

THE COMPARE OF ANGLES OF REPOSE WITH DISCRETE ELEMENT METHOD AND MEASUREMENT

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Abstract: In this study the angle of repose of hulled millet with different moisture content was investigated with measurement and discrete element method. The result of the simulation and measurement were compared. The aim of this study to define the unknown micromechanical parameter set of the discrete element model. In further studies the calibrated parameter set of the modelling of equipment could be used for the chosen granular material.

Key words: angle of repose, discrete element method, cohesion, moisture, hulled millet

1. INTRODUCTION

There are several technologies in the food and pharmaceutical industry which include mixing, drying and the transportation of granular materials.

The aim of this study is to compare angle of repose measurements on granular material processing equipment with simulations using the discrete element method. The angle of repose has a role in the design and maintenance of agricultural machines. These include equipment and vessels used in sowing, harvest, the transportation of material, drying and rationing. By determining the angle of repose we can describe the internal friction inside granular material. This is the preferred method to provide fast, cost-efficient approximation of the kinetic and thermodynamic processes in granular materials with sufficient accuracy. The material properties of the test material (hulled millet) and some simulation parameters were unknown, e.g. the cohesion forces and dimensionless numbers referring to the moisture value of the material. Hulled millet was chosen as a granular material for the purpose of this study because of being nearly homogeneous in the scale of the study and the fact that the geometry of the granules can be approximated by spheres made it a good choice as test material.

2. THE DISCRETE ELEMENT METHOD

Currently there are a number of discrete element method software available for research, e.g. EDEM [1], SAMADII [2], Yade [3], PFC3D [4].

2.1. The DEM simulation software

In this study, the Yade [5] open source discrete element method software was used, in which the simulation model was built in Python.

Several models are available to describe the rheological processes between granules in the program (e.g. friction, cohesive, capillary). During the simulation the software uses an explicit integral system with respect to time, which iterates between the movement and the force-displacement law, the cycle can be seen in Fig. 1.

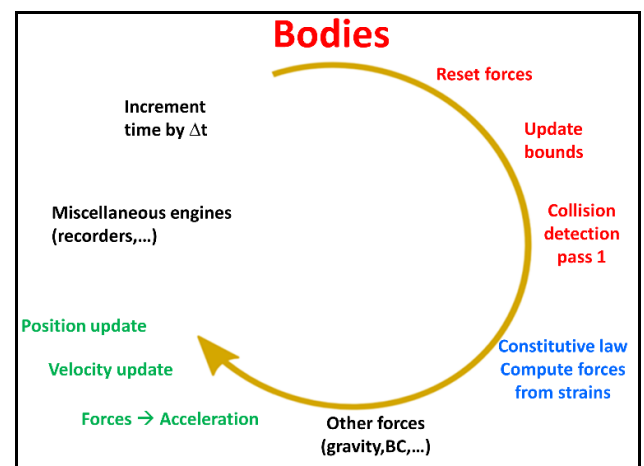


Fig.1: Calculation cycle in Yade discrete element software [5]

The first step in explicit time integral is to set the value of forces to default, then refresh the bonds. Afterwards the collisions are detected. The software then calculates the forces from displacements based on the engine settings. From these forces the acceleration is calculated, the velocity is refreshed, then these data (forces, displacements) are recorded. Finally, the time step is increased by Δt_i and the bodies on the program display are refreshed, then the cycle starts again. To provide the stability of the integral, an upper limit for Δt_i is calculated [6]:

$$\Delta t_{cr}^{(p)} = R_i \cdot \sqrt{\frac{\rho_i}{E_i}} \quad (1)$$

where:

$\Delta t_{cr}^{(p)}$ – critical time step (PWaveTimeStep) [s]

R_i – radius of particles [m]

ρ_i – particle density [kg/m³]

E_i – micromechanical Young modulus of particles [Pa]

The time step which was calculated is only an estimation, so it has to be validated.

