



Gravitational waves and scalar perturbations in spherically symmetric scalar-tensor gravity

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Preliminaries

- The recent detection of **gravitational waves**¹⁻⁶ (both from the study of the dispersion relations⁷ and from the mere 1.7s delay⁸ of the soft GRB 170817A following GW170817) has confirmed that gravitational waves **propagate with the speed of light**
- Many modified gravity theories** were **disriled** (certain galileon models, vector-tensor, generalised Proca or bigravity theories)
- Many scalar-tensor gravity theories** however **survived** the test
- The study of **perturbations** of such models is important to **establish the ghost-free and instability-free parameter regimes**
- This has been investigated⁹ for a wide range of scalar-tensor theories in the **spherically symmetric setup**, exploring a **2+1+1 decomposition of space-time based on an orthogonal double foliation**¹⁰⁻¹¹

A non-orthogonal 2+1+1 decomposition

- In order to be able to discuss the even sector perturbations, we worked out a novel 2+1+1 decomposition of space-time and gravitational dynamics, based on a **non-orthogonal double foliation**¹²
- temporal and spatial **evolutions**:

$$\left(\frac{\partial}{\partial t}\right)^a = Nn^a + N^a + \mathcal{N}m^a = \frac{N}{c}k^a + N^a$$

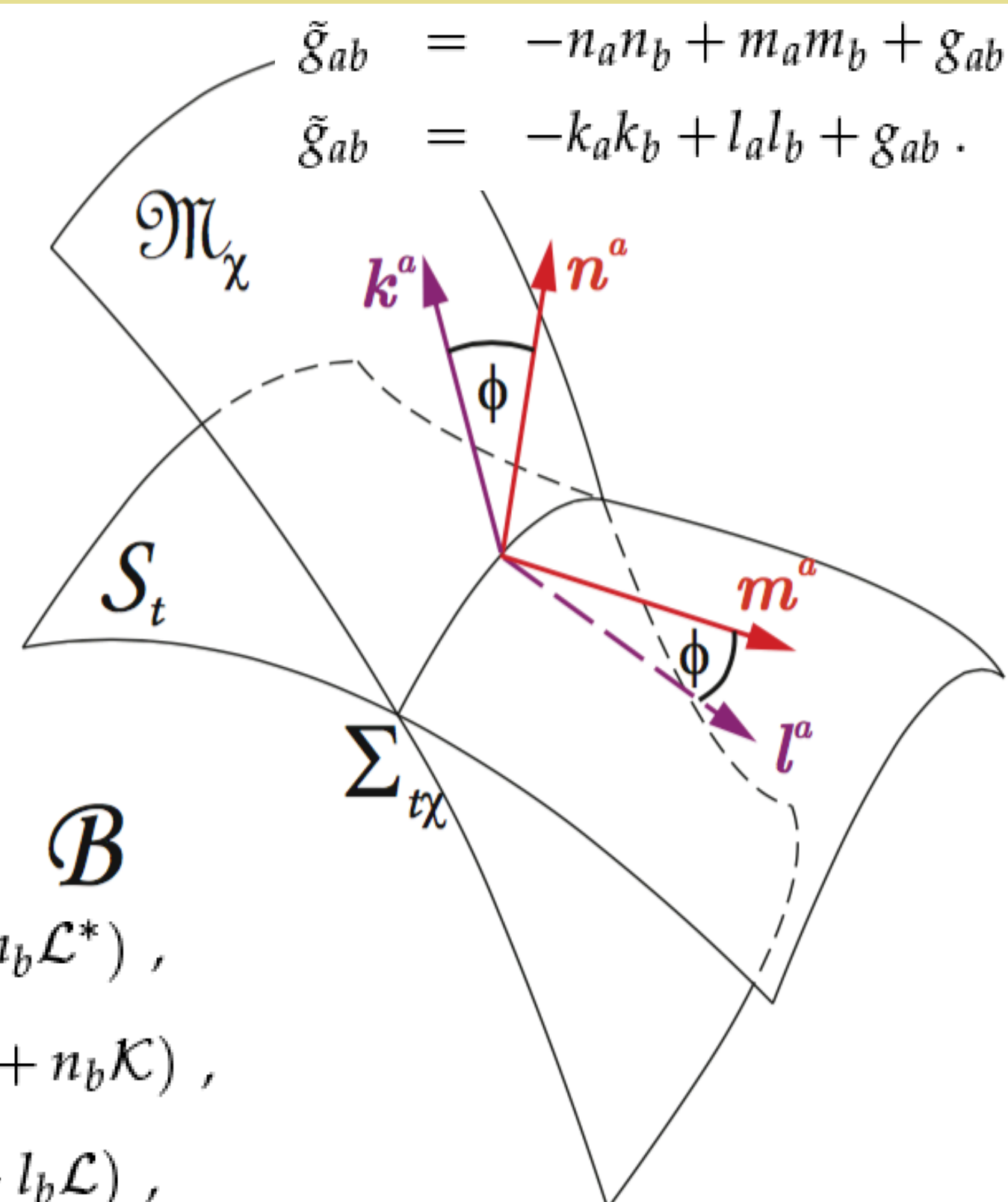
$$\left(\frac{\partial}{\partial \chi}\right)^a = Mm^a + M^a = M(-sk^a + cl^a) + M^a$$
- k^a and m^a have **vorticity**, both vanishing with $\mathcal{N}$
- angle of **rotation** ϕ related to $\mathcal{N}$
- 2+1+1 decomposition of the covariant derivatives:

