

Doubly-foliable space-times and gauge-fixing of perturbations in scalar-tensor gravity theories

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The direct detection of gravitational waves and gamma-ray counterparts has confirmed that gravitational waves propagate with the speed of light, disruling some of the scalar-tensor gravity models. A huge class of Horndeski theories (those with generic G2 and G3, and with G4 depending only of the scalar field) however survived the test. The study of perturbations of such models is important to establish the ghost-free and instability-free parameter regimes. This has been investigated for a wide range of scalar-tensor theories in the spherically symmetric setup, exploring a 2+1+1 decomposition of space-time based on an orthogonal double foliation. The orthogonality however consumed one gauge degree of freedom, allowing the discussion of only the odd sector of the scalar part of perturbations. In order to describe the even sector perturbations, we worked out a novel 2+1+1 decomposition of space-time and gravitational dynamics, based on a non-orthogonal double foliation. We explore this new formalism for the perturbations of both the spherically symmetric metric tensor and scalar field in generic scalar-tensor theories, achieving an unambiguous gauge-fixing. This opens up the way for the discussion of the full spectrum of perturbations of spherically symmetric scalar-tensor gravity, including both the odd and even sectors of the scalar part of perturbations.

Keywords: nonorthogonal 2+1+1 decomposition; scalar-tensor gravity, gauge-fixing.

1. Introduction

While General Relativity has been a tremendously successful theory in establishing the compatibility of the gravitational interaction with the Special Relativistic constraint for signal propagation and has been tested successfully in all sorts of observations both on the Solar System scale and beyond, its extensions have been also systematically investigated.

Their necessity emerged due to A) the expectation to combine the geometric theory of gravity with quantum considerations, B) the need to understand the mysterious, yet at present dominating component of the Universe, dark energy, which drives its accelerating expansion, and C) the yet unsuccessful attempts to identify dark matter through its weak interaction, rendering it at present entirely in the gravitational sector.

Possible extensions arise from both modifying the dynamics of the metric tensor and from including secondary fields. Such extensions could be tested, constrained, or falsified through precision cosmological observations or analysis of detected gravitational waves.

Indeed, the recent eleven-fold detection of gravitational waves by the LIGO Scientific Collaboration and Virgo Collaboration^{1–7} confirmed the propagation speed of gravitational waves (tensorial perturbations) to be the speed of light within -3×10^{-15} and $+7 \times 10^{-16}$ accuracy.⁸ The surviving theories could be further sub-

jected to stability analysis, demanding the avoidance of Ostrogradsky-instabilities, ghosts, Laplacian instabilities or tachyons.

In the framework of scalar-tensor gravity the Horndeski theories^{9,10} are known to ensure second-order dynamics for both the metric tensor and the scalar field. Demanding the speed of light to be the speed of propagation of tensorial modes, the L_5 contribution to the Lagrangian has been disruled, together with the kinetic term dependence of L_4 .^{11–14} The latter could be kept for a class of the beyond Horndeski models,^{15,16} which allow for higher-than-second order spatial derivatives, but no such time derivatives.

Both classes of theories were excessively analysed in a cosmological context. Black hole perturbations were also explored. For perturbations of spherically symmetric, static black holes the odd and even sector perturbations decouple. Both were analysed for the Horndeski class of models,^{17,18} while for the beyond Horndeski models only the stability analysis of the odd sector has been carried on,¹⁹ based on an orthogonal double foliation.^{20,21} The orthogonality requirement however resulted in a gauge fixing, which was unambiguous for the odd sector, nevertheless generated an arbitrary function of all coordinates but the radial for the even sector, hampering the physical interpretation of the even sector perturbations.

In order to avoid this shortcoming we proposed²² to develop another decomposition of space-time and all affected geometrical quantities along a nonorthogonal double foliation. The details of this work involve a quadruple set of embedding variables (extrinsic curvatures, normal fundamental forms and normal fundamental scalars).²³ These variables provide the set of velocity phase-space variables complementing the induced 2-metric, radial shift and radial lapse. A Hamiltonian analysis established the canonical coordinates and momenta in terms of these geometric variables in a General Relativistic setup, also the Hamiltonian and diffeomorphism constraints.²³ The complementary metric variables, the temporal shift and lapse are left arbitrary, hence an unambiguous gauge-fixing became possible.