2.2. The setup of the DEM simulation

The granular material used in this study was hulled millet, that can be seen in Fig.2., with a characteristic diameter of $d_p=1.7$ mm

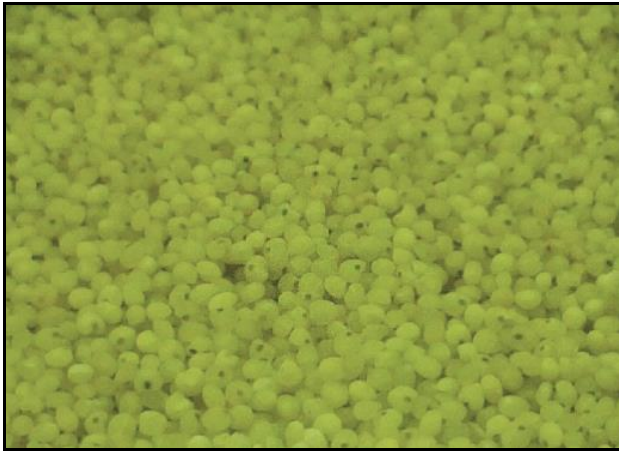


Fig.2: Hulled millet

The aim of the study is to determine the effect of moisture on the mechanical properties of the material. The shape of granules was approximated using spheres. Because the moisture content varied during the study, a cohesion engine was used in simulations, which takes adhesion into account, as this cohesion contact model can show the effect of different moisture contents. For cohesion contact models in DEM, the forces are calculated for each degree of freedom using stiffness coefficients [7]. The normal force can be described by the following equation:

$$F_n = k_n \cdot u_n \quad (2)$$

where:

F_n – normal force [N]

k_n – normal elastic stiffness [Pa]

u_n – normal displacement [m].

While macroscale stiffness is not related to the granule size, the normal stiffness is a function of the granule size:

$$k_n = 2E_c \cdot \frac{R_1 R_2}{R_1 + R_2}, \quad (3)$$

where:

E_c – macroscale stiffness [Pa]

R_1 – radius of an interacting particle [m]

R_2 – radius of the other interacting particle [m].

Shear and normal elastic stiffness can be scaled [7] using the following equation:

$$k_s = \alpha \cdot k_n \quad (4)$$

where:

k_s – shear elastic strength [Pa]

α – non-dimensional factor [1]

The DEM also calculates the maximal forces (5)-(6) and torques (7)-(8) in comparison with the normal and shear strengths [7]. If the tension exceeds the maximum strength during the simulation, the bond breaks, the connection ceases to exist. On certain settings a new a connection automatically forms on next contact. The maximal normal force is calculated as following in DEM model:

$$F_n \leq \sigma_n^l \cdot A \quad (5)$$

The maximal shear force:

$$\|F_s\| \leq F_n \cdot \tan \varphi_c + \sigma_s^l \cdot A \quad (6)$$

The bending moment:

$$M_b \leq \frac{\sigma_n^l \cdot I_b}{R_c} \quad (7)$$

A twisting moment:

$$M_t \leq \frac{\sigma_s^l \cdot I_t}{R_c} \quad (8)$$

where:

A – the reference surface area [m²]

F_s – shear force [N]

σ_n^l – tensile strength [Pa]

σ_s^l – shear strength [Pa]

φ_c – friction angle [°]

I_b – the reference bending moment of inertia [m⁴]

I_t – the reference polar moment of inertia [m⁴]

R_c – the reference radius of the contact [m]

In Table 1. the computer specifications used in the study and the maximal time requirements for simulations are shown.

Table 1. Computer specifications and time requirements of simulations

No	Processor Number	Processor base frequency	Memory	Max time
		[GHz]	[Gbyte]	[min]
1.	Intel® Core™ i7-4710HQ	2.5	8	52
2.	Intel® Core™ i5-6500	3.2	16	40

In Fig 3. the model of the measuring equipment built in Yade can be seen.

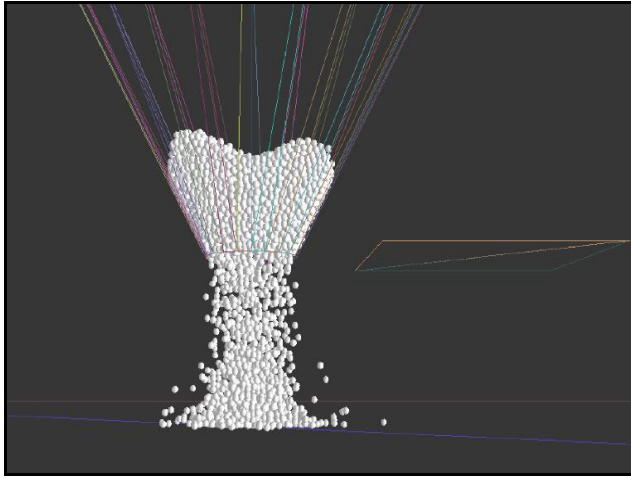


Fig. 3. The measuring equipment in Yade software

The model consists of a cylinder and two sides of a box in the z direction, which make up the upper and the lower surfaces. The first step in the simulation is the sedimentation of particles in the cylinder. When the unbalanced forced decreased to a sufficient value, cohesion was turned on with the setCohesionNow command. Then the outlet opened and the millet started flowing, while a harmonic engine was resonating the cylinder. After a number of iterations, the outlet closed because of the particles stuck on the surface of the cylinder, this way the simulation time decreased and the poured granule pile stabilized quicker. When the particles reached a sufficiently low velocity, the simulation stopped and the screen display was saved.

