

Model-based Motion Compensation for Dynamic PET

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Abstract. Dynamic tomography reconstructs the time activity function of points of the human or animal body. The self-motion of the examined patient often cannot be avoided, thus the motion should be measured and this information should be incorporated into the reconstruction process. The ML-EM approach can be generalized to consider subject movement by concatenating a geometric transformation with the system matrix multiplication, which establishes the correspondence between subject-space and tomograph-space voxels. However, a geometric transformation is valid only in a single point of time, thus frames must be small enough to allow the assumption that the same transformation can be used. Higher speed motions, which occur in studying neurological disorders, would require very high number of frames, which leads to unacceptable reconstruction times. To attack this problem, we re-formulate the ML-EM scheme to incorporate continuous motion during the frames that are defined according to the tracer dynamics. Instead of geometric transformation, we apply 3D motion blur. For the efficient implementation, the paths of subject-space voxels are approximated by poly-lines and anti-aliasing line drawing methods are adopted to efficiently calculate the blurred result. Having executed the continuous motion blur on the voxel array before the forward projection and after the back projection, all other components of the simulation can be reused from a dynamic reconstruction system developed without motion compensation.

1 Introduction

In dynamic Positron Emission Tomography (PET), we analyze the dynamic nature of biological processes. Direct dynamic tomography algorithms usually iterate two steps, first an ML-EM [6] static forward and back projection are executed for each frame, then the parameters of a time function of pre-defined form are fit in each of the voxels [10]. Thus, the number of projections is proportional to the number of frames, which should be kept under control to avoid prohibiting reconstruction times and memory requirements.

The self-motion of the examined patient often cannot be avoided, e.g. when studying neurological disorders like epilepsy, thus the motion should be measured and this information should be incorporated into the reconstruction process [4].

Accurate reconstruction requires projections that simulate all relevant phenomena, including positron range, gamma-photon scattering and absorption both in the measured object and in detector crystals, etc., which are all affected by the motion of the body. Classic forward and back projections should be executed for the transformed data of each frame if we can assume that inside a frame, the subject movement is negligible, i.e. the motion is slow. However, this is usually not the case when neurological disorders are examined, subject motion has much quicker dynamics than the tracer diffusion. Gated approaches [9, 3] keep hits close to discrete time samples to solve the problem of motion inside the frame, but they ignore significant information and thus less reliable for low statistics or low dose experiments.

The other option is to re-bin the list-mode data and find another event list that would be measured if the subject did not move, and execute a conventional reconstruction for the modified sinogram [5, 1, 2]. Such sinogram filtering techniques are simple to implement and efficient, but cannot be applied for effects happening in the detectors. They either add noise or result in filtered measured data that are not of Poisson distribution, and suffer from the missing or lost data problem.

2 Reconstruction algorithm

Radiotracer isotopes randomly decay generating events in the detectors of the tomograph. An event is a composition of the identification of the involved detectors, also called *Line Of Response* or *LOR*, and its time of occurrence. To get an efficient numerical method, the space of the measured object is discretized into voxels and the measurement time is discretized by interval boundaries $t_1, t_2, \dots, t_{N_F}$ and events are binned in *frames* (t_F, t_{F+1}) . The number of voxels and frames are critical because the computational complexity of the reconstruction is proportional to their product.

Using the radiotracer concentration function $C(\mathbf{p}_V, t)$ for voxel V represented by parameter vector $\mathbf{p}_V = (\mathbf{p}_{V,1}, \dots, \mathbf{p}_{V,N_P})$, the expected number of decays in time interval $[t_F, t_{F+1}]$ is

$$\tilde{x}_F(\mathbf{p}_V) = \int_{t_F}^{t_{F+1}} C(\mathbf{p}_V, t) e^{-\lambda t} dt \quad (1)$$

where λ is the decay rate of the radiotracer.

