theta + \dot{\phi}S_\phi^2 C_\theta) + (I_{yy} - I_{zz})\dot{\phi}C_\phi^2 C_\theta + I_{xx}\dot{\psi}S_\theta C_\theta - I_{yy}\dot{\psi}C_\phi^2 S_\theta C_\theta \\
c_{33} &= (I_{yy} - I_{zz})\dot{\phi}C_\phi S_\phi C_\theta^2 - I_{yy}\dot{\theta}S_\phi^2 C_\theta S_\theta - I_{zz}\dot{\theta}C_\phi^2 C_\theta S_\theta + I_{xx}\dot{\theta}C_\theta S_\theta
\end{aligned} \tag{17}$$

III. ROBUST FIXED POINT TRANSFORMATION

The Robust Fixed Point Transformation based adaptive control is a novel method which approaches the control problem iteratively [3]. The underlying idea is based on the realised-response scheme [14]. It means that by inverting the model - which is called the inverse model - a proper control signal can be obtained for a desired trajectory. If r^d denotes the desired trajectory then $u = \varphi(r^d)$ where φ is the inverse model. Since the inverse model is “neither complete nor exact”, the behaviour of the system will differ from the desired when the control signal is applied [14]. This phenomenon can be written in the form of $r^r = \psi(\varphi(r^d)) \neq r^d$ where ψ denotes the system. The aim of the controller is to deform the input signal in a way that $r^d = \psi(r_*^d)$ where r_*^d is the response of the system to the deformed input. The deform process can be achieved by the use of a deform function. In the case of Single Input Single Output systems the deform function holds the following form:

$$\begin{aligned}
G(r|r^d) &\stackrel{def}{=} (r + K)[1 + B \tanh(A[f(r) - r^d])] - K \\
G(r_*^d|r^d) &= r_*^d \text{ if } f(r_*^d) = r^d \\
G(-K|r^d) &= -K \text{ if } r = -K
\end{aligned} \tag{18}$$

The technique uses the deform function in the n th control cycle so that it can evaluate the next r value hence the iterative approach. This means that $r_{n+1} = G(r_n|r^d)$.

This idea can be extended to Multiple Input Multiple Output (MIMO) systems. According to *Tar et al.* [14], the deform function will be the following in MIMO case:

$$\begin{aligned}
\vec{h} &= \vec{f}(\vec{x}_n) - \vec{x}^d \\
\vec{e} &= \frac{\vec{h}}{||\vec{h}||} \\
\tilde{B} &= \sigma(A||\vec{h}||) \\
\vec{x}_{n+1} &= (1 + B)\vec{x}_n + \tilde{B}K\vec{e}
\end{aligned} \tag{19}$$

where $\vec{h}$ denotes the error between the desired and current states, $\vec{e}$ the normalized error, $\sigma(x)$ is a sigmoid function and K, B, A are control parameters. Furthermore, the control loop contains a kinematic block which is defined as

$$\begin{aligned}
e_{int}(t) &= \int_{t_0}^t (q_n(\tau) - q(\tau)) d\tau \\
\left(\frac{d}{dt} + \Lambda\right)^{n+1} e_{int} &= 0
\end{aligned} \tag{20}$$

where Λ is a control parameter. The above defined expression is similar to an error relaxation prescription that can be found in VS/SM controllers [15] but in this case, the power of the description is different ($n + 1$). Designing an RFPT controller requires only three design steps: determining the kinematic description, deducing the inverse model and tuning the control parameters.

The kinematic description can be expressed by using the binomial theorem. Since the order of the system is two (because the generalized coordinates appear with their second derivative) the kinematic block takes the form of

$$\ddot{q}_d = \ddot{q}_n + \Lambda^3 e_{int} + 3\Lambda^2 \dot{e} + 3\Lambda \dot{e} \tag{21}$$

In the next step, the inverse model must be expressed by using the mathematical model. First, a design constraint is introduced to angle ψ by setting its value to zero. This restriction must be applied because quadcopters only need two type of manoeuvres to move freely in 3D space thus limiting the yaw movement eliminates the ambiguity. Using (8) the following three equations can be obtained:

$$\begin{aligned}
\ddot{x} &= \frac{F_\xi}{m}(S_\theta C_\phi) - \frac{1}{m}A_x \dot{x} \\
\ddot{y} &= -\frac{F_\xi}{m}S_\phi - \frac{1}{m}A_y \dot{y} \\
\ddot{z} &= \frac{F_\xi}{m}(C_\theta C_\phi) - g - \frac{1}{m}A_z \dot{z}
\end{aligned} \tag{22}$$

