

BIFURCATION ANALYSIS OF AEROELASTIC SYSTEM WITH A LINEAR TUNED VIBRATION ABSORBER

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Introduction

Aeroelasticity is the study of the interaction of aerodynamic, elastic and inertia forces. When flexible structures, such as an airfoil, undergo wind excitation, flutter instability may arise. The flutter speed can be increased utilizing a Linear Tuned Vibration Absorber [1]. The influence of such an absorber on an aeroelastic system is investigated in this study.

The Model

The investigated model is a pitch-plunge aeroelastic system with a Linear Tuned Vibration Absorber (see Figure 1a). The nondimensional system equations are given by (for derivation see [2])

$$\begin{aligned} y'' + p_1 y' + y - (\xi p_1 (h' - y') + \eta (h - y)) &= -p_2 \mu^2 \left(c_0 (\alpha + y'/\mu) + c_1 (\alpha + y'/\mu)^3 \right), \\ \alpha'' + p_3 \alpha' + p_4 \alpha &= \mu^2 \left(c_0 (\alpha + y'/\mu) + c_1 (\alpha + y'/\mu)^3 \right), \\ \varepsilon h'' + \xi p_1 (h' - y') + \eta (h - y) &= 0, \end{aligned} \quad (1)$$

where the variable y describes the vertical (plunge) displacement, α is the angular (pitch) displacement of the airfoil. The variable h describes the vertical displacement of the absorber, which is attached to the center of gravity of the airfoil, see Figure 1b. The nondimensional system airfoil parameters are p_1 , p_2 , p_3 , and p_4 . The nondimensional parameters ε , ξ and η are the mass ratio, stiffness ratio and damping ratio of the wing and absorber. The aerodynamic lift and moment acting on the airfoil depend on the dimensionless wind velocity μ and the lift coefficient C_l . The lift coefficient of the airfoil depends on the effective angle of attack $\alpha_{eff} = \alpha + y'/\mu$, which takes into account the instantaneous motion of the system. In this paper, we used a symmetric NACA0012 airfoil [2]. To approximate the lift coefficient we used a cubic function of the effective angle of attack, i.e., $C_l(\alpha_{eff}) = c_0 \alpha_{eff} + c_1 \alpha_{eff}^3$, see Figure 1c. The coefficients c_0 and c_1 were determined by the least square method in the range $[-\alpha_{lim}, \alpha_{lim}]$, where $\alpha_{lim} = 0.3$.