The correspondence between decays and detected events is established by *system matrix* $\mathbf{A}$ that expresses the probability of generating an event in LOR L given that a decay happened in voxel V . If the measured object is moving, then the system matrix is also time dependent since a point of the object will be at different places. To handle this, we introduce two voxel arrays, an *object-space* array that follows the measured object, and a *tomograph-space* array that is fixed to the tomograph. To make the distinction clear, object-space voxels are denoted

by index V or V' , while tomograph-space voxels by W . The system matrix describing the correspondence between tomograph-space voxels and LORs is not time dependent, but tomograph-space voxels have dynamic activity due to both the dynamic radiotracer concentration and the time-dependent transformation between object-space and tomograph-space.

Let us denote the transformation at time t by $\mathbf{r}_t = T(\mathbf{r}_o, t)$ where $\mathbf{r}_t$ is the point in the tomograph's coordinate system, and $\mathbf{r}_o$ is the point in the object's coordinate system. Using this transformation, we can obtain an *overlap factor* $O_{V',W}(t)$ between object-space voxel V' and tomograph-space voxel W that expresses the ratio of the volume in voxel W that actually comes from object-space voxel V' .

The expected number of decays in tomograph-space voxel W during time interval $[t_F, t_{F+1}]$ is

$$\tilde{x}_{W,F} = \sum_{V'} \int_{t_F}^{t_{F+1}} C(\mathbf{p}_{V'}, t) e^{-\lambda t} O_{V',W}(t) dt = \sum_{V'} \tilde{x}_F(\mathbf{p}_{V'}) o_{V',W,F}. \quad (2)$$

where $o_{V',W,F}$ is the average overlapping of object-space voxel V' and tomograph-space voxel W during frame F :

$$o_{V',W,F} = \frac{\int_{t_F}^{t_{F+1}} C(\mathbf{p}_{V'}, t) e^{-\lambda t} O_{V',W}(t) dt}{\int_{t_F}^{t_{F+1}} C(\mathbf{p}_{V'}, t) e^{-\lambda t} dt}. \quad (3)$$

From now on, we assume that this average overlapping is independent of parameter vector $\mathbf{p}_V$, which is the case when either $C(\mathbf{p}_V, t) e^{-\lambda t}$ or the overlapping factor are constant in time, otherwise, it is a good approximation. In this case, we get

$$o_{V',W,F} \approx \frac{1}{t_{F+1} - t_F} \int_{t_F}^{t_{F+1}} O_{V',W}(t) dt. \quad (4)$$

The *expected number of events* $\tilde{y}_{L,F}$ in LOR L during frame F is the sum of the contributions of all voxels:

$$\tilde{y}_{L,F} = \sum_W \mathbf{A}_{L,W} \tilde{x}_{W,F}, \quad (5)$$

which is the conventional *forward projection*.

The measured number of hits in LOR L in frame F follows a Poisson distribution of expectation $\tilde{y}_{L,F}$. Because of the statistical independence of different LORs and different frames, the combined probability considering all LORs and all frames is the product of elementary probabilities. According to the concept of *maximum-likelihood reconstruction*, unknown parameters are found to maximize

the following log-likelihood:

$$\log \mathcal{L} = \sum_L \sum_F (y_{L,F} \log \tilde{y}_{L,F} - \tilde{y}_{L,F}) \quad (6)$$

where $y_{L,F}$ is the measured number of hits in LOR L during frame F .

The likelihood has an extremum where all partial derivatives are zero:

$$\sum_F \sum_W \frac{\partial \tilde{x}_{W,F}}{\partial \mathbf{p}_{V,P}} \sum_L \left(\mathbf{A}_{L,W} \frac{y_{L,F}}{\tilde{y}_{L,F}} - \mathbf{A}_{L,W} \right) = 0 \quad (7)$$

for $V = 1, 2, \dots, N_V$ and $P = 1, \dots, N_P$.