By rearranging the terms in equation three, C_ϕ can be determined:

$$C_\phi = \frac{\ddot{z}m + gm + A_z \dot{z}}{F_\xi C_\theta} \tag{23}$$

Substituting this into the first equation, angle θ can be found:

$$\begin{aligned}
f &= \frac{\ddot{x} + \frac{1}{m}A_x \dot{x}}{\ddot{z} + g + \frac{1}{m}A_z \dot{z}} \\
\theta &= \text{atan}(f)
\end{aligned} \tag{24}$$

If the second equation has F_ξ on one of its side, it can be substituted into equation three in conjunction with angle θ which leads to

$$\ddot{z} = \frac{(\ddot{y} + \frac{1}{m}A_y\dot{y})m}{-S_\phi} \frac{C_\phi}{\sqrt{f^2 + 1}} - g - \frac{1}{m}A_z\dot{z} \quad (25)$$

By a simple rearrangement, one can obtain angle ϕ :

$$h = \frac{\ddot{y} + \frac{1}{m}A_y\dot{y}}{\ddot{z} + g + \frac{1}{m}A_z\dot{z}\sqrt{f^2 + 1}} \quad (26)$$

$$\phi = \text{atan}(h)$$

By substituting ϕ into the second equation, F_ξ can be determined:

$$F_\xi = -\frac{(\ddot{y} + \frac{1}{m}A_y\dot{y})m\sqrt{h^2 + 1}}{h} \quad (27)$$

One final touch is to obtain the values of the motors' based on ϕ, θ, F_ξ . If the value of ϕ and θ are known then τ can be calculated by rearranging the terms in (9). By combining the values with (12) the angular velocities can be expressed as:

$$\begin{aligned} \omega_1^2 &= \frac{F_\xi}{4k} - \frac{\tau_\theta}{2kl} - \frac{\tau_\psi}{4b} \\ \omega_2^2 &= \frac{F_\xi}{4k} - \frac{\tau_\theta}{2kl} + \frac{\tau_\psi}{4b} \\ \omega_3^2 &= \frac{F_\xi}{4k} + \frac{\tau_\theta}{2kl} - \frac{\tau_\psi}{4b} \\ \omega_4^2 &= \frac{F_\xi}{4k} + \frac{\tau_\theta}{2kl} + \frac{\tau_\psi}{4b} \end{aligned} \quad (28)$$

The last step of the controller design is tuning the control parameters. According to [16], the initial value of the control parameters can be easily determined to the vast majority of systems by simple rules: K is often a great positive number, A is a small positive number, and the value of B is 1 or -1 .

IV. SIMULATION RESULTS

Multiple simulations were conducted so that the behaviour of the system could be observed and the controller parameters could be experimentally determined. The same model parameters were used that can be seen in the work of *T. Luukkonen* [13], hence $g = 9.81\text{m/s}^2$, $m = 0.468\text{kg}$, $l = 0.225\text{m}$, $k = 2.980 \cdot 10^{-6}$, $b = 1.140 \cdot 10^{-7}$, $I_{xx} = 4.856 \cdot 10^{-3}\text{kg} \cdot \text{m}^2$, $I_{yy} = 4.856 \cdot 10^{-3}\text{kg} \cdot \text{m}^2$, $I_{zz} = 8.801 \cdot 10^{-3}\text{kg} \cdot \text{m}^2$, $A_x = 0.25\text{kg/s}$, $A_y = 0.25\text{kg/s}$, $A_z = 0.25\text{kg/s}$. The initial value of the control parameters were the following: $K = 10^7$, $A = 10^{-8}$, $B = 1$, $\Lambda = 5$. The tests were conducted in Simulink. Two metrics were used to show the quality characteristics of the control algorithm; the maximal absolute value of the tracking error ($\max(|e|)$) and the mean of the absolute value of the tracking error ($|\bar{e}|$). The reference trajectories were the same in each direction (x, y, z). The classical impulse and step response were analysed at first. The parameter Λ had to be modified to $\Lambda = 3$ because the system did not converge to a solution. After the adjustment the controller behaved perfectly. It is worth to mention that these

trajectory prescriptions cause problem to the controller because they have discontinuities in their derivatives. To provide a remedy to this phenomenon, three different methods were tested. According to *T. Luukkonen* [13], a heuristic trajectory generation was made for the jounce of the quadcopter. The prescription was defined as