To further lower the simulation time requirement, the height of the cone outlet was decreased to 0.4 m, as in a 1:1 scale the simulation would have taken days with the specifications listed in Table 1.

2.3. The parameter sets of the simulation

Some of the unknown parameters have been previously determined by measurements. Friction angles have been determined for different moisture contents using direct shear box test [2]. Sieve analysis was used to determine the granule sizes and distribution, and granule density was calculated from porosity. The micromechanical Young modulus used in the DEM, and the shear and tensile strengths from the shear box test was based on previous simulation experience and literature [9-11]. According to previous shear box measurements [8], the increase in moisture content resulted in the decrease of cohesion forces and the friction angle. The parameters which have to be calculated are the dimensionless bending (etaRoll) and twisting (etaTwist) strengths, which were made equal and changed together during the simulation. [4]. These are marked as eta in the following pages. These settings can be seen in Table 2.

Table 2. Settings for the DEM system

No	x $\frac{[\text{kg H}_2\text{O} \cdot 100\%]}{[\text{kg wet product}]}$	d _p [mm]	ρ _p $\frac{[\text{kg}]}{[\text{m}^3]}$	φ _p [°]	E _p [MPa]	ν _p [1]	τ _{shear} [kPa]	σ _{normal} [kPa]
1.	12.5	1.7±0.1	1381	42.2	20	0.2	6.1	6
2.	14.5	1.7±0.1	1385	42.2	20	0.2	5.2	5
3.	19.4	1.7±0.1	1394	41.5	20	0.2	4.2	4
4.	21.9	1.7±0.1	1399	40.5	20	0.2	3.4	3
5.	24.7	1.7±0.1	1404	39.3	20	0.2	1.7	1.5
6.	29.8	1.7±0.1	1413	36.1	20	0.2	2.2	2

where:

x – moisture content based on the wet material $\frac{[\text{kg H}_2\text{O} \cdot 100\%]}{[\text{kg wet product}]}$

d_p – particle diameter [mm]

ρ_p – particle density $\frac{[\text{kg}]}{[\text{m}^3]}$

E_p – micromechanical Young modulus [MPa]

ν_p – particle Poisson's ratio [1]

φ_p – particle friction angle [°]

τ_{shear} – the shear strength of the cohesion [kPa]

σ_{normal} – the tensile strength of the cohesion [kPa]

There are two ways in the software to simulate cohesion forces. The first, which is used in this study is to make cohesion connections once in a moment in time between the currently particles in contact (setCohesionNow). This way, no new connections are formed after a time step. The other way is the continuous making of cohesion connections with every new contact (setCohesionOnNewContacts). The global damping which was used to decrease particle acceleration was set to 0.4 for each run. The time step value was $1.42 \cdot 10^{-5}$ s, which is the double of the value from the PWaveTimeStep (1) equation. These parameter sets provided numerically stable particles.

The cylinder and surfaces used in the DEM model are based on the properties of steel, which can be seen in Table 3. Between the wall and the particles the Mohr-Coulomb plasticity [6] was used instead of the cohesion model. In this case, only the equation (2)-(4) shown in section 2.2 were used by the DEM software.

Table 3. Material properties of the measuring equipments in the DEM software

No	ρ _E $\frac{[\text{kg}]}{[\text{m}^3]}$	E _E [GPa]	ν _E [1]	φ _E [°]
1.	7750	200	0.3	40.1

where:

ρ_E – equipment density $\frac{[\text{kg}]}{[\text{m}^3]}$

E_E – equipment Young modulus [GPa]

ν_E – equipment Poisson's ratio [1]

φ_E – equipment friction angle [°]

3. MEASUREMENT AND METHOD

The calculation of angle of repose is regulated by a standard. Different measuring equipment are recommended for

different granule sizes. The equipment for 1.7 mm hulled millet is described below.