Using Eq. 2 and also exploiting our simplifying assumption that the average overlapping is independent of the parameter vector, we get

$$\frac{\partial \tilde{x}_{W,F}}{\partial \mathbf{p}_{V,P}} = \sum_{V'} \frac{\partial \tilde{x}_F(\mathbf{p}_{V'})}{\partial \mathbf{p}_{V,P}} o_{V',W,F} = \frac{\partial \tilde{x}_{V,F}}{\partial \mathbf{p}_{V,P}} o_{V,W,F}. \quad (8)$$

since derivative $\frac{\partial \tilde{x}_F(\mathbf{p}_{V'})}{\partial \mathbf{p}_{V,P}}$ is non-zero only if $V = V'$. Thus, Eq. 7 simplifies to

$$\sum_F \frac{\partial \tilde{x}_F(\mathbf{p}_V)}{\partial \mathbf{p}_{V,P}} \sum_L \sum_W o_{V,W,F} \left(\mathbf{A}_{L,W} \frac{y_{L,F}}{\tilde{y}_{L,F}} - \mathbf{A}_{L,W} \right) = 0. \quad (9)$$

The computation of the derivatives of the log-likelihood requires a forward projection and an update operation in each frame F . Indeed, in frame F , the expected number of radioactive decays in voxel V is $\tilde{x}_F(\mathbf{p}_V)$, which is forward projected via all overlapped tomograph-space voxels to obtain $\tilde{y}_{L,F}$ according to Eq. (5). The update operation is called the *back projection* and obtains a new estimate of the activity as

$$x_{V,F} = \tilde{x}_F(\mathbf{p}_V) \cdot \frac{\sum_W o_{V,W,F} \sum_L \mathbf{A}_{L,W} \frac{y_{L,F}}{\tilde{y}_{L,F}}}{\sum_W o_{V,W,F} \sum_L \mathbf{A}_{L,W}}. \quad (10)$$

From this equation, we can express

$$\sum_W o_{V,W,F} \sum_L \mathbf{A}_{L,W} \frac{y_{L,F}}{\tilde{y}_{L,F}} = \frac{x_{V,F}}{\tilde{x}_F(\mathbf{p}_V)} \cdot \sum_W o_{V,W,F} \sum_L \mathbf{A}_{L,W}.$$

In this formula, we can recognise the *sensitivity* $S_{V,F}$ of voxel V during frame F :

$$S_{V,F} = \sum_W o_{V,W,F} \sum_L \mathbf{A}_{L,W}$$

Substituting these into Eq. (7), we obtain:

$$\sum_F S_{V,F} \frac{\partial \tilde{x}_F(\mathbf{p}_V)}{\partial \mathbf{p}_{V,P}} \left(\frac{x_{V,F}}{\tilde{x}_F(\mathbf{p}_V)} - 1 \right) = 0. \quad (11)$$

In this equation $\tilde{x}_F(\mathbf{p}_V)$ depends on the unknown parameter vector $\mathbf{p}_V$ of voxel V , while $x_{V,F}$ involves forward and back projections and depends on the

parameter vectors of all voxels. Thus, if $x_{V,F}$ were known, then the computation could be decoupled for different voxels, where a system of equations with N_P unknowns needs to be solved. In this way, forward/backward projection can be separated from the calculation of the parameter values, thus the complexity of the algorithm will be the sum of the complexities of the two steps and not their product [10]. Taking $x_{V,F}$ from the previous iteration, which is similar to the one-step-late strategy, the non-linear equation is solved, which can also be imagined as a *curve-fitting* process.