$$f(t) = \begin{cases} a \sin(\frac{1}{b}\pi t), & 0 \leq t \leq b, \\ -a \sin(\frac{1}{b}\pi t - \pi), & b \leq t \leq 3b, \\ a \sin(\frac{1}{b}\pi t - 3\pi), & 3b \leq t \leq 4b, \end{cases} \quad (29)$$

where $a = 1$, $b = 0.5$, $c = 4$. Integrating the above expression four times results in a 1m displacement of the quadcopter. Simulations showed that the controller was capable of a precise trajectory tracking in this case. The next reference trajectory was an exponential type based on the work of *Z. Rymansaib et al.* [17]. The definition of the reference trajectory in this case was

$$f(t) = 1 - e^{-(\alpha t)^3} \quad (30)$$

where $\alpha = 1$. By changing the α value, steeper trajectories can be obtained. It was shown that the controller was also able to follow this type of path. One last investigated trajectory was a sigmoid function based prescription which was defined as

$$f(t) = \frac{\tanh(t - 3) + 1}{2} \quad (31)$$

The controller could follow this trajectory as well. In table I., one can observe the tracking capabilities of the controller. From all of the simulations, those were chosen that had the least mean absolute error.

TABLE I
THE TRACKING PROPERTIES OF THE CONTROLLER

Type of trajectory	Λ	$\max(e)$	$ \bar{e} $
Impulse	3	1	0.0016
Step	3	1	0.0554
Heuristic	5	0.0018	0.0003
Exponential	5	0.0473	0.0031
Tanh	5	0.0062	0.0012

It can be concluded that the tracking error remains low in each case and does not exceed 0.1m . Aside from the trivial 1m error at the step and impulse response, the exponential method caused the maximal error because it was steeper than the other two. The other interesting aspect of the controller is that changing the value of K, A, B does not have any effect on the tracking quality. By changing the parameter Λ the response time can be decreased in exchange of increasing absolute maximal error. By switching off the adaptive part of the controller, one can obtain a PID type controller. For demonstration purposes this PID type controller was compared with the adaptive controller in the case of a tanh trajectory. On Fig. 2. one can observe a slight deviation of the tracking error in different directions, however this is not the case with the adaptive version that can be seen on Fig. 3. It is also

interesting to examine that significant oscillations occur in the control signal with the PID case which was smoothed out by the adaptive controller. This difference can be seen on Fig. 4. and Fig. 5. On Fig. 6. and Fig. 7. one can observe the difference between the output of the kinematic block and the output of the adaptive block. There is a small difference between them which improves the tracking capability of the controller.

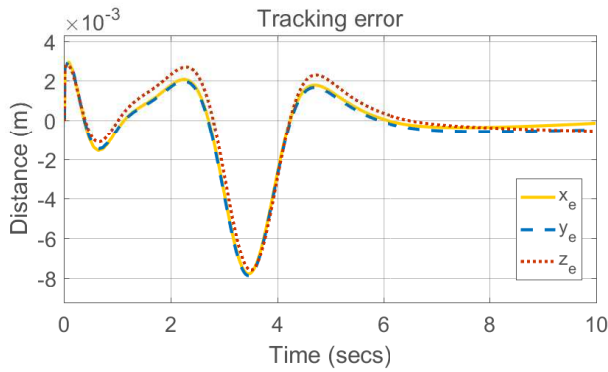


Fig. 2. The tracking error of the non-adaptive case.

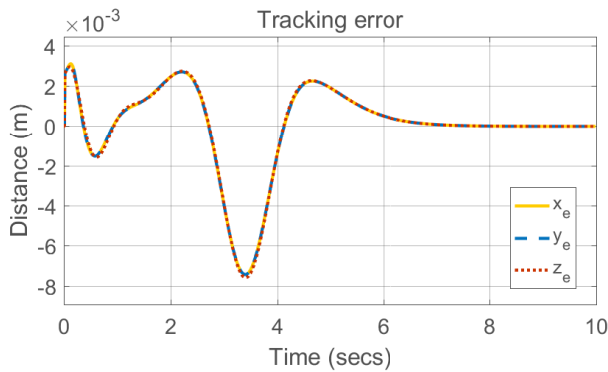


Fig. 3. The tracking error of the adaptive controller.