3.1. Measurement equipment

The measurement equipment was built according to MSZ EN standard [12] both the sematic and real device can be seen in Fig. 4.

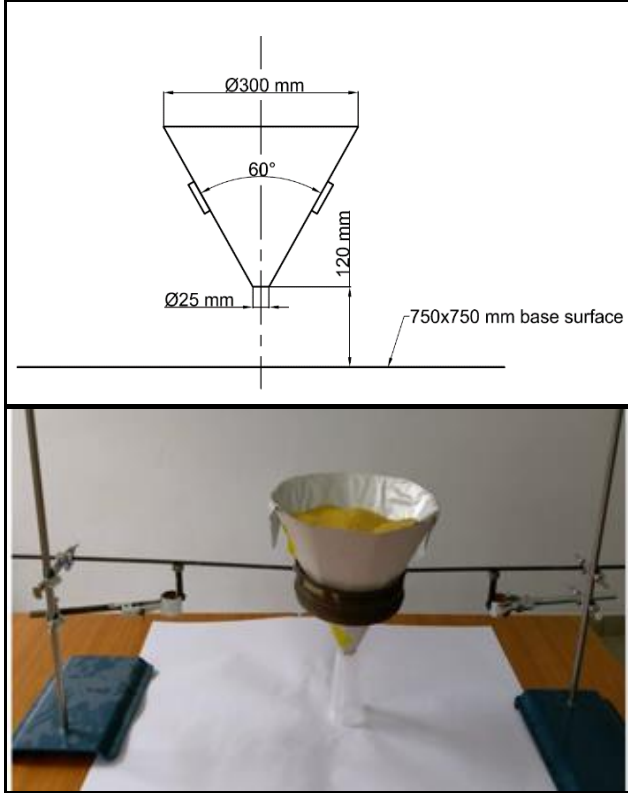


Fig. 4. In the upper part the size according to standard is shown, the real device is seen at the bottom

The cone is made of 1.5 mm thick cardboard, the inside is covered with aluminum foil, because of the moisture content of the material. It was fixed to the post using a retaining ring.

3.2. Measurements

First the hulled millet was put in a wind sorter to provide nearly constant granule size. Then water is added and the mixture is stirred every hour for 24 hours before measuring (the moisture content were 12,5%, 14,5%, 19,4%, 21,9%, 24,7%, 29,8%). The measurement was done according to the standard [12]. The millet had to be poured out of the cone until the pile of millet reached the outlet.

3.3. Method

There are two ways to evaluate the collected data. One is shown in Fig 5. where the tangent line is drawn on the pile of granules, and the angle of repose can be determined from the enclosed angle between the tangent and a horizontal line. This was used in the simulation evaluation because the calculating capacity only allowed the use of less granules, and the height of the outlet was lowered.

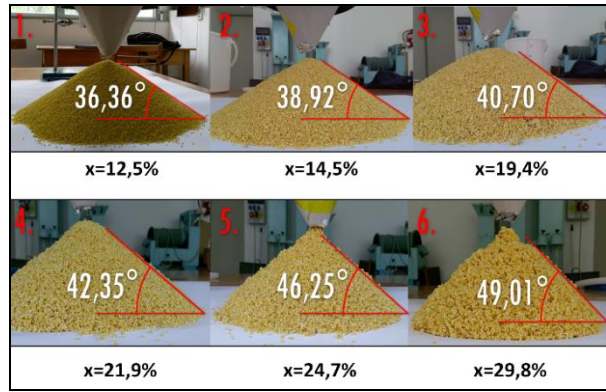


Fig. 5: The drawing of tangents for different moisture values

The other way to evaluate is stated in the standard [12], this was used for the data of a laboratory measurements. A circle is drawn on the paper under the cone, and a series of 45° lines are drawn in the circle, then the diameters of the pile are measured along the lines and the average value was taken. Then Eq. 2 was taken from the standard to calculate the angle of repose:

$$\alpha = \tan^{-1} \left(\frac{240}{\bar{d} - 25} \right) \quad (2)$$

where:

α – the calculated angle of repose [°]

$\bar{d}$ – the average diameter [mm].

4. RESULTS

Using result from a measurement regulated by standard, the dimensionless bending and twisting strengths were calculated which are used in the discrete element method.