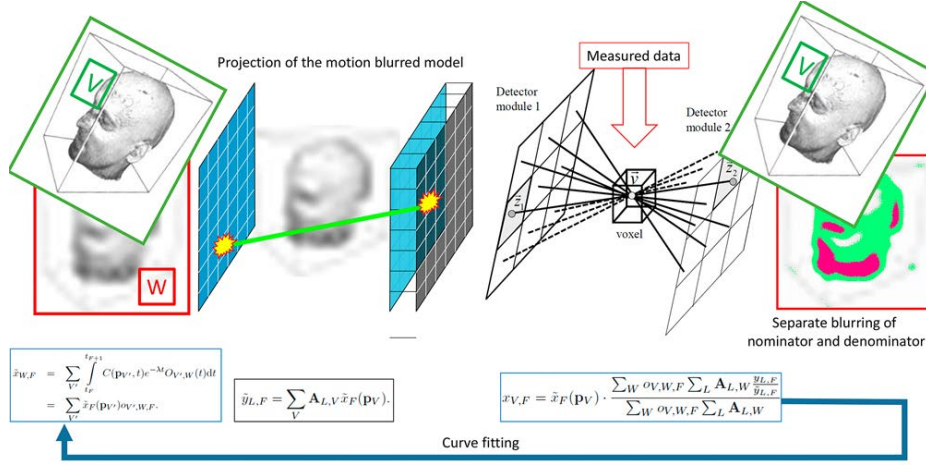


Fig. 1. System overview

Summarizing, an iteration of the motion compensated reconstruction consists of the following steps (Fig. 1):

1. For each frame we compute $x_{V,F}$, which in turn, includes the following steps:
 - (a) Evaluation of the integral of the radiotracer for each object-space voxel V for this frame to get $\tilde{x}_{V,F}$ (Eq. 1).
 - (b) Motion blur computation to compute the activity of tomograph-space voxels W to get $\tilde{x}_{W,F}$ (Eq. 2).
 - (c) Forward projection of the tomograph-space voxels to obtain expected LOR hits $\tilde{y}_{L,F}$ (Eq. 5).
 - (d) Computation of numerator $n_{W,F} = \sum_L \mathbf{A}_{L,W} y_{L,F} / \tilde{y}_{L,F}$ and denominator $S_{W,F} = \sum_L \mathbf{A}_{L,W}$.
 - (e) Motion blur of the tomograph-space numerator and denominator voxel arrays into object-space voxel arrays $n_{V,F}$ and $S_{V,F}$.
 - (f) Executing the division to compute $x_{V,F} = n_{V,F} / S_{V,F}$ (Eq. 10).
2. Fitting the parameters on $x_{V,F}$ for each object-space voxel V by the solution of Eq. 11.

3 Efficient motion blur computation

In our model we assume that in a frame the movement of a point of the subject is piece-wise linear, thus mapping between object-space and tomograph-space voxels can be described with a geometric transformation of the start and end points of a frame. With $\tilde{x}_{V,F}$ already computed for object-space voxel V in frame F , the next step is to determine which tomograph-space voxels W overlap with voxel V during this frame. At the beginning and end of the frame, we calculate the position of voxel V in tomograph-space with the geometric transformation describing the motion of V . Let W_{start} denote the tomograph-space voxel that contains the transformed center of voxel V at the beginning of the frame and W_{end} the voxel that contains the transformed center of V at the end of the frame. As motion is assumed to be linear, each tomograph-space voxel that is overlapped by V will be located on the line segment connecting W_{start} and W_{end} .

Based on our previous research, potentially overlapping voxels along the line segment are determined by the Antialiased Bresenham line drawing algorithm. The algorithm iterates through the direction in which the line changes the most rapidly in unit steps. In every iteration the closest and its three neighbor voxels are visited and the overlap factors $o_{V,W,F}$ are calculated as the weights of the Antialiased Bresenham algorithm. Coordinates and weights are calculated incrementally, and the total sum of weights along the line is 1.

In backward projection motion blur, the numerator $n_{W,F}$ and denominator $S_{W,F}$ of tomograph-space voxels W are known, and we want to compute $n_{V,F}$ and $S_{V,F}$ for object-space voxel V . This is achieved in the same manner as during forward projection, by determining tomograph-space starting and end points W_{start} and W_{end} of the motion of voxel V , and visiting tomograph-space voxels along the line segment with the Antialiased Bresenham algorithm. Numerator and denominator $n_{V,F}$ and $S_{V,F}$ for object-space voxel V are increased in each visited tomograph-space voxel W taking into account the overlapping between the voxels.

4 Results

To examine the proposed methods, first we use a 2D phantom [8] where the number of LORs is $N_L = 2115$ and the number of voxels is $N_V = 1024$ (Fig. 2) and reconstruct the activity curves of a 2D brain model.