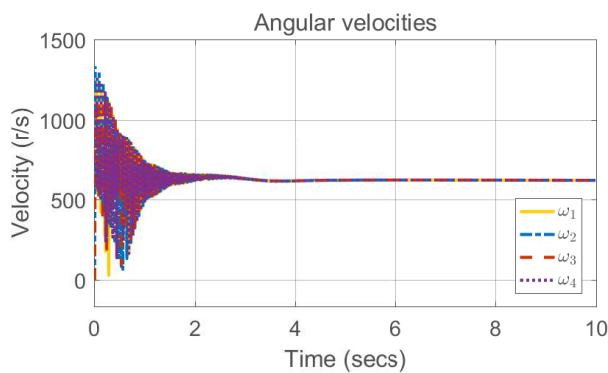


Fig. 4. The angular velocities of the motors' in the non-adaptive case.

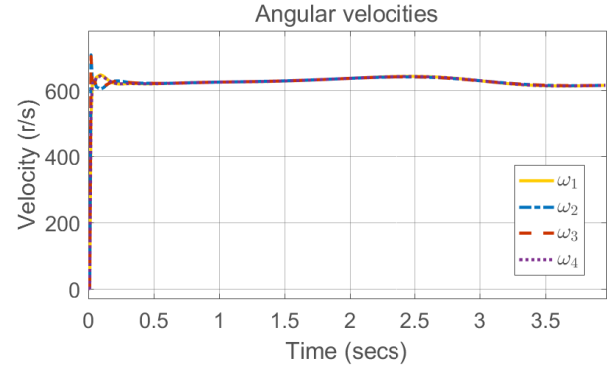


Fig. 5. The angular velocities of the motors' in the adaptive case.

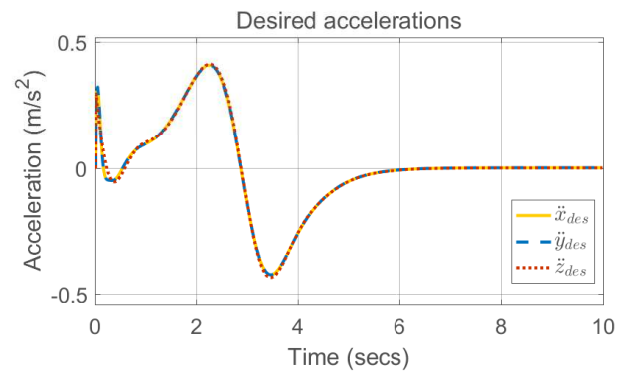


Fig. 6. The desired accelerations.

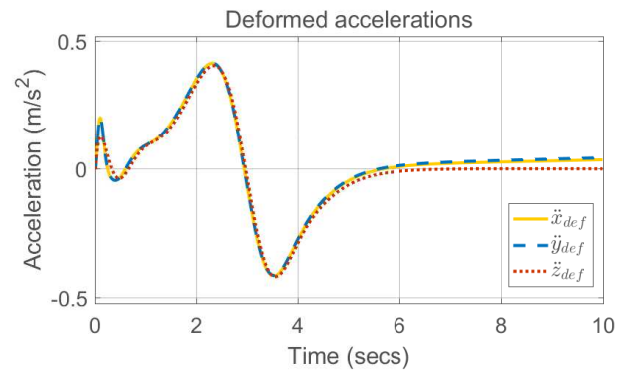


Fig. 7. The deformed accelerations.

V. CONCLUSION

In this paper, a trajectory tracking controller was designed for a quadcopter with the RFPT based adaptive control technique. A mathematical model was introduced which describes the kinematic and dynamic properties of the vehicle. After that, the RFPT method was briefly discussed, and the inverse model of the system was deduced. At the end of the paper, simulation results showed the efficacy of the controller with multiple trajectories.

The theoretical results yield a successful control design which tracks the desired trajectories precisely. One could also see that the underlying idea and the design steps are easy to follow thus the method could gain a wider recognition among control engineers. In the future research a huge emphasis has to be placed on the physical implementation procedure. The model is going to be tested with several modifications: real model parameters have to be evaluated, control signals have to be limited, the sensor effects have to be taken into consideration, and a bifurcation analysis has to be conducted. A physical validation of the technique might help to facilitate the spread of the RFPT method among control engineers which could have a significant impact on modern nonlinear control.

ACKNOWLEDGMENT

The authors would like to say thanks for the help of the Hungarian National Talent Program (NTP-HHTDK-0016 and NTP-SZKOLL-16).