2. The nonorthogonal double foliation

The space-time $\mathcal{B}$ is foliated with the 3-dimensional hypersurfaces $\mathcal{S}_t$ (spacelike, with normal n^a) and $\mathfrak{M}_\chi$ (timelike, with normal l^a), their intersection being the 2-dimensional surface $\Sigma_{t\chi}$ (with g_{ab} the induced metric, normals k^a along $\mathfrak{M}_\chi$ and m^a along $\mathcal{S}_t$). The 4-dimensional metric is 2+1+1 decomposed in two equivalent ways, employing the orthonormal bases (n^a, m^a) and (k^a, l^a) :

$$\tilde{g}_{ab} = -n_a n_b + m_a m_b + g_{ab} , \quad (1)$$

$$\tilde{g}_{ab} = -k_a k_b + l_a l_b + g_{ab} . \quad (2)$$

The evolution vectors along a temporal coordinate t and a spatial one χ running in $\mathfrak{M}_\chi$ can also be decomposed in these bases:

$$\left(\frac{\partial}{\partial t} \right)^a = N n^a + N^a + \mathcal{N} m^a = \frac{N}{\epsilon} k^a + N^a , \quad (3)$$

$$\left(\frac{\partial}{\partial\chi}\right)^a = Mm^a + M^a = M(-\mathfrak{s}k^a + \mathfrak{c}l^a) + M^a, \quad (4)$$

Here N^a is a 2-dimensional shift, M^a is a radial shift; N and M the lapse and radial lapse, $\mathfrak{s} = \sinh \phi$, $\mathfrak{c} = \cosh \phi$. The new element of the formalism is the third component $\mathcal{N} = N \tanh \phi$ of the 3-dimensional shift vector, which in the orthogonal limit vanishes.

The 2+1+1 decomposition of the covariant derivatives of the basis vectors

$$\tilde{\nabla}_a n_b = K_{ab} + 2m_{(a}\mathcal{K}_{b)} + m_a m_b \mathcal{K} + n_a m_b \mathcal{L}^* - n_a \mathfrak{a}_b, \quad (5)$$

$$\tilde{\nabla}_a l_b = L_{ab} + 2k_{(a}\mathcal{L}_{b)} + k_a k_b \mathcal{L} + l_a k_b \mathcal{K}^* + l_a \mathfrak{b}_b, \quad (6)$$

$$\tilde{\nabla}_a k_b = K_{ab}^* + l_a \mathcal{K}_b^* + l_b \mathcal{L}_a + l_a l_b \mathcal{K}^* + k_a l_b \mathcal{L} - k_a \mathfrak{a}_b^*, \quad (7)$$

$$\tilde{\nabla}_a m_b = L_{ab}^* + n_a \mathcal{L}_b^* + n_b \mathcal{K}_a + n_a n_b \mathcal{L}^* + m_a n_b \mathcal{K} + m_a \mathfrak{b}_b^*, \quad (8)$$

define a set of geometrical quantities, which characterize the embedding of $\Sigma_{t\chi}$: extrinsic curvatures (K_{ab} , L_{ab} , K_{ab}^* , L_{ab}^*); accelerations ($\mathfrak{a}_a$, $\mathfrak{a}_a^*$) of the timelike congruences (n^a , k^a), and similar “accelerations” ($\mathfrak{b}_a$, $\mathfrak{b}_a^*$) of the spacelike congruences (l^a , m^a); normal fundamental scalars ($\mathcal{K}$, $\mathcal{L}$, $\mathcal{K}^*$, $\mathcal{L}^*$); normal fundamental forms ($\mathcal{K}_a$, $\mathcal{L}_a$) and similarly defined quantities ($\mathcal{K}_a^*$, $\mathcal{L}_a^*$) for the basis vectors (m^a , k^a) with vorticities

$$\begin{aligned} \hat{\omega}_{ab}^{(\mathbf{k})} l^b &= \frac{1}{2} D_a \phi + \frac{\mathfrak{s}}{2\mathfrak{c}} (\mathfrak{a}_a + \mathfrak{b}_a), \\ \hat{\omega}_{ab}^{(\mathbf{m})} n^b &= \frac{1}{2} D_a \phi - \frac{\mathfrak{s}}{2\mathfrak{c}} (\mathfrak{a}_a + \mathfrak{b}_a). \end{aligned} \quad (9)$$

These are all depicted on Fig. 1. They exhibit a complex set of interdependence relations, meticulously explored.²³