4.1 Blood only model

In our first measurement, we examined the reconstruction of a small fraction of blood moving along a circular path with constant angular velocity. Measured events were binned into 5 frames, during which the object completed one full 360 degree turn around the center of the measured volume. Fig. 3 shows the blurred snapshots of its motion in each of the five frames, and the reconstructed time-activity function.

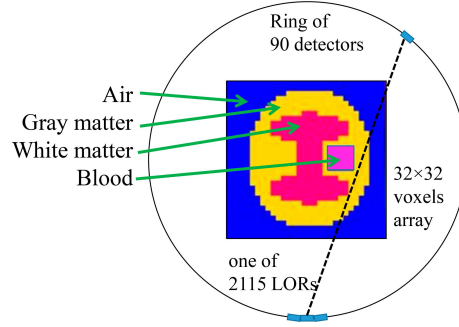


Fig. 2. 2D tomograph model: The detector ring contains 90 detector crystals and each of them is of size 2.2 in voxel units and participates in 47 LORs connecting this crystal to crystals being in the opposite half circle, thus the total number of LORs is $90 \times 47/2 = 2115$. The voxel array to be reconstructed is in the middle of the ring and has 32×32 resolution, i.e. 1024 voxels.

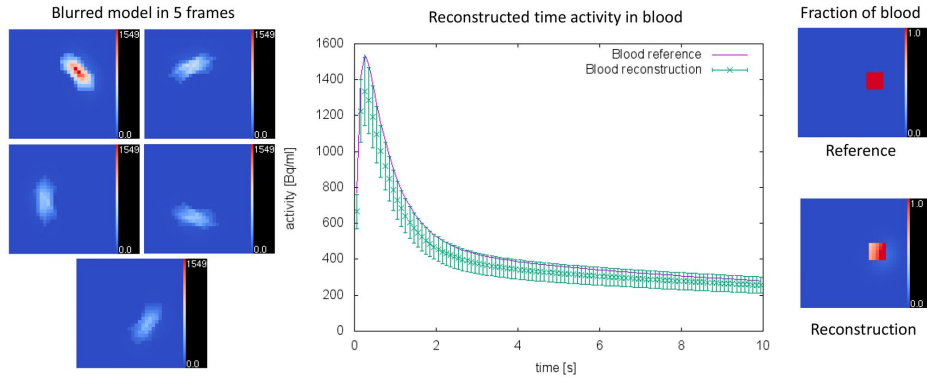


Fig. 3. Reconstruction of the blood with 5 frames. Note that the blood object is sharply reconstructed despite to the strong blurring inside frames.

4.2 Brain phantom

In our second measurement, we used a brain phantom consisting of three regions: gray matter, white matter and blood. Measured events were grouped into 10 frames, during which the phantom completed one full 360 degree rotation around its center point (Fig. 5). Reconstruction was first performed without motion compensation to demonstrate the high degree of blurring in the reconstructed image caused by the movement of the object. Applying motion compensation eliminated this blurring and significantly reduced reconstruction error, as shown in Fig.4.