REFERENCES

- [1] A. Lyapunov, *A general task about the stability of motion. (in Russian)*. PhD Thesis, University of Kazan, 1892.
- [2] A. Lyapunov, *Stability of motion*. Academic Press, New-York and London, 1966.
- [3] J.K.Tar, J. Bito, L. Nadai, and J. T. Machado, "Robust Fixed Point Transformations in adaptive control using local basin of attraction," *Acta Polytechnica Hungarica*, vol. 6, no. 1, pp. 21–37, 2009.
- [4] J. K. Tar, J. F. Bito, and I. J. Rudas, "Contradiction resolution in the adaptive control of underactuated mechanical systems evading the framework of optimal controllers," *Acta Polytechnica Hungarica*, vol. 13, no. 1, jan 2016. [Online]. Available: <http://dx.doi.org/10.12700/APH.13.1.2016.1.8>
- [5] G. Eigner, J. K. Tar, I. Rudas, and L. Kovacs, "LPV-based quality interpretations on modeling and control of diabetes," *Acta Polytechnica Hungarica*, vol. 13, no. 1, jan 2016. [Online]. Available: <http://dx.doi.org/10.12700/APH.13.1.2016.1.12>
- [6] A. Takacs, L. Kovacs, I. J. Rudas, R.-E. Precup, and T. Haidegger, "Models for force control in telesurgical robot systems," *Acta Polytechnica Hungarica*, vol. 12, no. 08, dec 2015. [Online]. Available: <http://dx.doi.org/10.12700/APH.12.8.2015.8.6>
- [7] B. Takarics and P. Baranyi, "TP model-based robust stabilization of the 3 degrees-of-freedom aeroelastic wing section," *Acta Polytechnica Hungarica*, vol. 12, no. 1, feb 2014. [Online]. Available: <http://dx.doi.org/10.12700/APH.12.1.2015.1.13>
- [8] K. Szell and P. Korondi, "Mathematical basis of sliding mode control of an uninterruptible power supply," *Acta Polytechnica Hungarica*, vol. 11, no. 03, feb 2014. [Online]. Available: <http://dx.doi.org/10.12700/APH.11.03.2014.03.6>
- [9] S. Molnar, "On the reachability of linear time varying systems," *Acta Polytechnica Hungarica*, vol. 11, no. 03, feb 2014. [Online]. Available: <http://dx.doi.org/10.12700/APH.11.03.2014.03.13>
- [10] E. Hernandez-Martinez, G. Fernandez-Anaya, E. Ferreira, J. Flores-Godoy, and A. Lopez-Gonzalez, "Trajectory tracking of a quadcopter UAV with optimal translational control," *IFAC-PapersOnLine*, vol. 48, no. 19, pp. 226–231, 2015. [Online]. Available: <http://dx.doi.org/10.1016/j.ifacol.2015.12.038>
- [11] Y. Yali, S. Feng, and W. Yuanxi, "Controller design of quadrotor aerial robot," *Physics Procedia*, vol. 33, pp. 1254–1260, 2012. [Online]. Available: <http://dx.doi.org/10.1016/j.phpro.2012.05.207>
- [12] G. V. Raffo, M. G. Ortega, and F. R. Rubio, "Path tracking of a UAV via an underactuated control strategy," *European Journal of Control*, vol. 17, no. 2, pp. 194–213, jan 2011. [Online]. Available: <http://dx.doi.org/10.3166/ejc.17.194-213>
- [13] T. Luukkonen, "Modelling and control of a quadcopter," *Aalto University*, 2011.
- [14] J. K. Tar, L. Nadai, and I. J. Rudas, *System and Control Theory with Especial Emphasis on Nonlinear Systems*. Typotex, 2012.
- [15] J.-J. Slotine and W. Li, *Applied Nonlinear Control*. Prentice Hall, 1991.
- [16] K. Kosi, S. Hajdu, J. F. Bito, and J. K. Tar, "Chaos formation and reduction in robust fixed point transformations based adaptive control," in *2012 IEEE 4th International Conference on Nonlinear Science and Complexity (NSC)*. Institute of Electrical & Electronics Engineers (IEEE), aug 2012, pp. 211 – 216. [Online]. Available: <http://dx.doi.org/10.1109/NSC.2012.6304756>
- [17] Z. Rymansaib, P. Iravani, and M. N. Sahinkaya, "Exponential trajectory generation for point to point motions," in *2013 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*. Institute of Electrical & Electronics Engineers (IEEE), jul 2013, pp. 906 – 911. [Online]. Available: <http://dx.doi.org/10.1109/AIM.2013.6584209>