$$\nabla_a n_b = K_{ab} + 2m_{(a}K_{b)} + m_a m_b \mathcal{K} - n_a (a_b - m_b \mathcal{L}^*)$$

$$\tilde{\nabla}_a m_b = L_{ab}^* + n_a \mathcal{L}_b^* + n_b \mathcal{K}_a + n_a n_b \mathcal{L}^* + m_a (b_b^* + n_b \mathcal{K})$$

$$\tilde{\nabla}_a k_b = K_{ab}^* + l_a \mathcal{K}_b^* + l_b \mathcal{L}_a + l_a l_b \mathcal{K}^* - k_a (a_b^* - l_b \mathcal{L})$$

$$\tilde{\nabla}_a l_b = L_{ab} + 2k_{(a} \mathcal{L}_{b)} + k_a k_b \mathcal{L} + l_a (b_b + k_b \mathcal{K}^*)$$
- Set of independent quantities: $(K_{ab}, \mathcal{K}^a, \mathcal{K}), (L_{ab}^*, \mathcal{L}^*), (a_a, b_a^*)$
 dynamical embedding vars. non-dynamical embedding vars.



Gauge-fixing

- We explore this new formalism in discussing the **gauge transformations** of the scalar perturbations of both the **metric tensor** and **scalar field** on a spherically symmetric background, achieving **unambiguous gauge-fixing**¹³
- The **perturbed metric** (overbar=unperturbed quantities)

$$ds^2 = -(\bar{N}^2 + 2\bar{N}\delta N)dt^2 + 2\bar{M}\delta N dt d\chi + 2\delta N_a dt dx^a + (\bar{g}_{ab} + \delta g_{ab})dx^a dx^b + 2\delta M_a dx^a d\chi + (\bar{M}^2 + 2\bar{M}\delta M)d\chi^2$$
 all shifts vanish on the background: $\bar{N}^a = \bar{M}^a = 0$
 the double foliation on the background chosen as perpendicular: $\bar{\mathcal{N}} = 0$
 the scalar field is constant on $\bar{\mathfrak{M}}_\chi$, thus $\bar{\phi} = \bar{\phi}(\chi)$
- Helmholtz-like decompositions** of vectors and symmetric tensors on spherically symmetric background:

$$V_a = \bar{D}_a V_{\text{rotfree}} + E^b_a \bar{D}_b V_{\text{divfree}}$$

$$S_{ab} = S_{ba} = \bar{h}_{ab} S_{\text{scalar}} + \bar{D}_a \bar{D}_b S_{\text{rotfree}} + \frac{1}{2} (E^c_a \bar{D}_c \bar{D}_b + E^c_b \bar{D}_c \bar{D}_a) S_{\text{divfree}}$$
 scalars (even sector) pseudo-scalars (odd sector)
- thus:

$$\delta N_a = \bar{D}_a P + E^b_a \bar{D}_b Q,$$

$$\delta M_a = \bar{D}_a V + E^b_a \bar{D}_b W,$$

$$\delta g_{ab} = \bar{g}_{ab} A + \bar{D}_a \bar{D}_b B + \frac{1}{2} (E^c_a \bar{D}_c \bar{D}_b + E^c_b \bar{D}_c \bar{D}_a) C$$
- Transformations of the metric and scalar under diffeomorphisms:

$$(\xi^t, \xi^\chi, \xi^a = \bar{D}^a \xi + E^{ba} \bar{D}_b \eta), \quad (a = \theta, \varphi) \quad (\text{overhat=perturbation after diffeomorphism})$$

$$\mathfrak{L}_\xi \tilde{g}_{ab} = \delta \tilde{g}_{ab} - \widehat{\delta \tilde{g}_{ab}}$$

$$\mathfrak{L}_\xi \phi = \delta \phi - \widehat{\delta \phi}$$
 in detail:

$$\widehat{\delta N} = \delta N - \bar{N} \dot{\xi}^t - \bar{N}' \xi^\chi,$$

$$\widehat{\delta \mathcal{N}} = \delta \mathcal{N} - \frac{\bar{N}^2}{2\bar{M}} \xi^{t'} + \frac{\bar{M}}{2} \dot{\xi}^\chi,$$