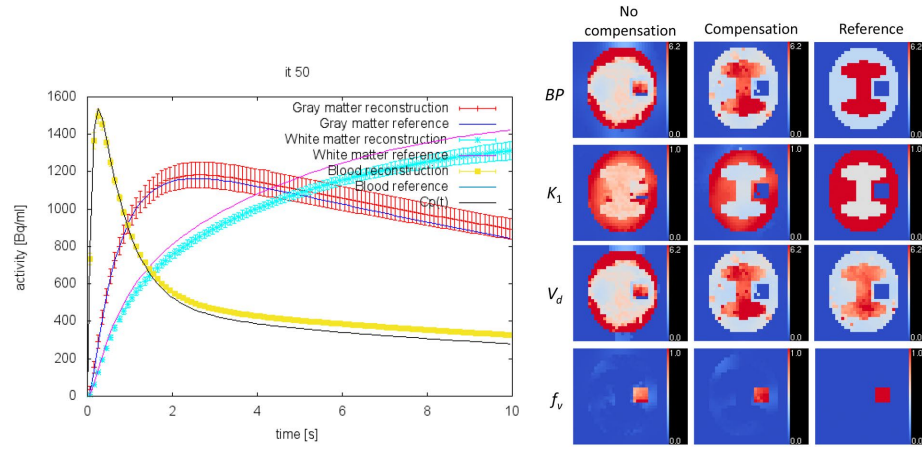


Fig. 4. Reconstruction of the brain with 10 frames

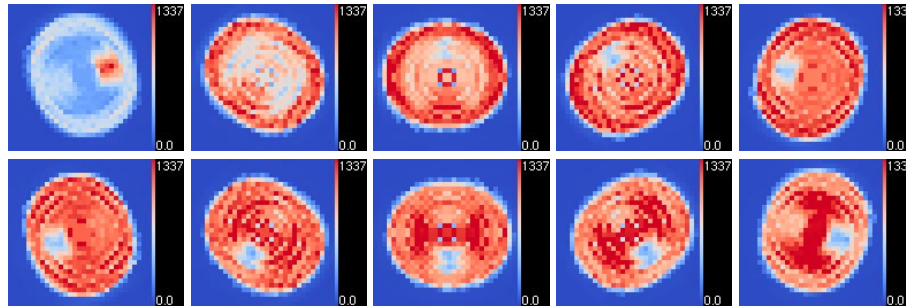


Fig. 5. Blurred images before forward projection in 10 frames

Dividing measured events into 25 frames instead of the previous 10 allows finer motion compensation, as in smaller time spans the motion of a voxel can be estimated with a geometric transformation more precisely. Fig. 6 shows that with more frames, motion compensation could reduce reconstruction error further.

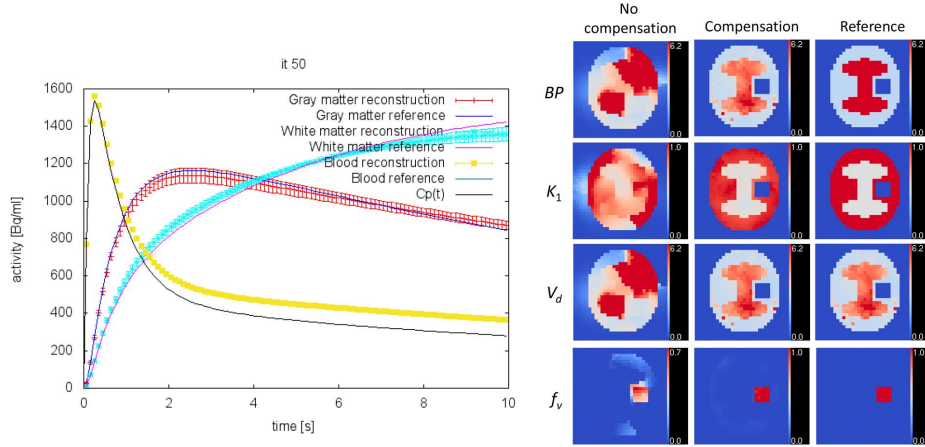


Fig. 6. Reconstruction of the brain with 25 frames

5 Conclusions

This paper proposed a model based motion compensation algorithm for emission tomography. Based on the generalization of the ML-EM algorithm seeking for the maximum likelihood estimate, we concluded that the motion can be captured by three motion blur operations in every iteration cycle. First, the current activity estimate should be motion blurred before executing forward projection. Then the numerator and the denominator of the back-projection formula should be motion blurred before computing their ratios. This is a work in progress, so the algorithm is tested only on 2D mathematical phantoms and not on real 3D data. The demonstration shows that artifacts can be eliminated by the proposed method.

Acknowledgements

This work has been supported by OTKA K-124124, EFOP 4.2.1-16-2017-00021, and EFOP-3.6.2-16-2017-00013.

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