4.1. Results of the simulation and measurement

First sensitivity tests were simulated with the smallest and largest moisture values based on Table 2., changing eta values on a logarithmical scale, the effect of this can be seen on Fig. 6. The reason of these tests was to lower the time of the simulations. From this we can approximate the range of eta values worth using for simulation, as they are between the minimum ($x=12.5\%$) and maximum ($x=29.8\%$) moisture values.

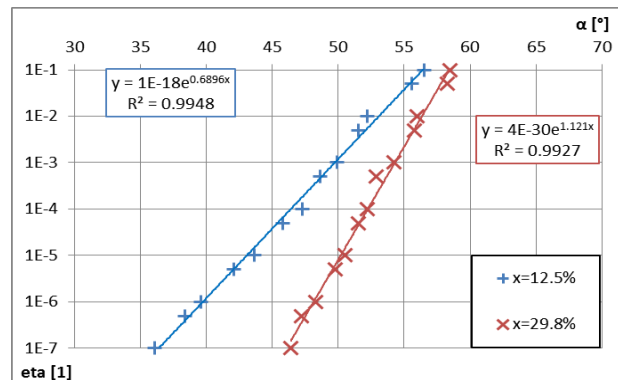


Fig 6: Eta values on a logarithmic scale as a function of angles of repose, for moisture values of $x=12,5\%$ and $x=29,8\%$

For both moisture values $R^2 > 0.99$, which means the trendlines fit the values well. Statistical analysis [13]-[14] was conducted on the data seen in Table 6. During the Abbe-test for 95%, 99%, and 99.9% the null-hypothesis was false, the measured data are not constant, so there is a tendency between them. During regression testing the error of sum squares (SSE) was 2.6 and 1.4 for moisture contents of 12.5% and 29.8% respectively. As both values are larger than 1, the results are sufficient. A confidence interval was determined for the data, which is shown in Fig. 7 for two moisture values.

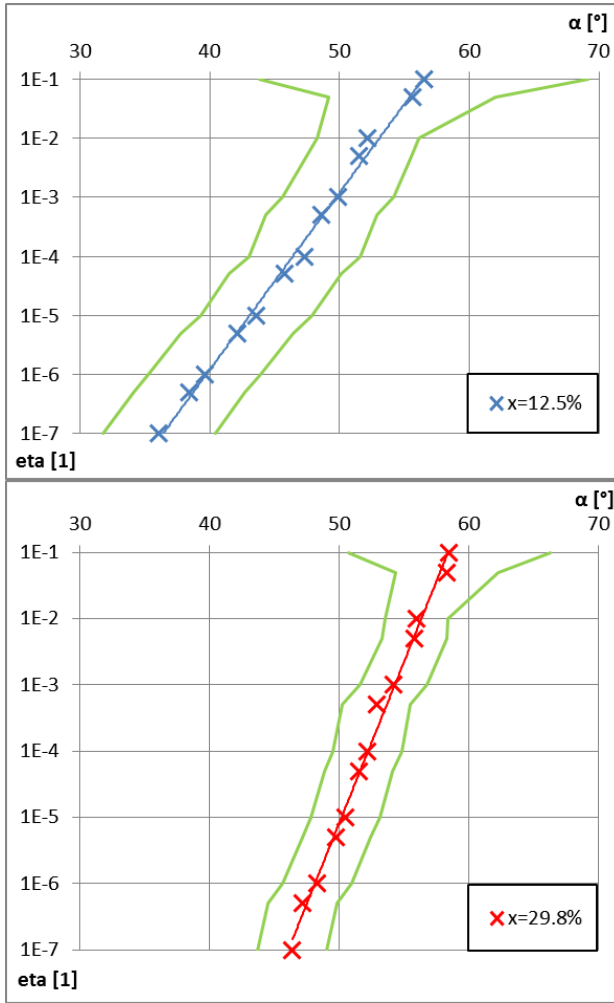


Fig 7. Eta parameter as a function of the angle of repose with confidence intervals (green), for moisture values of 12.5% (upper) and 29.8% (lower)

The simulation results are inside the confidence intervals with a 95% probability. The measured and simulated angles of repose are shown in Table 4., where the modeled values were given in intervals due to uncertainties in evaluation and reading.