$$\widehat{\delta M} = \delta M + \bar{M}' \xi^\chi + \bar{M} \xi^{\chi'},$$

$$\widehat{P} = P - \bar{N}^2 \xi^t + \dot{\xi},$$

$$\widehat{Q} = Q + \dot{\eta},$$

$$\widehat{V} = V + \bar{M}^2 \xi^\chi + \xi' - \frac{2}{\chi} \xi,$$

$$\widehat{W} = W + \eta' - \frac{2}{\chi} \eta,$$

$$\widehat{A} = A + \frac{2}{\chi} \xi^\chi,$$

$$\widehat{B} = B + 2\xi,$$

$$\widehat{C} = C + 2\eta.$$

$$\widehat{\delta \phi} = \delta \phi - \bar{\phi}' \xi^\chi$$
- Gauge choices:
 - $\rightarrow \xi^\chi$ to fix $\widehat{\delta \phi} = 0$
 - $\rightarrow \xi$ to fix $\widehat{B} = 0$
 - $\rightarrow \eta$ to fix $\widehat{C} = 0$
$$\widehat{g}_{ab} = (1 + \widehat{A}) \bar{g}_{ab}$$
 perturbation of 2-metric= conformal rescaling
- choice of ξ^t :
 - ① for orthogonal foliations $\rightarrow$ to fix $\widehat{\delta \mathcal{N}} = 0$

$$\xi^t = \int d\chi \frac{2\bar{M}}{\bar{N}^2} \left(\delta \mathcal{N} + \frac{\bar{M}}{2} \dot{\xi}^\chi \right) + F(t, \theta, \varphi)$$

$$\rightarrow$$
 contains an arbitrary function, hampering the physical interpretation of perturbations
 - ② for **non-orthogonal foliations** $\rightarrow$ to fix $\widehat{P} = 0$

$$\rightarrow$$
 unambiguous gauge-choice:
- After gauge-fixing the discussion of perturbations possible:
 - odd sector** $\widehat{Q}, \widehat{W}$ (Regge-Wheeler¹⁴-type)
 - even sector** $\widehat{V}, \widehat{A}, \widehat{\delta N}, \widehat{\delta \mathcal{N}}, \widehat{\delta M}$ (Zerilli¹⁵-type)

Gravitational dynamics

- 6 dynamical metric + 6 dynamical embedding variables: (g_{ab}, M^a, M)
- Canonical momenta**: $\pi^{ab} = \sqrt{g} M [K^{ab} - g^{ab} (K + \mathcal{K})],$
 $p_a = 2\sqrt{g} \mathcal{K}_a,$
 $p = -2\sqrt{g} K.$
- Einstein-Hilbert action** (with boundary terms Q , Lagrange-multipliers $N, N^a, \mathcal{N}$; written in “already Hamiltonian form”)

$$S_{EH} = \int dt \int d\chi \int_{\Sigma_{t\chi}} d^2x \left[\pi^{ab} \dot{g}_{ab} + p_a \dot{M}^a + p \dot{M} - N \mathcal{H}_\perp^G - N^a \mathcal{H}_a^G - \mathcal{N} \mathcal{H}_\mathcal{N}^G + Q \right]$$
- Hamiltonian and diffeomorphism constraints**¹³:

$$\mathcal{H}_\perp^G = \sqrt{g} \left[-M \left(R + 3L^{*ab} L_{ab}^* - L^{*2} \right) + 2g^{ab} \partial_\chi L_{ab}^* - 2 \left(M^c D_c L^* + 2L_{ab}^* D^a M^b \right) + 2D^a D_a M + \frac{M}{\sqrt{g}} \left[\frac{1}{M^2} \left(\pi_{ab} \pi^{ab} - \frac{\pi^2}{2} \right) + \frac{1}{2} p_a p^a + \frac{1}{8} p^2 - \frac{\pi p}{2M} \right] \right]$$

$$\mathcal{H}_a^G = -2D_b \pi_b^a + p D_a M - \partial_\chi p_a + p_a D_b M^b + M^b D_b p_a + p_b D_a M^b,$$

$$\mathcal{H}_\mathcal{N}^G = 2L_{ab}^* \pi^{ab} - 2p^a D_a M - M D_a p^a - \partial_\chi p + D_a (p M^a)$$

Conclusion

- Metric perturbation formalism adapted to the non-orthogonal double foliation, Hamiltonian formalism and gauge-fixing** worked out
- Discussion of the **full spectrum of perturbations (including gravitational waves) in spherically symmetric scalar-tensor gravity** becomes possible

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- Acknowledgements:** This work has been supported by the Hungarian National Research and Development Office Grant NFKI-123996. The work of C.G. was further supported by the UNKP-17-2 New National Excellence Program of the Ministry of Human Capacities. The work of Z.K. was further supported by the UNKP-17-4 New National Excellence Program of the Ministry of Human Capacities.