3. Gauge fixing in scalar-tensor gravitational theories

In this section we summarize the main result²³ in the gauge-fixing of the perturbations of the spherically symmetric, static background in an unspecified scalar-tensor theory. Similarly to the unitary gauge in cosmology, here we aim to achieve a “radial unitary” gauge, thus to absorb the perturbation of the scalar field in a suitable choice of coordinates. On the spherically symmetric, static background (where quantities carry an overbar) Helmholtz-like theorems assure the decompositions

$$\delta N_a = \bar{D}_a P + E^b{}_a \bar{D}_b Q, \quad (10a)$$

$$\delta M_a = \bar{D}_a V + E^b{}_a \bar{D}_b W, \quad (10b)$$

$$\delta g_{ab} = \bar{g}_{ab} A + \bar{D}_a \bar{D}_b B + \frac{1}{2} (E^c{}_a \bar{D}_c \bar{D}_b + E^c{}_b \bar{D}_c \bar{D}_a) C, \quad (10c)$$

with $E_{ab} = \sqrt{\bar{g}} \varepsilon_{ab}$ the 2-dimensional Levi-Civita tensor and $\varepsilon_{\theta\varphi} = 1$. The scalars ($N, M, \mathcal{N}, P, V, A, B$) and (Q, W, C) form the even and odd sectors of the perturbations, respectively, and they decouple. The latter represent the tensor degrees of freedom.¹⁷

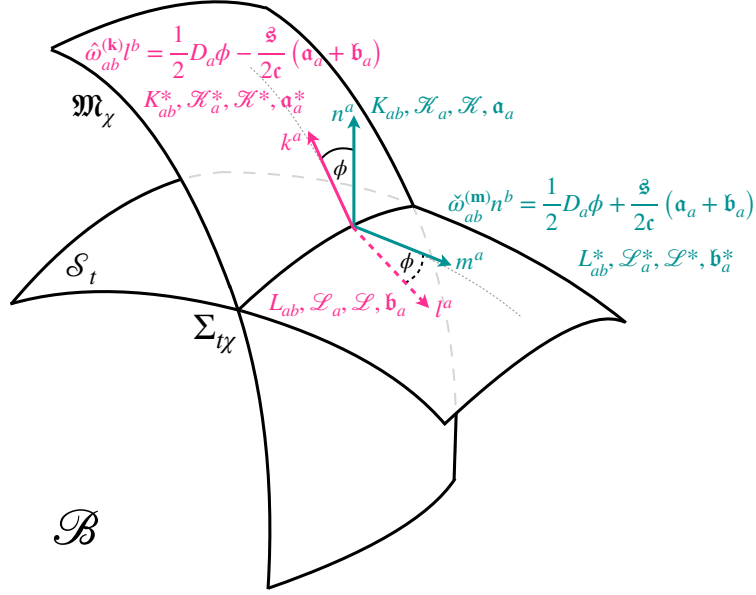


Fig. 1. The bases adapted to the two foliations, their embedding variables and the nonvanishing vorticities.

Next, we explore the diffeomorphism gauge freedom, with generators

$$(\xi^t, \xi^\chi, \xi^a = \bar{D}^a \xi + E^{ba} \bar{D}_b \eta) \quad , \quad (a = \theta, \varphi) \quad (11)$$

also Helmholtz-decomposed. The initial metric and scalar perturbations are related to those (denoted by overhat) emerging after the diffeomorphism as

$$\mathfrak{L}_\xi \tilde{g}_{ab} = \delta \tilde{g}_{ab} - \widehat{\delta g}_{ab} \quad , \quad \mathfrak{L}_\xi \phi = \delta \phi - \widehat{\delta \phi} \quad , \quad (12)$$

which generate the respective variations of the even and odd sector variables.¹⁹ In the framework of the nonorthogonal double foliation we managed to unambiguously fix the gauge through the choice

$$\xi^t = \frac{P + \dot{\xi}}{\bar{N}^2} \quad , \quad \xi^\chi = \frac{\delta \phi}{\phi'} \quad , \quad \xi = -\frac{B}{2} \quad , \quad \eta = -\frac{C}{2} \quad , \quad (13)$$

which ensure the radial unitary gauge $\widehat{\delta \phi} = 0$, the perturbation of the 2-metric being a conformal rescaling ($\widehat{B} = 0 = \widehat{C}$) and $\widehat{P} = 0$, a condition impossible to achieve in the orthogonal double foliation formalism, which consumed the respective degree of freedom.

Hence the odd sector consist of $(\widehat{Q}, \widehat{W})$, while the even sector is formed by $(\widehat{N}, \widehat{M}, \widehat{\mathcal{N}}, \widehat{V}, \widehat{A})$. The former has been extensively discussed,¹⁹ while the study of the latter became possible with the unambiguous gauge choice emerging in the nonorthogonal double foliation formalism and will be presented elsewhere.

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