Table 4. Results of measurement and simulation

No	1.	2.	3.	4.	5.	6.
x [%]	12.5	14.5	19.4	21.9	24.7	29.8

Measurement	α_m [°]	36.4	38.9	40.7	42.4	46.3	49
Simulation	α_{sz} [°]	36.1-38.4	37.6-39.4	39.8-41.9	41-43.3	45.4-47.3	48.3-49.8
	eta [10 ⁻⁶]	1-5	0.1-0.5	0.1-0.5	0.5-1	0.5-1	0.1-0.5

where:

α_m – angle of repose by measurement [°]

α_{sz} – angle of repose by DEM simulation [°]

eta – dimensionless bending and twisting strengths [10⁻⁶]

The measured angles of repose can be seen in Fig. 8., these were also analyzed using statistical methods. [12-13].

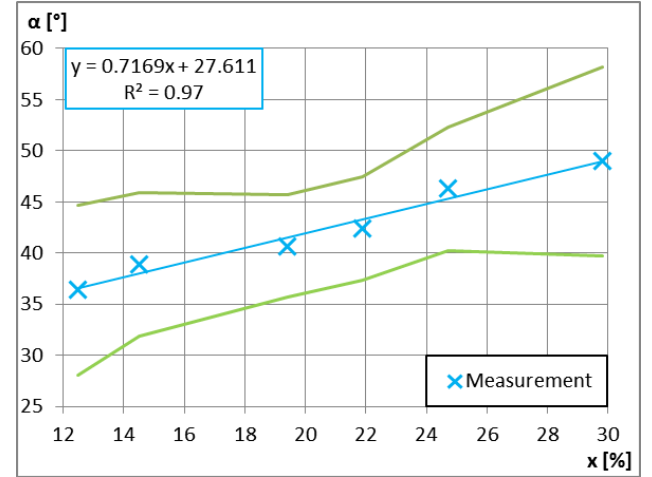


Fig 8. Angles of repose by measurement with confidence interval (green), in case of 95% probability

During the Abbe-test the null hypothesis was false for 95%, 99%, 99.9% cases, so the data series show a tendency. The error of sum squares (SSE) is 3.3, which is sufficient because it is larger than 1. Confidence intervals for 95% probability are shown in Fig. 8.

4.2. Comparison of angles of repose by measurement and simulation

The angles of repose by measurement and simulation are approved by using statistical methods in Chapter 4.1. The results of the angles of repose can be seen in Fig. 9., where blue and red marking the measured and simulated data respectively.

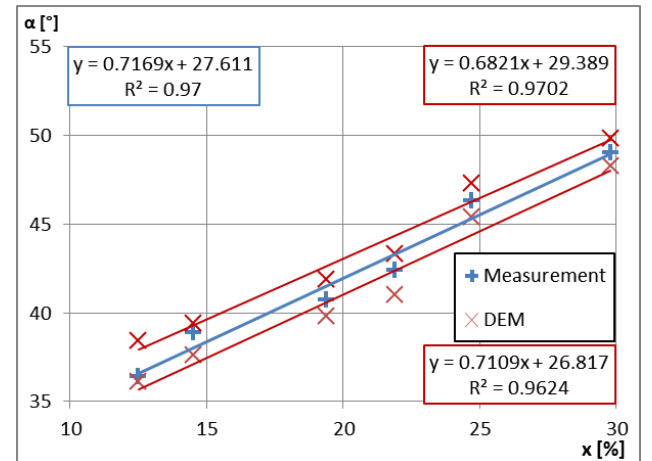


Fig 9: Angles of repose by measurement and simulation in the function of moisture content based on wet material

The results show that with the increase in moisture content the angle of repose increased, and this can also be seen in the simulated results as expected. While the granules stuck together more when increasing the shear and tensile strengths using sphere shaped granules, the increase in moisture content resulted in decreasing strengths. We can clearly state that the eta parameters are the main factors influencing angles of repose.

5. CONCLUSION

During this study the angles of repose were given for hulled millet using standard-regulated measurements and discrete element modeling (DEM) for different moisture contents. The geometry of the granules was approximated by a sphere for the purpose of this study. Increasing the moisture content resulted to an increase in the angle of repose, which was modeled with good accuracy on computer using the correct simulation parameters. Using the data from measurements, the dimensionless bending and twisting strengths were given (eta) for different moisture contents. The results of the sensitivity tests can be used in future studies. The study also showed that increasing the micromechanical stiffness in the simulation resulted the increase in the angles of repose. During the DEM simulations the cohesion between granules provided by shear and normal stiffness did not influence the results considerably, while changing the eta parameter had a larger effect on the results. Finally, statistical methods were used to analyze the measured angles of repose and the results of the sensitivity tests, and both proved to be satisfactory.

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