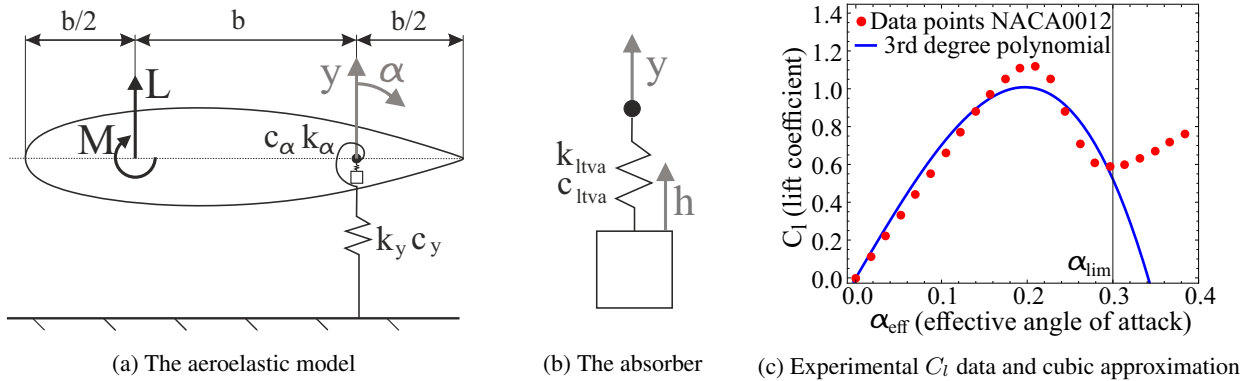


Figure 1: Experimental lift coefficient data and cubic approximation

Bifurcation Analysis with Matcont

The η stiffness ratio and μ the dimensionless wind velocity were the two bifurcation parameters. We used Matcont [3] for the numerical bifurcation analysis of Eq. (1). The other system parameters were $p_1 = 0.148$, $p_2 = 0.0147$, $p_3 = 0.054$, $p_4 = 0.275$, $\varepsilon = 0.3$, $c_0 = 7.69$, $c_1 = -65.8$. We determined the Hopf bifurcation curves of the angular displacement α and of the vertical displacement of the absorber h , for different ξ damping ratios (see Figure 2). On the Hopf curves we observed generalized Hopf points (where the first Poincaré-Lyapunov coefficient vanishes) and double Hopf bifurcation points (where the critical equilibrium has two pairs of purely imaginary eigenvalues). The generalized Hopf bifurcation point separates branches of subcritical and supercritical Hopf bifurcations. For nearby parameter values, the fold bifurcation of limit cycles occurs [4]. The double Hopf bifurcation can imply quasi-periodicity or a local birth of chaos. By increasing the value of ξ the double Hopf bifurcation vanishes and the two Hopf curves merge into one.

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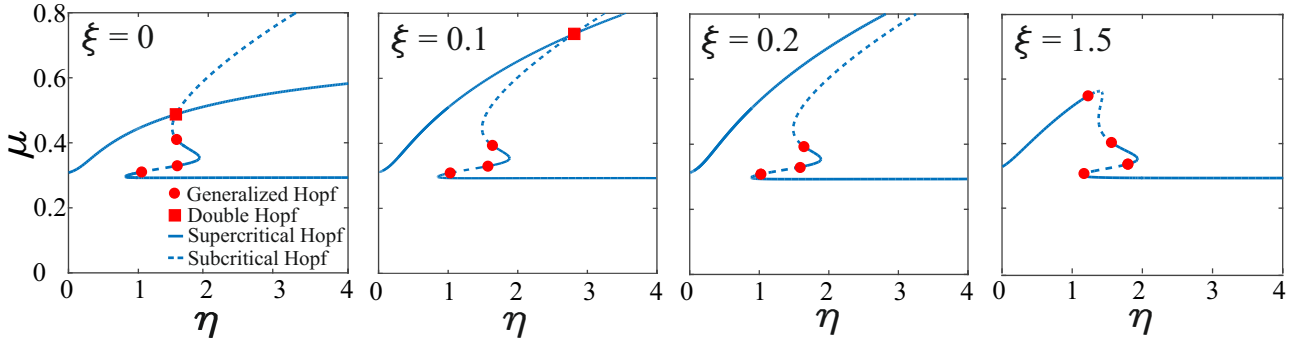


Figure 2: Bifurcation diagrams of the aeroelastic system for $\xi = \{0, 0.1, 0.2, 1.5\}$.

The nondimensional flutter velocity without absorber was $\mu = 0.31$. If we apply the Linear Tuned Vibration Absorber we can increase the flutter speed by increasing the η stiffness ratio. The stiffness ratio η can be increased safely until the absorber undergoes a Hopf bifurcation. Figure 3 illustrates the bifurcation diagrams with limit cycles of the aeroelastic system. Flutter velocity using the parameters $\xi = 0.11$, $\eta = 0.5$ was increased to $\mu = 0.41$. For larger η values more complex bifurcations also appear (Neimark-Sacker, period doubling of limit cycles, fold of limit cycles).

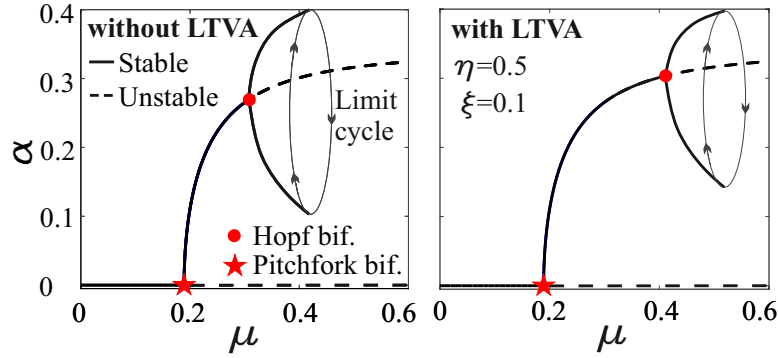


Figure 3: Bifurcation diagrams with and without the absorber.

Conclusions

The application of the Linear Tuned Vibration Absorber the flutter velocity can be increased by up to 71%. We observed the Hopf bifurcation of the airfoil and also the absorber. To reduce the amplitude of the limit cycles a Nonlinear Tuned Vibration Absorber [1, 5] have to be applied.

Acknowledgement

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